Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.4-d+e-x-^m-f+g-x-^n-a+b-x+c-x^2-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [958]. This is test number [35].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (958)	% 0.00 (0)
Mathematica	% 98.33 (942)	% 1.67 (16)
Maple	% 76.10 (729)	% 23.90 (229)
Maxima	% 34.55 (331)	% 65.45 (627)
Fricas	% 61.27 (587)	% 38.73 (371)
Sympy	% 27.14 (260)	% 72.86 (698)
Giac	% 29.12 (279)	% 70.88 (679)
Mupad	% 28.81 (276)	% 71.19 (682)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

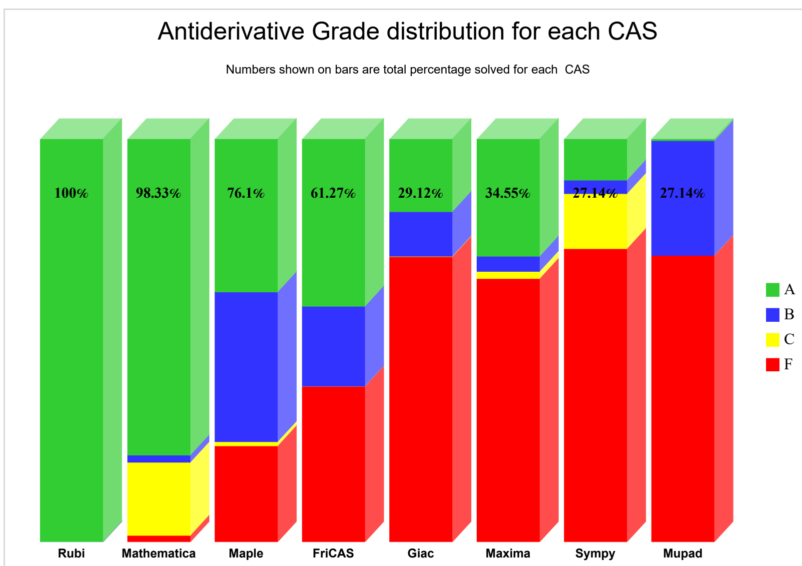
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

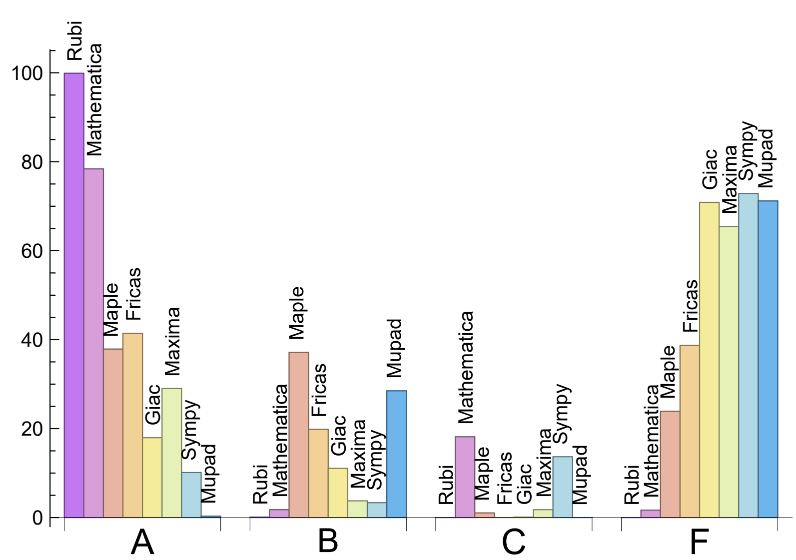
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.90	0.10	0.00	0.00
Mathematica	78.39	1.77	18.16	1.67
Maple	37.89	37.16	1.04	23.90
Maxima	29.02	3.76	1.77	65.45
Fricas	41.44	19.83	0.00	38.73
Sympy	10.13	3.34	13.67	72.86
Giac	17.95	11.06	0.10	70.88
Mupad	0.31	28.50	0.00	71.19

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	16	100.00 %	0.00 %	0.00 %
Maple	229	100.00 %	0.00 %	0.00 %
Maxima	627	86.44 %	0.48 %	13.08 %
Fricas	371	77.63 %	22.37 %	0.00 %
Sympy	698	67.62 %	30.09 %	2.29 %
Giac	679	54.93 %	14.43 %	30.63 %
Mupad	682	98.97 %	1.03 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

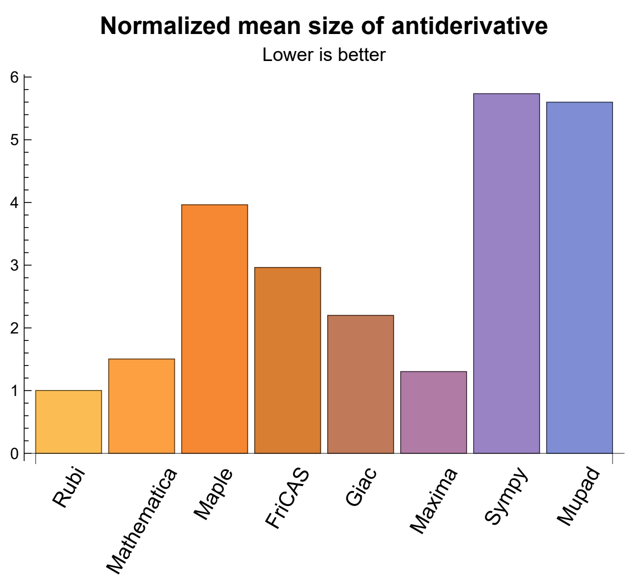
1.3 Performance

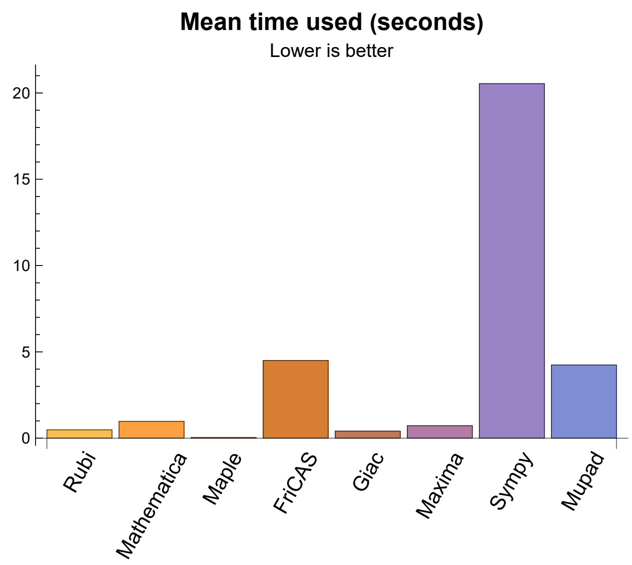
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.48	234.01	1.00	175.00	1.00
Mathematica	0.97	649.46	1.50	149.00	0.91
Maple	0.04	1938.63	3.96	288.00	1.77
Maxima	0.72	198.58	1.30	150.00	1.19
Fricas	4.50	656.92	2.96	251.00	1.88
Sympy	20.54	906.97	5.73	375.00	3.21
Giac	0.41	512.22	2.20	185.00	1.45
Mupad	4.24	2029.61	5.60	161.00	1.28

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{948, 952, 957}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {420}

Mathematica {267, 275, 276, 314, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 479, 498, 501, 507, 514, 521, 604, 656, 802, 833, 886, 887, 890, 891, 892, 893, 899, 900, 905, 906, 907, 908, 914, 915, 917, 918, 947, 951, 956}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `sleliuevre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

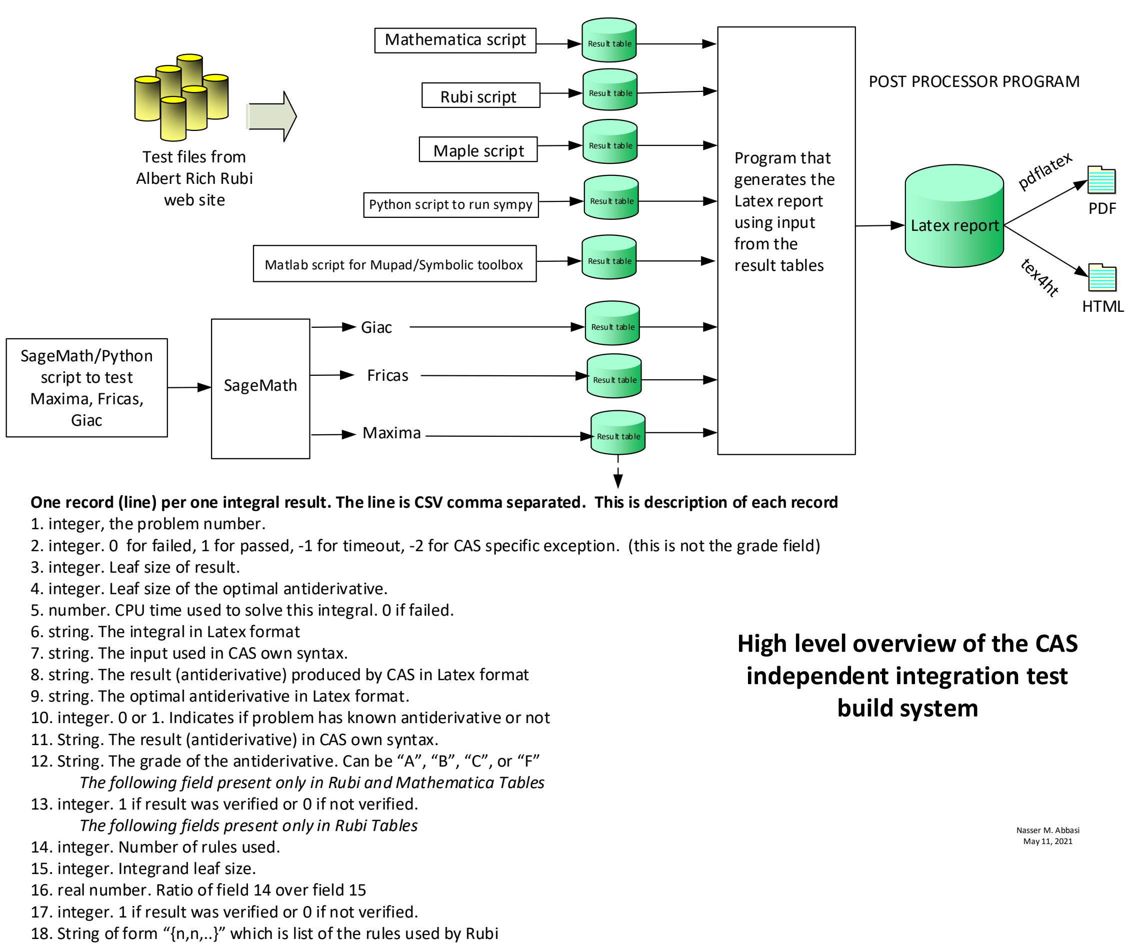
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958 }

B grade: { 833 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 269, 270, 271, 272, 273, 278, 279, 280, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 497, 504, 511, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 657, 658, 659, 660, 661, 662, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 684, 685, 689, 690, 691, 692, 693, 694, 696, 700, 701, 702, 703, 704, 705, 708, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 726, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 919, 920, 921, 922, 923, 924, 925, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 947, 948, 951, 952, 953, 956, 957 }

B grade: { 263, 268, 274, 277, 284, 285, 294, 295, 301, 304, 305, 360, 656, 802, 907, 918, 926 }

C grade: { 8, 9, 10, 11, 13, 14, 15, 42, 43, 50, 51, 52, 53, 62, 63, 64, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 267, 275, 276, 315, 325, 339, 340, 341, 479, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 587, 588, 595, 600, 601, 602, 604, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 663, 664, 669, 670, 671, 676, 677, 678, 686, 687, 688, 695, 697, 698, 699, 706, 707, 709, 710, 711, 720, 721, 727, 728, 745, 754, 755, 791, 838, 839, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 958 }

F grade: { 379, 380, 408, 416, 434, 435, 436, 541, 803, 804, 945, 946, 949, 950, 954, 955 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 96, 103, 107, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 200, 201, 211, 212, 213, 214, 215, 216, 218, 219, 224, 315, 326, 327, 328, 332, 333, 341, 350, 352, 353, 440, 470, 471, 472, 473, 474, 475, 476, 480, 481, 482, 487, 489, 490, 491, 493, 494, 496, 497, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 511, 512, 514, 515, 516, 518, 521, 524, 548, 549, 550, 551, 552, 553, 557, 558, 559, 560, 561, 562, 563, 566, 569, 570, 571, 572, 573, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 596, 597, 598, 599, 600, 604, 619, 640, 648, 649, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 696, 697, 700, 701, 702, 703, 704, 708, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 735, 736, 737, 738, 739, 740, 741, 744, 746, 747, 748, 749, 750, 753, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 788, 808, 814, 820, 821, 822, 823, 827, 828, 829, 840, 841, 848, 849, 875, 912, 913, 918, 919, 948, 952, 957, 958 }

B grade: { 9, 10, 11, 45, 78, 79, 84, 85, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 136, 137, 138, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 191, 194, 195, 196, 197, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 220, 221, 222, 223, 225, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 351, 355, 356, 357, 359, 360, 361, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 477, 478, 479, 483, 484, 485, 486, 488, 492, 495, 503, 509, 510, 513, 517, 519, 520, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 554, 555, 556, 564, 565, 567, 568, 574, 575, 576, 579, 580, 581, 585, 586, 587, 594, 595, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 646, 647, 650, 651, 652, 653, 654, 678, 688, 698, 699, 705, 706, 707, 709, 710, 711, 720, 721, 727, 733, 734, 742, 743, 745, 751, 752, 754, 755, 756, 789, 790, 791, 805, 806, 807, 815, 816, 817, 818, 819, 824, 825, 826, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 914, 915, 916, 917, 920, 921, 925, 926 }

C grade: { 792, 793, 794, 795, 796, 797, 798, 799, 800, 801 }

F grade: { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 137, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 184, 202, 203, 220, 224, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 342, 343, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 490, 497, 504, 511, 518, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 589, 590, 591, 592, 596, 597, 598, 599, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 768, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 792, 793, 794, 797, 798, 799, 807, 808, 814, 815, 816, 819, 820, 821, 822, 826, 827, 828, 829, 948, 952, 957 }

B grade: { 11, 19, 20, 21, 22, 44, 45, 83, 84, 85, 136, 138, 192, 193, 211, 212, 213, 214, 215, 222, 359, 360, 361, 449, 460, 579, 580, 581, 582, 583, 805, 806, 920, 921, 925, 926 }

C grade: { 102, 103, 104, 105, 106, 107, 157, 158, 159, 160, 161, 162, 198, 199, 200, 201, 604 }

F grade: { 98, 99, 100, 101, 124, 125, 126, 133, 134, 135, 144, 145, 146, 147, 154, 155, 156, 176, 177, 178, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 204, 205, 206, 207, 208, 209, 210, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 322, 323, 324, 325, 331, 332, 333, 339, 340, 341, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 585, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 817, 818, 823, 824, 825, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 222, 224, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 341, 348, 349, 350, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 473, 474, 475, 476, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 548, 549, 550, 551, 552, 557, 558, 559, 560, 562, 569, 570, 572, 573, 583, 584, 589, 590, 591, 592, 593, 596, 597, 598, 599, 603, 604, 620, 621, 657, 658, 659, 660, 661, 665, 666, 667, 668, 672, 673, 674, 679, 680, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 700, 701, 702, 703, 705, 706, 707, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 788, 792, 793, 794, 797, 798, 814, 815, 816, 819, 820, 821, 822, 823, 826, 827, 828, 829, 834, 835, 836, 837, 840, 842, 843, 844, 845, 857, 948, 952, 957 }

B grade: { 18, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 131, 132, 138, 139, 140, 141, 142, 143, 144, 148, 149, 216, 221, 223, 225, 330, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 359, 360, 361, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 553, 554, 555, 556, 561, 563, 564, 565, 566, 567, 568, 571, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 586, 587, 588, 594, 595, 600, 601, 602, 607, 608, 611, 612, 618, 619, 662, 663, 664, 669, 670, 671, 675, 676, 677, 678, 685, 686, 687, 688, 696, 697, 698, 699, 704, 708, 709, 710, 711, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 789, 790, 791, 795, 799, 805, 806, 807, 808, 824, 825, 830, 831, 832, 833, 838, 839, 841, 846, 847, 848, 849, 851, 873, 874, 875, 880, 881, 882, 920, 921, 925, 926 }

C grade: { }

F grade: { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 468, 469, 486, 488, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 532, 540, 541, 542, 543, 544, 545, 546, 547, 605, 606, 609, 610, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 817, 818, 850, 852, 853, 854, 855, 856, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.6 Sympy

A grade: { 3, 5, 6, 25, 32, 34, 36, 37, 54, 55, 56, 57, 58, 62, 65, 67, 69, 104, 106, 117, 157, 159, 161, 220, 222, 224, 244, 245, 253, 254, 255, 262, 264, 265, 315, 351, 352, 353, 355, 356, 357, 385, 386, 394, 395, 396, 403, 405, 406, 524, 529, 548, 549, 550, 551, 557, 558, 559, 560, 561, 562, 566, 569, 570, 571, 572, 573, 576, 577, 578, 588, 590, 591, 592, 593, 596, 597, 598, 599, 600, 806, 807, 808, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 920, 921, 948, 952 }

B grade: { 19, 21, 23, 240, 241, 242, 243, 249, 250, 251, 252, 258, 259, 260, 261, 263, 354, 358, 362, 552, 553, 554, 555, 556, 563, 564, 565, 567, 568, 574, 575, 814 }

C grade: { 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 33, 35, 38, 39, 40, 41, 42, 43, 59, 60, 61, 63, 64, 66, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 226, 227, 228, 229, 230, 231, 235, 236, 246, 247, 248, 256, 257, 266, 267, 268, 269, 270, 271, 272, 273, 274, 306, 307, 308, 309, 310, 313, 377, 378, 381, 382, 383, 384, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 401, 402, 404, 407, 431, 432, 433, 604, 794 }

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2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 117, 130, 134, 151, 152, 153, 154, 186, 187, 195, 196, 197, 219, 315, 316, 317, 318, 319, 320, 322, 323, 324, 326, 327, 328, 329, 330, 332, 333, 335, 336, 337, 338, 340, 341, 348, 504, 524, 529, 539, 548, 552, 557, 558, 563, 564, 575, 576, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 621, 771, 780, 792, 793, 794, 814, 815, 816, 817, 820, 821, 822, 823, 824, 828, 829, 830, 831, 833, 834, 835, 836, 837, 838, 842, 845, 874, 948, 952, 957 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 40, 41, 42, 43, 60, 61, 62, 63, 64, 75, 76, 77, 78, 79, 80, 81, 82, 98, 118, 156, 325, 334, 351, 352, 353, 355, 356, 357, 359, 360, 361, 490, 497, 525, 526, 527, 528, 530, 532, 533, 534, 535, 536, 537, 538, 540, 549, 550, 551, 559, 560, 561, 562, 569, 570, 571, 572, 573, 574, 579, 580, 585, 587, 596, 619, 620, 768, 769, 770, 795, 797, 798, 799, 805, 806, 807, 808, 818, 819, 825, 826, 827, 832, 839, 840, 841, 843, 844, 846, 860, 877, 880, 881, 882, 885, 920, 921, 925, 926 }

C grade: { 586 }

F grade: { 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 321, 331, 339, 342, 343, 344, 345, 346, 347, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 531, 541, 542, 543, 544, 545, 546, 547, 553, 554, 555, 556, 565, 566, 567, 568, 577, 578, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 875, 876, 878, 879, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.8 Mupad

A grade: { 948, 952, 957 }

B grade: { 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 117, 118, 123, 130, 131, 132, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 175, 182, 183, 184, 192, 193, 210, 211, 212, 213, 214, 215, 220, 222, 224, 245, 315, 351, 352, 353, 355, 356, 357, 359, 360, 361, 386, 473, 480, 481, 482, 487, 488, 490, 497, 504, 511, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 792, 793, 794, 795, 805, 806, 807, 808, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 837, 840, 841, 844, 845, 848, 849, 920, 921, 925, 926 }

C grade: { }

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2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	112	125	104	95	279	74	-1
normalized size	1	1.00	0.85	0.95	0.79	0.72	2.11	0.56	-0.01
time (sec)	N/A	0.076	0.124	0.059	0.983	0.707	5.478	0.258	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	157	198	177	138	830	117	-1
normalized size	1	1.00	0.78	0.99	0.88	0.69	4.13	0.58	-0.00
time (sec)	N/A	0.149	0.214	0.045	0.989	0.864	17.835	0.220	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	146	173	152	127	775	106	-1
normalized size	1	1.00	0.85	1.01	0.88	0.74	4.51	0.62	-0.01
time (sec)	N/A	0.101	0.191	0.023	0.977	0.889	17.170	0.226	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	135	148	127	116	653	96	-1
normalized size	1	1.00	0.85	0.93	0.80	0.73	4.11	0.60	-0.01
time (sec)	N/A	0.108	0.175	0.045	0.949	0.959	12.267	0.227	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	124	123	102	105	580	84	-1
normalized size	1	1.00	1.07	1.06	0.88	0.91	5.00	0.72	-0.01
time (sec)	N/A	0.034	0.142	0.015	0.984	1.016	12.059	0.212	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	124	123	102	105	580	84	-1
normalized size	1	1.00	1.07	1.06	0.88	0.91	5.00	0.72	-0.01
time (sec)	N/A	0.033	0.042	0.000	0.980	0.891	12.180	0.340	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	124	151	124	107	469	99	107
normalized size	1	1.00	1.10	1.34	1.10	0.95	4.15	0.88	0.95
time (sec)	N/A	0.098	0.184	0.019	0.985	0.907	22.411	0.273	2.904

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	124	182	129	124	386	157	114
normalized size	1	1.00	1.06	1.56	1.10	1.06	3.30	1.34	0.97
time (sec)	N/A	0.092	0.174	0.033	0.990	0.655	8.182	0.254	3.513

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	110	212	160	133	461	217	120
normalized size	1	1.00	0.91	1.75	1.32	1.10	3.81	1.79	0.99
time (sec)	N/A	0.094	0.077	0.022	0.988	0.856	9.533	0.276	3.735

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	235	184	129	457	261	-1
normalized size	1	1.00	0.92	1.96	1.53	1.08	3.81	2.18	-0.01
time (sec)	N/A	0.092	0.060	0.022	0.982	0.649	8.932	0.234	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	133	260	210	119	541	297	-1
normalized size	1	1.00	1.13	2.20	1.78	1.01	4.58	2.52	-0.01
time (sec)	N/A	0.092	0.089	0.025	0.983	0.917	11.056	0.232	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	133	158	155	98	774	368	93
normalized size	1	1.00	1.23	1.46	1.44	0.91	7.17	3.41	0.86
time (sec)	N/A	0.063	0.061	0.033	0.994	1.050	11.365	0.246	4.264

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	59	186	180	109	918	431	118
normalized size	1	1.00	0.41	1.30	1.26	0.76	6.42	3.01	0.83
time (sec)	N/A	0.095	0.021	0.031	0.988	0.920	15.254	0.276	4.660

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	72	211	205	120	1037	494	192
normalized size	1	1.00	0.42	1.23	1.19	0.70	6.03	2.87	1.12
time (sec)	N/A	0.125	0.020	0.042	0.990	0.974	16.620	0.256	5.334

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	73	236	230	131	1159	431	212
normalized size	1	1.00	0.36	1.17	1.14	0.65	5.77	2.14	1.05
time (sec)	N/A	0.156	0.022	0.073	0.995	0.905	22.545	0.256	6.044

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	102	81	72	177	54	112
normalized size	1	1.00	0.68	0.99	0.79	0.70	1.72	0.52	1.09
time (sec)	N/A	0.053	0.038	0.019	0.985	0.863	5.254	0.245	3.138

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	77	99	78	87	184	66	87
normalized size	1	1.00	1.05	1.36	1.07	1.19	2.52	0.90	1.19
time (sec)	N/A	0.042	0.031	0.019	0.979	0.758	9.708	0.252	2.956

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	52	55	88	104	231	51	55
normalized size	1	1.00	0.90	0.95	1.52	1.79	3.98	0.88	0.95
time (sec)	N/A	0.026	0.021	0.009	0.444	0.832	9.883	0.295	2.590

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	155	227	312	278	2004	120	-1
normalized size	1	1.00	0.96	1.41	1.94	1.73	12.45	0.75	-0.01
time (sec)	N/A	0.139	0.097	0.074	1.029	0.982	66.418	0.279	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	142	195	278	263	1821	109	-1
normalized size	1	1.00	0.97	1.33	1.89	1.79	12.39	0.74	-0.01
time (sec)	N/A	0.120	0.087	0.025	1.032	0.900	62.081	0.272	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	130	166	250	247	1739	97	-1
normalized size	1	1.00	1.07	1.36	2.05	2.02	14.25	0.80	-0.01
time (sec)	N/A	0.081	0.081	0.021	1.011	0.871	73.140	0.308	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	77	159	171	418	64	78
normalized size	1	1.00	0.98	0.92	1.89	2.04	4.98	0.76	0.93
time (sec)	N/A	0.052	0.025	0.010	0.453	0.916	63.869	0.272	2.701

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	77	134	172	337	58	78
normalized size	1	1.00	0.91	0.86	1.49	1.91	3.74	0.64	0.87
time (sec)	N/A	0.042	0.023	0.010	0.448	0.725	20.548	0.278	2.659

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	77	112	173	513	64	78
normalized size	1	1.00	0.87	0.82	1.19	1.84	5.46	0.68	0.83
time (sec)	N/A	0.046	0.025	0.009	0.438	0.932	21.308	0.269	2.615

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	77	87	172	432	57	78
normalized size	1	1.00	0.99	0.93	1.05	2.07	5.20	0.69	0.94
time (sec)	N/A	0.023	0.035	0.009	0.436	0.926	22.679	0.267	2.621

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	77	80	171	604	65	78
normalized size	1	1.00	1.02	0.96	1.00	2.14	7.55	0.81	0.98
time (sec)	N/A	0.021	0.029	0.009	0.435	0.898	24.406	0.260	2.584

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	131	163	157	244	2378	122	127
normalized size	1	1.00	1.12	1.39	1.34	2.09	20.32	1.04	1.09
time (sec)	N/A	0.103	0.063	0.013	0.445	0.951	41.141	0.274	3.079

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	195	189	270	2404	189	141
normalized size	1	1.00	0.96	1.27	1.24	1.76	15.71	1.24	0.92
time (sec)	N/A	0.127	0.073	0.016	0.463	1.063	31.225	0.286	3.305

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	183	227	221	291	2691	260	181
normalized size	1	1.00	0.99	1.23	1.20	1.58	14.62	1.41	0.98
time (sec)	N/A	0.160	0.135	0.016	0.468	0.917	35.060	0.359	3.428

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	104	99	135	239	903	77	164
normalized size	1	1.00	0.86	0.82	1.12	1.98	7.46	0.64	1.36
time (sec)	N/A	0.053	0.040	0.012	0.440	0.770	22.733	0.274	2.690

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	126	121	158	305	1401	90	202
normalized size	1	1.00	0.85	0.82	1.07	2.06	9.47	0.61	1.36
time (sec)	N/A	0.062	0.049	0.014	0.448	1.656	48.458	0.298	2.737

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	85	63	66	102	70	84
normalized size	1	1.00	0.93	1.57	1.17	1.22	1.89	1.30	1.56
time (sec)	N/A	0.034	0.029	0.016	0.964	0.768	8.328	0.211	0.088

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	103	174	153	105	558	84	-1
normalized size	1	1.00	0.60	1.01	0.88	0.61	3.23	0.49	-0.01
time (sec)	N/A	0.227	0.099	0.028	0.978	0.960	13.484	0.269	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	92	149	128	94	357	73	-1
normalized size	1	1.00	0.64	1.03	0.89	0.65	2.48	0.51	-0.01
time (sec)	N/A	0.186	0.090	0.010	0.973	0.914	7.871	0.263	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	124	103	83	386	63	-1
normalized size	1	1.00	0.70	1.08	0.90	0.72	3.36	0.55	-0.01
time (sec)	N/A	0.145	0.070	0.011	0.968	0.805	9.327	0.252	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	69	98	77	71	218	49	-1
normalized size	1	1.00	0.83	1.18	0.93	0.86	2.63	0.59	-0.01
time (sec)	N/A	0.085	0.054	0.009	0.970	0.868	5.617	0.253	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	71	53	60	269	40	-1
normalized size	1	1.00	0.70	0.86	0.64	0.72	3.24	0.48	-0.01
time (sec)	N/A	0.028	0.036	0.008	0.970	0.733	5.044	0.251	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	91	62	73	184	65	-1
normalized size	1	1.00	1.00	1.38	0.94	1.11	2.79	0.98	-0.02
time (sec)	N/A	0.111	0.027	0.010	0.971	0.785	6.958	0.257	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	93	64	79	207	107	-1
normalized size	1	1.00	1.00	1.37	0.94	1.16	3.04	1.57	-0.01
time (sec)	N/A	0.115	0.030	0.011	0.962	0.710	4.296	0.256	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	122	86	83	63	214	170	-1
normalized size	1	1.00	1.52	1.08	1.04	0.79	2.68	2.12	-0.01
time (sec)	N/A	0.109	0.243	0.012	0.951	0.940	6.721	0.269	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	87	114	108	74	303	239	-1
normalized size	1	1.00	0.81	1.07	1.01	0.69	2.83	2.23	-0.01
time (sec)	N/A	0.137	0.150	0.013	0.970	0.928	6.090	0.305	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	155	139	133	87	449	305	-1
normalized size	1	1.00	1.11	0.99	0.95	0.62	3.21	2.18	-0.01
time (sec)	N/A	0.172	0.148	0.015	0.970	0.671	10.316	0.290	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	79	164	158	98	510	365	-1
normalized size	1	1.00	0.47	0.97	0.93	0.58	3.02	2.16	-0.01
time (sec)	N/A	0.195	0.040	0.016	0.984	0.767	8.957	0.272	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	111	193	276	188	0	106	-1
normalized size	1	1.00	0.78	1.35	1.93	1.31	0.00	0.74	-0.01
time (sec)	N/A	0.271	0.212	0.010	1.016	0.765	0.000	0.298	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	96	236	298	172	0	95	-1
normalized size	1	1.00	0.79	1.95	2.46	1.42	0.00	0.79	-0.01
time (sec)	N/A	0.208	0.202	0.018	1.004	0.689	0.000	0.280	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	63	65	155	116	0	63	66
normalized size	1	1.00	0.65	0.67	1.60	1.20	0.00	0.65	0.68
time (sec)	N/A	0.173	0.060	0.008	0.445	0.854	0.000	0.287	2.893

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	66	131	117	0	61	67
normalized size	1	1.00	0.72	0.76	1.51	1.34	0.00	0.70	0.77
time (sec)	N/A	0.120	0.055	0.010	0.445	0.868	0.000	0.284	2.865

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	62	64	109	117	0	64	65
normalized size	1	1.00	0.70	0.72	1.22	1.31	0.00	0.72	0.73
time (sec)	N/A	0.033	0.051	0.008	0.438	0.858	0.000	0.276	2.862

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	65	78	116	0	61	66
normalized size	1	1.00	0.82	0.84	1.01	1.51	0.00	0.79	0.86
time (sec)	N/A	0.020	0.042	0.008	0.438	0.614	0.000	0.315	2.814

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	81	160	154	169	0	118	-1
normalized size	1	1.00	0.69	1.37	1.32	1.44	0.00	1.01	-0.01
time (sec)	N/A	0.159	0.042	0.010	0.456	0.862	0.000	0.288	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	90	193	187	195	0	188	-1
normalized size	1	1.00	0.62	1.33	1.29	1.34	0.00	1.30	-0.01
time (sec)	N/A	0.275	0.050	0.011	0.466	0.679	0.000	0.288	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	117	224	218	216	0	260	-1
normalized size	1	1.00	0.64	1.23	1.20	1.19	0.00	1.43	-0.01
time (sec)	N/A	0.358	0.056	0.016	0.469	0.857	0.000	0.298	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	105	249	243	227	0	325	-1
normalized size	1	1.00	0.50	1.19	1.16	1.09	0.00	1.56	-0.00
time (sec)	N/A	0.474	0.054	0.020	0.478	1.041	0.000	0.351	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	42	71	70	50	73	34	36
normalized size	1	1.00	0.52	0.88	0.86	0.62	0.90	0.42	0.44
time (sec)	N/A	0.093	0.036	0.012	0.968	0.860	1.409	0.182	2.501

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	37	57	56	45	60	30	31
normalized size	1	1.00	0.59	0.90	0.89	0.71	0.95	0.48	0.49
time (sec)	N/A	0.081	0.028	0.007	0.969	0.843	0.810	0.192	0.029

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	26	41	40	38	37	21	22
normalized size	1	1.00	0.63	1.00	0.98	0.93	0.90	0.51	0.54
time (sec)	N/A	0.048	0.017	0.005	0.971	0.954	0.408	0.220	0.029

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	29	28	33	27	19	21
normalized size	1	1.00	0.62	0.72	0.70	0.82	0.68	0.48	0.52
time (sec)	N/A	0.012	0.015	0.003	0.961	0.807	0.242	0.195	0.030

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	41	46	31	34	32
normalized size	1	1.00	1.00	0.91	1.28	1.44	0.97	1.06	1.00
time (sec)	N/A	0.059	0.010	0.005	0.979	0.865	6.295	0.180	0.047

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	42	53	51	55	35
normalized size	1	1.00	1.00	0.91	1.27	1.61	1.55	1.67	1.06
time (sec)	N/A	0.063	0.016	0.008	0.971	0.828	4.684	0.184	0.079

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	42	54	43	116	91	47
normalized size	1	1.00	0.78	0.82	1.06	0.84	2.27	1.78	0.92
time (sec)	N/A	0.061	0.020	0.007	0.969	0.842	7.033	0.179	2.487

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	56	68	48	128	125	67
normalized size	1	1.00	0.64	0.84	1.01	0.72	1.91	1.87	1.00
time (sec)	N/A	0.074	0.024	0.008	0.964	0.721	8.348	0.186	0.032

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	70	82	53	223	163	77
normalized size	1	1.00	0.82	0.79	0.92	0.60	2.51	1.83	0.87
time (sec)	N/A	0.090	0.038	0.009	0.972	0.850	11.059	0.184	0.032

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	50	84	96	58	201	199	90
normalized size	1	1.00	0.47	0.79	0.90	0.54	1.88	1.86	0.84
time (sec)	N/A	0.103	0.017	0.008	0.969	0.966	12.693	0.198	0.036

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	196	212	159	111	544	295	-1
normalized size	1	1.00	1.46	1.58	1.19	0.83	4.06	2.20	-0.01
time (sec)	N/A	0.215	0.243	0.022	0.982	0.942	10.027	0.273	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	212	291	270	194	2273	170	-1
normalized size	1	1.00	0.68	0.94	0.87	0.63	7.33	0.55	-0.00
time (sec)	N/A	0.487	0.358	0.107	1.002	0.849	101.431	0.258	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	200	266	245	183	2028	160	-1
normalized size	1	1.00	0.71	0.95	0.87	0.65	7.22	0.57	-0.00
time (sec)	N/A	0.406	0.330	0.018	0.987	0.920	64.638	0.256	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	189	241	220	172	1919	149	-1
normalized size	1	1.00	0.75	0.96	0.87	0.68	7.62	0.59	-0.00
time (sec)	N/A	0.364	0.297	0.015	0.992	0.968	59.744	0.242	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	178	216	195	161	1681	139	-1
normalized size	1	1.00	0.80	0.97	0.87	0.72	7.54	0.62	-0.00
time (sec)	N/A	0.306	0.267	0.014	0.993	0.918	40.610	0.292	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	167	191	170	150	1554	128	-1
normalized size	1	1.00	0.73	0.83	0.74	0.65	6.76	0.56	-0.00
time (sec)	N/A	0.121	0.379	0.010	0.987	0.811	40.183	0.240	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	156	154	136	139	1284	117	-1
normalized size	1	1.00	0.83	0.82	0.72	0.74	6.83	0.62	-0.01
time (sec)	N/A	0.082	0.326	0.010	0.985	0.898	25.884	0.249	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	168	231	204	151	1263	143	-1
normalized size	1	1.00	0.88	1.22	1.07	0.79	6.65	0.75	-0.01
time (sec)	N/A	0.307	0.355	0.011	1.004	0.885	47.722	0.260	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	221	243	217	167	1057	199	-1
normalized size	1	1.00	1.15	1.26	1.12	0.87	5.48	1.03	-0.01
time (sec)	N/A	0.305	0.525	0.013	0.998	1.024	19.884	0.245	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	259	252	229	179	1059	262	-1
normalized size	1	1.00	1.25	1.22	1.11	0.86	5.12	1.27	-0.00
time (sec)	N/A	0.313	0.640	0.017	1.004	1.141	22.217	0.252	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	251	277	226	179	911	318	-1
normalized size	1	1.00	1.20	1.32	1.08	0.85	4.34	1.51	-0.00
time (sec)	N/A	0.314	0.272	0.017	1.008	0.894	15.742	0.289	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	195	302	250	180	1028	374	-1
normalized size	1	1.00	0.93	1.44	1.20	0.86	4.92	1.79	-0.00
time (sec)	N/A	0.319	0.100	0.021	0.993	0.957	20.103	0.267	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	199	327	278	180	1178	430	-1
normalized size	1	1.00	0.92	1.51	1.29	0.83	5.45	1.99	-0.00
time (sec)	N/A	0.313	0.094	0.027	0.994	0.950	20.702	0.276	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	286	352	303	179	1397	485	-1
normalized size	1	1.00	1.34	1.64	1.42	0.84	6.53	2.27	-0.00
time (sec)	N/A	0.312	0.223	0.034	1.001	0.941	21.706	0.311	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	247	377	326	173	1513	510	-1
normalized size	1	1.00	1.20	1.83	1.58	0.84	7.34	2.48	-0.00
time (sec)	N/A	0.311	0.152	0.046	1.010	0.876	22.293	0.311	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	245	402	352	163	1719	538	-1
normalized size	1	1.00	1.20	1.97	1.73	0.80	8.43	2.64	-0.00
time (sec)	N/A	0.304	0.149	0.065	1.009	0.861	31.403	0.313	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	218	250	247	142	1889	620	-1
normalized size	1	1.00	1.17	1.34	1.32	0.76	10.10	3.32	-0.01
time (sec)	N/A	0.260	0.165	0.098	0.997	0.760	36.712	0.466	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	102	278	272	153	2159	683	-1
normalized size	1	1.00	0.45	1.24	1.21	0.68	9.60	3.04	-0.00
time (sec)	N/A	0.298	0.064	0.152	1.013	0.942	49.867	0.361	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	112	303	297	164	2397	746	-1
normalized size	1	1.00	0.44	1.19	1.17	0.65	9.44	2.94	-0.00
time (sec)	N/A	0.329	0.061	0.241	1.005	1.206	74.516	0.363	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	131	222	305	192	0	118	-1
normalized size	1	1.00	0.75	1.28	1.75	1.10	0.00	0.68	-0.01
time (sec)	N/A	0.405	0.237	0.013	1.028	0.932	0.000	0.301	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	119	262	324	177	0	107	-1
normalized size	1	1.00	0.84	1.85	2.28	1.25	0.00	0.75	-0.01
time (sec)	N/A	0.325	0.199	0.010	1.019	0.894	0.000	0.293	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	234	296	161	0	95	-1
normalized size	1	1.00	0.95	1.98	2.51	1.36	0.00	0.81	-0.01
time (sec)	N/A	0.216	0.151	0.013	1.017	0.877	0.000	0.292	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	58	55	154	106	0	72	49
normalized size	1	1.00	0.62	0.59	1.66	1.14	0.00	0.77	0.53
time (sec)	N/A	0.126	0.079	0.006	0.453	1.069	0.000	0.284	2.690

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	55	52	128	104	0	60	46
normalized size	1	1.00	0.64	0.60	1.49	1.21	0.00	0.70	0.53
time (sec)	N/A	0.036	0.176	0.009	0.443	0.896	0.000	0.308	2.656

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	58	55	101	106	0	70	49
normalized size	1	1.00	0.56	0.53	0.98	1.03	0.00	0.68	0.48
time (sec)	N/A	0.048	0.059	0.008	0.439	0.918	0.000	0.291	2.657

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	81	158	152	158	0	117	-1
normalized size	1	1.00	0.71	1.39	1.33	1.39	0.00	1.03	-0.01
time (sec)	N/A	0.159	0.059	0.014	0.458	0.900	0.000	0.291	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	96	190	184	184	0	185	-1
normalized size	1	1.00	0.66	1.31	1.27	1.27	0.00	1.28	-0.01
time (sec)	N/A	0.287	0.056	0.012	0.472	0.853	0.000	0.290	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	119	222	216	205	0	259	-1
normalized size	1	1.00	0.65	1.22	1.19	1.13	0.00	1.42	-0.01
time (sec)	N/A	0.362	0.069	0.014	0.475	0.684	0.000	0.331	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	91	208	125	95	0	77	-1
normalized size	1	1.00	0.62	1.41	0.85	0.65	0.00	0.52	-0.01
time (sec)	N/A	0.142	0.140	0.018	0.990	0.874	0.000	0.205	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	185	101	83	0	66	-1
normalized size	1	1.00	0.68	1.57	0.86	0.70	0.00	0.56	-0.01
time (sec)	N/A	0.099	0.101	0.013	0.990	0.806	0.000	0.211	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	160	77	73	0	54	-1
normalized size	1	1.00	0.80	1.86	0.90	0.85	0.00	0.63	-0.01
time (sec)	N/A	0.110	0.073	0.012	0.988	0.770	0.000	0.203	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	140	56	60	0	43	-1
normalized size	1	1.00	0.92	2.26	0.90	0.97	0.00	0.69	-0.02
time (sec)	N/A	0.041	0.059	0.010	0.971	0.853	0.000	0.220	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	77	31	52	0	31	-1
normalized size	1	1.00	0.93	1.67	0.67	1.13	0.00	0.67	-0.02
time (sec)	N/A	0.016	0.023	0.004	0.972	0.889	0.000	0.214	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	137	56	54	0	48	-1
normalized size	1	1.00	1.00	2.98	1.22	1.17	0.00	1.04	-0.02
time (sec)	N/A	0.059	0.038	0.013	0.988	0.885	0.000	0.212	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	222	0	50	0	102	-1
normalized size	1	1.00	1.04	4.35	0.00	0.98	0.00	2.00	-0.02
time (sec)	N/A	0.057	0.064	0.012	0.000	0.905	0.000	0.211	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	254	0	63	0	0	-1
normalized size	1	1.00	0.85	3.10	0.00	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.104	0.011	0.000	0.942	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	84	280	0	75	0	0	-1
normalized size	1	1.00	0.74	2.46	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.108	0.012	0.000	0.591	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	95	304	0	86	0	0	-1
normalized size	1	1.00	0.66	2.13	0.00	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.127	0.013	0.000	0.893	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	112	222	174	94	279	0	-1
normalized size	1	1.00	0.99	1.96	1.54	0.83	2.47	0.00	-0.01
time (sec)	N/A	0.131	0.118	0.015	1.011	0.877	7.848	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	135	330	246	139	830	0	-1
normalized size	1	1.00	0.67	1.64	1.22	0.69	4.13	0.00	-0.00
time (sec)	N/A	0.160	0.174	0.016	1.042	0.808	25.151	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	124	305	221	127	775	0	-1
normalized size	1	1.00	0.72	1.77	1.28	0.74	4.51	0.00	-0.01
time (sec)	N/A	0.123	0.131	0.012	1.033	0.903	23.006	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	113	282	198	117	653	0	-1
normalized size	1	1.00	0.81	2.01	1.41	0.84	4.66	0.00	-0.01
time (sec)	N/A	0.150	0.102	0.014	1.045	0.865	16.657	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	260	176	105	580	0	-1
normalized size	1	1.00	0.88	2.24	1.52	0.91	5.00	0.00	-0.01
time (sec)	N/A	0.059	0.087	0.010	1.016	0.958	16.117	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	147	109	95	435	0	-1
normalized size	1	1.00	0.91	1.47	1.09	0.95	4.35	0.00	-0.01
time (sec)	N/A	0.031	0.055	0.006	0.992	0.950	10.461	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	108	245	124	107	469	0	-1
normalized size	1	1.00	0.96	2.17	1.10	0.95	4.15	0.00	-0.01
time (sec)	N/A	0.116	0.088	0.011	0.990	0.863	25.649	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	114	380	131	123	386	0	-1
normalized size	1	1.00	0.99	3.30	1.14	1.07	3.36	0.00	-0.01
time (sec)	N/A	0.118	0.125	0.011	0.989	0.670	10.205	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	411	138	135	461	0	-1
normalized size	1	1.00	0.98	3.40	1.14	1.12	3.81	0.00	-0.01
time (sec)	N/A	0.113	0.155	0.012	0.975	0.919	13.347	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	116	439	132	130	457	0	-1
normalized size	1	1.00	0.97	3.66	1.10	1.08	3.81	0.00	-0.01
time (sec)	N/A	0.114	0.154	0.013	0.986	0.821	11.684	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	463	159	119	541	0	-1
normalized size	1	1.00	0.93	3.89	1.34	1.00	4.55	0.00	-0.01
time (sec)	N/A	0.115	0.185	0.014	1.009	0.948	14.371	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	493	153	97	774	0	-1
normalized size	1	1.00	0.98	4.56	1.42	0.90	7.17	0.00	-0.01
time (sec)	N/A	0.089	0.147	0.016	1.002	0.716	13.939	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	521	178	108	918	0	-1
normalized size	1	1.00	0.82	3.64	1.24	0.76	6.42	0.00	-0.01
time (sec)	N/A	0.120	0.177	0.014	0.996	0.914	18.691	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	128	546	203	119	1037	0	-1
normalized size	1	1.00	0.74	3.17	1.18	0.69	6.03	0.00	-0.01
time (sec)	N/A	0.154	0.190	0.015	0.994	0.886	18.790	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	139	571	228	130	1159	0	-1
normalized size	1	1.00	0.69	2.84	1.13	0.65	5.77	0.00	-0.00
time (sec)	N/A	0.190	0.212	0.017	0.998	1.047	27.712	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	34	28	31	29	19	20
normalized size	1	1.00	0.96	1.26	1.04	1.15	1.07	0.70	0.74
time (sec)	N/A	0.017	0.038	0.007	0.976	0.813	3.350	0.164	0.039

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	238	68	74	170	125	74
normalized size	1	1.00	0.96	4.67	1.33	1.45	3.33	2.45	1.45
time (sec)	N/A	0.072	0.037	0.018	0.989	0.888	6.533	0.195	0.051

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	91	147	113	112	0	0	-1
normalized size	1	1.00	0.77	1.25	0.96	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.088	0.014	1.007	0.618	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	80	120	86	101	0	0	-1
normalized size	1	1.00	0.88	1.32	0.95	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.059	0.011	0.997	0.899	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	97	63	85	0	0	-1
normalized size	1	1.00	0.77	1.26	0.82	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.070	0.010	0.976	0.840	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	74	40	67	0	0	-1
normalized size	1	1.00	0.94	1.42	0.77	1.29	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.029	0.010	0.974	0.802	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	29	30	35	0	0	29
normalized size	1	1.00	1.03	0.94	0.97	1.13	0.00	0.00	0.94
time (sec)	N/A	0.011	0.006	0.006	0.976	0.596	0.000	0.000	2.643

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	88	0	62	0	0	-1
normalized size	1	1.00	0.96	1.63	0.00	1.15	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.037	0.012	0.000	1.116	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	108	0	88	0	0	-1
normalized size	1	1.00	0.77	1.33	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.053	0.012	0.000	0.851	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	127	133	0	113	0	0	-1
normalized size	1	1.00	1.12	1.18	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.317	0.014	0.000	0.817	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	106	208	151	190	0	0	-1
normalized size	1	1.00	0.83	1.62	1.18	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.170	0.021	1.015	0.820	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	93	179	124	175	0	0	-1
normalized size	1	1.00	0.82	1.58	1.10	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.135	0.010	1.010	0.906	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	153	99	157	0	0	-1
normalized size	1	1.00	0.90	1.72	1.11	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.122	0.011	1.003	0.810	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	86	103	0	1	56
normalized size	1	1.00	1.00	0.80	1.43	1.72	0.00	0.02	0.93
time (sec)	N/A	0.035	0.052	0.008	0.464	0.672	0.000	0.245	2.712

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	44	67	101	0	0	52
normalized size	1	1.00	0.97	0.76	1.16	1.74	0.00	0.00	0.90
time (sec)	N/A	0.020	0.045	0.008	0.458	1.048	0.000	0.000	2.712

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	46	65	102	0	0	56
normalized size	1	1.00	1.00	0.79	1.12	1.76	0.00	0.00	0.97
time (sec)	N/A	0.015	0.033	0.007	0.453	0.565	0.000	0.000	2.714

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	142	0	155	0	0	-1
normalized size	1	1.00	0.94	1.61	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.103	0.012	0.000	0.901	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	101	188	0	181	0	1	-1
normalized size	1	1.00	0.84	1.57	0.00	1.51	0.00	0.01	-0.01
time (sec)	N/A	0.103	0.119	0.017	0.000	0.652	0.000	0.252	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	115	216	0	201	0	0	-1
normalized size	1	1.00	0.76	1.42	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.104	0.016	0.000	0.866	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	128	318	289	274	0	0	-1
normalized size	1	1.00	0.79	1.96	1.78	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.240	0.049	1.091	0.917	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	115	288	259	258	0	0	-1
normalized size	1	1.00	0.78	1.95	1.75	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.192	0.011	1.091	0.884	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	103	259	234	241	0	0	-1
normalized size	1	1.00	0.84	2.12	1.92	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.145	0.010	1.060	0.870	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	134	168	0	0	78
normalized size	1	1.00	0.96	0.82	1.58	1.98	0.00	0.00	0.92
time (sec)	N/A	0.074	0.086	0.008	0.499	0.733	0.000	0.000	2.952

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	110	171	0	0	78
normalized size	1	1.00	0.90	0.77	1.21	1.88	0.00	0.00	0.86
time (sec)	N/A	0.069	0.071	0.008	0.470	0.884	0.000	0.000	2.835

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	70	110	170	0	0	78
normalized size	1	1.00	0.86	0.74	1.16	1.79	0.00	0.00	0.82
time (sec)	N/A	0.053	0.060	0.009	0.478	0.807	0.000	0.000	2.788

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	90	171	0	0	78
normalized size	1	1.00	0.96	0.82	1.06	2.01	0.00	0.00	0.92
time (sec)	N/A	0.029	0.055	0.008	0.490	0.687	0.000	0.000	2.784

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	70	85	168	0	0	78
normalized size	1	1.00	1.00	0.85	1.04	2.05	0.00	0.00	0.95
time (sec)	N/A	0.022	0.039	0.008	0.452	0.924	0.000	0.000	2.759

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	106	196	0	237	0	0	-1
normalized size	1	1.00	0.89	1.65	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.090	0.015	0.000	0.879	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	122	268	0	265	0	0	-1
normalized size	1	1.00	0.79	1.74	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.124	0.016	0.000	0.731	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	137	298	0	286	0	0	-1
normalized size	1	1.00	0.74	1.60	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.130	0.017	0.000	1.056	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	148	326	0	297	0	0	-1
normalized size	1	1.00	0.69	1.52	0.00	1.38	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.162	0.021	0.000	1.448	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	92	133	239	0	0	161
normalized size	1	1.00	0.88	0.78	1.13	2.03	0.00	0.00	1.36
time (sec)	N/A	0.077	0.120	0.010	0.508	1.017	0.000	0.000	2.948

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	104	92	133	238	0	0	161
normalized size	1	1.00	0.85	0.75	1.08	1.93	0.00	0.00	1.31
time (sec)	N/A	0.058	0.080	0.010	0.485	1.012	0.000	0.000	2.883

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	100	68	75	0	0	116
normalized size	1	1.00	0.82	1.52	1.03	1.14	0.00	0.00	1.76
time (sec)	N/A	0.054	0.051	0.014	0.975	0.673	0.000	0.000	0.071

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	37	84	52	66	0	70	84
normalized size	1	1.00	0.67	1.53	0.95	1.20	0.00	1.27	1.53
time (sec)	N/A	0.086	0.053	0.008	0.978	0.872	0.000	0.198	0.067

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	65	33	58	0	52	57
normalized size	1	1.00	0.91	1.91	0.97	1.71	0.00	1.53	1.68
time (sec)	N/A	0.017	0.024	0.009	0.986	0.876	0.000	0.215	2.605

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	22	23	28	0	34	23
normalized size	1	1.00	0.96	0.85	0.88	1.08	0.00	1.31	0.88
time (sec)	N/A	0.010	0.006	0.006	0.975	0.717	0.000	0.200	2.588

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	58	0	52	0	74	58
normalized size	1	1.00	1.00	1.41	0.00	1.27	0.00	1.80	1.41
time (sec)	N/A	0.040	0.028	0.014	0.000	0.873	0.000	0.208	2.652

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	73	0	76	0	0	81
normalized size	1	1.00	0.78	1.14	0.00	1.19	0.00	0.00	1.27
time (sec)	N/A	0.053	0.041	0.013	0.000	0.887	0.000	0.000	2.592

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	63	94	0	97	0	213	105
normalized size	1	1.00	0.70	1.04	0.00	1.08	0.00	2.37	1.17
time (sec)	N/A	0.079	0.055	0.013	0.000	0.898	0.000	0.208	2.611

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	135	375	299	138	571	0	-1
normalized size	1	1.00	0.59	1.64	1.31	0.60	2.49	0.00	-0.00
time (sec)	N/A	0.315	0.213	0.020	1.072	0.883	17.476	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	124	350	275	128	690	0	-1
normalized size	1	1.00	0.62	1.75	1.38	0.64	3.45	0.00	-0.00
time (sec)	N/A	0.270	0.155	0.015	1.039	0.818	21.396	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	113	327	251	116	450	0	-1
normalized size	1	1.00	0.66	1.91	1.47	0.68	2.63	0.00	-0.01
time (sec)	N/A	0.215	0.127	0.016	1.038	0.880	12.029	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	102	303	230	106	541	0	-1
normalized size	1	1.00	0.72	2.13	1.62	0.75	3.81	0.00	-0.01
time (sec)	N/A	0.177	0.114	0.014	1.030	0.636	14.453	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	91	198	167	94	321	0	-1
normalized size	1	1.00	0.67	1.46	1.23	0.69	2.36	0.00	-0.01
time (sec)	N/A	0.057	0.076	0.013	1.002	0.807	8.566	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	194	119	84	350	0	-1
normalized size	1	1.00	0.74	1.80	1.10	0.78	3.24	0.00	-0.01
time (sec)	N/A	0.042	0.048	0.006	0.991	0.582	9.349	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	290	103	95	267	0	-1
normalized size	1	1.00	1.00	3.02	1.07	0.99	2.78	0.00	-0.01
time (sec)	N/A	0.162	0.096	0.012	0.987	0.840	14.833	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	100	425	112	111	347	0	-1
normalized size	1	1.00	0.95	4.05	1.07	1.06	3.30	0.00	-0.01
time (sec)	N/A	0.161	0.127	0.013	1.118	0.621	9.854	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	456	111	119	347	0	-1
normalized size	1	1.00	0.93	4.15	1.01	1.08	3.15	0.00	-0.01
time (sec)	N/A	0.163	0.140	0.014	0.986	0.979	10.175	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	479	134	106	338	0	-1
normalized size	1	1.00	0.94	4.70	1.31	1.04	3.31	0.00	-0.01
time (sec)	N/A	0.163	0.178	0.016	0.991	0.928	9.735	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	95	513	130	86	422	0	-1
normalized size	1	1.00	0.88	4.75	1.20	0.80	3.91	0.00	-0.01
time (sec)	N/A	0.147	0.175	0.016	1.008	0.841	12.355	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	541	155	97	660	0	-1
normalized size	1	1.00	0.76	3.86	1.11	0.69	4.71	0.00	-0.01
time (sec)	N/A	0.177	0.168	0.016	0.991	0.877	13.422	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	117	566	180	108	808	0	-1
normalized size	1	1.00	0.69	3.35	1.07	0.64	4.78	0.00	-0.01
time (sec)	N/A	0.208	0.234	0.017	1.010	0.918	19.721	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	128	591	205	119	835	0	-1
normalized size	1	1.00	0.65	2.98	1.04	0.60	4.22	0.00	-0.01
time (sec)	N/A	0.236	0.234	0.018	0.988	0.898	18.183	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	106	198	170	171	0	0	-1
normalized size	1	1.00	0.86	1.61	1.38	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.158	0.017	1.015	0.948	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	65	157	116	0	0	66
normalized size	1	1.00	0.71	0.66	1.59	1.17	0.00	0.00	0.67
time (sec)	N/A	0.204	0.077	0.007	0.468	0.861	0.000	0.000	2.970

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	65	136	118	0	0	66
normalized size	1	1.00	0.79	0.73	1.53	1.33	0.00	0.00	0.74
time (sec)	N/A	0.142	0.064	0.009	0.468	0.877	0.000	0.000	2.896

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	69	64	138	116	0	0	65
normalized size	1	1.00	0.76	0.70	1.52	1.27	0.00	0.00	0.71
time (sec)	N/A	0.036	0.054	0.010	0.464	0.902	0.000	0.000	2.879

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	66	136	115	0	0	66
normalized size	1	1.00	0.77	0.73	1.49	1.26	0.00	0.00	0.73
time (sec)	N/A	0.031	0.037	0.008	0.445	0.716	0.000	0.000	2.847

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	95	187	0	168	0	0	-1
normalized size	1	1.00	0.81	1.58	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.087	0.012	0.000	1.001	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	112	234	0	194	0	0	-1
normalized size	1	1.00	0.77	1.60	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.108	0.012	0.000	1.168	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	127	259	0	215	0	0	-1
normalized size	1	1.00	0.69	1.42	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.375	0.131	0.017	0.000	0.985	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	98	212	185	190	0	0	-1
normalized size	1	1.00	0.55	1.20	1.05	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.439	0.193	0.019	1.000	1.036	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	85	187	160	174	0	0	-1
normalized size	1	1.00	0.58	1.28	1.10	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.365	0.150	0.016	0.984	0.864	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	73	163	136	157	0	0	-1
normalized size	1	1.00	0.61	1.36	1.13	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.113	0.013	0.985	0.820	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	52	55	125	104	0	0	48
normalized size	1	1.00	0.55	0.58	1.32	1.09	0.00	0.00	0.51
time (sec)	N/A	0.128	0.061	0.008	0.983	0.980	0.000	0.000	2.762

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	49	52	129	100	0	0	45
normalized size	1	1.00	0.51	0.54	1.33	1.03	0.00	0.00	0.46
time (sec)	N/A	0.045	0.049	0.008	0.984	0.957	0.000	0.000	2.590

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	55	128	104	0	0	48
normalized size	1	1.00	0.52	0.55	1.28	1.04	0.00	0.00	0.48
time (sec)	N/A	0.037	0.029	0.007	0.984	0.914	0.000	0.000	2.619

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	76	179	0	153	0	0	-1
normalized size	1	1.00	0.66	1.56	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.120	0.012	0.000	0.954	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	92	199	0	181	0	1	-1
normalized size	1	1.00	0.63	1.36	0.00	1.24	0.00	0.01	-0.01
time (sec)	N/A	0.305	0.177	0.013	0.000	0.883	0.000	0.302	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	107	222	0	202	0	1	-1
normalized size	1	1.00	0.58	1.21	0.00	1.10	0.00	0.01	-0.01
time (sec)	N/A	0.380	0.173	0.014	0.000	0.793	0.000	0.293	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	109	297	0	200	0	0	-1
normalized size	1	1.00	0.53	1.46	0.00	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.593	0.184	0.025	0.000	0.957	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	98	273	0	190	0	0	-1
normalized size	1	1.00	0.61	1.71	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.415	0.179	0.014	0.000	0.979	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	85	212	0	174	0	0	-1
normalized size	1	1.00	0.57	1.43	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.130	0.011	0.000	1.078	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	73	214	0	157	0	0	-1
normalized size	1	1.00	0.63	1.86	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.122	0.011	0.000	0.941	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	42	125	102	0	0	46
normalized size	1	1.00	0.78	0.66	1.95	1.59	0.00	0.00	0.72
time (sec)	N/A	0.027	0.049	0.007	0.447	0.953	0.000	0.000	2.904

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	123	104	0	0	47
normalized size	1	1.00	0.76	0.64	1.84	1.55	0.00	0.00	0.70
time (sec)	N/A	0.024	0.028	0.006	0.445	1.028	0.000	0.000	2.776

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	76	196	0	153	0	0	-1
normalized size	1	1.00	0.69	1.78	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.137	0.013	0.000	0.833	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	361	0	181	0	1	-1
normalized size	1	1.00	0.64	2.52	0.00	1.27	0.00	0.01	-0.01
time (sec)	N/A	0.306	0.209	0.012	0.000	0.937	0.000	0.283	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	107	389	0	202	0	1	-1
normalized size	1	1.00	0.58	2.13	0.00	1.10	0.00	0.01	-0.01
time (sec)	N/A	0.392	0.229	0.013	0.000	0.958	0.000	0.303	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	118	412	0	213	0	1	-1
normalized size	1	1.00	0.56	1.96	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.494	0.266	0.019	0.000	0.818	0.000	0.368	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	131	416	478	156	0	0	-1
normalized size	1	1.00	0.52	1.65	1.90	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.663	0.232	0.026	1.059	0.806	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	125	393	456	146	0	0	-1
normalized size	1	1.00	0.56	1.75	2.04	0.65	0.00	0.00	-0.00
time (sec)	N/A	0.534	0.163	0.015	1.040	0.969	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	109	285	407	134	0	0	-1
normalized size	1	1.00	0.57	1.48	2.12	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.435	0.136	0.017	1.029	0.959	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	103	288	363	124	0	0	-1
normalized size	1	1.00	0.57	1.58	1.99	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.119	0.012	1.023	0.796	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	83	290	235	111	0	0	-1
normalized size	1	1.00	0.64	2.23	1.81	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.101	0.011	0.985	0.992	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	75	284	134	99	0	0	-1
normalized size	1	1.00	0.66	2.51	1.19	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.065	0.007	0.978	0.928	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	378	0	111	0	0	-1
normalized size	1	1.00	0.89	4.25	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.147	0.012	0.000	0.926	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	515	0	127	0	0	-1
normalized size	1	1.00	0.89	5.48	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.187	0.013	0.000	0.900	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	85	504	0	112	0	0	-1
normalized size	1	1.00	0.77	4.58	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.221	0.013	0.000	0.657	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	94	575	0	123	0	0	-1
normalized size	1	1.00	0.69	4.20	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.297	0.247	0.018	0.000	0.560	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	107	600	0	136	0	0	-1
normalized size	1	1.00	0.63	3.53	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.394	0.268	0.017	0.000	0.907	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	118	628	0	147	0	0	-1
normalized size	1	1.00	0.60	3.20	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.517	0.331	0.018	0.000	1.097	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	50	200	0	126	0	0	220
normalized size	1	1.00	0.53	2.11	0.00	1.33	0.00	0.00	2.32
time (sec)	N/A	0.132	0.114	0.018	0.000	0.956	0.000	0.000	2.703

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	44	153	102	0	0	287
normalized size	1	1.00	0.57	0.50	1.74	1.16	0.00	0.00	3.26
time (sec)	N/A	0.123	0.072	0.007	0.444	0.884	0.000	0.000	0.061

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	399	316	0	0	252
normalized size	1	1.00	0.66	0.63	1.91	1.51	0.00	0.00	1.21
time (sec)	N/A	0.315	0.176	0.011	0.496	1.922	0.000	0.000	3.217

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	401	317	0	0	252
normalized size	1	1.00	0.66	0.63	1.92	1.52	0.00	0.00	1.21
time (sec)	N/A	0.207	0.100	0.013	0.493	2.057	0.000	0.000	3.189

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	137	132	405	316	0	0	252
normalized size	1	1.00	0.65	0.63	1.92	1.50	0.00	0.00	1.19
time (sec)	N/A	0.105	0.088	0.011	0.490	2.338	0.000	0.000	3.194

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	137	132	393	314	0	0	242
normalized size	1	1.00	0.67	0.64	1.92	1.53	0.00	0.00	1.18
time (sec)	N/A	0.091	0.066	0.010	0.500	2.330	0.000	0.000	3.118

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	161	385	0	432	0	0	-1
normalized size	1	1.00	0.69	1.65	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.172	0.028	0.000	2.402	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	183	484	0	458	0	0	-1
normalized size	1	1.00	0.68	1.79	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.680	0.210	0.018	0.000	3.668	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	93	95	0	217	0	0	-1
normalized size	1	1.00	0.91	0.93	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.095	0.049	0.000	0.898	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	67	58	0	110	0	57	-1
normalized size	1	1.00	1.72	1.49	0.00	2.82	0.00	1.46	-0.03
time (sec)	N/A	0.040	0.035	0.018	0.000	0.908	0.000	0.172	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	62	48	92	83	0	38
normalized size	1	1.00	1.00	1.77	1.37	2.63	2.37	0.00	1.09
time (sec)	N/A	0.010	0.015	0.008	0.957	0.699	1.904	0.000	2.985

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	76	0	199	0	0	-1
normalized size	1	1.00	1.00	2.17	0.00	5.69	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.009	0.016	0.000	0.776	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	57	68	90	29	0	36
normalized size	1	1.00	1.00	1.68	2.00	2.65	0.85	0.00	1.06
time (sec)	N/A	0.009	0.012	0.006	0.955	0.892	1.966	0.000	3.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	86	0	208	0	0	-1
normalized size	1	1.00	1.00	2.53	0.00	6.12	0.00	0.00	-0.03
time (sec)	N/A	0.028	0.012	0.014	0.000	1.029	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	79	82	111	148	0	54
normalized size	1	1.00	0.78	1.25	1.30	1.76	2.35	0.00	0.86
time (sec)	N/A	0.015	0.020	0.005	0.959	0.749	3.391	0.000	2.597

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	92	0	221	0	0	-1
normalized size	1	1.00	0.78	1.46	0.00	3.51	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.007	0.014	0.000	0.707	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	199	0	0	0	513	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	2.05	0.00	-0.00
time (sec)	N/A	0.386	0.197	0.061	0.000	1.000	45.914	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	174	0	0	0	442	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	2.15	0.00	-0.00
time (sec)	N/A	0.209	0.109	0.025	0.000	0.783	33.194	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	121	0	0	0	374	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	2.31	0.00	-0.01
time (sec)	N/A	0.084	0.050	0.018	0.000	0.720	24.577	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	61	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.76	0.00	-0.01
time (sec)	N/A	0.025	0.018	0.017	0.000	0.930	10.074	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	248	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	1.52	0.00	-0.01
time (sec)	N/A	0.141	0.056	0.032	0.000	0.567	25.455	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	173	0	0	0	185	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.91	0.00	-0.00
time (sec)	N/A	0.215	0.111	0.025	0.000	0.962	91.469	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	245	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.176	0.025	0.000	0.772	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	199	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.186	0.024	0.000	0.965	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.109	0.025	0.000	0.957	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	162	121	0	0	0	117	0	-1
normalized size	1	1.31	0.98	0.00	0.00	0.00	0.94	0.00	-0.01
time (sec)	N/A	0.082	0.053	0.019	0.000	0.854	63.251	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	60	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.025	0.018	0.018	0.000	0.793	11.402	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.061	0.025	0.000	1.018	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	176	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.123	0.025	0.000	0.979	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	200	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.207	0.026	0.000	0.906	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	972	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	6.57	0.00	-0.01
time (sec)	N/A	0.094	0.091	0.077	0.000	0.798	7.456	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	129	0	0	0	972	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	6.61	0.00	-0.01
time (sec)	N/A	0.091	0.082	0.055	0.000	0.943	7.101	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	0	0	0	382	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	3.18	0.00	-0.01
time (sec)	N/A	0.071	0.076	0.050	0.000	0.915	5.457	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	103	0	0	0	382	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	3.21	0.00	-0.01
time (sec)	N/A	0.067	0.074	0.042	0.000	0.757	5.073	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	85	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.96	0.00	-0.01
time (sec)	N/A	0.034	0.041	0.037	0.000	0.940	3.524	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	82	0	78
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.99	0.00	0.94
time (sec)	N/A	0.023	0.046	0.036	0.000	0.856	4.177	0.000	4.345

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	78	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.055	0.036	0.027	0.000	1.011	9.367	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	0	82	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	-0.01
time (sec)	N/A	0.057	0.049	0.033	0.000	0.767	5.527	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	106	0	0	0	85	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.059	0.050	0.041	0.000	0.813	5.073	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	159	0	0	0	2924	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	16.43	0.00	-0.01
time (sec)	N/A	0.145	0.128	0.071	0.000	0.918	14.933	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	186	0	0	0	1015	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	5.49	0.00	-0.01
time (sec)	N/A	0.173	0.124	0.065	0.000	0.815	8.903	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	0	0	0	1328	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	8.91	0.00	-0.01
time (sec)	N/A	0.130	0.119	0.058	0.000	0.940	9.142	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	168	0	0	0	425	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	2.74	0.00	-0.01
time (sec)	N/A	0.139	0.107	0.052	0.000	0.832	6.428	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	110	0	0	0	440	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	3.73	0.00	-0.01
time (sec)	N/A	0.091	0.074	0.043	0.000	0.862	5.924	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	134	0	0	0	124	0	-1
normalized size	1	1.00	1.89	0.00	0.00	0.00	1.75	0.00	-0.01
time (sec)	N/A	0.031	0.057	0.038	0.000	0.856	4.950	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	103	0	0	0	136	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	1.06	0.00	-0.01
time (sec)	N/A	0.095	0.064	0.029	0.000	0.968	9.046	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	153	0	0	0	116	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.91	0.00	-0.01
time (sec)	N/A	0.116	0.072	0.033	0.000	0.931	6.895	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	131	0	0	0	139	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.125	0.082	0.043	0.000	0.919	6.917	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	205	0	0	0	2966	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	13.36	0.00	-0.00
time (sec)	N/A	0.191	0.237	0.062	0.000	0.925	16.246	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	219	0	0	0	2966	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	13.61	0.00	-0.00
time (sec)	N/A	0.177	0.243	0.079	0.000	0.785	16.046	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	187	0	0	0	1370	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	7.10	0.00	-0.01
time (sec)	N/A	0.182	0.181	0.071	0.000	0.699	11.265	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	187	0	0	0	1370	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	7.25	0.00	-0.01
time (sec)	N/A	0.169	0.190	0.063	0.000	1.049	10.462	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	159	0	0	0	479	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	4.13	0.00	-0.01
time (sec)	N/A	0.066	0.262	0.070	0.000	0.747	7.579	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	476	0	-1
normalized size	1	1.00	2.12	0.00	0.00	0.00	6.52	0.00	-0.01
time (sec)	N/A	0.026	0.163	0.044	0.000	0.938	6.579	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	169	0	0	0	178	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	1.04	0.00	-0.01
time (sec)	N/A	0.129	0.140	0.029	0.000	0.806	12.289	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	158	0	0	0	177	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	1.11	0.00	-0.01
time (sec)	N/A	0.185	0.087	0.036	0.000	0.865	7.450	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	182	0	0	0	177	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	1.07	0.00	-0.01
time (sec)	N/A	0.215	0.102	0.044	0.000	0.914	8.367	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	66	0	0	0	4442	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	30.01	0.00	-0.01
time (sec)	N/A	0.109	0.107	0.062	0.000	0.914	16.535	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	245	0	0	0	5090	0	-1
normalized size	1	1.00	2.02	0.00	0.00	0.00	42.07	0.00	-0.01
time (sec)	N/A	0.093	0.290	0.057	0.000	1.022	10.517	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	198	0	0	0	4124	0	-1
normalized size	1	1.00	1.66	0.00	0.00	0.00	34.66	0.00	-0.01
time (sec)	N/A	0.085	0.250	0.050	0.000	0.968	8.392	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	0	0	0	427	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	4.74	0.00	-0.01
time (sec)	N/A	0.058	0.103	0.041	0.000	0.822	7.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	321	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	4.40	0.00	-0.01
time (sec)	N/A	0.033	0.034	0.048	0.000	0.871	7.324	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	151	0	0	0	355	0	-1
normalized size	1	1.00	1.45	0.00	0.00	0.00	3.41	0.00	-0.01
time (sec)	N/A	0.068	0.110	0.034	0.000	0.663	6.901	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	450	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	4.25	0.00	-0.01
time (sec)	N/A	0.074	0.178	0.044	0.000	0.607	7.865	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	219	0	0	0	498	0	-1
normalized size	1	1.00	2.03	0.00	0.00	0.00	4.61	0.00	-0.01
time (sec)	N/A	0.080	0.565	0.056	0.000	0.623	9.477	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	66	0	0	0	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.147	0.080	0.000	1.012	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	66	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.116	0.073	0.000	0.666	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	332	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.290	0.063	0.000	0.697	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	177	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.162	0.054	0.000	0.914	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.086	0.054	0.000	0.932	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.040	0.053	0.000	0.747	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	201	0	0	0	0	0	-1
normalized size	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.156	0.035	0.000	0.714	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	223	0	0	0	0	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.334	0.048	0.000	0.820	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	283	0	0	0	0	0	-1
normalized size	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.500	0.061	0.000	0.954	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	334	0	0	0	0	0	-1
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.411	0.072	0.000	0.954	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	389	0	0	0	0	0	-1
normalized size	1	1.00	2.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.501	0.089	0.000	1.002	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	245	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.307	0.092	0.000	0.938	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	202	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.242	0.082	0.000	0.607	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.115	0.080	0.000	0.677	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.085	0.077	0.000	0.631	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.046	0.077	0.000	0.696	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	328	0	0	0	0	0	-1
normalized size	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.229	0.039	0.000	0.990	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	280	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.427	0.068	0.000	0.836	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	341	0	0	0	0	0	-1
normalized size	1	1.00	1.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.649	0.082	0.000	0.832	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	393	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.499	0.125	0.000	0.930	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	446	0	0	0	0	0	-1
normalized size	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.577	0.063	0.000	0.886	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	231	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.289	0.113	0.000	0.948	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	156	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.199	0.107	0.000	0.699	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	130	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.114	0.109	0.000	0.925	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.086	0.109	0.000	0.916	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.040	0.107	0.000	1.039	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	417	0	0	0	0	0	-1
normalized size	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.279	0.041	0.000	0.809	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	337	0	0	0	0	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.437	0.084	0.000	0.873	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	399	0	0	0	0	0	-1
normalized size	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.773	0.107	0.000	1.051	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	452	0	0	0	0	0	-1
normalized size	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.391	0.616	0.063	0.000	0.909	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	505	0	0	0	0	0	-1
normalized size	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.738	0.076	0.000	0.830	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	194	0	0	0	262	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.99	0.00	-0.00
time (sec)	N/A	0.373	0.167	0.140	0.000	0.680	25.295	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	169	0	0	0	192	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.93	0.00	-0.00
time (sec)	N/A	0.168	0.085	0.075	0.000	0.828	16.858	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	116	0	0	0	122	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.068	0.033	0.069	0.000	0.960	9.944	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	61	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.020	0.009	0.081	0.000	0.655	3.587	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	124	0	0	0	337	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	2.07	0.00	-0.01
time (sec)	N/A	0.136	0.051	0.072	0.000	0.831	13.152	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	180	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.105	0.081	0.000	0.987	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	206	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	0.193	0.100	0.000	0.716	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	308	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	3.46	0.00	-0.01
time (sec)	N/A	0.056	0.041	0.123	0.000	0.925	10.814	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	90	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.112	0.135	0.000	0.734	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	60	240	0	307	68	167	201
normalized size	1	1.00	0.28	1.12	0.00	1.43	0.32	0.78	0.94
time (sec)	N/A	0.247	0.043	0.117	0.000	1.005	11.157	0.896	0.111

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	259	560	249	1104	0	252	-1
normalized size	1	1.00	1.02	2.20	0.98	4.33	0.00	0.99	-0.00
time (sec)	N/A	0.629	0.613	0.021	0.682	13.721	0.000	0.210	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	225	515	207	963	0	201	-1
normalized size	1	1.00	1.07	2.44	0.98	4.56	0.00	0.95	-0.00
time (sec)	N/A	0.390	0.400	0.011	0.584	13.974	0.000	0.219	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	193	448	144	776	0	157	-1
normalized size	1	1.00	1.26	2.93	0.94	5.07	0.00	1.03	-0.01
time (sec)	N/A	0.211	0.319	0.013	0.524	1.495	0.000	0.210	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	175	423	122	684	0	135	-1
normalized size	1	1.00	1.38	3.33	0.96	5.39	0.00	1.06	-0.01
time (sec)	N/A	0.105	0.275	0.008	0.505	1.391	0.000	0.199	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	99	381	84	574	0	109	-1
normalized size	1	1.00	0.96	3.70	0.82	5.57	0.00	1.06	-0.01
time (sec)	N/A	0.071	0.024	0.006	0.483	1.147	0.000	0.191	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	420	103	1316	0	0	-1
normalized size	1	1.00	0.97	3.62	0.89	11.34	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.048	0.010	0.499	2.492	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	178	486	0	599	0	145	-1
normalized size	1	1.00	1.70	4.63	0.00	5.70	0.00	1.38	-0.01
time (sec)	N/A	0.167	0.238	0.012	0.000	1.122	0.000	0.220	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	283	567	0	726	0	230	-1
normalized size	1	1.00	1.77	3.54	0.00	4.54	0.00	1.44	-0.01
time (sec)	N/A	0.211	0.397	0.013	0.000	0.979	0.000	0.217	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	301	600	0	824	0	309	-1
normalized size	1	1.00	1.58	3.14	0.00	4.31	0.00	1.62	-0.01
time (sec)	N/A	0.232	1.020	0.015	0.000	1.001	0.000	0.212	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	344	703	0	1007	0	596	-1
normalized size	1	1.00	1.26	2.57	0.00	3.68	0.00	2.18	-0.00
time (sec)	N/A	0.297	1.097	0.016	0.000	1.185	0.000	0.260	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	149	260	171	1060	0	163	-1
normalized size	1	1.00	0.76	1.33	0.88	5.44	0.00	0.84	-0.01
time (sec)	N/A	0.482	0.227	0.016	0.547	5.995	0.000	0.213	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	131	217	130	924	0	129	-1
normalized size	1	1.00	0.86	1.43	0.86	6.08	0.00	0.85	-0.01
time (sec)	N/A	0.274	0.222	0.010	0.518	5.569	0.000	0.224	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	172	90	745	0	105	-1
normalized size	1	1.00	0.96	1.58	0.83	6.83	0.00	0.96	-0.01
time (sec)	N/A	0.128	0.074	0.009	0.491	1.213	0.000	0.226	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	151	71	631	0	88	-1
normalized size	1	1.00	1.00	1.76	0.83	7.34	0.00	1.02	-0.01
time (sec)	N/A	0.044	0.026	0.010	0.485	1.154	0.000	0.201	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	127	52	211	0	59	-1
normalized size	1	1.00	1.00	2.35	0.96	3.91	0.00	1.09	-0.02
time (sec)	N/A	0.017	0.007	0.005	0.463	0.895	0.000	0.189	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	158	0	634	0	0	-1
normalized size	1	1.00	1.00	1.84	0.00	7.37	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.045	0.012	0.000	0.889	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	107	180	0	767	0	142	-1
normalized size	1	1.00	0.96	1.62	0.00	6.91	0.00	1.28	-0.01
time (sec)	N/A	0.095	0.083	0.012	0.000	1.051	0.000	0.221	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	163	236	0	956	0	239	-1
normalized size	1	1.00	0.97	1.40	0.00	5.69	0.00	1.42	-0.01
time (sec)	N/A	0.140	0.642	0.014	0.000	1.041	0.000	0.222	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	179	396	251	1525	0	299	-1
normalized size	1	1.00	1.23	2.71	1.72	10.45	0.00	2.05	-0.01
time (sec)	N/A	0.312	0.444	0.020	0.621	9.361	0.000	0.265	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	153	354	211	1323	0	219	-1
normalized size	1	1.00	1.24	2.88	1.72	10.76	0.00	1.78	-0.01
time (sec)	N/A	0.165	0.280	0.010	0.587	9.784	0.000	0.235	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	311	171	455	0	174	-1
normalized size	1	1.00	1.00	3.27	1.80	4.79	0.00	1.83	-0.01
time (sec)	N/A	0.111	0.084	0.012	0.546	1.111	0.000	0.217	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	283	148	425	0	162	-1
normalized size	1	1.00	1.00	3.22	1.68	4.83	0.00	1.84	-0.01
time (sec)	N/A	0.052	0.053	0.007	0.540	1.042	0.000	0.205	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	260	123	456	0	172	-1
normalized size	1	1.00	1.00	2.77	1.31	4.85	0.00	1.83	-0.01
time (sec)	N/A	0.046	0.045	0.006	0.502	1.066	0.000	0.232	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	132	318	0	1325	0	0	-1
normalized size	1	1.00	0.90	2.16	0.00	9.01	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.168	0.013	0.000	1.550	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	163	363	0	1556	0	266	-1
normalized size	1	1.00	0.84	1.87	0.00	8.02	0.00	1.37	-0.01
time (sec)	N/A	0.167	0.405	0.012	0.000	1.474	0.000	0.251	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	275	203	439	0	1943	0	358	-1
normalized size	1	1.00	0.74	1.59	0.00	7.04	0.00	1.30	-0.00
time (sec)	N/A	0.236	0.342	0.014	0.000	2.125	0.000	0.289	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	230	474	274	2025	0	0	-1
normalized size	1	1.00	0.94	1.94	1.12	8.30	0.00	0.00	-0.00
time (sec)	N/A	0.890	0.513	0.019	0.604	91.203	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	208	435	233	1786	0	0	-1
normalized size	1	1.00	1.02	2.13	1.14	8.75	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.366	0.013	0.571	152.354	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	184	386	193	1449	0	0	-1
normalized size	1	1.00	1.15	2.41	1.21	9.06	0.00	0.00	-0.01
time (sec)	N/A	0.327	0.274	0.013	0.543	16.100	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	172	368	171	1260	0	0	-1
normalized size	1	1.00	1.26	2.69	1.25	9.20	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.323	0.011	0.533	13.027	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	340	148	382	0	0	-1
normalized size	1	1.00	1.00	3.78	1.64	4.24	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.046	0.009	0.515	1.063	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	115	210	93	381	0	0	-1
normalized size	1	1.00	1.26	2.31	1.02	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.074	0.006	0.490	1.266	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	178	364	0	1261	0	126	-1
normalized size	1	1.00	0.99	2.03	0.00	7.04	0.00	0.70	-0.01
time (sec)	N/A	0.137	0.222	0.011	0.000	1.810	0.000	0.566	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	197	395	0	1512	0	0	-1
normalized size	1	1.00	0.93	1.86	0.00	7.13	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.328	0.012	0.000	1.627	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	229	452	0	1867	0	0	-1
normalized size	1	1.00	0.85	1.69	0.00	6.97	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.448	0.012	0.000	2.791	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	114	328	210	368	4134	624	363
normalized size	1	1.00	0.84	2.43	1.56	2.73	30.62	4.62	2.69
time (sec)	N/A	0.082	0.092	0.007	0.472	0.904	6.697	0.194	2.816

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	195	146	250	2181	410	255
normalized size	1	1.00	1.07	1.91	1.43	2.45	21.38	4.02	2.50
time (sec)	N/A	0.052	0.104	0.005	0.467	0.895	3.535	0.189	2.701

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	100	89	148	952	237	163
normalized size	1	1.00	0.93	1.43	1.27	2.11	13.60	3.39	2.33
time (sec)	N/A	0.031	0.041	0.004	0.457	0.592	2.073	0.164	2.629

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	0	0	0	345	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	4.48	0.00	-0.01
time (sec)	N/A	0.051	0.040	0.031	0.000	0.898	7.222	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	199	1000	447	1027	14317	1750	932
normalized size	1	1.00	0.86	4.31	1.93	4.43	61.71	7.54	4.02
time (sec)	N/A	0.139	0.181	0.016	0.505	0.963	21.587	0.215	3.119

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	323	677	335	757	8940	1266	723
normalized size	1	1.00	1.75	3.66	1.81	4.09	48.32	6.84	3.91
time (sec)	N/A	0.100	0.498	0.012	0.500	0.961	13.801	0.215	3.051

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	160	420	235	519	5097	851	496
normalized size	1	1.00	1.14	3.00	1.68	3.71	36.41	6.08	3.54
time (sec)	N/A	0.068	0.178	0.010	0.477	0.889	7.128	0.200	2.837

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	1678	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	11.34	0.00	-0.01
time (sec)	N/A	0.210	0.134	0.030	0.000	0.994	11.189	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	302	2232	795	2165	0	3713	1796
normalized size	1	1.00	0.88	6.51	2.32	6.31	0.00	10.83	5.24
time (sec)	N/A	0.211	0.248	0.024	0.550	1.033	0.000	0.304	3.809

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	709	1639	625	1675	0	2851	1459
normalized size	1	1.00	2.51	5.81	2.22	5.94	0.00	10.11	5.17
time (sec)	N/A	0.168	1.468	0.018	0.532	0.955	0.000	0.253	3.478

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	347	1140	472	1244	0	2085	1144
normalized size	1	1.00	1.56	5.11	2.12	5.58	0.00	9.35	5.13
time (sec)	N/A	0.126	0.498	0.014	0.507	0.827	0.000	0.235	3.164

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	226	0	0	0	5692	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	23.14	0.00	-0.00
time (sec)	N/A	0.345	0.197	0.033	0.000	0.782	17.733	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	217	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.496	0.063	0.000	0.968	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	168	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.207	0.047	0.000	0.654	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	170	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.128	0.043	0.000	0.935	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.064	0.041	0.000	1.021	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	145	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.070	0.041	0.000	0.662	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	189	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	0.121	0.036	0.000	0.860	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	167	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.251	0.054	0.000	0.918	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	413	0	0	0	0	0	-1
normalized size	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	0.827	0.094	0.000	0.754	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	247	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.918	0.092	0.000	0.797	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	403	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.530	0.667	0.061	0.000	0.972	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	230	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	0.340	0.060	0.000	0.868	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	253	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	0.375	0.055	0.000	0.922	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	391	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.605	0.887	0.044	0.000	0.979	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	437	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.698	0.668	0.077	0.000	1.031	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	377	275	0	0	0	131	0	-1
normalized size	1	0.94	0.69	0.00	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.765	0.136	0.073	0.000	0.967	38.734	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	113	0	0	0	82	0	-1
normalized size	1	0.91	0.69	0.00	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.133	0.061	0.053	0.000	0.661	15.754	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.104	0.053	0.000	0.615	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.203	0.060	0.000	0.837	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	1012	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	8.10	0.00	-0.01
time (sec)	N/A	0.084	0.083	0.060	0.000	0.785	29.684	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	1012	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	8.10	0.00	-0.01
time (sec)	N/A	0.085	0.066	0.050	0.000	0.647	21.959	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	394	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	3.94	0.00	-0.01
time (sec)	N/A	0.065	0.072	0.050	0.000	0.884	16.782	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	394	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	3.94	0.00	-0.01
time (sec)	N/A	0.059	0.066	0.036	0.000	0.626	11.877	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	65	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.87	0.00	-0.01
time (sec)	N/A	0.033	0.032	0.033	0.000	0.856	9.008	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	65
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.87	0.00	0.93
time (sec)	N/A	0.020	0.072	0.027	0.000	0.601	6.414	0.000	3.363

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	65	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.74	0.00	-0.01
time (sec)	N/A	0.053	0.051	0.023	0.000	0.736	9.673	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	68	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.053	0.047	0.028	0.000	0.627	10.867	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	71	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.053	0.046	0.037	0.000	0.992	13.917	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	205	0	0	0	3046	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	16.20	0.00	-0.01
time (sec)	N/A	0.180	0.203	0.074	0.000	0.844	46.589	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	169	156	0	0	0	1047	0	-1
normalized size	1	0.95	0.88	0.00	0.00	0.00	5.92	0.00	-0.01
time (sec)	N/A	0.163	0.160	0.069	0.000	0.790	38.893	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	152	0	0	0	1386	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	9.30	0.00	-0.01
time (sec)	N/A	0.138	0.138	0.058	0.000	0.990	25.946	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	144	139	0	0	0	430	0	-1
normalized size	1	0.95	0.91	0.00	0.00	0.00	2.83	0.00	-0.01
time (sec)	N/A	0.137	0.146	0.053	0.000	0.868	21.178	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	184	0	0	0	439	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	3.88	0.00	-0.01
time (sec)	N/A	0.097	0.182	0.045	0.000	0.976	12.909	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	125	133	0	0	0	97	0	-1
normalized size	1	0.94	1.00	0.00	0.00	0.00	0.73	0.00	-0.01
time (sec)	N/A	0.077	0.115	0.040	0.000	0.702	11.609	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	101	0	0	0	109	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.92	0.00	-0.01
time (sec)	N/A	0.093	0.101	0.034	0.000	0.827	12.064	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	134	0	0	0	95	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.123	0.080	0.033	0.000	0.978	13.567	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	0	119	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.94	0.00	-0.01
time (sec)	N/A	0.125	0.089	0.046	0.000	0.966	18.621	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	241	249	0	0	0	3082	0	-1
normalized size	1	0.98	1.01	0.00	0.00	0.00	12.48	0.00	-0.00
time (sec)	N/A	0.247	0.216	0.073	0.000	0.941	74.611	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	241	249	0	0	0	3082	0	-1
normalized size	1	0.97	1.00	0.00	0.00	0.00	12.38	0.00	-0.00
time (sec)	N/A	0.235	0.200	0.085	0.000	0.912	57.337	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	201	196	0	0	0	1421	0	-1
normalized size	1	0.97	0.95	0.00	0.00	0.00	6.86	0.00	-0.00
time (sec)	N/A	0.201	0.148	0.070	0.000	0.893	42.196	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	202	196	0	0	0	1421	0	-1
normalized size	1	0.96	0.93	0.00	0.00	0.00	6.77	0.00	-0.00
time (sec)	N/A	0.204	0.142	0.064	0.000	0.933	29.714	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	159	228	0	0	0	471	0	-1
normalized size	1	0.95	1.37	0.00	0.00	0.00	2.82	0.00	-0.01
time (sec)	N/A	0.150	0.231	0.053	0.000	0.986	22.796	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	169	223	0	0	0	468	0	-1
normalized size	1	0.96	1.27	0.00	0.00	0.00	2.66	0.00	-0.01
time (sec)	N/A	0.149	0.213	0.046	0.000	0.952	16.257	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	165	170	0	0	0	144	0	-1
normalized size	1	0.96	0.99	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.133	0.131	0.031	0.000	0.914	17.623	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	154	0	0	0	143	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.90	0.00	-0.01
time (sec)	N/A	0.189	0.117	0.037	0.000	0.895	14.908	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	174	0	0	0	150	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.221	0.117	0.048	0.000	0.823	21.011	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.649	0.053	0.000	0.926	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	260	0	0	0	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.329	0.047	0.000	0.941	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	227	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.239	0.044	0.000	0.745	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	172	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.186	0.037	0.000	0.858	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.046	0.046	0.000	0.920	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	170	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.178	0.061	0.000	0.878	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	214	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.349	0.048	0.000	0.908	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	256	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.279	0.066	0.000	0.855	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.888	0.705	0.064	0.000	0.977	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	343	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.545	0.687	0.056	0.000	0.996	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	277	300	0	0	0	0	0	-1
normalized size	1	0.99	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.334	0.053	0.000	0.994	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	223	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.183	0.048	0.000	0.923	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	191	141	0	0	0	0	0	-1
normalized size	1	0.78	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	0.086	0.049	0.000	0.943	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	303	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.347	0.041	0.000	0.837	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	342	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.457	0.061	0.000	1.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	462	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.938	0.933	0.088	0.000	0.781	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	436	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.592	0.742	0.081	0.000	0.744	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	290	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	0.344	0.075	0.000	1.053	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	229	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.208	0.070	0.000	0.939	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.112	0.063	0.000	0.822	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	434	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.818	0.716	0.054	0.000	0.935	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	478	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.860	0.766	0.097	0.000	1.067	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	254	182	0	0	0	0	0	-1
normalized size	1	0.92	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.468	0.206	0.063	0.000	0.979	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	158	0	0	0	172	0	-1
normalized size	1	0.95	0.77	0.00	0.00	0.00	0.84	0.00	-0.00
time (sec)	N/A	0.197	0.104	0.056	0.000	0.651	170.872	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	106	0	0	0	109	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.063	0.033	0.050	0.000	0.937	88.937	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.019	0.010	0.064	0.000	0.676	22.602	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.090	0.053	0.000	0.924	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.089	0.058	0.000	1.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	0.218	0.069	0.000	1.164	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	304	946	0	678	0	0	-1
normalized size	1	1.00	0.88	2.74	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.506	1.593	0.028	0.000	1.241	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	245	713	0	536	0	0	-1
normalized size	1	1.00	0.98	2.84	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.260	0.825	0.014	0.000	1.087	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	197	516	0	418	0	0	-1
normalized size	1	1.00	0.95	2.49	0.00	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.661	0.010	0.000	1.014	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	155	205	0	337	0	0	-1
normalized size	1	1.00	1.18	1.56	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.752	0.006	0.000	0.984	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	210	439	0	947	0	0	-1
normalized size	1	1.00	1.25	2.61	0.00	5.64	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.182	0.015	0.000	1.309	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	594	0	355	0	0	-1
normalized size	1	1.00	0.85	4.34	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.139	0.015	0.000	1.082	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	162	882	0	442	0	0	-1
normalized size	1	1.00	0.80	4.37	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.160	0.017	0.000	1.944	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	210	1165	0	558	0	0	-1
normalized size	1	1.00	0.73	4.07	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.404	0.239	0.019	0.000	4.389	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	273	1494	0	702	0	0	-1
normalized size	1	1.00	0.70	3.84	0.00	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.594	0.362	0.023	0.000	21.116	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	425	1883	0	1044	0	0	-1
normalized size	1	1.00	0.95	4.19	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.570	2.288	0.026	0.000	1.156	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	497	1560	0	846	0	0	-1
normalized size	1	1.00	1.41	4.43	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.328	2.771	0.016	0.000	0.853	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	276	1279	0	676	0	0	-1
normalized size	1	1.00	0.94	4.34	0.00	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.282	1.240	0.012	0.000	1.162	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	264	566	462	532	0	0	-1
normalized size	1	1.00	1.31	2.82	2.30	2.65	0.00	0.00	-0.00
time (sec)	N/A	0.117	0.675	0.010	0.503	1.259	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	275	1130	0	1327	0	0	-1
normalized size	1	1.00	1.10	4.50	0.00	5.29	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.853	0.015	0.000	11.018	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	263	1310	0	1221	0	0	-1
normalized size	1	1.00	1.10	5.46	0.00	5.09	0.00	0.00	-0.00
time (sec)	N/A	0.273	1.201	0.017	0.000	4.370	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	285	1604	0	1375	0	0	-1
normalized size	1	1.00	1.11	6.27	0.00	5.37	0.00	0.00	-0.00
time (sec)	N/A	0.283	2.446	0.019	0.000	5.629	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	188	1945	0	558	0	0	-1
normalized size	1	1.00	0.89	9.22	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.236	0.277	0.022	0.000	3.682	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	253	2427	0	704	0	0	-1
normalized size	1	1.00	0.86	8.23	0.00	2.39	0.00	0.00	-0.00
time (sec)	N/A	0.386	0.303	0.027	0.000	17.383	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	310	2888	0	872	0	0	-1
normalized size	1	1.00	0.78	7.31	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.514	0.487	0.033	0.000	40.540	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	380	3387	0	1072	0	0	-1
normalized size	1	1.00	0.76	6.80	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.725	0.775	0.045	0.000	112.218	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	681	3178	0	1524	0	0	-1
normalized size	1	1.00	1.19	5.54	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.695	3.608	0.028	0.000	1.235	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	562	2731	0	1272	0	0	-1
normalized size	1	1.00	1.24	6.04	0.00	2.81	0.00	0.00	-0.00
time (sec)	N/A	0.410	5.706	0.018	0.000	0.835	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	506	2411	0	1046	0	0	-1
normalized size	1	1.00	1.33	6.33	0.00	2.75	0.00	0.00	-0.00
time (sec)	N/A	0.386	2.735	0.013	0.000	1.034	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	384	1123	915	844	0	0	-1
normalized size	1	1.00	1.40	4.10	3.34	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.185	1.217	0.010	0.574	0.999	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	390	2180	0	1873	0	0	-1
normalized size	1	1.00	0.99	5.53	0.00	4.75	0.00	0.00	-0.00
time (sec)	N/A	0.450	2.007	0.017	0.000	93.323	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	350	2364	0	1717	0	0	-1
normalized size	1	1.00	0.99	6.72	0.00	4.88	0.00	0.00	-0.00
time (sec)	N/A	0.432	2.073	0.018	0.000	30.735	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	334	2688	0	1569	0	0	-1
normalized size	1	1.00	0.99	7.93	0.00	4.63	0.00	0.00	-0.00
time (sec)	N/A	0.388	2.245	0.021	0.000	14.800	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	357	3144	0	1741	0	0	-1
normalized size	1	1.00	0.96	8.47	0.00	4.69	0.00	0.00	-0.00
time (sec)	N/A	0.470	3.038	0.027	0.000	18.605	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	404	3646	0	1917	0	0	-1
normalized size	1	1.00	1.00	9.02	0.00	4.75	0.00	0.00	-0.00
time (sec)	N/A	0.456	3.482	0.033	0.000	48.640	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	295	3991	0	872	0	0	-1
normalized size	1	1.00	1.02	13.81	0.00	3.02	0.00	0.00	-0.00
time (sec)	N/A	0.328	0.939	0.045	0.000	39.549	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	344	4735	0	1072	0	0	-1
normalized size	1	1.00	0.89	12.27	0.00	2.78	0.00	0.00	-0.00
time (sec)	N/A	0.495	0.994	0.064	0.000	114.748	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	408	5353	0	0	0	0	-1
normalized size	1	1.00	0.82	10.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.639	0.747	0.086	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	512	6030	0	0	0	0	-1
normalized size	1	1.00	0.82	9.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.885	1.456	0.128	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	298	331	391	0	758	0	0	-1
normalized size	1	1.10	1.22	1.44	0.00	2.80	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.514	0.021	0.000	2.289	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	255	241	0	586	0	0	-1
normalized size	1	1.00	1.31	1.24	0.00	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.346	0.364	0.013	0.000	1.111	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	189	131	0	443	0	0	-1
normalized size	1	1.00	1.36	0.94	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.416	0.010	0.000	1.375	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	51	0	59	0	0	50
normalized size	1	1.00	0.81	0.98	0.00	1.13	0.00	0.00	0.96
time (sec)	N/A	0.024	0.016	0.008	0.000	1.242	0.000	0.000	2.645

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	131	136	0	454	0	0	-1
normalized size	1	1.00	0.92	0.95	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.126	0.016	0.000	2.015	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	201	253	0	610	0	0	-1
normalized size	1	1.00	0.88	1.10	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.123	0.016	0.000	3.298	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	283	414	0	792	0	0	-1
normalized size	1	1.00	0.86	1.26	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.508	0.189	0.017	0.000	7.604	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	296	1680	0	2120	0	0	-1
normalized size	1	1.00	0.57	3.26	0.00	4.12	0.00	0.00	-0.00
time (sec)	N/A	0.619	5.632	0.030	0.000	10.286	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	387	1266	0	1782	0	0	-1
normalized size	1	1.00	0.88	2.89	0.00	4.07	0.00	0.00	-0.00
time (sec)	N/A	0.541	1.342	0.013	0.000	4.633	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	1443	977	0	1466	0	0	-1
normalized size	1	1.00	4.86	3.29	0.00	4.94	0.00	0.00	-0.00
time (sec)	N/A	0.293	4.640	0.014	0.000	5.331	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	99	145	0	308	0	0	1071
normalized size	1	1.00	0.79	1.15	0.00	2.44	0.00	0.00	8.50
time (sec)	N/A	0.108	0.065	0.011	0.000	4.271	0.000	0.000	3.603

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	100	149	0	314	0	0	499
normalized size	1	1.00	0.72	1.08	0.00	2.28	0.00	0.00	3.62
time (sec)	N/A	0.094	0.035	0.010	0.000	4.126	0.000	0.000	3.319

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	306	0	0	120
normalized size	1	1.00	0.79	1.14	0.00	2.53	0.00	0.00	0.99
time (sec)	N/A	0.042	0.032	0.010	0.000	3.726	0.000	0.000	2.885

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	262	682	0	1476	0	0	-1
normalized size	1	1.00	0.97	2.52	0.00	5.45	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.415	0.016	0.000	10.615	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	370	912	0	1812	0	0	-1
normalized size	1	1.00	0.94	2.31	0.00	4.60	0.00	0.00	-0.00
time (sec)	N/A	0.590	0.569	0.016	0.000	30.792	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	467	1319	0	2162	0	0	-1
normalized size	1	1.00	0.89	2.53	0.00	4.14	0.00	0.00	-0.00
time (sec)	N/A	0.800	0.935	0.020	0.000	64.968	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	593	1705	0	0	0	0	-1
normalized size	1	1.00	0.89	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.170	1.395	0.021	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	235	366	0	820	0	0	3099
normalized size	1	1.00	0.91	1.41	0.00	3.17	0.00	0.00	11.97
time (sec)	N/A	0.236	0.123	0.014	0.000	88.308	0.000	0.000	4.327

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	433	663	0	0	0	0	11469
normalized size	1	1.00	1.27	1.94	0.00	0.00	0.00	0.00	33.63
time (sec)	N/A	0.289	0.195	0.020	0.000	0.000	0.000	0.000	7.725

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	221	257	0	0	0	0	-1
normalized size	1	1.00	1.30	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.920	0.140	0.000	1.275	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	22	0	67	22
normalized size	1	1.00	1.00	0.78	0.96	0.96	0.00	2.91	0.96
time (sec)	N/A	0.018	0.032	0.003	0.955	1.019	0.000	0.241	2.620

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	347	361	0	0	0	0	-1
normalized size	1	1.00	1.18	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	0.508	0.034	0.000	0.957	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	169	252	0	0	0	0	-1
normalized size	1	1.00	1.17	1.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.481	0.029	0.000	1.132	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	197	43	0	60	0	0	-1
normalized size	1	1.00	2.98	0.65	0.00	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.407	0.026	0.000	1.080	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	349	363	0	0	0	0	-1
normalized size	1	1.00	1.22	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.399	0.030	0.000	0.735	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	185	259	0	0	0	0	-1
normalized size	1	1.00	1.27	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.412	0.032	0.000	1.145	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	235	262	0	0	0	0	-1
normalized size	1	1.00	1.17	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.076	0.883	0.030	0.000	1.296	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	27	27	0	173	25
normalized size	1	1.00	1.00	0.78	1.17	1.17	0.00	7.52	1.09
time (sec)	N/A	0.021	0.042	0.005	0.974	1.086	0.000	0.610	0.117

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	244	366	0	0	0	0	-1
normalized size	1	1.00	0.75	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.107	0.491	0.031	0.000	1.208	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	176	257	0	0	0	0	-1
normalized size	1	1.00	1.02	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.636	0.016	0.000	1.062	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	201	57	0	65	0	0	-1
normalized size	1	1.00	2.14	0.61	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.296	0.011	0.000	0.694	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	244	368	0	0	0	0	-1
normalized size	1	1.00	0.76	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.493	0.020	0.000	1.032	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	192	264	0	0	0	0	-1
normalized size	1	1.00	1.10	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.350	0.019	0.000	1.161	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	169	248	0	0	0	0	-1
normalized size	1	1.00	1.19	1.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.605	0.043	0.000	0.995	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	17	0	18	9
normalized size	1	1.00	1.00	0.78	0.96	0.74	0.00	0.78	0.39
time (sec)	N/A	0.019	0.024	0.005	0.970	0.975	0.000	0.179	0.150

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	375	275	0	0	0	0	-1
normalized size	1	1.00	1.48	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.996	0.035	0.000	0.947	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	0	0	0	-1
normalized size	1	1.00	1.35	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.142	0.019	0.000	1.007	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	2463	33	0	43	0	0	-1
normalized size	1	1.00	58.64	0.79	0.00	1.02	0.00	0.00	-0.02
time (sec)	N/A	0.026	12.801	0.028	0.000	0.703	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	400	363	0	0	0	0	-1
normalized size	1	1.00	1.42	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.832	0.024	0.000	1.199	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	144	171	259	0	0	0	0	-1
normalized size	1	0.99	1.17	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.650	0.022	0.000	0.801	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	245	0	0	0	0	-1
normalized size	1	1.00	1.18	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.570	0.066	0.000	0.907	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	24	0	0	17
normalized size	1	1.00	1.00	0.78	0.74	1.04	0.00	0.00	0.74
time (sec)	N/A	0.021	0.030	0.006	0.978	0.945	0.000	0.000	2.692

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	402	356	0	0	0	0	-1
normalized size	1	1.00	1.43	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	0.707	0.051	0.000	1.024	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	216	247	0	0	0	0	-1
normalized size	1	1.00	1.58	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.360	0.036	0.000	0.777	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	2511	43	0	78	0	0	-1
normalized size	1	1.00	38.05	0.65	0.00	1.18	0.00	0.00	-0.02
time (sec)	N/A	0.034	6.077	0.041	0.000	1.323	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	409	363	0	0	0	0	-1
normalized size	1	1.00	1.29	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.771	0.040	0.000	1.143	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	170	259	0	0	0	0	-1
normalized size	1	1.00	1.00	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.395	0.039	0.000	0.824	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	467	0	0	0	0	-1
normalized size	1	1.00	1.06	2.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.498	0.072	0.000	0.945	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	24	29	0	0	82
normalized size	1	1.00	1.00	0.78	1.04	1.26	0.00	0.00	3.57
time (sec)	N/A	0.021	0.040	0.005	0.971	1.011	0.000	0.000	2.875

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	409	688	0	0	0	0	-1
normalized size	1	1.00	1.29	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	0.759	0.056	0.000	1.010	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	469	0	0	0	0	-1
normalized size	1	1.00	1.06	2.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.474	0.053	0.000	1.150	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	2539	69	0	101	0	0	-1
normalized size	1	1.00	26.45	0.72	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.037	6.089	0.063	0.000	1.035	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	414	695	0	0	0	0	-1
normalized size	1	1.00	1.19	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.859	0.061	0.000	1.192	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	183	481	0	0	0	0	-1
normalized size	1	1.00	0.90	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.072	0.648	0.063	0.000	0.988	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	75	134	88	71	84
normalized size	1	1.00	0.80	0.70	0.77	1.38	0.91	0.73	0.87
time (sec)	N/A	0.071	0.052	0.013	0.960	1.013	0.243	0.183	0.129

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	568	2218	0	5507	0	1171	13879
normalized size	1	1.00	1.16	4.53	0.00	11.24	0.00	2.39	28.32
time (sec)	N/A	14.847	0.746	0.110	0.000	2.524	0.000	0.827	4.857

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	397	466	1764	0	4245	0	1045	11143
normalized size	1	1.22	1.43	5.41	0.00	13.02	0.00	3.21	34.18
time (sec)	N/A	7.466	0.531	0.058	0.000	1.883	0.000	0.564	4.366

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	375	1329	0	2966	0	868	8171
normalized size	1	1.00	1.19	4.21	0.00	9.39	0.00	2.75	25.86
time (sec)	N/A	3.158	0.442	0.045	0.000	1.023	0.000	0.447	3.911

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	301	926	0	1721	0	753	5664
normalized size	1	1.00	1.05	3.23	0.00	6.00	0.00	2.62	19.74
time (sec)	N/A	3.200	0.406	0.038	0.000	1.280	0.000	0.412	3.820

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	175	545	0	715	155	223	709
normalized size	1	1.00	0.88	2.75	0.00	3.61	0.78	1.13	3.58
time (sec)	N/A	0.271	0.397	0.027	0.000	1.177	50.096	0.268	2.992

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	267	581	0	2446	0	712	10894
normalized size	1	1.00	0.97	2.11	0.00	8.89	0.00	2.59	39.61
time (sec)	N/A	1.129	0.938	0.058	0.000	2.148	0.000	0.392	7.410

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	356	364	999	0	4860	0	0	19887
normalized size	1	0.97	0.99	2.71	0.00	13.21	0.00	0.00	54.04
time (sec)	N/A	3.664	1.591	0.050	0.000	25.227	0.000	0.000	6.814

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	516	1486	0	0	0	1041	33838
normalized size	1	1.00	0.97	2.80	0.00	0.00	0.00	1.96	63.73
time (sec)	N/A	3.569	2.335	0.051	0.000	0.000	0.000	0.585	8.089

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	808	3685	0	14340	0	1577	31485
normalized size	1	1.00	1.24	5.67	0.00	22.06	0.00	2.43	48.44
time (sec)	N/A	2.679	1.116	0.081	0.000	33.462	0.000	0.686	7.969

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	680	2988	0	11459	0	1362	25497
normalized size	1	1.00	1.17	5.14	0.00	19.72	0.00	2.34	43.88
time (sec)	N/A	15.247	0.904	0.063	0.000	12.124	0.000	0.584	7.139

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	538	2358	0	8530	0	1160	19465
normalized size	1	1.00	1.22	5.35	0.00	19.34	0.00	2.63	44.14
time (sec)	N/A	2.150	0.675	0.059	0.000	5.093	0.000	0.526	5.724

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	779	1714	0	5572	0	978	13841
normalized size	1	1.00	1.72	3.78	0.00	12.30	0.00	2.16	30.55
time (sec)	N/A	4.529	1.489	0.052	0.000	2.314	0.000	0.460	4.723

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	317	1138	0	2770	0	783	8334
normalized size	1	1.00	0.98	3.53	0.00	8.60	0.00	2.43	25.88
time (sec)	N/A	1.244	0.705	0.038	0.000	1.107	0.000	0.422	4.435

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	331	944	0	5167	0	822	20897
normalized size	1	1.00	0.97	2.78	0.00	15.20	0.00	2.42	61.46
time (sec)	N/A	1.578	1.160	0.043	0.000	15.304	0.000	0.405	8.163

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	402	393	1215	0	8653	0	425	29890
normalized size	1	1.00	0.98	3.01	0.00	21.47	0.00	1.05	74.17
time (sec)	N/A	3.075	1.540	0.048	0.000	73.172	0.000	0.547	7.365

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	587	1880	0	0	0	1121	44649
normalized size	1	1.00	0.97	3.10	0.00	0.00	0.00	1.85	73.56
time (sec)	N/A	3.931	2.857	0.056	0.000	0.000	0.000	0.612	8.194

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.319	0.208	0.000	1.225	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	261	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	0.820	0.125	0.000	1.069	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	202	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.382	0.373	0.115	0.000	0.996	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	183	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.288	0.111	0.000	0.901	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	163	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.252	0.111	0.000	0.669	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	207	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.408	0.101	0.000	0.794	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	246	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	0.433	0.129	0.000	1.088	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	134	186	175	176	150	249	351
normalized size	1	1.00	0.95	1.32	1.24	1.25	1.06	1.77	2.49
time (sec)	N/A	0.183	0.077	0.006	0.472	0.823	0.604	0.164	0.111

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	103	145	138	139	109	211	197
normalized size	1	1.00	0.94	1.33	1.27	1.28	1.00	1.94	1.81
time (sec)	N/A	0.136	0.054	0.005	0.446	0.679	0.479	0.209	2.586

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	73	110	97	98	70	172	127
normalized size	1	1.00	1.12	1.69	1.49	1.51	1.08	2.65	1.95
time (sec)	N/A	0.060	0.034	0.003	0.445	0.979	0.381	0.155	0.071

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	82	63	64	46	134	65
normalized size	1	1.00	0.86	1.64	1.26	1.28	0.92	2.68	1.30
time (sec)	N/A	0.034	0.020	0.003	0.439	0.902	0.291	0.170	2.605

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	107	82	76	112	81	81
normalized size	1	1.00	0.89	1.73	1.32	1.23	1.81	1.31	1.31
time (sec)	N/A	0.080	0.022	0.008	0.445	0.905	0.645	0.152	0.154

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	149	113	165	182	0	109
normalized size	1	1.00	0.95	1.73	1.31	1.92	2.12	0.00	1.27
time (sec)	N/A	0.086	0.047	0.010	0.456	0.889	1.002	0.000	2.696

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	206	149	271	185	0	100
normalized size	1	1.00	1.00	2.37	1.71	3.11	2.13	0.00	1.15
time (sec)	N/A	0.097	0.075	0.011	0.450	0.774	1.027	0.000	0.129

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	122	259	206	400	248	0	152
normalized size	1	1.00	1.08	2.29	1.82	3.54	2.19	0.00	1.35
time (sec)	N/A	0.116	0.059	0.010	0.478	0.824	1.416	0.000	2.650

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	312	236	511	282	0	180
normalized size	1	1.00	1.02	2.24	1.70	3.68	2.03	0.00	1.29
time (sec)	N/A	0.133	0.090	0.012	0.494	0.887	1.925	0.000	0.145

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	226	286	258	328	250	367	1029
normalized size	1	1.00	1.04	1.31	1.18	1.50	1.15	1.68	4.72
time (sec)	N/A	0.284	0.121	0.012	0.453	0.820	1.203	0.183	2.641

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	185	245	218	288	199	327	565
normalized size	1	1.00	1.05	1.38	1.23	1.63	1.12	1.85	3.19
time (sec)	N/A	0.232	0.120	0.010	0.448	0.898	1.014	0.182	2.615

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	154	204	182	251	162	291	316
normalized size	1	1.00	1.05	1.40	1.25	1.72	1.11	1.99	2.16
time (sec)	N/A	0.181	0.093	0.010	0.460	0.829	0.854	0.188	0.094

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	167	141	206	119	250	185
normalized size	1	1.00	1.07	1.56	1.32	1.93	1.11	2.34	1.73
time (sec)	N/A	0.136	0.085	0.010	0.450	0.748	0.743	0.174	0.070

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	138	104	157	94	212	116
normalized size	1	1.00	1.06	1.77	1.33	2.01	1.21	2.72	1.49
time (sec)	N/A	0.096	0.058	0.007	0.442	0.718	0.586	0.184	2.535

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	96	69	95	61	160	72
normalized size	1	1.00	0.92	1.92	1.38	1.90	1.22	3.20	1.44
time (sec)	N/A	0.057	0.049	0.007	0.439	0.742	0.402	0.170	2.561

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	156	114	168	182	159	111
normalized size	1	1.00	1.06	1.81	1.33	1.95	2.12	1.85	1.29
time (sec)	N/A	0.082	0.049	0.009	0.453	0.554	1.040	0.185	2.644

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	85	180	111	155	156	101	115
normalized size	1	1.00	1.15	2.43	1.50	2.09	2.11	1.36	1.55
time (sec)	N/A	0.029	0.038	0.012	0.439	0.882	0.714	0.161	2.606

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	139	253	212	417	279	0	198
normalized size	1	1.00	1.15	2.09	1.75	3.45	2.31	0.00	1.64
time (sec)	N/A	0.136	0.114	0.013	0.466	0.798	1.264	0.000	0.146

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	171	270	197	337	241	0	148
normalized size	1	1.00	1.17	1.85	1.35	2.31	1.65	0.00	1.01
time (sec)	N/A	0.155	0.095	0.015	0.479	0.842	1.360	0.000	2.632

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	195	341	298	648	376	0	274
normalized size	1	1.00	1.10	1.92	1.67	3.64	2.11	0.00	1.54
time (sec)	N/A	0.200	0.144	0.016	0.504	0.847	1.928	0.000	2.700

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	229	394	342	693	427	0	314
normalized size	1	1.00	1.09	1.88	1.63	3.30	2.03	0.00	1.50
time (sec)	N/A	0.242	0.182	0.018	0.534	0.917	2.152	0.000	2.718

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	193	263	227	336	219	364	375
normalized size	1	1.00	1.08	1.47	1.27	1.88	1.22	2.03	2.09
time (sec)	N/A	0.241	0.100	0.010	0.468	0.833	1.551	0.194	0.141

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	157	228	188	294	178	324	240
normalized size	1	1.00	1.05	1.53	1.26	1.97	1.19	2.17	1.61
time (sec)	N/A	0.197	0.083	0.009	0.460	0.855	1.372	0.196	0.105

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	198	149	241	151	273	161
normalized size	1	1.00	1.00	1.68	1.26	2.04	1.28	2.31	1.36
time (sec)	N/A	0.143	0.090	0.009	0.457	0.883	1.212	0.377	2.597

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	93	151	105	159	102	227	107
normalized size	1	1.00	1.15	1.86	1.30	1.96	1.26	2.80	1.32
time (sec)	N/A	0.101	0.040	0.009	0.451	1.029	0.872	0.205	2.598

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	105	81	100	83	195	80
normalized size	1	1.00	0.80	1.72	1.33	1.64	1.36	3.20	1.31
time (sec)	N/A	0.060	0.026	0.007	0.443	1.193	0.540	0.204	0.069

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	218	150	271	185	197	103
normalized size	1	1.00	1.02	2.48	1.70	3.08	2.10	2.24	1.17
time (sec)	N/A	0.099	0.077	0.011	0.463	0.833	1.011	0.174	0.134

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	140	257	211	417	277	191	198
normalized size	1	1.00	1.15	2.11	1.73	3.42	2.27	1.57	1.62
time (sec)	N/A	0.115	0.102	0.030	0.471	1.070	1.325	0.168	2.638

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	110	298	152	252	144	127	114
normalized size	1	1.00	0.87	2.35	1.20	1.98	1.13	1.00	0.90
time (sec)	N/A	0.061	0.045	0.015	0.449	1.264	0.998	0.166	0.103

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	197	348	308	662	321	0	249
normalized size	1	1.00	1.05	1.85	1.64	3.52	1.71	0.00	1.32
time (sec)	N/A	0.210	0.155	0.015	0.497	0.676	1.831	0.000	2.679

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	244	421	359	793	372	0	296
normalized size	1	1.00	1.04	1.79	1.53	3.37	1.58	0.00	1.26
time (sec)	N/A	0.273	0.174	0.017	0.510	0.908	2.146	0.000	2.639

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	193	1308	1579	807	0	537	-1
normalized size	1	1.00	0.72	4.86	5.87	3.00	0.00	2.00	-0.00
time (sec)	N/A	0.975	0.968	0.059	1.054	1.073	0.000	0.661	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	168	1030	1178	624	0	411	-1
normalized size	1	1.00	0.78	4.79	5.48	2.90	0.00	1.91	-0.00
time (sec)	N/A	0.666	0.741	0.014	1.044	1.257	0.000	0.405	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	182	713	891	454	0	309	-1
normalized size	1	1.00	0.99	3.90	4.87	2.48	0.00	1.69	-0.01
time (sec)	N/A	0.403	0.807	0.013	1.032	1.004	0.000	0.380	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	110	131	583	279	0	198	125
normalized size	1	1.00	0.76	0.90	4.02	1.92	0.00	1.37	0.86
time (sec)	N/A	0.224	0.393	0.011	0.467	0.963	0.000	0.340	2.871

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	83	85	373	183	0	139	79
normalized size	1	1.00	0.71	0.73	3.19	1.56	0.00	1.19	0.68
time (sec)	N/A	0.059	0.237	0.009	0.464	0.580	0.000	0.332	2.791

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	58	55	101	106	0	70	49
normalized size	1	1.00	0.56	0.53	0.98	1.03	0.00	0.68	0.48
time (sec)	N/A	0.049	0.060	0.008	0.440	0.646	0.000	0.304	2.701

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	225	3961	0	1767	0	2966	-1
normalized size	1	1.00	0.93	16.37	0.00	7.30	0.00	12.26	-0.00
time (sec)	N/A	0.618	0.394	0.050	0.000	1.138	0.000	0.456	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	341	6760	0	3305	0	4343	-1
normalized size	1	1.00	1.10	21.74	0.00	10.63	0.00	13.96	-0.00
time (sec)	N/A	1.264	0.610	0.029	0.000	2.093	0.000	2.968	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	387	9593	0	5361	0	6017	-1
normalized size	1	1.00	0.97	24.10	0.00	13.47	0.00	15.12	-0.00
time (sec)	N/A	2.568	1.141	0.032	0.000	7.146	0.000	2.091	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	114	0	499	107	116	124
normalized size	1	1.00	0.81	1.02	0.00	4.46	0.96	1.04	1.11
time (sec)	N/A	0.215	0.089	0.019	0.000	0.915	87.879	0.187	0.234

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	207	365	326	324	0	378	222
normalized size	1	1.00	0.86	1.52	1.36	1.35	0.00	1.58	0.92
time (sec)	N/A	0.345	0.245	0.007	0.448	0.866	0.000	0.181	0.119

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	149	215	197	197	673	243	159
normalized size	1	1.00	0.85	1.23	1.13	1.13	3.85	1.39	0.91
time (sec)	N/A	0.237	0.150	0.007	0.448	0.802	108.501	0.172	2.580

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	101	104	100	374	134	100
normalized size	1	1.00	0.83	0.89	0.92	0.88	3.31	1.19	0.88
time (sec)	N/A	0.074	0.087	0.005	0.444	0.909	61.129	0.179	0.073

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	41	53	40	150	53	44
normalized size	1	1.00	0.72	0.67	0.87	0.66	2.46	0.87	0.72
time (sec)	N/A	0.026	0.026	0.004	0.433	0.592	13.098	0.154	2.558

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	132	0	297	100	107	107
normalized size	1	1.00	0.88	1.27	0.00	2.86	0.96	1.03	1.03
time (sec)	N/A	0.124	0.160	0.015	0.000	0.944	48.655	0.200	0.107

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	171	237	0	539	0	148	128
normalized size	1	1.00	1.40	1.94	0.00	4.42	0.00	1.21	1.05
time (sec)	N/A	0.203	0.288	0.017	0.000	0.741	0.000	0.174	2.682

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	207	384	0	896	0	278	224
normalized size	1	1.00	1.16	2.16	0.00	5.03	0.00	1.56	1.26
time (sec)	N/A	0.303	0.816	0.021	0.000	1.073	0.000	0.209	2.909

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	207	365	334	333	328	453	292
normalized size	1	1.00	0.87	1.53	1.40	1.40	1.38	1.90	1.23
time (sec)	N/A	0.268	0.240	0.008	0.460	0.934	110.873	0.209	0.091

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	149	215	205	206	204	275	199
normalized size	1	1.00	0.86	1.24	1.18	1.19	1.18	1.59	1.15
time (sec)	N/A	0.204	0.154	0.006	0.449	0.927	50.851	0.326	2.658

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	101	112	110	112	143	111
normalized size	1	1.00	0.83	0.91	1.01	0.99	1.01	1.29	1.00
time (sec)	N/A	0.066	0.083	0.004	0.445	1.030	25.285	0.213	0.075

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	41	54	49	58	56	44
normalized size	1	1.00	0.73	0.69	0.92	0.83	0.98	0.95	0.75
time (sec)	N/A	0.026	0.028	0.004	0.443	0.608	10.144	0.181	0.054

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	90	165	0	492	104	101	141
normalized size	1	1.00	0.80	1.47	0.00	4.39	0.93	0.90	1.26
time (sec)	N/A	0.174	0.068	0.014	0.000	0.676	41.279	0.200	0.136

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	118	269	0	906	0	225	187
normalized size	1	1.00	0.82	1.87	0.00	6.29	0.00	1.56	1.30
time (sec)	N/A	0.271	0.082	0.021	0.000	0.671	0.000	0.191	3.287

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	140	546	0	1539	0	361	310
normalized size	1	1.00	0.65	2.55	0.00	7.19	0.00	1.69	1.45
time (sec)	N/A	0.505	0.096	0.025	0.000	1.034	0.000	0.221	3.367

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	155	306	0	336	0	155	569
normalized size	1	1.00	1.05	2.08	0.00	2.29	0.00	1.05	3.87
time (sec)	N/A	0.141	0.558	0.038	0.000	1.048	0.000	0.270	20.128

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	66	13	9	12	129	12	16
normalized size	1	1.00	4.12	0.81	0.56	0.75	8.06	0.75	1.00
time (sec)	N/A	0.009	0.094	0.005	0.430	0.660	43.482	0.169	2.798

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	410	2497	0	0	0	0	-1
normalized size	1	1.00	1.00	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.511	2.436	0.111	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	339	1569	0	0	0	0	-1
normalized size	1	1.00	0.99	4.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.086	1.273	0.028	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1383	0	1921	0	0	-1
normalized size	1	1.00	0.95	5.76	0.00	8.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.347	0.038	0.000	24.207	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	265	5383	0	5816	0	0	-1
normalized size	1	1.00	0.75	15.34	0.00	16.57	0.00	0.00	-0.00
time (sec)	N/A	2.153	0.696	0.062	0.000	129.495	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	613	353	14861	0	0	0	0	-1
normalized size	1	1.00	0.58	24.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.160	2.864	0.087	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	339	2336	0	0	0	0	-1
normalized size	1	1.00	1.01	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.463	1.091	0.037	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1387	0	1913	0	0	-1
normalized size	1	1.00	0.95	5.78	0.00	7.97	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.334	0.033	0.000	20.731	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	225	1415	0	4325	0	0	-1
normalized size	1	1.00	0.98	6.15	0.00	18.80	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.189	0.042	0.000	38.214	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	287	10977	0	0	0	0	-1
normalized size	1	1.00	0.81	31.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	0.695	0.093	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	336	8264	0	0	0	0	-1
normalized size	1	1.00	0.54	13.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.531	0.732	0.072	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	265	5383	0	0	0	0	-1
normalized size	1	1.00	0.75	15.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.806	0.641	0.056	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	287	10977	0	0	0	0	-1
normalized size	1	1.00	0.81	31.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.763	0.778	0.090	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	543	521	30656	0	0	0	0	-1
normalized size	1	0.99	0.95	55.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.324	2.065	0.227	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	305	0	744	0	0	1610
normalized size	1	1.00	0.97	4.69	0.00	11.45	0.00	0.00	24.77
time (sec)	N/A	0.049	0.061	0.174	0.000	0.749	0.000	0.000	8.490

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	91	125	0	193	0	266	164
normalized size	1	1.00	1.14	1.56	0.00	2.41	0.00	3.32	2.05
time (sec)	N/A	0.143	0.136	0.018	0.000	1.215	0.000	0.350	2.955

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	148	1178	0	318	0	208	148
normalized size	1	1.00	1.38	11.01	0.00	2.97	0.00	1.94	1.38
time (sec)	N/A	0.241	0.296	0.038	0.000	1.473	0.000	0.588	0.286

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	120	524	0	318	0	131	148
normalized size	1	1.00	1.12	4.90	0.00	2.97	0.00	1.22	1.38
time (sec)	N/A	0.179	0.108	0.021	0.000	1.293	0.000	0.461	0.125

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	851	851	1034	6457	0	0	0	0	-1
normalized size	1	1.00	1.22	7.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.697	10.471	0.161	0.000	1.272	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	809	4351	0	0	0	0	-1
normalized size	1	1.00	1.27	6.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.622	7.261	0.038	0.000	1.004	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	610	2549	0	0	0	0	-1
normalized size	1	1.00	1.41	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	4.578	0.026	0.000	0.821	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	536	1162	0	0	0	0	-1
normalized size	1	1.00	1.48	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	2.912	0.022	0.000	1.198	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	683	683	1216	2496	0	0	0	0	-1
normalized size	1	1.00	1.78	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.122	8.976	0.047	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	1331	6044	0	0	0	0	-1
normalized size	1	1.00	2.05	9.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.661	7.027	0.059	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1205	1205	2703	19181	0	0	0	0	-1
normalized size	1	1.00	2.24	15.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.199	11.229	0.100	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	864	5079	0	0	0	0	-1
normalized size	1	1.00	1.30	7.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.571	7.881	0.058	0.000	1.969	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	503	712	3278	0	0	0	0	-1
normalized size	1	0.99	1.40	6.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.977	5.260	0.035	0.000	2.173	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	545	1828	0	0	0	0	-1
normalized size	1	1.00	1.50	5.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	4.834	0.025	0.000	0.989	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	456	688	0	0	0	0	-1
normalized size	1	1.00	1.42	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	1.986	0.020	0.000	1.897	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1096	1216	0	0	0	0	-1
normalized size	1	1.00	2.32	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.639	4.311	0.033	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	1336	6034	0	0	0	0	-1
normalized size	1	1.00	1.93	8.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.722	6.409	0.039	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1241	1241	2197	19187	0	0	0	0	-1
normalized size	1	1.00	1.77	15.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.237	10.637	0.078	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	527	747	3924	0	0	0	0	-1
normalized size	1	0.99	1.41	7.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.122	6.160	0.056	0.000	0.901	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	596	2470	0	0	0	0	-1
normalized size	1	1.00	1.45	6.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	4.297	0.049	0.000	1.307	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	464	1286	0	0	0	0	-1
normalized size	1	1.00	1.40	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	3.348	0.039	0.000	1.307	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	294	396	0	0	0	0	-1
normalized size	1	1.00	2.16	2.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.470	0.036	0.000	0.775	0.000	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	300	439	0	0	0	0	-1
normalized size	1	1.00	0.94	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	1.182	0.048	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	698	698	1330	5743	0	0	0	0	-1
normalized size	1	1.00	1.91	8.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.933	6.009	0.058	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1246	1246	2450	20359	0	0	0	0	-1
normalized size	1	1.00	1.97	16.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.402	10.967	0.101	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	808	1440	3164	0	0	0	0	-1
normalized size	1	1.35	2.40	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.946	9.576	0.054	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	927	959	0	0	0	0	-1
normalized size	1	1.00	1.98	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	1.079	0.048	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	625	2949	0	0	0	0	-1
normalized size	1	1.00	1.37	6.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.613	4.404	0.057	0.000	1.633	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	473	1769	0	0	0	0	-1
normalized size	1	1.00	1.33	4.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	3.543	0.050	0.000	1.036	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	439	520	0	0	0	0	-1
normalized size	1	1.00	1.52	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	1.676	0.043	0.000	1.583	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	186	200	0	0	0	0	-1
normalized size	1	1.00	1.37	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.214	0.041	0.000	1.646	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	311	235	0	0	0	0	-1
normalized size	1	1.00	1.86	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	0.912	0.050	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	1349	5738	0	0	0	0	-1
normalized size	1	1.00	1.81	7.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.109	6.743	0.062	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1257	1257	2491	20366	0	0	0	0	-1
normalized size	1	1.00	1.98	16.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.333	11.762	0.131	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	468	2011	0	0	0	0	-1
normalized size	1	1.00	1.21	5.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	3.226	0.084	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	1917	9409	0	0	0	0	-1
normalized size	1	1.00	2.34	11.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.996	8.973	0.116	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	261	215	0	0	0	0	-1
normalized size	1	1.00	2.37	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.945	0.116	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	344	433	0	0	0	0	-1
normalized size	1	1.00	0.76	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	1.404	0.203	0.000	0.969	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	107	58	0	0	0	0	-1
normalized size	1	1.00	2.06	1.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.236	0.030	0.000	1.387	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	136	188	218	193	0	0	218
normalized size	1	1.00	0.51	0.70	0.81	0.72	0.00	0.00	0.81
time (sec)	N/A	0.423	0.127	0.009	0.656	0.915	0.000	0.000	3.658

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	89	116	133	123	0	0	142
normalized size	1	1.00	0.44	0.58	0.66	0.62	0.00	0.00	0.71
time (sec)	N/A	0.233	0.076	0.008	0.613	0.937	0.000	0.000	3.404

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	53	67	65	71	0	0	88
normalized size	1	1.00	0.42	0.54	0.52	0.57	0.00	0.00	0.70
time (sec)	N/A	0.092	0.047	0.007	0.551	1.051	0.000	0.000	3.226

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	18	49	0	0	54
normalized size	1	1.00	0.76	1.09	0.39	1.07	0.00	0.00	1.17
time (sec)	N/A	0.021	0.017	0.005	0.503	0.774	0.000	0.000	3.196

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	87	0	252	0	0	-1
normalized size	1	1.00	1.16	1.09	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.042	0.030	0.000	1.385	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	136	168	0	703	0	0	-1
normalized size	1	1.00	0.97	1.20	0.00	5.02	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.106	0.023	0.000	1.268	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	77	285	0	1283	0	0	-1
normalized size	1	1.00	0.36	1.34	0.00	6.02	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.045	0.023	0.000	0.782	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	77	450	0	2027	0	0	-1
normalized size	1	1.00	0.28	1.61	0.00	7.24	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.044	0.024	0.000	0.897	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	134	187	165	216	0	0	252
normalized size	1	1.00	0.52	0.73	0.64	0.84	0.00	0.00	0.98
time (sec)	N/A	0.331	0.099	0.009	0.697	1.012	0.000	0.000	3.614

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	88	116	98	147	0	0	178
normalized size	1	1.00	0.49	0.64	0.54	0.81	0.00	0.00	0.98
time (sec)	N/A	0.185	0.068	0.007	0.632	1.173	0.000	0.000	3.433

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	51	66	48	96	0	0	118
normalized size	1	1.00	0.34	0.44	0.32	0.64	0.00	0.00	0.79
time (sec)	N/A	0.144	0.040	0.007	0.570	1.033	0.000	0.000	3.368

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	18	74	0	0	82
normalized size	1	1.00	0.76	1.09	0.39	1.61	0.00	0.00	1.78
time (sec)	N/A	0.022	0.012	0.003	0.512	1.198	0.000	0.000	3.265

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	71	128	0	553	0	0	-1
normalized size	1	1.00	0.53	0.96	0.00	4.16	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.028	0.026	0.000	0.985	0.000	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	73	225	0	1067	0	0	-1
normalized size	1	1.00	0.36	1.11	0.00	5.28	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.033	0.030	0.000	0.806	0.000	0.000	0.000

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	77	379	0	1863	0	0	-1
normalized size	1	1.00	0.28	1.38	0.00	6.80	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.038	0.035	0.000	1.114	0.000	0.000	0.000

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	131	187	219	251	0	0	278
normalized size	1	1.00	0.55	0.78	0.92	1.05	0.00	0.00	1.16
time (sec)	N/A	0.280	0.110	0.009	0.744	0.755	0.000	0.000	3.773

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	87	116	138	180	0	0	206
normalized size	1	1.00	0.41	0.55	0.65	0.85	0.00	0.00	0.98
time (sec)	N/A	0.220	0.070	0.009	0.673	1.140	0.000	0.000	3.609

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	52	66	73	129	0	0	149
normalized size	1	1.00	0.34	0.43	0.47	0.84	0.00	0.00	0.97
time (sec)	N/A	0.134	0.048	0.007	0.602	1.197	0.000	0.000	3.504

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	28	107	0	0	110
normalized size	1	1.00	0.77	1.04	0.58	2.23	0.00	0.00	2.29
time (sec)	N/A	0.022	0.029	0.003	0.519	1.350	0.000	0.000	3.316

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	73	219	0	1015	0	0	-1
normalized size	1	1.00	0.39	1.16	0.00	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.036	0.030	0.000	1.244	0.000	0.000	0.000

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	75	424	0	1907	0	0	-1
normalized size	1	1.00	0.28	1.58	0.00	7.12	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.045	0.035	0.000	1.473	0.000	0.000	0.000

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	79	670	0	2935	0	0	-1
normalized size	1	1.00	0.23	1.96	0.00	8.58	0.00	0.00	-0.00
time (sec)	N/A	0.538	0.051	0.038	0.000	1.338	0.000	0.000	0.000

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	195	283	320	375	0	0	347
normalized size	1	1.00	0.58	0.84	0.95	1.12	0.00	0.00	1.03
time (sec)	N/A	0.607	0.177	0.009	0.708	1.088	0.000	0.000	3.599

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	136	188	218	264	0	0	242
normalized size	1	1.00	0.51	0.70	0.81	0.98	0.00	0.00	0.90
time (sec)	N/A	0.390	0.117	0.010	0.646	1.002	0.000	0.000	3.371

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	116	133	173	0	0	157
normalized size	1	1.00	0.45	0.58	0.66	0.86	0.00	0.00	0.78
time (sec)	N/A	0.228	0.079	0.007	0.592	1.138	0.000	0.000	3.255

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	54	67	65	102	0	0	93
normalized size	1	1.00	0.43	0.54	0.52	0.82	0.00	0.00	0.74
time (sec)	N/A	0.095	0.049	0.006	0.541	1.280	0.000	0.000	3.131

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	18	57	0	0	49
normalized size	1	1.00	0.77	1.04	0.38	1.19	0.00	0.00	1.02
time (sec)	N/A	0.021	0.024	0.003	0.493	0.779	0.000	0.000	3.047

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	153	0	318	0	0	-1
normalized size	1	1.00	0.81	1.23	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.131	0.022	0.000	1.334	0.000	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	161	0	562	0	0	-1
normalized size	1	1.00	0.83	1.22	0.00	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.192	0.025	0.000	0.940	0.000	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	79	285	0	1056	0	0	-1
normalized size	1	1.00	0.38	1.38	0.00	5.10	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.040	0.030	0.000	0.977	0.000	0.000	0.000

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	79	453	0	1732	0	0	-1
normalized size	1	1.00	0.29	1.64	0.00	6.25	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.043	0.036	0.000	1.077	0.000	0.000	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	79	696	0	2610	0	0	-1
normalized size	1	1.00	0.23	2.01	0.00	7.52	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.043	0.035	0.000	1.019	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	195	283	413	472	0	0	445
normalized size	1	1.00	0.58	0.84	1.23	1.40	0.00	0.00	1.32
time (sec)	N/A	0.607	0.231	0.010	0.741	0.964	0.000	0.000	3.800

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	137	188	294	340	0	0	310
normalized size	1	1.00	0.51	0.70	1.09	1.26	0.00	0.00	1.15
time (sec)	N/A	0.407	0.166	0.009	0.687	0.944	0.000	0.000	3.646

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	116	192	230	0	0	206
normalized size	1	1.00	0.45	0.58	0.96	1.15	0.00	0.00	1.03
time (sec)	N/A	0.233	0.119	0.009	0.625	0.867	0.000	0.000	3.426

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	54	67	107	137	0	0	109
normalized size	1	1.00	0.43	0.54	0.86	1.10	0.00	0.00	0.87
time (sec)	N/A	0.101	0.074	0.005	0.571	0.821	0.000	0.000	3.250

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	43	74	0	0	62
normalized size	1	1.00	0.77	1.04	0.90	1.54	0.00	0.00	1.29
time (sec)	N/A	0.023	0.032	0.005	0.505	0.963	0.000	0.000	3.077

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	132	263	0	408	0	0	-1
normalized size	1	1.00	0.74	1.47	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.264	0.023	0.000	0.966	0.000	0.000	0.000

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	75	306	0	444	0	0	-1
normalized size	1	1.00	0.42	1.72	0.00	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.051	0.030	0.000	1.015	0.000	0.000	0.000

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	135	276	0	840	0	0	-1
normalized size	1	1.00	0.69	1.42	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.323	0.029	0.000	1.154	0.000	0.000	0.000

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	79	453	0	1434	0	0	-1
normalized size	1	1.00	0.30	1.71	0.00	5.41	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.062	0.032	0.000	0.949	0.000	0.000	0.000

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	79	665	0	2238	0	0	-1
normalized size	1	1.00	0.24	1.99	0.00	6.68	0.00	0.00	-0.00
time (sec)	N/A	0.452	0.061	0.041	0.000	1.096	0.000	0.000	0.000

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	79	955	0	3204	0	0	-1
normalized size	1	1.00	0.20	2.36	0.00	7.91	0.00	0.00	-0.00
time (sec)	N/A	0.563	0.064	0.039	0.000	1.031	0.000	0.000	0.000

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	205	283	498	567	0	0	523
normalized size	1	1.00	0.61	0.84	1.48	1.69	0.00	0.00	1.56
time (sec)	N/A	0.618	0.209	0.010	0.761	0.756	0.000	0.000	4.086

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	147	188	362	416	0	0	379
normalized size	1	1.00	0.55	0.70	1.35	1.55	0.00	0.00	1.41
time (sec)	N/A	0.397	0.159	0.011	0.697	0.862	0.000	0.000	3.805

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	100	116	243	284	0	0	259
normalized size	1	1.00	0.50	0.58	1.22	1.42	0.00	0.00	1.30
time (sec)	N/A	0.235	0.112	0.009	0.637	1.264	0.000	0.000	3.562

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	64	67	141	173	0	0	134
normalized size	1	1.00	0.51	0.54	1.13	1.38	0.00	0.00	1.07
time (sec)	N/A	0.100	0.079	0.005	0.567	1.111	0.000	0.000	3.373

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	60	91	0	0	79
normalized size	1	1.00	0.77	1.04	1.25	1.90	0.00	0.00	1.65
time (sec)	N/A	0.021	0.040	0.005	0.508	0.867	0.000	0.000	3.160

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	145	431	0	587	0	0	-1
normalized size	1	1.00	0.61	1.83	0.00	2.49	0.00	0.00	-0.00
time (sec)	N/A	0.467	0.363	0.023	0.000	1.017	0.000	0.000	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	75	523	0	672	0	0	-1
normalized size	1	1.00	0.32	2.23	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.068	0.031	0.000	1.267	0.000	0.000	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	79	526	0	683	0	0	-1
normalized size	1	1.00	0.32	2.14	0.00	2.78	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.072	0.033	0.000	0.994	0.000	0.000	0.000

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	171	441	0	1140	0	0	-1
normalized size	1	1.00	0.68	1.74	0.00	4.51	0.00	0.00	-0.00
time (sec)	N/A	0.336	0.371	0.033	0.000	0.684	0.000	0.000	0.000

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	79	665	0	1862	0	0	-1
normalized size	1	1.00	0.24	2.06	0.00	5.76	0.00	0.00	-0.00
time (sec)	N/A	0.473	0.075	0.035	0.000	0.927	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	79	924	0	2750	0	0	-1
normalized size	1	1.00	0.20	2.35	0.00	7.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	0.080	0.043	0.000	1.069	0.000	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	79	1261	0	3872	0	0	-1
normalized size	1	1.00	0.17	2.72	0.00	8.36	0.00	0.00	-0.00
time (sec)	N/A	0.716	0.082	0.042	0.000	1.424	0.000	0.000	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	269	511	0	841	0	0	-1
normalized size	1	1.00	0.86	1.63	0.00	2.69	0.00	0.00	-0.00
time (sec)	N/A	0.562	0.606	0.045	0.000	2.698	0.000	0.000	0.000

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	234	328	0	655	0	0	-1
normalized size	1	1.00	0.96	1.34	0.00	2.68	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.473	0.029	0.000	2.473	0.000	0.000	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	213	201	0	521	0	0	-1
normalized size	1	1.00	1.26	1.19	0.00	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.192	0.025	0.000	2.515	0.000	0.000	0.000

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	160	120	0	343	0	0	-1
normalized size	1	1.00	1.52	1.14	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.112	0.030	0.000	2.023	0.000	0.000	0.000

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	63	0	114	0	0	100
normalized size	1	1.00	0.82	1.03	0.00	1.87	0.00	0.00	1.64
time (sec)	N/A	0.065	0.031	0.007	0.000	0.891	0.000	0.000	4.638

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	98	0	288	0	0	147
normalized size	1	1.00	0.53	0.76	0.00	2.23	0.00	0.00	1.14
time (sec)	N/A	0.143	0.055	0.008	0.000	0.714	0.000	0.000	4.898

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	572	0	0	242
normalized size	1	1.00	0.53	0.85	0.00	2.89	0.00	0.00	1.22
time (sec)	N/A	0.219	0.086	0.010	0.000	1.009	0.000	0.000	5.170

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	953	0	0	357
normalized size	1	1.00	0.57	0.97	0.00	3.57	0.00	0.00	1.34
time (sec)	N/A	0.313	0.120	0.012	0.000	1.040	0.000	0.000	5.511

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	100	648	0	971	0	0	-1
normalized size	1	1.00	0.33	2.15	0.00	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.118	0.038	0.000	2.360	0.000	0.000	0.000

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	100	396	0	725	0	0	-1
normalized size	1	1.00	0.44	1.74	0.00	3.19	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.084	0.030	0.000	2.330	0.000	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	176	210	0	569	0	0	-1
normalized size	1	1.00	1.09	1.30	0.00	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.369	0.027	0.000	2.371	0.000	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	63	0	125	0	0	147
normalized size	1	1.00	0.82	1.03	0.00	2.05	0.00	0.00	2.41
time (sec)	N/A	0.069	0.027	0.006	0.000	0.977	0.000	0.000	4.678

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	64	97	0	325	0	0	151
normalized size	1	1.00	0.52	0.78	0.00	2.62	0.00	0.00	1.22
time (sec)	N/A	0.154	0.049	0.007	0.000	0.894	0.000	0.000	4.976

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	105	168	0	649	0	0	268
normalized size	1	1.00	0.55	0.88	0.00	3.38	0.00	0.00	1.40
time (sec)	N/A	0.247	0.068	0.010	0.000	1.020	0.000	0.000	5.333

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	150	259	0	1062	0	0	414
normalized size	1	1.00	0.57	0.99	0.00	4.05	0.00	0.00	1.58
time (sec)	N/A	0.329	0.094	0.013	0.000	0.993	0.000	0.000	5.701

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	102	652	0	1055	0	0	-1
normalized size	1	1.00	0.35	2.26	0.00	3.65	0.00	0.00	-0.00
time (sec)	N/A	0.432	0.140	0.044	0.000	2.385	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	102	343	0	755	0	0	-1
normalized size	1	1.00	0.47	1.57	0.00	3.45	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.105	0.030	0.000	2.216	0.000	0.000	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	193	0	0	169
normalized size	1	1.00	0.83	1.00	0.00	3.06	0.00	0.00	2.68
time (sec)	N/A	0.066	0.031	0.008	0.000	0.899	0.000	0.000	4.322

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	0	318	0	0	246
normalized size	1	1.00	0.53	0.77	0.00	2.48	0.00	0.00	1.92
time (sec)	N/A	0.141	0.055	0.008	0.000	1.083	0.000	0.000	5.059

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	103	169	0	667	0	0	255
normalized size	1	1.00	0.53	0.87	0.00	3.44	0.00	0.00	1.31
time (sec)	N/A	0.220	0.066	0.010	0.000	0.994	0.000	0.000	5.282

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	152	258	0	1065	0	0	416
normalized size	1	1.00	0.58	0.99	0.00	4.10	0.00	0.00	1.60
time (sec)	N/A	0.313	0.097	0.012	0.000	1.175	0.000	0.000	5.859

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	300	870	0	1065	0	0	-1
normalized size	1	1.00	0.78	2.26	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	0.718	1.156	0.031	0.000	5.543	0.000	0.000	0.000

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	255	602	0	847	0	0	-1
normalized size	1	1.00	0.81	1.92	0.00	2.71	0.00	0.00	-0.00
time (sec)	N/A	0.518	0.844	0.028	0.000	3.265	0.000	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	215	385	0	657	0	0	-1
normalized size	1	1.00	0.89	1.60	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.594	0.022	0.000	2.463	0.000	0.000	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	173	198	0	516	0	0	-1
normalized size	1	1.00	1.04	1.19	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.814	0.023	0.000	2.289	0.000	0.000	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	169	197	0	521	0	0	-1
normalized size	1	1.00	1.07	1.25	0.00	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.789	0.031	0.000	2.159	0.000	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	169	0	0	136
normalized size	1	1.00	0.83	1.00	0.00	2.68	0.00	0.00	2.16
time (sec)	N/A	0.065	0.033	0.007	0.000	0.945	0.000	0.000	3.923

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	402	0	0	187
normalized size	1	1.00	0.53	0.77	0.00	3.12	0.00	0.00	1.45
time (sec)	N/A	0.139	0.054	0.008	0.000	0.924	0.000	0.000	4.084

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	748	0	0	289
normalized size	1	1.00	0.53	0.85	0.00	3.78	0.00	0.00	1.46
time (sec)	N/A	0.220	0.092	0.009	0.000	0.992	0.000	0.000	4.290

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	1179	0	0	409
normalized size	1	1.00	0.57	0.97	0.00	4.42	0.00	0.00	1.53
time (sec)	N/A	0.308	0.139	0.013	0.000	0.914	0.000	0.000	4.501

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	302	870	0	1059	0	0	-1
normalized size	1	1.00	0.79	2.28	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	0.711	1.170	0.025	0.000	5.256	0.000	0.000	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	254	602	0	847	0	0	-1
normalized size	1	1.00	0.82	1.94	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.829	0.024	0.000	2.997	0.000	0.000	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	193	325	0	651	0	0	-1
normalized size	1	1.00	0.81	1.37	0.00	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.771	0.027	0.000	2.352	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	102	383	0	663	0	0	-1
normalized size	1	1.00	0.46	1.73	0.00	2.99	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.158	0.031	0.000	2.071	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	188	331	0	685	0	0	-1
normalized size	1	1.00	0.88	1.55	0.00	3.20	0.00	0.00	-0.00
time (sec)	N/A	0.278	1.063	0.034	0.000	2.037	0.000	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	232	0	0	232
normalized size	1	1.00	0.83	1.00	0.00	3.68	0.00	0.00	3.68
time (sec)	N/A	0.070	0.052	0.008	0.000	0.870	0.000	0.000	4.067

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	526	0	0	247
normalized size	1	1.00	0.53	0.77	0.00	4.08	0.00	0.00	1.91
time (sec)	N/A	0.148	0.085	0.008	0.000	0.745	0.000	0.000	4.305

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	918	0	0	377
normalized size	1	1.00	0.53	0.85	0.00	4.64	0.00	0.00	1.90
time (sec)	N/A	0.228	0.129	0.010	0.000	1.034	0.000	0.000	4.478

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	1420	0	0	519
normalized size	1	1.00	0.57	0.97	0.00	5.32	0.00	0.00	1.94
time (sec)	N/A	0.324	0.176	0.013	0.000	1.072	0.000	0.000	4.833

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	285	1191	0	1331	0	0	-1
normalized size	1	1.00	0.64	2.66	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.889	6.013	0.031	0.000	11.970	0.000	0.000	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	299	870	0	1065	0	0	-1
normalized size	1	1.00	0.80	2.31	0.00	2.83	0.00	0.00	-0.00
time (sec)	N/A	0.678	1.135	0.025	0.000	5.349	0.000	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	229	508	0	837	0	0	-1
normalized size	1	1.00	0.75	1.67	0.00	2.75	0.00	0.00	-0.00
time (sec)	N/A	0.487	0.998	0.031	0.000	3.123	0.000	0.000	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	112	635	0	915	0	0	-1
normalized size	1	1.00	0.38	2.16	0.00	3.11	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.104	0.033	0.000	2.499	0.000	0.000	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	112	638	0	973	0	0	-1
normalized size	1	1.00	0.39	2.25	0.00	3.43	0.00	0.00	-0.00
time (sec)	N/A	0.404	0.123	0.032	0.000	2.308	0.000	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	224	511	0	933	0	0	-1
normalized size	1	1.00	0.82	1.86	0.00	3.41	0.00	0.00	-0.00
time (sec)	N/A	0.366	1.369	0.037	0.000	2.294	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	299	0	0	325
normalized size	1	1.00	0.83	1.00	0.00	4.75	0.00	0.00	5.16
time (sec)	N/A	0.069	0.077	0.008	0.000	0.948	0.000	0.000	4.343

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	79	99	0	639	0	0	315
normalized size	1	1.00	0.61	0.77	0.00	4.95	0.00	0.00	2.44
time (sec)	N/A	0.149	0.082	0.009	0.000	0.913	0.000	0.000	4.543

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	115	169	0	1101	0	0	465
normalized size	1	1.00	0.58	0.85	0.00	5.56	0.00	0.00	2.35
time (sec)	N/A	0.230	0.108	0.011	0.000	0.944	0.000	0.000	4.822

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	162	260	0	1648	0	0	627
normalized size	1	1.00	0.61	0.97	0.00	6.17	0.00	0.00	2.35
time (sec)	N/A	0.324	0.141	0.014	0.000	1.054	0.000	0.000	5.119

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	-1
normalized size	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.083	0.128	0.000	0.937	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	98	0	0	0	0	0	-1
normalized size	1	1.06	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.043	0.108	0.000	0.759	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	118	98	0	0	0	0	0	-1
normalized size	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.043	0.104	0.000	0.859	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	120	100	0	0	0	0	0	-1
normalized size	1	1.15	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.066	0.109	0.000	0.851	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	-1
normalized size	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.101	0.105	0.000	0.669	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	110	0	0	0	0	0	-1
normalized size	1	1.17	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.071	0.105	0.000	0.926	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	95	0	0	0	0	0	-1
normalized size	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.052	0.205	0.000	0.886	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	134	527	331	705	0	2024	615
normalized size	1	1.00	0.39	1.54	0.97	2.06	0.00	5.90	1.79
time (sec)	N/A	0.449	0.172	0.011	0.606	0.806	0.000	0.331	3.753

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	131	235	193	350	0	981	327
normalized size	1	1.00	0.53	0.96	0.78	1.42	0.00	3.99	1.33
time (sec)	N/A	0.205	0.108	0.010	0.556	0.919	0.000	0.280	3.518

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	67	89	94	145	0	369	139
normalized size	1	1.00	0.45	0.59	0.63	0.97	0.00	2.46	0.93
time (sec)	N/A	0.082	0.062	0.005	0.517	0.905	0.000	0.254	3.362

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	33	57	0	87	57
normalized size	1	1.00	0.78	1.06	0.61	1.06	0.00	1.61	1.06
time (sec)	N/A	0.015	0.022	0.003	0.473	0.946	0.000	0.225	3.248

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.027	0.118	0.000	0.787	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	84	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.033	0.123	0.000	0.986	0.000	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.036	0.129	0.000	0.934	0.000	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.053	0.107	0.000	0.939	0.000	0.000	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.039	0.113	0.000	1.036	0.000	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.033	0.107	0.000	0.923	0.000	0.000	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.034	0.107	0.000	0.841	0.000	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.037	0.106	0.000	0.875	0.000	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	64	49	66	0	114	63
normalized size	1	1.00	0.82	0.98	0.75	1.02	0.00	1.75	0.97
time (sec)	N/A	0.044	0.029	0.003	0.495	0.738	0.000	0.247	3.543

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	32	35	0	0	-1
normalized size	1	1.00	0.82	0.00	0.41	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.035	0.242	0.504	0.923	0.000	0.000	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	222	145	0	0	0	0	0	-1
normalized size	1	1.04	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.125	0.106	0.000	0.976	0.000	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	246	641	693	597	0	0	653
normalized size	1	1.00	0.49	1.28	1.38	1.19	0.00	0.00	1.30
time (sec)	N/A	0.894	0.419	0.011	0.739	0.673	0.000	0.000	4.088

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	264	425	484	408	0	0	438
normalized size	1	1.00	0.64	1.03	1.17	0.99	0.00	0.00	1.06
time (sec)	N/A	0.627	0.273	0.010	0.685	0.964	0.000	0.000	3.864

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	169	255	309	256	0	0	279
normalized size	1	1.00	0.53	0.79	0.96	0.80	0.00	0.00	0.87
time (sec)	N/A	0.420	0.180	0.010	0.627	0.904	0.000	0.000	3.711

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	96	131	168	141	0	0	152
normalized size	1	1.00	0.46	0.63	0.80	0.67	0.00	0.00	0.73
time (sec)	N/A	0.198	0.096	0.005	0.570	0.927	0.000	0.000	3.484

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	54	69	65	73	0	0	85
normalized size	1	1.00	0.50	0.63	0.60	0.67	0.00	0.00	0.78
time (sec)	N/A	0.059	0.042	0.003	0.504	0.884	0.000	0.000	3.364

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	140	163	0	511	0	0	-1
normalized size	1	1.00	1.01	1.17	0.00	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.108	0.024	0.000	0.670	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	155	347	0	896	0	0	-1
normalized size	1	1.00	0.91	2.04	0.00	5.27	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.147	0.028	0.000	1.063	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	189	673	0	1704	0	0	-1
normalized size	1	1.00	0.72	2.58	0.00	6.53	0.00	0.00	-0.00
time (sec)	N/A	0.355	0.423	0.042	0.000	0.721	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	132	1142	0	2736	0	0	-1
normalized size	1	1.00	0.38	3.25	0.00	7.79	0.00	0.00	-0.00
time (sec)	N/A	0.558	0.103	0.047	0.000	1.075	0.000	0.000	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	229	602	365	251	0	412	1768
normalized size	1	1.00	0.71	1.86	1.13	0.77	0.00	1.27	5.46
time (sec)	N/A	0.934	0.264	0.059	0.978	0.834	0.000	0.618	31.330

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	114	291	171	134	0	196	897
normalized size	1	1.00	0.69	1.75	1.03	0.81	0.00	1.18	5.40
time (sec)	N/A	0.319	0.116	0.027	0.973	0.684	0.000	0.422	13.854

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	57	67	282	76	232
normalized size	1	1.00	0.71	1.86	0.90	1.06	4.48	1.21	3.68
time (sec)	N/A	0.063	0.033	0.018	0.970	0.996	49.711	0.268	7.760

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	260	1759	0	4313	0	684	33018
normalized size	1	1.00	0.92	6.24	0.00	15.29	0.00	2.43	117.09
time (sec)	N/A	0.522	0.558	0.141	0.000	1.195	0.000	1.505	82.367

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	508	41837	0	0	0	0	-1
normalized size	1	1.00	0.89	73.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.235	1.282	0.814	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	239	755	371	376	0	732	-1
normalized size	1	1.00	0.87	2.74	1.34	1.36	0.00	2.65	-0.00
time (sec)	N/A	0.598	0.241	0.040	0.980	0.700	0.000	0.611	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	127	380	176	204	0	387	-1
normalized size	1	1.00	0.94	2.81	1.30	1.51	0.00	2.87	-0.01
time (sec)	N/A	0.195	0.117	0.030	0.969	0.888	0.000	0.394	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	39	151	61	101	0	182	-1
normalized size	1	1.00	0.98	3.78	1.52	2.52	0.00	4.55	-0.02
time (sec)	N/A	0.051	0.048	0.026	0.967	0.968	0.000	0.303	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	335	11141	0	0	0	0	-1
normalized size	1	1.00	0.76	25.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.443	2.394	0.125	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	939	938	890	108974	0	0	0	0	-1
normalized size	1	1.00	0.95	116.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.844	9.219	4.138	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0	-1
normalized size	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.203	0.164	0.000	1.313	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.093	0.168	0.000	1.181	0.000	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.080	0.165	0.000	1.041	0.000	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	249	2017	811	2032	0	3760	1943
normalized size	1	1.00	0.91	7.33	2.95	7.39	0.00	13.67	7.07
time (sec)	N/A	0.263	0.313	0.019	0.597	0.853	0.000	0.585	3.901

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	187	1048	512	1122	11946	2114	1133
normalized size	1	1.00	0.90	5.04	2.46	5.39	57.43	10.16	5.45
time (sec)	N/A	0.188	0.199	0.013	0.543	1.319	11.821	0.230	3.522

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	141	449	289	549	4952	1018	572
normalized size	1	1.00	0.97	3.08	1.98	3.76	33.92	6.97	3.92
time (sec)	N/A	0.112	0.280	0.007	0.505	1.179	5.269	0.387	3.288

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	147	135	218	1489	373	211
normalized size	1	1.00	0.87	1.75	1.61	2.60	17.73	4.44	2.51
time (sec)	N/A	0.063	0.100	0.005	0.475	1.076	2.212	0.188	3.072

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	93	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.150	0.058	0.000	1.250	0.000	0.000	0.000

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.091	0.065	0.000	1.008	0.000	0.000	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	106	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.095	0.090	0.000	1.103	0.000	0.000	0.000

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	106	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.104	0.167	0.000	1.049	0.000	0.000	0.000

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	227	179	0	0	0	0	0	-1
normalized size	1	0.98	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.185	0.072	0.000	1.269	0.000	0.000	0.000

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	142	87	99	420	88	84
normalized size	1	1.00	1.02	1.71	1.05	1.19	5.06	1.06	1.01
time (sec)	N/A	0.100	0.049	0.008	0.445	1.215	9.516	0.176	3.420

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	177	444	255	313	0	281	266
normalized size	1	1.00	0.96	2.41	1.39	1.70	0.00	1.53	1.45
time (sec)	N/A	0.312	0.148	0.012	0.450	1.684	0.000	0.169	3.505

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	476	1232	721	736	0	907	794
normalized size	1	1.00	0.90	2.32	1.36	1.39	0.00	1.71	1.50
time (sec)	N/A	0.988	0.423	0.020	0.488	9.833	0.000	0.171	4.195

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	246	606	0	0	0	392	12173
normalized size	1	1.00	1.00	2.46	0.00	0.00	0.00	1.59	49.48
time (sec)	N/A	0.468	0.324	0.012	0.000	0.000	0.000	0.185	19.247

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	710	9103	0	0	0	3315	130035
normalized size	1	1.00	1.10	14.14	0.00	0.00	0.00	5.15	201.92
time (sec)	N/A	2.052	2.632	0.050	0.000	0.000	0.000	0.276	32.634

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	249	540	429	429	1544	565	283
normalized size	1	1.00	0.87	1.88	1.49	1.49	5.38	1.97	0.99
time (sec)	N/A	0.495	0.412	0.009	0.459	1.260	164.697	0.200	0.150

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	184	315	261	260	1001	363	204
normalized size	1	1.00	0.87	1.49	1.23	1.23	4.72	1.71	0.96
time (sec)	N/A	0.339	0.341	0.009	0.449	0.743	105.539	0.233	3.169

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	144	129	125	549	199	125
normalized size	1	1.00	0.96	1.05	0.94	0.91	4.01	1.45	0.91
time (sec)	N/A	0.109	0.193	0.006	0.457	0.904	55.828	0.192	0.077

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	77	54	223	77	58
normalized size	1	1.00	0.74	0.73	1.05	0.74	3.05	1.05	0.79
time (sec)	N/A	0.042	0.048	0.003	0.442	0.796	10.865	0.171	3.119

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	118	189	0	341	112	128	117
normalized size	1	1.00	1.02	1.63	0.00	2.94	0.97	1.10	1.01
time (sec)	N/A	0.168	0.192	0.013	0.000	0.870	37.629	0.167	0.142

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	371	0	637	0	175	146
normalized size	1	1.00	1.07	2.65	0.00	4.55	0.00	1.25	1.04
time (sec)	N/A	0.290	0.560	0.019	0.000	1.240	0.000	0.182	0.233

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	297	538	0	1096	0	373	270
normalized size	1	1.00	1.44	2.61	0.00	5.32	0.00	1.81	1.31
time (sec)	N/A	0.386	0.646	0.021	0.000	0.976	0.000	0.200	0.281

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	249	540	437	438	452	669	394
normalized size	1	1.00	0.87	1.89	1.53	1.54	1.59	2.35	1.38
time (sec)	N/A	0.405	0.729	0.009	0.462	0.911	158.555	0.235	0.116

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	184	315	269	269	272	404	270
normalized size	1	1.00	0.88	1.50	1.28	1.28	1.30	1.92	1.29
time (sec)	N/A	0.289	0.356	0.008	0.445	0.991	79.487	0.230	3.127

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	128	144	137	135	141	204	147
normalized size	1	1.00	0.95	1.07	1.01	1.00	1.04	1.51	1.09
time (sec)	N/A	0.098	0.179	0.005	0.443	0.951	34.525	0.181	3.134

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	53	66	63	70	74	58
normalized size	1	1.00	0.76	0.75	0.93	0.89	0.99	1.04	0.82
time (sec)	N/A	0.042	0.055	0.004	0.441	0.677	13.121	0.153	0.060

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	124	237	0	540	116	112	162
normalized size	1	1.00	1.02	1.94	0.00	4.43	0.95	0.92	1.33
time (sec)	N/A	0.221	0.291	0.014	0.000	0.941	52.234	0.255	3.209

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	176	418	0	1088	0	282	218
normalized size	1	1.00	1.07	2.53	0.00	6.59	0.00	1.71	1.32
time (sec)	N/A	0.367	0.407	0.022	0.000	0.931	0.000	0.229	0.300

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	290	847	0	1883	0	462	363
normalized size	1	1.00	1.17	3.42	0.00	7.59	0.00	1.86	1.46
time (sec)	N/A	0.629	1.102	0.026	0.000	1.065	0.000	0.257	3.409

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	191	113	231	0	214	0	16	916
normalized size	1	2.10	1.24	2.54	0.00	2.35	0.00	0.18	10.07
time (sec)	N/A	0.141	0.212	0.086	0.000	0.963	0.000	0.198	5.018

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	173	425	0	380	0	179	833
normalized size	1	1.00	1.05	2.59	0.00	2.32	0.00	1.09	5.08
time (sec)	N/A	0.179	0.775	0.029	0.000	1.107	0.000	0.261	22.379

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	313	1207	0	852	0	448	-1
normalized size	1	1.00	0.94	3.62	0.00	2.56	0.00	1.35	-0.00
time (sec)	N/A	0.353	1.538	0.032	0.000	1.319	0.000	0.415	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	225	763	0	576	0	291	1832
normalized size	1	1.00	0.91	3.10	0.00	2.34	0.00	1.18	7.45
time (sec)	N/A	0.256	1.008	0.023	0.000	1.154	0.000	0.349	74.336

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	173	425	0	380	0	179	833
normalized size	1	1.00	1.05	2.59	0.00	2.32	0.00	1.09	5.08
time (sec)	N/A	0.143	0.622	0.000	0.000	1.098	0.000	0.252	0.002

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	222	697	0	588	0	201	-1
normalized size	1	1.00	1.72	5.40	0.00	4.56	0.00	1.56	-0.01
time (sec)	N/A	0.135	0.592	0.029	0.000	3.101	0.000	0.387	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	173	773	0	792	0	504	-1
normalized size	1	1.00	1.08	4.83	0.00	4.95	0.00	3.15	-0.01
time (sec)	N/A	0.180	0.230	0.028	0.000	7.058	0.000	0.685	0.000

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	178	238	0	353	0	1080	260
normalized size	1	1.00	0.90	1.20	0.00	1.78	0.00	5.45	1.31
time (sec)	N/A	0.212	0.228	0.010	0.000	25.370	0.000	0.853	4.304

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	332	468	0	641	0	1868	452
normalized size	1	1.00	1.18	1.67	0.00	2.28	0.00	6.65	1.61
time (sec)	N/A	0.291	0.353	0.010	0.000	99.863	0.000	1.272	4.654

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	196	834	0	580	0	237	-1
normalized size	1	1.00	0.79	3.35	0.00	2.33	0.00	0.95	-0.00
time (sec)	N/A	0.280	1.091	0.042	0.000	3.234	0.000	0.440	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	204	571	0	546	0	717	-1
normalized size	1	1.00	0.85	2.38	0.00	2.28	0.00	2.99	-0.00
time (sec)	N/A	0.225	0.848	0.044	0.000	2.020	0.000	0.519	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	163	392	0	414	0	441	1797
normalized size	1	1.00	0.93	2.23	0.00	2.35	0.00	2.51	10.21
time (sec)	N/A	0.171	0.437	0.025	0.000	1.300	0.000	0.382	73.154

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	135	247	0	308	0	145	893
normalized size	1	1.00	1.11	2.02	0.00	2.52	0.00	1.19	7.32
time (sec)	N/A	0.124	0.408	0.029	0.000	1.422	0.000	0.244	20.635

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	134	438	0	463	0	193	-1
normalized size	1	1.00	1.24	4.06	0.00	4.29	0.00	1.79	-0.01
time (sec)	N/A	0.120	0.337	0.032	0.000	2.033	0.000	0.364	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	128	601	0	665	0	0	-1
normalized size	1	1.00	1.10	5.18	0.00	5.73	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.296	0.032	0.000	1.592	0.000	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	110	150	0	293	0	0	268
normalized size	1	1.00	0.83	1.13	0.00	2.20	0.00	0.00	2.02
time (sec)	N/A	0.130	0.084	0.012	0.000	3.271	0.000	0.000	4.301

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	173	248	0	487	0	0	389
normalized size	1	1.00	0.92	1.31	0.00	2.58	0.00	0.00	2.06
time (sec)	N/A	0.192	0.121	0.013	0.000	6.619	0.000	0.000	4.506

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	401	11688	0	0	0	0	-1
normalized size	1	1.00	0.96	28.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.141	1.789	0.179	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	266	5482	0	4471	0	0	-1
normalized size	1	1.00	0.93	19.24	0.00	15.69	0.00	0.00	-0.00
time (sec)	N/A	0.526	0.874	0.047	0.000	100.729	0.000	0.000	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	267	5507	0	0	0	0	-1
normalized size	1	1.00	0.93	19.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.752	0.065	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	334	47351	0	0	0	0	-1
normalized size	1	1.00	0.78	110.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.356	2.062	0.343	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	559	3941	0	0	0	0	-1
normalized size	1	1.00	1.05	7.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.704	1.013	0.038	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	372	2602	0	0	0	0	-1
normalized size	1	1.00	1.14	8.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	0.415	0.016	0.000	0.000	0.000	0.000	0.000

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	216	1559	0	0	0	0	-1
normalized size	1	1.00	0.99	7.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.347	0.010	0.000	0.000	0.000	0.000	0.000

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	145	715	0	992	0	0	-1
normalized size	1	1.00	0.95	4.70	0.00	6.53	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.155	0.006	0.000	5.704	0.000	0.000	0.000

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	218	1529	0	0	0	0	-1
normalized size	1	1.00	0.96	6.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.341	0.031	0.000	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	222	3162	0	0	0	0	-1
normalized size	1	1.00	0.45	6.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	0.517	0.020	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	609	6714	0	0	0	1844	-1
normalized size	1	1.00	0.90	9.98	0.00	0.00	0.00	2.74	-0.00
time (sec)	N/A	0.862	1.373	0.021	0.000	0.000	0.000	4.636	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	933	933	858	11995	0	0	0	0	-1
normalized size	1	1.00	0.92	12.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.225	4.159	0.024	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1098	1098	743	10058	0	0	0	0	-1
normalized size	1	1.00	0.68	9.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.864	2.385	0.027	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	536	6860	0	0	0	0	-1
normalized size	1	1.00	0.81	10.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.554	1.249	0.017	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	420	4188	0	0	0	0	-1
normalized size	1	1.00	0.95	9.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.853	1.128	0.009	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	236	1946	0	0	0	0	-1
normalized size	1	1.00	0.94	7.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.412	0.007	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	323	4226	0	0	0	0	-1
normalized size	1	1.00	0.66	8.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	1.068	0.018	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	357	7959	0	0	0	0	-1
normalized size	1	1.00	0.45	10.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.376	1.399	0.021	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1066	1066	1036	15927	0	0	0	0	-1
normalized size	1	1.00	0.97	14.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.706	3.273	0.024	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	886	886	647	9052	0	0	0	0	-1
normalized size	1	1.00	0.73	10.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.800	2.539	0.022	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	553	1597	0	0	0	0	-1
normalized size	1	1.00	1.28	3.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.370	0.888	0.029	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	358	1007	0	0	0	0	-1
normalized size	1	1.00	1.33	3.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	0.380	0.016	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	170	613	0	0	0	0	-1
normalized size	1	1.00	0.97	3.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.483	0.015	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	126	349	0	1071	0	0	-1
normalized size	1	1.00	0.96	2.66	0.00	8.18	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.151	0.008	0.000	66.096	0.000	0.000	0.000

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	157	0	343	0	72	-1
normalized size	1	1.00	0.99	1.99	0.00	4.34	0.00	0.91	-0.01
time (sec)	N/A	0.038	0.014	0.006	0.000	1.075	0.000	0.229	0.000

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	169	327	0	1952	0	0	-1
normalized size	1	1.00	0.93	1.80	0.00	10.73	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.249	0.017	0.000	176.227	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	256	788	0	0	0	0	-1
normalized size	1	1.00	0.75	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	0.955	0.020	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	549	1817	0	0	0	2256	-1
normalized size	1	1.00	0.94	3.10	0.00	0.00	0.00	3.84	-0.00
time (sec)	N/A	0.810	2.429	0.024	0.000	0.000	0.000	3.260	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	587	4453	0	0	0	0	-1
normalized size	1	1.00	1.18	8.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.199	2.457	0.025	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	373	3127	0	0	0	0	-1
normalized size	1	1.00	1.04	8.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.528	1.044	0.016	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	265	2123	0	2023	0	757	-1
normalized size	1	1.00	1.10	8.85	0.00	8.43	0.00	3.15	-0.00
time (sec)	N/A	0.305	0.633	0.016	0.000	10.422	0.000	0.351	0.000

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	183	1261	0	1663	0	568	-1
normalized size	1	1.00	0.98	6.74	0.00	8.89	0.00	3.04	-0.01
time (sec)	N/A	0.135	0.161	0.009	0.000	9.557	0.000	0.323	0.000

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	162	603	0	1349	0	447	-1
normalized size	1	1.00	1.05	3.89	0.00	8.70	0.00	2.88	-0.01
time (sec)	N/A	0.101	0.256	0.008	0.000	1.398	0.000	0.318	0.000

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	317	1343	0	0	0	0	-1
normalized size	1	1.00	0.90	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	1.224	0.021	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	623	2807	0	0	0	0	-1
normalized size	1	1.00	0.97	4.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	5.084	0.026	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1013	5459	0	0	0	14731	-1
normalized size	1	1.00	0.95	5.13	0.00	0.00	0.00	13.84	-0.00
time (sec)	N/A	1.898	5.738	0.035	0.000	0.000	0.000	21.913	0.000

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1551	1551	26600	32647	0	0	0	0	-1
normalized size	1	1.00	17.15	21.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.093	17.750	0.260	0.000	0.848	0.000	0.000	0.000

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1015	1015	15781	20224	0	0	0	0	-1
normalized size	1	1.00	15.55	19.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.775	15.511	0.090	0.000	0.790	0.000	0.000	0.000

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	8432	10711	0	0	0	0	-1
normalized size	1	1.00	12.93	16.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.112	13.949	0.056	0.000	0.501	0.000	0.000	0.000

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	697	4356	0	0	0	0	-1
normalized size	1	1.00	1.36	8.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	10.686	0.037	0.000	0.838	0.000	0.000	0.000

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	764	969	35245	6812	0	0	0	0	-1
normalized size	1	1.27	46.13	8.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.099	15.170	0.062	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	934	16573	16688	0	0	0	0	-1
normalized size	1	1.26	22.31	22.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.316	13.844	0.086	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1034	1705	33765	55368	0	0	0	0	-1
normalized size	1	1.65	32.65	53.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.091	16.525	0.209	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1098	1098	17771	22215	0	0	0	0	-1
normalized size	1	1.00	16.18	20.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.764	15.883	0.108	0.000	1.428	0.000	0.000	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	755	755	10030	12923	0	0	0	0	-1
normalized size	1	1.00	13.28	17.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.931	14.140	0.075	0.000	1.420	0.000	0.000	0.000

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	911	6207	0	0	0	0	-1
normalized size	1	1.00	1.76	11.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	10.513	0.046	0.000	1.109	0.000	0.000	0.000

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	936	1854	0	0	0	0	-1
normalized size	1	1.00	2.11	4.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	8.310	0.030	0.000	0.936	0.000	0.000	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	16471	3126	0	0	0	0	-1
normalized size	1	1.00	23.53	4.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.953	13.918	0.039	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	957	6911	13872	0	0	0	0	-1
normalized size	1	1.30	9.39	18.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.230	13.389	0.071	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	1747	36616	57841	0	0	0	0	-1
normalized size	1	1.67	34.91	55.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.184	16.678	0.221	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	10649	14978	0	0	0	0	-1
normalized size	1	1.00	13.76	19.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.106	14.674	0.092	0.000	1.254	0.000	0.000	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	1002	8248	0	0	0	0	-1
normalized size	1	1.00	1.77	14.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	11.438	0.067	0.000	0.955	0.000	0.000	0.000

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	638	3805	0	0	0	0	-1
normalized size	1	1.00	1.41	8.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	7.143	0.049	0.000	0.746	0.000	0.000	0.000

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	365	747	0	0	0	0	-1
normalized size	1	1.00	1.94	3.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.777	0.035	0.000	0.617	0.000	0.000	0.000

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	379	834	0	0	0	0	-1
normalized size	1	1.00	0.81	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.602	1.623	0.044	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	994	994	18563	13017	0	0	0	0	-1
normalized size	1	1.00	18.68	13.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.701	13.734	0.080	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1786	1786	36634	59522	0	0	0	0	-1
normalized size	1	1.00	20.51	33.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.396	17.665	0.249	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	1385	1879	0	0	0	0	-1
normalized size	1	1.00	2.05	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.655	4.550	0.048	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1138	1138	37137	7464	0	0	0	0	-1
normalized size	1	1.00	32.63	6.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.138	16.254	0.066	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	12746	8755	0	0	0	0	-1
normalized size	1	1.00	20.20	13.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.090	13.692	0.078	0.000	1.072	0.000	0.000	0.000

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	1080	4295	0	0	0	0	-1
normalized size	1	1.00	2.25	8.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	12.789	0.058	0.000	1.044	0.000	0.000	0.000

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	814	1014	0	0	0	0	-1
normalized size	1	1.00	2.07	2.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	5.878	0.044	0.000	1.563	0.000	0.000	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	0	0	0	-1
normalized size	1	1.00	1.63	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.635	0.039	0.000	1.141	0.000	0.000	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	499	330	0	0	0	0	-1
normalized size	1	1.00	1.78	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.247	1.750	0.048	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1037	1037	10881	14048	0	0	0	0	-1
normalized size	1	1.00	10.49	13.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.527	13.948	0.098	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1114	1762	40396	64947	0	0	0	0	-1
normalized size	1	1.58	36.26	58.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.012	17.882	0.344	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	950	4757	0	0	0	0	-1
normalized size	1	1.00	1.72	8.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.613	6.117	0.095	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	1125	14759	27597	0	0	0	0	-1
normalized size	1	1.00	13.12	24.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.310	15.523	0.214	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	1118	645	0	0	0	0	-1
normalized size	1	1.00	2.35	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	9.471	0.198	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	375	605	0	0	0	0	-1
normalized size	1	1.00	0.64	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.167	3.521	0.102	0.000	2.235	0.000	0.000	0.000

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	198	1347	684	1381	15757	2740	1354
normalized size	1	1.00	0.90	6.12	3.11	6.28	71.62	12.45	6.15
time (sec)	N/A	0.215	0.273	0.016	0.577	1.266	14.705	0.271	3.945

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	180	503	352	613	5930	1162	602
normalized size	1	1.00	1.25	3.49	2.44	4.26	41.18	8.07	4.18
time (sec)	N/A	0.113	0.331	0.008	0.520	0.965	5.863	0.198	3.594

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.140	0.062	0.000	0.749	0.000	0.000	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.146	0.063	0.000	0.947	0.000	0.000	0.000

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	243	157	0	0	0	0	0	-1
normalized size	1	0.99	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.160	0.097	0.000	1.430	0.000	0.000	0.000

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	492	5890	2034	4747	0	10489	4871
normalized size	1	1.00	0.94	11.22	3.87	9.04	0.00	19.98	9.28
time (sec)	N/A	0.615	0.773	0.035	0.787	1.379	0.000	0.547	5.383

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	655	2563	1118	2368	0	4940	2307
normalized size	1	1.00	2.11	8.24	3.59	7.61	0.00	15.88	7.42
time (sec)	N/A	0.392	1.523	0.020	0.669	1.328	0.000	0.355	4.380

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	265	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	0.415	0.125	0.000	1.371	0.000	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	261	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.141	0.391	0.128	0.000	1.103	0.000	0.000	0.000

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	257	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.487	0.345	0.171	0.000	0.918	0.000	0.000	0.000

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	151	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.330	0.186	0.000	1.133	0.000	0.000	0.000

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	117	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.138	0.125	0.000	1.045	0.000	0.000	0.000

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	91	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.099	0.123	0.000	1.014	0.000	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.086	0.119	0.000	0.784	0.000	0.000	0.000

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.116	0.138	0.000	1.011	0.000	0.000	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	110	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.194	0.126	0.000	1.141	0.000	0.000	0.000

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	152	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.159	0.133	0.000	0.989	0.000	0.000	0.000

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	251	0	0	0	0	0	-1
normalized size	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.444	0.158	0.000	0.991	0.000	0.000	0.000

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	252	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.377	0.149	0.000	1.038	0.000	0.000	0.000

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	156	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.384	0.145	0.000	1.052	0.000	0.000	0.000

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	149	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.243	0.139	0.000	0.956	0.000	0.000	0.000

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.203	0.137	0.000	1.130	0.000	0.000	0.000

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	274	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	0.773	0.159	0.000	1.151	0.000	0.000	0.000

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	287	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	0.636	0.167	0.000	1.054	0.000	0.000	0.000

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	230	171	0	0	0	0	0	-1
normalized size	1	0.97	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.285	0.063	0.000	1.119	0.000	0.000	0.000

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	509	506	0	0	0	0	0	0	-1
normalized size	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	1.392	0.139	0.000	1.031	0.000	0.000	0.000

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.745	0.120	0.000	1.377	0.000	0.000	0.000

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.020	0.121	0.000	1.006	0.000	0.000	0.000

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.114	0.129	0.000	1.377	0.000	0.000	0.000

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	500	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.681	1.302	0.131	0.000	1.300	0.000	0.000	0.000

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.776	0.133	0.000	1.327	0.000	0.000	0.000

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.025	0.130	0.000	1.276	0.000	0.000	0.000

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.121	0.131	0.000	1.083	0.000	0.000	0.000

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	263	187	0	0	0	0	0	-1
normalized size	1	0.99	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.216	0.074	0.000	1.129	0.000	0.000	0.000

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	525	523	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	2.245	0.140	0.000	1.536	0.000	0.000	0.000

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	1.321	0.119	0.000	1.079	0.000	0.000	0.000

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.101	0.172	0.000	1.171	0.000	0.000	0.000

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.170	0.133	0.000	1.137	0.000	0.000	0.000

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	188	148	0	0	0	0	-1
normalized size	1	1.00	2.11	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.636	0.143	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [429] had the largest ratio of [.7000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	5	1.00	25	0.200
2	A	8	5	1.00	25	0.200
3	A	7	5	1.00	25	0.200
4	A	12	5	1.00	25	0.200
5	A	5	4	1.00	23	0.174
6	A	5	4	1.00	23	0.174
7	A	8	7	1.00	25	0.280
8	A	8	8	1.00	25	0.320
9	A	8	7	1.00	25	0.280
10	A	8	8	1.00	25	0.320
11	A	8	7	1.00	25	0.280
12	A	6	5	1.00	25	0.200
13	A	7	6	1.00	25	0.240
14	A	8	6	1.00	25	0.240
15	A	9	6	1.00	25	0.240
16	A	8	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	3	3	1.00	25	0.120
19	A	6	4	1.00	25	0.160
20	A	6	4	1.00	25	0.160
21	A	5	4	1.00	25	0.160
22	A	4	3	1.00	25	0.120
23	A	3	3	1.00	25	0.120
24	A	3	3	1.00	25	0.120
25	A	3	3	1.00	23	0.130
26	A	3	3	1.00	22	0.136
27	A	7	5	1.00	25	0.200
28	A	7	5	1.00	25	0.200
29	A	8	6	1.00	25	0.240
30	A	4	4	1.00	25	0.160
31	A	5	4	1.00	25	0.160
32	A	4	4	1.00	24	0.167
33	A	7	5	1.00	27	0.185
34	A	6	5	1.00	27	0.185
35	A	5	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	4	4	1.00	25	0.160
37	A	4	4	1.00	24	0.167
38	A	7	7	1.00	27	0.259
39	A	7	7	1.00	27	0.259
40	A	5	5	1.00	27	0.185
41	A	6	6	1.00	27	0.222
42	A	7	6	1.00	27	0.222
43	A	8	6	1.00	27	0.222
44	A	6	5	1.00	27	0.185
45	A	6	5	1.00	27	0.185
46	A	3	2	1.00	27	0.074
47	A	3	3	1.00	27	0.111
48	A	3	3	1.00	25	0.120
49	A	3	3	1.00	24	0.125
50	A	7	6	1.00	27	0.222
51	A	7	5	1.00	27	0.185
52	A	8	6	1.00	27	0.222
53	A	9	6	1.00	27	0.222
54	A	5	4	1.00	20	0.200
55	A	4	4	1.00	20	0.200
56	A	3	3	1.00	18	0.167
57	A	3	3	1.00	17	0.176
58	A	6	6	1.00	20	0.300
59	A	6	6	1.00	20	0.300
60	A	5	5	1.00	20	0.250
61	A	6	6	1.00	20	0.300
62	A	7	6	1.00	20	0.300
63	A	8	6	1.00	20	0.300
64	A	9	8	1.00	27	0.296
65	A	12	6	1.00	27	0.222
66	A	11	6	1.00	27	0.222
67	A	10	6	1.00	27	0.222
68	A	9	6	1.00	27	0.222
69	A	9	6	1.00	25	0.240
70	A	8	5	1.00	24	0.208
71	A	11	8	1.00	27	0.296

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	11	9	1.00	27	0.333
73	A	11	8	1.00	27	0.296
74	A	11	9	1.00	27	0.333
75	A	11	9	1.00	27	0.333
76	A	11	8	1.00	27	0.296
77	A	11	9	1.00	27	0.333
78	A	11	9	1.00	27	0.333
79	A	11	8	1.00	27	0.296
80	A	9	6	1.00	27	0.222
81	A	10	7	1.00	27	0.259
82	A	11	7	1.00	27	0.259
83	A	7	5	1.00	27	0.185
84	A	6	4	1.00	27	0.148
85	A	5	4	1.00	27	0.148
86	A	3	3	1.00	27	0.111
87	A	3	3	1.00	25	0.120
88	A	4	3	1.00	24	0.125
89	A	7	6	1.00	27	0.222
90	A	7	5	1.00	27	0.185
91	A	8	6	1.00	27	0.222
92	A	7	5	1.00	27	0.185
93	A	6	5	1.00	27	0.185
94	A	6	6	1.00	27	0.222
95	A	4	4	1.00	25	0.160
96	A	3	3	1.00	24	0.125
97	A	7	7	1.00	27	0.259
98	A	5	5	1.00	27	0.185
99	A	6	6	1.00	27	0.222
100	A	7	6	1.00	27	0.222
101	A	8	6	1.00	27	0.222
102	A	7	7	1.00	27	0.259
103	A	9	6	1.00	27	0.222
104	A	8	6	1.00	27	0.222
105	A	8	7	1.00	27	0.259
106	A	6	5	1.00	25	0.200
107	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	9	8	1.00	27	0.296
109	A	9	9	1.00	27	0.333
110	A	9	8	1.00	27	0.296
111	A	9	9	1.00	27	0.333
112	A	9	8	1.00	27	0.296
113	A	7	6	1.00	27	0.222
114	A	8	7	1.00	27	0.259
115	A	9	7	1.00	27	0.259
116	A	10	7	1.00	27	0.259
117	A	3	3	1.00	18	0.167
118	A	7	7	1.00	26	0.269
119	A	6	6	1.00	27	0.222
120	A	5	5	1.00	27	0.185
121	A	5	5	1.00	27	0.185
122	A	3	3	1.00	25	0.120
123	A	1	1	1.00	24	0.042
124	A	5	5	1.00	27	0.185
125	A	5	5	1.00	27	0.185
126	A	6	6	1.00	27	0.222
127	A	6	5	1.00	27	0.185
128	A	6	5	1.00	27	0.185
129	A	5	5	1.00	27	0.185
130	A	3	3	1.00	27	0.111
131	A	2	2	1.00	25	0.080
132	A	2	2	1.00	24	0.083
133	A	6	6	1.00	27	0.222
134	A	6	6	1.00	27	0.222
135	A	7	7	1.00	27	0.259
136	A	7	5	1.00	27	0.185
137	A	7	5	1.00	27	0.185
138	A	6	5	1.00	27	0.185
139	A	5	4	1.00	27	0.148
140	A	4	4	1.00	27	0.148
141	A	3	3	1.00	27	0.111
142	A	3	3	1.00	25	0.120
143	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	7	6	1.00	27	0.222
145	A	7	6	1.00	27	0.222
146	A	8	7	1.00	27	0.259
147	A	9	7	1.00	27	0.259
148	A	5	5	1.00	27	0.185
149	A	4	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	4	4	1.00	25	0.160
152	A	2	2	1.00	23	0.087
153	A	1	1	1.00	22	0.045
154	A	5	5	1.00	26	0.192
155	A	5	5	1.00	26	0.192
156	A	6	6	1.00	26	0.231
157	A	10	7	1.00	27	0.259
158	A	9	7	1.00	27	0.259
159	A	8	7	1.00	27	0.259
160	A	7	7	1.00	27	0.259
161	A	6	5	1.00	25	0.200
162	A	6	6	1.00	24	0.250
163	A	9	9	1.00	27	0.333
164	A	9	9	1.00	27	0.333
165	A	9	9	1.00	27	0.333
166	A	9	9	1.00	27	0.333
167	A	7	7	1.00	27	0.259
168	A	8	8	1.00	27	0.296
169	A	9	8	1.00	27	0.296
170	A	10	8	1.00	27	0.296
171	A	7	6	1.00	27	0.222
172	A	4	3	1.00	27	0.111
173	A	4	4	1.00	27	0.148
174	A	3	3	1.00	25	0.120
175	A	3	2	1.00	24	0.083
176	A	8	7	1.00	27	0.259
177	A	8	6	1.00	27	0.222
178	A	9	7	1.00	27	0.259
179	A	8	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	7	5	1.00	27	0.185
181	A	6	5	1.00	27	0.185
182	A	4	4	1.00	27	0.148
183	A	3	3	1.00	25	0.120
184	A	3	2	1.00	24	0.083
185	A	8	7	1.00	27	0.259
186	A	8	6	1.00	27	0.222
187	A	9	7	1.00	27	0.259
188	A	9	6	1.00	27	0.222
189	A	7	5	1.00	27	0.185
190	A	9	7	1.00	27	0.259
191	A	8	6	1.00	27	0.222
192	A	2	2	1.00	25	0.080
193	A	2	2	1.00	24	0.083
194	A	8	7	1.00	27	0.259
195	A	8	6	1.00	27	0.222
196	A	9	7	1.00	27	0.259
197	A	10	7	1.00	27	0.259
198	A	11	6	1.00	27	0.222
199	A	10	6	1.00	27	0.222
200	A	9	6	1.00	27	0.222
201	A	8	7	1.00	27	0.259
202	A	6	6	1.00	25	0.240
203	A	5	4	1.00	24	0.167
204	A	9	9	1.00	27	0.333
205	A	9	9	1.00	27	0.333
206	A	7	7	1.00	27	0.259
207	A	8	7	1.00	27	0.259
208	A	9	7	1.00	27	0.259
209	A	10	7	1.00	27	0.259
210	A	7	5	1.00	26	0.192
211	A	4	4	1.00	26	0.154
212	A	9	5	1.00	27	0.185
213	A	8	5	1.00	27	0.185
214	A	7	4	1.00	25	0.160
215	A	7	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	12	7	1.00	27	0.259
217	A	12	6	1.00	27	0.222
218	A	4	4	1.00	29	0.138
219	A	2	2	1.00	29	0.069
220	A	3	3	1.00	16	0.188
221	A	4	4	1.00	29	0.138
222	A	3	3	1.00	15	0.200
223	A	4	4	1.00	30	0.133
224	A	4	3	1.00	16	0.188
225	A	5	4	1.00	29	0.138
226	A	7	4	1.00	29	0.138
227	A	6	4	1.00	29	0.138
228	A	5	3	1.00	27	0.111
229	A	2	2	1.00	22	0.091
230	A	8	5	1.00	29	0.172
231	A	7	5	1.00	29	0.172
232	A	8	5	1.00	29	0.172
233	A	6	4	1.00	29	0.138
234	A	6	4	1.00	29	0.138
235	A	5	3	1.31	27	0.111
236	A	2	2	1.00	22	0.091
237	A	8	5	1.00	29	0.172
238	A	7	5	1.00	29	0.172
239	A	7	5	1.00	29	0.172
240	A	6	5	1.00	23	0.217
241	A	6	5	1.00	23	0.217
242	A	6	5	1.00	23	0.217
243	A	6	5	1.00	23	0.217
244	A	4	4	1.00	21	0.190
245	A	3	3	1.00	20	0.150
246	A	5	5	1.00	23	0.217
247	A	5	5	1.00	23	0.217
248	A	5	5	1.00	23	0.217
249	A	7	6	1.00	25	0.240
250	A	8	7	1.00	25	0.280
251	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
252	A	8	7	1.00	25	0.280
253	A	7	6	1.00	23	0.261
254	A	2	2	1.00	22	0.091
255	A	7	7	1.00	25	0.280
256	A	6	6	1.00	25	0.240
257	A	6	6	1.00	25	0.240
258	A	7	6	1.00	25	0.240
259	A	7	6	1.00	25	0.240
260	A	7	6	1.00	25	0.240
261	A	7	6	1.00	25	0.240
262	A	3	3	1.00	23	0.130
263	A	2	2	1.00	22	0.091
264	A	7	7	1.00	25	0.280
265	A	8	8	1.00	25	0.320
266	A	7	6	1.00	25	0.240
267	A	7	6	1.00	25	0.240
268	A	7	6	1.00	25	0.240
269	A	7	6	1.00	25	0.240
270	A	5	5	1.00	23	0.217
271	A	2	2	1.00	22	0.091
272	A	6	6	1.00	25	0.240
273	A	6	6	1.00	25	0.240
274	A	6	6	1.00	25	0.240
275	A	8	7	1.00	25	0.280
276	A	9	8	1.00	25	0.320
277	A	8	7	1.00	25	0.280
278	A	9	8	1.00	25	0.320
279	A	3	3	1.00	23	0.130
280	A	2	2	1.00	22	0.091
281	A	8	8	1.00	25	0.320
282	A	7	7	1.00	25	0.280
283	A	7	7	1.00	25	0.280
284	A	7	7	1.00	25	0.280
285	A	7	7	1.00	25	0.280
286	A	8	7	1.00	25	0.280
287	A	8	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	4	4	1.00	25	0.160
289	A	3	3	1.00	23	0.130
290	A	2	2	1.00	22	0.091
291	A	8	8	1.00	25	0.320
292	A	9	9	1.00	25	0.360
293	A	8	7	1.00	25	0.280
294	A	8	7	1.00	25	0.280
295	A	8	7	1.00	25	0.280
296	A	9	8	1.00	25	0.320
297	A	5	4	1.00	25	0.160
298	A	4	4	1.00	25	0.160
299	A	3	3	1.00	23	0.130
300	A	2	2	1.00	22	0.091
301	A	9	9	1.00	25	0.360
302	A	9	9	1.00	25	0.360
303	A	10	9	1.00	25	0.360
304	A	9	7	1.00	25	0.280
305	A	9	7	1.00	25	0.280
306	A	7	4	1.00	27	0.148
307	A	6	4	1.00	27	0.148
308	A	5	3	1.00	25	0.120
309	A	2	2	1.00	20	0.100
310	A	8	5	1.00	27	0.185
311	A	7	5	1.00	27	0.185
312	A	8	5	1.00	27	0.185
313	A	6	4	1.00	25	0.160
314	A	4	3	1.00	27	0.111
315	A	11	7	1.00	16	0.438
316	A	9	6	1.00	22	0.273
317	A	8	6	1.00	22	0.273
318	A	8	7	1.00	22	0.318
319	A	6	5	1.00	20	0.250
320	A	6	5	1.00	19	0.263
321	A	9	8	1.00	22	0.364
322	A	15	11	1.00	22	0.500
323	A	19	12	1.00	22	0.546

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	20	13	1.00	22	0.591
325	A	25	14	1.00	22	0.636
326	A	8	5	1.00	22	0.227
327	A	7	5	1.00	22	0.227
328	A	7	6	1.00	22	0.273
329	A	5	4	1.00	20	0.200
330	A	2	2	1.00	19	0.105
331	A	7	6	1.00	22	0.273
332	A	8	7	1.00	22	0.318
333	A	12	8	1.00	22	0.364
334	A	7	6	1.00	22	0.273
335	A	6	5	1.00	22	0.227
336	A	4	4	1.00	22	0.182
337	A	4	4	1.00	20	0.200
338	A	4	4	1.00	19	0.210
339	A	10	9	1.00	22	0.409
340	A	12	11	1.00	22	0.500
341	A	17	11	1.00	22	0.500
342	A	9	6	1.00	22	0.273
343	A	8	6	1.00	22	0.273
344	A	7	6	1.00	22	0.273
345	A	6	5	1.00	22	0.227
346	A	3	3	1.00	20	0.150
347	A	3	3	1.00	19	0.158
348	A	10	7	1.00	22	0.318
349	A	11	8	1.00	22	0.364
350	A	15	9	1.00	22	0.409
351	A	2	1	1.00	18	0.056
352	A	2	1	1.00	16	0.062
353	A	2	1	1.00	15	0.067
354	A	3	3	1.00	18	0.167
355	A	2	1	1.00	20	0.050
356	A	2	1	1.00	18	0.056
357	A	2	1	1.00	17	0.059
358	A	4	3	1.00	20	0.150
359	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	2	1	1.00	18	0.056
361	A	2	1	1.00	17	0.059
362	A	4	3	1.00	20	0.150
363	A	6	3	1.00	20	0.150
364	A	6	3	1.00	20	0.150
365	A	6	3	1.00	20	0.150
366	A	4	2	1.00	18	0.111
367	A	4	2	1.00	17	0.118
368	A	7	4	1.00	20	0.200
369	A	7	4	1.00	20	0.200
370	A	5	3	1.00	20	0.150
371	A	5	3	1.00	20	0.150
372	A	5	3	1.00	20	0.150
373	A	5	3	1.00	18	0.167
374	A	5	3	1.00	17	0.176
375	A	12	5	1.00	20	0.250
376	A	12	6	1.00	20	0.300
377	A	6	5	0.94	22	0.227
378	A	4	4	0.91	20	0.200
379	A	6	3	1.00	22	0.136
380	A	12	4	1.00	22	0.182
381	A	6	5	1.00	18	0.278
382	A	6	5	1.00	18	0.278
383	A	6	5	1.00	18	0.278
384	A	6	5	1.00	18	0.278
385	A	4	4	1.00	16	0.250
386	A	3	3	1.00	15	0.200
387	A	5	5	1.00	18	0.278
388	A	5	5	1.00	18	0.278
389	A	5	5	1.00	18	0.278
390	A	7	6	1.00	20	0.300
391	A	8	7	0.95	20	0.350
392	A	7	6	1.00	20	0.300
393	A	8	7	0.95	20	0.350
394	A	7	6	1.00	18	0.333
395	A	4	4	0.94	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	7	7	1.00	20	0.350
397	A	6	6	1.00	20	0.300
398	A	6	6	1.00	20	0.300
399	A	7	6	0.98	20	0.300
400	A	7	6	0.97	20	0.300
401	A	7	6	0.97	20	0.300
402	A	7	6	0.96	20	0.300
403	A	7	6	0.95	18	0.333
404	A	4	4	0.96	17	0.235
405	A	7	7	0.96	20	0.350
406	A	8	8	1.00	20	0.400
407	A	7	6	1.00	20	0.300
408	A	7	6	1.00	20	0.300
409	A	6	6	1.00	20	0.300
410	A	6	6	1.00	20	0.300
411	A	9	8	1.00	18	0.444
412	A	6	5	1.00	17	0.294
413	A	7	7	1.00	20	0.350
414	A	7	7	1.00	20	0.350
415	A	8	8	1.00	20	0.400
416	A	12	10	1.00	20	0.500
417	A	11	10	1.00	20	0.500
418	A	10	9	0.99	20	0.450
419	A	10	9	1.00	18	0.500
420	A	8	7	0.78	17	0.412
421	A	18	10	1.00	20	0.500
422	A	20	12	1.00	20	0.600
423	A	12	10	1.00	20	0.500
424	A	11	9	1.00	20	0.450
425	A	11	10	1.00	20	0.500
426	A	11	9	1.00	18	0.500
427	A	11	9	1.00	17	0.529
428	A	29	12	1.00	20	0.600
429	A	31	14	1.00	20	0.700
430	A	7	4	0.92	22	0.182
431	A	6	4	0.95	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	5	3	1.00	20	0.150
433	A	2	2	1.00	15	0.133
434	A	5	3	1.00	22	0.136
435	A	8	3	1.00	22	0.136
436	A	10	3	1.00	22	0.136
437	A	6	5	1.00	40	0.125
438	A	5	5	1.00	40	0.125
439	A	4	4	1.00	38	0.105
440	A	3	3	1.00	37	0.081
441	A	6	5	1.00	40	0.125
442	A	4	4	1.00	40	0.100
443	A	5	5	1.00	40	0.125
444	A	6	5	1.00	40	0.125
445	A	7	5	1.00	40	0.125
446	A	7	6	1.00	40	0.150
447	A	6	6	1.00	40	0.150
448	A	5	5	1.00	38	0.132
449	A	4	4	1.00	37	0.108
450	A	7	6	1.00	40	0.150
451	A	7	6	1.00	40	0.150
452	A	7	6	1.00	40	0.150
453	A	5	5	1.00	40	0.125
454	A	6	6	1.00	40	0.150
455	A	7	6	1.00	40	0.150
456	A	8	6	1.00	40	0.150
457	A	8	6	1.00	40	0.150
458	A	7	6	1.00	40	0.150
459	A	6	5	1.00	38	0.132
460	A	5	4	1.00	37	0.108
461	A	8	6	1.00	40	0.150
462	A	8	7	1.00	40	0.175
463	A	8	6	1.00	40	0.150
464	A	8	7	1.00	40	0.175
465	A	8	6	1.00	40	0.150
466	A	6	5	1.00	40	0.125
467	A	7	6	1.00	40	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	8	6	1.00	40	0.150
469	A	9	6	1.00	40	0.150
470	A	5	5	1.10	40	0.125
471	A	4	4	1.00	40	0.100
472	A	3	3	1.00	38	0.079
473	A	1	1	1.00	37	0.027
474	A	5	5	1.00	40	0.125
475	A	5	5	1.00	40	0.125
476	A	6	6	1.00	40	0.150
477	A	6	5	1.00	40	0.125
478	A	6	5	1.00	40	0.125
479	A	5	5	1.00	40	0.125
480	A	3	3	1.00	40	0.075
481	A	2	2	1.00	38	0.053
482	A	2	2	1.00	37	0.054
483	A	6	5	1.00	40	0.125
484	A	6	5	1.00	40	0.125
485	A	7	6	1.00	40	0.150
486	A	8	6	1.00	40	0.150
487	A	3	3	1.00	40	0.075
488	A	4	4	1.00	40	0.100
489	A	4	4	1.00	23	0.174
490	A	1	1	1.00	23	0.043
491	A	5	5	1.00	21	0.238
492	A	3	3	1.00	20	0.150
493	A	5	5	1.00	23	0.217
494	A	5	5	1.00	23	0.217
495	A	3	3	1.00	23	0.130
496	A	5	4	1.00	23	0.174
497	A	1	1	1.00	23	0.043
498	A	6	5	1.00	21	0.238
499	A	4	3	1.00	20	0.150
500	A	6	5	1.00	23	0.217
501	A	6	6	1.00	23	0.261
502	A	4	4	1.00	23	0.174
503	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	1	1	1.00	23	0.043
505	A	4	4	1.00	21	0.190
506	A	2	2	1.00	20	0.100
507	A	4	4	1.00	23	0.174
508	A	5	5	1.00	23	0.217
509	A	3	3	0.99	23	0.130
510	A	3	3	1.00	23	0.130
511	A	1	1	1.00	23	0.043
512	A	5	5	1.00	21	0.238
513	A	3	3	1.00	20	0.150
514	A	5	5	1.00	23	0.217
515	A	6	6	1.00	23	0.261
516	A	4	4	1.00	23	0.174
517	A	4	4	1.00	23	0.174
518	A	1	1	1.00	23	0.043
519	A	6	5	1.00	21	0.238
520	A	4	3	1.00	20	0.150
521	A	6	5	1.00	23	0.217
522	A	7	6	1.00	23	0.261
523	A	5	4	1.00	23	0.174
524	A	7	6	1.00	19	0.316
525	A	6	4	1.00	25	0.160
526	A	6	4	1.22	25	0.160
527	A	6	4	1.00	25	0.160
528	A	5	4	1.00	23	0.174
529	A	4	3	1.00	22	0.136
530	A	7	5	1.00	25	0.200
531	A	9	6	0.97	25	0.240
532	A	12	6	1.00	25	0.240
533	A	6	4	1.00	25	0.160
534	A	6	4	1.00	25	0.160
535	A	6	4	1.00	25	0.160
536	A	6	4	1.00	23	0.174
537	A	5	4	1.00	22	0.182
538	A	7	5	1.00	25	0.200
539	A	9	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
540	A	12	6	1.00	25	0.240
541	A	6	3	1.00	23	0.130
542	A	4	2	1.00	23	0.087
543	A	4	2	1.00	23	0.087
544	A	4	2	1.00	21	0.095
545	A	4	2	1.00	20	0.100
546	A	7	4	1.00	23	0.174
547	A	8	4	1.00	23	0.174
548	A	3	2	1.00	29	0.069
549	A	3	2	1.00	29	0.069
550	A	3	2	1.00	29	0.069
551	A	3	2	1.00	27	0.074
552	A	5	3	1.00	22	0.136
553	A	3	2	1.00	29	0.069
554	A	4	3	1.00	29	0.103
555	A	4	3	1.00	29	0.103
556	A	4	3	1.00	29	0.103
557	A	3	2	1.00	29	0.069
558	A	3	2	1.00	29	0.069
559	A	3	2	1.00	29	0.069
560	A	3	2	1.00	29	0.069
561	A	3	2	1.00	29	0.069
562	A	3	2	1.00	29	0.069
563	A	3	2	1.00	27	0.074
564	A	2	2	1.00	22	0.091
565	A	4	3	1.00	29	0.103
566	A	4	3	1.00	29	0.103
567	A	4	3	1.00	29	0.103
568	A	4	3	1.00	29	0.103
569	A	3	2	1.00	29	0.069
570	A	3	2	1.00	29	0.069
571	A	3	2	1.00	29	0.069
572	A	3	2	1.00	29	0.069
573	A	3	2	1.00	29	0.069
574	A	4	3	1.00	29	0.103
575	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
576	A	3	3	1.00	22	0.136
577	A	4	3	1.00	29	0.103
578	A	4	3	1.00	29	0.103
579	A	7	5	1.00	31	0.161
580	A	6	4	1.00	31	0.129
581	A	5	4	1.00	31	0.129
582	A	3	3	1.00	31	0.097
583	A	3	3	1.00	29	0.103
584	A	4	3	1.00	24	0.125
585	A	6	5	1.00	31	0.161
586	A	6	4	1.00	31	0.129
587	A	7	5	1.00	31	0.161
588	A	4	3	1.00	24	0.125
589	A	3	2	1.00	24	0.083
590	A	3	2	1.00	24	0.083
591	A	2	1	1.00	22	0.045
592	A	2	1	1.00	17	0.059
593	A	4	3	1.00	24	0.125
594	A	4	4	1.00	24	0.167
595	A	4	4	1.00	24	0.167
596	A	3	2	1.00	24	0.083
597	A	3	2	1.00	24	0.083
598	A	2	1	1.00	22	0.045
599	A	2	1	1.00	17	0.059
600	A	4	3	1.00	24	0.125
601	A	4	4	1.00	24	0.167
602	A	5	5	1.00	24	0.208
603	A	5	5	1.00	26	0.192
604	A	1	1	1.00	22	0.045
605	A	11	8	1.00	28	0.286
606	A	10	7	1.00	28	0.250
607	A	6	3	1.00	28	0.107
608	A	8	5	1.00	28	0.179
609	A	11	7	1.00	28	0.250
610	A	11	7	1.00	28	0.250
611	A	6	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
612	A	6	3	1.00	28	0.107
613	A	8	4	1.00	28	0.143
614	A	19	9	1.00	28	0.321
615	A	8	5	1.00	28	0.179
616	A	8	4	1.00	28	0.143
617	A	12	6	0.99	28	0.214
618	A	6	3	1.00	20	0.150
619	A	5	5	1.00	26	0.192
620	A	6	6	1.00	30	0.200
621	A	5	5	1.00	29	0.172
622	A	10	6	1.00	28	0.214
623	A	9	6	1.00	28	0.214
624	A	7	6	1.00	26	0.231
625	A	7	6	1.00	21	0.286
626	A	14	10	1.00	28	0.357
627	A	14	10	1.00	28	0.357
628	A	23	11	1.00	28	0.393
629	A	9	6	1.00	28	0.214
630	A	8	6	0.99	28	0.214
631	A	6	5	1.00	26	0.192
632	A	6	5	1.00	21	0.238
633	A	10	9	1.00	28	0.321
634	A	14	10	1.00	28	0.357
635	A	23	11	1.00	28	0.393
636	A	8	6	0.99	28	0.214
637	A	7	6	1.00	28	0.214
638	A	6	5	1.00	26	0.192
639	A	2	2	1.00	21	0.095
640	A	7	7	1.00	28	0.250
641	A	14	10	1.00	28	0.357
642	A	23	11	1.00	28	0.393
643	A	16	10	1.35	28	0.357
644	A	10	8	1.00	28	0.286
645	A	7	6	1.00	28	0.214
646	A	7	6	1.00	28	0.214
647	A	5	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
648	A	2	2	1.00	21	0.095
649	A	4	4	1.00	28	0.143
650	A	14	10	1.00	28	0.357
651	A	23	10	1.00	28	0.357
652	A	10	9	1.00	28	0.321
653	A	17	12	1.00	28	0.429
654	A	4	4	1.00	28	0.143
655	A	2	2	1.00	30	0.067
656	A	4	3	1.00	26	0.115
657	A	4	3	1.00	46	0.065
658	A	3	3	1.00	46	0.065
659	A	2	2	1.00	44	0.045
660	A	1	1	1.00	39	0.026
661	A	2	2	1.00	46	0.043
662	A	3	3	1.00	46	0.065
663	A	4	3	1.00	46	0.065
664	A	5	3	1.00	46	0.065
665	A	4	4	1.00	46	0.087
666	A	3	3	1.00	46	0.065
667	A	2	2	1.00	44	0.045
668	A	1	1	1.00	39	0.026
669	A	3	3	1.00	46	0.065
670	A	4	4	1.00	46	0.087
671	A	5	4	1.00	46	0.087
672	A	4	3	1.00	46	0.065
673	A	3	3	1.00	46	0.065
674	A	2	2	1.00	44	0.045
675	A	1	1	1.00	39	0.026
676	A	4	3	1.00	46	0.065
677	A	5	4	1.00	46	0.087
678	A	6	4	1.00	46	0.087
679	A	5	3	1.00	46	0.065
680	A	4	3	1.00	46	0.065
681	A	3	3	1.00	46	0.065
682	A	2	2	1.00	44	0.045
683	A	1	1	1.00	39	0.026

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
684	A	3	3	1.00	46	0.065
685	A	3	3	1.00	46	0.065
686	A	4	4	1.00	46	0.087
687	A	5	4	1.00	46	0.087
688	A	6	4	1.00	46	0.087
689	A	5	3	1.00	46	0.065
690	A	4	3	1.00	46	0.065
691	A	3	3	1.00	46	0.065
692	A	2	2	1.00	44	0.045
693	A	1	1	1.00	39	0.026
694	A	4	3	1.00	46	0.065
695	A	4	4	1.00	46	0.087
696	A	4	3	1.00	46	0.065
697	A	5	4	1.00	46	0.087
698	A	6	4	1.00	46	0.087
699	A	7	4	1.00	46	0.087
700	A	5	3	1.00	46	0.065
701	A	4	3	1.00	46	0.065
702	A	3	3	1.00	46	0.065
703	A	2	2	1.00	44	0.045
704	A	1	1	1.00	39	0.026
705	A	5	3	1.00	46	0.065
706	A	5	4	1.00	46	0.087
707	A	5	4	1.00	46	0.087
708	A	5	3	1.00	46	0.065
709	A	6	4	1.00	46	0.087
710	A	7	4	1.00	46	0.087
711	A	8	4	1.00	46	0.087
712	A	7	5	1.00	48	0.104
713	A	6	5	1.00	48	0.104
714	A	5	5	1.00	48	0.104
715	A	4	4	1.00	48	0.083
716	A	1	1	1.00	48	0.021
717	A	2	2	1.00	48	0.042
718	A	3	2	1.00	48	0.042
719	A	4	2	1.00	48	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
720	A	7	6	1.00	48	0.125
721	A	6	6	1.00	48	0.125
722	A	5	5	1.00	48	0.104
723	A	1	1	1.00	48	0.021
724	A	2	2	1.00	48	0.042
725	A	3	3	1.00	48	0.062
726	A	4	3	1.00	48	0.062
727	A	7	6	1.00	48	0.125
728	A	6	5	1.00	48	0.104
729	A	1	1	1.00	48	0.021
730	A	2	2	1.00	48	0.042
731	A	3	2	1.00	48	0.042
732	A	4	3	1.00	48	0.062
733	A	8	6	1.00	48	0.125
734	A	7	6	1.00	48	0.125
735	A	6	6	1.00	48	0.125
736	A	5	5	1.00	48	0.104
737	A	5	5	1.00	48	0.104
738	A	1	1	1.00	48	0.021
739	A	2	2	1.00	48	0.042
740	A	3	2	1.00	48	0.042
741	A	4	2	1.00	48	0.042
742	A	8	6	1.00	48	0.125
743	A	7	6	1.00	48	0.125
744	A	6	5	1.00	48	0.104
745	A	6	6	1.00	48	0.125
746	A	6	5	1.00	48	0.104
747	A	1	1	1.00	48	0.021
748	A	2	2	1.00	48	0.042
749	A	3	2	1.00	48	0.042
750	A	4	2	1.00	48	0.042
751	A	9	6	1.00	48	0.125
752	A	8	6	1.00	48	0.125
753	A	7	5	1.00	48	0.104
754	A	7	6	1.00	48	0.125
755	A	7	6	1.00	48	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
756	A	7	5	1.00	48	0.104
757	A	1	1	1.00	48	0.021
758	A	2	2	1.00	48	0.042
759	A	3	2	1.00	48	0.042
760	A	4	2	1.00	48	0.042
761	A	3	3	1.17	46	0.065
762	A	3	3	1.06	46	0.065
763	A	3	3	1.13	46	0.065
764	A	3	3	1.15	46	0.065
765	A	3	3	1.17	46	0.065
766	A	3	3	1.17	46	0.065
767	A	3	3	1.04	44	0.068
768	A	4	3	1.00	44	0.068
769	A	3	3	1.00	44	0.068
770	A	2	2	1.00	42	0.048
771	A	1	1	1.00	37	0.027
772	A	2	2	1.00	44	0.045
773	A	2	2	1.00	44	0.045
774	A	2	2	1.00	44	0.045
775	A	3	3	1.00	46	0.065
776	A	3	3	1.00	46	0.065
777	A	3	3	1.00	46	0.065
778	A	3	3	1.00	46	0.065
779	A	3	3	1.00	46	0.065
780	A	1	1	1.00	47	0.021
781	A	3	3	1.00	73	0.041
782	A	4	4	1.04	46	0.087
783	A	6	4	1.00	46	0.087
784	A	5	4	1.00	46	0.087
785	A	4	4	1.00	46	0.087
786	A	3	3	1.00	44	0.068
787	A	2	2	1.00	39	0.051
788	A	3	3	1.00	46	0.065
789	A	3	3	1.00	46	0.065
790	A	4	4	1.00	46	0.087
791	A	5	4	1.00	46	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
792	A	8	4	1.00	32	0.125
793	A	6	4	1.00	32	0.125
794	A	4	4	1.00	30	0.133
795	A	6	4	1.00	32	0.125
796	A	7	5	1.00	32	0.156
797	A	7	5	1.00	32	0.156
798	A	5	5	1.00	32	0.156
799	A	4	4	1.00	30	0.133
800	A	7	5	1.00	32	0.156
801	A	8	6	1.00	32	0.188
802	A	3	3	1.00	25	0.120
803	A	4	3	1.00	25	0.120
804	A	4	3	1.00	28	0.107
805	A	2	1	1.00	28	0.036
806	A	2	1	1.00	28	0.036
807	A	2	1	1.00	26	0.038
808	A	2	1	1.00	21	0.048
809	A	3	3	1.00	28	0.107
810	A	3	2	1.00	28	0.071
811	A	3	3	1.00	28	0.107
812	A	3	3	1.00	28	0.107
813	A	4	4	0.98	28	0.143
814	A	2	1	1.00	25	0.040
815	A	2	1	1.00	27	0.037
816	A	2	1	1.00	27	0.037
817	A	6	5	1.00	27	0.185
818	A	9	6	1.00	27	0.222
819	A	3	2	1.00	27	0.074
820	A	3	2	1.00	27	0.074
821	A	2	1	1.00	25	0.040
822	A	2	1	1.00	20	0.050
823	A	4	3	1.00	27	0.111
824	A	4	4	1.00	27	0.148
825	A	4	4	1.00	27	0.148
826	A	3	2	1.00	27	0.074
827	A	3	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
828	A	2	1	1.00	25	0.040
829	A	2	1	1.00	20	0.050
830	A	4	3	1.00	27	0.111
831	A	4	4	1.00	27	0.148
832	A	5	5	1.00	27	0.185
833	B	9	7	2.10	25	0.280
834	A	5	5	1.00	29	0.172
835	A	7	6	1.00	29	0.207
836	A	6	6	1.00	29	0.207
837	A	5	5	1.00	29	0.172
838	A	5	5	1.00	29	0.172
839	A	5	5	1.00	29	0.172
840	A	3	3	1.00	29	0.103
841	A	4	4	1.00	29	0.138
842	A	6	6	1.00	29	0.207
843	A	7	6	1.00	38	0.158
844	A	6	6	1.00	38	0.158
845	A	5	5	1.00	38	0.132
846	A	5	5	1.00	38	0.132
847	A	5	5	1.00	38	0.132
848	A	3	3	1.00	38	0.079
849	A	4	4	1.00	38	0.105
850	A	11	7	1.00	31	0.226
851	A	6	3	1.00	31	0.097
852	A	6	3	1.00	31	0.097
853	A	8	4	1.00	31	0.129
854	A	8	6	1.00	29	0.207
855	A	7	6	1.00	29	0.207
856	A	6	5	1.00	27	0.185
857	A	6	5	1.00	22	0.227
858	A	8	5	1.00	29	0.172
859	A	20	7	1.00	29	0.241
860	A	23	8	1.00	29	0.276
861	A	27	9	1.00	29	0.310
862	A	9	6	1.00	29	0.207
863	A	8	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
864	A	7	5	1.00	27	0.185
865	A	7	6	1.00	22	0.273
866	A	13	7	1.00	29	0.241
867	A	23	8	1.00	29	0.276
868	A	30	9	1.00	29	0.310
869	A	15	7	1.00	29	0.241
870	A	8	5	1.00	29	0.172
871	A	7	5	1.00	29	0.172
872	A	6	5	1.00	29	0.172
873	A	5	4	1.00	27	0.148
874	A	2	2	1.00	22	0.091
875	A	6	3	1.00	29	0.103
876	A	9	4	1.00	29	0.138
877	A	13	6	1.00	29	0.207
878	A	7	6	1.00	29	0.207
879	A	6	5	1.00	29	0.172
880	A	4	4	1.00	29	0.138
881	A	4	4	1.00	27	0.148
882	A	4	4	1.00	22	0.182
883	A	10	5	1.00	29	0.172
884	A	14	6	1.00	29	0.207
885	A	19	7	1.00	29	0.241
886	A	10	6	1.00	31	0.194
887	A	9	6	1.00	31	0.194
888	A	7	6	1.00	29	0.207
889	A	7	6	1.00	24	0.250
890	A	15	10	1.27	31	0.323
891	A	15	10	1.26	31	0.323
892	A	25	11	1.65	31	0.355
893	A	9	6	1.00	31	0.194
894	A	8	6	1.00	31	0.194
895	A	6	5	1.00	29	0.172
896	A	6	5	1.00	24	0.208
897	A	11	9	1.00	31	0.290
898	A	15	10	1.30	31	0.323
899	A	25	11	1.67	31	0.355

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	8	6	1.00	31	0.194
901	A	7	6	1.00	31	0.194
902	A	6	5	1.00	29	0.172
903	A	2	2	1.00	24	0.083
904	A	8	7	1.00	31	0.226
905	A	15	10	1.00	31	0.323
906	A	25	11	1.00	31	0.355
907	A	11	8	1.00	31	0.258
908	A	17	10	1.00	31	0.323
909	A	7	6	1.00	31	0.194
910	A	7	6	1.00	31	0.194
911	A	5	4	1.00	29	0.138
912	A	2	2	1.00	24	0.083
913	A	5	4	1.00	31	0.129
914	A	15	10	1.00	31	0.323
915	A	25	10	1.58	31	0.323
916	A	11	9	1.00	31	0.290
917	A	18	12	1.00	31	0.387
918	A	1	1	1.00	33	0.030
919	A	2	2	1.00	33	0.061
920	A	2	1	1.00	25	0.040
921	A	2	1	1.00	23	0.043
922	A	3	3	1.00	25	0.120
923	A	3	3	1.00	25	0.120
924	A	3	3	0.99	25	0.120
925	A	2	1	1.00	27	0.037
926	A	2	1	1.00	25	0.040
927	A	4	3	1.00	27	0.111
928	A	4	3	1.00	27	0.111
929	A	5	5	1.00	27	0.185
930	A	4	2	1.00	27	0.074
931	A	4	2	1.00	27	0.074
932	A	4	2	1.00	27	0.074
933	A	4	2	1.00	25	0.080
934	A	4	2	1.00	20	0.100
935	A	7	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	8	3	1.00	27	0.111
937	A	5	3	1.00	27	0.111
938	A	5	3	1.00	27	0.111
939	A	5	3	1.00	27	0.111
940	A	5	3	1.00	25	0.120
941	A	5	3	1.00	20	0.150
942	A	12	4	1.00	27	0.148
943	A	13	4	1.00	27	0.148
944	A	4	4	0.97	27	0.148
945	A	6	4	0.99	29	0.138
946	A	5	3	1.00	27	0.111
947	A	2	2	1.00	22	0.091
948	A	0	0	0.00	0	0.000
949	A	6	4	1.00	29	0.138
950	A	5	3	1.00	27	0.111
951	A	2	2	1.00	22	0.091
952	A	0	0	0.00	0	0.000
953	A	4	4	0.99	25	0.160
954	A	6	4	1.00	27	0.148
955	A	5	3	1.00	25	0.120
956	A	2	2	1.00	20	0.100
957	A	0	0	0.00	0	0.000
958	A	5	5	1.00	27	0.185

Chapter 3

Listing of integrals

3.1 $\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=132

$$-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

```
[Out] -1/3*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/5*(-e^
2*x^2+d^2)^(5/2)/e^3+1/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 195, 217, 203}

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - (d^2*(d^2 - e^2*x^2)^(3/2))/(3*e^3) -
 (d*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx &= -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}+\frac{d^2\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= -\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{d\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}+\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\left(3d^3\right)\int}{}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\left(3d^5\right)\int}{}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}}{}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}}{}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^4\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4\right)\right)}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2
*e^2*x^2 + 30*d*e^3*x^3 + 24*e^4*x^4) + 15*d^4*ArcSin[(e*x)/d]))/(120*e^3*S
qrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.71, size = 95, normalized size = 0.72

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4+30\,de^3x^3-8\,d^2e^2x^2-15\,d^3ex-16\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 + 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 - 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.26, size = 74, normalized size = 0.56

$$\frac{1}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d) - \frac{1}{120}\left(16\,d^4e^{(-3)} + \left(15\,d^3e^{(-2)} + 2\left(4\,d^2e^{(-1)} - 3\,(4\,xe + 5\,d)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^5*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/120*(16*d^4*e^(-3) + (15*d^3*e^(-2)
 + 2*(4*d^2*e^(-1) - 3*(4*x*e + 5*d)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.06, size = 125, normalized size = 0.95

$$\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2} + \frac{\sqrt{-e^2x^2+d^2}\,d^3x}{8e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4e^2} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*x^2*(-e^2*x^2+d^2)^(3/2)/e-2/15*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(
-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/8*d^5/e^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

maxima [A] time = 0.98, size = 104, normalized size = 0.79

$$\frac{d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^3} + \frac{\sqrt{-e^2x^2+d^2}\,d^3x}{8\,e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^2} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*d^5*arcsin(e*x/d)/e^3 + 1/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^2 - 1/5*(-e^2*
x^2 + d^2)^(3/2)*x^2/e - 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^2 - 2/15*(-e^2*x^
2 + d^2)^(3/2)*d^2/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x), x)
```

sympy [C] time = 5.48, size = 279, normalized size = 2.11

$$d\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\left\{\begin{matrix} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \\ \frac{x^4\sqrt{d^2}}{4} \end{matrix}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piece
wise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2
*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**
2)/4, True))
```

3.2 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}+\frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2\right.}{6}$$

```
[Out] 1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-1
/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-1/5040*d^3
*(315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2+d^
2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.15, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {833, 780, 195, 217, 203}

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx &= -\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{\int x^3\left(-4d^2e-9de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2}\\
&= -\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}+\frac{\int x^2\left(27d^3e^2+32d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{72e^4}\\
&= -\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{\int x\left(-64d^4e^3-\right.}{}\\
&= -\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{d^3(128d+315e}{50}\\
&= \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/}}{9e}\\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/}}{8e^2}\\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/}}{8e^2}\\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/}}{8e^2}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 157, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\left(945d^8\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(1024d^8+945d^7ex+512d^6e^2x^2+630d^5e^3x^3+384d^4e^4x^4-7560d^3e^5\right.\right.}{40320e^5\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(1024*d^8 + 945*d^7*e*x + 5
12*d^6*e^2*x^2 + 630*d^5*e^3*x^3 + 384*d^4*e^4*x^4 - 7560*d^3*e^5*x^5 - 640
0*d^2*e^6*x^6 + 5040*d*e^7*x^7 + 4480*e^8*x^8)) + 945*d^8*ArcSin[(e*x)/d]))
/(40320*e^5*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.86, size = 138, normalized size = 0.69

$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4480\,e^8x^8+5040\,de^7x^7-6400\,d^2e^6x^6-7560\,d^3e^5x^5+384\,d^4e^4x^4+630\,d^5e^3\right.}{40320\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4480*e^8*x^
8 + 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 - 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
+ 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 + 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

giac [A] time = 0.22, size = 117, normalized size = 0.58

$$\frac{3}{128}\,d^9\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{40320}\left(1024\,d^8e^{(-5)}+\left(945\,d^7e^{(-4)}+2\left(256\,d^6e^{(-3)}+\left(315\,d^5e^{(-2)}+4\left(48\,d^4e^{(-\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/40320*(1024*d^8*e^(-5) + (945*d^7
*e^(-4) + 2*(256*d^6*e^(-3) + (315*d^5*e^(-2) + 4*(48*d^4*e^(-1) - 5*(189*d
^3 + 2*(80*d^2*e - 7*(8*x*e^3 + 9*d*e^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

maple [A] time = 0.04, size = 198, normalized size = 0.99

$$\frac{3d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}\,e^4}+\frac{3\sqrt{-e^2x^2+d^2}\,d^7x}{128e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^4}{9e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{64e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^3}{8e^2}-\frac{4\left(-e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-8/315
*d^4/e^5*(-e^2*x^2+d^2)^(5/2)-1/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/16*d^3/e
^4*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+3/128*d^7*x*(
-e^2*x^2+d^2)^(1/2)/e^4+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*
x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 177, normalized size = 0.88

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^4}{9\,e}+\frac{3\,d^9\arcsin\left(\frac{ex}{d}\right)}{128\,e^5}+\frac{3\sqrt{-e^2x^2+d^2}\,d^7x}{128\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^3}{8\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{64\,e^4}-\frac{4\left(-e^2x^2+d\right.}{63\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*(-e^2*x^2 + d^2)^(5/2)*x^4/e + 3/128*d^9*arcsin(e*x/d)/e^5 + 3/128*sqr
t(-e^2*x^2 + d^2)*d^7*x/e^4 - 1/8*(-e^2*x^2 + d^2)^(5/2)*d*x^3/e^2 + 1/64*(
-e^2*x^2 + d^2)^(3/2)*d^5*x/e^4 - 4/63*(-e^2*x^2 + d^2)^(5/2)*d^2*x^2/e^3 -
 1/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 8/315*(-e^2*x^2 + d^2)^(5/2)*d^4/e
^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [C] time = 17.83, size = 830, normalized size = 4.13

$$d^3\left(\begin{cases} -\frac{id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + d^2e\left(\left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x}}{105e^6} \\ \frac{x^6\sqrt{d^2}}{6} \end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) + d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7
) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e*
*4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e*
*7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) +
 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2
/d**2)), True)) - e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8
) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 -
 e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*
sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.3 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}+\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}$$

```
[Out] 1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2-1/8*
x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e^4
+3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+3/128*d^6*x*(-e^2*x^2+d^2)^
(1/2)/e^3
```

Rubi [A] time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 780, 195, 217, 203}

$$\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}+3d$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))
/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{\int x^2\left(-3d^2e-8de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$

$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} + \frac{\int x\left(16d^3e^2+21d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{56e^4}$$

$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{d^4\int (}{}$$

$$= \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2\right.}{560e^4}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} -$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} -$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} -$$

Mathematica [A] time = 0.19, size = 146, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(105d^7\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(256d^7+105d^6ex+128d^5e^2x^2+70d^4e^3x^3-1024d^3e^4x^4-840d^2e^5x^5\right.\right.}{4480e^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(256*d^7 + 105*d^6*e*x + 12
8*d^5*e^2*x^2 + 70*d^4*e^3*x^3 - 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 + 640*d
*e^6*x^6 + 560*e^7*x^7)) + 105*d^7*ArcSin[(e*x)/d]))/(4480*e^4*Sqrt[1 - (e^
2*x^2)/d^2])
```

fricas [A] time = 0.89, size = 127, normalized size = 0.74

$$-\frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(560\,e^7x^7+640\,de^6x^6-840\,d^2e^5x^5-1024\,d^3e^4x^4+70\,d^4e^3x^3+128\,d^5e^2x^2+1\right.}{4480\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 +
 640*d*e^6*x^6 - 840*d^2*e^5*x^5 - 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 + 128*
d^5*e^2*x^2 + 105*d^6*e*x + 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.23, size = 106, normalized size = 0.62

$$\frac{3}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{4480}\left(256\,d^7e^{(-4)}+\left(105\,d^6e^{(-3)}+2\left(64\,d^5e^{(-2)}+\left(35\,d^4e^{(-1)}-4\left(128\,d^3+5\left(2\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^8*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/4480*(256*d^7*e^(-4) + (105*d^6*e
^(-3) + 2*(64*d^5*e^(-2) + (35*d^4*e^(-1) - 4*(128*d^3 + 5*(21*d^2*e - 2*(7
*x*e^3 + 8*d*e^2)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 173, normalized size = 1.01

$$\frac{3d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}\,e^3}+\frac{3\sqrt{-e^2x^2+d^2}\,d^6x}{128e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^2}{7e^2}-\frac{\left(-e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/8*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/16*d^2/e^3*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^
4*x*(-e^2*x^2+d^2)^(3/2)/e^3+3/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^3+3/128*d^8
/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/7*d*x^2*(-e^2
*x^2+d^2)^(5/2)/e^2-2/35*d^3/e^4*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.98, size = 152, normalized size = 0.88

$$\frac{3\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^4}+\frac{3\,\sqrt{-e^2x^2+d^2}\,d^6x}{128\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}{7\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{16\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*d^8*arcsin(e*x/d)/e^4 + 3/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^3 - 1/8*(-
e^2*x^2 + d^2)^(5/2)*x^3/e + 1/64*(-e^2*x^2 + d^2)^(3/2)*d^4*x/e^3 - 1/7*(-
e^2*x^2 + d^2)^(5/2)*d*x^2/e^2 - 1/16*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^3 - 2/
35*(-e^2*x^2 + d^2)^(5/2)*d^3/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(d^2-e^2x^2\right)^{3/2}(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [A] time = 17.17, size = 775, normalized size = 4.51

$$d^3\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\end{cases}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e
*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48
*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) -
e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-8*d**
6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e*
*2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - e**3*Piecewise((-5*I*d*
*8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e
**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5
*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2))
 + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sq
rt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.4 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}$$

```
[Out] 1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d*
x*(-e^2*x^2+d^2)^(5/2)/e^2+1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 195, 217, 203}

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) - (d^2*(d^2 - e^2*x^2)^(5/2))/(5*e^3) - (d*x*(d^2 - e^2*x^2)^(5/2))/(6*e
^2) + (d^2 - e^2*x^2)^(7/2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(16*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx = -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}dx}{e^2}+\frac{d^2\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}dx}{e^2}$$
$$= -\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{d\int \left(d^2-e^2x^2\right)^{5/2}dx}{e^2}+\frac{d^3\int \left(d^2-e^2x^2\right)^{3/}}{e^2}$$
$$= \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{(5a}{}$$
$$= \frac{3d^5x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+$$

Mathematica [A] time = 0.17, size = 135, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(105d^6\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(96d^6+105d^5ex+48d^4e^2x^2-490d^3e^3x^3-384d^2e^4x^4+280de^5x^5+2\right.\right.}{1680e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(96*d^6 + 105*d^5*e*x + 48*
d^4*e^2*x^2 - 490*d^3*e^3*x^3 - 384*d^2*e^4*x^4 + 280*d*e^5*x^5 + 240*e^6*x
^6)) + 105*d^6*ArcSin[(e*x)/d]))/(1680*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.96, size = 116, normalized size = 0.73

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(240\,e^6x^6+280\,de^5x^5-384\,d^2e^4x^4-490\,d^3e^3x^3+48\,d^4e^2x^2+105\,d^5ex+96\,d\right.}{1680\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (240*e^6*x^6 +
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 - 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 + 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.23, size = 96, normalized size = 0.60

$$\frac{1}{16}\,d^7\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{1680}\left(96\,d^6e^{(-3)}+\left(105\,d^5e^{(-2)}+2\left(24\,d^4e^{(-1)}-\left(245\,d^3+4\left(48\,d^2e-5\left(6\,xe^3+7\,a\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

[Out] 1/16*d^7*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/1680*(96*d^6*e^(-3) + (105*d^5*e^(
-2) + 2*(24*d^4*e^(-1) - (245*d^3 + 4*(48*d^2*e - 5*(6*x*e^3 + 7*d*e^2)*x)*
x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)

maple [A] time = 0.04, size = 148, normalized size = 0.93

$$\frac{d^7 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,d^5x}{16e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6e^2}-\frac{2\left(-e^2x^2\right.}{3\!\!\;}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)

[Out] -1/7*x^2*(-e^2*x^2+d^2)^(5/2)/e-2/35*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d*x*(
-e^2*x^2+d^2)^(5/2)/e^2+1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/16*d^5*x*(-e^
2*x^2+d^2)^(1/2)/e^2+1/16*d^7/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)

maxima [A] time = 0.95, size = 127, normalized size = 0.80

$$\frac{d^7 \arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,d^5x}{16\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{35\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")

[Out] 1/16*d^7*arcsin(e*x/d)/e^3 + 1/16*sqrt(-e^2*x^2 + d^2)*d^5*x/e^2 + 1/24*(-e
^2*x^2 + d^2)^(3/2)*d^3*x/e^2 - 1/7*(-e^2*x^2 + d^2)^(5/2)*x^2/e - 1/6*(-e^
2*x^2 + d^2)^(5/2)*d*x/e^2 - 2/35*(-e^2*x^2 + d^2)^(5/2)*d^2/e^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)

[Out] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)

sympy [C] time = 12.27, size = 653, normalized size = 4.11

$$d^3\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+d^2e\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} \\ \frac{x^4\sqrt{d^2}}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)

[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2

```
*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**
2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*
sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I
*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x
/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e
**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e*
*2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((-8*d**6*sq
rt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x
**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.5 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

```
[Out] 1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^2
+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^(1
/2)/e
```

Rubi [A] time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {780, 195, 217, 203}

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}$$
$$=\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}}{16e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\text{Subst}\left(\int\frac{}{1}\right.}{}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{e}{\sqrt{d^2}}\right.}{16e^2}$$

Mathematica [A] time = 0.14, size = 124, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^5\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5\right)\right)}{240e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(48*d^5 + 15*d^4*e*x - 96*d
^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5)) + 15*d^5*ArcSin[(
e*x)/d]))/(240*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 1.02, size = 105, normalized size = 0.91

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

giac [A] time = 0.21, size = 84, normalized size = 0.72

$$\frac{1}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\text{sgn}(d)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

maple [A] time = 0.02, size = 123, normalized size = 1.06

$$\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e}+\frac{\sqrt{-e^2x^2+d^2}\,d^4x}{16e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/6*x*(-e^2*x^2+d^2)^(5/2)/e+1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e+1/16*d^4*x*
(-e^2*x^2+d^2)^(1/2)/e+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-1/5*d/e^2*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.98, size = 102, normalized size = 0.88

$$\frac{d^6 \arcsin\left(\frac{ex}{d}\right)}{16\,e^2} + \frac{\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e*x/d)/e^2 + 1/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e + 1/24*(-e^2
*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2
+ d^2)^(5/2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [A] time = 12.06, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\; e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + d^2e\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\; \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.6 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

```
[Out] 1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^2
+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^(1
/2)/e
```

Rubi [A] time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {780, 195, 217, 203}

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx = -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}dx}{6e}$$
$$=\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2}}}{16e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\mathrm{Subst}\left(\right.}{}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\right.}{16e}$$

Mathematica [A] time = 0.04, size = 124, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^5\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5\right)\right)}{240e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(48*d^5 + 15*d^4*e*x - 96*d
^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5)) + 15*d^5*ArcSin[(
e*x)/d]))/(240*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.89, size = 105, normalized size = 0.91

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

giac [A] time = 0.34, size = 84, normalized size = 0.72

$$\frac{1}{16}d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

maple [A] time = 0.00, size = 123, normalized size = 1.06

$$\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e}+\frac{\sqrt{-e^2x^2+d^2}\,d^4x}{16e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/16/(e^2)^(1/2)*d^6/e*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/16*(-e^
2*x^2+d^2)^(1/2)*d^4/e*x+1/24*(-e^2*x^2+d^2)^(3/2)*d^2/e*x-1/6*(-e^2*x^2+d^
2)^(5/2)/e*x-1/5*(-e^2*x^2+d^2)^(5/2)*d/e^2
```

maxima [A] time = 0.98, size = 102, normalized size = 0.88

$$\frac{d^6 \arcsin\left(\frac{ex}{d}\right)}{16\,e^2} + \frac{\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e*x/d)/e^2 + 1/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e + 1/24*(-e^2
*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2
+ d^2)^(5/2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [A] time = 12.18, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2 = 0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + d^2e\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.7 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/12*(3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(3*e*x+8*d)*(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (d^2*(8*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 + (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2-3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4+3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-}}\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.$$

Mathematica [A] time = 0.18, size = 124, normalized size = 1.10

$$d^4\left(-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{3d^3\sqrt{d^2-e^2x^2}\,\sin^{-1}\left(\frac{ex}{d}\right)}{8\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3+15d^2ex-8de^2x^2-6e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 +
(3*d^3*Sqrt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(8*Sqrt[1 - (e^2*x^2)/d^2]) - d
^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

fricas [A] time = 0.91, size = 107, normalized size = 0.95

$$-\frac{3}{4}d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\frac{1}{24}\left(6e^3x^3+8de^2x^2-15d^2ex-32d^3\right)\sqrt{-e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) - 1/24*(6*e^3*x^3 + 8*d*e^2*x^2 - 15*d^2*e*x - 32*d^3)*sqr
t(-e^2*x^2 + d^2)
```

giac [A] time = 0.27, size = 99, normalized size = 0.88

$$\frac{3}{8}d^4\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d) - d^4\log\left(\frac{\left|-2\,de - 2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{1}{24}\left(32\,d^3 + \left(15\,d^2e - 2\left(3\,xe^3 + 4\,de^2\right)x\right)x\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 3/8*d^4*arcsin(x*e/d)*sgn(d) - d^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d
^2)*e)*e^(-2)/abs(x)) + 1/24*(32*d^3 + (15*d^2*e - 2*(3*x*e^3 + 4*d*e^2)*x)
*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 151, normalized size = 1.34

$$-\frac{d^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} + \frac{3d^4e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}} + \frac{3\sqrt{-e^2x^2+d^2}\,d^2ex}{8} + \sqrt{-e^2x^2+d^2}\,d^3 + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x)
```

```
[Out] 1/4*e*x*(-e^2*x^2+d^2)^(3/2)+3/8*e*d^2*x*(-e^2*x^2+d^2)^(1/2)+3/8*e*d^4/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/3*d*(-e^2*x^2+d^2)^(3
/2)+d^3*(-e^2*x^2+d^2)^(1/2)-d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 124, normalized size = 1.10

$$\frac{3}{8}d^4\arcsin\left(\frac{ex}{d}\right) - d^4\log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right) + \frac{3}{8}\sqrt{-e^2x^2+d^2}\,d^2ex + \sqrt{-e^2x^2+d^2}\,d^3 + \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*d^4*arcsin(e*x/d) - d^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs
(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(-e^2*x^2 + d^2)*d^3 + 1/4*(-
e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2)^(3/2)*d
```

mupad [B] time = 2.90, size = 107, normalized size = 0.95

$$\frac{d\left(d^2-e^2\,x^2\right)^{3/2}}{3} - d^4\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right) + d^3\sqrt{d^2-e^2\,x^2} + \frac{e\,x\left(d^2-e^2\,x^2\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x,x)
```

```
[Out] (d*(d^2 - e^2*x^2)^(3/2))/3 - d^4*atanh((d^2 - e^2*x^2)^(1/2)/d) + d^3*(d^2
 - e^2*x^2)^(1/2) + (e*x*(d^2 - e^2*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2,
(e^2*x^2)/d^2))/(1 - (e^2*x^2)/d^2)^(3/2)
```

sympy [C] time = 22.41, size = 469, normalized size = 4.15

$$d^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{fo}\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{o}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x,x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d
**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) - e**3*Piecewise((-I*d**4*acosh(e*x/d
)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
 e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.8 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=117

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*d*e*(-3*e*x+2*d)*(-e^2*x^2
+d^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {813, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(3
/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{1}{2}\int \frac{\left(-2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\frac{\int \frac{4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\frac{1}{2}\left(d^4e\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e}}\right.$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-$$

Mathematica [C] time = 0.17, size = 124, normalized size = 1.06

$$-\frac{d^5\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{d^2-e^2x^2}}-\frac{1}{3}e\left(\sqrt{d^2-e^2x^2}\left(e^2x^2-4d^2\right)+3d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] -1/3*(e*(Sqrt[d^2 - e^2*x^2]*(-4*d^2 + e^2*x^2) + 3*d^3*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])) - (d^5*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[-3/2, -1/2,
1/2, (e^2*x^2)/d^2])/(x*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 0.66, size = 124, normalized size = 1.06

$$\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+8\,d^3ex-\left(2\,e^3x^3+3\,de^2x^2-8\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) + 8*d^3*e*x - (2*e^3*x^3 + 3*d*e^2*x^2 - 8*d^
2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

giac [A] time = 0.25, size = 157, normalized size = 1.34

$$-\frac{3}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}(d)-d^3e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^3xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^{(-1)}}{2\,x}+\frac{1}{6}\sqrt{-x^2e^2+d^2}\left(8\,d^2e-\left(2\,xe^3+3\,de^2\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e*sgn(d) - d^3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^
2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^3*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) -
 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^(-1)/x + 1/6*sqrt(-x^2*e^2 + d^2)
*(8*d^2*e - (2*x*e^3 + 3*d*e^2)*x)
```

maple [A] time = 0.03, size = 182, normalized size = 1.56

$$-\frac{d^4e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{3d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{3\sqrt{-e^2x^2+d^2}\,d\,e^2x}{2}+\sqrt{-e^2x^2+d^2}\,d^2e-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2x}{d}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x)
```

```
[Out] -1/d/x*(-e^2*x^2+d^2)^(5/2)-e^2/d*x*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2*x*(-e^2*
x^2+d^2)^(1/2)-3/2*e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1
/2)*x)+1/3*e*(-e^2*x^2+d^2)^(3/2)+e*d^2*(-e^2*x^2+d^2)^(1/2)-e*d^4/(d^2)^(1
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 129, normalized size = 1.10

$$-\frac{3}{2}\,d^3e\arcsin\left(\frac{ex}{d}\right)-d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,de^2x+\sqrt{-e^2x^2+d^2}\,d^2e+\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*d^3*e*arcsin(e*x/d) - d^3*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e +
 1/3*(-e^2*x^2 + d^2)^(3/2)*e - (-e^2*x^2 + d^2)^(3/2)*d/x
```

mupad [B] time = 3.51, size = 114, normalized size = 0.97

$$\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3}+d^2e\sqrt{d^2-e^2x^2}-d^3e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{d^3\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^2,x)
```

```
[Out] (e*(d^2 - e^2*x^2)^(3/2))/3 + d^2*e*(d^2 - e^2*x^2)^(1/2) - d^3*e*atanh((d^
2 - e^2*x^2)^(1/2)/d) - (d^3*(d^2 - e^2*x^2)^(1/2)*hypergeom([-3/2, -1/2],
1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(1/2))
```

sympy [C] time = 8.18, size = 386, normalized size = 3.30

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\,\mathrm{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\,\mathrm{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\mathrm{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{fo}\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\mathrm{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{ot}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**2,x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
- I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 -
e**2*x**2/d**2)/2, True)) - e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0))
, (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

3.9 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=121

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2-3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2
)^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)-3/2*d*e*(e*x+d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {813, 844, 217, 203, 266, 63, 208}

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(2*x^2) - (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^
2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(-4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2-8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e}}\right.$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) +$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) +$$

Mathematica [C] time = 0.08, size = 110, normalized size = 0.91

$$-\frac{d^2e\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}\;{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] -((d^2*e*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (e^2*x^2)/d
^2])/(x*Sqrt[1 - (e^2*x^2)/d^2])) - (e^2*(d^2 - e^2*x^2)^(5/2)*Hypergeometr
ic2F1[2, 5/2, 7/2, 1 - (e^2*x^2)/d^2])/(5*d^3)
```

fricas [A] time = 0.86, size = 133, normalized size = 1.10

$$\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - 2\,d^2e^2x^2 - \left(e^3x^3+2\,de^2x^2+2\,d^2ex+d^3\right)\sqrt{-e^2x^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 3*d^2*e^2*x^
2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d^2*e^2*x^2 - (e^3*x^3 + 2*d*e^2*x
^2 + 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [B] time = 0.28, size = 217, normalized size = 1.79

$$-\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}(d)+\frac{3}{2}\,d^2e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{1}{8}\left(\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^8}{x}+\frac{(de$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/2*d^2*arcsin(x*e/d)*e^2*sgn(d) + 3/2*d^2*e^2*log(1/2*abs(-2*d*e - 2*sqrt
(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d^2*e^8/x + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^6/x^2)*e^(-8) - 1/2*sqrt
(-x^2*e^2 + d^2)*(x*e^3 + 2*d*e^2) + 1/8*(d^2*e^6 + 4*(d*e + sqrt(-x^2*e^2
+ d^2)*e)*d^2*e^4/x)*x^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
```

maple [B] time = 0.02, size = 212, normalized size = 1.75

$$\frac{3d^3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{3d^2e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{3\sqrt{-e^2x^2+d^2}\,e^3x}{2}-\frac{3\sqrt{-e^2x^2+d^2}\,d\,e^2}{2}-\frac{\left(-e^2x^2+d^2\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x)
```

```
[Out] -e/d^2/x*(-e^2*x^2+d^2)^(5/2)-e^3/d^2*x*(-e^2*x^2+d^2)^(3/2)-3/2*e^3*x*(-e^
2*x^2+d^2)^(1/2)-3/2*e^3*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^
(1/2)*x)-1/2/d/x^2*(-e^2*x^2+d^2)^(5/2)-1/2*e^2/d*(-e^2*x^2+d^2)^(3/2)-3/2*
d*e^2*(-e^2*x^2+d^2)^(1/2)+3/2*e^2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 160, normalized size = 1.32

$$-\frac{3}{2}\,d^2e^2\arcsin\left(\frac{ex}{d}\right)+\frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,e^3x-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,de^2-\frac{\left(-e^2x^2+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*d^2*e^2*arcsin(e*x/d) + 3/2*d^2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
 + d^2)*d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*e^3*x - 3/2*sqrt(-e^2*x^2 + d^
2)*d*e^2 - 1/2*(-e^2*x^2 + d^2)^(3/2)*e^2/d - (-e^2*x^2 + d^2)^(3/2)*e/x -
1/2*(-e^2*x^2 + d^2)^(5/2)/(d*x^2)
```

mupad [B] time = 3.74, size = 120, normalized size = 0.99

$$\frac{3\,d^2\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2}-\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{2\,x^2}-d\,e^2\,\sqrt{d^2-e^2\,x^2}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,{\left(1-\frac{e^2\,x^2}{d^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^3,x)
```

```
[Out] (3*d^2*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/2 - (d^3*(d^2 - e^2*x^2)^(1/2))/
(2*x^2) - d*e^2*(d^2 - e^2*x^2)^(1/2) - (e*(d^2 - e^2*x^2)^(3/2)*hypergeom(
[-3/2, -1/2], 1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(3/2))
```

sympy [C] time = 9.53, size = 461, normalized size = 3.81

$$d^3\left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) + d^2e\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{fo} \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{ot} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) + d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d
/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
- d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
> 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*
x/sqrt(-d**2/(e**2*x**2) + 1), True)) - e**3*Piecewise((-I*d**2*acosh(e*x/d
)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*s
qrt(1 - e**2*x**2/d**2)/2, True))
```

3.10 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/6*(3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))+3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(-3*e*x+2*d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (e^2*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d + 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = -\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{\int \frac{\left(4d^3e^2+6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{\int \frac{-12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{1}{4}\left(3d^2e^3\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-}}\right.$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) +$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) +$$

Mathematica [C] time = 0.06, size = 111, normalized size = 0.92

$$-\frac{e^3\left(d^2-e^2x^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)}{5d^4} - \frac{d^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] -1/3*(d^3*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (e^2*x^2)
/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (e^3*(d^2 - e^2*x^2)^(5/2)*Hypergeom
etric2F1[2, 5/2, 7/2, 1 - (e^2*x^2)/d^2])/(5*d^4)
```

fricas [A] time = 0.65, size = 129, normalized size = 1.08

$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+6\,de^3x^3+\left(6\,e^3x^3-8\,de^2x^2+3\,d^2ex+2\,d^3\right)\sqrt{}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 6*d*e^3*x^3 + (6*e^3*x^3 - 8*d*e^2*x^2
 + 3*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

giac [B] time = 0.23, size = 261, normalized size = 2.18

$$d\arcsin\left(\frac{xe}{d}\right)e^3\mathrm{sgn}(d)+\frac{3}{2}\,de^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(de^8+\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^6}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d}\right.}{x^2}\right.}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^3*sgn(d) + 3/2*d*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d*e^8 + 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d*e^6/x - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^4/x^2)*x^3*e/(d*e + sqrt(
-x^2*e^2 + d^2)*e)^3 + 1/24*(15*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^16/x - 3
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^14/x^2 - (d*e + sqrt(-x^2*e^2 + d^2)*
e)^3*d*e^12/x^3)*e^(-15) - sqrt(-x^2*e^2 + d^2)*e^3
```

maple [B] time = 0.02, size = 235, normalized size = 1.96

$$\frac{3d^2e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}+\frac{d\,e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{-e^2x^2+d^2}\,e^4x}{d}-\frac{3\sqrt{-e^2x^2+d^2}\,e^3}{2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x)
```

```
[Out] -1/3/d/x^3*(-e^2*x^2+d^2)^(5/2)+2/3*e^2/d^3/x*(-e^2*x^2+d^2)^(5/2)+2/3*e^4/
d^3*x*(-e^2*x^2+d^2)^(3/2)+e^4/d*x*(-e^2*x^2+d^2)^(1/2)+d*e^4/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2*e/d^2/x^2*(-e^2*x^2+d^2)^(5/2
)-1/2*e^3/d^2*(-e^2*x^2+d^2)^(3/2)-3/2*e^3*(-e^2*x^2+d^2)^(1/2)+3/2*e^3*d^2
/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.98, size = 184, normalized size = 1.53

$$de^3\arcsin\left(\frac{ex}{d}\right)+\frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{\sqrt{-e^2x^2+d^2}\,e^4x}{d}-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,e^3-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{2\,d^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] d*e^3*arcsin(e*x/d) + 3/2*d*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^4*x/d - 3/2*sqrt(-e^2*x^2 + d^2)*e^3 - 1/
2*(-e^2*x^2 + d^2)^(3/2)*e^3/d^2 + 2/3*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x) - 1
/2*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^2) - 1/3*(-e^2*x^2 + d^2)^(5/2)/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4, x)
```

sympy [C] time = 8.93, size = 457, normalized size = 3.81

$$d^{3}\left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 x^{2}} + \frac{e^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 d^{2}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 x^{2}} + \frac{i e^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 d^{2}} & \text{otherwise} \end{cases}\right) + d^{2} e \left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**4,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 d**2*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d)
, True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(
e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
(-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*
x**2/d**2)), True)) - e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
 - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
 > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e
*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.11 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=118

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*(4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4+e^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(8*e*x+3*d)*(-e^2*x^2+
d^2)^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {811, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (e^2*(3*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d + 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) + e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx &= -\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}-\frac{\int \frac{\left(6d^3e^2+8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+\frac{\int \frac{12d^5e^4+32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+\frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+\frac{1}{16}\left(3de^4\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2}}\right.\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{8}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 133, normalized size = 1.13

$$-\frac{\sqrt{d^2-e^2x^2}\left(3d^2\left(2d^2-5e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}+9e^4x^4\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+8d^3ex\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)\right)}{24dx^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(3*d^2*(2*d^2 - 5*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^
2] + 9*e^4*x^4*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]] + 8*d^3*e*x*Hypergeometric2
F1[-3/2, -3/2, -1/2, (e^2*x^2)/d^2]))/(d*x^4*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.92, size = 119, normalized size = 1.01

$$-\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3+15\,de^2x^2-8\,d^2ex-6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*e^4*x^4*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 15*d*e^2*x^2 - 8*d^2*e*x -
6*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

giac [B] time = 0.23, size = 297, normalized size = 2.52

$$\arcsin\left(\frac{xe}{d}\right)e^4\operatorname{sgn}(d)+\frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^8}{x}-\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^6}{x^2}-\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^4}{x^3}+3\,e^{10}\right)e^2}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}+\frac{1}{192}\left(120\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^4*sgn(d) + 1/192*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/
x - 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^6/x^2 - 120*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*e^4/x^3 + 3*e^10)*e^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 + 1/192*
(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^26/x + 24*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^2*e^24/x^2 - 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^22/x^3 - 3*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^4*e^20/x^4)*e^(-24) - 3/8*e^4*log(1/2*abs(-2*d*e - 2*s
qrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))
```

maple [B] time = 0.02, size = 260, normalized size = 2.20

$$-\frac{3d\,e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^2}+\frac{3\sqrt{-e^2x^2+d^2}\,e^4}{8d}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3d^4}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x)
```

```
[Out] -1/4/d/x^4*(-e^2*x^2+d^2)^(5/2)+1/8*e^2/d^3/x^2*(-e^2*x^2+d^2)^(5/2)+1/8*e^
4/d^3*(-e^2*x^2+d^2)^(3/2)+3/8*e^4/d*(-e^2*x^2+d^2)^(1/2)-3/8*d*e^4/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3*e/d^2/x^3*(-e^2*x
^2+d^2)^(5/2)+2/3*e^3/d^4/x*(-e^2*x^2+d^2)^(5/2)+2/3*e^5/d^4*x*(-e^2*x^2+d^
2)^(3/2)+e^5/d^2*x*(-e^2*x^2+d^2)^(1/2)+e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/
(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [B] time = 0.98, size = 210, normalized size = 1.78

$$e^4\arcsin\left(\frac{ex}{d}\right)-\frac{3}{8}e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^2}+\frac{3\sqrt{-e^2x^2+d^2}\,e^4}{8\,d}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{8\,d^3}+\frac{2\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] e^4*arcsin(e*x/d) - 3/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs
(x)) + sqrt(-e^2*x^2 + d^2)*e^5*x/d^2 + 3/8*sqrt(-e^2*x^2 + d^2)*e^4/d + 1/
8*(-e^2*x^2 + d^2)^(3/2)*e^4/d^3 + 2/3*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x) +
 1/8*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^2) - 1/3*(-e^2*x^2 + d^2)^(5/2)*e/(d
^2*x^3) - 1/4*(-e^2*x^2 + d^2)^(5/2)/(d*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5, x)
```

sympy [C] time = 11.06, size = 541, normalized size = 4.58

$$d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)+d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) -
1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d
**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asi
n(d/(e*x))/(2*d), True)) - e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)
) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.12 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6} dx$

Optimal. Leaf size=108

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}+\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

```
[Out] -1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-3/8*e^5*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d+3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 266, 47, 63, 208}

$$\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) - (
d^2 - e^2*x^2)^(5/2)/(5*d*x^5) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*
d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{16}\left(3e^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2 \ldots}}\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\ldots\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 133, normalized size = 1.23

$$-\frac{8d^6+10d^5ex-24d^4e^2x^2-35d^3e^3x^3+24d^2e^4x^4+15de^5x^5\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+25de^5x^5-8e^6x^6}{40dx^5\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] -1/40*(8*d^6 + 10*d^5*e*x - 24*d^4*e^2*x^2 - 35*d^3*e^3*x^3 + 24*d^2*e^4*x^
4 + 25*d*e^5*x^5 - 8*e^6*x^6 + 15*d*e^5*x^5*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh
[Sqrt[1 - (e^2*x^2)/d^2]])/(d*x^5*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 1.05, size = 98, normalized size = 0.91

$$\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(8\,e^4x^4-25\,de^3x^3-16\,d^2e^2x^2+10\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 - 25*d*e^3
*x^3 - 16*d^2*e^2*x^2 + 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

giac [B] time = 0.25, size = 368, normalized size = 3.41

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{10}}{x}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^8}{x^2}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^6}{x^3}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^4}{x^4}+2\,e^{12}\right)e^3}{320\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d}-\frac{3\,e^5\log\left(\frac{|-2d\ldots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/320*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x - 10*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^8/x^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^6/x^3 + 20*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + 2*e^12)*e^3/((d*e + sqrt(-x^2*e^2
+ d^2)*e)^5*d) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(
-2)/abs(x))/d - 1/320*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^38/x - 40*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^36/x^2 - 10*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^3*d^4*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^32/x^4 + 2*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*e^30/x^5)*e^(-35)/d^5
```

maple [A] time = 0.03, size = 158, normalized size = 1.46

$$-\frac{3e^5 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{3\sqrt{-e^2x^2+d^2}\,e^5}{8d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{8d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{4d^2x^4}-\frac{\left(-e^2x\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-1/4*e/d^2/x^4*(-e^2*x^2+d^2)^(5/2)+1/8*e^3/
d^4/x^2*(-e^2*x^2+d^2)^(5/2)+1/8*e^5/d^4*(-e^2*x^2+d^2)^(3/2)+3/8*e^5/d^2*(
-e^2*x^2+d^2)^(1/2)-3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 155, normalized size = 1.44

$$-\frac{3\,e^5 \log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d}+\frac{3\,\sqrt{-e^2x^2+d^2}\,e^5}{8\,d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8\,d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{8\,d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{4\,d^2x^4}-\frac{\left(-e^2x\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d + 3/8*sqrt(-
e^2*x^2 + d^2)*e^5/d^2 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/d^4 + 1/8*(-e^2*x^2
 + d^2)^(5/2)*e^3/(d^4*x^2) - 1/4*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^4) - 1/5*
(-e^2*x^2 + d^2)^(5/2)/(d*x^5)
```

mupad [B] time = 4.26, size = 93, normalized size = 0.86

$$\frac{3\,d^2\,e\,\sqrt{d^2-e^2\,x^2}}{8\,x^4}-\frac{\left(d^2-e^2\,x^2\right)^{5/2}}{5\,d\,x^5}-\frac{3\,e^5\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{8\,d}-\frac{5\,e\,\left(d^2-e^2\,x^2\right)^{3/2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^6,x)
```

```
[Out] (3*d^2*e*(d^2 - e^2*x^2)^(1/2))/(8*x^4) - (d^2 - e^2*x^2)^(5/2)/(5*d*x^5) -
 (3*e^5*atanh((d^2 - e^2*x^2)^(1/2)/d))/(8*d) - (5*e*(d^2 - e^2*x^2)^(3/2))
/(8*x^4)
```

sympy [C] time = 11.36, size = 774, normalized size = 7.17

$$d^3\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)+d^2e\left(\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+ \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**6,x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - e**3*Piecewise((-d**2/(2*e*x**
3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*a
cosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**
2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

3.13 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=143

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

```
[Out] -1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/5*e*(
-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/
16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{\int \frac{\left(-6d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)}{12d}\\
&= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,\right.}{16d}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^6\,\text{Subst}\left(\int\right.}{}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\text{Subst}\left(\int\right.}{}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^6\tanh^{-1}\left(\right.}{16d}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.41

$$-\frac{e\left(d^2-e^2x^2\right)^{5/2}\left(d^5+e^5x^5\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{5d^7x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] -1/5*(e*(d^2 - e^2*x^2)^(5/2)*(d^5 + e^5*x^5*Hypergeometric2F1[5/2, 4, 7/2,
 1 - (e^2*x^2)/d^2]))/(d^7*x^5)
```

fricas [A] time = 0.92, size = 109, normalized size = 0.76

$$\frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(48\,e^5x^5+15\,de^4x^4-96\,d^2e^3x^3-70\,d^3e^2x^2+48\,d^4ex+40\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x^5 + 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 - 70*d^3*e^2*x^2 + 48*d^4*e*x + 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

giac [B] time = 0.28, size = 431, normalized size = 3.01

$$\frac{x^6\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{12}}{x} - \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2 e^{10}}{x^2} - \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3 e^8}{x^3} - \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4 e^6}{x^4} + \frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5 \cdots}{x^5}\cdots\right.}{1920\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*x^6*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^12/x - 15*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2*e^10/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^8/x^3 - 15
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^6/x^4 + 120*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^5*e^4/x^5 + 5*e^14)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2) - 1/16*e
^6*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/19
20*(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^10*e^52/x - 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d^10*e^50/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^10*e^48
/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^10*e^46/x^4 + 12*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*d^10*e^44/x^5 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^10
*e^42/x^6)*e^(-48)/d^12
```

maple [A] time = 0.03, size = 186, normalized size = 1.30

$$-\frac{e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}\,d}+\frac{\sqrt{-e^2x^2+d^2}\,e^6}{16d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}{48d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{48d^5x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{24d^3x^4}-\frac{\left(-e^2x^2\cdots\right.}{5d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x)
```

```
[Out] -1/5*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/24*e^2
/d^3/x^4*(-e^2*x^2+d^2)^(5/2)+1/48*e^4/d^5/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^
6/d^5*(-e^2*x^2+d^2)^(3/2)+1/16*e^6/d^3*(-e^2*x^2+d^2)^(1/2)-1/16*e^6/d/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 180, normalized size = 1.26

$$-\frac{e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{16\,d^2}+\frac{\sqrt{-e^2x^2+d^2}\,e^6}{16\,d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}{48\,d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{48\,d^5x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{24\,d^3x^4}-\frac{\left(-e^2x^2\cdots\right.}{5\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^3 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^6/d^5 + 1/48*(-e
^2*x^2 + d^2)^(5/2)*e^4/(d^5*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^
4) - 1/5*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2)/(d
*x^6)
```

mupad [B] time = 4.66, size = 118, normalized size = 0.83

$$\frac{d^3\sqrt{d^2-e^2x^2}}{16x^6}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{6x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{16\,d\,x^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5\,d^2x^5}+\frac{e^6\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2x^2}\,1\mathrm{i}}{d}\right)1\mathrm{i}}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^7,x)
```

```
[Out] (d^3*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (d*(d^2 - e^2*x^2)^(3/2))/(6*x^6) -
(d^2 - e^2*x^2)^(5/2)/(16*d*x^6) + (e^6*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*
1i)/(16*d^2) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5)
```

sympy [C] time = 15.25, size = 918, normalized size = 6.42

$$d^3\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\,\mathrm{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\,\mathrm{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{3id^3\sqrt{-1\cdots}}{-15d^2x^5+\cdots}\\ \frac{3d^3\sqrt{1-\cdots}}{-15d^2x^5+\cdots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) - e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

3.14 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=172

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}+\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] -1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-1/6*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5-1/16*e^
7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d^
2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) - (e*(d^2 - e^2*x^2)^(5/2))/(6*d^2
*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) - (e^7*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/(16*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(-7d^2e-2de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} + \frac{\int \frac{\left(12d^3e^2+7d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^3\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^3\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,\right.}{12d^2}$$

$$= -\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^5\,\text{Sul}}{}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^{2}\right.}{35d^3x}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^{2}\right.}{35d^3x}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^{2}\right.}{35d^3x}$$

Mathematica [C] time = 0.02, size = 72, normalized size = 0.42

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}\left(5d^7+2d^5e^2x^2+7e^7x^7\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{35d^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] -1/35*((d^2 - e^2*x^2)^(5/2)*(5*d^7 + 2*d^5*e^2*x^2 + 7*e^7*x^7*Hypergeomet
ric2F1[5/2, 4, 7/2, 1 - (e^2*x^2)/d^2]))/(d^8*x^7)
```

fricas [A] time = 0.97, size = 120, normalized size = 0.70

$$\frac{105\,e^{7}x^{7}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(96\,e^{6}x^{6}+105\,de^{5}x^{5}+48\,d^{2}e^{4}x^{4}-490\,d^{3}e^{3}x^{3}-384\,d^{4}e^{2}x^{2}+280\,d^{5}ex+240\,d^{\cdots}\right.}{1680\,d^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (96*e^6*x^6 + 105*
d*e^5*x^5 + 48*d^2*e^4*x^4 - 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 + 280*d^5*e*
x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

giac [B] time = 0.26, size = 494, normalized size = 2.87

$$\frac{x^{7}\left(\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{14}}{x}-\frac{21\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{12}}{x^{2}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{10}}{x^{3}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{8}}{x^{4}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\cdots\right.}{x^{5}}\cdots\right.}{13440\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{7}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/13440*x^7*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^14/x - 21*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2*e^12/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^10/x^3 -
 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^8/x^4 - 105*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^5*e^6/x^5 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^4/x^6 + 15*e^16)
*e^5/((d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^3) - 1/16*e^7*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/13440*(315*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^18*e^68/x - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^18*e^
66/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^64/x^3 - 105*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^4*d^18*e^62/x^4 - 21*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5
*d^18*e^60/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^18*e^58/x^6 + 15*(d*
e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^56/x^7)*e^(-63)/d^21
```

maple [A] time = 0.04, size = 211, normalized size = 1.23

$$-\frac{e^{7}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}\,d^{2}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{16d^{4}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}{48d^{6}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{5}}{48d^{6}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}{24d^{4}x^{4}}-\frac{2\left(-e^{2}x\cdots\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x)
```

```
[Out] -1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5-1/6*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-1/24*e^3/d^4/x^4*(-e^2*x^2+d^2)^(5/2)+1/48*e^
5/d^6/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^7/d^6*(-e^2*x^2+d^2)^(3/2)+1/16*e^7/d
^4*(-e^2*x^2+d^2)^(1/2)-1/16*e^7/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 205, normalized size = 1.19

$$-\frac{e^{7}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{3}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{16\,d^{4}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}{48\,d^{6}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{5}}{48\,d^{6}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}{24\,d^{4}x^{4}}-\frac{2\left(-e^{2}x^{2}\cdots\right.}{35\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^7/d^4 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^7/d^6 + 1/48*(-e
^2*x^2 + d^2)^(5/2)*e^5/(d^6*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^
4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2)
*e/(d^2*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)/(d*x^7)
```

mupad [B] time = 5.33, size = 192, normalized size = 1.12

$$\frac{8\,d\,e^2\,\sqrt{d^2-e^2\,x^2}}{35\,x^5}-\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{7\,x^7}-\frac{e^4\,\sqrt{d^2-e^2\,x^2}}{35\,d\,x^3}-\frac{2\,e^6\,\sqrt{d^2-e^2\,x^2}}{35\,d^3\,x}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{6\,x^6}+\frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{16\,x^6}-\frac{e\,\left(d^2-\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^8,x)
```

```
[Out] (e^7*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*1i)/(16*d^3) - (d^3*(d^2 - e^2*x^2)
^(1/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(3/2))/(6*x^6) - (e^4*(d^2 - e^2*x^2)^
(1/2))/(35*d*x^3) - (2*e^6*(d^2 - e^2*x^2)^(1/2))/(35*d^3*x) + (8*d*e^2*(d^
2 - e^2*x^2)^(1/2))/(35*x^5) + (d^2*e*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (e*
(d^2 - e^2*x^2)^(5/2))/(16*d^2*x^6)
```

sympy [C] time = 16.62, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**8,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

3.15 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}$$

```
[Out] -1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/7*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-2/35*e^
3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^
4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A] time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8 \text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) - (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) - (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-8d^2e-3de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{\left(21d^3e^2+16d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{\left(-96d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{3/}}{x^6}}{336d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\int}{}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\,\mathrm{Sul}}{}$$

$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2\right.}{35}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2\right.}{16d^3x}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2\right.}{16d^3x}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2\right.}{16d^3x}$$

Mathematica [C] time = 0.02, size = 73, normalized size = 0.36

$$-\frac{e\left(d^2-e^2x^2\right)^{5/2}\left(5d^7+2d^5e^2x^2+7e^7x^7\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{35d^9x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] -1/35*(e*(d^2 - e^2*x^2)^(5/2)*(5*d^7 + 2*d^5*e^2*x^2 + 7*e^7*x^7*Hypergeom
etric2F1[5/2, 5, 7/2, 1 - (e^2*x^2)/d^2]))/(d^9*x^7)
```

fricas [A] time = 0.90, size = 131, normalized size = 0.65

$$\frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(256\,e^7x^7+105\,de^6x^6+128\,d^2e^5x^5+70\,d^3e^4x^4-1024\,d^4e^3x^3-840\,d^5e^2x^2+6\cdots\right.}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (256*e^7*x^7 + 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 + 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 - 840*d^5*
e^2*x^2 + 640*d^6*e*x + 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

giac [B] time = 0.26, size = 431, normalized size = 2.14

$$\frac{x^8\left(\frac{80\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{16}}{x}-\frac{112\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{12}}{x^3}-\frac{280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{10}}{x^4}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{8}}{x^5}+\frac{1680\left(de+\sqrt{-x^2e^2+\cdots}\right.}{x^7}\cdots\right.}{71680\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/71680*x^8*(80*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^16/x - 112*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^3*e^12/x^3 - 280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^10/x^4
- 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^8/x^5 + 1680*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^7*e^4/x^7 + 35*e^18)*e^6/((d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^4) -
 3/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^
4 - 1/71680*(1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^28*e^86/x - 560*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^3*d^28*e^82/x^3 - 280*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^4*d^28*e^80/x^4 - 112*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^28*e^78/x^5 + 80*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^28*e^74/x^7 + 35*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^8*d^28*e^72/x^8)*e^(-80)/d^32
```

maple [A] time = 0.07, size = 236, normalized size = 1.17

$$-\frac{3e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{128\sqrt{d^2}\,d^3}+\frac{3\sqrt{-e^2x^2+d^2}\,e^8}{128d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8}{128d^7}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{128d^7x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{64d^5x^4}-\frac{2\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-1/64*
e^4/d^5/x^4*(-e^2*x^2+d^2)^(5/2)+1/128*e^6/d^7/x^2*(-e^2*x^2+d^2)^(5/2)+1/1
28*e^8/d^7*(-e^2*x^2+d^2)^(3/2)+3/128*e^8/d^5*(-e^2*x^2+d^2)^(1/2)-3/128*e^
8/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/7*e*(-
e^2*x^2+d^2)^(5/2)/d^2/x^7-2/35*e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5
```

maxima [A] time = 0.99, size = 230, normalized size = 1.14

$$-\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{128\,d^4}+\frac{3\,\sqrt{-e^2x^2+d^2}\,e^8}{128\,d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8}{128\,d^7}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{128\,d^7x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{64\,d^5x^4}-\frac{2\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 1/128*(-e^2*x^2 + d^2)^(3/2)*e^8/d^7 + 1/128
*(-e^2*x^2 + d^2)^(5/2)*e^6/(d^7*x^2) - 1/64*(-e^2*x^2 + d^2)^(5/2)*e^4/(d^
5*x^4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^5) - 1/16*(-e^2*x^2 + d^2)^
(5/2)*e^2/(d^3*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^7) - 1/8*(-e^2*x^
2 + d^2)^(5/2)/(d*x^8)
```

mupad [B] time = 6.04, size = 212, normalized size = 1.05

$$\frac{3\,d^3\,\sqrt{d^2-e^2\,x^2}}{128\,x^8}-\frac{11\,d\,\left(d^2-e^2\,x^2\right)^{3/2}}{128\,x^8}-\frac{11\,\left(d^2-e^2\,x^2\right)^{5/2}}{128\,d\,x^8}+\frac{3\,\left(d^2-e^2\,x^2\right)^{7/2}}{128\,d^3\,x^8}+\frac{8\,e^3\,\sqrt{d^2-e^2\,x^2}}{35\,x^5}-\frac{e^5\,\sqrt{d^2-e^2\,x^2}}{35\,d^2\,x^3}-\frac{2\,e^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^9,x)
```

```
[Out] (3*d^3*(d^2 - e^2*x^2)^(1/2))/(128*x^8) - (11*d*(d^2 - e^2*x^2)^(3/2))/(128
*x^8) - (11*(d^2 - e^2*x^2)^(5/2))/(128*d*x^8) + (3*(d^2 - e^2*x^2)^(7/2))/
(128*d^3*x^8) + (8*e^3*(d^2 - e^2*x^2)^(1/2))/(35*x^5) + (e^8*atan(((d^2 -
e^2*x^2)^(1/2)*1i)/d)*3i)/(128*d^4) - (e^5*(d^2 - e^2*x^2)^(1/2))/(35*d^2*x
^3) - (2*e^7*(d^2 - e^2*x^2)^(1/2))/(35*d^4*x) - (d^2*e*(d^2 - e^2*x^2)^(1/
2))/(7*x^7)
```

sympy [C] time = 22.55, size = 1159, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
```

```
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

3.16 $\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] 1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d
^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 195, 217, 203}

$$-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((d^2*Sqrt[d^2 - e^2*x^2])/e^3) - (d*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2
 - e^2*x^2)^(3/2)/(3*e^3) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$
$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{d\int \sqrt{d^2-e^2x^2}\,dx}{e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$
$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{1+e^2x}\right.}{e}$$
$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3} - \frac{d^3\,\text{Subst}\left(\int \frac{}{1-}\right.}{}$$
$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.68

$$\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(4d^2+3dex+2e^2x^2\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(4*d^2 + 3*d*e*x + 2*e^2*x^2)) + 3*d^3*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(6*e^3)
```

fricas [A] time = 0.86, size = 72, normalized size = 0.70

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(2\,e^2x^2+3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.24, size = 54, normalized size = 0.52

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d) - \frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(4\,d^2e^{(-3)} + \left(2\,xe^{(-1)} + 3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) + 3*d*e^(-2))*x)
```

maple [A] time = 0.02, size = 102, normalized size = 0.99

$$\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2} - \frac{\sqrt{-e^2x^2+d^2}\,x^2}{3e} - \frac{\sqrt{-e^2x^2+d^2}\,dx}{2e^2} - \frac{2\sqrt{-e^2x^2+d^2}\,d^2}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2/e*(-e^2*x^2+d^2)^(1/2)-2/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-
e^2*x^2+d^2)^(1/2)/e^2+1/2*d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 81, normalized size = 0.79

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3\,e}+\frac{d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,dx}{2\,e^2}-\frac{2\,\sqrt{-e^2x^2+d^2}\,d^2}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2/e + 1/2*d^3*arcsin(e*x/d)/e^3 - 1/2*sqrt(-e^2
*x^2 + d^2)*d*x/e^2 - 2/3*sqrt(-e^2*x^2 + d^2)*d^2/e^3
```

mupad [B] time = 3.14, size = 112, normalized size = 1.09

$$\begin{cases} \frac{d\,x^3}{3\,\sqrt{d^2}} & \text{if } e=0 \\ -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3}-\frac{d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}}-\frac{d\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (d*x^3)/(3*(d^2)^(1/2)), e ~= 0, - ((d^2 - e^2*x^2)^(1/2)
*(2*d^2 + e^2*x^2))/(3*e^3) - (d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)
^(1/2)))/(2*(-e^2)^(3/2)) - (d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

sympy [C] time = 5.25, size = 177, normalized size = 1.72

$$d\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} -\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for } e\neq 0 \\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2
)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e
**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
 + e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

3.17 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+d*(e*x+d)/e^3/(-e^2*x^2+d^2)^(1/2)+
(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 217, 203, 637}

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e^3 - (d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{e^2}$$

$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.05

$$\frac{-d\sqrt{d^2-e^2x^2}\,\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2+dex-e^2x^2}{e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
[Out] (2*d^2 + d*e*x - e^2*x^2 - d*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/(e^3*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 0.76, size = 87, normalized size = 1.19

$$\frac{2\,dex-2\,d^2+2\left(dex-d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}\,(ex-2\,d)}{e^4x-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] (2*d*e*x - 2*d^2 + 2*(d*e*x - d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(e^4*x - d*e^3)
```

giac [A] time = 0.25, size = 66, normalized size = 0.90

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\text{sgn}(d)-\frac{\sqrt{-x^2e^2+d^2}\left(2\,d^2e^{(-3)}-\left(xe^{(-1)}-de^{(-2)}\right)x\right)}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] -d*arcsin(x*e/d)*e^(-3)*sgn(d) - sqrt(-x^2*e^2 + d^2)*(2*d^2*e^(-3) - (x*e^
(-1) - d*e^(-2))*x)/(x^2*e^2 - d^2)
```

maple [A] time = 0.02, size = 99, normalized size = 1.36

$$-\frac{x^2}{\sqrt{-e^2x^2+d^2}\,e}+\frac{dx}{\sqrt{-e^2x^2+d^2}\,e^2}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^2}+\frac{2d^2}{\sqrt{-e^2x^2+d^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -x^2/e/(-e^2*x^2+d^2)^(1/2)+2*d^2/e^3/(-e^2*x^2+d^2)^(1/2)+d*x/e^2/(-e^2*x^
2+d^2)^(1/2)-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 78, normalized size = 1.07

$$-\frac{x^2}{\sqrt{-e^2x^2+d^2}\,e}+\frac{dx}{\sqrt{-e^2x^2+d^2}\,e^2}-\frac{d\arcsin\left(\frac{ex}{d}\right)}{e^3}+\frac{2\,d^2}{\sqrt{-e^2x^2+d^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(-e^2*x^2 + d^2)*e) + d*x/(sqrt(-e^2*x^2 + d^2)*e^2) - d*arcsin(e
*x/d)/e^3 + 2*d^2/(sqrt(-e^2*x^2 + d^2)*e^3)
```

mupad [B] time = 2.96, size = 87, normalized size = 1.19

$$\frac{2\,d^2-e^2\,x^2}{e^3\,\sqrt{d^2-e^2\,x^2}}+\frac{d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\left(-e^2\right)^{3/2}}+\frac{d\,x}{e^2\,\sqrt{d^2-e^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x)
```

```
[Out] (2*d^2 - e^2*x^2)/(e^3*(d^2 - e^2*x^2)^(1/2)) + (d*log(x*(-e^2)^(1/2) + (d^
2 - e^2*x^2)^(1/2)))/(-e^2)^(3/2) + (d*x)/(e^2*(d^2 - e^2*x^2)^(1/2))
```

sympy [C] time = 9.71, size = 184, normalized size = 2.52

$$d\left(\begin{cases}\frac{i\operatorname{acosh}\left(\frac{ex}{d}\right)}{e^3}-\frac{ix}{de^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{\operatorname{asin}\left(\frac{ex}{d}\right)}{e^3}+\frac{x}{de^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\tilde{\infty}x^4 & \text{for}\ \left(d=0\vee d=-\sqrt{e^2x^2}\vee d=\right.\\ \frac{x^4}{4\left(d^2\right)^{\frac{3}{2}}} & \text{for}\ e=0\\ \frac{2d^2}{e^4\sqrt{d^2-e^2x^2}}-\frac{x^2}{e^2\sqrt{d^2-e^2x^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d*Piecewise((I*acosh(e*x/d)/e**3 - I*x/(d*e**2*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (-asin(e*x/d)/e**3 + x/(d*e**2*sqrt(1 - e**2*x**2
/d**2)), True)) + e*Piecewise((zoo*x**4, (Eq(d, 0) | Eq(d, sqrt(e**2*x**2))
 | Eq(d, -sqrt(e**2*x**2))) & (Eq(e, 0) | Eq(d, sqrt(e**2*x**2)) | Eq(d, -s
qrt(e**2*x**2)))), (x**4/(4*(d**2)**(3/2)), Eq(e, 0)), (2*d**2/(e**4*sqrt(d
**2 - e**2*x**2)) - x**2/(e**2*sqrt(d**2 - e**2*x**2)), True))
```

3.18 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(3/2)-2/3/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {796, 12, 261}

$$\frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (x^2*(d + e*x))/(3*d*e*(d^2 - e^2*x^2)^(3/2)) - 2/(3*e^3*Sqrt[d^2 - e^2*x^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{2d^2ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}\\ &= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.90

$$\frac{-2d^2+2dex+e^2x^2}{3de^3(d-ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
[Out] (-2*d^2 + 2*d*e*x + e^2*x^2)/(3*d*e^3*(d - e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.83, size = 104, normalized size = 1.79

$$-\frac{2\,e^3x^3-2\,de^2x^2-2\,d^2ex+2\,d^3-\left(e^2x^2+2\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(de^6x^3-d^2e^5x^2-d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
[Out] -1/3*(2*e^3*x^3 - 2*d*e^2*x^2 - 2*d^2*e*x + 2*d^3 - (e^2*x^2 + 2*d*e*x - 2*
d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - d^2*e^5*x^2 - d^3*e^4*x + d^4*e^3)
```

giac [A] time = 0.30, size = 51, normalized size = 0.88

$$\frac{\left(x^2\left(\frac{x}{d}+3\,e^{(-1)}\right)-2\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{3\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] 1/3*(x^2*(x/d + 3*e^(-1)) - 2*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d
^2)^2
```

maple [A] time = 0.01, size = 55, normalized size = 0.95

$$-\frac{(-ex+d)\,(ex+d)^2\left(-e^2x^2-2dex+2d^2\right)}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
[Out] -1/3*(-e*x+d)*(e*x+d)^2*(-e^2*x^2-2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.44, size = 88, normalized size = 1.52

$$\frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}+\frac{dx}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{x}{3\sqrt{-e^2x^2+d^2}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] x^2/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

mupad [B] time = 2.59, size = 55, normalized size = 0.95

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^2+2\,d\,e\,x+e^2\,x^2\right)}{3\,d\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 2*d^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)*(d -
 e*x)^2)
```

sympy [C] time = 9.88, size = 231, normalized size = 3.98

$$d\left(\begin{cases}\frac{ix^3}{-3d^5\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{x^3}{-3d^5\sqrt{1-\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\frac{2d^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}}-\frac{3e^2x^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} \\ \frac{x^4}{4\left(d^2\right)^{\frac{5}{2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d*Piecewise((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2) + 3*d**3*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-x**3/(-3*d**5*sqrt(1
 - e**2*x**2/d**2) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)) + e
*Piecewise((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e**6*x**2*sqrt(
d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e
**6*x**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(5/2)), True)
)
```

3.19 $\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=161

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^6*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(7*e*x+6*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(35*e*x+2
4*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e^8
```

Rubi [A] time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {819, 780, 217, 203}

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^6*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d + 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d + 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d + 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) - (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3+7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5+35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7+105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \cdots$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \cdots$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - 7\cdots$$

Mathematica [A] time = 0.10, size = 155, normalized size = 0.96

$$\frac{96d^6+9d^5ex-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-105d^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-15de^5x\cdots}{30e^8(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (96*d^6 + 9*d^5*e*x - 249*d^4*e^2*x^2 + 4*d^3*e^3*x^3 + 176*d^2*e^4*x^4 - 1
5*d*e^5*x^5 - 15*e^6*x^6 - 105*d^2*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2
]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8*(d - e*x)^2*(d + e*x)*Sqrt[d^2
 - e^2*x^2])
```

fricas [A] time = 0.98, size = 278, normalized size = 1.73

$$\frac{96\,d^2e^5x^5-96\,d^3e^4x^4-192\,d^4e^3x^3+192\,d^5e^2x^2+96\,d^6ex-96\,d^7+210\left(d^2e^5x^5-d^3e^4x^4-2\,d^4e^3x^3+2\,d^5e^2x^2+\cdots\right.}{30\left(e^{13}x^5-de^{12}x^4-2\,d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 - 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 + 192*d^5*e^2*x^2 +
 96*d^6*e*x - 96*d^7 + 210*(d^2*e^5*x^5 - d^3*e^4*x^4 - 2*d^4*e^3*x^3 + 2*d
^5*e^2*x^2 + d^6*e*x - d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15
*e^6*x^6 + 15*d*e^5*x^5 - 176*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 - 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 - d*e^12*x^4 - 2*d^2
*e^11*x^3 + 2*d^3*e^10*x^2 + d^4*e^9*x - d^5*e^8)
```

giac [A] time = 0.28, size = 120, normalized size = 0.75

$$-\frac{7}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-8)}\mathrm{sgn}(d)-\frac{\left(96\,d^7e^{(-8)}+\left(105\,d^6e^{(-7)}-\left(240\,d^5e^{(-6)}+\left(245\,d^4e^{(-5)}-\left(180\,d^3e^{(-4)}+\left(161\,d^2e\right.\right.\right.\right.\right.\right.}{30\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -7/2*d^2*arcsin(x*e/d)*e^(-8)*sgn(d) - 1/30*(96*d^7*e^(-8) + (105*d^6*e^(-7
) - (240*d^5*e^(-6) + (245*d^4*e^(-5) - (180*d^3*e^(-4) + (161*d^2*e^(-3) -
 15*(x*e^(-1) + 2*d*e^(-2))*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2
 - d^2)^3
```

maple [A] time = 0.07, size = 227, normalized size = 1.41

$$-\frac{x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{d\,x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{7d^2x^5}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{6d^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{8d^5x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}-\frac{7d^2x^3}{6\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2*x^7/e/(-e^2*x^2+d^2)^(5/2)+7/10*d^2/e^3*x^5/(-e^2*x^2+d^2)^(5/2)-7/6*d
^2/e^5*x^3/(-e^2*x^2+d^2)^(3/2)+7/2*d^2/e^7*x/(-e^2*x^2+d^2)^(1/2)-7/2*d^2/
e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-d*x^6/e^2/(-e^2*
x^2+d^2)^(5/2)+6*d^3/e^4*x^4/(-e^2*x^2+d^2)^(5/2)-8*d^5/e^6*x^2/(-e^2*x^2+d
^2)^(5/2)+16/5*d^7/e^8/(-e^2*x^2+d^2)^(5/2)
```

maxima [B] time = 1.03, size = 312, normalized size = 1.94

$$-\frac{x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{7\,d^2x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)}{30\,e}-\frac{dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{7\,d^2x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\right.}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^7/((-e^2*x^2 + d^2)^(5/2)*e) + 7/30*d^2*x*(15*x^4/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d
^2)^(5/2)*e^6))/e - d*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/6*d^2*x*(3*x^2/(
(-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^3 + 6*d
^3*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) - 8*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6
) + 16/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e^8) + 14/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^7) - 49/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^7) - 7/2*d^2*arcsin(e*x/d)/
e^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,(d+e\,x)}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [B] time = 66.42, size = 2004, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 40*d**4*
e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*e**4*x**4/(
5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x*
*2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4*e**8*sqrt(
d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**
4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), True)) + e*P
iecewise((-210*I*d**7*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-60*d**5*e**9
*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2)
- 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 105*pi*d**7*sqrt(-1 + e**2*x
**2/d**2)/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sq
rt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 210*
I*d**6*e*x/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*s
qrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 420
*I*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-60*d**5*e**9*sqr
t(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60
*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 210*pi*d**5*e**2*x**2*sqrt(-1 +
e**2*x**2/d**2)/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x
**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2))
- 490*I*d**4*e**3*x**3/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*
e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/
d**2)) - 210*I*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-60*d
**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**
2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 105*pi*d**3*e**4*x**
4*sqrt(-1 + e**2*x**2/d**2)/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*
d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*
x**2/d**2)) + 322*I*d**2*e**5*x**5/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2)
 + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1
+ e**2*x**2/d**2)) - 30*I*e**7*x**7/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2
) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1
 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-105*d**7*sqrt(1 - e**2*x**
2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*
x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) +
 105*d**6*e*x/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*s
qrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 210*d
**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e
**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x
**4*sqrt(1 - e**2*x**2/d**2)) - 245*d**4*e**3*x**3/(30*d**5*e**9*sqrt(1 - e
**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x
**4*sqrt(1 - e**2*x**2/d**2)) - 105*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)
*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sq
rt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 161*d*
*2*e**5*x**5/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sq
rt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*e**
7*x**7/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 -
 e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.20 $\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(6*e*x+5*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(8*e*x+5*d)/e^6/(-
e^2*x^2+d^2)^(1/2)+16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {819, 641, 217, 203}

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^5*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d + 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d + 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) + (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4\left(5d^3+6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(15d^5+24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7+48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}}{e^6}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right.}{e^7}$$

Mathematica [A] time = 0.09, size = 142, normalized size = 0.97

$$\frac{48d^5-33d^4ex-87d^3e^2x^2+52d^2e^3x^3-15d(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+38de^4x^4-15e^5x^5}{15e^7(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (48*d^5 - 33*d^4*e*x - 87*d^3*e^2*x^2 + 52*d^2*e^3*x^3 + 38*d*e^4*x^4 - 15*
e^5*x^5 - 15*d*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(15*e^7*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.90, size = 263, normalized size = 1.79

$$\frac{48\,de^5x^5-48\,d^2e^4x^4-96\,d^3e^3x^3+96\,d^4e^2x^2+48\,d^5ex-48\,d^6+30\left(de^5x^5-d^2e^4x^4-2\,d^3e^3x^3+2\,d^4e^2x^2+d^5ex\right.}{15\left(e^{12}x^5-de^{11}x^4-2\,d^2e^{10}x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(48*d*e^5*x^5 - 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 + 96*d^4*e^2*x^2 + 48*
d^5*e*x - 48*d^6 + 30*(d*e^5*x^5 - d^2*e^4*x^4 - 2*d^3*e^3*x^3 + 2*d^4*e^2*
x^2 + d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x^
5 - 38*d*e^4*x^4 - 52*d^2*e^3*x^3 + 87*d^3*e^2*x^2 + 33*d^4*e*x - 48*d^5)*s
qrt(-e^2*x^2 + d^2))/(e^12*x^5 - d*e^11*x^4 - 2*d^2*e^10*x^3 + 2*d^3*e^9*x^
2 + d^4*e^8*x - d^5*e^7)
```

giac [A] time = 0.27, size = 109, normalized size = 0.74

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-7)}\text{sgn}(d)-\frac{\left(48\,d^6e^{(-7)}+\left(15\,d^5e^{(-6)}-\left(120\,d^4e^{(-5)}+\left(35\,d^3e^{(-4)}-\left(90\,d^2e^{(-3)}-\left(15\,xe^{(-1)}-23\,de^{(\right.\right.\right.\right.\right.\right.}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-7)*sgn(d) - 1/15*(48*d^6*e^(-7) + (15*d^5*e^(-6) - (12
0*d^4*e^(-5) + (35*d^3*e^(-4) - (90*d^2*e^(-3) - (15*x*e^(-1) - 23*d*e^(-2)
)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.02, size = 195, normalized size = 1.33

$$-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e}+\frac{d\,x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^2}+\frac{6d^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^3}-\frac{8d^4x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^5}-\frac{d\,x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^4}+\frac{16d^6}{5\left(-e^2x^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -x^6/e/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^3*x^4/(-e^2*x^2+d^2)^(5/2)-8*d^4/e^5*x^
2/(-e^2*x^2+d^2)^(5/2)+16/5*d^6/e^7/(-e^2*x^2+d^2)^(5/2)+1/5*d*x^5/e^2/(-e^
2*x^2+d^2)^(5/2)-1/3*d/e^4*x^3/(-e^2*x^2+d^2)^(3/2)+d/e^6*x/(-e^2*x^2+d^2)^
(1/2)-d/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [B] time = 1.03, size = 278, normalized size = 1.89

$$\frac{1}{15}\,dx\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{dx\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right.}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*d*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/((-e^2*x^2 + d^2)^
(5/2)*e) - 1/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 +
 d^2)^(3/2)*e^4))/e^2 + 6*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^3) - 8*d^4*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^5) + 16/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^7) + 4/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^6) - 7/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) -
 d*arcsin(e*x/d)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(d+e\,x\right)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [C] time = 62.08, size = 1821, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e*
*7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2)
- 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d**5*sqrt(-1 + e**2*x*
*2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(
-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*I*d*
*4*e*x/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1
 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 60*I*d**3
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7*sqrt(-1 +
e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*
x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/
d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1
+ e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 70*I*d**2*
e**3*x**3/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt
(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*I*d
*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7*sqrt(-1 +
e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*
x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**
2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 46*I*e**5*x**
5/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e*
*2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d
**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqr
t(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e
**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e*
*2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**
4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*as
in(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1
 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e*
*3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 -
 e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x*
*4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d*
*2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 -
 e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 3
0*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x
**2/d**2)), True)) + e*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**
2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e
**2*x**2)) - 40*d**4*e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**
2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2))
+ 30*d**2*e**4*x**4/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**
2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x*
*6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**
2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)*
*(7/2)), True))
```

3.21 $\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^4*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(5*e*x+4*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(15*e*x+8*d)/e^6/(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {819, 778, 217, 203}

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) - Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^5(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3+5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5+15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x}}\right.}{e^5}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] time = 0.08, size = 130, normalized size = 1.07

$$\frac{8d^4+7d^3ex-27d^2e^2x^2-15(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\,\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-8de^3x^3+23e^4x^4}{15e^6(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^3 + 23*e^4*x^4 - 15*(d - e*
x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e
^6*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.87, size = 247, normalized size = 2.02

$$\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5+30\left(e^5x^5-de^4x^4-2\,d^2e^3x^3+2\,d^3e^2x^2+d^4ex-d^5\right)\text{arc}}{15\left(e^{11}x^5-de^{10}x^4-2\,d^2e^9x^3+2\,d^3e^8x^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 + 30*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^4*e*
x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23*e^4*x^4 - 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 + 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 -
 d*e^10*x^4 - 2*d^2*e^9*x^3 + 2*d^3*e^8*x^2 + d^4*e^7*x - d^5*e^6)
```

giac [A] time = 0.31, size = 97, normalized size = 0.80

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\text{sgn}(d)-\frac{\left(8\,d^5e^{(-6)}+\left(15\,d^4e^{(-5)}-\left(20\,d^3e^{(-4)}+\left(35\,d^2e^{(-3)}-\left(23\,xe^{(-1)}+15\,de^{(-2)}\right)x\right)x\right)x\right)x\right)\sqrt{-x}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-6)*sgn(d) - 1/15*(8*d^5*e^(-6) + (15*d^4*e^(-5) - (20*d^
3*e^(-4) + (35*d^2*e^(-3) - (23*x*e^(-1) + 15*d*e^(-2))*x)*x)*x)*x)*sqrt(-x
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.02, size = 166, normalized size = 1.36

$$\frac{x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{4d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+\frac{8d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}+\frac{x}{\sqrt{-e^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*x^5/e/(-e^2*x^2+d^2)^(5/2)-1/3/e^3*x^3/(-e^2*x^2+d^2)^(3/2)+1/e^5*x/(-e
^2*x^2+d^2)^(1/2)-1/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)+d*x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4/3*d^3/e^4*x^2/(-e^2*x^2+d^2)^(5/2)+8/1
5*d^5/e^6/(-e^2*x^2+d^2)^(5/2)
```

maxima [B] time = 1.01, size = 250, normalized size = 2.05

$$\frac{1}{15}\,ex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{3\,e}+\frac{dx^4}{\left(-e^2x^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*x^
2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + d*x^4/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 4/3*d^3*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8/15*d^5
/((-e^2*x^2 + d^2)^(5/2)*e^6) + 4/15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 7
/15*x/(sqrt(-e^2*x^2 + d^2)*e^5) - arcsin(e*x/d)/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(d+e\,x\right)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [B] time = 73.14, size = 1739, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**
2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) - 20*d**2*
e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**
```

```
2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e**4*x**4/(15*d
**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2)
+ 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**(7/2))
, True)) + e*Piecewise((-30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(
-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*
x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d**5*sqrt(-
1 + e**2*x**2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9
*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)
) + 30*I*d**4*e*x/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x
**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2))
+ 60*I*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) -
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*pi*d**3*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x*
*2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) -
 70*I*d**2*e**3*x**3/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**
9*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2
)) - 30*I*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) -
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 46
*I*e**5*x**5/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*s
qrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2/d**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d*
*5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**
2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*
sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*
d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x*
*2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*
x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) -
 35*d**2*e**3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x*
*2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 1
5*d*e**4*x**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e
**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x*
*4*sqrt(1 - e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**
2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt
(1 - e**2*x**2/d**2)), True))
```

3.22 $\int \frac{x^4(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^4*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2)+
4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {805, 266, 43}

$$\frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2 - e^2*
x^2)^(3/2)) + 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{x^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)^{5/2}}\,dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \left(\frac{d^2}{e^2\left(d^2-e^2x\right)^{5/2}} - \frac{1}{e^2\left(d^2-e^2x\right)^{3/2}}\right)dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.98

$$\frac{8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4}{15de^5(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 + 3*e^4*x^4)/(15*d*e^5*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.92, size = 171, normalized size = 2.04

$$\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5-\left(3\,e^4x^4+12\,de^3x^3-12\,d^2e^2x^2-8\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2}}{15\left(de^{10}x^5-d^2e^9x^4-2\,d^3e^8x^3+2\,d^4e^7x^2+d^5e^6x-d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 - (3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 - 8*d^3*e*x + 8*d^4)*
sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 - d^2*e^9*x^4 - 2*d^3*e^8*x^3 + 2*d^4*e^7
*x^2 + d^5*e^6*x - d^6*e^5)
```

giac [A] time = 0.27, size = 64, normalized size = 0.76

$$-\frac{\left(8\,d^4e^{(-5)}+\left(3\,x^2\left(\frac{x}{d}+5\,e^{(-1)}\right)-20\,d^2e^{(-3)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(8*d^4*e^(-5) + (3*x^2*(x/d + 5*e^(-1)) - 20*d^2*e^(-3))*x^2)*sqrt(-x
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.92

$$\frac{(-ex+d)\,(ex+d)^2\left(3x^4e^4+12x^3d\,e^3-12d^2x^2e^2-8d^3xe+8d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(3*e^4*x^4+12*d*e^3*x^3-12*d^2*e^2*x^2-8*d^3*e*x+8*
d^4)/d/e^5/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.45, size = 159, normalized size = 1.89

$$\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{4\,d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{3\,d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}+\frac{dx}{10\left(-e^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4
/3*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^3*x/((-e^2*x^2 + d^2)^(5/2
)*e^4) + 8/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 1/10*d*x/((-e^2*x^2 + d^2)
^(3/2)*e^4) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

mupad [B] time = 2.70, size = 78, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4-8\,d^3\,e\,x-12\,d^2\,e^2\,x^2+12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 8*d^3*e*x))/(15*d*e^5*(d + e*x)^2*(d - e*x)^3)
```

sympy [C] time = 63.87, size = 418, normalized size = 4.98

$$d\left(\begin{cases}-\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\cdots} \\ \frac{x^6}{6\left(d^2\right)^{\frac{7}{2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d**5*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2
)), True)) + e*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*
d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2
)) - 20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*
x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e*
*4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2
- e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(
d**2)**(7/2)), True))
```

3.23 $\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=90

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(-3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(3/2)+1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{819, 778, 191\}$

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d + 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) + x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2d^3+3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$
$$= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}$$
$$= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.91

$$\frac{-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4}{15d^2e^4(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*e^4*x^4)/(15*d^2*e^4*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.73, size = 172, normalized size = 1.91

$$-\frac{2\,e^5x^5-2\,de^4x^4-4\,d^2e^3x^3+4\,d^3e^2x^2+2\,d^4ex-2\,d^5+\left(3\,e^4x^4-3\,de^3x^3+3\,d^2e^2x^2+2\,d^3ex-2\,d^4\right)\sqrt{-e^2x^2}}{15\left(d^2e^9x^5-d^3e^8x^4-2\,d^4e^7x^3+2\,d^5e^6x^2+d^6e^5x-d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 + (3*e^4*x^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 - d^3*e^8*x^4 - 2*d^4*e^7*x^3 + 2*d^5*e^6*x
^2 + d^6*e^5*x - d^7*e^4)
```

giac [A] time = 0.28, size = 58, normalized size = 0.64

$$\frac{\left(2\,d^3e^{(-4)}-\left(\frac{3\,x^3e}{d^2}+5\,de^{(-2)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*d^3*e^(-4) - (3*x^3*e/d^2 + 5*d*e^(-2))*x^2)*sqrt(-x^2*e^2 + d^2)/(
x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.86

$$-\frac{(-ex+d)\,(ex+d)^2\left(-3x^4e^4+3x^3d\,e^3-3d^2x^2e^2-2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(-3*e^4*x^4+3*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*
d^4)/d^2/e^4/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.45, size = 134, normalized size = 1.49

$$\frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{2\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+\frac{x}{5\sqrt{-e^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) + 1/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2)
 - 3/10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 2/15*d^3/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/10*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 1/5*x/(sqrt(-e^2*x^2 + d^2)
*d^2*e^3)
```

mupad [B] time = 2.66, size = 78, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2-3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^2*(d - e*x)^3)
```

sympy [B] time = 20.55, size = 337, normalized size = 3.74

$$d\left(\begin{cases}-\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}}+\frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0\\ \frac{x^4}{4\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x*
*2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2*x
**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e*
*2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)
**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) -
 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2
/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt
(1 - e**2*x**2/d**2)), True))
```

3.24 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-2/15*(-e*x+d)/d/e^3/(-e^2*x^2+d^2)
^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {796, 778, 191}

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d - e*x))/(15*d*e^3*(d^
2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x\left(2d^2e-2de^2x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\ &= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de^2}\\ &= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.87

$$\frac{-2d^4+2d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4}{15d^3e^3(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4)/(15*d^3*e^3*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.93, size = 173, normalized size = 1.84

$$-\frac{2e^5x^5-2de^4x^4-4d^2e^3x^3+4d^3e^2x^2+2d^4ex-2d^5-\left(2e^4x^4-2de^3x^3-3d^2e^2x^2-2d^3ex+2d^4\right)\sqrt{-e^2x^2+d}}{15\left(d^3e^8x^5-d^4e^7x^4-2d^5e^6x^3+2d^6e^5x^2+d^7e^4x-d^8e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 - (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 2*d^3*e*x + 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^8*x^5 - d^4*e^7*x^4 - 2*d^5*e^6*x^3 + 2*d^6*e^5*x
^2 + d^7*e^4*x - d^8*e^3)
```

giac [A] time = 0.27, size = 64, normalized size = 0.68

$$\frac{\left(\left(x\left(\frac{2x^2e^2}{d^3}-\frac{5}{d}\right)-5e^{(-1)}\right)x^2+2d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x*(2*x^2*e^2/d^3 - 5/d) - 5*e^(-1))*x^2 + 2*d^2*e^(-3))*sqrt(-x^2*e^
2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$-\frac{(-ex+d)(ex+d)^2\left(2x^4e^4-2x^3de^3-3d^2x^2e^2-2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(2*e^4*x^4-2*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*d
^4)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 112, normalized size = 1.19

$$\frac{x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{2d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2}-\frac{2x}{15\sqrt{-e^2x^2+d^2}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e^2) -
 2/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e
^2) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

mupad [B] time = 2.61, size = 78, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d*e^3*x^3 - 2*e^4*x^4 - 2*d^4 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^2*(d - e*x)^3)
```

sympy [C] time = 21.31, size = 513, normalized size = 5.46

$$d\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left| \right. \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{other} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*x**
2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**9*sqrt(1 - e**2*x**2/d**2) -
 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**
2)), True)) + e*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 3
0*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**
2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*
sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (
x**4/(4*(d**2)**(7/2)), True))
```

3.25 $\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)-2/15
*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {778, 192, 191}

$$-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)
) - (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\
&= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e}\\
&= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.99

$$\frac{3d^4 - 3d^3ex + 3d^2e^2x^2 + 2de^3x^3 - 2e^4x^4}{15d^4e^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4)/(15*d^4*e^2*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.93, size = 172, normalized size = 2.07

$$\frac{3e^5x^5 - 3de^4x^4 - 6d^2e^3x^3 + 6d^3e^2x^2 + 3d^4ex - 3d^5 + \left(2e^4x^4 - 2de^3x^3 - 3d^2e^2x^2 + 3d^3ex - 3d^4\right)\sqrt{-e^2x^2 +}}{15\left(d^4e^7x^5 - d^5e^6x^4 - 2d^6e^5x^3 + 2d^7e^4x^2 + d^8e^3x - d^9e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 + (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 - d^5*e^6*x^4 - 2*d^6*e^5*x^3 + 2*d^7*e^4*x^
2 + d^8*e^3*x - d^9*e^2)
```

giac [A] time = 0.27, size = 57, normalized size = 0.69

$$\frac{\left(x^3\left(\frac{2x^2e^3}{d^4} - \frac{5e}{d^2}\right) - 3de^{(-2)}\right)\sqrt{-x^2e^2 + d^2}}{15\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^3*(2*x^2*e^3/d^4 - 5*e/d^2) - 3*d*e^(-2))*sqrt(-x^2*e^2 + d^2)/(x^2
*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.93

$$\frac{(-ex+d)(ex+d)^2\left(-2x^4e^4 + 2x^3de^3 + 3d^2x^2e^2 - 3d^3xe + 3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(-2*e^4*x^4+2*d*e^3*x^3+3*d^2*e^2*x^2-3*d^3*e*x+3*d
^4)/d^4/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 87, normalized size = 1.05

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e} - \frac{2x}{15\sqrt{-e^2x^2+d^2}\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

mupad [B] time = 2.62, size = 78, normalized size = 0.94

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(3\,d^4 - 3\,d^3\,e\,x + 3\,d^2\,e^2\,x^2 + 2\,d\,e^3\,x^3 - 2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d + e\,x\right)}^2\,{\left(d - e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 + 2*d*e^3*x^3 + 3*d^2*e^2*x^2 - 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^2*(d - e*x)^3)
```

sympy [A] time = 22.68, size = 432, normalized size = 5.20

$$d\left(\begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e \neq 0 \\ \frac{x^2}{2\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x}{d^2}} \cdots} \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \cdots \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqrt
(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/
(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 +
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x
**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**
9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1
5*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 -
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4
*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.26 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)+8/15*x
/d^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {639, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}\\
&= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.02

$$\frac{3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4}{15d^5e(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8*e^4*x^4)/(15*d^5*e*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.90, size = 171, normalized size = 2.14

$$\frac{3\,e^5x^5-3\,de^4x^4-6\,d^2e^3x^3+6\,d^3e^2x^2+3\,d^4ex-3\,d^5-\left(8\,e^4x^4-8\,de^3x^3-12\,d^2e^2x^2+12\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^6x^5-d^6e^5x^4-2\,d^7e^4x^3+2\,d^8e^3x^2+d^9e^2x-d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e^3*
x^2 + d^9*e^2*x - d^10*e)
```

giac [A] time = 0.26, size = 65, normalized size = 0.81

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(4\,x^2\left(\frac{2\,x^2e^4}{d^5}-\frac{5\,e^2}{d^3}\right)+\frac{15}{d}\right)x+3\,e^{(-1)}\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((4*x^2*(2*x^2*e^4/d^5 - 5*e^2/d^3) + 15/d)*x +
3*e^(-1))/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.96

$$\frac{(-ex+d)\,(ex+d)^2\left(8x^4e^4-8x^3d\,e^3-12d^2x^2e^2+12d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3*
d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.43, size = 80, normalized size = 1.00

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

mupad [B] time = 2.58, size = 78, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+12\,d^3\,e\,x-12\,d^2\,e^2\,x^2-8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 +
12*d^3*e*x))/(15*d^5*e*(d + e*x)^2*(d - e*x)^3)
```

sympy [C] time = 24.41, size = 604, normalized size = 7.55

$$d\left(\begin{cases} -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{20id^2e^2x^3}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{}{15d^1} \\ \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d**2) -
 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*
d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x
**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*
x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**
4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e,
0)), (x**2/(2*(d**2)**(7/2)), True))
```

3.27 $\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*(4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(8*e*x+15*d)/d^6/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {823, 12, 266, 63, 208}

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (5*d + 4*e*x)/(15*d^4*(d^2 - e^2*
x^2)^(3/2)) + (15*d + 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^
2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{5d^3e^2+4d^2e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^5e^4+8d^4e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^7e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x\right.}{2d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{\right.}{d^5e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 1.12

$$\frac{23d^4-8d^3ex-27d^2e^2x^2-15(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+7de^3x^3+8e^4x^4}{15d^6(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3 + 8*e^4*x^4 - 15*(d - e*
x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(15*d^6*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.95, size = 244, normalized size = 2.09

$$\frac{23\,e^5x^5-23\,de^4x^4-46\,d^2e^3x^3+46\,d^3e^2x^2+23\,d^4ex-23\,d^5+15\left(e^5x^5-de^4x^4-2\,d^2e^3x^3+2\,d^3e^2x^2+d^4ex-\right.}{15\left(d^6e^5x^5-d^7e^4x^4-2\,d^8e^3x^3+2\,d^9e^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 - 23*d*e^4*x^4 - 46*d^2*e^3*x^3 + 46*d^3*e^2*x^2 + 23*d^4*
e*x - 23*d^5 + 15*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^
4*e*x - d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 + 7*d*e^3*x^3
- 27*d^2*e^2*x^2 - 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 -
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 + 2*d^9*e^2*x^2 + d^10*e*x - d^11)
```

giac [A] time = 0.27, size = 122, normalized size = 1.04

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{8\,xe^5}{d^6}+\frac{15\,e^4}{d^5}\right)-\frac{20\,e^3}{d^4}\right)x-\frac{35\,e^2}{d^3}\right)x+\frac{15\,e}{d^2}\right)x+\frac{23}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(8*x*e^5/d^6 + 15*e^4/d^5) - 20*e^3/d^4)*x
 - 35*e^2/d^3)*x + 15*e/d^2)*x + 23/d)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d
*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6
```

maple [A] time = 0.01, size = 163, normalized size = 1.39

$$\frac{ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{4ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5}+\frac{}{15\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/15*e/d^4*x/(-e^2*x^2+d^2)^(3/2)+8/15*e/d
^6*x/(-e^2*x^2+d^2)^(1/2)+1/5/d/(-e^2*x^2+d^2)^(5/2)+1/3/d^3/(-e^2*x^2+d^2)
^(3/2)+1/d^5/(-e^2*x^2+d^2)^(1/2)-1/d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.45, size = 157, normalized size = 1.34

$$\frac{ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{4\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,ex}{15\,\sqrt{-e^2x^2+d^2}\,d^6}-\frac{\log\left(\frac{2\,d^2}{|x|}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*d) + 4/1
5*e*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/1
5*e*x/(sqrt(-e^2*x^2 + d^2)*d^6) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2
)*d/abs(x))/d^6 + 1/(sqrt(-e^2*x^2 + d^2)*d^5)
```

mupad [B] time = 3.08, size = 127, normalized size = 1.09

$$\frac{\frac{d^2-e^2\,x^2}{3\,d^3}+\frac{\left(d^2-e^2\,x^2\right)^2}{d^5}+\frac{1}{5\,d}}{\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^6}+\frac{e\,x\,\left(15\,d^4-20\,d^2\,e^2\,x^2+8\,e^4\,x^4\right)}{15\,d^6\,\left(d^2-e^2\,x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] ((d^2 - e^2*x^2)/(3*d^3) + (d^2 - e^2*x^2)^2/d^5 + 1/(5*d))/(d^2 - e^2*x^2)
^(5/2) - atanh((d^2 - e^2*x^2)^(1/2)/d)/d^6 + (e*x*(15*d^4 + 8*e^4*x^4 - 20
*d^2*e^2*x^2))/(15*d^6*(d^2 - e^2*x^2)^(5/2))
```

sympy [C] time = 41.14, size = 2378, normalized size = 20.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*
x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 15*d**6*log(e**2*x**2/d**2)
/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) -
30*d**6*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*
d**7*e**6*x**6) + 30*I*d**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 +
90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 70*I*d**4*e**2*x**2*sqrt(-1 + e**2
*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e*
*6*x**6) - 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*
x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2*log(e*x/d
)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) -
 90*I*d**4*e**2*x**2*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9
*e**4*x**4 - 30*d**7*e**6*x**6) + 30*I*d**2*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6
) + 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 +
90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(e*x/d)/(30*d
**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*I*d
**2*e**4*x**4*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x
**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d**13 - 90*
d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**6*x**6*log
(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x
**6) - 30*I*e**6*x**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**
9*e**4*x**4 - 30*d**7*e**6*x**6), Abs(e**2*x**2/d**2) > 1), (46*d**6*sqrt(1
 - e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*
d**7*e**6*x**6) + 15*d**6*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**
2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 30*d**6*log(sqrt(1 - e**2*x**2
/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e*
*6*x**6) + 15*I*pi*d**6/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4
- 30*d**7*e**6*x**6) - 70*d**4*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(30*d**13
 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 45*d**4*e*
*2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x
**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2*log(sqrt(1 - e**2*x**2/d**2) +
 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6)
 - 45*I*pi*d**4*e**2*x**2/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**
4 - 30*d**7*e**6*x**6) + 30*d**2*e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(30*d**
13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 45*d**2*
e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4
*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2)
 + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**
6) + 45*I*pi*d**2*e**4*x**4/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x
**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d**13 - 90*
d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**6*x**6*log
(sqrt(1 - e**2*x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**
4*x**4 - 30*d**7*e**6*x**6) - 15*I*pi*e**6*x**6/(30*d**13 - 90*d**11*e**2*x
**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6), True)) + e*Piecewise((-15*I*d
**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e**
2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 20*I*d**2*e**
2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e*
```

```
*2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)) - 8*I*e**4*x**
5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e**2*x*
*2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*s
qrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 20*
d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(
1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) + 8*e**4*
x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x
**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.28 $\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=153

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*(e*x+d)/d^2/x/(-e^2*x^2+d^2)^(5/2)+1/15*(5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2
)^(3/2)-e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(5*e*x+8*d)/d^6/x/(-e^2*x
^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {823, 807, 266, 63, 208}

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x*(d^2 - e^2*x^2)^(5/2)) + (6*d + 5*e*x)/(15*d^4*x*(d^2 -
e^2*x^2)^(3/2)) + (8*d + 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[d^
2 - e^2*x^2])/(5*d^7*x) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{6d^3e^2+5d^2e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{24d^5e^4+15d^4e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{\int \frac{48d^7e^6+15d^6e^7x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\int \frac{}{x\sqrt{}}}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\,\text{Sub}}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{\text{Subst}}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\tanh}{}$$

Mathematica [A] time = 0.07, size = 147, normalized size = 0.96

$$\frac{-15d^5+38d^4ex+52d^3e^2x^2-87d^2e^3x^3-15ex(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-33de^4x^4+48e^5x^5}{15d^7x(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-15*d^5 + 38*d^4*e*x + 52*d^3*e^2*x^2 - 87*d^2*e^3*x^3 - 33*d*e^4*x^4 + 48
*e^5*x^5 - 15*e*x*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^
2 - e^2*x^2]/d])/(15*d^7*x*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 1.06, size = 270, normalized size = 1.76

$$\frac{23e^6x^6-23de^5x^5-46d^2e^4x^4+46d^3e^3x^3+23d^4e^2x^2-23d^5ex+15\left(e^6x^6-de^5x^5-2d^2e^4x^4+2d^3e^3x^3+d^4e^2\right.}{15\left(d^7e^5x^6-d^8e^4x^5-2d^9e^3x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^6*x^6 - 23*d*e^5*x^5 - 46*d^2*e^4*x^4 + 46*d^3*e^3*x^3 + 23*d^4*
e^2*x^2 - 23*d^5*e*x + 15*(e^6*x^6 - d*e^5*x^5 - 2*d^2*e^4*x^4 + 2*d^3*e^3*
x^3 + d^4*e^2*x^2 - d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x
^5 - 33*d*e^4*x^4 - 87*d^2*e^3*x^3 + 52*d^3*e^2*x^2 + 38*d^4*e*x - 15*d^5)*
sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 - d^8*e^4*x^5 - 2*d^9*e^3*x^4 + 2*d^10*e
^2*x^3 + d^11*e*x^2 - d^12*x)
```

giac [A] time = 0.29, size = 189, normalized size = 1.24

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(3\left(x\left(\frac{11\,xe^6}{d^7}+\frac{5\,e^5}{d^6}\right)-\frac{25\,e^4}{d^5}\right)x-\frac{35\,e^3}{d^4}\right)x+\frac{45\,e^2}{d^3}\right)x+\frac{23\,e}{d^2}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^7}+\frac{}{2\left(de+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((3*(x*(11*x*e^6/d^7 + 5*e^5/d^6) - 25*e^4/d^5)
*x - 35*e^3/d^4)*x + 45*e^2/d^3)*x + 23*e/d^2)/(x^2*e^2 - d^2)^3 - e*log(1/
2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^7 + 1/2*x*e^3/((d
*e + sqrt(-x^2*e^2 + d^2)*e)*d^7) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-
1)/(d^7*x)
```

maple [A] time = 0.02, size = 195, normalized size = 1.27

$$\frac{6e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}-\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}+\frac{8e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{e\ln\left(\frac{2d^2}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e/d^2/(-e^2*x^2+d^2)^(5/2)+1/3*e/d^4/(-e^2*x^2+d^2)^(3/2)+e/d^6/(-e^2*x
^2+d^2)^(1/2)-e/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2
))/x)-1/d/x/(-e^2*x^2+d^2)^(5/2)+6/5*e^2/d^3*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^2
/d^5*x/(-e^2*x^2+d^2)^(3/2)+16/5*e^2/d^7*x/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.46, size = 189, normalized size = 1.24

$$\frac{6\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{8\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}+\frac{16\,e^2}{5\,\sqrt{-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 1/5*e/((-e^2*x^2 + d^2)^(5/2)*d^2)
 + 8/5*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 1/3*e/((-e^2*x^2 + d^2)^(3/2)*d
^4) - 1/((-e^2*x^2 + d^2)^(5/2)*d*x) + 16/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^7
) - e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 + e/(sqrt(-e^
2*x^2 + d^2)*d^6)
```

mupad [B] time = 3.31, size = 141, normalized size = 0.92

$$\frac{\frac{e}{5d^2}+\frac{e\left(d^2-e^2x^2\right)^2}{d^6}+\frac{e\left(d^2-e^2x^2\right)}{3d^4}}{\left(d^2-e^2x^2\right)^{5/2}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}-\frac{d^6-6d^4e^2x^2+8d^2e^4x^4-\frac{16e^6x^6}{5}}{d^7x\left(d^2-e^2x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] (e/(5*d^2) + (e*(d^2 - e^2*x^2)^2)/d^6 + (e*(d^2 - e^2*x^2))/(3*d^4))/(d^2
- e^2*x^2)^(5/2) - (e*atanh((d^2 - e^2*x^2)^(1/2)/d))/d^7 - (d^6 - (16*e^6*
x^6)/5 - 6*d^4*e^2*x^2 + 8*d^2*e^4*x^4)/(d^7*x*(d^2 - e^2*x^2)^(5/2))
```

sympy [C] time = 31.22, size = 2404, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*
x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2
/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d
**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 1
5*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*s
qrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x*
*4 + 5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/
(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d*
*8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 +
 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e*
*5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**
10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) +
1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6),
 True)) + e*Piecewise((46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(30*d**13 - 90*d
**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 15*d**6*log(e**2*
x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**
6*x**6) - 30*d**6*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*
x**4 - 30*d**7*e**6*x**6) + 30*I*d**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e*
*2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 70*I*d**4*e**2*x**2*sqrt
(-1 + e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 -
30*d**7*e**6*x**6) - 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d
**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2
*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e*
*6*x**6) - 90*I*d**4*e**2*x**2*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2
 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*I*d**2*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7
*e**6*x**6) + 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e*
*2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(e*
x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6
) + 90*I*d**2*e**4*x**4*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d
**9*e**4*x**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d
**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**
6*x**6*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d
**7*e**6*x**6) - 30*I*e**6*x**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**
2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6), Abs(e**2*x**2/d**2) > 1), (46*d
**6*sqrt(1 - e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*
x**4 - 30*d**7*e**6*x**6) + 15*d**6*log(e**2*x**2/d**2)/(30*d**13 - 90*d**1
1*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 30*d**6*log(sqrt(1 -
```

```
 e**2*x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 -
30*d**7*e**6*x**6) + 15*I*pi*d**6/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*
e**4*x**4 - 30*d**7*e**6*x**6) - 70*d**4*e**2*x**2*sqrt(1 - e**2*x**2/d**2)
/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) -
45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d
**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2*log(sqrt(1 - e**2*x*
*2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*
e**6*x**6) - 45*I*pi*d**4*e**2*x**2/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**
9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*d**2*e**4*x**4*sqrt(1 - e**2*x**2/d**
2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6)
+ 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90
*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(sqrt(1 - e**2*
x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**
7*e**6*x**6) + 45*I*pi*d**2*e**4*x**4/(30*d**13 - 90*d**11*e**2*x**2 + 90*d
**9*e**4*x**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d
**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**
6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 9
0*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 15*I*pi*e**6*x**6/(30*d**13 - 90*d*
*11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6), True))
```

3.29 $\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=184

$$\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2\right.}$$

```
[Out] 1/5*(e*x+d)/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+1/15*(6*e*x+7*d)/d^4/x^2/(-e^2*x^2
+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(24*e*x+35*d)/
d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2-16/5*e*(-e^2*
x^2+d^2)^(1/2)/d^8/x
```

Rubi [A] time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {823, 835, 807, 266, 63, 208}

$$-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7e^2\tanh^{-1}\left(\sqrt{\right.}}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x^2*(d^2 - e^2*x^2)^(5/2)) + (7*d + 6*e*x)/(15*d^4*x^2*(d^
2 - e^2*x^2)^(3/2)) + (35*d + 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) - (7
*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) - (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x) -
(7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{7d^3e^2+6d^2e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{35d^5e^4+24d^4e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{\int \frac{105d^7e^6+48d^6e^7x}{x^3\sqrt{d^2-e^2x^2}}}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-$$

Mathematica [A] time = 0.13, size = 183, normalized size = 0.99

$$\frac{105e^2x^2(d+ex)^2(ex-d)^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+d\sqrt{1-\frac{e^2x^2}{d^2}}\left(-15d^6-15d^5ex+176d^4e^2x^2+4d^3e^3x^3-249d^2e^4x^4\right.}{30d^9x^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d*Sqrt[1 - (e^2*x^2)/d^2]*(-15*d^6 - 15*d^5*e*x + 176*d^4*e^2*x^2 + 4*d^3*
e^3*x^3 - 249*d^2*e^4*x^4 + 9*d*e^5*x^5 + 96*e^6*x^6) + 105*e^2*x^2*(-d + e
*x)^3*(d + e*x)^2*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(30*d^9*x^2*(d - e*x)^2
*(d + e*x)*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.92, size = 291, normalized size = 1.58

$$\frac{116\,e^7x^7-116\,de^6x^6-232\,d^2e^5x^5+232\,d^3e^4x^4+116\,d^4e^3x^3-116\,d^5e^2x^2+105\left(e^7x^7-de^6x^6-2\,d^2e^5x^5+2\,d^3e^4\right.}{30\left(d^8e^5x^7-d^9e^4x^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 - 116*d*e^6*x^6 - 232*d^2*e^5*x^5 + 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 - 116*d^5*e^2*x^2 + 105*(e^7*x^7 - d*e^6*x^6 - 2*d^2*e^5*x^5 +
 2*d^3*e^4*x^4 + d^4*e^3*x^3 - d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) - (96*e^6*x^6 + 9*d*e^5*x^5 - 249*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 - 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 - d^9*e^
4*x^6 - 2*d^10*e^3*x^5 + 2*d^11*e^2*x^4 + d^12*e*x^3 - d^13*x^2)
```

giac [A] time = 0.36, size = 260, normalized size = 1.41

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(3\left(x\left(\frac{11\,xe^7}{d^8}+\frac{15\,e^6}{d^7}\right)-\frac{25\,e^5}{d^6}\right)x-\frac{100\,e^4}{d^5}\right)x+\frac{45\,e^3}{d^4}\right)x+\frac{58\,e^2}{d^3}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{7\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^8}+\frac{x^2\left(\frac{4(\cdots}{\cdots}\right.}{8\left(de\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((3*(x*(11*x*e^7/d^8 + 15*e^6/d^7) - 25*e^5/d^6
)*x - 100*e^4/d^5)*x + 45*e^3/d^4)*x + 58*e^2/d^3)/(x^2*e^2 - d^2)^3 - 7/2*
e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^8 + 1/8
*x^2*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d^8) - 1/8*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^8/x + (d*e +
sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^6/x^2)*e^(-8)/d^16
```

maple [A] time = 0.02, size = 227, normalized size = 1.23

$$\frac{6e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{8e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}-\frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^2}+\frac{\cdots}{6\left(-e^2x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2/d/x^2/(-e^2*x^2+d^2)^(5/2)+7/10*e^2/d^3/(-e^2*x^2+d^2)^(5/2)+7/6*e^2/d
^5/(-e^2*x^2+d^2)^(3/2)+7/2*e^2/d^7/(-e^2*x^2+d^2)^(1/2)-7/2*e^2/d^7/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-e/d^2/x/(-e^2*x^2+d^
2)^(5/2)+6/5*e^3/d^4*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^3/d^6*x/(-e^2*x^2+d^2)^(3
/2)+16/5*e^3/d^8*x/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.47, size = 221, normalized size = 1.20

$$\frac{6\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7\,e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{8\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}+\frac{7\,e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{16}{5\sqrt{-e^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/10*e^2/((-e^2*x^2 + d^2)^(5/2)*d
^3) + 8/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/6*e^2/((-e^2*x^2 + d^2)^(3
/2)*d^5) - e/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 16/5*e^3*x/(sqrt(-e^2*x^2 + d
^2)*d^8) - 7/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^8
+ 7/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^7) - 1/2/((-e^2*x^2 + d^2)^(5/2)*d*x^2)
```

mupad [B] time = 3.43, size = 181, normalized size = 0.98

$$\frac{161\,e^2}{30\,d^3\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{1}{2\,d\,x^2\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{7\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2\,d^8}-\frac{49\,e^4\,x^2}{6\,d^5\left(d^2-e^2\,x^2\right)^{5/2}}+\frac{7\,e^6\,x^4}{2\,d^7\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] (161*e^2)/(30*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(2*d*x^2*(d^2 - e^2*x^2)^(5/2)
) - (7*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/(2*d^8) - (49*e^4*x^2)/(6*d^5*(d
^2 - e^2*x^2)^(5/2)) + (7*e^6*x^4)/(2*d^7*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d^
6 - 16*e^6*x^6 - 30*d^4*e^2*x^2 + 40*d^2*e^4*x^4))/(5*d^8*x*(d^2 - e^2*x^2)
^(5/2))
```

sympy [C] time = 35.06, size = 2691, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-30*I*d**8*sqrt(-1 + e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**1
3*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) + 322*I*d**6*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**
11*e**4*x**6 - 60*d**9*e**6*x**8) + 105*d**6*e**2*x**2*log(e**2*x**2/d**2)/
(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x
**8) - 210*d**6*e**2*x**2*log(e*x/d)/(60*d**15*x**2 - 180*d**13*e**2*x**4 +
 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) + 210*I*d**6*e**2*x**2*asin(d/(e*
x))/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e*
*6*x**8) - 490*I*d**4*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(60*d**15*x**2 -
180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 315*d**4*e
**4*x**4*log(e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**
11*e**4*x**6 - 60*d**9*e**6*x**8) + 630*d**4*e**4*x**4*log(e*x/d)/(60*d**15
*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 63
0*I*d**4*e**4*x**4*asin(d/(e*x))/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180
```

```
*d**11*e**4*x**6 - 60*d**9*e**6*x**8) + 210*I*d**2*e**6*x**6*sqrt(-1 + e**2
*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60
*d**9*e**6*x**8) + 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(60*d**15*x**2 -
180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 630*d**2*e
**6*x**6*log(e*x/d)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x
**6 - 60*d**9*e**6*x**8) + 630*I*d**2*e**6*x**6*asin(d/(e*x))/(60*d**15*x**
2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 105*e*
*8*x**8*log(e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**1
1*e**4*x**6 - 60*d**9*e**6*x**8) + 210*e**8*x**8*log(e*x/d)/(60*d**15*x**2
- 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 210*I*e*
*8*x**8*asin(d/(e*x))/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4
*x**6 - 60*d**9*e**6*x**8), Abs(e**2*x**2/d**2) > 1), (30*d**8*sqrt(1 - e**
2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 +
60*d**9*e**6*x**8) - 322*d**6*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-60*d**15
*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 10
5*d**6*e**2*x**2*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4
- 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 210*d**6*e**2*x**2*log(sqrt(1
- e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e*
*4*x**6 + 60*d**9*e**6*x**8) - 105*I*pi*d**6*e**2*x**2/(-60*d**15*x**2 + 18
0*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 490*d**4*e**
4*x**4*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180
*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d**4*e**4*x**4*log(e**2*x**2/d*
*2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e
**6*x**8) - 630*d**4*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15
*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 31
5*I*pi*d**4*e**4*x**4/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**
4*x**6 + 60*d**9*e**6*x**8) - 210*d**2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(
-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x
**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e
**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 630*d**2*e**6*x**6*lo
g(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180
*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*I*pi*d**2*e**6*x**6/(-60*d**15*
x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 105
*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*
d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(sqrt(1 - e**2*x**2
/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 6
0*d**9*e**6*x**8) + 105*I*pi*e**8*x**8/(-60*d**15*x**2 + 180*d**13*e**2*x**
4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), True)) + e*Piecewise((5*d**6*
e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4
*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2/(e**2*x**2) - 1)/(-
5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*
d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 -
15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*sqrt(d**2/(e**2*x**2)
 - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**
6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/(e**2*x**2) + 1)/(-5
*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*I
*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2
- 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e**5*x**4*sqrt(-d**2/(
e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**
8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d*
*12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6), True))
```

3.30 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/7*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)-2/35*(-2*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = $\{796, 778, 192, 191\}$

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(
d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(
105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx = \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^2e-4de^2x\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{105d^3e^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.04, size = 104, normalized size = 0.86

$$\frac{-6d^6+6d^5ex+15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4-8de^5x^5+8e^6x^6}{105d^5e^3(d-ex)^3(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 - 20*d^2*e^4*x^4 - 8*
d*e^5*x^5 + 8*e^6*x^6)/(105*d^5*e^3*(d - e*x)^3*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2])
```

fricas [B] time = 0.77, size = 239, normalized size = 1.98

$$-\frac{6\,e^7x^7-6\,de^6x^6-18\,d^2e^5x^5+18\,d^3e^4x^4+18\,d^4e^3x^3-18\,d^5e^2x^2-6\,d^6ex+6\,d^7-\left(8\,e^6x^6-8\,de^5x^5-20\,d^2e^4x^4\right.}{105\left(d^5e^{10}x^7-d^6e^9x^6-3\,d^7e^8x^5+3\,d^8e^7x^4+3\,d^9e^6x^3-3\,d^{10}e^5x^2-d^{11}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(6*e^7*x^7 - 6*d*e^6*x^6 - 18*d^2*e^5*x^5 + 18*d^3*e^4*x^4 + 18*d^4*
e^3*x^3 - 18*d^5*e^2*x^2 - 6*d^6*e*x + 6*d^7 - (8*e^6*x^6 - 8*d*e^5*x^5 - 2
0*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x - 6*d^6)*sqrt(-
e^2*x^2 + d^2))/(d^5*e^10*x^7 - d^6*e^9*x^6 - 3*d^7*e^8*x^5 + 3*d^8*e^7*x^4
 + 3*d^9*e^6*x^3 - 3*d^10*e^5*x^2 - d^11*e^4*x + d^12*e^3)
```

giac [A] time = 0.27, size = 77, normalized size = 0.64

$$\frac{\left(\left(\left(4\,x^2\left(\frac{2\,x^2e^4}{d^5}-\frac{7\,e^2}{d^3}\right)+\frac{35}{d}\right)x+21\,e^{(-1)}\right)x^2-6\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{105\left(x^2e^2-d^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*x^2*e^4/d^5 - 7*e^2/d^3) + 35/d)*x + 21*e^(-1))*x^2 - 6*d
^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^4
```

maple [A] time = 0.01, size = 99, normalized size = 0.82

$$-\frac{(-ex+d)(ex+d)^2\left(-8e^6x^6+8e^5x^5d+20e^4x^4d^2-20x^3d^3e^3-15x^2d^4e^2-6x\,d^5e+6d^6\right)}{105\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x)
```

```
[Out] -1/105*(-e*x+d)*(e*x+d)^2*(-8*e^6*x^6+8*d*e^5*x^5+20*d^2*e^4*x^4-20*d^3*e^3
*x^3-15*d^4*e^2*x^2-6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(9/2)
```

maxima [A] time = 0.44, size = 135, normalized size = 1.12

$$\frac{x^2}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}+\frac{dx}{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}-\frac{2\,d^2}{35\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}-\frac{4\,x}{105\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}-\frac{}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/((-e^2*x^2 + d^2)^(7/2)*e) + 1/7*d*x/((-e^2*x^2 + d^2)^(7/2)*e^2) -
 2/35*d^2/((-e^2*x^2 + d^2)^(7/2)*e^3) - 1/35*x/((-e^2*x^2 + d^2)^(5/2)*d*e
^2) - 4/105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d
^2)*d^5*e^2)
```

mupad [B] time = 2.69, size = 164, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,(d-e\,x)^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}-\frac{3\,x}{70\,d\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}+\frac{4\,x}{105\,d^3\,e^2}\right)}{(d+e\,x)^2\,(d-e\,x)^2}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,(d+e\,x)\,(d-e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d - e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(2/
(35*e^3) - (3*x)/(70*d*e^2)))/((d + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^
(1/2)*(1/(56*d^2*e^3) + (4*x)/(105*d^3*e^2)))/((d + e*x)^2*(d - e*x)^2) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

sympy [C] time = 22.73, size = 903, normalized size = 7.46

$$d\left(\begin{cases}\frac{35id^4x^3}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{28}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ -\frac{35d^4x^3}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{28d^2e^2x^5}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(9/2),x)
```

```
[Out] d*Piecewise((35*I*d**4*x**3/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**
11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*
x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) - 28*I*d**2*e**2
*x**5/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) + 8*I*e**4*x**7/(-105*d**13*sqrt(-1 +
```

```
e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*
e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (-35*d**4*x**3/(-105*d**13*sqrt(1 - e**
2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4
*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2
)) + 28*d**2*e**2*x**5/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**
2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**
2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2)) - 8*e**4*x**7/(-105*d**13
*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) -
315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e
**2*x**2/d**2)), True)) + e*Piecewise((2*d**2/(-35*d**6*e**4*sqrt(d**2 - e*
*2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**2*x**2) - 105*d**2*e**8*x**4*s
qrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d**2 - e**2*x**2)) - 7*e**2*x**2
/(-35*d**6*e**4*sqrt(d**2 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**
2*x**2) - 105*d**2*e**8*x**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d*
*2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(9/2)), True))
```

3.31 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] 1/9*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(9/2)-2/63*(-3*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(7/2)-2/105*x/d^3/e^2/(-e^2*x^2+d^2)^(5/2)-8/315*x/d^5/e^2/(-e^2*x^2+d^2
)^(3/2)-16/315*x/d^7/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {796, 778, 192, 191}

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(
d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(
315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2
])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{x\left(2d^2e-6de^2x\right)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9d^2e^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{21de^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{105d^3e^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/}}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/}}$$

Mathematica [A] time = 0.05, size = 126, normalized size = 0.85

$$\frac{-10d^8+10d^7ex+35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2e^6x^6+16de^7x^7-16e^8x^8}{315d^7e^3(d-ex)^4(d+ex)^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x^3 - 70*d^4*e^4*x^4 -
56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*e^8*x^8)/(315*d^7*e^3*(
d - e*x)^4*(d + e*x)^3*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 1.66, size = 305, normalized size = 2.06

$$-\frac{10\,e^9x^9-10\,de^8x^8-40\,d^2e^7x^7+40\,d^3e^6x^6+60\,d^4e^5x^5-60\,d^5e^4x^4-40\,d^6e^3x^3+40\,d^7e^2x^2+10\,d^8ex-10\,d^9-}{315\left(d^7e^{12}x^9-d^8e^{11}x^8-4\,d^9e^{10}x^7+4\,d^{10}e^9x^6+6\,d^{11}e^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/315*(10*e^9*x^9 - 10*d*e^8*x^8 - 40*d^2*e^7*x^7 + 40*d^3*e^6*x^6 + 60*d^
4*e^5*x^5 - 60*d^5*e^4*x^4 - 40*d^6*e^3*x^3 + 40*d^7*e^2*x^2 + 10*d^8*e*x -
 10*d^9 - (16*e^8*x^8 - 16*d*e^7*x^7 - 56*d^2*e^6*x^6 + 56*d^3*e^5*x^5 + 70
*d^4*e^4*x^4 - 70*d^5*e^3*x^3 - 35*d^6*e^2*x^2 - 10*d^7*e*x + 10*d^8)*sqrt(
-e^2*x^2 + d^2))/(d^7*e^12*x^9 - d^8*e^11*x^8 - 4*d^9*e^10*x^7 + 4*d^10*e^9
*x^6 + 6*d^11*e^8*x^5 - 6*d^12*e^7*x^4 - 4*d^13*e^6*x^3 + 4*d^14*e^5*x^2 +
d^15*e^4*x - d^16*e^3)
```

giac [A] time = 0.30, size = 90, normalized size = 0.61

$$\frac{\left(\left(\left(2\left(4\,x^2\left(\frac{2\,x^2e^6}{d^7}-\frac{9\,e^4}{d^5}\right)+\frac{63\,e^2}{d^3}\right)x^2-\frac{105}{d}\right)x-45\,e^{(-1)}\right)x^2+10\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{315\left(x^2e^2-d^2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((2*(4*x^2*(2*x^2*e^6/d^7 - 9*e^4/d^5) + 63*e^2/d^3)*x^2 - 105/d)*x
- 45*e^(-1))*x^2 + 10*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^5
```

maple [A] time = 0.01, size = 121, normalized size = 0.82

$$-\frac{(-ex+d)(ex+d)^2\left(16e^8x^8-16e^7x^7d-56e^6x^6d^2+56e^5x^5d^3+70e^4x^4d^4-70x^3d^5e^3-35x^2d^6e^2-10x\,d^7e+\right.}{315\left(-e^2x^2+d^2\right)^{\frac{11}{2}}d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x)
```

```
[Out] -1/315*(-e*x+d)*(e*x+d)^2*(16*e^8*x^8-16*d*e^7*x^7-56*d^2*e^6*x^6+56*d^3*e^
5*x^5+70*d^4*e^4*x^4-70*d^5*e^3*x^3-35*d^6*e^2*x^2-10*d^7*e*x+10*d^8)/d^7/e
^3/(-e^2*x^2+d^2)^(11/2)
```

maxima [A] time = 0.45, size = 158, normalized size = 1.07

$$\frac{x^2}{7\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e}+\frac{dx}{9\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e^2}-\frac{2\,d^2}{63\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e^3}-\frac{x}{63\left(-e^2x^2+d^2\right)^{\frac{7}{2}}de^2}-\frac{2\,x}{105\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^2}-\frac{}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/7*x^2/((-e^2*x^2 + d^2)^(9/2)*e) + 1/9*d*x/((-e^2*x^2 + d^2)^(9/2)*e^2) -
 2/63*d^2/((-e^2*x^2 + d^2)^(9/2)*e^3) - 1/63*x/((-e^2*x^2 + d^2)^(7/2)*d*e
^2) - 2/105*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^2) - 8/315*x/((-e^2*x^2 + d^2)^
(3/2)*d^5*e^2) - 16/315*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^2)
```

mupad [B] time = 2.74, size = 202, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{144\,d^3\,e^3\,(d-e\,x)^5}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{252\,e^3}-\frac{17\,x}{252\,d\,e^2}\right)}{(d+e\,x)^4\,(d-e\,x)^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{5}{144\,d^2\,e^3}+\frac{131\,x}{5040\,d^3\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^5\,e^2\,(d+e\,x)^2\,(d-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(144*d^3*e^3*(d - e*x)^5) - ((d^2 - e^2*x^2)^(1/2)*(1
/(252*e^3) - (17*x)/(252*d*e^2)))/((d + e*x)^4*(d - e*x)^4) - ((d^2 - e^2*x
^2)^(1/2)*(5/(144*d^2*e^3) + (131*x)/(5040*d^3*e^2)))/((d + e*x)^3*(d - e*x
)^3) - (8*x*(d^2 - e^2*x^2)^(1/2))/(315*d^5*e^2*(d + e*x)^2*(d - e*x)^2) -
(16*x*(d^2 - e^2*x^2)^(1/2))/(315*d^7*e^2*(d + e*x)*(d - e*x))
```

sympy [C] time = 48.46, size = 1401, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

```
[Out] d*Piecewise((-105*I*d**6*x**3/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d
**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*
e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 126*I*d**4*e**2*x**5/(315*d**17*sqrt
(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 18
```

```
90*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1
 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 72*I*d
**2*e**4*x**7/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) -
 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-
1 + e**2*x**2/d**2)) + 16*I*e**6*x**9/(315*d**17*sqrt(-1 + e**2*x**2/d**2)
- 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 3
15*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (10
5*d**6*x**3/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt
(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260
*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**
2*x**2/d**2)) - 126*d**4*e**2*x**5/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 12
60*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 -
 e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9
*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) + 72*d**2*e**4*x**7/(315*d**17*sqrt(1
- e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d*
*13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2
*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) - 16*e**6*x**9/(
315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**
2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x
**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((-2*d**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*
d**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2
*x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d*
*2 - e**2*x**2)) + 9*e**2*x**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*d
**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2*
x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d**
2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(11/2)), True))
```

3.32 $\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\sin^{-1}(ax)}{a^3}-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

```
[Out] -arcsin(a*x)/a^3+(a*x-1)/a^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {797, 641, 216, 637}

$$-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/a^3 - ArcSin[a*x]/
a^3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2}-\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.93

$$\frac{a^2x^2 - \sqrt{1-a^2x^2}\,\sin^{-1}(ax) + ax - 2}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-2 + a*x + a^2*x^2 - Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a^3*Sqrt[1 - a^2*x^2]
)
```

fricas [A] time = 0.77, size = 66, normalized size = 1.22

$$-\frac{2\,ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax+2) + 2}{a^4x + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

giac [A] time = 0.21, size = 70, normalized size = 1.30

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.02, size = 85, normalized size = 1.57

$$\frac{x^2}{\sqrt{-a^2x^2+1}\;a} + \frac{x}{\sqrt{-a^2x^2+1}\;a^2} - \frac{\arctan\left(\frac{\sqrt{a^2}\;x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\;a^2} - \frac{2}{\sqrt{-a^2x^2+1}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] x^2/a/(-a^2*x^2+1)^(1/2)-2/a^3/(-a^2*x^2+1)^(1/2)+x/a^2/(-a^2*x^2+1)^(1/2)-
1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 63, normalized size = 1.17

$$\frac{x^2}{\sqrt{-a^2x^2+1}\,a} + \frac{x}{\sqrt{-a^2x^2+1}\,a^2} - \frac{\arcsin(ax)}{a^3} - \frac{2}{\sqrt{-a^2x^2+1}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(-a^2*x^2 + 1)*a) + x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3 -
 2/(sqrt(-a^2*x^2 + 1)*a^3)
```

mupad [B] time = 0.09, size = 84, normalized size = 1.56

$$\frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x - 1))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

sympy [A] time = 8.33, size = 102, normalized size = 1.89

$$-a\left(\begin{cases}-\frac{x^2}{a^2\sqrt{-a^2x^2+1}}+\frac{2}{a^4\sqrt{-a^2x^2+1}} & \text{for}\ a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix}{a^2\sqrt{a^2x^2-1}}+\frac{i\,\mathrm{acosh}\,(ax)}{a^3} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\mathrm{asin}\,(ax)}{a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a*Piecewise((-x**2/(a**2*sqrt(-a**2*x**2 + 1)) + 2/(a**4*sqrt(-a**2*x**2 +
 1)), Ne(a, 0)), (x**4/4, True)) + Piecewise((-I*x/(a**2*sqrt(a**2*x**2 - 1
)) + I*acosh(a*x)/a**3, Abs(a**2*x**2) > 1), (x/(a**2*sqrt(-a**2*x**2 + 1))
 - asin(a*x)/a**3, True))
```

3.33 $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$

Optimal. Leaf size=173

$$-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}+\frac{11d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}-8$$

```
[Out] 11/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-8/15*d^3*x^2*(-e^2*x^2+d^2)^
(1/2)/e^3-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-2/5*d*x^4*(-e^2*x^2+d^2)^(
1/2)/e-1/6*x^5*(-e^2*x^2+d^2)^(1/2)-1/240*d^4*(165*e*x+256*d)*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A] time = 0.23, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}-\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}+\frac{11d^6 t}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-8*d^3*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*Sqrt[d^2 - e^2*x^2]
)/(24*e^2) - (2*d*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (x^5*Sqrt[d^2 - e^2*x^2]
)/6 - (d^4*(256*d + 165*e*x)*Sqrt[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^4\left(-11d^2e^2-12de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2}$$

$$= -\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^3\left(48d^3e^3+55d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{30e^4}$$

$$= -\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2\left(-165d^4e^4-192d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{120e^6}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x(384d^5 \cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d \cdots}{\cdots}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d \cdots}{\cdots}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d \cdots}{\cdots}$$

Mathematica [A] time = 0.10, size = 103, normalized size = 0.60

$$\frac{165d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(256d^5+165d^4ex+128d^3e^2x^2+110d^2e^3x^3+96de^4x^4+40e^5x^5\right)}{240e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(256*d^5 + 165*d^4*e*x + 128*d^3*e^2*x^2 + 110*d^2*e
^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5)) + 165*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*
x^2]])/(240*e^5)
```

fricas [A] time = 0.96, size = 105, normalized size = 0.61

$$-\frac{330\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(40\,e^5x^5 + 96\,de^4x^4 + 110\,d^2e^3x^3 + 128\,d^3e^2x^2 + 165\,d^4ex + 256\,d^5\right)\sqrt{-e^2x^2 \cdots}}{240\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(330*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 9
6*d*e^4*x^4 + 110*d^2*e^3*x^3 + 128*d^3*e^2*x^2 + 165*d^4*e*x + 256*d^5)*sq
rt(-e^2*x^2 + d^2))/e^5
```

giac [A] time = 0.27, size = 84, normalized size = 0.49

$$\frac{11}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{240}\left(256\,d^5e^{(-5)}+\left(165\,d^4e^{(-4)}+2\left(64\,d^3e^{(-3)}+\left(55\,d^2e^{(-2)}+4\left(12\,de^{(-1)}+5\,x\right)x\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 11/16*d^6*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/240*(256*d^5*e^(-5) + (165*d^4*e^
(-4) + 2*(64*d^3*e^(-3) + (55*d^2*e^(-2) + 4*(12*d*e^(-1) + 5*x)*x)*x)*x)*x
)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.03, size = 174, normalized size = 1.01

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^5}{6}-\frac{2\sqrt{-e^2x^2+d^2}\,d\,x^4}{5e}+\frac{11d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^4}-\frac{11\sqrt{-e^2x^2+d^2}\,d^2x^3}{24e^2}-\frac{8\sqrt{-e^2x^2+d^2}\,d^3x^2}{15e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/6*x^5*(-e^2*x^2+d^2)^(1/2)-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-11/16*
d^4*x*(-e^2*x^2+d^2)^(1/2)/e^4+11/16/e^4*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2)*x)-2/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e-8/15*d^3*x^2*(-e^
2*x^2+d^2)^(1/2)/e^3-16/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5
```

maxima [A] time = 0.98, size = 153, normalized size = 0.88

$$-\frac{1}{6}\sqrt{-e^2x^2+d^2}\,x^5-\frac{2\sqrt{-e^2x^2+d^2}\,dx^4}{5\,e}-\frac{11\sqrt{-e^2x^2+d^2}\,d^2x^3}{24\,e^2}-\frac{8\sqrt{-e^2x^2+d^2}\,d^3x^2}{15\,e^3}+\frac{11\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*x^5 - 2/5*sqrt(-e^2*x^2 + d^2)*d*x^4/e - 11/24*sq
rt(-e^2*x^2 + d^2)*d^2*x^3/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*d^3*x^2/e^3 + 11
/16*d^6*arcsin(e*x/d)/e^5 - 11/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^4 - 16/15*sq
rt(-e^2*x^2 + d^2)*d^5/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [C] time = 13.48, size = 558, normalized size = 3.23

$$d^2\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6}-\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4} \\ \frac{x^6}{6\sqrt{d^2}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5
/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*
x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2
*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) +
2*d*e*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqr
t(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e,
 0)), (x**6/(6*sqrt(d**2)), True)) + e**2*Piecewise((-5*I*d**6*acosh(e*x/d)
/(16*e**7) + 5*I*d**5*x/(16*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**3*x**3
/(48*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**5/(24*e**2*sqrt(-1 + e**2*x**
2/d**2)) - I*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (5*d**6*asin(e*x/d)/(16*e**7) - 5*d**5*x/(16*e**6*sqrt(1 - e**2*x**2/d**2
)) + 5*d**3*x**3/(48*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**5/(24*e**2*sqrt(
1 - e**2*x**2/d**2)) + x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.34 $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$

Optimal. Leaf size=144

$$-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}$$

```
[Out] 3/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1/
2)/e^2-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/20*d
^3*(5*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(2*e
) - (x^4*Sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*Sqrt[d^2 - e^2*x^2])
/(20*e^4) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^3\left(-9d^2e^2-10de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}$$

$$= -\frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^2\left(30d^3e^3+36d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x\left(-72d^4e^4-90d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{(3}{}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{(3}{}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{3d}{}$$

Mathematica [A] time = 0.09, size = 92, normalized size = 0.64

$$\frac{15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}\left(24d^4+15d^3ex+12d^2e^2x^2+10de^3x^3+4e^4x^4\right)}{20e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(24*d^4 + 15*d^3*e*x + 12*d^2*e^2*x^2 + 10*d*e^3*x^3
 + 4*e^4*x^4)) + 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(20*e^4)
```

fricas [A] time = 0.91, size = 94, normalized size = 0.65

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^4x^4+10\,de^3x^3+12\,d^2e^2x^2+15\,d^3ex+24\,d^4\right)\sqrt{-e^2x^2+d^2}}{20\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^4*x^4 + 10*d
*e^3*x^3 + 12*d^2*e^2*x^2 + 15*d^3*e*x + 24*d^4)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.26, size = 73, normalized size = 0.51

$$\frac{3}{4}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{20}\left(24\,d^4e^{(-4)}+\left(15\,d^3e^{(-3)}+2\left(6\,d^2e^{(-2)}+\left(5\,de^{(-1)}+2\,x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^5*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/20*(24*d^4*e^(-4) + (15*d^3*e^(-3)
+ 2*(6*d^2*e^(-2) + (5*d*e^(-1) + 2*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 149, normalized size = 1.03

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^4}{5}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,d\,x^3}{2e}-\frac{3\sqrt{-e^2x^2+d^2}\,d^2x^2}{5e^2}-\frac{3\sqrt{-e^2x^2+d^2}\,d^3x}{4e^3}-\frac{6\sqrt{-e^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-6/5*d^4*
(-e^2*x^2+d^2)^(1/2)/e^4-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-3/4*d^3*x*(-e^2*x
^2+d^2)^(1/2)/e^3+3/4*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)
^(1/2)*x)
```

maxima [A] time = 0.97, size = 128, normalized size = 0.89

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}\,x^4-\frac{\sqrt{-e^2x^2+d^2}\,dx^3}{2\,e}-\frac{3\,\sqrt{-e^2x^2+d^2}\,d^2x^2}{5\,e^2}+\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{4\,e^4}-\frac{3\,\sqrt{-e^2x^2+d^2}\,d^3x}{4\,e^3}-\frac{6\,\sqrt{-e^2x^2+d^2}}{5\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*x^4 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x^3/e - 3/5*sqrt
(-e^2*x^2 + d^2)*d^2*x^2/e^2 + 3/4*d^5*arcsin(e*x/d)/e^4 - 3/4*sqrt(-e^2*x^
2 + d^2)*d^3*x/e^3 - 6/5*sqrt(-e^2*x^2 + d^2)*d^4/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 7.87, size = 357, normalized size = 2.48

$$d^2\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{f}\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{o}\end{cases}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + 2*d*e*Piecew
ise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(
```

```
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*x/d)/(8*e**
5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e
**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecew
ise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqrt(d**2 - e**
2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e, 0)), (x**6/
(6*sqrt(d**2)), True))
```

3.35 $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] 7/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2/3*d*x^2*(-e^2*x^2+d^2)^(1/2)
/e-1/4*x^3*(-e^2*x^2+d^2)^(1/2)-1/24*d^2*(21*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)
/e^3
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-2*d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (x^3*Sqrt[d^2 - e^2*x^2])/4 - (d^2*(
32*d + 21*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{\int \frac{x^2\left(-7d^2e^2-8de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{\int \frac{x\left(16d^3e^3+21d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}\\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{\left(7d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{\left(7d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x}\right.}{8e^2}\\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.70

$$\frac{21d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}\left(32d^3+21d^2ex+16de^2x^2+6e^3x^3\right)}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)) +
21*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^3)
```

fricas [A] time = 0.80, size = 83, normalized size = 0.72

$$-\frac{42\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3+16\,de^2x^2+21\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(42*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 16*d
*e^2*x^2 + 21*d^2*e*x + 32*d^3)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.25, size = 63, normalized size = 0.55

$$\frac{7}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\text{sgn}(d)-\frac{1}{24}\left(32\,d^3e^{(-3)}+\left(21\,d^2e^{(-2)}+2\left(8\,de^{(-1)}+3\,x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 7/8*d^4*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(32*d^3*e^(-3) + (21*d^2*e^(-2)
+ 2*(8*d*e^(-1) + 3*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 124, normalized size = 1.08

$$\frac{7d^4 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,x^3}{4}-\frac{2\sqrt{-e^2x^2+d^2}\,d\,x^2}{3e}-\frac{7\sqrt{-e^2x^2+d^2}\,d^2x}{8e^2}-\frac{4\sqrt{-e^2x^2+d^2}\,d^3}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*x^3*(-e^2*x^2+d^2)^(1/2)-7/8/e^2*d^2*x*(-e^2*x^2+d^2)^(1/2)+7/8/e^2*d^
4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-2/3*d*x^2*(-e^2*x^
2+d^2)^(1/2)/e-4/3*d^3*(-e^2*x^2+d^2)^(1/2)/e^3
```

maxima [A] time = 0.97, size = 103, normalized size = 0.90

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}\,x^3-\frac{2\sqrt{-e^2x^2+d^2}\,dx^2}{3\,e}+\frac{7\,d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}-\frac{7\sqrt{-e^2x^2+d^2}\,d^2x}{8\,e^2}-\frac{4\sqrt{-e^2x^2+d^2}\,d^3}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*x^3 - 2/3*sqrt(-e^2*x^2 + d^2)*d*x^2/e + 7/8*d^4*
arcsin(e*x/d)/e^3 - 7/8*sqrt(-e^2*x^2 + d^2)*d^2*x/e^2 - 4/3*sqrt(-e^2*x^2
+ d^2)*d^3/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(d+e\,x)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [C] time = 9.33, size = 386, normalized size = 3.36

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d
**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(
2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), Tru
e)) + 2*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(
d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + e**2*
Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e
**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*x/d)/
(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt
(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.36 $\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=83

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/3*x^2*(-e^2*x^2+d^2)^(1/2)-1/3*d
*(3*e*x+5*d)*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1809, 780, 217, 203}

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(x^2*Sqrt[d^2 - e^2*x^2])/3 - (d*(5*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(3*e^2
) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{\int \frac{x(-5d^2e^2-6de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}$$
$$= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}$$
$$= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}$$
$$= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.83

$$\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(5d^2+3dex+e^2x^2\right)}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
[Out] (-(Sqrt[d^2 - e^2*x^2]*(5*d^2 + 3*d*e*x + e^2*x^2)) + 3*d^3*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(3*e^2)
```

fricas [A] time = 0.87, size = 71, normalized size = 0.86

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(e^2x^2+3\,dex+5\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (e^2*x^2 + 3*d*e*x
+ 5*d^2)*sqrt(-e^2*x^2 + d^2))/e^2
```

giac [A] time = 0.25, size = 49, normalized size = 0.59

$$d^3\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\text{sgn}(d) - \frac{1}{3}\sqrt{-x^2e^2+d^2}\left(5\,d^2e^{(-2)} + \left(3\,de^{(-1)}+x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] d^3*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/3*sqrt(-x^2*e^2 + d^2)*(5*d^2*e^(-2) +
(3*d*e^(-1) + x)*x)
```

maple [A] time = 0.01, size = 98, normalized size = 1.18

$$\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e} - \frac{\sqrt{-e^2x^2+d^2}\,x^2}{3} - \frac{\sqrt{-e^2x^2+d^2}\,dx}{e} - \frac{5\sqrt{-e^2x^2+d^2}\,d^2}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2*(-e^2*x^2+d^2)^(1/2)-5/3*d^2/e^2*(-e^2*x^2+d^2)^(1/2)-d*x*(-e^2*x^
2+d^2)^(1/2)/e+d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 77, normalized size = 0.93

$$-\frac{1}{3}\sqrt{-e^2x^2+d^2}\,x^2+\frac{d^3\arcsin\left(\frac{ex}{d}\right)}{e^2}-\frac{\sqrt{-e^2x^2+d^2}\,dx}{e}-\frac{5\sqrt{-e^2x^2+d^2}\,d^2}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2 + d^3*arcsin(e*x/d)/e^2 - sqrt(-e^2*x^2 + d^2
)*d*x/e - 5/3*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(d+e\,x)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 5.62, size = 218, normalized size = 2.63

$$d^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4} \\ \frac{x^4}{4\sqrt{d^2}}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)
/e**2, True)) + 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt
(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)
/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2
*x**2/d**2)), True)) + e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e*
*4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2))
, True))
```

3.37 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] 3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-3/2*d*(-e^2*x^2+d^2)^(1/2)/e-1/2
*(e*x+d)*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{671, 641, 217, 203\}$

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}(3d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [**A**] time = 0.04, size = 58, normalized size = 0.70

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - (4d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-((4*d + e*x)*Sqrt[d^2 - e^2*x^2]) + 3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(2*e)
```

fricas [**A**] time = 0.73, size = 60, normalized size = 0.72

$$-\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \sqrt{-e^2x^2+d^2}\,(ex+4\,d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2
)*(e*x + 4*d))/e
```

giac [**A**] time = 0.25, size = 40, normalized size = 0.48

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\text{sgn}(d) - \frac{1}{2}\,\sqrt{-x^2e^2+d^2}\left(4\,de^{(-1)}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(4*d*e^(-1)
+ x)
```

maple [**A**] time = 0.01, size = 71, normalized size = 0.86

$$\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}\,x}{2} - \frac{2\sqrt{-e^2x^2+d^2}\,d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^
2+d^2)^(1/2)*x)-2*d*(-e^2*x^2+d^2)^(1/2)/e
```

maxima [A] time = 0.97, size = 53, normalized size = 0.64

$$\frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e}-\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,x-\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*x - 2*sqrt(-e^2*x^2 + d^
2)*d/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 5.04, size = 269, normalized size = 3.24

$$d^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+2de\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+ & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 2*d*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(
e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + e**2*Piecewise((-I*d**2*
acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*
x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**
2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.38 $\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx &= -\sqrt{d^2-e^2x^2}-\frac{\int \frac{-d^2e^2-2de^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\sqrt{d^2-e^2x^2}+d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+(2de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\sqrt{d^2-e^2x^2}+\frac{1}{2}d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+(2de)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{}{\sqrt{d^2}}\right.\\
&= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^2\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.00

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

fricas [A] time = 0.78, size = 73, normalized size = 1.11

$$-4\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d*log(-(d - sqrt(-e^2*x^2
+ d^2))/x) - sqrt(-e^2*x^2 + d^2)
```

giac [A] time = 0.26, size = 65, normalized size = 0.98

$$2\,d\arcsin\left(\frac{xe}{d}\right)\text{sgn}(d)-d\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d*arcsin(x*e/d)*sgn(d) - d*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) - sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 91, normalized size = 1.38

$$-\frac{d^2 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{2de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)+2*e*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(
1/2)*x)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.97, size = 62, normalized size = 0.94

$$2\,d\arcsin\left(\frac{ex}{d}\right)-d\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*d*arcsin(e*x/d) - d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) -
 sqrt(-e^2*x^2 + d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d+e\,x)^2}{x\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 6.96, size = 184, normalized size = 2.79

$$d^2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0\end{cases}\right)+e^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2 =\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwi}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e
*x))/d, True)) + 2*d*e*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/s
```

```
qrt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d
**2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e
**2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0))) + e**2*Piecewise((x**2/(2
*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
```

3.39 $\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e^
2*x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{x}-\frac{\int \frac{-2d^3e-d^2e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+(2de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+(de)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+e^2\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-}}\right.\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

fricas [A] time = 0.71, size = 79, normalized size = 1.16

$$-\frac{2\,ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-2\,ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*e*x*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/x
```

giac [A] time = 0.26, size = 107, normalized size = 1.57

$$\arcsin\left(\frac{xe}{d}\right)e\,\mathrm{sgn}(d)-2\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e*sgn(d) - 2*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) + 1/2*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) - 1/2*(d*e + sqr
t(-x^2*e^2 + d^2)*e)*e^(-1)/x
```

maple [A] time = 0.01, size = 93, normalized size = 1.37

$$-\frac{2de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-(-e^2*x^2+d^2)^(
1/2)/x-2*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.96, size = 64, normalized size = 0.94

$$e\arcsin\left(\frac{ex}{d}\right)-2\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{\sqrt{-e^2x^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e*arcsin(e*x/d) - 2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) -
 sqrt(-e^2*x^2 + d^2)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases}\frac{e^2\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{\sqrt{d^2-e^2\,x^2}}{x}-\frac{2\,d\,e\,\ln\left(\frac{\sqrt{d^2}+\sqrt{d^2-e^2\,x^2}}{x}\right)}{\sqrt{d^2}} & \text{if}\ \ e^2<0\\ \int \frac{e^2}{\sqrt{d^2-e^2\,x^2}}+\frac{d^2}{x^2\,\sqrt{d^2-e^2\,x^2}}+\frac{2\,d\,e}{x\,\sqrt{d^2-e^2\,x^2}}\,dx & \text{if}\ \ \neg e^2<0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] piecewise(e^2 < 0, - (d^2 - e^2*x^2)^(1/2)/x + (e^2*log(x*(-e^2)^(1/2) + (d
^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (2*d*e*log(((d^2)^(1/2) + (d^2 - e^2*x
^2)^(1/2))/x))/(d^2)^(1/2), ~e^2 < 0, int(e^2/(d^2 - e^2*x^2)^(1/2) + d^2/(
x^2*(d^2 - e^2*x^2)^(1/2)) + (2*d*e)/(x*(d^2 - e^2*x^2)^(1/2)), x))
```

sympy [C] time = 4.30, size = 207, normalized size = 3.04

$$d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) + 2*d*e*Piecewise((-aco
sh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e*x))/d, True)) + e**
2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0)))
```

3.40 $\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-1/2*(-e^2*x^2+d^2)^(1/2)/x^2-2*e
*(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1807, 807, 266, 63, 208}

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{\int \frac{-4d^3e-3d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{2}\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{4}\left(3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 122, normalized size = 1.52

$$\frac{e\left(-\frac{4d\sqrt{d^2-e^2x^2}}{x}-2de\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-e\sqrt{d^2-e^2x^2}\left(\frac{d^2}{e^2x^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (e*((-4*d*Sqrt[d^2 - e^2*x^2])/x - 2*d*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d] - e
*Sqrt[d^2 - e^2*x^2]*(d^2/(e^2*x^2) + ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]]/Sqrt
[1 - (e^2*x^2)/d^2])))/(2*d^2)
```

fricas [A] time = 0.94, size = 63, normalized size = 0.79

$$\frac{3\,e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}\,(4\,ex+d)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - sqrt(-e^2*x^2 + d^2)*(4
*e*x + d))/(d*x^2)
```

giac [B] time = 0.27, size = 170, normalized size = 2.12

$$-\frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d}+\frac{x^2\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^4}{x}+e^6\right)}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d}-\frac{\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^6}{x^2}\right)e^{(-8)}}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d +
1/8*x^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d) - 1/8*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^8/x + (d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*d*e^6/x^2)*e^(-8)/d^2
```

maple [A] time = 0.01, size = 86, normalized size = 1.08

$$-\frac{3e^2 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{2\sqrt{-e^2x^2+d^2}\,e}{dx}-\frac{\sqrt{-e^2x^2+d^2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)/x^2-3/2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)-2*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

maxima [A] time = 0.95, size = 83, normalized size = 1.04

$$-\frac{3\,e^2 \log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{2\,d}-\frac{2\,\sqrt{-e^2x^2+d^2}\,e}{dx}-\frac{\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 2*sqrt(-e^
2*x^2 + d^2)*e/(d*x) - 1/2*sqrt(-e^2*x^2 + d^2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^3\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 6.72, size = 214, normalized size = 2.68

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right)+2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases}\right)+e^2\left(\begin{cases} -\frac{\text{a}}{\ } \\ \frac{i\,\text{as}}{\ } \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*
x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e*x)
)/(2*d**3), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, A
bs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) +
 e**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(
e*x))/d, True))
```

3.41 $\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/3*(-e^2*x^2+d^2)^(1/2)/x^3-e*(-e
^2*x^2+d^2)^(1/2)/d/x^2-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [**A**] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (5*e^2*Sqr
t[d^2 - e^2*x^2])/(3*d^2*x) - (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{\int \frac{-6d^3e-5d^2e^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}+\frac{\int \frac{10d^4e^2+6d^3e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{e^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 87, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{d\left(d^2+3dex+5e^2x^2\right)}{x^3}-\frac{3e^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-((d*(d^2 + 3*d*e*x + 5*e^2*x^2))/x^3) - (3*e^3*ArcTa
nh[Sqrt[1 - (e^2*x^2)/d^2]])/Sqrt[1 - (e^2*x^2)/d^2]))/(3*d^3)
```

fricas [A] time = 0.93, size = 74, normalized size = 0.69

$$\frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(5\,e^2x^2+3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

[Out] 1/3*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (5*e^2*x^2 + 3*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*x^3)

giac [B] time = 0.30, size = 239, normalized size = 2.23

$$\frac{x^3\left(\frac{6\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^6}{x}+\frac{21\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^4}{x^2}+e^8\right)e}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2}-\frac{e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^2}-\frac{\left(\frac{21\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e^{16}}{x}+\frac{6\left(de+\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] 1/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x + 21*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^4/x^2 + e^8)*e/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2) - e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/24*(21*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^12/x^3)*e^(-15)/d^6

maple [A] time = 0.01, size = 114, normalized size = 1.07

$$-\frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d}-\frac{5\sqrt{-e^2x^2+d^2}\,e^2}{3d^2x}-\frac{\sqrt{-e^2x^2+d^2}\,e}{d\,x^2}-\frac{\sqrt{-e^2x^2+d^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x)

[Out] -e*(-e^2*x^2+d^2)^(1/2)/d/x^2-1/d*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x-1/3*(-e^2*x^2+d^2)^(1/2)/x^3

maxima [A] time = 0.97, size = 108, normalized size = 1.01

$$-\frac{e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^2}-\frac{5\,\sqrt{-e^2x^2+d^2}\,e^2}{3\,d^2x}-\frac{\sqrt{-e^2x^2+d^2}\,e}{dx^2}-\frac{\sqrt{-e^2x^2+d^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out] -e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 5/3*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x) - sqrt(-e^2*x^2 + d^2)*e/(d*x^2) - 1/3*sqrt(-e^2*x^2 + d^2)/x^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^4\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)),x)

[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)), x)

<u>**sympy**</u> **[C]** time = 6.09, size = 303, normalized size = 2.83

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2}-\frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2}-\frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)+2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwi} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d
**4), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) -
e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e
**2*asin(d/(e*x))/(2*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**
2) - 1)/d**2, Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)
/d**2, True))
```

3.42 $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=140

$$-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

```
[Out] -7/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/4*(-e^2*x^2+d^2)^(1/2)/x^4-2
/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-7/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-4/3*e^
3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A] time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*x^4) - (2*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (7*e^2
*Sqrt[d^2 - e^2*x^2])/(8*d^2*x^2) - (4*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) -
 (7*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
```

```
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{\int \frac{-8d^3e-7d^2e^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{\int \frac{21d^4e^2+16d^3e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{\int \frac{-32d^5e^3-21d^4e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{(7e^4)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d}{8d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{(7e^4)\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{16d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{(7e^2)\,\mathrm{Subst}\left(\int \frac{}{\frac{d^2}{e^2}}\right.}{}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2}}{d}\right.}{8d^3}$$

Mathematica [C] time = 0.15, size = 155, normalized size = 1.11

$$-\frac{e\sqrt{d^2-e^2x^2}\left(d\left(4d^2+3dex+8e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}+6e^3x^3\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+3e^3x^3\tanh^{-1}\left(\sqrt{1\right.\right.}{6d^4x^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/6*(e*Sqrt[d^2 - e^2*x^2]*(d*(4*d^2 + 3*d*e*x + 8*e^2*x^2)*Sqrt[1 - (e^2*
x^2)/d^2] + 3*e^3*x^3*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]] + 6*e^3*x^3*Sqrt[1 -
 (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (e^2*x^2)/d^2]))/(d^4*x^
3*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.67, size = 87, normalized size = 0.62

$$\frac{21\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3+21\,de^2x^2+16\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 21*d*e^
2*x^2 + 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*x^4)
```

giac [B] time = 0.29, size = 305, normalized size = 2.18

$$\frac{x^4\left(\frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^8}{x}+\frac{48\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^6}{x^2}+\frac{144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^4}{x^3}+3\,e^{10}\right)e^2}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3}-\frac{7\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/x + 48*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^6/x^2 + 144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^4/x^3 + 3*e^
10)*e^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3) - 7/8*e^4*log(1/2*abs(-2*d*e
 - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/192*(144*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^9*e^26/x + 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9*e^24/
x^2 + 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*e^22/x^3 + 3*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^4*d^9*e^20/x^4)*e^(-24)/d^12
```

maple [A] time = 0.02, size = 139, normalized size = 0.99

$$-\frac{7e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\,d^2}-\frac{4\sqrt{-e^2x^2+d^2}\,e^3}{3d^3x}-\frac{7\sqrt{-e^2x^2+d^2}\,e^2}{8d^2x^2}-\frac{2\sqrt{-e^2x^2+d^2}\,e}{3d\,x^3}-\frac{\sqrt{-e^2x^2+d^2}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -2/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-4/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x-7/8*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-7/8*e^4/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1
/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4*(-e^2*x^2+d^2)^(1/2)/x^4
```

maxima [A] time = 0.97, size = 133, normalized size = 0.95

$$-\frac{7\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d^3}-\frac{4\,\sqrt{-e^2x^2+d^2}\,e^3}{3\,d^3x}-\frac{7\,\sqrt{-e^2x^2+d^2}\,e^2}{8\,d^2x^2}-\frac{2\,\sqrt{-e^2x^2+d^2}\,e}{3\,dx^3}-\frac{\sqrt{-e^2x^2+d^2}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 4/3*sqrt
(-e^2*x^2 + d^2)*e^3/(d^3*x) - 7/8*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^2) - 2/3
*sqrt(-e^2*x^2 + d^2)*e/(d*x^3) - 1/4*sqrt(-e^2*x^2 + d^2)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^5\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 10.32, size = 449, normalized size = 3.21

$$d^2\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}}}{3d^4} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-}}{3} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*s
qrt(d**2/(e**2*x**2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) -
 3*e**4*acosh(d/(e*x))/(8*d**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 - 3*I*e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))
/(8*d**5), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*
x**2) - 2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/
(e**2*x**2) + 1)/(3*d**4), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2
) - 1)/(2*d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1
), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3), True))
```

3.43 $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}$$

```
[Out] -3/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/5*(-e^2*x^2+d^2)^(1/2)/x^5-1
/2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/4*e^
3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*x^5) - (e*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) - (3*e^2*S
qrt[d^2 - e^2*x^2])/(5*d^2*x^3) - (3*e^3*Sqrt[d^2 - e^2*x^2])/(4*d^3*x^2) -
 (6*e^4*Sqrt[d^2 - e^2*x^2])/(5*d^4*x) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(4*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
```

```
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{\int \frac{-10d^3e-9d^2e^2x}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} + \frac{\int \frac{36d^4e^2+30d^3e^3x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{\int \frac{-90d^5e^3-72d^4e^4x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} + \frac{\int \frac{144d^6e^4+90d^5e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{120d^8}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \ldots$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \ldots$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \ldots$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \ldots$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.47

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^5+3d^3e^2x^2+10e^5x^5\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+6de^4x^4\right)}{5d^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^5 + 3*d^3*e^2*x^2 + 6*d*e^4*x^4 + 10*e^5*x^5*H
ypergeometric2F1[1/2, 3, 3/2, 1 - (e^2*x^2)/d^2]))/(d^5*x^5)
```

fricas [A] time = 0.77, size = 98, normalized size = 0.58

$$\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(24\,e^4x^4+15\,de^3x^3+12\,d^2e^2x^2+10\,d^3ex+4\,d^4\right)\sqrt{-e^2x^2+d^2}}{20\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (24*e^4*x^4 + 15*d*e^
3*x^3 + 12*d^2*e^2*x^2 + 10*d^3*e*x + 4*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*x^5
)
```

giac [B] time = 0.27, size = 365, normalized size = 2.16

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{10}}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^8}{x^2}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^6}{x^3}+\frac{110\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^4}{x^4}+e^{12}\right)e^3}{160\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4}-\frac{3\,e^5\log\left(\frac{|-2\,de}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x + 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^8/x^2 + 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^6/x^3 + 110*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + e^12)*e^3/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)^5*d^4) - 3/4*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^
(-2)/abs(x))/d^4 - 1/160*(110*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^38/x +
40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^16*e^36/x^2 + 15*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^3*d^16*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^16*e^32/x
^4 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^16*e^30/x^5)*e^(-35)/d^20
```

maple [A] time = 0.02, size = 164, normalized size = 0.97

$$-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4\sqrt{d^2}\,d^3}-\frac{6\sqrt{-e^2x^2+d^2}\,e^4}{5d^4x}-\frac{3\sqrt{-e^2x^2+d^2}\,e^3}{4d^3x^2}-\frac{3\sqrt{-e^2x^2+d^2}\,e^2}{5d^2x^3}-\frac{\sqrt{-e^2x^2+d^2}\,e}{2d\,x^4}-\frac{\sqrt{-e^2x^2+}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x-1/
2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/4*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-3/4/d^3
*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5*(-e^2
*x^2+d^2)^(1/2)/x^5
```

maxima [A] time = 0.98, size = 158, normalized size = 0.93

$$-\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{4\,d^4}-\frac{6\,\sqrt{-e^2x^2+d^2}\,e^4}{5\,d^4x}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^3}{4\,d^3x^2}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^2}{5\,d^2x^3}-\frac{\sqrt{-e^2x^2+d^2}\,e}{2\,dx^4}-\frac{\sqrt{-e^2x^2}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 - 6/5*sqrt
(-e^2*x^2 + d^2)*e^4/(d^4*x) - 3/4*sqrt(-e^2*x^2 + d^2)*e^3/(d^3*x^2) - 3/5
```

```
*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^3) - 1/2*sqrt(-e^2*x^2 + d^2)*e/(d*x^4) -
1/5*sqrt(-e^2*x^2 + d^2)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d + e\,x)^2}{x^6\,\sqrt{d^2 - e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 8.96, size = 510, normalized size = 3.02

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{5d^2x^4} - \frac{4e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^4x^2} - \frac{8e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{5d^2x^4} - \frac{4ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^4x^2} - \frac{8ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^6} & \text{otherwise} \end{cases}\right) + 2de\left(\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \cdots \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \cdots \end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**6/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(5*d**2*x**4) - 4*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(15*d**4*x**2) - 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(15
*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(5*d*
*2*x**4) - 4*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**4*x**2) - 8*I*e**5*s
qrt(-d**2/(e**2*x**2) + 1)/(15*d**6), True)) + 2*d*e*Piecewise((-1/(4*e*x**
5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1))
+ 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) - 3*e**4*acosh(d/(e*x))/(8*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
 + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e**3/(8*d**4*x*sqrt(
-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))/(8*d**5), True)) + e**2*Pi
ecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**4), T
rue))
```

3.44 $\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*d^4*(e*x+d)^2/e^6/(-e^2*x^2+d^2)^(5/2)-22/15*d^3*(e*x+d)/e^6/(-e^2*x^2+
d^2)^(3/2)-2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+2/15*d*(23*e*x+30*d)/e^
6/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^6
```

Rubi [A] time = 0.27, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1814, 641, 217, 203}

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (22*d^3*(d + e*x))/(15*e^
6*(d^2 - e^2*x^2)^(3/2)) + (2*d*(30*d + 23*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + Sqrt[d^2 - e^2*x^2]/e^6 - (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{16d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{30d^5}{e^5}+\frac{15d^4x}{e^4}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\int \frac{1}{\sqrt{d^2-}}}{e^5}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\,\text{Subst}}{}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d\tan^{-1}\left(\right.}{e^6}$$

Mathematica [A] time = 0.21, size = 111, normalized size = 0.78

$$\frac{56d^4-82d^3ex-32d^2e^2x^2-\frac{30(d-ex)^3(d+ex)\sin^{-1}\left(\frac{ex}{d}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}+76de^3x^3-15e^4x^4}{15e^6(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (56*d^4 - 82*d^3*e*x - 32*d^2*e^2*x^2 + 76*d*e^3*x^3 - 15*e^4*x^4 - (30*(d
- e*x)^3*(d + e*x)*ArcSin[(e*x)/d])/Sqrt[1 - (e^2*x^2)/d^2])/(15*e^6*(d - e
*x)^2*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 0.76, size = 188, normalized size = 1.31

$$\frac{56\,de^4x^4-112\,d^2e^3x^3+112\,d^4ex-56\,d^5+60\left(de^4x^4-2\,d^2e^3x^3+2\,d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^4\right.}{15\left(e^{10}x^4-2\,de^9x^3+2\,d^3e^7x-d^4e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(56*d*e^4*x^4 - 112*d^2*e^3*x^3 + 112*d^4*e*x - 56*d^5 + 60*(d*e^4*x^4
 - 2*d^2*e^3*x^3 + 2*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + (15*e^4*x^4 - 76*d*e^3*x^3 + 32*d^2*e^2*x^2 + 82*d^3*e*x - 56*d^4)*sqr
t(-e^2*x^2 + d^2))/(e^10*x^4 - 2*d*e^9*x^3 + 2*d^3*e^7*x - d^4*e^6)
```

giac [A] time = 0.30, size = 106, normalized size = 0.74

$$-2\,d\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{\left(56\,d^6e^{(-6)}+\left(30\,d^5e^{(-5)}-\left(140\,d^4e^{(-4)}+\left(70\,d^3e^{(-3)}-\left(105\,d^2e^{(-2)}+\left(46\,de^{(-1)}-15\right.\right.\right.\right.\right.\right.}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*d*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/15*(56*d^6*e^(-6) + (30*d^5*e^(-5) - (
140*d^4*e^(-4) + (70*d^3*e^(-3) - (105*d^2*e^(-2) + (46*d*e^(-1) - 15*x)*x)
*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 193, normalized size = 1.35

$$-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2d\,x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{7d^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{28d^4x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{2d\,x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+\frac{56d^6}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -x^6/(-e^2*x^2+d^2)^(5/2)+7/e^2*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-28/3/e^4*d^4*x
^2/(-e^2*x^2+d^2)^(5/2)+56/15/e^6*d^6/(-e^2*x^2+d^2)^(5/2)+2/5/e*d*x^5/(-e^
2*x^2+d^2)^(5/2)-2/3/e^3*d*x^3/(-e^2*x^2+d^2)^(3/2)+2/e^5*d*x/(-e^2*x^2+d^2
)^(1/2)-2/e^5*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [B] time = 1.02, size = 276, normalized size = 1.93

$$\frac{2}{15}\,dex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2\,dx\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*d*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/(-e^2*x^2 + d^2)
^(5/2) - 2/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d
^2)^(3/2)*e^4))/e + 7*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 28/3*d^4*x^2/(
(-e^2*x^2 + d^2)^(5/2)*e^4) + 56/15*d^6/((-e^2*x^2 + d^2)^(5/2)*e^6) + 8/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 14/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^5)
- 2*d*arcsin(e*x/d)/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,{\left(d+e\,x\right)}^2}{{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.45 $\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] 1/5*d^3*(e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)-17/15*d^2*(e*x+d)/e^5/(-e^2*x^2+
d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(13*e*x+15*d)/e^5/(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1814, 12, 217, 203}

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (17*d^2*(d + e*x))/(15*e^
5*(d^2 - e^2*x^2)^(3/2)) + (2*(15*d + 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2])
 - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.79

$$\frac{16d^3-15d(d-ex)^2\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)-17d^2ex-22de^2x^2+26e^3x^3}{15e^5(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (16*d^3 - 17*d^2*e*x - 22*d*e^2*x^2 + 26*e^3*x^3 - 15*d*(d - e*x)^2*Sqrt[1
- (e^2*x^2)/d^2]*ArcSin[(e*x)/d])/(15*e^5*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 0.69, size = 172, normalized size = 1.42

$$\frac{16\,e^4x^4-32\,de^3x^3+32\,d^3ex-16\,d^4+30\left(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(26\,e^3x^3-22\right.}{15\left(e^9x^4-2\,de^8x^3+2\,d^3e^6x-d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*e^4*x^4 - 32*d*e^3*x^3 + 32*d^3*e*x - 16*d^4 + 30*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (26*e
```

```
^3*x^3 - 22*d*e^2*x^2 - 17*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^4
 - 2*d*e^8*x^3 + 2*d^3*e^6*x - d^4*e^5)
```

giac [A] time = 0.28, size = 95, normalized size = 0.79

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{\left(16\,d^5e^{(-5)}+\left(15\,d^4e^{(-4)}-\left(40\,d^3e^{(-3)}+\left(35\,d^2e^{(-2)}-2\left(15\,de^{(-1)}+13\,x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-5)*sgn(d) - 1/15*(16*d^5*e^(-5) + (15*d^4*e^(-4) - (40*d
^3*e^(-3) + (35*d^2*e^(-2) - 2*(15*d*e^(-1) + 13*x)*x)*x)*x)*x)*sqrt(-x^2*e
^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.02, size = 236, normalized size = 1.95

$$\frac{x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2d\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{8d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{3d^4x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*x^5/(-e^2*x^2+d^2)^(5/2)-1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)+6/5/e^4*x/(-e
^2*x^2+d^2)^(1/2)-1/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)+2*d/e*x^4/(-e^2*x^2+d^2)^(5/2)-8/3*d^3/e^3*x^2/(-e^2*x^2+d^2)^(5/2)+16/
15*d^5/e^5/(-e^2*x^2+d^2)^(5/2)+1/2*d^2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d
^4/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^2/e^4*x/(-e^2*x^2+d^2)^(3/2)
```

maxima [B] time = 1.00, size = 298, normalized size = 2.46

$$\frac{1}{15}e^2x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{1}{3}x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)+\frac{}{\left(-e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*
x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 2*d*x^4/((-e^
2*x^2 + d^2)^(5/2)*e) + 1/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8/3*d^3*
x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 16/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^5) + 11/30*d^2*x/((-e^2*x^2 + d^2)^(3
/2)*e^4) - 4/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e*x/d)/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(d+e\,x)^2}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.46 $\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d^2*(e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(e*x+d)/e^4/(-e^2*x^2+d^2)
^(3/2)+1/5*(2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1635, 637}

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d + e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d + 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}\\
&= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.65

$$\frac{2d^3 - 4d^2ex + de^2x^2 + 2e^3x^3}{5de^4(d - ex)^2\sqrt{d^2 - e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3)/(5*d*e^4*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

fricas [A] time = 0.85, size = 116, normalized size = 1.20

$$\frac{2\,e^4x^4 - 4\,de^3x^3 + 4\,d^3ex - 2\,d^4 - \left(2\,e^3x^3 + de^2x^2 - 4\,d^2ex + 2\,d^3\right)\sqrt{-e^2x^2 + d^2}}{5\left(de^8x^4 - 2\,d^2e^7x^3 + 2\,d^4e^5x - d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 + d*e^2*x^2 -
 4*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 - 2*d^2*e^7*x^3 + 2*d^
4*e^5*x - d^5*e^4)
```

giac [A] time = 0.29, size = 63, normalized size = 0.65

$$-\frac{\left(2\,d^4e^{(-4)} + \left(x^2\left(\frac{2\,xe}{d} + 5\right) - 5\,d^2e^{(-2)}\right)x^2\right)\sqrt{-x^2e^2 + d^2}}{5\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(2*d^4*e^(-4) + (x^2*(2*x*e/d + 5) - 5*d^2*e^(-2))*x^2)*sqrt(-x^2*e^2
+ d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 65, normalized size = 0.67

$$\frac{(-ex + d)\,(ex + d)^3\left(2e^3x^3 + d\,e^2x^2 - 4d^2ex + 2d^3\right)}{5\left(-e^2x^2 + d^2\right)^{\frac{7}{2}}\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3+d*e^2*x^2-4*d^2*e*x+2*d^3)/d/e^4/(-e^2*x^
2+d^2)^(7/2)
```

maxima [A] time = 0.45, size = 155, normalized size = 1.60

$$\frac{x^4}{\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}} + \frac{dx^3}{\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}e} - \frac{d^2x^2}{\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}e^2} - \frac{3\,d^3x}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}e^3} + \frac{2\,d^4}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}e^4} + \frac{dx}{5\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/(-e^2*x^2 + d^2)^(5/2) + d*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - d^2*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/5*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2/5*d^
```

```
4/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 2/5
*x/(sqrt(-e^2*x^2 + d^2)*d*e^3)
```

mupad [B] time = 2.89, size = 66, normalized size = 0.68

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(2\,d^3 - 4\,d^2\,e\,x + d\,e^2\,x^2 + 2\,e^3\,x^3\right)}{5\,d\,e^4\,\left(d + e\,x\right)\,\left(d - e\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 + d*e^2*x^2 - 4*d^2*e*x))/(5*d*e^
4*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(d + ex)^2}{(-(-d + ex)\,(d + ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.47 $\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}+\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*d*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^(5/2)-7/15*(e*x+d)/e^3/(-e^2*x^2+d^2)^(3
/2)+1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1635, 778, 191}

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (7*(d + e*x))/(15*e^3*(d^2
- e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$
$$= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}$$
$$= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.72

$$\frac{-4d^3+8d^2ex-2de^2x^2+e^3x^3}{15d^2e^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)/(15*d^2*e^3*(d - e*x)^2*Sqrt[d
^2 - e^2*x^2])
```

fricas [A] time = 0.87, size = 117, normalized size = 1.34

$$-\frac{4\,e^4x^4-8\,de^3x^3+8\,d^3ex-4\,d^4+\left(e^3x^3-2\,de^2x^2+8\,d^2ex-4\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^7x^4-2\,d^3e^6x^3+2\,d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(4*e^4*x^4 - 8*d*e^3*x^3 + 8*d^3*e*x - 4*d^4 + (e^3*x^3 - 2*d*e^2*x^2
 + 8*d^2*e*x - 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 - 2*d^3*e^6*x^3 +
2*d^5*e^4*x - d^6*e^3)
```

giac [A] time = 0.28, size = 61, normalized size = 0.70

$$\frac{\left(4\,d^3e^{(-3)}-\left(x\left(\frac{x^2e^2}{d^2}+5\right)+10\,de^{(-1)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*d^3*e^(-3) - (x*(x^2*e^2/d^2 + 5) + 10*d*e^(-1))*x^2)*sqrt(-x^2*e^2
 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 66, normalized size = 0.76

$$-\frac{(-ex+d)\,(ex+d)^3\left(-e^3x^3+2d\,e^2x^2-8d^2ex+4d^3\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^3*(-e^3*x^3+2*d*e^2*x^2-8*d^2*e*x+4*d^3)/d^2/e^3/(-e
^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.45, size = 131, normalized size = 1.51

$$\frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2\,dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{4\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}+\frac{\cdots}{15\sqrt{-e^2x^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 1/1
0*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e^3
) + 1/30*x/((-e^2*x^2 + d^2)^(3/2)*e^2) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*
e^2)
```

mupad [B] time = 2.87, size = 67, normalized size = 0.77

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3-8\,d^2\,e\,x+2\,d\,e^2\,x^2-e^3\,x^3\right)}{15\,d^2\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(4*d^3 - e^3*x^3 + 2*d*e^2*x^2 - 8*d^2*e*x))/(15*d^
2*e^3*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.48 $\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=89

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*(e*x+d)/d/e^2/(-e^2*x^2+d^2)^(3
/2)-4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {789, 639, 191}

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(15*d*e^2*(d^2 -
e^2*x^2)^(3/2)) - (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\
&= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\
&= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.70

$$\frac{d^3 - 2d^2ex + 8de^2x^2 - 4e^3x^3}{15d^3e^2(d - ex)^2\sqrt{d^2 - e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3)/(15*d^3*e^2*(d - e*x)^2*Sqrt[d^
2 - e^2*x^2])
```

fricas [A] time = 0.86, size = 117, normalized size = 1.31

$$\frac{e^4x^4 - 2\,de^3x^3 + 2\,d^3ex - d^4 + \left(4\,e^3x^3 - 8\,de^2x^2 + 2\,d^2ex - d^3\right)\sqrt{-e^2x^2 + d^2}}{15\,\left(d^3e^6x^4 - 2\,d^4e^5x^3 + 2\,d^6e^3x - d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4 + (4*e^3*x^3 - 8*d*e^2*x^2 +
2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 - 2*d^4*e^5*x^3 + 2*d^6
*e^3*x - d^7*e^2)
```

giac [A] time = 0.28, size = 64, normalized size = 0.72

$$\frac{\left(\left(2\,x\left(\frac{2\,x^2e^3}{d^3} - \frac{5\,e}{d}\right) - 5\right)x^2 - d^2e^{(-2)}\right)\sqrt{-x^2e^2 + d^2}}{15\,\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((2*x*(2*x^2*e^3/d^3 - 5*e/d) - 5)*x^2 - d^2*e^(-2))*sqrt(-x^2*e^2 + d
^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 64, normalized size = 0.72

$$\frac{(-ex + d)\,(ex + d)^3\,\left(-4e^3x^3 + 8d\,e^2x^2 - 2d^2ex + d^3\right)}{15\,\left(-e^2x^2 + d^2\right)^{\frac{7}{2}}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^3*(-4*e^3*x^3+8*d*e^2*x^2-2*d^2*e*x+d^3)/d^3/e^2/(-e^
2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 109, normalized size = 1.22

$$\frac{x^2}{3\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}} + \frac{2\,dx}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}e} + \frac{d^2}{15\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}e^2} - \frac{2\,x}{15\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}de} - \frac{4\,x}{15\,\sqrt{-e^2x^2 + d^2}\,d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(-e^2*x^2 + d^2)^(5/2) + 2/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*
d^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 2/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e) - 4/
15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

mupad [B] time = 2.86, size = 65, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3-2\,d^2\,e\,x+8\,d\,e^2\,x^2-4\,e^3\,x^3\right)}{15\,d^3\,e^2\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 - 4*e^3*x^3 + 8*d*e^2*x^2 - 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex\right)^{2}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.49 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=77

$$\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*(e*x+d)/e/(-e^2*x^2+d^2)^(5/2)+1/5*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/5*x/d^4
/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {653, 192, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{3}{5}\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\ &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}\\ &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.82

$$\frac{2d^3+d^2ex-4de^2x^2+2e^3x^3}{5d^4e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)/(5*d^4*e*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

fricas [A] time = 0.61, size = 116, normalized size = 1.51

$$\frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-\left(2\,e^3x^3-4\,de^2x^2+d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^4e^5x^4-2\,d^5e^4x^3+2\,d^7e^2x-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^2
 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 + 2*
d^7*e^2*x - d^8*e)
```

giac [A] time = 0.31, size = 61, normalized size = 0.79

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(x^2\left(\frac{2\,x^2e^4}{d^4}-\frac{5\,e^2}{d^2}\right)+5\right)x+2\,de^{(-1)}\right)}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((x^2*(2*x^2*e^4/d^4 - 5*e^2/d^2) + 5)*x + 2*d*e^
(-1))/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 65, normalized size = 0.84

$$\frac{(-ex+d)\,(ex+d)^3\left(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3\right)}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x^
2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 78, normalized size = 1.01

$$\frac{2\,x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2\,d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{2\,x}{5\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((-
e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

mupad [B] time = 2.81, size = 66, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+d^2\,e\,x-4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 - 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.50 $\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 2/5*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3/2
)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(16*e*x+15*d)/d^5/(-e^2*x^2+d^2)
^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d + 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d + 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2-16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,\right.}{2d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{c\right.}{d^4e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.69

$$\frac{3d^5+30d^4ex-40d^2e^3x^3+3d^5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+16e^5x^5}{15d^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (3*d^5 + 30*d^4*e*x - 40*d^2*e^3*x^3 + 16*e^5*x^5 + 3*d^5*Hypergeometric2F1
[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^5*(d^2 - e^2*x^2)^(5/2))
```

fricas [A] time = 0.86, size = 169, normalized size = 1.44

$$\frac{26\,e^4x^4 - 52\,de^3x^3 + 52\,d^3ex - 26\,d^4 + 15\left(e^4x^4 - 2\,de^3x^3 + 2\,d^3ex - d^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(16\,e^3x^3 - 17\,de^2x^2\right.}{15\left(d^5e^4x^4 - 2\,d^6e^3x^3 + 2\,d^8ex - d^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 - 52*d*e^3*x^3 + 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (16*e^3*x^3
- 17*d*e^2*x^2 - 22*d^2*e*x + 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 -
2*d^6*e^3*x^3 + 2*d^8*e*x - d^9)
```

giac [A] time = 0.29, size = 118, normalized size = 1.01

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{16\,xe^5}{d^5}+\frac{15\,e^4}{d^4}\right)-\frac{40\,e^3}{d^3}\right)x-\frac{35\,e^2}{d^2}\right)x+\frac{30\,e}{d}\right)x+26\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(16*x*e^5/d^5 + 15*e^4/d^4) - 40*e^3/d^3)*
x - 35*e^2/d^2)*x + 30*e/d)*x + 26)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d*e
- 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5
```

maple [A] time = 0.01, size = 160, normalized size = 1.37

$$\frac{2ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{8ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}+\frac{16ex}{15\sqrt{-e^2x^2+d^2}\,d^5}+\frac{\ }{\sqrt{-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 2/5/(-e^2*x^2+d^2)^(5/2)+2/5/d*e*x/(-e^2*x^2+d^2)^(5/2)+8/15/d^3*e*x/(-e^2*
x^2+d^2)^(3/2)+16/15/d^5*e*x/(-e^2*x^2+d^2)^(1/2)+1/3/d^2/(-e^2*x^2+d^2)^(3
/2)+1/d^4/(-e^2*x^2+d^2)^(1/2)-1/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.46, size = 154, normalized size = 1.32

$$\frac{2\,ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{8\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{16\,ex}{15\,\sqrt{-e^2x^2+d^2}\,d^5}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2}{\ }\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d) + 2/5/(-e^2*x^2 + d^2)^(5/2) + 8/15*e*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^2) + 16/15*e*x
/(sqrt(-e^2*x^2 + d^2)*d^5) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/a
bs(x))/d^5 + 1/(sqrt(-e^2*x^2 + d^2)*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.51 $\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 2/5*e*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e*(13*e*x+10*d)/d^4/(-e^2*x^2+d
^2)^(3/2)-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e*(41*e*x+30*d)/d^6/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1805, 807, 266, 63, 208}

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e*(10*d + 13*e*x))/(15*d^4
*(d^2 - e^2*x^2)^(3/2)) + (e*(30*d + 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/(d^6*x) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^2-10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^2+30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^2-30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{(2e)\int -}{}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e\,\mathrm{Subst}}{}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{2\,\mathrm{Subst}}{}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{2e\tanh}{}$$

Mathematica [C] time = 0.05, size = 90, normalized size = 0.62

$$\frac{-15d^6+105d^4e^2x^2-140d^2e^4x^4+6d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+56e^6x^6}{15d^6x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-15*d^6 + 105*d^4*e^2*x^2 - 140*d^2*e^4*x^4 + 56*e^6*x^6 + 6*d^5*e*x*Hyper
geometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^6*x*(d^2 - e^2*x^2)^(
5/2))
```

fricas [A] time = 0.68, size = 195, normalized size = 1.34

$$\frac{46\,e^5x^5-92\,de^4x^4+92\,d^3e^2x^2-46\,d^4ex+30\left(e^5x^5-2\,de^4x^4+2\,d^3e^2x^2-d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(56\,e^4x\right.}{15\left(d^6e^4x^5-2\,d^7e^3x^4+2\,d^9ex^2-d^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(46*e^5*x^5 - 92*d*e^4*x^4 + 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^5
 - 2*d*e^4*x^4 + 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) - (56*e^4*x^4 - 82*d*e^3*x^3 - 32*d^2*e^2*x^2 + 76*d^3*e*x - 15*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^6*e^4*x^5 - 2*d^7*e^3*x^4 + 2*d^9*e*x^2 - d^10*x)
```

giac [A] time = 0.29, size = 188, normalized size = 1.30

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{41\,xe^6}{d^6}+\frac{30\,e^5}{d^5}\right)-\frac{95\,e^4}{d^4}\right)x-\frac{70\,e^3}{d^3}\right)x+\frac{60\,e^2}{d^2}\right)x+\frac{46\,e}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{2\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^6}+\frac{\quad}{2\left(de+\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(41*x*e^6/d^6 + 30*e^5/d^5) - 95*e^4/d^4)*
x - 70*e^3/d^3)*x + 60*e^2/d^2)*x + 46*e/d)/(x^2*e^2 - d^2)^3 - 2*e*log(1/2
*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 + 1/2*x*e^3/((d*
e + sqrt(-x^2*e^2 + d^2)*e)*d^6) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1
)/(d^6*x)
```

maple [A] time = 0.01, size = 193, normalized size = 1.33

$$\frac{7e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{2e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{28e^2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}+\frac{2e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}-\frac{2e\ln\left(\frac{2d^2+2\sqrt{}}{}\right.}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 7/5*e^2*x/d^2/(-e^2*x^2+d^2)^(5/2)+28/15*e^2/d^4*x/(-e^2*x^2+d^2)^(3/2)+56/
15*e^2/d^6*x/(-e^2*x^2+d^2)^(1/2)+2/5/d*e/(-e^2*x^2+d^2)^(5/2)+2/3/d^3*e/(-
e^2*x^2+d^2)^(3/2)+2/d^5*e/(-e^2*x^2+d^2)^(1/2)-2/d^5*e/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/x/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.47, size = 187, normalized size = 1.29

$$\frac{7\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{2\,e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{28\,e^2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}+\frac{2\,e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}-\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}+\frac{56\,e^2x}{15\sqrt{-e^2x^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 7/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 2/5*e/((-e^2*x^2 + d^2)^(5/2)*d) +
 28/15*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 2/3*e/((-e^2*x^2 + d^2)^(3/2)*d
^3) - 1/((-e^2*x^2 + d^2)^(5/2)*x) + 56/15*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^6)
 - 2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^6 + 2*e/(sqrt(
-e^2*x^2 + d^2)*d^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^2\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.52 $\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] 2/5*e^2*(e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(6*e*x+5*d)/d^5/(-e^2*x^2+
d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(11*e*x+10*d
)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2-2*e*(-e^2*x^2+d
^2)^(1/2)/d^7/x
```

Rubi [A] time = 0.36, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d + 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d + 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int \frac{60d^3e+1\ldots}{x^2\sqrt{d^2\ldots}}}{30\ldots}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-\ldots}}{d^7x}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-\ldots}}{d^7x}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-\ldots}}{d^7x}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-\ldots}}{d^7x}$$

Mathematica [C] time = 0.06, size = 117, normalized size = 0.64

$$\frac{e\left(-10d^6+60d^4e^2x^2-80d^2e^4x^4+d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+d^5ex\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+32e^6x^6\right)}{5d^7x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (e*(-10*d^6 + 60*d^4*e^2*x^2 - 80*d^2*e^4*x^4 + 32*e^6*x^6 + d^5*e*x*Hyperg
eometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2] + d^5*e*x*Hypergeometric2F1[-
5/2, 2, -3/2, 1 - (e^2*x^2)/d^2]))/(5*d^7*x*(d^2 - e^2*x^2)^(5/2))
```

fricas [A] time = 0.86, size = 216, normalized size = 1.19

$$\frac{54\,e^6x^6-108\,de^5x^5+108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6-2\,de^5x^5+2\,d^3e^3x^3-d^4e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(64}{10\left(d^7e^4x^6-2\,d^8e^3x^5+2\,d^{10}ex^3-d^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 - 108*d*e^5*x^5 + 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 - 2*d*e^5*x^5 + 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) - (64*e^5*x^5 - 83*d*e^4*x^4 - 58*d^2*e^3*x^3 + 94*d^3*e^2*x^2
- 10*d^4*e*x - 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 - 2*d^8*e^3*x^5 +
2*d^10*e*x^3 - d^11*x^2)
```

giac [A] time = 0.30, size = 260, normalized size = 1.43

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(2\left(x\left(\frac{11\,xe^7}{d^7}+\frac{10\,e^6}{d^6}\right)-\frac{25\,e^5}{d^5}\right)x-\frac{45\,e^4}{d^4}\right)x+\frac{30\,e^3}{d^3}\right)x+\frac{27\,e^2}{d^2}\right)}{5\left(x^2e^2-d^2\right)^3}-\frac{9\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^7}+\frac{x^2\left(8\left(\right.\right.}{8\left(de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*(((2*(x*(11*x*e^7/d^7 + 10*e^6/d^6) - 25*e^5/d^5)
*x - 45*e^4/d^4)*x + 30*e^3/d^3)*x + 27*e^2/d^2)/(x^2*e^2 - d^2)^3 - 9/2*e^
2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^7 + 1/8*x
^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2 + d
^2)*e)^2*d^7) - 1/8*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^8/x + (d*e + sq
rt(-x^2*e^2 + d^2)*e)^2*d^7*e^6/x^2)*e^(-8)/d^14
```

maple [A] time = 0.02, size = 224, normalized size = 1.23

$$\frac{12e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{9e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}-\frac{2e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}+\frac{16e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{3e^2}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{1}{2\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 9/10*e^2/d^2/(-e^2*x^2+d^2)^(5/2)+3/2*e^2/d^4/(-e^2*x^2+d^2)^(3/2)+9/2*e^2/
d^6/(-e^2*x^2+d^2)^(1/2)-9/2*e^2/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
```

```
e^2*x^2+d^2)^(1/2))/x)-2/d*e/x/(-e^2*x^2+d^2)^(5/2)+12/5/d^3*e^3*x/(-e^2*x^
2+d^2)^(5/2)+16/5/d^5*e^3*x/(-e^2*x^2+d^2)^(3/2)+32/5/d^7*e^3*x/(-e^2*x^2+d
^2)^(1/2)-1/2/x^2/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.47, size = 218, normalized size = 1.20

$$\frac{12\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{9\,e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{16\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{3\,e^2}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{2\,e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}+\frac{32\,e}{5\sqrt{-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 12/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 9/10*e^2/((-e^2*x^2 + d^2)^(5/2)*
d^2) + 16/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 3/2*e^2/((-e^2*x^2 + d^2)^
(3/2)*d^4) - 2*e/((-e^2*x^2 + d^2)^(5/2)*d*x) + 32/5*e^3*x/(sqrt(-e^2*x^2 +
 d^2)*d^7) - 9/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^
7 + 9/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^6) - 1/2/((-e^2*x^2 + d^2)^(5/2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^3\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.53 $\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{}{5d}$$

```
[Out] 2/5*e^3*(e*x+d)/d^4/(-e^2*x^2+d^2)^(5/2)+1/15*e^3*(23*e*x+20*d)/d^6/(-e^2*x
^2+d^2)^(3/2)-7*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+2/15*e^3*(53*e*x+45
*d)/d^8/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^6/x^3-e*(-e^2*x^2+d
^2)^(1/2)/d^7/x^2-14/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^8/x
```

Rubi [A] time = 0.47, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{7e^3\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + (e^3*(20*d + 23*e*x))/(15
*d^6*(d^2 - e^2*x^2)^(3/2)) + (2*e^3*(45*d + 53*e*x))/(15*d^8*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^6*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d^7*x^
2) - (14*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^8*x) - (7*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{10e^3x^3}{d}-\frac{8e^4x^4}{d^2}}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{60e^3x^3}{d}+\frac{46e^4x^4}{d^2}}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2-\frac{90e\ldots}{\ldots}}{x^4\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{\int \frac{90d^3e\ldots}{\ldots}}{\ldots}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-\ldots}}{d^7x\ldots}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-\ldots}}{d^7x\ldots}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-\ldots}}{d^7x\ldots}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-\ldots}}{d^7x\ldots}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-\ldots}}{d^7x\ldots}$$

Mathematica [C] time = 0.05, size = 105, normalized size = 0.50

$$\frac{-5d^8-55d^6e^2x^2+330d^4e^4x^4-440d^2e^6x^6+6d^5e^3x^3\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+176e^8x^8}{15d^8x^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-5*d^8 - 55*d^6*e^2*x^2 + 330*d^4*e^4*x^4 - 440*d^2*e^6*x^6 + 176*e^8*x^8
+ 6*d^5*e^3*x^3*Hypergeometric2F1[-5/2, 2, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^
8*x^3*(d^2 - e^2*x^2)^(5/2))
```

fricas [A] time = 1.04, size = 227, normalized size = 1.09

$$\frac{116\,e^7x^7-232\,de^6x^6+232\,d^3e^4x^4-116\,d^4e^3x^3+105\left(e^7x^7-2\,de^6x^6+2\,d^3e^4x^4-d^4e^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(}{15\left(d^8e^4x^7-2\,d^9e^3x^6+2\,d^{11}ex^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(116*e^7*x^7 - 232*d*e^6*x^6 + 232*d^3*e^4*x^4 - 116*d^4*e^3*x^3 + 105
*(e^7*x^7 - 2*d*e^6*x^6 + 2*d^3*e^4*x^4 - d^4*e^3*x^3)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (176*e^6*x^6 - 247*d*e^5*x^5 - 122*d^2*e^4*x^4 + 246*d^3*e
^3*x^3 - 40*d^4*e^2*x^2 - 5*d^5*e*x - 5*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^4
*x^7 - 2*d^9*e^3*x^6 + 2*d^11*e*x^4 - d^12*x^3)
```

giac [A] time = 0.35, size = 325, normalized size = 1.56

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(2\,x\left(\frac{53\,xe^8}{d^8}+\frac{45\,e^7}{d^7}\right)-\frac{235\,e^6}{d^6}\right)x-\frac{200\,e^5}{d^5}\right)x+\frac{135\,e^4}{d^4}\right)x+\frac{116\,e^3}{d^3}\right)}{15\left(x^2e^2-d^2\right)^3}+\frac{x^3\left(\frac{6\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^6}{x}+\frac{57\left(de+\sqrt{-x^2e^2}\right.}{x^2}\right.}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((2*x*(53*x*e^8/d^8 + 45*e^7/d^7) - 235*e^6/d^
6)*x - 200*e^5/d^5)*x + 135*e^4/d^4)*x + 116*e^3/d^3)/(x^2*e^2 - d^2)^3 + 1
/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x + 57*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*e^4/x^2 + e^8)*e/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^8) - 7*e^3*
log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^8 - 1/24*(5
7*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^2*d^16*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^16*e^12/x^3)*e^(-
15)/d^24
```

maple [A] time = 0.02, size = 249, normalized size = 1.19

$$\frac{22e^4x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7e^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}-\frac{11e^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{88e^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^2}+\frac{}{3\left(-e^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x)
```

[Out] -1/d*e/x^2/(-e^2*x^2+d^2)^(5/2)+7/5/d^3*e^3/(-e^2*x^2+d^2)^(5/2)+7/3/d^5*e^3/(-e^2*x^2+d^2)^(3/2)+7/d^7*e^3/(-e^2*x^2+d^2)^(1/2)-7/d^7*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-11/3*e^2/d^2/x/(-e^2*x^2+d^2)^(5/2)+22/5*e^4/d^4*x/(-e^2*x^2+d^2)^(5/2)+88/15*e^4/d^6*x/(-e^2*x^2+d^2)^(3/2)+176/15*e^4/d^8*x/(-e^2*x^2+d^2)^(1/2)-1/3/x^3/(-e^2*x^2+d^2)^(5/2)

maxima [A] time = 0.48, size = 243, normalized size = 1.16

$$\frac{22\,e^4x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7\,e^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{88\,e^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}+\frac{7\,e^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{11\,e^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{1}{15\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 22/5*e^4*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/5*e^3/((-e^2*x^2 + d^2)^(5/2)*d^3) + 88/15*e^4*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/3*e^3/((-e^2*x^2 + d^2)^(3/2)*d^5) - 11/3*e^2/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 176/15*e^4*x/(sqrt(-e^2*x^2 + d^2)*d^8) - 7*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^8 + 7*e^3/(sqrt(-e^2*x^2 + d^2)*d^7) - e/((-e^2*x^2 + d^2)^(5/2)*d*x^2) - 1/3/((-e^2*x^2 + d^2)^(5/2)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{x^4\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^4\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**4*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.54 $\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{5}\sqrt{1-x^2}\,x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}-\frac{1}{5}\sqrt{1-x^2}\,x^4-\frac{1}{2}\sqrt{1-x^2}\,x^3+\frac{3}{4}\sin^{-1}(x)$$

```
[Out] 3/4*arcsin(x)-3/5*x^2*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)-1/5*x^4*(-x^2+1
)^(1/2)-3/20*(8+5*x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1809, 833, 780, 216}

$$-\frac{1}{5}\sqrt{1-x^2}\,x^4-\frac{1}{2}\sqrt{1-x^2}\,x^3-\frac{3}{5}\sqrt{1-x^2}\,x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2*Sqrt[1 - x^2])/5 - (x^3*Sqrt[1 - x^2])/2 - (x^4*Sqrt[1 - x^2])/5 -
(3*(8 + 5*x)*Sqrt[1 - x^2])/20 + (3*ArcSin[x])/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{5}x^4\sqrt{1-x^2} - \frac{1}{5}\int \frac{(-9-10x)x^3}{\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} + \frac{1}{20}\int \frac{x^2(30+36x)}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3}{5}x^2\sqrt{1-x^2} - \frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} - \frac{1}{60}\int \frac{(-72-90x)x}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3}{5}x^2\sqrt{1-x^2} - \frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} - \frac{3}{20}(8+5x)\sqrt{1-x^2} + \frac{3}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3}{5}x^2\sqrt{1-x^2} - \frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} - \frac{3}{20}(8+5x)\sqrt{1-x^2} + \frac{3}{4}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.52

$$\frac{3}{4}\sin^{-1}(x) - \frac{1}{20}\sqrt{1-x^2}\left(4x^4+10x^3+12x^2+15x+24\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] -1/20*(Sqrt[1 - x^2]*(24 + 15*x + 12*x^2 + 10*x^3 + 4*x^4)) + (3*ArcSin[x])
/4
```

fricas [A] time = 0.86, size = 50, normalized size = 0.62

$$-\frac{1}{20}\left(4x^4+10x^3+12x^2+15x+24\right)\sqrt{-x^2+1} - \frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/20*(4*x^4 + 10*x^3 + 12*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 3/2*arctan((sq
rt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.18, size = 34, normalized size = 0.42

$$-\frac{1}{20}\left(\left(2\left(\left(2x+5\right)x+6\right)x+15\right)x+24\right)\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

maple [A] time = 0.01, size = 71, normalized size = 0.88

$$-\frac{\sqrt{-x^2+1}\,x^4}{5} - \frac{\sqrt{-x^2+1}\,x^3}{2} - \frac{3\sqrt{-x^2+1}\,x^2}{5} - \frac{3\sqrt{-x^2+1}\,x}{4} + \frac{3\arcsin(x)}{4} - \frac{6\sqrt{-x^2+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^2/(-x^2+1)^(1/2),x)
[Out] -1/5*x^4*(-x^2+1)^(1/2)-3/5*x^2*(-x^2+1)^(1/2)-6/5*(-x^2+1)^(1/2)-1/2*x^3*(
-x^2+1)^(1/2)-3/4*x*(-x^2+1)^(1/2)+3/4*arcsin(x)
```

maxima [**A**] time = 0.97, size = 70, normalized size = 0.86

$$-\frac{1}{5}\sqrt{-x^2+1}\,x^4-\frac{1}{2}\sqrt{-x^2+1}\,x^3-\frac{3}{5}\sqrt{-x^2+1}\,x^2-\frac{3}{4}\sqrt{-x^2+1}\,x-\frac{6}{5}\sqrt{-x^2+1}+\frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)*x^4 - 1/2*sqrt(-x^2 + 1)*x^3 - 3/5*sqrt(-x^2 + 1)*x^2 -
 3/4*sqrt(-x^2 + 1)*x - 6/5*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

mupad [**B**] time = 2.50, size = 36, normalized size = 0.44

$$\frac{3\,\mathrm{asin}(x)}{4}-\sqrt{1-x^2}\,\left(\frac{x^4}{5}+\frac{x^3}{2}+\frac{3\,x^2}{5}+\frac{3\,x}{4}+\frac{6}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (3*asin(x))/4 - (1 - x^2)^(1/2)*((3*x)/4 + (3*x^2)/5 + x^3/2 + x^4/5 + 6/5)
```

sympy [**A**] time = 1.41, size = 73, normalized size = 0.90

$$-\frac{x^4\sqrt{1-x^2}}{5}-\frac{x^3\sqrt{1-x^2}}{2}-\frac{3x^2\sqrt{1-x^2}}{5}-\frac{3x\sqrt{1-x^2}}{4}-\frac{6\sqrt{1-x^2}}{5}+\frac{3\,\mathrm{asin}\,(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**4*sqrt(1 - x**2)/5 - x**3*sqrt(1 - x**2)/2 - 3*x**2*sqrt(1 - x**2)/5 -
3*x*sqrt(1 - x**2)/4 - 6*sqrt(1 - x**2)/5 + 3*asin(x)/4
```

3.55 $\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=63

$$-\frac{2}{3}\sqrt{1-x^2}\,x^2 - \frac{1}{24}(21x+32)\sqrt{1-x^2} - \frac{1}{4}\sqrt{1-x^2}\,x^3 + \frac{7}{8}\sin^{-1}(x)$$

```
[Out] 7/8*arcsin(x)-2/3*x^2*(-x^2+1)^(1/2)-1/4*x^3*(-x^2+1)^(1/2)-1/24*(32+21*x)*
(-x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1809, 833, 780, 216}

$$-\frac{1}{4}\sqrt{1-x^2}\,x^3 - \frac{2}{3}\sqrt{1-x^2}\,x^2 - \frac{1}{24}(21x+32)\sqrt{1-x^2} + \frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-2*x^2*Sqrt[1 - x^2])/3 - (x^3*Sqrt[1 - x^2])/4 - ((32 + 21*x)*Sqrt[1 - x^
2])/24 + (7*ArcSin[x])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{4}\int \frac{(-7-8x)x^2}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} + \frac{1}{12}\int \frac{x(16+21x)}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{24}(32+21x)\sqrt{1-x^2} + \frac{7}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{24}(32+21x)\sqrt{1-x^2} + \frac{7}{8}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.59

$$\frac{7}{8}\sin^{-1}(x) - \frac{1}{24}\sqrt{1-x^2}\left(6x^3+16x^2+21x+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/24*(Sqrt[1 - x^2]*(32 + 21*x + 16*x^2 + 6*x^3)) + (7*ArcSin[x])/8
```

fricas [A] time = 0.84, size = 45, normalized size = 0.71

$$-\frac{1}{24}\left(6\,x^3+16\,x^2+21\,x+32\right)\sqrt{-x^2+1} - \frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 + 16*x^2 + 21*x + 32)*sqrt(-x^2 + 1) - 7/4*arctan((sqrt(-x^2 +
 1) - 1)/x)
```

giac [A] time = 0.19, size = 30, normalized size = 0.48

$$-\frac{1}{24}\left(\left(2\left(3\,x+8\right)x+21\right)x+32\right)\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

maple [A] time = 0.01, size = 57, normalized size = 0.90

$$-\frac{\sqrt{-x^2+1}\,x^3}{4} - \frac{2\sqrt{-x^2+1}\,x^2}{3} - \frac{7\sqrt{-x^2+1}\,x}{8} + \frac{7\arcsin(x)}{8} - \frac{4\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-x^2+1)^(1/2)*x^3-7/8*(-x^2+1)^(1/2)*x+7/8*arcsin(x)-2/3*(-x^2+1)^(1/
2)*x^2-4/3*(-x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 56, normalized size = 0.89

$$-\frac{1}{4}\sqrt{-x^2+1}\,x^3 - \frac{2}{3}\sqrt{-x^2+1}\,x^2 - \frac{7}{8}\sqrt{-x^2+1}\,x - \frac{4}{3}\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - 2/3*sqrt(-x^2 + 1)*x^2 - 7/8*sqrt(-x^2 + 1)*x - 4
/3*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

mupad [B] time = 0.03, size = 31, normalized size = 0.49

$$\frac{7\,\mathrm{asin}(x)}{8}-\sqrt{1-x^2}\,\left(\frac{x^3}{4}+\frac{2\,x^2}{3}+\frac{7\,x}{8}+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (7*asin(x))/8 - (1 - x^2)^(1/2)*((7*x)/8 + (2*x^2)/3 + x^3/4 + 4/3)
```

sympy [A] time = 0.81, size = 60, normalized size = 0.95

$$-\frac{x^3\sqrt{1-x^2}}{4}-\frac{2x^2\sqrt{1-x^2}}{3}-\frac{7x\sqrt{1-x^2}}{8}-\frac{4\sqrt{1-x^2}}{3}+\frac{7\,\mathrm{asin}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**3*sqrt(1 - x**2)/4 - 2*x**2*sqrt(1 - x**2)/3 - 7*x*sqrt(1 - x**2)/8 - 4
*sqrt(1 - x**2)/3 + 7*asin(x)/8
```

3.56 $\int \frac{x(1+x)^2}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=41

$$-\frac{1}{3}\sqrt{1-x^2}\,x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}+\sin^{-1}(x)$$

```
[Out] arcsin(x)-1/3*x^2*(-x^2+1)^(1/2)-1/3*(5+3*x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1809, 780, 216}

$$-\frac{1}{3}\sqrt{1-x^2}\,x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -(x^2*Sqrt[1 - x^2])/3 - ((5 + 3*x)*Sqrt[1 - x^2])/3 + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+x)^2}{\sqrt{1-x^2}} \, dx &= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}\int \frac{(-5-6x)x}{\sqrt{1-x^2}} \, dx\\ &= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}} \, dx\\ &= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.63

$$\sin^{-1}(x)-\frac{1}{3}\sqrt{1-x^2}\left(x^2+3x+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] -1/3*(Sqrt[1 - x^2]*(5 + 3*x + x^2)) + ArcSin[x]
```

fricas [A] time = 0.95, size = 38, normalized size = 0.93

$$-\frac{1}{3}\left(x^2+3x+5\right)\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.22, size = 21, normalized size = 0.51

$$-\frac{1}{3}\left((x+3)x+5\right)\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

maple [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{\sqrt{-x^2+1}\,x^2}{3}-\sqrt{-x^2+1}\,x+\arcsin(x)-\frac{5\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^2/(-x^2+1)^(1/2),x)
[Out] -1/3*(-x^2+1)^(1/2)*x^2-5/3*(-x^2+1)^(1/2)-(-x^2+1)^(1/2)*x+arcsin(x)
```

maxima [A] time = 0.97, size = 40, normalized size = 0.98

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}\,x-\frac{5}{3}\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.54

$$\operatorname{asin}(x)-\sqrt{1-x^2}\,\left(\frac{x^2}{3}+x+\frac{5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^2)/(1 - x^2)^(1/2),x)
[Out] asin(x) - (1 - x^2)^(1/2)*(x + x^2/3 + 5/3)
```

sympy [A] time = 0.41, size = 37, normalized size = 0.90

$$-\frac{x^2\sqrt{1-x^2}}{3}-x\sqrt{1-x^2}-\frac{5\sqrt{1-x^2}}{3}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

3.57 $\int \frac{(1+x)^2}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=40

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+1)-\frac{3\sqrt{1-x^2}}{2}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)-3/2*(-x^2+1)^(1/2)-1/2*(1+x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {671, 641, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+1)-\frac{3\sqrt{1-x^2}}{2}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-3*Sqrt[1 - x^2])/2 - ((1 + x)*Sqrt[1 - x^2])/2 + (3*ArcSin[x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int \frac{1+x}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.62

$$\frac{1}{2}\left(3\sin^{-1}(x)-(x+4)\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/Sqrt[1 - x^2],x]
[Out] (-((4 + x)*Sqrt[1 - x^2]) + 3*ArcSin[x])/2
```

fricas [A] time = 0.81, size = 33, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4)-3\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.19, size = 19, normalized size = 0.48

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4)+\frac{3}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.72

$$-\frac{\sqrt{-x^2+1}\,x}{2}+\frac{3\arcsin(x)}{2}-2\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x)
[Out] -1/2*(-x^2+1)^(1/2)*x+3/2*arcsin(x)-2*(-x^2+1)^(1/2)
```

maxima [A] time = 0.96, size = 28, normalized size = 0.70

$$-\frac{1}{2}\sqrt{-x^2+1}\,x-2\sqrt{-x^2+1}+\frac{3}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.52

$$\frac{3\,\mathrm{asin}(x)}{2}-\left(\frac{x}{2}+2\right)\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(1 - x^2)^(1/2),x)
[Out] (3*asin(x))/2 - (x/2 + 2)*(1 - x^2)^(1/2)
```

sympy [A] time = 0.24, size = 27, normalized size = 0.68

$$-\frac{x\sqrt{1-x^2}}{2}-2\sqrt{1-x^2}+\frac{3\,\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

3.58 $\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=32

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

```
[Out] 2*arcsin(x)-arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1809, 844, 216, 266, 63, 206}

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2} - \int \frac{-1-2x}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2} + 2\int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right) \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\sqrt{1-x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + 2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

fricas [A] time = 0.86, size = 46, normalized size = 1.44

$$-\sqrt{-x^2+1} - 4\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1) - 4*arctan((sqrt(-x^2 + 1) - 1)/x) + log((sqrt(-x^2 + 1) -
1)/x)
```

giac [A] time = 0.18, size = 34, normalized size = 1.06

$$-\sqrt{-x^2+1} + 2\arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + 2\arcsin(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x/(-x^2+1)^(1/2),x)
```

```
[Out] -(-x^2+1)^(1/2)+2*arcsin(x)-arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.98, size = 41, normalized size = 1.28

$$-\sqrt{-x^2+1}+2\arcsin(x)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.05, size = 32, normalized size = 1.00

$$2\,\mathrm{asin}(x)+\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x*(1 - x^2)^(1/2)),x)
```

```
[Out] 2*asin(x) + log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)
```

sympy [A] time = 6.30, size = 31, normalized size = 0.97

$$-\sqrt{1-x^2}+\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}+2\,\mathrm{asin}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), T
rue)) + 2*asin(x)
```

3.59 $\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

```
[Out] arcsin(x)-2*arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 844, 216, 266, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{x} - \int \frac{-2-x}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{x} + 2\int \frac{1}{x\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) - 2\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{\sqrt{1-x^2}}{x} - 2\tanh^{-1}\left(\sqrt{1-x^2}\right) + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

fricas [A] time = 0.83, size = 53, normalized size = 1.61

$$-\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - 2\,x\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - 2*x*log((sqrt(-x^2 + 1) - 1)/x) + sq
rt(-x^2 + 1))/x
```

giac [A] time = 0.18, size = 55, normalized size = 1.67

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2\,x} + \arcsin(x) + 2\,\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*log
(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$-2\,\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \arcsin(x) - \frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^2/(-x^2+1)^(1/2),x)
[Out] arcsin(x)-(-x^2+1)^(1/2)/x-2*arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.97, size = 42, normalized size = 1.27

$$-\frac{\sqrt{-x^2+1}}{x}+\arcsin(x)-2\,\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/x + arcsin(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.08, size = 35, normalized size = 1.06

$$\mathrm{asin}(x)+2\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\frac{\sqrt{1-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^2*(1 - x^2)^(1/2)),x)
[Out] asin(x) + 2*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)/x
```

sympy [C] time = 4.68, size = 51, normalized size = 1.55

$$\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}+2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}\right)+\mathrm{asin}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
+ 2*Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) + asin(x
)
```

3.60 $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -3/2*arctanh((-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)/x^2-2*(-x^2+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1807, 807, 266, 63, 206}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(2*x^2) - (2*Sqrt[1 - x^2])/x - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{1}{2}\int \frac{-4-3x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{2}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.78

$$-\frac{\sqrt{1-x^2}\,(4x+1)}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
[Out] -1/2*((1 + 4*x)*Sqrt[1 - x^2])/x^2 - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

fricas [A] time = 0.84, size = 43, normalized size = 0.84

$$\frac{3\,x^2\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\,(4\,x+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(3*x^2*log((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(4*x + 1))/x^2
```

giac [B] time = 0.18, size = 91, normalized size = 1.78

$$\frac{x^2\left(\frac{8\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{8\left(\sqrt{-x^2+1}-1\right)^2}-\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{8\,x^2}+\frac{3}{2}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/8*x^2*(8*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2
+ 1) - 1)/x + 1/8*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2*log(-(sqrt(-x^2 + 1) - 1
)/abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.82

$$-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}-\frac{2\sqrt{-x^2+1}}{x}-\frac{\sqrt{-x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^3/(-x^2+1)^(1/2),x)
```

```
[Out] -2*(-x^2+1)^(1/2)/x-3/2*arctanh(1/(-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)/x^2
```

maxima [A] time = 0.97, size = 54, normalized size = 1.06

$$-\frac{2\sqrt{-x^2+1}}{x} - \frac{\sqrt{-x^2+1}}{2x^2} - \frac{3}{2}\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 1/2*sqrt(-x^2 + 1)/x^2 - 3/2*log(2*sqrt(-x^2 + 1)/abs
(x) + 2/abs(x))
```

mupad [B] time = 2.49, size = 47, normalized size = 0.92

$$\frac{3\ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right)}{2} - \frac{2\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^3*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/2 - (2*(1 - x^2)^(1/2))/x - (1 -
 x^2)^(1/2)/(2*x^2)
```

sympy [C] time = 7.03, size = 116, normalized size = 2.27

$$2\left(\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} - \frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i}{2x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True)
) + Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2) > 1),
(I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**2)), Tr
ue)) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True))
```

3.61 $\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

```
[Out] -arctanh((-x^2+1)^(1/2))-1/3*(-x^2+1)^(1/2)/x^3-(-x^2+1)^(1/2)/x^2-5/3*(-x^
2+1)^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\frac{\sqrt{1-x^2}}{3x^3}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(3*x^3) - Sqrt[1 - x^2]/x^2 - (5*Sqrt[1 - x^2])/(3*x) - ArcT
anh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{1}{3}\int \frac{-6-5x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{6}\int \frac{10+6x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.64

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}\left(5x^2+3x+1\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
[Out] -1/3*(Sqrt[1 - x^2]*(1 + 3*x + 5*x^2))/x^3 - ArcTanh[Sqrt[1 - x^2]]
```

fricas [A] time = 0.72, size = 48, normalized size = 0.72

$$\frac{3\,x^3\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(5\,x^2+3\,x+1\right)\sqrt{-x^2+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(3*x^3*log((sqrt(-x^2 + 1) - 1)/x) - (5*x^2 + 3*x + 1)*sqrt(-x^2 + 1))/
x^3
```

giac [B] time = 0.19, size = 125, normalized size = 1.87

$$-\frac{x^3\left(\frac{6\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{21\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+1}-1\right)^3}-\frac{7\left(\sqrt{-x^2+1}-1\right)}{8\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{24\,x^3}+\log\left(-\frac{\sqrt{-x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*x^3*(6*(sqrt(-x^2 + 1) - 1)/x - 21*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(s
qrt(-x^2 + 1) - 1)^3 - 7/8*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1
)^2/x^2 - 1/24*(sqrt(-x^2 + 1) - 1)^3/x^3 + log(-(sqrt(-x^2 + 1) - 1)/abs(x
))
```

maple [A] time = 0.01, size = 56, normalized size = 0.84

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{5\sqrt{-x^2+1}}{3x}-\frac{\sqrt{-x^2+1}}{x^2}-\frac{\sqrt{-x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^4/(-x^2+1)^(1/2),x)
```

```
[Out] -5/3*(-x^2+1)^(1/2)/x-1/3*(-x^2+1)^(1/2)/x^3-(-x^2+1)^(1/2)/x^2-arctanh(1/(
-x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 68, normalized size = 1.01

$$-\frac{5\sqrt{-x^2+1}}{3x}-\frac{\sqrt{-x^2+1}}{x^2}-\frac{\sqrt{-x^2+1}}{3x^3}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/3*sqrt(-x^2 + 1)/x - sqrt(-x^2 + 1)/x^2 - 1/3*sqrt(-x^2 + 1)/x^3 - log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.03, size = 67, normalized size = 1.00

$$\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}\,\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\frac{\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^4*(1 - x^2)^(1/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)*(2/(3*x) + 1/(3*x^
3)) - (1 - x^2)^(1/2)/x - (1 - x^2)^(1/2)/x^2
```

sympy [C] time = 8.35, size = 128, normalized size = 1.91

$$\begin{cases} -\frac{\sqrt{1-x^2}}{x}-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1 \end{cases}+\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}+2\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for}\ \frac{1}{|x^2|} \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherw} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x,
True)) + 2*Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2)
 > 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**
2)), True))
```

3.62 $\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}$$

[Out] -7/8*arctanh((-x^2+1)^(1/2))-1/4*(-x^2+1)^(1/2)/x^4-2/3*(-x^2+1)^(1/2)/x^3-7/8*(-x^2+1)^(1/2)/x^2-4/3*(-x^2+1)^(1/2)/x

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{4x^4}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]

[Out] -Sqrt[1 - x^2]/(4*x^4) - (2*Sqrt[1 - x^2])/(3*x^3) - (7*Sqrt[1 - x^2])/(8*x^2) - (4*Sqrt[1 - x^2])/(3*x) - (7*ArcTanh[Sqrt[1 - x^2]])/8

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{1}{4}\int \frac{-8-7x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}+\frac{1}{12}\int \frac{21+16x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{1}{24}\int \frac{-32-21x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{8}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{16}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.82

$$-\sqrt{1-x^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}\,\left(8x^2+3x+4\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -1/6*(Sqrt[1 - x^2]*(4 + 3*x + 8*x^2))/x^3 - ArcTanh[Sqrt[1 - x^2]]/2 - Sqr
t[1 - x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^2]
```

fricas [A] time = 0.85, size = 53, normalized size = 0.60

$$\frac{21\,x^4\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(32\,x^3+21\,x^2+16\,x+6\right)\sqrt{-x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*x^4*log((sqrt(-x^2 + 1) - 1)/x) - (32*x^3 + 21*x^2 + 16*x + 6)*sqr
t(-x^2 + 1))/x^4
```

giac [B] time = 0.18, size = 163, normalized size = 1.83

$$\frac{x^4\left(\frac{16\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{48\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{144\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-3\right)}{192\left(\sqrt{-x^2+1}-1\right)^4}-\frac{3\left(\sqrt{-x^2+1}-1\right)}{4\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/x^2 + 144*
(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4 - 3/4*(sqrt(-x^2 + 1
) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/12*(sqrt(-x^2 + 1) - 1)^3/x^3
 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4 + 7/8*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.01, size = 70, normalized size = 0.79

$$-\frac{7\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}-\frac{4\sqrt{-x^2+1}}{3x}-\frac{7\sqrt{-x^2+1}}{8x^2}-\frac{2\sqrt{-x^2+1}}{3x^3}-\frac{\sqrt{-x^2+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^5/(-x^2+1)^(1/2),x)
```

```
[Out] -7/8*(-x^2+1)^(1/2)/x^2-7/8*arctanh(1/(-x^2+1)^(1/2))-2/3*(-x^2+1)^(1/2)/x^
3-4/3*(-x^2+1)^(1/2)/x-1/4*(-x^2+1)^(1/2)/x^4
```

maxima [A] time = 0.97, size = 82, normalized size = 0.92

$$-\frac{4\sqrt{-x^2+1}}{3\,x}-\frac{7\sqrt{-x^2+1}}{8\,x^2}-\frac{2\sqrt{-x^2+1}}{3\,x^3}-\frac{\sqrt{-x^2+1}}{4\,x^4}-\frac{7}{8}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(-x^2 + 1)/x - 7/8*sqrt(-x^2 + 1)/x^2 - 2/3*sqrt(-x^2 + 1)/x^3 - 1
/4*sqrt(-x^2 + 1)/x^4 - 7/8*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.03, size = 77, normalized size = 0.87

$$\frac{7\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{8}-\sqrt{1-x^2}\,\left(\frac{4}{3\,x}+\frac{2}{3\,x^3}\right)-\sqrt{1-x^2}\,\left(\frac{3}{8\,x^2}+\frac{1}{4\,x^4}\right)-\frac{\sqrt{1-x^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^5*(1 - x^2)^(1/2)),x)
```

```
[Out] (7*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/8 - (1 - x^2)^(1/2)*(4/(3*x) + 2
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(8*x^2) + 1/(4*x^4)) - (1 - x^2)^(1/2)/(2*x^
2)
```

sympy [A] time = 11.06, size = 223, normalized size = 2.51

$$2\left(\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3x^3} & \text{for}\: x>-1\wedge x<1\end{cases}\right)+\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for}\: \frac{1}{|x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}+\begin{cases}-\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8}+\ldots\\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8}-\frac{\ldots}{8x\sqrt{\ldots}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x
< 1))) + Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2) >
 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**2)
), True)) + Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*
x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1),
 (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)
) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

3.63 $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}$$

```
[Out] -3/4*arctanh((-x^2+1)^(1/2))-1/5*(-x^2+1)^(1/2)/x^5-1/2*(-x^2+1)^(1/2)/x^4-
3/5*(-x^2+1)^(1/2)/x^3-3/4*(-x^2+1)^(1/2)/x^2-6/5*(-x^2+1)^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{\sqrt{1-x^2}}{5x^5}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(5*x^5) - Sqrt[1 - x^2]/(2*x^4) - (3*Sqrt[1 - x^2])/(5*x^3)
- (3*Sqrt[1 - x^2])/(4*x^2) - (6*Sqrt[1 - x^2])/(5*x) - (3*ArcTanh[Sqrt[1 -
x^2]])/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{1}{5}\int \frac{-10-9x}{x^5\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}+\frac{1}{20}\int \frac{36+30x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{1}{60}\int \frac{-90-72x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}+\frac{1}{120}\int \frac{144+90x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{8}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.47

$$-\frac{\sqrt{1-x^2}\left(10x^5\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^2\right)+6x^4+3x^2+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -1/5*(Sqrt[1 - x^2]*(1 + 3*x^2 + 6*x^4 + 10*x^5*Hypergeometric2F1[1/2, 3, 3
/2, 1 - x^2]))/x^5
```

fricas [A] time = 0.97, size = 58, normalized size = 0.54

$$\frac{15\,x^5\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(24\,x^4+15\,x^3+12\,x^2+10\,x+4\right)\sqrt{-x^2+1}}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*x^5*log((sqrt(-x^2 + 1) - 1)/x) - (24*x^4 + 15*x^3 + 12*x^2 + 10*x
 + 4)*sqrt(-x^2 + 1))/x^5
```

giac [B] time = 0.20, size = 199, normalized size = 1.86

$$-\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{15\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{40\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-\frac{110\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{160\left(\sqrt{-x^2+1}-1\right)^5}-\frac{11\left(\sqrt{-x^2+1}-1\right)}{16\,x}+\frac{\left(\sqrt{-x^2+1}-1\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/160*x^5*(5*(sqrt(-x^2 + 1) - 1)/x - 15*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40*(
sqrt(-x^2 + 1) - 1)^3/x^3 - 110*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2
+ 1) - 1)^5 - 11/16*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2
 - 3/32*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1/16
0*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.01, size = 84, normalized size = 0.79

$$-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{4}-\frac{6\sqrt{-x^2+1}}{5x}-\frac{3\sqrt{-x^2+1}}{4x^2}-\frac{3\sqrt{-x^2+1}}{5x^3}-\frac{\sqrt{-x^2+1}}{2x^4}-\frac{\sqrt{-x^2+1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^6/(-x^2+1)^(1/2),x)
```

```
[Out] -1/5*(-x^2+1)^(1/2)/x^5-3/5*(-x^2+1)^(1/2)/x^3-6/5*(-x^2+1)^(1/2)/x-1/2*(-x
^2+1)^(1/2)/x^4-3/4*(-x^2+1)^(1/2)/x^2-3/4*arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.97, size = 96, normalized size = 0.90

$$-\frac{6\sqrt{-x^2+1}}{5\,x}-\frac{3\sqrt{-x^2+1}}{4\,x^2}-\frac{3\sqrt{-x^2+1}}{5\,x^3}-\frac{\sqrt{-x^2+1}}{2\,x^4}-\frac{\sqrt{-x^2+1}}{5\,x^5}-\frac{3}{4}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -6/5*sqrt(-x^2 + 1)/x - 3/4*sqrt(-x^2 + 1)/x^2 - 3/5*sqrt(-x^2 + 1)/x^3 - 1
/2*sqrt(-x^2 + 1)/x^4 - 1/5*sqrt(-x^2 + 1)/x^5 - 3/4*log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

mupad [B] time = 0.04, size = 90, normalized size = 0.84

$$\frac{3\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{4}-\sqrt{1-x^2}\,\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\sqrt{1-x^2}\,\left(\frac{3}{4\,x^2}+\frac{1}{2\,x^4}\right)-\sqrt{1-x^2}\,\left(\frac{8}{15\,x}+\frac{4}{15\,x^3}+\frac{1}{5\,x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^6*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/4 - (1 - x^2)^(1/2)*(2/(3*x) + 1
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(4*x^2) + 1/(2*x^4)) - (1 - x^2)^(1/2)*(8/(1
5*x) + 4/(15*x^3) + 1/(5*x^5))
```

sympy [C] time = 12.69, size = 201, normalized size = 1.88

$$\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} + \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{2(1-x^2)^{\frac{3}{2}}}{3x^3} - \frac{(1-x^2)^{\frac{5}{2}}}{5x^5} & \text{for } x > -1 \wedge x < 1 \end{cases} + 2 \left(\begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \cdots \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{\cdots}{8x\sqrt{\cdots}} \end{cases} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-sqrt(1 - x**2)/x - 2*(1 - x**2)**(3/2)/(3*x**3) - (1 - x
**2)**(5/2)/(5*x**5), (x > -1) & (x < 1))) + 2*Piecewise((-3*acosh(1/x)/8 +
 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqr
t(-1 + x**(-2))), 1/Abs(x**2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/
x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

3.64 $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

Optimal. Leaf size=134

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.$$

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-e*(-e^2*x^2+d^2)^(3/2)/x^3-e^4*arctan(e*x/(
-e^2*x^2+d^2)^(1/2))+13/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/8*e^2*(8*e*
x+13*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -(e^2*(13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d*(d^2 - e^2*x^2)^(3/2
))/(4*x^4) - (e*(d^2 - e^2*x^2)^(3/2))/x^3 - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]] + (13*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(-12d^4e-13d^3e^2x-4d^2e^3x^2\right)}{x^4}\,dx}{4d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}+\frac{\int \frac{\left(39d^5e^2+12d^4e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{12d^4}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{\int \frac{78d^7e^4+48d^6e^5x}{x\sqrt{d^2-e^2x^2}}\,d
\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{1}{8}\left(13de^4\right)\int \frac{}{x\sqrt{d
\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{1}{16}\left(13de^4\right)\text{Subst}
\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e
\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e
\end{aligned}
$$

Mathematica [C] time = 0.24, size = 196, normalized size = 1.46

$$
-\frac{e\sqrt{d^2-e^2x^2}\left(6d^2e^3x^3\sin^{-1}\left(\frac{ex}{d}\right)+2e^3x^3\left(d^2-e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{e^2x^2}{d^2}\right)-9d^2e^3x^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x
}{6d^3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -1/6*(e*Sqrt[d^2 - e^2*x^2]*(6*d^5*Sqrt[1 - (e^2*x^2)/d^2] + 9*d^4*e*x*Sqrt
[1 - (e^2*x^2)/d^2] + 6*d^2*e^3*x^3*ArcSin[(e*x)/d] - 9*d^2*e^3*x^3*ArcTanh
[Sqrt[1 - (e^2*x^2)/d^2]] + 2*e^3*x^3*(d^2 - e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^
2]*Hypergeometric2F1[3/2, 3, 5/2, 1 - (e^2*x^2)/d^2]))/(d^3*x^3*Sqrt[1 - (e
^2*x^2)/d^2])
```

fricas [A] time = 0.94, size = 111, normalized size = 0.83

$$\frac{16\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(11\,de^2x^2+8\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(16*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (11*d*e^2*x^2 + 8*d^2*e*x + 2*d^3)*sqrt(-e
^2*x^2 + d^2))/x^4
```

giac [B] time = 0.27, size = 295, normalized size = 2.20

$$-\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}(d)+\frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^8}{x}+\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^6}{x^2}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^4}{x^3}+e^{10}\right)e^2}{64\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}-\frac{1}{64}\left(\frac{8\left(de\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^4*sgn(d) + 1/64*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/
x + 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^6/x^2 + 8*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^3*e^4/x^3 + e^10)*e^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 - 1/64*(8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*e^26/x + 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e
^24/x^2 + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^22/x^3 + (d*e + sqrt(-x^2*e^
2 + d^2)*e)^4*e^20/x^4)*e^(-24) + 13/8*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2
*e^2 + d^2)*e)*e^(-2)/abs(x))
```

maple [A] time = 0.02, size = 212, normalized size = 1.58

$$\frac{13d\,e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}-\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^2}-\frac{13\sqrt{-e^2x^2+d^2}\,e^4}{8d}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-13/8/d*e^2/x^2*(-e^2*x^2+d^2)^(3/2)-13/8*(-
e^2*x^2+d^2)^(1/2)/d*e^4+13/8/(d^2)^(1/2)*d*e^4*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-e^3/d^2/x*(-e^2*x^2+d^2)^(3/2)-(-e^2*x^2+d^2)^(1/2)/d
^2*e^5*x-1/(e^2)^(1/2)*e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-e*(-e
^2*x^2+d^2)^(3/2)/x^3
```

maxima [A] time = 0.98, size = 159, normalized size = 1.19

$$-e^4\arcsin\left(\frac{ex}{d}\right)+\frac{13}{8}\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{13\,\sqrt{-e^2x^2+d^2}\,e^4}{8\,d}-\frac{\sqrt{-e^2x^2+d^2}\,e^3}{x}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{8\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -e^4*arcsin(e*x/d) + 13/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/a
bs(x)) - 13/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^2*x^2 + d^2)*e^3/x - 13/
8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) - (-e^2*x^2 + d^2)^(3/2)*e/x^3 - 1/4*(
-e^2*x^2 + d^2)^(3/2)*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5, x)
```

sympy [C] time = 10.03, size = 544, normalized size = 4.06

$$d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right)+3d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + 3*d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2
) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), A
bs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2
*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d
**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2
*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x
/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.65 $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=310

$$-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}+\frac{35d^{14}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}+\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}$$

```
[Out] 35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+7/768*d^8*x*(-e^2*x^2+d^2)^(5/2)/e^
5-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-7/48*d^4*x^3*(-e^2*x^2+d^2)^(7/
2)/e^3-31/143*d^3*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-7/24*d^2*x^5*(-e^2*x^2+d^2)^
(7/2)/e-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)-1/1
153152*d^6*(63063*e*x+31744*d)*(-e^2*x^2+d^2)^(7/2)/e^6+35/2048*d^14*arctan
(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+35/2048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.49, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}-124d^5x$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (35*d^12*x*Sqrt[d^2 - e^2*x^2])/(2048*e^5) + (35*d^10*x*(d^2 - e^2*x^2)^(3/
2))/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^(5/2))/(768*e^5) - (124*d^5*x^2*(
d^2 - e^2*x^2)^(7/2))/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^(7/2))/(48*e^
3) - (31*d^3*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x
^2)^(7/2))/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (e*x^7*(d^2 - e^2*
x^2)^(7/2))/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^(7/2))/(1153152
*e^6) + (35*d^14*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^5\left(d^2-e^2x^2\right)^{5/2}\left(-14d^3e^2-49d^2e^3x-42de^4x^2\right)dx}{14e^2}$$

$$= -\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^5\left(434d^3e^4+637d^2e^5x\right)\left(d^2\right.}{182e^4}$$

$$= -\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^4\left(-3\right.}{}$$

$$= -\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(\right.$$

$$= -\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(\right.$$

$$= -\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x}{}$$

$$= -\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x}{}$$

$$= \frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(\right.}{1}$$

$$= \frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(\right.}{}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d\right.}{128}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d\right.}{128}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d\right.}{128}$$

Mathematica [A] time = 0.36, size = 212, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\left(315315d^{13}\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(507904d^{13}+315315d^{12}ex+253952d^{11}e^2x^2+210210d^{10}e^3x^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(507904*d^13 + 315315*d^12*
e*x + 253952*d^11*e^2*x^2 + 210210*d^10*e^3*x^3 + 190464*d^9*e^4*x^4 + 1681
68*d^8*e^5*x^5 - 2916352*d^7*e^6*x^6 - 7763184*d^6*e^7*x^7 - 2551808*d^5*e^
8*x^8 + 9499776*d^4*e^9*x^9 + 8773632*d^3*e^10*x^10 - 1427712*d^2*e^11*x^11
 - 4257792*d*e^12*x^12 - 1317888*e^13*x^13)) + 315315*d^13*ArcSin[(e*x)/d])
)/(18450432*e^6*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.85, size = 194, normalized size = 0.63

$$-\frac{630630\,d^{14}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(1317888\,e^{13}x^{13}+4257792\,de^{12}x^{12}+1427712\,d^2e^{11}x^{11}-8773632\,d^3e^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/18450432*(630630*d^14*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (13178
88*e^13*x^13 + 4257792*d*e^12*x^12 + 1427712*d^2*e^11*x^11 - 8773632*d^3*e^
10*x^10 - 9499776*d^4*e^9*x^9 + 2551808*d^5*e^8*x^8 + 7763184*d^6*e^7*x^7 +
 2916352*d^7*e^6*x^6 - 168168*d^8*e^5*x^5 - 190464*d^9*e^4*x^4 - 210210*d^1
0*e^3*x^3 - 253952*d^11*e^2*x^2 - 315315*d^12*e*x - 507904*d^13)*sqrt(-e^2*
x^2 + d^2))/e^6
```

giac [A] time = 0.26, size = 170, normalized size = 0.55

$$\frac{35}{2048}d^{14}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{1}{18450432}\left(507904\,d^{13}e^{(-6)}+\left(315315\,d^{12}e^{(-5)}+2\left(126976\,d^{11}e^{(-4)}+\left(10510\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/2048*d^14*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/18450432*(507904*d^13*e^(-6) +
 (315315*d^12*e^(-5) + 2*(126976*d^11*e^(-4) + (105105*d^10*e^(-3) + 4*(238
08*d^9*e^(-2) + (21021*d^8*e^(-1) - 2*(182272*d^7 + (485199*d^6*e + 8*(1993
6*d^5*e^2 - 3*(24739*d^4*e^3 + 2*(11424*d^3*e^4 - 11*(169*d^2*e^5 + 12*(13*
x*e^7 + 42*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.11, size = 291, normalized size = 0.94

$$\frac{35d^{14}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2048\sqrt{e^2}\,e^5}+\frac{35\sqrt{-e^2x^2+d^2}\,d^{12}x}{2048e^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^7}{14}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,x^6}{13}+\frac{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3072e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)-7/24*d^2*x^5*(-e^2*x^2+d^2)^(7/2)/e-7/48*d
^4*x^3*(-e^2*x^2+d^2)^(7/2)/e^3-7/128/e^5*d^6*x*(-e^2*x^2+d^2)^(7/2)+7/768*
```

```
d^8*x*(-e^2*x^2+d^2)^(5/2)/e^5+35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+35/2
048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^5+35/2048/e^5*d^14/(e^2)^(1/2)*arctan((e^
2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-31/143*d^3
*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-248
/9009/e^6*d^7*(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 1.00, size = 270, normalized size = 0.87

$$-\frac{1}{14}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^7-\frac{3}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^6-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^5}{24\,e}+\frac{35\,d^{14}\arcsin\left(\frac{ex}{d}\right)}{2048\,e^6}+\frac{35\,\sqrt{-e^2x^2+d^2}\,d^{12}x}{2048\,e^5}-\frac{31}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/14*(-e^2*x^2 + d^2)^(7/2)*e*x^7 - 3/13*(-e^2*x^2 + d^2)^(7/2)*d*x^6 - 7/
24*(-e^2*x^2 + d^2)^(7/2)*d^2*x^5/e + 35/2048*d^14*arcsin(e*x/d)/e^6 + 35/2
048*sqrt(-e^2*x^2 + d^2)*d^12*x/e^5 - 31/143*(-e^2*x^2 + d^2)^(7/2)*d^3*x^4
/e^2 + 35/3072*(-e^2*x^2 + d^2)^(3/2)*d^10*x/e^5 - 7/48*(-e^2*x^2 + d^2)^(7
/2)*d^4*x^3/e^3 + 7/768*(-e^2*x^2 + d^2)^(5/2)*d^8*x/e^5 - 124/1287*(-e^2*x
^2 + d^2)^(7/2)*d^5*x^2/e^4 - 7/128*(-e^2*x^2 + d^2)^(7/2)*d^6*x/e^5 - 248/
9009*(-e^2*x^2 + d^2)^(7/2)*d^7/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\left(d^2-e^2\,x^2\right)^{5/2}\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [A] time = 101.43, size = 2273, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + 3
*d**6*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6
*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d
**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6
*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)
) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e
**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**5*e
**2*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqr
t(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e*
*4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x*
*2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) - 5*d**4*e**3*Piecewise((-7*I*
d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d*
*2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(
1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2
*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10
*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/
```

```
d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/
(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*
x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*
sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), Tru
e)) - 5*d**3*e**4*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10)
 - 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2
 - e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) -
 d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/
11, Ne(e, 0)), (x**10*sqrt(d**2)/10, True)) + d**2*e**5*Piecewise((-21*I*d*
*12*acosh(e*x/d)/(1024*e**11) + 21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**
2/d**2)) - 7*I*d**9*x**3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x
**5/(2560*e**6*sqrt(-1 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 +
 e**2*x**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**2)) - 11*I
*d*x**11/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**13/(12*d*sqrt(-1 + e**
2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (21*d**12*asin(e*x/d)/(1024*e**11)
 - 21*d**11*x/(1024*e**10*sqrt(1 - e**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**
8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**
2)) + d**5*x**7/(640*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*s
qrt(1 - e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e**2
*x**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-256*d
**12*sqrt(d**2 - e**2*x**2)/(9009*e**12) - 128*d**10*x**2*sqrt(d**2 - e**2*
x**2)/(9009*e**10) - 32*d**8*x**4*sqrt(d**2 - e**2*x**2)/(3003*e**8) - 80*d
**6*x**6*sqrt(d**2 - e**2*x**2)/(9009*e**6) - 10*d**4*x**8*sqrt(d**2 - e**2
*x**2)/(1287*e**4) - d**2*x**10*sqrt(d**2 - e**2*x**2)/(143*e**2) + x**12*s
qrt(d**2 - e**2*x**2)/13, Ne(e, 0)), (x**12*sqrt(d**2)/12, True)) + e**7*Pi
ecewise((-33*I*d**14*acosh(e*x/d)/(2048*e**13) + 33*I*d**13*x/(2048*e**12*s
qrt(-1 + e**2*x**2/d**2)) - 11*I*d**11*x**3/(2048*e**10*sqrt(-1 + e**2*x**2
/d**2)) - 11*I*d**9*x**5/(5120*e**8*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d**7*
x**7/(8960*e**6*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d**5*x**9/(13440*e**4*sqr
t(-1 + e**2*x**2/d**2)) - I*d**3*x**11/(1680*e**2*sqrt(-1 + e**2*x**2/d**2)
) - 13*I*d*x**13/(168*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**15/(14*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (33*d**14*asin(e*x/d)/(204
8*e**13) - 33*d**13*x/(2048*e**12*sqrt(1 - e**2*x**2/d**2)) + 11*d**11*x**3
/(2048*e**10*sqrt(1 - e**2*x**2/d**2)) + 11*d**9*x**5/(5120*e**8*sqrt(1 - e
**2*x**2/d**2)) + 11*d**7*x**7/(8960*e**6*sqrt(1 - e**2*x**2/d**2)) + 11*d*
*5*x**9/(13440*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**11/(1680*e**2*sqrt(
1 - e**2*x**2/d**2)) + 13*d*x**13/(168*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
15/(14*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.66 $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=281

$$-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}+\frac{27d^{13}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}+\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+$$

```
[Out] 9/512*d^9*x*(-e^2*x^2+d^2)^(3/2)/e^4+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4-2
0/143*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/e^
2-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-1/4*d*x^5*(-e^2*x^2+d^2)^(7/2)-1/13
*e*x^6*(-e^2*x^2+d^2)^(7/2)-1/320320*d^5*(27027*e*x+12800*d)*(-e^2*x^2+d^2)
^(7/2)/e^5+27/1024*d^13*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+27/1024*d^11*x
*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.41, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}-\frac{20d^4x^2\left(d^2-\right.}{143e}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (27*d^11*x*Sqrt[d^2 - e^2*x^2])/(1024*e^4) + (9*d^9*x*(d^2 - e^2*x^2)^(3/2)
)/(512*e^4) + (9*d^7*x*(d^2 - e^2*x^2)^(5/2))/(640*e^4) - (20*d^4*x^2*(d^2
- e^2*x^2)^(7/2))/(143*e^3) - (9*d^3*x^3*(d^2 - e^2*x^2)^(7/2))/(40*e^2) -
(45*d^2*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e) - (d*x^5*(d^2 - e^2*x^2)^(7/2))/
4 - (e*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (d^5*(12800*d + 27027*e*x)*(d^2 - e^
2*x^2)^(7/2))/(320320*e^5) + (27*d^13*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1
024*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^4\left(d^2-e^2x^2\right)^{5/2}\left(-13d^3e^2-45d^2e^3x-39de^4x^2\right)}{13e^2}$$

$$= -\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^4\left(351d^3e^4+540d^2e^5x\right)(d}{156e^4}$$

$$= -\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^3}{}$$

$$= -\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex$$

$$= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}d$$

$$= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}d$$

$$= \frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4}{}$$

$$= \frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3(}{}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2(d}{14}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2(d}{14}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2(d}{14}$$

Mathematica [A] time = 0.33, size = 200, normalized size = 0.71

$$\sqrt{d^2-e^2x^2}\left(135135d^{12}\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-204800d^{12}-135135d^{11}ex-102400d^{10}e^2x^2-90090d^9e^3x^3-76\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-204800*d^12 - 135135*d^11*e
*x - 102400*d^10*e^2*x^2 - 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d
^7*e^5*x^5 + 2498560*d^6*e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8
 - 2690688*d^3*e^9*x^9 + 430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 39424
0*e^12*x^12) + 135135*d^12*ArcSin[(e*x)/d]))/(5125120*e^5*Sqrt[1 - (e^2*x^2
)/d^2])
```

fricas [A] time = 0.92, size = 183, normalized size = 0.65

$$-\frac{270270\,d^{13}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(394240\,e^{12}x^{12}+1281280\,de^{11}x^{11}+430080\,d^2e^{10}x^{10}-2690688\,d^3e^9x^9-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5125120*(270270*d^13*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (394240
*e^12*x^12 + 1281280*d*e^11*x^11 + 430080*d^2*e^10*x^10 - 2690688*d^3*e^9*x
^9 - 2938880*d^4*e^8*x^8 + 816816*d^5*e^7*x^7 + 2498560*d^6*e^6*x^6 + 95295
2*d^7*e^5*x^5 - 76800*d^8*e^4*x^4 - 90090*d^9*e^3*x^3 - 102400*d^10*e^2*x^2
 - 135135*d^11*e*x - 204800*d^12)*sqrt(-e^2*x^2 + d^2))/e^5
```

giac [A] time = 0.26, size = 160, normalized size = 0.57

$$\frac{27}{1024}\,d^{13}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{5125120}\left(204800\,d^{12}e^{(-5)}+\left(135135\,d^{11}e^{(-4)}+2\left(51200\,d^{10}e^{(-3)}+\left(45045\,d^9e^{(\cdot}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 27/1024*d^13*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5125120*(204800*d^12*e^(-5) +
(135135*d^11*e^(-4) + 2*(51200*d^10*e^(-3) + (45045*d^9*e^(-2) + 4*(9600*d^
8*e^(-1) - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^
3 + (3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*x*e^7 + 13*d*e^6)*x)*x)*x)*x)*x)
*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 266, normalized size = 0.95

$$\frac{27d^{13}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{1024\sqrt{e^2}\,e^4}+\frac{27\sqrt{-e^2x^2+d^2}\,d^{11}x}{1024e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^6}{13}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^9x}{512e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,x^5}{4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/13*e*x^6*(-e^2*x^2+d^2)^(7/2)-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-20/1
43*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-40/1001/e^5*d^6*(-e^2*x^2+d^2)^(7/2)-1/
4*d*x^5*(-e^2*x^2+d^2)^(7/2)-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/e^2-27/320/e
```

```
^4*d^5*x*(-e^2*x^2+d^2)^(7/2)+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4+9/512*d^
9*x*(-e^2*x^2+d^2)^(3/2)/e^4+27/1024*d^11*x*(-e^2*x^2+d^2)^(1/2)/e^4+27/102
4/e^4*d^13/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 245, normalized size = 0.87

$$-\frac{1}{13}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}ex^{6}-\frac{1}{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}dx^{5}+\frac{27\,d^{13}\arcsin\left(\frac{ex}{d}\right)}{1024\,e^{5}}+\frac{27\,\sqrt{-e^{2}x^{2}+d^{2}}\,d^{11}x}{1024\,e^{4}}-\frac{45\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{2}x^{4}}{143\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/13*(-e^2*x^2 + d^2)^(7/2)*e*x^6 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d*x^5 + 27/
1024*d^13*arcsin(e*x/d)/e^5 + 27/1024*sqrt(-e^2*x^2 + d^2)*d^11*x/e^4 - 45/
143*(-e^2*x^2 + d^2)^(7/2)*d^2*x^4/e + 9/512*(-e^2*x^2 + d^2)^(3/2)*d^9*x/e
^4 - 9/40*(-e^2*x^2 + d^2)^(7/2)*d^3*x^3/e^2 + 9/640*(-e^2*x^2 + d^2)^(5/2)
*d^7*x/e^4 - 20/143*(-e^2*x^2 + d^2)^(7/2)*d^4*x^2/e^3 - 27/320*(-e^2*x^2 +
 d^2)^(7/2)*d^5*x/e^4 - 40/1001*(-e^2*x^2 + d^2)^(7/2)*d^6/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(d^2-e^2\,x^2\right)^{5/2}\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [C] time = 64.64, size = 2028, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) + 3*d**6*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/
(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt
(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x
**6*sqrt(d**2)/6, True)) + d**5*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128
*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(3
84*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x
**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(1
28*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*
e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**
2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2
*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**
2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4
*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e
**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
 - 5*d**3*e**4*Piecewise((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(
256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**
```

```
2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**
3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2
*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 -
e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5
*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e
**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d
*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewise((-128*d**10*sqrt(d
**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e
**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d
**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) +
x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10*sqrt(d**2)/10, True)) +
3*d*e**6*Piecewise((-21*I*d**12*acosh(e*x/d)/(1024*e**11) + 21*I*d**11*x/(1
024*e**10*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**9*x**3/(1024*e**8*sqrt(-1 + e
**2*x**2/d**2)) - 7*I*d**7*x**5/(2560*e**6*sqrt(-1 + e**2*x**2/d**2)) - I*d
**5*x**7/(640*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(
-1 + e**2*x**2/d**2)) - 11*I*d*x**11/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e*
*2*x**13/(12*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (21*d*
*12*asin(e*x/d)/(1024*e**11) - 21*d**11*x/(1024*e**10*sqrt(1 - e**2*x**2/d*
*2)) + 7*d**9*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**5/(2560
*e**6*sqrt(1 - e**2*x**2/d**2)) + d**5*x**7/(640*e**4*sqrt(1 - e**2*x**2/d*
*2)) + d**3*x**9/(960*e**2*sqrt(1 - e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt
(1 - e**2*x**2/d**2)) - e**2*x**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True))
+ e**7*Piecewise((-256*d**12*sqrt(d**2 - e**2*x**2)/(9009*e**12) - 128*d**1
0*x**2*sqrt(d**2 - e**2*x**2)/(9009*e**10) - 32*d**8*x**4*sqrt(d**2 - e**2*
x**2)/(3003*e**8) - 80*d**6*x**6*sqrt(d**2 - e**2*x**2)/(9009*e**6) - 10*d*
*4*x**8*sqrt(d**2 - e**2*x**2)/(1287*e**4) - d**2*x**10*sqrt(d**2 - e**2*x*
*2)/(143*e**2) + x**12*sqrt(d**2 - e**2*x**2)/13, Ne(e, 0)), (x**12*sqrt(d*
*2)/12, True))
```

3.67 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}+\frac{41d^{12}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}+\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}$$

```
[Out] 41/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/e
^3-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-41/120*d^2*x^3*(-e^2*x^2+d^2)^(7/
2)/e-3/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-1/2217
60*d^4*(28413*e*x+14720*d)*(-e^2*x^2+d^2)^(7/2)/e^4+41/1024*d^12*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^4+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] time = 0.36, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3}+\frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}-\frac{23d^3x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (41*d^10*x*Sqrt[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^(3/2
))/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^(5/2))/(1920*e^3) - (23*d^3*x^2*(
d^2 - e^2*x^2)^(7/2))/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^(7/2))/(120*e)
 - (3*d*x^4*(d^2 - e^2*x^2)^(7/2))/11 - (e*x^5*(d^2 - e^2*x^2)^(7/2))/12 -
(d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^(7/2))/(221760*e^4) + (41*d^12*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1024*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^3\left(d^2-e^2x^2\right)^{5/2}\left(-12d^3e^2-41d^2e^3x-36de^4x^2\right)dx}{12e^2}$$

$$= -\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^3\left(276d^3e^4+451d^2e^5x\right)\left(d^2\right.}{132e^4}$$

$$= -\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^2\left(-\right.}{}$$

$$= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5$$

$$= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5$$

$$= \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(\right.$$

$$= \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(\right.}{}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2\right.}{99}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2\right.}{99}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2\right.}{99}$$

Mathematica [A] time = 0.30, size = 189, normalized size = 0.75

$$\frac{\sqrt{d^2-e^2x^2}\left(142065d^{11}\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-235520d^{11}-142065d^{10}ex-117760d^9e^2x^2-94710d^8e^3x^3+79\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-235520*d^11 - 142065*d^10*e
*x - 117760*d^9*e^2*x^2 - 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*
d^6*e^5*x^5 + 665600*d^5*e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^
8 + 325248*d^2*e^9*x^9 + 967680*d*e^10*x^10 + 295680*e^11*x^11) + 142065*d^
11*ArcSin[(e*x)/d]))/(3548160*e^4*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.97, size = 172, normalized size = 0.68

$$-\frac{284130\,d^{12}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(295680\,e^{11}x^{11}+967680\,de^{10}x^{10}+325248\,d^2e^9x^9-2078720\,d^3e^8x^8-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3548160*(284130*d^12*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (295680
*e^11*x^11 + 967680*d*e^10*x^10 + 325248*d^2*e^9*x^9 - 2078720*d^3*e^8*x^8
- 2295216*d^4*e^7*x^7 + 665600*d^5*e^6*x^6 + 2053128*d^6*e^5*x^5 + 798720*d
^7*e^4*x^4 - 94710*d^8*e^3*x^3 - 117760*d^9*e^2*x^2 - 142065*d^10*e*x - 235
520*d^11)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.24, size = 149, normalized size = 0.59

$$\frac{41}{1024}\,d^{12}\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{3548160}\left(235520\,d^{11}e^{(-4)}+\left(142065\,d^{10}e^{(-3)}+2\left(58880\,d^9e^{(-2)}+\left(47355\,d^8\cdots\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 41/1024*d^12*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/3548160*(235520*d^11*e^(-4) +
(142065*d^10*e^(-3) + 2*(58880*d^9*e^(-2) + (47355*d^8*e^(-1) - 4*(99840*d^
7 + (256641*d^6*e + 2*(41600*d^5*e^2 - 7*(20493*d^4*e^3 + 8*(2320*d^3*e^4 -
 3*(121*d^2*e^5 + 10*(11*x*e^7 + 36*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sq
rt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 241, normalized size = 0.96

$$\frac{41d^{12}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{1024\sqrt{e^2}\,e^3}+\frac{41\sqrt{-e^2x^2+d^2}\,d^{10}x}{1024e^3}+\frac{41\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8x}{1536e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^5}{12}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\cdots}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-41/120*d^2*x^3*(-e^2*x^2+d^2)^(7/2)/e-41/3
20/e^3*d^4*x*(-e^2*x^2+d^2)^(7/2)+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/e^3+41
/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^
3+41/1024/e^3*d^12/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3
/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-46/69
3/e^4*d^5*(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.99, size = 220, normalized size = 0.87

$$-\frac{1}{12}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^5+\frac{41\,d^{12}\arcsin\left(\frac{ex}{d}\right)}{1024\,e^4}+\frac{41\,\sqrt{-e^2x^2+d^2}\,d^{10}x}{1024\,e^3}-\frac{3}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^4+\frac{41\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8x}{1536\,e^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-e^2*x^2 + d^2)^(7/2)*e*x^5 + 41/1024*d^12*arcsin(e*x/d)/e^4 + 41/10
24*sqrt(-e^2*x^2 + d^2)*d^10*x/e^3 - 3/11*(-e^2*x^2 + d^2)^(7/2)*d*x^4 + 41
/1536*(-e^2*x^2 + d^2)^(3/2)*d^8*x/e^3 - 41/120*(-e^2*x^2 + d^2)^(7/2)*d^2*
x^3/e + 41/1920*(-e^2*x^2 + d^2)^(5/2)*d^6*x/e^3 - 23/99*(-e^2*x^2 + d^2)^(
7/2)*d^3*x^2/e^2 - 41/320*(-e^2*x^2 + d^2)^(7/2)*d^4*x/e^3 - 46/693*(-e^2*x
^2 + d^2)^(7/2)*d^5/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(d^2 - e^2 x^2\right)^{5/2} (d + e x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [A] time = 59.74, size = 1919, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + 3*d**6*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5)
 + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt
(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2
*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin
(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(
48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2))
- e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**5*e**2*Piecewise((-
8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**
2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2
 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**4*e**3*Piecew
ise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**
2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3
*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*
x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d*
*2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2
*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/
(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)
) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewis
e((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**
2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x
**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e,
 0)), (x**8*sqrt(d**2)/8, True)) + d**2*e**5*Piecewise((-7*I*d**10*acosh(e*
x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**
7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt
(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) -
 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9)
- 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt
(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*
x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6
*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*s
```

```
qrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(11
55*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d
**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x
**10*sqrt(d**2)/10, True)) + e**7*Piecewise((-21*I*d**12*acosh(e*x/d)/(1024
*e**11) + 21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**9*x*
*3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**5/(2560*e**6*sqrt(-1
 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*
d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d*x**11/(120*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**13/(12*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2/d**2) > 1), (21*d**12*asin(e*x/d)/(1024*e**11) - 21*d**11*x/(1024*e*
*10*sqrt(1 - e**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d
**2)) + 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**2)) + d**5*x**7/(640*e
**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*sqrt(1 - e**2*x**2/d**2
)) + 11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e**2*x**13/(12*d*sqrt(1 -
e**2*x**2/d**2)), True))
```

3.68 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=223

$$-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}+\frac{19d^{11}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3}+\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+$$

```
[Out] 19/384*d^7*x*(-e^2*x^2+d^2)^(3/2)/e^2+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^2
-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-1/11*
e*x^4*(-e^2*x^2+d^2)^(7/2)-1/55440*d^3*(13167*e*x+5920*d)*(-e^2*x^2+d^2)^(7
/2)/e^3+19/256*d^11*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+19/256*d^9*x*(-e^2
*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.31, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3}-\frac{37d^2x^2\left(d^2-\right.}{99e}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (19*d^9*x*Sqrt[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^(3/2))
/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^(5/2))/(480*e^2) - (37*d^2*x^2*(d^2
- e^2*x^2)^(7/2))/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^(7/2))/10 - (e*x^4*(d^2
 - e^2*x^2)^(7/2))/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^(7/2))/(5
5440*e^3) + (19*d^11*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^2\left(d^2-e^2x^2\right)^{5/2}\left(-11d^3e^2-37d^2e^3x-33de^4x^2\right) \ldots}{11e^2}$$

$$= -\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^2\left(209d^3e^4+370d^2e^5x\right)\left(\ldots\right.}{110e^4}$$

$$= -\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x \ldots}{\ldots}$$

$$= -\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{d^3(5\ldots}{\ldots}$$

$$= \frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2} - \frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4 \ldots$$

$$= \frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2} + \frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2} - \frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e} - \frac{3}{10}dx \ldots$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2} + \frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2} - \frac{37d^2x^2\left(\ldots\right.}{\ldots}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2} + \frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2} - \frac{37d^2x^2\left(\ldots\right.}{\ldots}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2} + \frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2} - \frac{37d^2x^2\left(\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.27, size = 178, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(65835d^{10}\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-94720d^{10}-65835d^9ex-47360d^8e^2x^2+251790d^7e^3x^3+629\ldots\right.\right.}{887040e^3\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-94720*d^10 - 65835*d^9*e*x
- 47360*d^8*e^2*x^2 + 251790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*
e^5*x^5 - 657920*d^4*e^6*x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266
112*d*e^9*x^9 + 80640*e^10*x^10) + 65835*d^10*ArcSin[(e*x)/d]))/(887040*e^3
*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.92, size = 161, normalized size = 0.72

$$-\frac{131670\,d^{11}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(80640\,e^{10}x^{10}+266112\,de^9x^9+89600\,d^2e^8x^8-587664\,d^3e^7x^7-657920\,d^4\cdots\right.}{887\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/887040*(131670*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (80640*e
^10*x^10 + 266112*d*e^9*x^9 + 89600*d^2*e^8*x^8 - 587664*d^3*e^7*x^7 - 6579
20*d^4*e^6*x^6 + 201432*d^5*e^5*x^5 + 629760*d^6*e^4*x^4 + 251790*d^7*e^3*x
^3 - 47360*d^8*e^2*x^2 - 65835*d^9*e*x - 94720*d^10)*sqrt(-e^2*x^2 + d^2))/
e^3
```

giac [A] time = 0.29, size = 139, normalized size = 0.62

$$\frac{19}{256}\,d^{11}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{887040}\left(94720\,d^{10}e^{(-3)}+\left(65835\,d^9e^{(-2)}+2\left(23680\,d^8e^{(-1)}-\left(125895\,d^7+4\left(7\cdots\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 19/256*d^11*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/887040*(94720*d^10*e^(-3) + (65
835*d^9*e^(-2) + 2*(23680*d^8*e^(-1) - (125895*d^7 + 4*(78720*d^6*e + (2517
9*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e^4 - 8*(100*d^2*e^5 + 9*(10*x*e
^7 + 33*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 216, normalized size = 0.97

$$\frac{19d^{11}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{256\sqrt{e^2}\,e^2}+\frac{19\sqrt{-e^2x^2+d^2}\,d^9x}{256e^2}+\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^4}{11}+\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x}{480e^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/11*e*x^4*(-e^2*x^2+d^2)^(7/2)-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-74/69
3/e^3*d^4*(-e^2*x^2+d^2)^(7/2)-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-19/80/e^2*d^
3*x*(-e^2*x^2+d^2)^(7/2)+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^2+19/384*d^7*x
*(-e^2*x^2+d^2)^(3/2)/e^2+19/256*d^9*x*(-e^2*x^2+d^2)^(1/2)/e^2+19/256/e^2*
d^11/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 195, normalized size = 0.87

$$\frac{19\,d^{11}\arcsin\left(\frac{ex}{d}\right)}{256\,e^3}+\frac{19\,\sqrt{-e^2x^2+d^2}\,d^9x}{256\,e^2}-\frac{1}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^4+\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384\,e^2}-\frac{3}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^3+\frac{19\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 19/256*d^11*arcsin(e*x/d)/e^3 + 19/256*sqrt(-e^2*x^2 + d^2)*d^9*x/e^2 - 1/1
1*(-e^2*x^2 + d^2)^(7/2)*e*x^4 + 19/384*(-e^2*x^2 + d^2)^(3/2)*d^7*x/e^2 -
```

```
3/10*(-e^2*x^2 + d^2)^(7/2)*d*x^3 + 19/480*(-e^2*x^2 + d^2)^(5/2)*d^5*x/e^2
 - 37/99*(-e^2*x^2 + d^2)^(7/2)*d^2*x^2/e - 19/80*(-e^2*x^2 + d^2)^(7/2)*d^
3*x/e^2 - 74/693*(-e^2*x^2 + d^2)^(7/2)*d^4/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(d^2 - e^2 x^2\right)^{5/2} (d + e x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [C] time = 40.61, size = 1681, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*
*6*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**5*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5
) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqr
t(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asi
n(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/
(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2))
 - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise
((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*
x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d
**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**3*e**4*Pie
cewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 +
e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d
**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e*
*2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e
**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x*
*5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d*
*2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewi
se((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e*
*2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*
x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e
, 0)), (x**8*sqrt(d**2)/8, True)) + 3*d*e**6*Piecewise((-7*I*d**10*acosh(e*
x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**
7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt
(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) -
 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9)
- 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt
(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*
x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Pie
cewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(
```

```
d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e
**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2
- e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10
*sqrt(d**2)/10, True))
```

3.69 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=230

$$-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^{10}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}+\frac{33d}{}$$

```
[Out] 11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+11/160*d^4*x*(-e^2*x^2+d^2)^(5/2)/e-33/
560*d^3*(-e^2*x^2+d^2)^(7/2)/e^2-11/240*d^2*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e^
2-1/30*d*(e*x+d)^2*(-e^2*x^2+d^2)^(7/2)/e^2-1/10*(e*x+d)^3*(-e^2*x^2+d^2)^(
7/2)/e^2+33/256*d^10*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+33/256*d^8*x*(-e^
2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {795, 671, 641, 195, 217, 203}

$$\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (33*d^8*x*Sqrt[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^(3/2))/(
128*e) + (11*d^4*x*(d^2 - e^2*x^2)^(5/2))/(160*e) - (33*d^3*(d^2 - e^2*x^2)
^(7/2))/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(240*e^2) - (d
*(d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)
^(7/2))/(10*e^2) + (33*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
```

```
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(3d)\int(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx}{10e}$$

$$= -\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{\left(11d^2\right)\int(d+ex)^2\left(d^2-e\right.}{30e}$$

$$= -\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}$$

$$= -\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-$$

$$= \frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+}{}$$

$$= \frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex}{2}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2\right.}{560e^2}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2\right.}{560e^2}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2\right.}{560e^2}$$

Mathematica [A] time = 0.38, size = 167, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\left(3465d^9\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-6400d^9-3465d^8ex+10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-\right.\right.}{26880e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-6400*d^9 - 3465*d^8*e*x + 1
0240*d^7*e^2*x^2 + 24570*d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5
 - 20480*d^3*e^6*x^6 + 3024*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9) +
3465*d^9*ArcSin[(e*x)/d]))/(26880*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.81, size = 150, normalized size = 0.65

$$-\frac{6930\,d^{10}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2688\,e^9x^9+8960\,de^8x^8+3024\,d^2e^7x^7-20480\,d^3e^6x^6-23352\,d^4e^5x^5+\right.}{26880\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/26880*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2688*e^9*x
^9 + 8960*d*e^8*x^8 + 3024*d^2*e^7*x^7 - 20480*d^3*e^6*x^6 - 23352*d^4*e^5*
x^5 + 7680*d^5*e^4*x^4 + 24570*d^6*e^3*x^3 + 10240*d^7*e^2*x^2 - 3465*d^8*e
*x - 6400*d^9)*sqrt(-e^2*x^2 + d^2))/e^2
```

giac [A] time = 0.24, size = 128, normalized size = 0.56

$$\frac{33}{256}\,d^{10}\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{26880}\left(6400\,d^9e^{(-2)}+\left(3465\,d^8e^{(-1)}-2\left(5120\,d^7+\left(12285\,d^6e+4\left(960\,d^5e^2\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 33/256*d^10*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/26880*(6400*d^9*e^(-2) + (3465*
d^8*e^(-1) - 2*(5120*d^7 + (12285*d^6*e + 4*(960*d^5*e^2 - (2919*d^4*e^3 +
2*(1280*d^3*e^4 - 7*(27*d^2*e^5 + 8*(3*x*e^7 + 10*d*e^6)*x)*x)*x)*x)*x)*x)*
x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 191, normalized size = 0.83

$$\frac{33d^{10}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{256\sqrt{e^2}\,e}+\frac{33\sqrt{-e^2x^2+d^2}\,d^8x}{256e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x}{128e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x}{160e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/10*e*x^3*(-e^2*x^2+d^2)^(7/2)-33/80/e*d^2*x*(-e^2*x^2+d^2)^(7/2)+11/160*
d^4*x*(-e^2*x^2+d^2)^(5/2)/e+11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+33/256*d^8
*x*(-e^2*x^2+d^2)^(1/2)/e+33/256/e*d^10/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^
2*x^2+d^2)^(1/2)*x)-1/3*d*x^2*(-e^2*x^2+d^2)^(7/2)-5/21*d^3*(-e^2*x^2+d^2)^
(7/2)/e^2
```

maxima [A] time = 0.99, size = 170, normalized size = 0.74

$$\frac{33\,d^{10}\arcsin\left(\frac{ex}{d}\right)}{256\,e^2}+\frac{33\,\sqrt{-e^2x^2+d^2}\,d^8x}{256\,e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x}{128\,e}-\frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^3+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x}{160\,e}-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 33/256*d^10*arcsin(e*x/d)/e^2 + 33/256*sqrt(-e^2*x^2 + d^2)*d^8*x/e + 11/12
8*(-e^2*x^2 + d^2)^(3/2)*d^6*x/e - 1/10*(-e^2*x^2 + d^2)^(7/2)*e*x^3 + 11/1
60*(-e^2*x^2 + d^2)^(5/2)*d^4*x/e - 1/3*(-e^2*x^2 + d^2)^(7/2)*d*x^2 - 33/8
0*(-e^2*x^2 + d^2)^(7/2)*d^2*x/e - 5/21*(-e^2*x^2 + d^2)^(7/2)*d^3/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(d^2-e^2\,x^2\right)^{5/2}\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [A] time = 40.18, size = 1554, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + 3*d**6*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*
d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2
)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x
**2/d**2)), True)) + d**5*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(1
5*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**
2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**4*e**3*Piecewise((-
I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)
) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-
1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e
**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(
24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), Tr
ue)) - 5*d**3*e**4*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4
*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x
**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)
/6, True)) + d**2*e**5*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d
**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 -
e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7
/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d*e**6*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8
*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2
*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(
d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) + e**7*Piecewise
((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*
x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5
*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*as
in(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**
7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1
- e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x*
*9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2
)), True))
```

3.70 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=188

$$-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{55d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}+\frac{55}{128}d^7x\sqrt{d^2}$$

```
[Out] 55/192*d^5*x*(-e^2*x^2+d^2)^(3/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)-11/56*d^
2*(-e^2*x^2+d^2)^(7/2)/e-11/72*d*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e-1/9*(e*x+d)
^2*(-e^2*x^2+d^2)^(7/2)/e+55/128*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+55/
128*d^7*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7}}{72e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (55*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (55*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (11*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 - (11*d^2*(d^2 - e^2*x^2)^(7/2))/(56*e
) - (11*d*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(72*e) - ((d + e*x)^2*(d^2 - e^2
*x^2)^(7/2))/(9*e) + (55*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
```

```
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx = -\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{9}(11d)\int (d+ex)^2\left(d^2-e^2x^2\right)^{5/2}dx$$

$$= -\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{8}\left(11d^2\right)\int (d+ex)\left(d^2-\right.$$

$$= -\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{8}($$

$$= \frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-(d+ex)$$

$$= \frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)}{7}$$

$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e\right.}{56e}$$

$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e\right.}{56e}$$

$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e\right.}{56e}$$

Mathematica [A] time = 0.33, size = 156, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2x^2}\left(3465d^8\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-3712d^8+4599d^7ex+10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7\right.\right.}{8064e\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-3712*d^8 + 4599*d^7*e*x + 1
0240*d^6*e^2*x^2 + 3066*d^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 +
 1024*d^2*e^6*x^6 + 3024*d*e^7*x^7 + 896*e^8*x^8) + 3465*d^8*ArcSin[(e*x)/d
]))/(8064*e*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.90, size = 139, normalized size = 0.74

$$-\frac{6930\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(896\,e^8x^8+3024\,de^7x^7+1024\,d^2e^6x^6-7224\,d^3e^5x^5-8448\,d^4e^4x^4+3066\,d^5e\right.}{8064\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(6930*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (896*e^8*x^8
+ 3024*d*e^7*x^7 + 1024*d^2*e^6*x^6 - 7224*d^3*e^5*x^5 - 8448*d^4*e^4*x^4 +
 3066*d^5*e^3*x^3 + 10240*d^6*e^2*x^2 + 4599*d^7*e*x - 3712*d^8)*sqrt(-e^2*
x^2 + d^2))/e
```

giac [A] time = 0.25, size = 117, normalized size = 0.62

$$\frac{55}{128}\,d^9\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)-\frac{1}{8064}\left(3712\,d^8e^{(-1)}-\left(4599\,d^7+2\left(5120\,d^6e+\left(1533\,d^5e^2-4\left(1056\,d^4e^3+\left(90\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 55/128*d^9*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/8064*(3712*d^8*e^(-1) - (4599*d^
7 + 2*(5120*d^6*e + (1533*d^5*e^2 - 4*(1056*d^4*e^3 + (903*d^3*e^4 - 2*(64*
d^2*e^5 + 7*(8*x*e^7 + 27*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 154, normalized size = 0.82

$$\frac{55d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}}+\frac{55\sqrt{-e^2x^2+d^2}\,d^7x}{128}+\frac{55\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{192}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x}{48}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/9*e*x^2*(-e^2*x^2+d^2)^(7/2)-29/63*d^2*(-e^2*x^2+d^2)^(7/2)/e-3/8*d*x*(-
e^2*x^2+d^2)^(7/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)+55/192*d^5*x*(-e^2*x^2+
d^2)^(3/2)+55/128*d^7*x*(-e^2*x^2+d^2)^(1/2)+55/128*d^9/(e^2)^(1/2)*arctan(
(e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 136, normalized size = 0.72

$$\frac{55\,d^9\arcsin\left(\frac{ex}{d}\right)}{128\,e}+\frac{55}{128}\sqrt{-e^2x^2+d^2}\,d^7x+\frac{55}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{11}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x-\frac{1}{9}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 55/128*d^9*arcsin(e*x/d)/e + 55/128*sqrt(-e^2*x^2 + d^2)*d^7*x + 55/192*(-e
^2*x^2 + d^2)^(3/2)*d^5*x + 11/48*(-e^2*x^2 + d^2)^(5/2)*d^3*x - 1/9*(-e^2*
x^2 + d^2)^(7/2)*e*x^2 - 3/8*(-e^2*x^2 + d^2)^(7/2)*d*x - 29/63*(-e^2*x^2 +
 d^2)^(7/2)*d^2/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d^2-e^2\,x^2\right)^{5/2}\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [C] time = 25.88, size = 1284, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
```

```
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 3*d*
*6*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**5*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I
*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**
2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*
x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)
/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 -
e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**3*e**4*Piecewise
((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d*
*2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqr
t(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1
- e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**
5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)),
 True)) + d**2*e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) -
4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*
x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2
)/6, True)) + 3*d*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d
**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 -
e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7
/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + e**7*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**
6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**
2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2
 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.71 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=190

$$\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{125}{128}d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^8\tanh^{-}$$

```
[Out] 1/192*d^4*(125*e*x+64*d)*(-e^2*x^2+d^2)^(3/2)+1/240*d^2*(125*e*x+48*d)*(-e^
2*x^2+d^2)^(5/2)-3/7*d*(-e^2*x^2+d^2)^(7/2)-1/8*e*x*(-e^2*x^2+d^2)^(7/2)+12
5/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^8*arctanh((-e^2*x^2+d^2)^(1/2)
/d)+1/128*d^6*(125*e*x+128*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (d^6*(128*d + 125*e*x)*Sqrt[d^2 - e^2*x^2])/128 + (d^4*(64*d + 125*e*x)*(d^
2 - e^2*x^2)^(3/2))/192 + (d^2*(48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/240
- (3*d*(d^2 - e^2*x^2)^(7/2))/7 - (e*x*(d^2 - e^2*x^2)^(7/2))/8 + (125*d^8*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/128 - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx = -\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-8d^3e^2-25d^2e^3x-24de^4x^2\right)}{x}\,dx}{8e^2}$$

$$= -\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{\int \frac{\left(56d^3e^4+175d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx}{56e^4}$$

$$= \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}-\frac{\int (}{}$$

$$= \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\,(d^2$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2($$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2($$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2($$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2($$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2($$

Mathematica [A] time = 0.36, size = 168, normalized size = 0.88

$$d^8\left(-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{125d^7\sqrt{d^2-e^2x^2}\sin^{-1}\left(\frac{ex}{d}\right)}{128\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{\sqrt{d^2-e^2x^2}\left(14848d^7+27195d^6ex+7424d^5e^2x^2-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710*
d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 1680*
e^7*x^7))/13440 + (125*d^7*Sqrt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(128*Sqrt[1
 - (e^2*x^2)/d^2]) - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

fricas [A] time = 0.88, size = 151, normalized size = 0.79

$$-\frac{125}{64}d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{13440}\left(1680e^7x^7+5760de^6x^6+1960d^2\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -125/64*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^8*log(-(d - sqrt(
-e^2*x^2 + d^2))/x) + 1/13440*(1680*e^7*x^7 + 5760*d*e^6*x^6 + 1960*d^2*e^5
*x^5 - 14592*d^3*e^4*x^4 - 17710*d^4*e^3*x^3 + 7424*d^5*e^2*x^2 + 27195*d^6
*e*x + 14848*d^7)*sqrt(-e^2*x^2 + d^2)
```

giac [A] time = 0.26, size = 143, normalized size = 0.75

$$\frac{125}{128}d^8\arcsin\left(\frac{xe}{d}\right)\mathrm{sgn}(d)-d^8\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{1}{13440}\left(14848\,d^7+\left(27195\,d^6e+2\left(3712\,d\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 125/128*d^8*arcsin(x*e/d)*sgn(d) - d^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/13440*(14848*d^7 + (27195*d^6*e + 2*(3712*d^5
*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*(49*d^2*e^5 + 6*(7*x*e^7 + 24*d*
e^6)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 231, normalized size = 1.22

$$-\frac{d^9\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{125d^8e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}}+\frac{125\sqrt{-e^2x^2+d^2}\,d^6ex}{128}+\sqrt{-e^2x^2+d^2}\,d^7+\frac{125\left(-e^2x^2\cdots\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x)
```

```
[Out] -1/8*e*x*(-e^2*x^2+d^2)^(7/2)+25/48*d^2*e*x*(-e^2*x^2+d^2)^(5/2)+125/192*e*
d^4*x*(-e^2*x^2+d^2)^(3/2)+125/128*e*d^6*x*(-e^2*x^2+d^2)^(1/2)+125/128*e*d
^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3/7*d*(-e^2*x^2+d
^2)^(7/2)+1/5*d^3*(-e^2*x^2+d^2)^(5/2)+1/3*d^5*(-e^2*x^2+d^2)^(3/2)+d^7*(-e
^2*x^2+d^2)^(1/2)-d^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)
```

maxima [A] time = 1.00, size = 204, normalized size = 1.07

$$\frac{125}{128}\,d^8\arcsin\left(\frac{ex}{d}\right)-d^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{125}{128}\,\sqrt{-e^2x^2+d^2}\,d^6ex+\sqrt{-e^2x^2+d^2}\,d^7+\frac{125}{192}\left(-e^2x^2+d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 125/128*d^8*arcsin(e*x/d) - d^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x)) + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*e*x + sqrt(-e^2*x^2 + d^2)*d^7
+ 125/192*(-e^2*x^2 + d^2)^(3/2)*d^4*e*x + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^5 +
 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*e*x + 1/5*(-e^2*x^2 + d^2)^(5/2)*d^3 - 1/
8*(-e^2*x^2 + d^2)^(7/2)*e*x - 3/7*(-e^2*x^2 + d^2)^(7/2)*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x, x)
```

sympy [C] time = 47.72, size = 1263, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x,x)
```

```
[Out] d**7*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + 3*d**6*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(
2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2
/d**2)/2, True)) + d**5*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-
(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**4*e**3*Piecewise((-I*d**4
*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*
d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e
**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**
2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((-2*d
**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15
*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True
)) + d**2*e**5*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**
4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**
2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1
+ e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) -
 d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e*
*2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sq
rt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-8*d**6*sqrt(d**2 - e
**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2
*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(
e, 0)), (x**6*sqrt(d**2)/6, True)) + e**7*Piecewise((-5*I*d**8*acosh(e*x/d)
/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x*
*3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e
```

```
**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(
8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/
d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/
(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 -
 e**2*x**2/d**2)), True))
```

3.72 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=193

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{15}{16}d^7e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.$$

```
[Out] 1/8*d^3*e*(-5*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)+1/10*d*e*(-5*e*x+6*d)*(-e^2*x^2
+d^2)^(5/2)-1/7*e*(-e^2*x^2+d^2)^(7/2)-d*(-e^2*x^2+d^2)^(7/2)/x-15/16*d^7*e
*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-3*d^7*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3
/16*d^5*e*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (3*d^5*e*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^3*e*(8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/8 + (d*e*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/10 - (e*(d
^2 - e^2*x^2)^(7/2))/7 - (d*(d^2 - e^2*x^2)^(7/2))/x - (15*d^7*e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/16 - 3*d^7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-3d^4e+3d^3e^2x-d^2e^3x^2\right)}{x}dx}{d^2}$$

$$= -\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} + \frac{\int \frac{\left(21d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{7d^2e^2}$$

$$= \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{(-126d^6e^5 \cdots}{\cdots}}{\cdots}$$

$$= \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} \cdots$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\cdots \right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\cdots \right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\cdots \right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\cdots \right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\cdots \right.$$

Mathematica [C] time = 0.53, size = 221, normalized size = 1.15

$$-\frac{d^7\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - 3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{15d^6e\sqrt{d^2-e^2x^2}\,\sin^{-1}\left(\frac{ex}{d}\right)}{16\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{1}{560}e\sqrt{d^2-e^2x^2}\,(\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(2496*d^6 + 1155*d^5*e*x - 992*d^4*e^2*x^2 - 910*d^3
*e^3*x^3 + 96*d^2*e^4*x^4 + 280*d*e^5*x^5 + 80*e^6*x^6))/560 + (15*d^6*e*Sq
rt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(16*Sqrt[1 - (e^2*x^2)/d^2]) - 3*d^7*e*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d] - (d^7*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[
-5/2, -1/2, 1/2, (e^2*x^2)/d^2])/(x*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 1.02, size = 167, normalized size = 0.87

$$\frac{1050\,d^7ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 1680\,d^7ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 2496\,d^7ex + \left(80\,e^7x^7 + 280\,de^6x^6 + 96\,d^2e^5x^5 \cdots\right.}{560\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/560*(1050*d^7*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1680*d^7*e*
x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2496*d^7*e*x + (80*e^7*x^7 + 280*d*e
```

```
^6*x^6 + 96*d^2*e^5*x^5 - 770*d^3*e^4*x^4 - 992*d^4*e^3*x^3 + 525*d^5*e^2*x
^2 + 2496*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/x
```

giac [A] time = 0.24, size = 199, normalized size = 1.03

$$-\frac{15}{16}d^7\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}(d) - 3\,d^7e\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{d^7xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)} - \frac{\left(de+\sqrt{-x^2e^2\right.}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*d^7*arcsin(x*e/d)*e*sgn(d) - 3*d^7*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^7*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*
e) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^(-1)/x + 1/560*(2496*d^6*e +
(525*d^5*e^2 - 2*(496*d^4*e^3 + (385*d^3*e^4 - 4*(12*d^2*e^5 + 5*(2*x*e^7 +
 7*d*e^6)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 243, normalized size = 1.26

$$-\frac{3d^8e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} - \frac{15d^7e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}} - \frac{15\sqrt{-e^2x^2+d^2}\,d^5e^2x}{16} + 3\sqrt{-e^2x^2+d^2}\,d^6e - \frac{5\left(-e^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x)
```

```
[Out] -1/7*e*(-e^2*x^2+d^2)^(7/2)-1/2*d*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/8*d^3*e^2*x*
(-e^2*x^2+d^2)^(3/2)-15/16*d^5*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/16*d^7*e^2/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-d*(-e^2*x^2+d^2)^(7/2)/
x+3/5*d^2*e*(-e^2*x^2+d^2)^(5/2)+d^4*e*(-e^2*x^2+d^2)^(3/2)+3*d^6*e*(-e^2*x
^2+d^2)^(1/2)-3*d^8*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)
```

maxima [A] time = 1.00, size = 217, normalized size = 1.12

$$-\frac{15}{16}d^7e\arcsin\left(\frac{ex}{d}\right) - 3\,d^7e\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right) - \frac{15}{16}\sqrt{-e^2x^2+d^2}\,d^5e^2x + 3\,\sqrt{-e^2x^2+d^2}\,d^6e - \frac{5}{8}\left(-e^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] -15/16*d^7*e*arcsin(e*x/d) - 3*d^7*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d
^2)*d/abs(x)) - 15/16*sqrt(-e^2*x^2 + d^2)*d^5*e^2*x + 3*sqrt(-e^2*x^2 + d^
2)*d^6*e - 5/8*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^
4*e + 1/2*(-e^2*x^2 + d^2)^(5/2)*d*e^2*x + 3/5*(-e^2*x^2 + d^2)^(5/2)*d^2*e
 - 1/7*(-e^2*x^2 + d^2)^(7/2)*e - (-e^2*x^2 + d^2)^(5/2)*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2, x)
```

sympy [C] time = 19.88, size = 1057, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

```
[Out] d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d**6*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acos
h(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-
I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-
d**2/(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(
2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**
2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt
(1 - e**2*x**2/d**2)/2, True)) - 5*d**4*e**3*Piecewise((x**2*sqrt(d**2)/2,
Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**3*e**4*Pi
ecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x*
*2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqr
t(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**
3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x
**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*P
iecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt
(d**2)/4, True)) + 3*d*e**6*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d
**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/
(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d
)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**
2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2
*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((-8*d**6*sqrt
(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**
4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**
2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.73 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=207

$$\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{85}{16}d^6e^2\tan$$

```
[Out] 1/24*d^2*e^2*(-85*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+1/30*e^2*(-85*e*x+3*d)*(-e^
2*x^2+d^2)^(5/2)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x^2-3*e*(-e^2*x^2+d^2)^(7/2)/x-
85/16*d^6*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^6*e^2*arctanh((-e^2*x^
2+d^2)^(1/2)/d)+1/16*d^4*e^2*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (d^4*e^2*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^2*e^2*(4*d - 85*e*x)*(
d^2 - e^2*x^2)^(3/2))/24 + (e^2*(3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/30 -
(d*(d^2 - e^2*x^2)^(7/2))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(7/2))/x - (85*d^6
*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/16 - (d^6*e^2*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-6d^4e-d^3e^2x-2d^2e^3x^2\right)}{x^2}dx}{2d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x} + \frac{\int \frac{\left(d^5e^2-34d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{2d^4}$$

$$= \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{(-6d^7}{}}{}$$

$$= \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d\left(d^2-e^2\right.}{2x^2}$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d$$

Mathematica [C] time = 0.64, size = 259, normalized size = 1.25

$$-\frac{e\left(5040d^9\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)+ex\left(240\left(d^2-e^2x^2\right)^4\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-7d\left(1104d^7+165d^6e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] -1/1680*(e*(5040*d^9*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[-5/2, -1/2,
1/2, (e^2*x^2)/d^2] + e*x*(-7*d*(1104*d^7 + 165*d^6*e*x - 1632*d^5*e^2*x^2
- 295*d^4*e^3*x^3 + 672*d^3*e^4*x^4 + 170*d^2*e^5*x^5 - 144*d*e^6*x^6 - 40*
e^7*x^7 + 75*d^7*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d] - 720*d^6*Sqrt[d^2
 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]) + 240*(d^2 - e^2*x^2)^4*Hyperge
ometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2])))/(d*x*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 1.14, size = 179, normalized size = 0.86

$$\frac{2550\,d^6e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+120\,d^6e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+544\,d^6e^2x^2+\left(40\,e^7x^7+144\,de^6x^6+50\right.}{240\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/240*(2550*d^6*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 120*d^6
*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 544*d^6*e^2*x^2 + (40*e^7*x^7
```

```
 + 144*d*e^6*x^6 + 50*d^2*e^5*x^5 - 448*d^3*e^4*x^4 - 645*d^4*e^3*x^3 + 544
*d^5*e^2*x^2 - 720*d^6*e*x - 120*d^7)*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [A] time = 0.25, size = 262, normalized size = 1.27

$$-\frac{85}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}(d)-\frac{1}{2}\,d^6e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{1}{8}\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6e^8}{x}+\frac{\left(de\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -85/16*d^6*arcsin(x*e/d)*e^2*sgn(d) - 1/2*d^6*e^2*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(12*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^6*e^8/x + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*e^6/x^2)*e^(-8) + 1/240
*(544*d^5*e^2 - (645*d^4*e^3 + 2*(224*d^3*e^4 - (25*d^2*e^5 + 4*(5*x*e^7 +
18*d*e^6)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2) + 1/8*(d^6*e^6 + 12*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*d^6*e^4/x)*x^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
```

maple [A] time = 0.02, size = 252, normalized size = 1.22

$$-\frac{d^7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{85d^6e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}}-\frac{85\sqrt{-e^2x^2+d^2}\,d^4e^3x}{16}+\frac{\sqrt{-e^2x^2+d^2}\,d^5e^2}{2}-\frac{85\left(-e^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x)
```

```
[Out] -17/6*e^3*x*(-e^2*x^2+d^2)^(5/2)-85/24*e^3*d^2*x*(-e^2*x^2+d^2)^(3/2)-85/16
*e^3*d^4*x*(-e^2*x^2+d^2)^(1/2)-85/16*e^3*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2
)/(-e^2*x^2+d^2)^(1/2)*x)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x^2+1/10*d*e^2*(-e^2*x
^2+d^2)^(5/2)+1/6*d^3*e^2*(-e^2*x^2+d^2)^(3/2)+1/2*d^5*e^2*(-e^2*x^2+d^2)^(
1/2)-1/2*d^7*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)-3*e*(-e^2*x^2+d^2)^(7/2)/x
```

maxima [A] time = 1.00, size = 229, normalized size = 1.11

$$-\frac{85}{16}\,d^6e^2\arcsin\left(\frac{ex}{d}\right)-\frac{1}{2}\,d^6e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{85}{16}\,\sqrt{-e^2x^2+d^2}\,d^4e^3x+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,d^5e^2-\frac{85}{24}\left(-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -85/16*d^6*e^2*arcsin(e*x/d) - 1/2*d^6*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x
^2 + d^2)*d/abs(x)) - 85/16*sqrt(-e^2*x^2 + d^2)*d^4*e^3*x + 1/2*sqrt(-e^2*
x^2 + d^2)*d^5*e^2 - 85/24*(-e^2*x^2 + d^2)^(3/2)*d^2*e^3*x + 1/6*(-e^2*x^2
 + d^2)^(3/2)*d^3*e^2 + 1/6*(-e^2*x^2 + d^2)^(5/2)*e^3*x + 1/10*(-e^2*x^2 +
 d^2)^(5/2)*d*e^2 - 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e/x - 1/2*(-e^2*x^2 + d^2)
^(7/2)*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3, x)
```

sympy [C] time = 22.22, size = 1059, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)
```

```
[Out] d**7*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) + 3*d**6*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True)) + d**5*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) -
 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x*
*2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) +
 I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**4*e**3*Piecewise((-I*d**2
*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*
d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(
2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 5*d**3*e**4*Piecewise((x**2
*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) +
 d**2*e**5*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt
(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*
x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(
e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sq
rt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
+ 3*d*e**6*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*
sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0))
, (x**4*sqrt(d**2)/4, True)) + e**7*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**
5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e*
*2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*as
in(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3
/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)
) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.74 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=210

$$-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{25}{8}d^5e^3\tan$$

```
[Out] -1/12*d*e^3*(25*e*x+26*d)*(-e^2*x^2+d^2)^(3/2)-1/30*e^2*(39*e*x+50*d)*(-e^2
*x^2+d^2)^(5/2)/x-1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-3/2*e*(-e^2*x^2+d^2)^(7/2)
/x^2-25/8*d^5*e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+13/2*d^5*e^3*arctanh((-e
^2*x^2+d^2)^(1/2)/d)-1/8*d^3*e^3*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] -(d^3*e^3*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/8 - (d*e^3*(26*d + 25*e*x)*(
d^2 - e^2*x^2)^(3/2))/12 - (e^2*(50*d + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*
x) - (d*(d^2 - e^2*x^2)^(7/2))/(3*x^3) - (3*e*(d^2 - e^2*x^2)^(7/2))/(2*x^2
) - (25*d^5*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (13*d^5*e^3*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-9d^4e-5d^3e^2x-3d^2e^3x^2\right)}{x^3}dx}{3d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} + \frac{\int \frac{\left(10d^5e^2-39d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^2}dx}{6d^4}$$

$$= -\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \int \frac{(78d^6e^3+}{}$$

$$= -\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right.}{3x^3}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{}$$

Mathematica [C] time = 0.27, size = 251, normalized size = 1.20

$$-\frac{3e^3\left(d^2-e^2x^2\right)^{7/2}\,_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^2} - \frac{d^7\sqrt{d^2-e^2x^2}\,_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3d^5e^2\sqrt{d^2-e^2x^2}\,_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\right.}{x\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] (e^3*(Sqrt[d^2 - e^2*x^2]*(23*d^4 - 11*d^2*e^2*x^2 + 3*e^4*x^4) - 15*d^5*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d]))/15 - (d^7*Sqrt[d^2 - e^2*x^2]*Hypergeometric
2F1[-5/2, -3/2, -1/2, (e^2*x^2)/d^2])/(3*x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (3*
d^5*e^2*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (e^2*x^2)/d^
2])/(x*Sqrt[1 - (e^2*x^2)/d^2]) - (3*e^3*(d^2 - e^2*x^2)^(7/2)*Hypergeometr
ic2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2])/(7*d^2)
```

fricas [A] time = 0.89, size = 179, normalized size = 0.85

$$\frac{750\,d^5e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 780\,d^5e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - 656\,d^5e^3x^3 + \left(24\,e^7x^7 + 90\,de^6x^6 + 32\,d^2e^5x\right.}{120\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/120*(750*d^5*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 780*d^5*
e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 656*d^5*e^3*x^3 + (24*e^7*x^7
```

```
+ 90*d*e^6*x^6 + 32*d^2*e^5*x^5 - 345*d^3*e^4*x^4 - 656*d^4*e^3*x^3 - 80*d^
5*e^2*x^2 - 180*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^3
```

giac [A] time = 0.29, size = 318, normalized size = 1.51

$$-\frac{25}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^3\mathrm{sgn}(d)+\frac{13}{2}\,d^5e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(d^5e^8+\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5e^6}{x}+\frac{9\left(de\right.}{}\right.}{24\left(de+\sqrt{-x^2e^2+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -25/8*d^5*arcsin(x*e/d)*e^3*sgn(d) + 13/2*d^5*e^3*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d^5*e^8 + 9*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^5*e^6/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^4/x^2)*x^
3*e/(d*e + sqrt(-x^2*e^2 + d^2)*e)^3 - 1/24*(9*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^5*e^16/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^14/x^2 + (d*e + sq
rt(-x^2*e^2 + d^2)*e)^3*d^5*e^12/x^3)*e^(-15) - 1/120*(656*d^4*e^3 + (345*d
^3*e^4 - 2*(16*d^2*e^5 + 3*(4*x*e^7 + 15*d*e^6)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)
```

maple [A] time = 0.02, size = 277, normalized size = 1.32

$$\frac{13d^6e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{25d^5e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{25\sqrt{-e^2x^2+d^2}\,d^3e^4x}{8}-\frac{13\sqrt{-e^2x^2+d^2}\,d^4e^3}{2}-\frac{25\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x)
```

```
[Out] -1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-5/3/d*e^2/x*(-e^2*x^2+d^2)^(7/2)-5/3/d*e^4*
x*(-e^2*x^2+d^2)^(5/2)-25/12*d*e^4*x*(-e^2*x^2+d^2)^(3/2)-25/8*d^3*e^4*x*(-
e^2*x^2+d^2)^(1/2)-25/8*d^5*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^
2)^(1/2)*x)-3/2*e*(-e^2*x^2+d^2)^(7/2)/x^2-13/10*e^3*(-e^2*x^2+d^2)^(5/2)-1
3/6*d^2*e^3*(-e^2*x^2+d^2)^(3/2)-13/2*d^4*e^3*(-e^2*x^2+d^2)^(1/2)+13/2*d^6
*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 1.01, size = 226, normalized size = 1.08

$$-\frac{25}{8}\,d^5e^3\arcsin\left(\frac{ex}{d}\right)+\frac{13}{2}\,d^5e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{25}{8}\,\sqrt{-e^2x^2+d^2}\,d^3e^4x-\frac{13}{2}\,\sqrt{-e^2x^2+d^2}\,d^4e^3-\frac{25}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -25/8*d^5*e^3*arcsin(e*x/d) + 13/2*d^5*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x
^2 + d^2)*d/abs(x)) - 25/8*sqrt(-e^2*x^2 + d^2)*d^3*e^4*x - 13/2*sqrt(-e^2*
x^2 + d^2)*d^4*e^3 - 25/12*(-e^2*x^2 + d^2)^(3/2)*d*e^4*x - 13/6*(-e^2*x^2
+ d^2)^(3/2)*d^2*e^3 - 13/10*(-e^2*x^2 + d^2)^(5/2)*e^3 - 5/3*(-e^2*x^2 + d
^2)^(5/2)*d*e^2/x - 3/2*(-e^2*x^2 + d^2)^(7/2)*e/x^2 - 1/3*(-e^2*x^2 + d^2)
^(7/2)*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4, x)
```

sympy [C] time = 15.74, size = 911, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 3*d**6*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2
)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*
d), True)) + d**5*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*a
cosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 -
e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2
*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/
(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/
(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**3*e**4*Piecewise(
(-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*
x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(
e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + d**2*e**5*Piecewise
((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), Tr
ue)) + 3*d*e**6*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2
*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*
asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/
(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), Tr
ue)) + e**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2
*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)
), (x**4*sqrt(d**2)/4, True))
```

3.75 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d\right.}{8x}$$

```
[Out] 15/8*d*e^3*(-e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x-3/8*e^2*(2*e*x+3*d)*(-e^2*x^2+
d^2)^(5/2)/x^2-1/4*d*(-e^2*x^2+d^2)^(7/2)/x^4-e*(-e^2*x^2+d^2)^(7/2)/x^3+45
/8*d^4*e^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+45/8*d^4*e^4*arctanh((-e^2*x^2+
d^2)^(1/2)/d)-45/8*d^2*e^4*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{d\left(d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (-45*d^2*e^4*(d - e*x)*Sqrt[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2
- e^2*x^2)^(3/2))/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^(5/2))/(8*x^
2) - (d*(d^2 - e^2*x^2)^(7/2))/(4*x^4) - (e*(d^2 - e^2*x^2)^(7/2))/x^3 + (4
5*d^4*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^5} dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-12d^4e-9d^3e^2x-4d^2e^3x^2\right)}{x^4} dx}{4d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} + \frac{\int \frac{\left(27d^5e^2-36d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^3} dx}{12d^4}$$

$$= -\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} - \frac{5\int \frac{(144d}{}}{}$$

$$= \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\right.}{8x^2}$$

Mathematica [C] time = 0.10, size = 195, normalized size = 0.93

$$\frac{e\sqrt{d^2-e^2x^2}\left(3\left(e^3x^2-d^2e\right)^3 {}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+\left(e^3x^2-d^2e\right)^3 {}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-\frac{7d^9 {}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3\sqrt{1-\frac{e^2x^2}{d^2}}}\right.}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*((-7*d^9*Hypergeometric2F1[-5/2, -3/2, -1/2, (e^2*x^
2)/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (7*d^7*e^2*Hypergeometric2F1[-5/2,
 -1/2, 1/2, (e^2*x^2)/d^2])/(x*Sqrt[1 - (e^2*x^2)/d^2]) + 3*(-(d^2*e) + e^3
*x^2)^3*Hypergeometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2] + (-(d^2*e) + e^3
*x^2)^3*Hypergeometric2F1[3, 7/2, 9/2, 1 - (e^2*x^2)/d^2]))/(7*d^3)
```

fricas [A] time = 0.96, size = 180, normalized size = 0.86

$$-\frac{90\,d^4e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+45\,d^4e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+48\,d^4e^4x^4-\left(2\,e^7x^7+8\,de^6x^6+3\,d^2e^5x^5\right.}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(90*d^4*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 45*d^4*e^4
*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 48*d^4*e^4*x^4 - (2*e^7*x^7 + 8*d
```

```
*e^6*x^6 + 3*d^2*e^5*x^5 - 48*d^3*e^4*x^4 + 48*d^4*e^3*x^3 - 3*d^5*e^2*x^2
- 8*d^6*e*x - 2*d^7)*sqrt(-e^2*x^2 + d^2))/x^4
```

giac [B] time = 0.27, size = 374, normalized size = 1.79

$$\frac{45}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}(d)+\frac{45}{8}\,d^4e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(d^4e^{10}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e^8}{x}+\frac{8\left(de+\sqrt{-}\right.}{}\right.}{64\left(de+\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 45/8*d^4*arcsin(x*e/d)*e^4*sgn(d) + 45/8*d^4*e^4*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/64*(d^4*e^10 + 8*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^4*e^8/x + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^6/x^2 - 1
84*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^4/x^3)*x^4*e^2/(d*e + sqrt(-x^2*e
^2 + d^2)*e)^4 + 1/64*(184*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^26/x - 8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^24/x^2 - 8*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^4*e^22/x^3 - (d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^20/x^4)*e^(-24)
 - 1/8*(48*d^3*e^4 - (3*d^2*e^5 + 2*(x*e^7 + 4*d*e^6)*x)*x)*sqrt(-x^2*e^2 +
 d^2)
```

maple [A] time = 0.02, size = 302, normalized size = 1.44

$$\frac{45d^5e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{45d^4e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{45\sqrt{-e^2x^2+d^2}\,d^2e^5x}{8}-\frac{45\sqrt{-e^2x^2+d^2}\,d^3e^4}{8}+\frac{15\left(-e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x)
```

```
[Out] -e*(-e^2*x^2+d^2)^(7/2)/x^3+3/d^2*e^3/x*(-e^2*x^2+d^2)^(7/2)+3/d^2*e^5*x*(-
e^2*x^2+d^2)^(5/2)+15/4*e^5*x*(-e^2*x^2+d^2)^(3/2)+45/8*d^2*e^5*x*(-e^2*x^2
+d^2)^(1/2)+45/8*d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2
)*x)-9/8/d*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-9/8/d*e^4*(-e^2*x^2+d^2)^(5/2)-15/8
*d*e^4*(-e^2*x^2+d^2)^(3/2)-45/8*d^3*e^4*(-e^2*x^2+d^2)^(1/2)+45/8*d^5*e^4/
(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4*d*(-e^2*x^
2+d^2)^(7/2)/x^4
```

maxima [A] time = 0.99, size = 250, normalized size = 1.20

$$\frac{45}{8}\,d^4e^4\arcsin\left(\frac{ex}{d}\right)+\frac{45}{8}\,d^4e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{45}{8}\,\sqrt{-e^2x^2+d^2}\,d^2e^5x-\frac{45}{8}\,\sqrt{-e^2x^2+d^2}\,d^3e^4+\frac{15}{4}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 45/8*d^4*e^4*arcsin(e*x/d) + 45/8*d^4*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^
2 + d^2)*d/abs(x)) + 45/8*sqrt(-e^2*x^2 + d^2)*d^2*e^5*x - 45/8*sqrt(-e^2*x
^2 + d^2)*d^3*e^4 + 15/4*(-e^2*x^2 + d^2)^(3/2)*e^5*x - 15/8*(-e^2*x^2 + d^
2)^(3/2)*d*e^4 - 9/8*(-e^2*x^2 + d^2)^(5/2)*e^4/d + 3*(-e^2*x^2 + d^2)^(5/2
)*e^3/x - 9/8*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^2) - (-e^2*x^2 + d^2)^(7/2)*e
/x^3 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5, x)
```

sympy [C] time = 20.10, size = 1028, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)
```

```
[Out] d**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + d**5*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**
2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**
2*asin(d/(e*x))/(2*d), True)) - 5*d**4*e**3*Piecewise((I*d/(x*sqrt(-1 + e**
2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d)
+ e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((d**2
/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*
x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2
) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + d
**2*e**5*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
3*d*e**6*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**
(3/2)/(3*e**2), True)) + e**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*
d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2
)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x
**2/d**2)), True))
```

3.76 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=216

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}+\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52e$$

```
[Out] 1/24*d*e^3*(-52*e*x+25*d)*(-e^2*x^2+d^2)^(3/2)/x^2-1/60*e^2*(25*e*x+52*d)*(
-e^2*x^2+d^2)^(5/2)/x^3-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5-3/4*e*(-e^2*x^2+d^2)
^(7/2)/x^4+13/2*d^3*e^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-25/8*d^3*e^5*arcta
nh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*e^4*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 813, 844, 217, 203, 266, 63, 208}

$$\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (d^2*e^4*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x) + (d*e^3*(25*d - 52*e*x
)*(d^2 - e^2*x^2)^(3/2))/(24*x^2) - (e^2*(52*d + 25*e*x)*(d^2 - e^2*x^2)^(5
/2))/(60*x^3) - (d*(d^2 - e^2*x^2)^(7/2))/(5*x^5) - (3*e*(d^2 - e^2*x^2)^(7
/2))/(4*x^4) + (13*d^3*e^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (25*d^3*e
^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-15d^4e-13d^3e^2x-5d^2e^3x^2\right)}{x^5}\,dx}{5d^2}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{\int \frac{\left(52d^5e^2-25d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx}{20d^4}$$
$$= -\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \int \frac{\left(150d^6e^3\right.}{}$$
$$= \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}$$
$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)}{60x}$$
$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)}{60x}$$
$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)}{60x}$$
$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)}{60x}$$
$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)}{60x}$$

Mathematica [C] time = 0.09, size = 199, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\left(5e^5\left(e^2x^2-d^2\right)^3\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+15e^5\left(e^2x^2-d^2\right)^3\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-\frac{7d^{11}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{x^5\sqrt{1-\frac{e^2x^2}{d^2}}}\right.}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((-7*d^11*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2
)/d^2])/(x^5*Sqrt[1 - (e^2*x^2)/d^2]) - (35*d^9*e^2*Hypergeometric2F1[-5/2,
 -3/2, -1/2, (e^2*x^2)/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) + 5*e^5*(-d^2 +
e^2*x^2)^3*Hypergeometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2] + 15*e^5*(-d^2
 + e^2*x^2)^3*Hypergeometric2F1[3, 7/2, 9/2, 1 - (e^2*x^2)/d^2]))/(35*d^4)
```

fricas [A] time = 0.95, size = 180, normalized size = 0.83

$$-\frac{1560\,d^3e^5x^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-375\,d^3e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-80\,d^3e^5x^5-\left(40\,e^7x^7+180\,de^6x^6+80\,d^2\right.}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(1560*d^3*e^5*x^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 375*d^
3*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 80*d^3*e^5*x^5 - (40*e^7*x^7
```

```
 + 180*d*e^6*x^6 + 80*d^2*e^5*x^5 + 656*d^3*e^4*x^4 + 345*d^4*e^3*x^3 - 32*
d^5*e^2*x^2 - 90*d^6*e*x - 24*d^7)*sqrt(-e^2*x^2 + d^2))/x^5
```

giac [B] time = 0.28, size = 430, normalized size = 1.99

$$\frac{13}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^5\mathrm{sgn}(d) - \frac{25}{8}\,d^3e^5\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{\left(6\,d^3e^{12} + \frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^{10}}{x} + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 13/2*d^3*arcsin(x*e/d)*e^5*sgn(d) - 25/8*d^3*e^5*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/960*(6*d^3*e^12 + 45*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^3*e^10/x + 50*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*e^8/x
^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^6/x^3 - 2580*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^4*d^3*e^4/x^4)*x^5*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e)^5 +
1/960*(2580*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^38/x + 600*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*d^3*e^36/x^2 - 50*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^
34/x^3 - 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3*e^32/x^4 - 6*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*d^3*e^30/x^5)*e^(-35) + 1/6*(4*d^2*e^5 + (2*x*e^7 + 9*d
*e^6)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.03, size = 327, normalized size = 1.51

$$-\frac{25d^4e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}} + \frac{13d^3e^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}} + \frac{13\sqrt{-e^2x^2+d^2}\,d\,e^6x}{2} + \frac{25\sqrt{-e^2x^2+d^2}\,d^2e^5}{8} + \frac{13}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x)
```

```
[Out] -3/4*e*(-e^2*x^2+d^2)^(7/2)/x^4+5/8/d^2*e^3/x^2*(-e^2*x^2+d^2)^(7/2)+5/8/d^
2*e^5*(-e^2*x^2+d^2)^(5/2)+25/24*e^5*(-e^2*x^2+d^2)^(3/2)+25/8*d^2*e^5*(-e^
2*x^2+d^2)^(1/2)-25/8*d^4*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)-13/15/d*e^2/x^3*(-e^2*x^2+d^2)^(7/2)+52/15/d^3*e^4/x*(-e^2*
x^2+d^2)^(7/2)+52/15/d^3*e^6*x*(-e^2*x^2+d^2)^(5/2)+13/3/d*e^6*x*(-e^2*x^2+
d^2)^(3/2)+13/2*d*e^6*x*(-e^2*x^2+d^2)^(1/2)+13/2*d^3*e^6/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5
```

maxima [A] time = 0.99, size = 278, normalized size = 1.29

$$\frac{13}{2}\,d^3e^5\arcsin\left(\frac{ex}{d}\right) - \frac{25}{8}\,d^3e^5\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right) + \frac{13}{2}\,\sqrt{-e^2x^2+d^2}\,de^6x + \frac{25}{8}\,\sqrt{-e^2x^2+d^2}\,d^2e^5 + \frac{13\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 13/2*d^3*e^5*arcsin(e*x/d) - 25/8*d^3*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^
2 + d^2)*d/abs(x)) + 13/2*sqrt(-e^2*x^2 + d^2)*d*e^6*x + 25/8*sqrt(-e^2*x^2
 + d^2)*d^2*e^5 + 13/3*(-e^2*x^2 + d^2)^(3/2)*e^6*x/d + 25/24*(-e^2*x^2 + d
^2)^(3/2)*e^5 + 5/8*(-e^2*x^2 + d^2)^(5/2)*e^5/d^2 + 52/15*(-e^2*x^2 + d^2)
^(5/2)*e^4/(d*x) + 5/8*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^2) - 13/15*(-e^2*x
^2 + d^2)^(7/2)*e^2/(d*x^3) - 3/4*(-e^2*x^2 + d^2)^(7/2)*e/x^4 - 1/5*(-e^2*
x^2 + d^2)^(7/2)*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)^3}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6, x)
```

sympy [C] time = 20.70, size = 1178, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**6,x)
```

```
[Out] d**7*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + 3*d**6*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e*
*2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x
**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)
) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**5*e**2*Piecewise((-e*sqrt(d*
*2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*d**4*e**3*Piecewise((-d
**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) -
1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d*
*2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**3*e**
4*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*
x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 -
e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), Tru
e)) + d**2*e**5*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) + 3*d*e**6*Piecewise((-I*d**2*acosh(e*x/d)/(2*e
) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x
**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1
- e**2*x**2/d**2)/2, True)) + e**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0
)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

3.77 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=214

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{1}{2}d^2e^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{85}{16}d^2e^6\tanh^{-}$$

```
[Out] 1/48*d*e^3*(85*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/120*e^2*(12*e*x+85*d)*(-
e^2*x^2+d^2)^(5/2)/x^4-1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-3/5*e*(-e^2*x^2+d^2)^
(7/2)/x^5-1/2*d^2*e^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-85/16*d^2*e^6*arctan
h((-e^2*x^2+d^2)^(1/2)/d)-1/16*d*e^5*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] -(d*e^5*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (d*e^3*(8*d + 85*e*x)*
(d^2 - e^2*x^2)^(3/2))/(48*x^3) - (e^2*(85*d + 12*e*x)*(d^2 - e^2*x^2)^(5/2
))/(120*x^4) - (d*(d^2 - e^2*x^2)^(7/2))/(6*x^6) - (3*e*(d^2 - e^2*x^2)^(7/
2))/(5*x^5) - (d^2*e^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (85*d^2*e^6*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/16
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7} dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-18d^4e-17d^3e^2x-6d^2e^3x^2\right)}{x^6} dx}{6d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} + \frac{\int \frac{\left(85d^5e^2-6d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^5} dx}{30d^4}$$

$$= -\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{\int \frac{(48d^6}{}}{}$$

$$= \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7}}{6x^6}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)}{12(}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)}{12(}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)}{12(}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)}{12(}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)}{12(}$$

Mathematica [C] time = 0.22, size = 286, normalized size = 1.34

$$-\frac{3d^6e\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3e^6\left(d^2-e^2x^2\right)^{7/2}\ {}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^5} - \frac{d^4e^3\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{5}{2},-\frac{3}{2}\right.}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] (-8*d^9 + 34*d^7*e^2*x^2 - 59*d^5*e^4*x^4 + 33*d^3*e^6*x^6 + 15*d^3*e^6*x^6
*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(48*x^6*Sqrt[d^2
 - e^2*x^2]) - (3*d^6*e*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -
3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1 - (e^2*x^2)/d^2]) - (d^4*e^3*Sqrt[d^2 -
e^2*x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (e^2*x^2)/d^2])/(3*x^3*Sqrt[1
- (e^2*x^2)/d^2]) - (3*e^6*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[3, 7/2,
9/2, 1 - (e^2*x^2)/d^2])/(7*d^5)
```

fricas [A] time = 0.94, size = 179, normalized size = 0.84

$$\frac{240\,d^2e^6x^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 1275\,d^2e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 720\,d^2e^6x^6 + \left(120\,e^7x^7 + 720\,de^6x^6 - 5\right.}{240\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(240*d^2*e^6*x^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1275*d^2
*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 720*d^2*e^6*x^6 + (120*e^7*x^
7 + 720*d*e^6*x^6 - 544*d^2*e^5*x^5 + 645*d^3*e^4*x^4 + 448*d^4*e^3*x^3 - 5
0*d^5*e^2*x^2 - 144*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^6
```

giac [B] time = 0.31, size = 485, normalized size = 2.27

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^6\mathrm{sgn}(d)-\frac{85}{16}\,d^2e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,d^2e^{14}+\frac{36\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^{12}}{x}+\frac{45\left(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^6*sgn(d) - 85/16*d^2*e^6*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/1920*(5*d^2*e^14 + 36*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*d^2*e^12/x + 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^1
0/x^2 - 340*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*e^8/x^3 - 1215*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^4*d^2*e^6/x^4 + 1800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d
^2*e^4/x^5)*x^6*e^4/(d*e + sqrt(-x^2*e^2 + d^2)*e)^6 - 1/1920*(1800*(d*e +
sqrt(-x^2*e^2 + d^2)*e)*d^2*e^52/x - 1215*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
d^2*e^50/x^2 - 340*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*e^48/x^3 + 45*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^46/x^4 + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e
)^5*d^2*e^44/x^5 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2*e^42/x^6)*e^(-48)
 + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^7 + 6*d*e^6)
```

maple [A] time = 0.03, size = 352, normalized size = 1.64

$$-\frac{85d^3e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}}-\frac{d^2e^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}\,e^7x}{2}+\frac{85\sqrt{-e^2x^2+d^2}\,d\,e^6}{16}-\frac{\left(-e^2x^2+d^2\right)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x)
```

```
[Out] 1/15*e^3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/15*e^5/d^4/x*(-e^2*x^2+d^2)^(7/2)-4
/15*e^7/d^4*x*(-e^2*x^2+d^2)^(5/2)-1/3*e^7/d^2*x*(-e^2*x^2+d^2)^(3/2)-1/2*e
^7*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-17/24/d*e^2/x
^4*(-e^2*x^2+d^2)^(7/2)+17/16/d^3*e^4/x^2*(-e^2*x^2+d^2)^(7/2)-85/16*d^3*e^
6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-3/5*e*(-e^2*
x^2+d^2)^(7/2)/x^5-1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-1/2*e^7*x*(-e^2*x^2+d^2)^
(1/2)+17/16/d^3*e^6*(-e^2*x^2+d^2)^(5/2)+85/48/d*e^6*(-e^2*x^2+d^2)^(3/2)+8
5/16*d*e^6*(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.00, size = 303, normalized size = 1.42

$$-\frac{1}{2}\,d^2e^6\arcsin\left(\frac{ex}{d}\right)-\frac{85}{16}\,d^2e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,e^7x+\frac{85}{16}\,\sqrt{-e^2x^2+d^2}\,de^6-\frac{\left(-e^2x^2+\right.}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*e^6*arcsin(e*x/d) - 85/16*d^2*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x
^2 + d^2)*d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*e^7*x + 85/16*sqrt(-e^2*x^2
+ d^2)*d*e^6 - 1/3*(-e^2*x^2 + d^2)^(3/2)*e^7*x/d^2 + 85/48*(-e^2*x^2 + d^2
```

```
)^(3/2)*e^6/d + 17/16*(-e^2*x^2 + d^2)^(5/2)*e^6/d^3 - 4/15*(-e^2*x^2 + d^2
)^(5/2)*e^5/(d^2*x) + 17/16*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^2) + 1/15*(-e
^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^3) - 17/24*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^4
) - 3/5*(-e^2*x^2 + d^2)^(7/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(7/2)*d/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2 - e^2 x^2\right)^{5/2} (d + e x)^3}{x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7, x)
```

sympy [C] time = 21.71, size = 1397, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)
[Out] d**7*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + 3*d**6*e*Piecewise((3*I*d**3*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*s
qrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d*
*2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) -
 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*
e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e*
*4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) +
 d**5*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*
x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e
*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))
/(8*d**3), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3
*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x*
*2) + 1)/(3*d**2), True)) - 5*d**3*e**4*Piecewise((-d**2/(2*e*x**3*sqrt(d**
2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*
x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2
*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + d**2*e**5*Piecewise((I*d/(x*sqrt
(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asi
n(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise
((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/
(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**
2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True
)) + e**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

3.78 $\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^8} \, dx$

Optimal. Leaf size=206

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-3de^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{15}{16}de^7\tanh^{-1}\left(\frac{\sqrt{d^2-\ldots}}{\ldots}\right.$$

```
[Out] 1/16*e^4*(5*e*x+16*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/40*e^2*(5*e*x+24*d)*(-e^2*
x^2+d^2)^(5/2)/x^5-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7-1/2*e*(-e^2*x^2+d^2)^(7/2
)/x^6-3*d*e^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-15/16*d*e^7*arctanh((-e^2*x^
2+d^2)^(1/2)/d)-3/16*e^6*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 811, 813, 844, 217, 203, 266, 63, 208}

$$-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\frac{d\left(d^2-\ldots\right.}{7\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (-3*e^6*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (e^4*(16*d + 5*e*x)*(d
^2 - e^2*x^2)^(3/2))/(16*x^3) - (e^2*(24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/
(40*x^5) - (d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(7/2))/(2
*x^6) - 3*d*e^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (15*d*e^7*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/16
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-21d^4e-21d^3e^2x-7d^2e^3x^2\right)}{x^7}dx}{7d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} + \frac{\int \frac{\left(126d^5e^2+21d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^6}dx}{42d^4}$$

$$= -\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} - \int \frac{(1008d^7e^4+2\ldots}{}$$

$$= \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \ldots$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\ldots\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\ldots\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\ldots\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\ldots\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\ldots\right.}{40x^5}$$

Mathematica [C] time = 0.15, size = 247, normalized size = 1.20

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e^7\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^6} - \frac{3d^5e^2\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{-8d^8e+34d^6\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] -1/7*(d*(d^2 - e^2*x^2)^(7/2))/x^7 + (-8*d^8*e + 34*d^6*e^3*x^2 - 59*d^4*e^
5*x^4 + 33*d^2*e^7*x^6 + 15*d^2*e^7*x^6*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]])/(16*x^6*Sqrt[d^2 - e^2*x^2]) - (3*d^5*e^2*Sqrt[d^2 -
 e^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2)/d^2]])/(5*x^5*Sqrt[1
 - (e^2*x^2)/d^2]) - (e^7*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9
/2, 1 - (e^2*x^2)/d^2])/(7*d^6)
```

fricas [A] time = 0.88, size = 173, normalized size = 0.84

$$\frac{3360\,de^7x^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 525\,de^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 560\,de^7x^7 + \left(560\,e^7x^7 - 2496\,de^6x^6 - 525\,d^2\ldots\right.}{560\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/560*(3360*d*e^7*x^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 525*d*e^7
*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 560*d*e^7*x^7 + (560*e^7*x^7 - 24
```

```
96*d*e^6*x^6 - 525*d^2*e^5*x^5 + 992*d^3*e^4*x^4 + 770*d^4*e^3*x^3 - 96*d^5
*e^2*x^2 - 280*d^6*e*x - 80*d^7)*sqrt(-e^2*x^2 + d^2))/x^7
```

giac [B] time = 0.31, size = 510, normalized size = 2.48

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{7}\mathrm{sgn}(d)-\frac{15}{16}\,de^{7}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,de^{16}+\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)de^{14}}{x}+\frac{49\left(de\cdots\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^7*sgn(d) - 15/16*d*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/4480*(5*d*e^16 + 35*(d*e + sqrt(-x^2*e^2
 + d^2)*e)*d*e^14/x + 49*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^12/x^2 - 245*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e^10/x^3 - 875*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^4*d*e^8/x^4 + 455*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d*e^6/x^5 + 9065*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d*e^4/x^6)*x^7*e^5/(d*e + sqrt(-x^2*e^2 + d
^2)*e)^7 - 1/4480*(9065*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^68/x + 455*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^2*d*e^66/x^2 - 875*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^3*d*e^64/x^3 - 245*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^62/x^4 + 49*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^5*d*e^60/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6
*d*e^58/x^6 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d*e^56/x^7)*e^(-63) + sqrt
(-x^2*e^2 + d^2)*e^7
```

maple [B] time = 0.05, size = 377, normalized size = 1.83

$$-\frac{15d^{2}e^{7}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}}-\frac{3d\,e^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}x}{d}+\frac{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{16}-\frac{2\left(-e^{2}x^{2}\cdots\right.}{d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x)
```

```
[Out] -3/5/d*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+2/5/d^3*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-8/
5/d^5*e^6/x*(-e^2*x^2+d^2)^(7/2)-8/5/d^5*e^8*x*(-e^2*x^2+d^2)^(5/2)-2/d^3*e
^8*x*(-e^2*x^2+d^2)^(3/2)-3/d*e^8*x*(-e^2*x^2+d^2)^(1/2)-3*d*e^8/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2*e*(-e^2*x^2+d^2)^(7/2)/x^6
-1/8/d^2*e^3/x^4*(-e^2*x^2+d^2)^(7/2)+3/16/d^4*e^5/x^2*(-e^2*x^2+d^2)^(7/2)
+3/16/d^4*e^7*(-e^2*x^2+d^2)^(5/2)+5/16/d^2*e^7*(-e^2*x^2+d^2)^(3/2)+15/16*
e^7*(-e^2*x^2+d^2)^(1/2)-15/16*d^2*e^7/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7
```

maxima [A] time = 1.01, size = 326, normalized size = 1.58

$$-3\,de^{7}\arcsin\left(\frac{ex}{d}\right)-\frac{15}{16}\,de^{7}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)-\frac{3\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}x}{d}+\frac{15}{16}\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}-\frac{2\left(-e^{2}x^{2}+\cdots\right.}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -3*d*e^7*arcsin(e*x/d) - 15/16*d*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d
^2)*d/abs(x)) - 3*sqrt(-e^2*x^2 + d^2)*e^8*x/d + 15/16*sqrt(-e^2*x^2 + d^2)
*e^7 - 2*(-e^2*x^2 + d^2)^(3/2)*e^8*x/d^3 + 5/16*(-e^2*x^2 + d^2)^(3/2)*e^7
/d^2 + 3/16*(-e^2*x^2 + d^2)^(5/2)*e^7/d^4 - 8/5*(-e^2*x^2 + d^2)^(5/2)*e^6
```

```
/(d^3*x) + 3/16*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^2) + 2/5*(-e^2*x^2 + d^2)
^(7/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^4) - 3/5*(-e^2
*x^2 + d^2)^(7/2)*e^2/(d*x^5) - 1/2*(-e^2*x^2 + d^2)^(7/2)*e/x^6 - 1/7*(-e^
2*x^2 + d^2)^(7/2)*d/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}(d+ex)^3}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8, x)
```

sympy [C] time = 22.29, size = 1513, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**8,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + 3*d**6*e
*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt
(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5)
, Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)
) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**5*e**2*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - 5*
d**4*e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x
**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)
) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*
x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*
x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1
), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**
2) + 1)/(3*d**2), True)) + d**2*e**5*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(
e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))
/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x)
 - I*e**2*asin(d/(e*x))/(2*d), True)) + 3*d*e**6*Piecewise((I*d/(x*sqrt(-1
+ e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**
2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*
x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((d**2/(
e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x*
```

```
*2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2)
+ 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.79 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=204

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}+e^8\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{125}{128}e^8\tanh^{-1}\left(\right.$$

```
[Out] 1/192*e^4*(64*e*x+125*d)*(-e^2*x^2+d^2)^(3/2)/x^4-1/240*e^2*(48*e*x+125*d)*
(-e^2*x^2+d^2)^(5/2)/x^6-1/8*d*(-e^2*x^2+d^2)^(7/2)/x^8-3/7*e*(-e^2*x^2+d^2
)^(7/2)/x^7-e^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+125/128*e^8*arctanh((-e^2*
x^2+d^2)^(1/2)/d)-1/128*e^6*(128*e*x+125*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] -(e^6*(125*d + 128*e*x)*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (e^4*(125*d + 64*e
*x)*(d^2 - e^2*x^2)^(3/2))/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2
)^(5/2))/(240*x^6) - (d*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (3*e*(d^2 - e^2*x^
2)^(7/2))/(7*x^7) - e^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (125*e^8*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/128
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = -\frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-24d^4e-25d^3e^2x-8d^2e^3x^2)}{x^8}\,dx}{8d^2}$$
$$= -\frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} + \frac{\int \frac{(175d^5e^2+56d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^7}\,dx}{56d^4}$$
$$= -\frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} - \int \frac{(1750d^{7}}{}$$
$$= \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - \frac{d(d^2-e^2x^2)^{7/}}{8x^8}$$
$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48e}{24}$$
$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48e}{24}$$
$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48e}{24}$$
$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48e}{24}$$
$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48e}{24}$$

Mathematica [C] time = 0.15, size = 245, normalized size = 1.20

$$-\frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} - \frac{e^8(d^2-e^2x^2)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^7} - \frac{d^4e^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{-8d^7e^2+34d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] (-3*e*(d^2 - e^2*x^2)^(7/2))/(7*x^7) + (-8*d^7*e^2 + 34*d^5*e^4*x^2 - 59*d^
3*e^6*x^4 + 33*d*e^8*x^6 + 15*d*e^8*x^6*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]])/(16*x^6*Sqrt[d^2 - e^2*x^2]) - (d^4*e^3*Sqrt[d^2 - e
^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1 -
 (e^2*x^2)/d^2]) - (e^8*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2
, 1 - (e^2*x^2)/d^2])/(7*d^7)
```

fricas [A] time = 0.86, size = 163, normalized size = 0.80

$$\frac{26880\,e^8x^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 13125\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(14848\,e^7x^7 + 27195\,de^6x^6 + 7424\,d^2e^5x^5 - \right.}{13440\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/13440*(26880*e^8*x^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13125*e^
8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (14848*e^7*x^7 + 27195*d*e^6*x^6
```

```
 + 7424*d^2*e^5*x^5 - 17710*d^3*e^4*x^4 - 14592*d^4*e^3*x^3 + 1960*d^5*e^2*
x^2 + 5760*d^6*e*x + 1680*d^7)*sqrt(-e^2*x^2 + d^2))/x^8
```

giac [B] time = 0.31, size = 538, normalized size = 2.64

$$-\arcsin\left(\frac{xe}{d}\right)e^8\mathrm{sgn}(d)+\frac{x^8\left(\frac{720\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{16}}{x}+\frac{1120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{14}}{x^2}-\frac{3696\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{12}}{x^3}-\frac{14280\left(de+\sqrt{\right.}}{}\right.}{215040\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^8*sgn(d) + 1/215040*x^8*(720*(d*e + sqrt(-x^2*e^2 + d^2)*e
)*e^16/x + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^14/x^2 - 3696*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^3*e^12/x^3 - 14280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^
10/x^4 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^8/x^5 + 77280*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^6*e^6/x^6 + 122640*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^4/x
^7 + 105*e^18)*e^6/(d*e + sqrt(-x^2*e^2 + d^2)*e)^8 - 1/215040*(122640*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)*e^86/x + 77280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
e^84/x^2 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^82/x^3 - 14280*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^4*e^80/x^4 - 3696*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^7
8/x^5 + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^76/x^6 + 720*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^7*e^74/x^7 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*e^72/x^8
)*e^(-80) + 125/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

maple [B] time = 0.06, size = 402, normalized size = 1.97

$$\frac{125d\,e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{128\sqrt{d^2}}-\frac{e^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}\,e^9x}{d^2}-\frac{125\sqrt{-e^2x^2+d^2}\,e^8}{128d}-\frac{2\left(-e^2x^2+d^2\right.}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x)
```

```
[Out] -1/8*d*(-e^2*x^2+d^2)^(7/2)/x^8-25/48/d*e^2/x^6*(-e^2*x^2+d^2)^(7/2)+25/192
/d^3*e^4/x^4*(-e^2*x^2+d^2)^(7/2)-25/128/d^5*e^6/x^2*(-e^2*x^2+d^2)^(7/2)-2
5/128/d^5*e^8*(-e^2*x^2+d^2)^(5/2)-125/384/d^3*e^8*(-e^2*x^2+d^2)^(3/2)-125
/128/d*e^8*(-e^2*x^2+d^2)^(1/2)+125/128*d*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-3/7*e*(-e^2*x^2+d^2)^(7/2)/x^7-1/5*e^3/d^2/
x^5*(-e^2*x^2+d^2)^(7/2)+2/15*e^5/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-8/15*e^7/d^6
/x*(-e^2*x^2+d^2)^(7/2)-8/15*e^9/d^6*x*(-e^2*x^2+d^2)^(5/2)-2/3*e^9/d^4*x*(
-e^2*x^2+d^2)^(3/2)-e^9/d^2*x*(-e^2*x^2+d^2)^(1/2)-e^9/(e^2)^(1/2)*arctan((
e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 1.01, size = 352, normalized size = 1.73

$$-e^8\arcsin\left(\frac{ex}{d}\right)+\frac{125}{128}e^8\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{\sqrt{-e^2x^2+d^2}\,e^9x}{d^2}-\frac{125\sqrt{-e^2x^2+d^2}\,e^8}{128\,d}-\frac{2\left(-e^2x^2+d^2\right)}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] -e^8*arcsin(e*x/d) + 125/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x)) - sqrt(-e^2*x^2 + d^2)*e^9*x/d^2 - 125/128*sqrt(-e^2*x^2 + d^2)*e
```

```
^8/d - 2/3*(-e^2*x^2 + d^2)^(3/2)*e^9*x/d^4 - 125/384*(-e^2*x^2 + d^2)^(3/2
)*e^8/d^3 - 25/128*(-e^2*x^2 + d^2)^(5/2)*e^8/d^5 - 8/15*(-e^2*x^2 + d^2)^(
5/2)*e^7/(d^4*x) - 25/128*(-e^2*x^2 + d^2)^(7/2)*e^6/(d^5*x^2) + 2/15*(-e^2
*x^2 + d^2)^(7/2)*e^5/(d^4*x^3) + 25/192*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^
4) - 1/5*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^5) - 25/48*(-e^2*x^2 + d^2)^(7/2
)*e^2/(d*x^6) - 3/7*(-e^2*x^2 + d^2)^(7/2)*e/x^7 - 1/8*(-e^2*x^2 + d^2)^(7/
2)*d/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2 x^2\right)^{5/2} (d+e x)^3}{x^9} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9, x)
```

sympy [C] time = 31.40, size = 1719, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9,x)
```

```
[Out] d**7*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6)
 + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*
x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*
e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**
6), True)) + d**5*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*ac
osh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 5*d**4*e**3
*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*
x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15
*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), True)) - 5*d**3*e**4*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**
2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sq
rt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x*
*2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
```

```
 - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**2*e**5*Piecewise((-e*sqrt(d**
2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs
(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**
3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + 3*d*e**6*Piecewise((-d**2/
(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1))
+ e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(
e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**7*Piecewise
((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d
**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.80 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=187

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{55e^9\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)}{192x^4}$$

```
[Out] 55/192*e^5*(-e^2*x^2+d^2)^(3/2)/x^4-11/48*e^3*(-e^2*x^2+d^2)^(5/2)/x^6-1/9*
d*(-e^2*x^2+d^2)^(7/2)/x^9-3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-29/63*e^2*(-e^2*x
^2+d^2)^(7/2)/d/x^7+55/128*e^9*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-55/128*e^7
*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 807, 266, 47, 63, 208}

$$-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{d\left(d^2-e^2x^2\right)^{7}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-55*e^7*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (55*e^5*(d^2 - e^2*x^2)^(3/2))/(1
92*x^4) - (11*e^3*(d^2 - e^2*x^2)^(5/2))/(48*x^6) - (d*(d^2 - e^2*x^2)^(7/2
))/(9*x^9) - (3*e*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (29*e^2*(d^2 - e^2*x^2)^
(7/2))/(63*d*x^7) + (55*e^9*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-27d^4e-29d^3e^2x-9d^2e^3x^2\right)}{x^9}\,dx}{9d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}+\frac{\int \frac{\left(232d^5e^2+99d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx}{72d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{1}{8}\left(11e^3\right)\int \frac{\left(d^2-\right.}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{1}{16}\left(11e^3\right)\text{Subst}\left(\right.$$

$$= -\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7}}{63dx^7}$$

$$= \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7}}{9x^9}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7}}{9x^9}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{d\left(d^2-e^2x^2\right)^{7}}{9x^9}$$

Mathematica [C] time = 0.17, size = 218, normalized size = 1.17

$$\frac{-112d^{10}-16d^8e^2x^2-168d^7e^3x^3+1184d^6e^4x^4+714d^5e^5x^5-2336d^4e^6x^6-1239d^3e^7x^7+1744d^2e^8x^8+315de}{1008dx^9\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-112*d^10 - 16*d^8*e^2*x^2 - 168*d^7*e^3*x^3 + 1184*d^6*e^4*x^4 + 714*d^5*
e^5*x^5 - 2336*d^4*e^6*x^6 - 1239*d^3*e^7*x^7 + 1744*d^2*e^8*x^8 + 693*d*e^
9*x^9 - 464*e^10*x^10 + 315*d*e^9*x^9*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[
1 - (e^2*x^2)/d^2]])/(1008*d*x^9*Sqrt[d^2 - e^2*x^2]) - (3*e^9*(d^2 - e^2*x
^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2])/(7*d^8)
```

fricas [A] time = 0.76, size = 142, normalized size = 0.76

$$-\frac{3465\,e^9x^9\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(3712\,e^8x^8-4599\,de^7x^7-10240\,d^2e^6x^6-3066\,d^3e^5x^5+8448\,d^4e^4x^4+7224\,d^5\right.}{8064\,dx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*e^9*x^9*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (3712*e^8*x^8 -
4599*d*e^7*x^7 - 10240*d^2*e^6*x^6 - 3066*d^3*e^5*x^5 + 8448*d^4*e^4*x^4 +
7224*d^5*e^3*x^3 - 1024*d^6*e^2*x^2 - 3024*d^7*e*x - 896*d^8)*sqrt(-e^2*x^2
 + d^2))/(d*x^9)
```

giac [B] time = 0.47, size = 620, normalized size = 3.32

$$\frac{x^9\left(\frac{189\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{18}}{x}+\frac{324\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{16}}{x^2}-\frac{672\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{14}}{x^3}-\frac{3024\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{12}}{x^4}-\frac{1512\left(de+\sqrt{-x^2e^2+\right.}}{x^5}\right.}{129024\left(de+\sqrt{-x^2e^2+d^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/129024*x^9*(189*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^18/x + 324*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^2*e^16/x^2 - 672*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^14/x^
3 - 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^12/x^4 - 1512*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^5*e^10/x^5 + 9744*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^8/x^6 +
18144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^6/x^7 - 16632*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^8*e^4/x^8 + 28*e^20)*e^7/((d*e + sqrt(-x^2*e^2 + d^2)*e)^9*d) +
55/128*e^9*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d
+ 1/129024*(16632*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^106/x - 18144*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^104/x^2 - 9744*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^8*e^102/x^3 + 1512*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^8*e^100/x^4 +
 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^8*e^98/x^5 + 672*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^6*d^8*e^96/x^6 - 324*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^8*e^9
4/x^7 - 189*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^8*e^92/x^8 - 28*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^9*d^8*e^90/x^9)*e^(-99)/d^9
```

maple [A] time = 0.10, size = 250, normalized size = 1.34

$$\frac{55e^9\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{128\sqrt{d^2}}-\frac{55\sqrt{-e^2x^2+d^2}\,e^9}{128d^2}-\frac{55\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^9}{384d^4}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^9}{128d^6}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^7}{128d^6x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x)
```

```
[Out] -29/63*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^7-3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-11/48/
d^2*e^3/x^6*(-e^2*x^2+d^2)^(7/2)+11/192/d^4*e^5/x^4*(-e^2*x^2+d^2)^(7/2)-11
/128/d^6*e^7/x^2*(-e^2*x^2+d^2)^(7/2)-11/128/d^6*e^9*(-e^2*x^2+d^2)^(5/2)-5
```

```
5/384/d^4*e^9*(-e^2*x^2+d^2)^(3/2)-55/128/d^2*e^9*(-e^2*x^2+d^2)^(1/2)+55/1
28*e^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/9*d*(
-e^2*x^2+d^2)^(7/2)/x^9
```

maxima [A] time = 1.00, size = 247, normalized size = 1.32

$$\frac{55\,e^9\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{128\,d}-\frac{55\,\sqrt{-e^2x^2+d^2}\,e^9}{128\,d^2}-\frac{55\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^9}{384\,d^4}-\frac{11\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^9}{128\,d^6}-\frac{11\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{128\,d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 55/128*e^9*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 55/128*s
qrt(-e^2*x^2 + d^2)*e^9/d^2 - 55/384*(-e^2*x^2 + d^2)^(3/2)*e^9/d^4 - 11/12
8*(-e^2*x^2 + d^2)^(5/2)*e^9/d^6 - 11/128*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x
^2) + 11/192*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^4) - 11/48*(-e^2*x^2 + d^2)^
(7/2)*e^3/(d^2*x^6) - 29/63*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^7) - 3/8*(-e^2*
x^2 + d^2)^(7/2)*e/x^8 - 1/9*(-e^2*x^2 + d^2)^(7/2)*d/x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{10}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10, x)
```

sympy [C] time = 36.71, size = 1889, normalized size = 10.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**10,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d*
*2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**
2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x
**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt
(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2)
+ 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e*
*2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*
x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x*
*2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d
/(e*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**
2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(
-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1))
 - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) + d**5*e**2*Piecewise((-e*sqrt
(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*
x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x*
*4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(
-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5*d**4*e**3*Piecewise((-d**2/(6
```

```
*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) -
1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x)
)/(16*d**5), True)) - 5*d**3*e**4*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**
2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqr
t(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1
 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e*
*2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2
*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + d**2*e**5*Piecewise(
(-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*
x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*
x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2
*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + 3
*d*e**6*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/
(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e*
*2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) + e**7*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sq
rt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)
) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d
), True))
```

3.81 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=225

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}-\frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}+\frac{33e^{10}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6\left(d^2-\right.}{128}$$

```
[Out] 11/128*e^6*(-e^2*x^2+d^2)^(3/2)/d/x^4-11/160*e^4*(-e^2*x^2+d^2)^(5/2)/d/x^6
-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10-1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-33/80*e^2*
(-e^2*x^2+d^2)^(7/2)/d/x^8-5/21*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7+33/256*e^1
0*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-33/256*e^8*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 835, 807, 266, 47, 63, 208}

$$-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4}-\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6}-\frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}-\frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}-\frac{e\left(d^2-e^2\right.}{3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (-33*e^8*Sqrt[d^2 - e^2*x^2])/(256*d*x^2) + (11*e^6*(d^2 - e^2*x^2)^(3/2))/
(128*d*x^4) - (11*e^4*(d^2 - e^2*x^2)^(5/2))/(160*d*x^6) - (d*(d^2 - e^2*x^
2)^(7/2))/(10*x^10) - (e*(d^2 - e^2*x^2)^(7/2))/(3*x^9) - (33*e^2*(d^2 - e^
2*x^2)^(7/2))/(80*d*x^8) - (5*e^3*(d^2 - e^2*x^2)^(7/2))/(21*d^2*x^7) + (33
*e^10*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(256*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{11}} dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-30d^4e-33d^3e^2x-10d^2e^3x^2\right)}{x^{10}} dx}{10d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} + \frac{\int \frac{\left(297d^5e^2+150d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^9} dx}{90d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{\int \frac{\left(-1200d^6e^3-297d^5e^4\right.}{x^8}}{720d}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}$$

$$= -\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}$$

$$= \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} -$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

Mathematica [C] time = 0.06, size = 102, normalized size = 0.45

$$-\frac{e\left(d^2-e^2x^2\right)^{7/2}\left(7d^9+5d^7e^2x^2+9e^9x^9\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+3e^9x^9\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{21d^9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] -1/21*(e*(d^2 - e^2*x^2)^(7/2)*(7*d^9 + 5*d^7*e^2*x^2 + 9*e^9*x^9*Hypergeom
etric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2] + 3*e^9*x^9*Hypergeometric2F1[7/2,
 6, 9/2, 1 - (e^2*x^2)/d^2]))/(d^9*x^9)
```

fricas [A] time = 0.94, size = 153, normalized size = 0.68

$$-\frac{3465\,e^{10}x^{10}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(6400\,e^9x^9+3465\,de^8x^8-10240\,d^2e^7x^7-24570\,d^3e^6x^6-7680\,d^4e^5x^5+2\right.}{26880\,d^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/26880*(3465*e^10*x^10*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (6400*e^9*x^9
 + 3465*d*e^8*x^8 - 10240*d^2*e^7*x^7 - 24570*d^3*e^6*x^6 - 7680*d^4*e^5*x^
5 + 23352*d^5*e^4*x^4 + 20480*d^6*e^3*x^3 - 3024*d^7*e^2*x^2 - 8960*d^8*e*x
 - 2688*d^9)*sqrt(-e^2*x^2 + d^2))/(d^2*x^10)
```

giac [B] time = 0.36, size = 683, normalized size = 3.04

$$\frac{x^{10}\left(\frac{280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{20}}{x}+\frac{525\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{18}}{x^2}-\frac{600\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{16}}{x^3}-\frac{3570\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{14}}{x^4}-\frac{3360\left(de+\sqrt{-x^2e^2-\right.}{x^5}}{430080\left(de+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/430080*x^10*(280*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^20/x + 525*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^2*e^18/x^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^16/x
^3 - 3570*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^14/x^4 - 3360*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^5*e^12/x^5 + 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^10/x^6
+ 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^8/x^7 + 10500*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^8*e^6/x^8 - 31920*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*e^4/x^9 + 4
2*e^22)*e^8/((d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^2) + 33/256*e^10*log(1/2*a
bs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 + 1/430080*(31920*
(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^18*e^128/x - 10500*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^2*d^18*e^126/x^2 - 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^12
4/x^3 - 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^18*e^122/x^4 + 3360*(d*e +
sqrt(-x^2*e^2 + d^2)*e)^5*d^18*e^120/x^5 + 3570*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^6*d^18*e^118/x^6 + 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^116/x^7
- 525*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^18*e^114/x^8 - 280*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^9*d^18*e^112/x^9 - 42*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^1
8*e^110/x^10)*e^(-120)/d^20
```

maple [A] time = 0.15, size = 278, normalized size = 1.24

$$\frac{33e^{10}\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{256\sqrt{d^2}\,d}-\frac{33\sqrt{-e^2x^2+d^2}\,e^{10}}{256d^3}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{10}}{256d^5}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{10}}{1280d^7}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{1280d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x)
```

```
[Out] -1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-5/21*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7-33/80
*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^8-11/160/d^3*e^4/x^6*(-e^2*x^2+d^2)^(7/2)+11/
640/d^5*e^6/x^4*(-e^2*x^2+d^2)^(7/2)-33/1280/d^7*e^8/x^2*(-e^2*x^2+d^2)^(7/
2)-33/1280/d^7*e^10*(-e^2*x^2+d^2)^(5/2)-11/256/d^5*e^10*(-e^2*x^2+d^2)^(3/
2)-33/256/d^3*e^10*(-e^2*x^2+d^2)^(1/2)+33/256/d*e^10/(d^2)^(1/2)*ln((2*d^2
+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10
```

maxima [A] time = 1.01, size = 272, normalized size = 1.21

$$\frac{33\,e^{10}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{256\,d^2}-\frac{33\,\sqrt{-e^2x^2+d^2}\,e^{10}}{256\,d^3}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{10}}{256\,d^5}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{10}}{1280\,d^7}-\frac{33\left(-e^2x^2+d^2\right)}{1280\,d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 33/256*e^10*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 33/25
6*sqrt(-e^2*x^2 + d^2)*e^10/d^3 - 11/256*(-e^2*x^2 + d^2)^(3/2)*e^10/d^5 -
33/1280*(-e^2*x^2 + d^2)^(5/2)*e^10/d^7 - 33/1280*(-e^2*x^2 + d^2)^(7/2)*e^
8/(d^7*x^2) + 11/640*(-e^2*x^2 + d^2)^(7/2)*e^6/(d^5*x^4) - 11/160*(-e^2*x^
2 + d^2)^(7/2)*e^4/(d^3*x^6) - 5/21*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^7) -
33/80*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^8) - 1/3*(-e^2*x^2 + d^2)^(7/2)*e/x^9
 - 1/10*(-e^2*x^2 + d^2)^(7/2)*d/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2 x^2\right)^{5/2} (d+e x)^3}{x^{11}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11, x)
```

sympy [C] time = 49.87, size = 2159, normalized size = 9.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

```
[Out] d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/(80*x**
9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*x**2) -
 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(768*d**
6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e**2*x**
2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 7
*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768*d**6*x
**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(e**2*x*
*2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + 3*d**6*e*Piecewise
((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**2) - 1)/
(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4) + 8*e**7
*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e**2*x**2)
 - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**
5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2/(e**2*x
**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**8
), True)) + d**5*e**2*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)
) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**
2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5
*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*
d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2)
 + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**
5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*a
sin(d/(e*x))/(128*d**7), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e
**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**
2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e
**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**6), True)) - 5*d**3*e**4*Piecewise((-d**2/(6*e*x**7*sqr
t(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/
(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2
```

```
*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e*
*5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5)
, True)) + d**2*e**5*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x*
*2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/
d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)
/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(
-15*d**3*x**5 + 15*d*e**2*x**7), True)) + 3*d*e**6*Piecewise((-d**2/(4*e*x*
*5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
- 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/
(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**7*Piecewise(
(-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(
3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

3.82 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=254

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9}-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x\right.}{480d^2x^6}$$

```
[Out] 19/384*e^7*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-19/480*e^5*(-e^2*x^2+d^2)^(5/2)/d^2
/x^6-1/11*d*(-e^2*x^2+d^2)^(7/2)/x^11-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10-37/9
9*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-74/
693*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x^7+19/256*e^11*arctanh((-e^2*x^2+d^2)^(1/
2)/d)/d^3-19/256*e^9*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] time = 0.33, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 835, 807, 266, 47, 63, 208}

$$-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6}-\frac{74e^4\left(d^2-e^2x^2\right)^{7/2}}{693d^3x^7}-\frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8}-\frac{37e^2\left(d^2\right.}{99}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] (-19*e^9*Sqrt[d^2 - e^2*x^2])/(256*d^2*x^2) + (19*e^7*(d^2 - e^2*x^2)^(3/2)
)/(384*d^2*x^4) - (19*e^5*(d^2 - e^2*x^2)^(5/2))/(480*d^2*x^6) - (d*(d^2 -
e^2*x^2)^(7/2))/(11*x^11) - (3*e*(d^2 - e^2*x^2)^(7/2))/(10*x^10) - (37*e^2
*(d^2 - e^2*x^2)^(7/2))/(99*d*x^9) - (19*e^3*(d^2 - e^2*x^2)^(7/2))/(80*d^2
*x^8) - (74*e^4*(d^2 - e^2*x^2)^(7/2))/(693*d^3*x^7) + (19*e^11*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/(256*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-33d^4e-37d^3e^2x-11d^2e^3x^2\right)}{x^{11}}dx}{11d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} + \frac{\int \frac{\left(370d^5e^2+209d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}dx}{110d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{\int \frac{\left(-1881d^6e^3-740d^5\right.}{x^9}}{990}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7}}{80d^2x^8}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7}}{80d^2x^8}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7}}{80d^2x^8}$$

$$= -\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7}}{99dx^9}$$

$$= \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/}}{11x^{11}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/}}{11x^{11}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/}}{11x^{11}}$$

Mathematica [C] time = 0.06, size = 112, normalized size = 0.44

$$-\frac{\left(d^2-e^2x^2\right)^{7/2}\left(63d^{11}+259d^9e^2x^2+74d^7e^4x^4+99e^{11}x^{11}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+297e^{11}x^{11}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};1-\frac{e^2}{d}\right.\right.}{693d^{10}x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] -1/693*((d^2 - e^2*x^2)^(7/2)*(63*d^11 + 259*d^9*e^2*x^2 + 74*d^7*e^4*x^4 +
 99*e^11*x^11*Hypergeometric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2] + 297*e^11*
x^11*Hypergeometric2F1[7/2, 6, 9/2, 1 - (e^2*x^2)/d^2]))/(d^10*x^11)
```

fricas [A] time = 1.21, size = 164, normalized size = 0.65

$$-\frac{65835\,e^{11}x^{11}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(94720\,e^{10}x^{10}+65835\,de^9x^9+47360\,d^2e^8x^8-251790\,d^3e^7x^7-629760\,d\right.}{887}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/887040*(65835*e^11*x^11*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (94720*e^10
*x^10 + 65835*d*e^9*x^9 + 47360*d^2*e^8*x^8 - 251790*d^3*e^7*x^7 - 629760*d
^4*e^6*x^6 - 201432*d^5*e^5*x^5 + 657920*d^6*e^4*x^4 + 587664*d^7*e^3*x^3 -
 89600*d^8*e^2*x^2 - 266112*d^9*e*x - 80640*d^10)*sqrt(-e^2*x^2 + d^2))/(d^
3*x^11)
```

giac [B] time = 0.36, size = 746, normalized size = 2.94

$$x^{11}\left(\frac{4158\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{22}}{x}+\frac{8470\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{20}}{x^2}-\frac{3465\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{18}}{x^3}-\frac{40590\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{16}}{x^4}-\frac{57750\left(de+\sqrt{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/14192640*x^11*(4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^22/x + 8470*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*e^20/x^2 - 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e
^18/x^3 - 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^16/x^4 - 57750*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^5*e^14/x^5 + 6930*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^
12/x^6 + 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^10/x^7 + 244860*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^8*e^8/x^8 + 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*
e^6/x^9 - 568260*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*e^4/x^10 + 630*e^24)*e^9
/((d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^3) + 19/256*e^11*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 + 1/14192640*(568260*(d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^30*e^152/x - 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^2*d^30*e^150/x^2 - 244860*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^30*e^148/x^3
- 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^30*e^146/x^4 - 6930*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^5*d^30*e^144/x^5 + 57750*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^6*d^30*e^142/x^6 + 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^30*e^140/x^7 +
 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^30*e^138/x^8 - 8470*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^9*d^30*e^136/x^9 - 4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*
d^30*e^134/x^10 - 630*(d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^30*e^132/x^11)*e^
(-143)/d^33
```

maple [A] time = 0.24, size = 303, normalized size = 1.19

$$\frac{19e^{11}\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{256\sqrt{d^2}\,d^2}-\frac{19\sqrt{-e^2x^2+d^2}\,e^{11}}{256d^4}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{11}}{768d^6}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{11}}{1280d^8}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\cdots}{1280d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x)
```

```
[Out] -37/99*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-74/693*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x
^7-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-19/480*e^5/d^4/x^6*(-e^2*x^2+d^2)
^(7/2)+19/1920*e^7/d^6/x^4*(-e^2*x^2+d^2)^(7/2)-19/1280*e^9/d^8/x^2*(-e^2*x
^2+d^2)^(7/2)-19/1280*e^11/d^8*(-e^2*x^2+d^2)^(5/2)-19/768*e^11/d^6*(-e^2*x
^2+d^2)^(3/2)-19/256*e^11/d^4*(-e^2*x^2+d^2)^(1/2)+19/256*e^11/d^2/(d^2)^(1
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/11*d*(-e^2*x^2+d^2)^
(7/2)/x^11-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10
```

maxima [A] time = 1.01, size = 297, normalized size = 1.17

$$\frac{19\,e^{11}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{256\,d^3}-\frac{19\,\sqrt{-e^2x^2+d^2}\,e^{11}}{256\,d^4}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{11}}{768\,d^6}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{11}}{1280\,d^8}-\frac{19\left(-e^2x^2+d^2\right)^{\cdots}}{1280\,d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 19/256*e^11*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 19/25
6*sqrt(-e^2*x^2 + d^2)*e^11/d^4 - 19/768*(-e^2*x^2 + d^2)^(3/2)*e^11/d^6 -
19/1280*(-e^2*x^2 + d^2)^(5/2)*e^11/d^8 - 19/1280*(-e^2*x^2 + d^2)^(7/2)*e^
9/(d^8*x^2) + 19/1920*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x^4) - 19/480*(-e^2*x
^2 + d^2)^(7/2)*e^5/(d^4*x^6) - 74/693*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^7)
 - 19/80*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^8) - 37/99*(-e^2*x^2 + d^2)^(7/2
)*e^2/(d*x^9) - 3/10*(-e^2*x^2 + d^2)^(7/2)*e/x^10 - 1/11*(-e^2*x^2 + d^2)^
(7/2)*d/x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12, x)
```

sympy [C] time = 74.52, size = 2397, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(11*x**10) + e**3*sqrt(d**2/(
e**2*x**2) - 1)/(99*d**2*x**8) + 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(693*d**
4*x**6) + 16*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1155*d**6*x**4) + 64*e**9*sqr
t(d**2/(e**2*x**2) - 1)/(3465*d**8*x**2) + 128*e**11*sqrt(d**2/(e**2*x**2)
- 1)/(3465*d**10), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
 + 1)/(11*x**10) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(99*d**2*x**8) + 8*I*
e**5*sqrt(-d**2/(e**2*x**2) + 1)/(693*d**4*x**6) + 16*I*e**7*sqrt(-d**2/(e*
*2*x**2) + 1)/(1155*d**6*x**4) + 64*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(346
5*d**8*x**2) + 128*I*e**11*sqrt(-d**2/(e**2*x**2) + 1)/(3465*d**10), True))
 + 3*d**6*e*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/
(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(
768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e
**2*x**2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2))
> 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqr
t(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) +
1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768
*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + d**5*e**2*P
iecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**
2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4)
+ 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e*
*2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) +
 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2
/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(
315*d**8), True)) - 5*d**4*e**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**
5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2)
```

```
 - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e
*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(19
2*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d*
*2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) -
5*I*e**8*asin(d/(e*x))/(128*d**7), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(
d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x
**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2
/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/
(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**
4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-
d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e**5*Piecewise((-d**2/(6*e*
x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1))
 + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d*
*2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2))
 > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)
) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(
16*d**5), True)) + 3*d*e**6*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-1
5*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 -
e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**
2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**
2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/
d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**7*Piecewise((-d**2/(4*e
*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1))
 - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3
), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1
)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True))
```

3.83 $\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=174

$$\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*d^4*(e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(e*x+d)^2/e^6/(-e^2*x^
2+d^2)^(3/2)-13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(e*x+
d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*x*(-e^2*x^2+d^
2)^(1/2)/e^5
```

Rubi [A] time = 0.40, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1815, 641, 217, 203}

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d + e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d + e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 + (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{-\frac{195d^5}{e^3}-\frac{90d^4x}{e^2}\cdots}{\sqrt{d^2-e^2x^2}}\,dx}{30d^3e^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}\cdots$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}\cdots$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}\cdots$$

Mathematica [A] time = 0.24, size = 131, normalized size = 0.75

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(304d^4-717d^3ex+479d^2e^2x^2-45de^3x^3-15e^4x^4\right)-195d(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{30e^6(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(304*d^4 - 717*d^3*e*x + 479*d^2*e^2*x^
2 - 45*d*e^3*x^3 - 15*e^4*x^4) - 195*d*(d - e*x)^3*ArcSin[(e*x)/d]))/(30*e^
6*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.93, size = 192, normalized size = 1.10

$$\frac{304\,d^2e^3x^3-912\,d^3e^2x^2+912\,d^4ex-304\,d^5+390\left(d^2e^3x^3-3\,d^3e^2x^2+3\,d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(15\cdots}{30\left(e^9x^3-3\,de^8x^2+3\,d^2e^7x-d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 - 912*d^3*e^2*x^2 + 912*d^4*e*x - 304*d^5 + 390*(d^2*
e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) + (15*e^4*x^4 + 45*d*e^3*x^3 - 479*d^2*e^2*x^2 + 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 - 3*d*e^8*x^2 + 3*d^2*e^7*x - d^3*e^6
)
```

giac [A] time = 0.30, size = 118, normalized size = 0.68

$$-\frac{13}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{\left(304\,d^7e^{(-6)}+\left(195\,d^6e^{(-5)}-\left(760\,d^5e^{(-4)}+\left(455\,d^4e^{(-3)}-\left(570\,d^3e^{(-2)}+\left(299\right.\right.\right.\right.\right.\right.}{30\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -13/2*d^2*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/30*(304*d^7*e^(-6) + (195*d^6*e^(
-5) - (760*d^5*e^(-4) + (455*d^4*e^(-3) - (570*d^3*e^(-2) + (299*d^2*e^(-1)
 - 15*(x*e + 6*d)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 222, normalized size = 1.28

$$-\frac{e\,x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{3d\,x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{13d^2x^5}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{19d^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{76d^5x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{13d^2x^3}{6\left(-e^2x^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2*e*x^7/(-e^2*x^2+d^2)^(5/2)+13/10/e*d^2*x^5/(-e^2*x^2+d^2)^(5/2)-13/6/e
^3*d^2*x^3/(-e^2*x^2+d^2)^(3/2)+13/2/e^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-13/2/e^
5*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3*d*x^6/(-e^2*
x^2+d^2)^(5/2)+19/e^2*d^3*x^4/(-e^2*x^2+d^2)^(5/2)-76/3/e^4*d^5*x^2/(-e^2*x
^2+d^2)^(5/2)+152/15/e^6*d^7/(-e^2*x^2+d^2)^(5/2)
```

maxima [B] time = 1.03, size = 305, normalized size = 1.75

$$-\frac{ex^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{13}{30}\,d^2ex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{3\,dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{13\,d^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e*x^7/(-e^2*x^2 + d^2)^(5/2) + 13/30*d^2*e*x*(15*x^4/((-e^2*x^2 + d^2)
^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 +
d^2)^(5/2)*e^6)) - 3*d*x^6/(-e^2*x^2 + d^2)^(5/2) - 13/6*d^2*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 19*d^3*x^
4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 76/3*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 152/15*d^7/((-e^2*x^2 + d^2)^(5/2)*e^6) + 26/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^5) - 91/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^5) - 13/2*d^2*arcsin(e*x/d)
/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,(d+e\,x)^3}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,(d+ex)^3}{\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**5*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.84 $\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] 1/5*d^3*(e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)-6/5*d^2*(e*x+d)^2/e^5/(-e^2*x^2+
d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+24/5*d*(e*x+d)/e^5/(-e^
2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.32, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1635, 641, 217, 203}

$$\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (6*d^2*(d + e*x)^2)/(5*e^
5*(d^2 - e^2*x^2)^(3/2)) + (24*d*(d + e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) + S
qrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{27d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}dx}{15d^2}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}+\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}dx}{15d^3}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int \frac{1}{\sqrt{d^2-e^2x^2}}d}{e^4}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\,\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.}{e^5}$$

Mathematica [A] time = 0.20, size = 119, normalized size = 0.84

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(24d^3-57d^2ex+39de^2x^2-5e^3x^3\right)-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{5e^5(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(24*d^3 - 57*d^2*e*x + 39*d*e^2*x^2 - 5
*e^3*x^3) - 15*(d - e*x)^3*ArcSin[(e*x)/d]))/(5*e^5*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.89, size = 177, normalized size = 1.25

$$\frac{24\,de^3x^3-72\,d^2e^2x^2+72\,d^3ex-24\,d^4+30\left(de^3x^3-3\,d^2e^2x^2+3\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^3x^3-3\right.}{5\left(e^8x^3-3\,de^7x^2+3\,d^2e^6x-d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*d*e^3*x^3 - 72*d^2*e^2*x^2 + 72*d^3*e*x - 24*d^4 + 30*(d*e^3*x^3 -
3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
+ (5*e^3*x^3 - 39*d*e^2*x^2 + 57*d^2*e*x - 24*d^3)*sqrt(-e^2*x^2 + d^2))/(e
^8*x^3 - 3*d*e^7*x^2 + 3*d^2*e^6*x - d^3*e^5)
```

giac [A] time = 0.29, size = 107, normalized size = 0.75

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\text{sgn}(d)-\frac{\left(24\,d^6e^{(-5)}+\left(15\,d^5e^{(-4)}-\left(60\,d^4e^{(-3)}+\left(35\,d^3e^{(-2)}-\left(45\,d^2e^{(-1)}-(5\,xe-24\,d)x\right)x\right)\right.\right.\right.}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5*(24*d^6*e^(-5) + (15*d^5*e^(-4) - (6
0*d^4*e^(-3) + (35*d^3*e^(-2) - (45*d^2*e^(-1) - (5*x*e - 24*d)*x)*x)*x)*x)
*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 262, normalized size = 1.85

$$-\frac{e\,x^{6}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{3d\,x^{5}}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{9d^{2}x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}+\frac{d^{3}x^{3}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{12d^{4}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}-\frac{3d^{5}x}{10\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -e*x^6/(-e^2*x^2+d^2)^(5/2)+9/e*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-12/e^3*d^4*x^2
/(-e^2*x^2+d^2)^(5/2)+24/5/e^5*d^6/(-e^2*x^2+d^2)^(5/2)+3/5*d*x^5/(-e^2*x^2
+d^2)^(5/2)-d/e^2*x^3/(-e^2*x^2+d^2)^(3/2)+16/5*d/e^4*x/(-e^2*x^2+d^2)^(1/2
)-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2*d^3*x^
3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^3/e^4
*x/(-e^2*x^2+d^2)^(3/2)
```

maxima [B] time = 1.02, size = 324, normalized size = 2.28

$$\frac{1}{5}de^{2}x\left(\frac{15\,x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{20\,d^{2}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}+\frac{8\,d^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}\right)-\frac{ex^{6}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-dx\left(\frac{3\,x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}-\frac{}{\left(-e^{2}x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*d*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e*x^6/(-e^2*x^2 + d
^2)^(5/2) - d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^
2)^(3/2)*e^4)) + 9*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^3*x^3/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 12*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^5*
x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 24/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^5) + 9/
10*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 6/5*d*x/(sqrt(-e^2*x^2 + d^2)*e^4)
- 3*d*arcsin(e*x/d)/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(d + ex\right)^{3}}{\left(- \left(-d + ex\right) \left(d + ex\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.85 $\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=118

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(e*x+d)^2/e^4/(-e^2*x^2+
d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(e*x+d)/e^4/(-e^2*x^2
+d^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1635, 778, 217, 203}

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d + e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d + e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) - Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] time = 0.15, size = 112, normalized size = 0.95

$$\frac{(d+ex)\left(d\left(22d^2-51dex+32e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{15de^4(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(d*(22*d^2 - 51*d*e*x + 32*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2] - 15
*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d*e^4*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sq
rt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.88, size = 161, normalized size = 1.36

$$\frac{22\,e^3x^3-66\,de^2x^2+66\,d^2ex-22\,d^3+30\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(32\,e^2x^2-51\,de\right.}{15\left(e^7x^3-3\,de^6x^2+3\,d^2e^5x-d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 - 66*d*e^2*x^2 + 66*d^2*e*x - 22*d^3 + 30*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (32*e
^2*x^2 - 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 - 3*d*e^6*x^2 +
3*d^2*e^5*x - d^3*e^4)
```

giac [A] time = 0.29, size = 95, normalized size = 0.81

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{\left(22\,d^5e^{(-4)}+\left(15\,d^4e^{(-3)}-\left(55\,d^3e^{(-2)}+\left(35\,d^2e^{(-1)}-(32\,xe+45\,d)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-4)*sgn(d) - 1/15*(22*d^5*e^(-4) + (15*d^4*e^(-3) - (55*d
^3*e^(-2) + (35*d^2*e^(-1) - (32*x*e + 45*d)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 234, normalized size = 1.98

$$\frac{e\,x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3d\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{11d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{9d^4x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{x^3}{3\left(-e^2x^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e*x^5/(-e^2*x^2+d^2)^(5/2)-1/3/e*x^3/(-e^2*x^2+d^2)^(3/2)+8/5/e^3*x/(-e
^2*x^2+d^2)^(1/2)-1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)+3*d*x^4/(-e^2*x^2+d^2)^(5/2)-11/3/e^2*d^3*x^2/(-e^2*x^2+d^2)^(5/2)+22/1
5/e^4*d^5/(-e^2*x^2+d^2)^(5/2)+3/2/e*d^2*x^3/(-e^2*x^2+d^2)^(5/2)-9/10/e^3*
d^4*x/(-e^2*x^2+d^2)^(5/2)+3/10/e^3*d^2*x/(-e^2*x^2+d^2)^(3/2)
```

maxima [B] time = 1.02, size = 296, normalized size = 2.51

$$\frac{1}{15}\,e^3x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{1}{3}\,ex\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 3*d*x^4/(-e
^2*x^2 + d^2)^(5/2) + 3/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - 11/3*d^3*x^2
/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2
2/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^4) + 17/30*d^2*x/((-e^2*x^2 + d^2)^(3/2)
*e^3) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e*x/d)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(d + e x\right)^{3}}{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.86 $\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=93

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d*(e*x+d)^3/e^3/(-e^2*x^2+d^2)^(5/2)-8/15*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^
(3/2)+7/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1635, 789, 637}

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (8*(d + e*x)^2)/(15*e^3*(d^
2 - e^2*x^2)^(3/2)) + (7*(d + e*x))/(15*d*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{3d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{7\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}$$

$$= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 0.62

$$\frac{(d+ex)\left(2d^2-6dex+7e^2x^2\right)}{15de^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

fricas [A] time = 1.07, size = 106, normalized size = 1.14

$$\frac{2\,e^3x^3-6\,de^2x^2+6\,d^2ex-2\,d^3-\left(7\,e^2x^2-6\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^6x^3-3\,d^2e^5x^2+3\,d^3e^4x-d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^3*x^3 - 6*d*e^2*x^2 + 6*d^2*e*x - 2*d^3 - (7*e^2*x^2 - 6*d*e*x +
2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - 3*d^2*e^5*x^2 + 3*d^3*e^4*x - d^4
*e^3)
```

giac [A] time = 0.28, size = 72, normalized size = 0.77

$$-\frac{\left(2\,d^4e^{(-3)}-\left(5\,d^2e^{(-1)}-\left(x\left(\frac{7\,xe^2}{d}+15\,e\right)+5\,d\right)x\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(2*d^4*e^(-3) - (5*d^2*e^(-1) - (x*(7*x*e^2/d + 15*e) + 5*d)*x)*x^2)*
sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.59

$$\frac{(-ex+d)\,(ex+d)^4\left(7e^2x^2-6dex+2d^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(7*e^2*x^2-6*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(7/2
)
```

maxima [A] time = 0.45, size = 154, normalized size = 1.66

$$\frac{ex^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,dx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{7\,d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{2\,d^4}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{7\,dx}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*x^4/(-e^2*x^2 + d^2)^(5/2) + 3/2*d*x^3/(-e^2*x^2 + d^2)^(5/2) - 1/3*d^2*x
^2/((-e^2*x^2 + d^2)^(5/2)*e) - 7/10*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 2
/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^3) + 7/30*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2
) + 7/15*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

mupad [B] time = 2.69, size = 49, normalized size = 0.53

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2-6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 - 6*d*e*x))/(15*d*e^3*(d - e*x)^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.87 $\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)^3/e^2/(-e^2*x^2+d^2)^(5/2)-2/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)
-1/5*x/d^2/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {789, 653, 191}

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(5*e^2*(d^2 - e^2
*x^2)^(3/2)) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\
&= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e}\\
&= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 55, normalized size = 0.64

$$-\frac{(d+ex)\left(d^2-3dex+e^2x^2\right)}{5d^2e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/5*((d + e*x)*(d^2 - 3*d*e*x + e^2*x^2))/(d^2*e^2*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

fricas [A] time = 0.90, size = 104, normalized size = 1.21

$$-\frac{e^3x^3-3\,de^2x^2+3\,d^2ex-d^3-\left(e^2x^2-3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^3-3\,d^3e^4x^2+3\,d^4e^3x-d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 - 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 - 3*d^3*e^4*x^2 + 3*d^4*e^3*x - d^5*e^2)
```

giac [A] time = 0.31, size = 60, normalized size = 0.70

$$\frac{\left(d^3e^{(-2)}+\left(x\left(\frac{x^2e^3}{d^2}-5\,e\right)-5\,d\right)x^2\right)\sqrt{-x^2e^2+d^2}}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/5*(d^3*e^(-2) + (x*(x^2*e^3/d^2 - 5*e) - 5*d)*x^2)*sqrt(-x^2*e^2 + d^2)/(
x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.60

$$-\frac{(-ex+d)\,(ex+d)^4\left(e^2x^2-3dex+d^2\right)}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/5*(-e*x+d)*(e*x+d)^4*(e^2*x^2-3*d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 128, normalized size = 1.49

$$\frac{ex^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{dx^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{d^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}-\frac{x}{5\sqrt{-e^2x^2+d^2}\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^3/(-e^2*x^2 + d^2)^(5/2) + d*x^2/(-e^2*x^2 + d^2)^(5/2) + 3/10*d^2*
x/((-e^2*x^2 + d^2)^(5/2)*e) - 1/5*d^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/10*
x/((-e^2*x^2 + d^2)^(3/2)*e) - 1/5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e)
```

mupad [B] time = 2.66, size = 46, normalized size = 0.53

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2-3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 - 3*d*e*x))/(5*d^2*e^2*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex\right)^{3}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.88 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-e
*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

fricas [A] time = 0.92, size = 106, normalized size = 1.03

$$\frac{7\,e^3x^3 - 21\,de^2x^2 + 21\,d^2ex - 7\,d^3 - \left(2\,e^2x^2 - 6\,dex + 7\,d^2\right)\sqrt{-e^2x^2 + d^2}}{15\left(d^3e^4x^3 - 3\,d^4e^3x^2 + 3\,d^5e^2x - d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

giac [A] time = 0.29, size = 70, normalized size = 0.68

$$-\frac{\sqrt{-x^2e^2+d^2}\left(7\,d^2e^{(-1)} + \left(\left(x\left(\frac{2\,x^2e^4}{d^3} - \frac{5\,e^2}{d}\right) + 5\,e\right)x + 15\,d\right)x\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.53

$$\frac{(-ex+d)\,(ex+d)^4\left(2e^2x^2-6dex+7d^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

maxima [A] time = 0.44, size = 101, normalized size = 0.98

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

mupad [B] time = 2.66, size = 49, normalized size = 0.48

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.89 $\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=114

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] 4/5*(e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/2)
-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(22*e*x+15*d)/d^4/(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d + 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d + 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2-22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-\right.}{d^3e^2}$$

$$= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [C] time = 0.06, size = 81, normalized size = 0.71

$$\frac{9d^5+45d^4ex-55d^2e^3x^3+3d^5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+22e^5x^5}{15d^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (9*d^5 + 45*d^4*e*x - 55*d^2*e^3*x^3 + 22*e^5*x^5 + 3*d^5*Hypergeometric2F1
[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^4*(d^2 - e^2*x^2)^(5/2))
```

fricas [A] time = 0.90, size = 158, normalized size = 1.39

$$\frac{32\,e^{3}x^{3}-96\,de^{2}x^{2}+96\,d^{2}ex-32\,d^{3}+15\left(e^{3}x^{3}-3\,de^{2}x^{2}+3\,d^{2}ex-d^{3}\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(22\,e^{2}x^{2}-51\,de\right.}{15\left(d^{4}e^{3}x^{3}-3\,d^{5}e^{2}x^{2}+3\,d^{6}ex-d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 - 96*d*e^2*x^2 + 96*d^2*e*x - 32*d^3 + 15*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (22*e^2*x^2
- 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 - 3*d^5*e^2*x^2 + 3
*d^6*e*x - d^7)
```

giac [A] time = 0.29, size = 117, normalized size = 1.03

$$-\frac{\sqrt{-x^{2}e^{2}+d^{2}}\left(\left(\left(\left(x\left(\frac{22\,xe^{5}}{d^{4}}+\frac{15\,e^{4}}{d^{3}}\right)-\frac{55\,e^{3}}{d^{2}}\right)x-\frac{35\,e^{2}}{d}\right)x+45\,e\right)x+32\,d\right)}{15\left(x^{2}e^{2}-d^{2}\right)^{3}}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(22*x*e^5/d^4 + 15*e^4/d^3) - 55*e^3/d^2)*
x - 35*e^2/d)*x + 45*e)*x + 32*d)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d*e -
2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4
```

maple [A] time = 0.01, size = 158, normalized size = 1.39

$$\frac{4ex}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{4d}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{11ex}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}}+\frac{1}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d}-\frac{\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}\,d^{3}}+\frac{2}{15\sqrt{-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*e*x/(-e^2*x^2+d^2)^(5/2)+11/15*e/d^2*x/(-e^2*x^2+d^2)^(3/2)+22/15*e/d^4
*x/(-e^2*x^2+d^2)^(1/2)+4/5*d/(-e^2*x^2+d^2)^(5/2)+1/3/d/(-e^2*x^2+d^2)^(3/
2)+1/d^3/(-e^2*x^2+d^2)^(1/2)-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.46, size = 152, normalized size = 1.33

$$\frac{4\,ex}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{4\,d}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{11\,ex}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}}+\frac{1}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d}+\frac{22\,ex}{15\,\sqrt{-e^{2}x^{2}+d^{2}}\,d^{4}}-\frac{\log\left(\frac{2\,d^{2}}{|x|}+\frac{2}{}\right.}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*e*x/(-e^2*x^2 + d^2)^(5/2) + 4/5*d/(-e^2*x^2 + d^2)^(5/2) + 11/15*e*x/(
(-e^2*x^2 + d^2)^(3/2)*d^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d) + 22/15*e*x/(s
qrt(-e^2*x^2 + d^2)*d^4) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(
x))/d^4 + 1/(sqrt(-e^2*x^2 + d^2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**3/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.90 $\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 4/5*e*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/5*e*(7*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(
3/2)-3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/5*e*(19*e*x+15*d)/d^5/(-e^2*
x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A] time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1805, 807, 266, 63, 208}

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (e*(5*d + 7*e*x))/(5*d^3*(d^2
 - e^2*x^2)^(3/2)) + (e*(15*d + 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqrt
[d^2 - e^2*x^2]/(d^5*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(3e)\int \frac{1}{x\sqrt{d^2-e^2}}}{d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(3e)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{3\,\mathrm{Subst}\left(\int \frac{}{\frac{d^2}{e^2}}\right.}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{3e\tanh^{-1}\left(\sqrt{d}\right.}{d^5}$$

Mathematica [C] time = 0.06, size = 96, normalized size = 0.66

$$\frac{-5d^6+d^5ex+45d^4e^2x^2-60d^2e^4x^4+3d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+24e^6x^6}{5d^5x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-5*d^6 + d^5*e*x + 45*d^4*e^2*x^2 - 60*d^2*e^4*x^4 + 24*e^6*x^6 + 3*d^5*e*
x*Hypergeometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(5*d^5*x*(d^2 - e^2*
x^2)^(5/2))
```

fricas [A] time = 0.85, size = 184, normalized size = 1.27

$$\frac{24\,e^4x^4-72\,de^3x^3+72\,d^2e^2x^2-24\,d^3ex+15\left(e^4x^4-3\,de^3x^3+3\,d^2e^2x^2-d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(24\,e^3x^3-\right.}{5\left(d^5e^3x^4-3\,d^6e^2x^3+3\,d^7ex^2-d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*e^4*x^4 - 72*d*e^3*x^3 + 72*d^2*e^2*x^2 - 24*d^3*e*x + 15*(e^4*x^4
- 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 - (24*e^3*x^3 - 57*d*e^2*x^2 + 39*d^2*e*x - 5*d^3)*sqrt(-e^2*x^2 + d^2))/(
d^5*e^3*x^4 - 3*d^6*e^2*x^3 + 3*d^7*e*x^2 - d^8*x)
```

giac [A] time = 0.29, size = 185, normalized size = 1.28

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{19\,xe^6}{d^5}+\frac{15\,e^5}{d^4}\right)-\frac{45\,e^4}{d^3}\right)x-\frac{35\,e^3}{d^2}\right)x+\frac{30\,e^2}{d}\right)x+24\,e\right)}{5\left(x^2e^2-d^2\right)^3}-\frac{3\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}+\frac{}{2\left(de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*(((((x*(19*x*e^6/d^5 + 15*e^5/d^4) - 45*e^4/d^3)*x
 - 35*e^3/d^2)*x + 30*e^2/d)*x + 24*e)/(x^2*e^2 - d^2)^3 - 3*e*log(1/2*abs(
-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5 + 1/2*x*e^3/((d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^5) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^
5*x)
```

maple [A] time = 0.01, size = 190, normalized size = 1.31

$$\frac{9e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{4e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}+\frac{12e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}-\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}}{}\right.}{\sqrt{d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*e/(-e^2*x^2+d^2)^(5/2)+9/5*e^2/d*x/(-e^2*x^2+d^2)^(5/2)+12/5*e^2/d^3*x/
(-e^2*x^2+d^2)^(3/2)+24/5*e^2/d^5*x/(-e^2*x^2+d^2)^(1/2)-d/x/(-e^2*x^2+d^2)
^(5/2)+e/d^2/(-e^2*x^2+d^2)^(3/2)+3*e/d^4/(-e^2*x^2+d^2)^(1/2)-3*e/d^4/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.47, size = 184, normalized size = 1.27

$$\frac{9\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{4\,e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{12\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}-\frac{d}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}+\frac{24\,e^2x}{5\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 9/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d) + 4/5*e/(-e^2*x^2 + d^2)^(5/2) + 12/5*
e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + e/((-e^2*x^2 + d^2)^(3/2)*d^2) - d/((-
e^2*x^2 + d^2)^(5/2)*x) + 24/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^5) - 3*e*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 + 3*e/(sqrt(-e^2*x^2 + d
^2)*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x^2\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.91 $\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 4/5*e^2*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(31*e*x+25*d)/d^4/(-e^2*x
^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(107*e*
x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2-3*e*(-e^2
*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] time = 0.36, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d + 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d + 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) - (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-20de^2x^2-16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+75de^2x^2+62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex-90de^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{90d^4e+195}{x^2\sqrt{d^2-e}}}{30d}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2}}{d^6x}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2}}{d^6x}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2}}{d^6x}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2}}{d^6x}$$

Mathematica [C] time = 0.07, size = 119, normalized size = 0.65

$$\frac{e\left(-45d^6+285d^4e^2x^2-380d^2e^4x^4+9d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+3d^5ex\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+152e^6x^6\right)}{15d^6x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (e*(-45*d^6 + 285*d^4*e^2*x^2 - 380*d^2*e^4*x^4 + 152*e^6*x^6 + 9*d^5*e*x*H
ypergeometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2] + 3*d^5*e*x*Hypergeometr
ic2F1[-5/2, 2, -3/2, 1 - (e^2*x^2)/d^2]))/(15*d^6*x*(d^2 - e^2*x^2)^(5/2))
```

fricas [A] time = 0.68, size = 205, normalized size = 1.13

$$\frac{254\,e^5x^5 - 762\,de^4x^4 + 762\,d^2e^3x^3 - 254\,d^3e^2x^2 + 195\left(e^5x^5 - 3\,de^4x^4 + 3\,d^2e^3x^3 - d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)}{30\left(d^6e^3x^5 - 3\,d^7e^2x^4 + 3\,d^8ex^3 - d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 - 762*d*e^4*x^4 + 762*d^2*e^3*x^3 - 254*d^3*e^2*x^2 + 195
*(e^5*x^5 - 3*d*e^4*x^4 + 3*d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (304*e^4*x^4 - 717*d*e^3*x^3 + 479*d^2*e^2*x^2 - 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 - 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 - d^9*x^2)
```

giac [A] time = 0.33, size = 259, normalized size = 1.42

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{107\,xe^7}{d^6}+\frac{90\,e^6}{d^5}\right)-\frac{245\,e^5}{d^4}\right)x-\frac{205\,e^4}{d^3}\right)x+\frac{150\,e^3}{d^2}\right)x+\frac{127\,e^2}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{13\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(107*x*e^7/d^6 + 90*e^6/d^5) - 245*e^5/d^4
)*x - 205*e^4/d^3)*x + 150*e^3/d^2)*x + 127*e^2/d)/(x^2*e^2 - d^2)^3 - 13/2
*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 + 1/
8*x^2*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*d^6) - 1/8*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*e^8/x + (d*e
 + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*e^6/x^2)*e^(-8)/d^12
```

maple [A] time = 0.01, size = 222, normalized size = 1.22

$$\frac{19e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{13e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{76e^3x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{3e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}-\frac{d}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}+\frac{1}{6\left(-e^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 19/5*e^3*x/d^2/(-e^2*x^2+d^2)^(5/2)+76/15*e^3/d^4*x/(-e^2*x^2+d^2)^(3/2)+15
2/15*e^3/d^6*x/(-e^2*x^2+d^2)^(1/2)-3*e/x/(-e^2*x^2+d^2)^(5/2)+13/10*e^2/d/
(-e^2*x^2+d^2)^(5/2)+13/6*e^2/d^3/(-e^2*x^2+d^2)^(3/2)+13/2*e^2/d^5/(-e^2*x
^2+d^2)^(1/2)-13/2*e^2/d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x)-1/2*d/x^2/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.48, size = 216, normalized size = 1.19

$$\frac{19\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{13\,e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{76\,e^3x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}+\frac{13\,e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}-\frac{3\,e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}+\frac{152\,e^3x}{15\sqrt{-e^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 19/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 13/10*e^2/((-e^2*x^2 + d^2)^(5/2)
*d) + 76/15*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 13/6*e^2/((-e^2*x^2 + d^2)
^(3/2)*d^3) - 3*e/((-e^2*x^2 + d^2)^(5/2)*x) + 152/15*e^3*x/(sqrt(-e^2*x^2
+ d^2)*d^6) - 13/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/
d^6 + 13/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^5) - 1/2*d/((-e^2*x^2 + d^2)^(5/2)*x
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x^3\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.92 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=147

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}+\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}$$

```
[Out] 3/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+4/15*d^2*x^2*(-e^2*x^2+d^2)^(1
/2)/e^3-1/4*d*x^3*(-e^2*x^2+d^2)^(1/2)/e^2+1/5*x^4*(-e^2*x^2+d^2)^(1/2)/e+1
/120*d^3*(-45*e*x+64*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 833, 780, 217, 203}

$$\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (4*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(4*e
^2) + (x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (d^3*(64*d - 45*e*x)*Sqrt[d^2 - e^2
*x^2])/(120*e^5) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \int \frac{x^4(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x^3\left(4d^2e-5de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\
&= -\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{\int \frac{x^2\left(15d^3e^2-16d^2e^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}\\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x\left(32d^4e^3-45d^3e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}\\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{(3d^5}{}\\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{(3d^5}{}\\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{3d^5}{}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 91, normalized size = 0.62

$$
\frac{45d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(64d^4-45d^3ex+32d^2e^2x^2-30de^3x^3+24e^4x^4\right)}{120e^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3 +
 24*e^4*x^4) + 45*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e^5)
```

fricas [A] time = 0.87, size = 95, normalized size = 0.65

$$
-\frac{90\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4-30\,de^3x^3+32\,d^2e^2x^2-45\,d^3ex+64\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(90*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 - 30
*d*e^3*x^3 + 32*d^2*e^2*x^2 - 45*d^3*e*x + 64*d^4)*sqrt(-e^2*x^2 + d^2))/e^
5
```

giac [A] time = 0.20, size = 77, normalized size = 0.52

$$
\frac{3}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{1}{120}\left(64\,d^4e^{(-5)}-\left(45\,d^3e^{(-4)}-2\left(16\,d^2e^{(-3)}+3\left(4\,xe^{(-1)}-5\,de^{(-2)}\right)x\right)x\right)x\right)\sqrt{-x^2e^2+}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/8*d^5*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/120*(64*d^4*e^(-5) - (45*d^3*e^(-4)
 - 2*(16*d^2*e^(-3) + 3*(4*x*e^(-1) - 5*d*e^(-2))*x)*x)*x)*sqrt(-x^2*e^2 +
d^2)
```

maple [A] time = 0.02, size = 208, normalized size = 1.41

$$\frac{d^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e^4}-\frac{5d^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^4}-\frac{5\sqrt{-e^2x^2+d^2}\,d^3x}{8e^4}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,d^4}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/5/e^3*x^2*(-e^2*x^2+d^2)^(3/2)-7/15*d^2/e^5*(-e^2*x^2+d^2)^(3/2)+1/4*d/e
^4*x*(-e^2*x^2+d^2)^(3/2)-5/8*d^3/e^4*x*(-e^2*x^2+d^2)^(1/2)-5/8*d^5/e^4/(e
^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+d^4/e^5*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)+d^5/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2))
```

maxima [A] time = 0.99, size = 125, normalized size = 0.85

$$\frac{3\,d^5 \arcsin\left(\frac{ex}{d}\right)}{8\,e^5}-\frac{5\,\sqrt{-e^2x^2+d^2}\,d^3x}{8\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,d^4}{e^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*d^5*arcsin(e*x/d)/e^5 - 5/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^4 - 1/5*(-e^2*
x^2 + d^2)^(3/2)*x^2/e^3 + sqrt(-e^2*x^2 + d^2)*d^4/e^5 + 1/4*(-e^2*x^2 + d
^2)^(3/2)*d*x/e^4 - 7/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.93 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=118

$$-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

```
[Out] -3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/3*d*x^2*(-e^2*x^2+d^2)^(1/2
)/e^2+1/4*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/24*d^2*(-9*e*x+16*d)*(-e^2*x^2+d^2)^
(1/2)/e^4
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 833, 780, 217, 203}

$$-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -(d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e^2) + (x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d
^2*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(8*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \int \frac{x^3(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{\int \frac{x^2\left(3d^2e-4de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} + \frac{\int \frac{x\left(8d^3e^2-9d^2e^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}\\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{\left(3d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^3}\\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{\left(3d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{8e^3}\\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^3+9d^2ex-8de^2x^2+6e^3x^3\right)-9d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) - 9*d^
4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^4)
```

fricas [A] time = 0.81, size = 83, normalized size = 0.70

$$\frac{18\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3-8\,de^2x^2+9\,d^2ex-16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(18*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 8*d*e
^2*x^2 + 9*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.21, size = 66, normalized size = 0.56

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\text{sgn}(d)-\frac{1}{24}\left(16\,d^3e^{(-4)}-\left(9\,d^2e^{(-3)}+2\left(3\,xe^{(-1)}-4\,de^{(-2)}\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/8*d^4*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/24*(16*d^3*e^(-4) - (9*d^2*e^(-3)
+ 2*(3*x*e^(-1) - 4*d*e^(-2))*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 185, normalized size = 1.57

$$-\frac{d^4 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, e^3}+\frac{5d^4 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\, e^3}+\frac{5\sqrt{-e^2x^2+d^2}\, d^2x}{8e^3}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\, d^3}{e^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/4/e^3*x*(-e^2*x^2+d^2)^(3/2)+5/8*d^2/e^3*x*(-e^2*x^2+d^2)^(1/2)+5/8/e^3*
d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/3*d/e^4*(-e^2*
x^2+d^2)^(3/2)-d^3/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-d^4/e^3/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 101, normalized size = 0.86

$$-\frac{3\, d^4 \arcsin\left(\frac{ex}{d}\right)}{8\, e^4}+\frac{5\, \sqrt{-e^2x^2+d^2}\, d^2x}{8\, e^3}-\frac{\sqrt{-e^2x^2+d^2}\, d^3}{e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} x}{4\, e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d}{3\, e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(e*x/d)/e^4 + 5/8*sqrt(-e^2*x^2 + d^2)*d^2*x/e^3 - sqrt(-e^2
*x^2 + d^2)*d^3/e^4 - 1/4*(-e^2*x^2 + d^2)^(3/2)*x/e^3 + 1/3*(-e^2*x^2 + d^
2)^(3/2)*d/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\, \sqrt{d^2-e^2\, x^2}}{d+e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.94 $\int \frac{x^2 \sqrt{d^2-e^2x^2}}{d+ex} dx$

Optimal. Leaf size=86

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+
1/2*d*(-e*x+2*d)*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1639, 12, 785, 780, 217, 203}

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (d^2 - e^2*x^2)^(3/2)/(3*e^3)
 + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{3de^3x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{d\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{x\left(d^2e-de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2-3dex+2e^2x^2\right)+3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2) + 3*d^3*ArcTan[(e*x)/Sqr
t[d^2 - e^2*x^2]])/(6*e^3)
```

fricas [A] time = 0.77, size = 73, normalized size = 0.85

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2\,e^2x^2-3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2*e^2*x^2 - 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.20, size = 54, normalized size = 0.63

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\text{sgn}(d)+\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(4\,d^2e^{(-3)}+\left(2\,xe^{(-1)}-3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) - 3*d*e^(-2))*x)
```

maple [B] time = 0.01, size = 160, normalized size = 1.86

$$\frac{d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e^2}-\frac{d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,dx}{2e^2}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,d^2}{e^3}-\frac{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3-1/2*(-e^2*x^2+d^2)^(1/2)*d/e^2*x-1/2/(e^2)^(1
/2)*d^3/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+d^2/e^3*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)+d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 77, normalized size = 0.90

$$\frac{d^3 \arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,dx}{2\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,d^2}{e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*arcsin(e*x/d)/e^3 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 + sqrt(-e^2*x^
2 + d^2)*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.95 $\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=62

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

```
[Out] -1/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/2*(-e*x+2*d)*(-e^2*x^2+d^2)
^(1/2)/e^2
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {785, 780, 217, 203}

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (d^2*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(2*e^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{de}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.92

$$\frac{(ex-2d)\sqrt{d^2-e^2x^2} - d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2] - d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(2*e^2)
```

fricas [A] time = 0.85, size = 60, normalized size = 0.97

$$\frac{2\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \sqrt{-e^2x^2+d^2}\,(ex-2\,d)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2)
*(e*x - 2*d))/e^2
```

giac [A] time = 0.22, size = 43, normalized size = 0.69

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\text{sgn}(d) + \frac{1}{2}\sqrt{-x^2e^2+d^2}\left(xe^{(-1)} - 2\,de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^(-2)*sgn(d) + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-1) -
 2*d*e^(-2))
```

maple [B] time = 0.01, size = 140, normalized size = 2.26

$$-\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e} + \frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e} + \frac{\sqrt{-e^2x^2+d^2}\,x}{2e} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2
*x^2+d^2)^(1/2)*x)-d/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-d^2/e/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 56, normalized size = 0.90

$$-\frac{d^2 \arcsin\left(\frac{ex}{d}\right)}{2\,e^2} + \frac{\sqrt{-e^2x^2 + d^2}\,x}{2\,e} - \frac{\sqrt{-e^2x^2 + d^2}\,d}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*arcsin(e*x/d)/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*x/e - sqrt(-e^2*x^2 +
 d^2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{d^2 - e^2\,x^2}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(-d + ex)\,(d + ex)}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.96 $\int \frac{\sqrt{d^2-e^2x^2}}{d+ex} \, dx$

Optimal. Leaf size=46

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {665, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{d+ex} \, dx &= \frac{\sqrt{d^2-e^2x^2}}{e}+d\int \frac{1}{\sqrt{d^2-e^2x^2}} \, dx\\ &= \frac{\sqrt{d^2-e^2x^2}}{e}+d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2} \, dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\ &= \frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}+d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
[Out] (Sqrt[d^2 - e^2*x^2] + d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

fricas [A] time = 0.89, size = 52, normalized size = 1.13

$$-\frac{2\, d \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] -(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - sqrt(-e^2*x^2 + d^2))/e
```

giac [A] time = 0.21, size = 31, normalized size = 0.67

$$d \arcsin\left(\frac{xe}{d}\right) e^{(-1)} \mathrm{sgn}(d) + \sqrt{-x^2e^2+d^2}\, e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] d*arcsin(x*e/d)*e^(-1)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-1)
```

maple [A] time = 0.00, size = 77, normalized size = 1.67

$$\frac{d \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
[Out] 1/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 31, normalized size = 0.67

$$\frac{d \arcsin\left(\frac{ex}{d}\right)}{e} + \frac{\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] d*arcsin(e*x/d)/e + sqrt(-e^2*x^2 + d^2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.97 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)} dx$

Optimal. Leaf size=46

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -arctan(e*x/(-e^2*x^2+d^2)^(1/2))-arctanh((-e^2*x^2+d^2)^(1/2)/d)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 844, 217, 203, 266, 63, 208}

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx &= \int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= d\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2} \\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\log\left(\sqrt{d^2-e^2x^2}+d\right) - \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + Log[x] - Log[d + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.88, size = 54, normalized size = 1.17

$$2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + log(-(d - sqrt(-e^2*x^2 + d^2
))/x)
```

giac [A] time = 0.21, size = 48, normalized size = 1.04

$$-\arcsin\left(\frac{xe}{d}\right)\text{sgn}(d) - \log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*sgn(d) - log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

maple [B] time = 0.01, size = 137, normalized size = 2.98

$$-\frac{d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{-e^2x^2+d^2}}{d}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x)
```

```
[Out] (-e^2*x^2+d^2)^(1/2)/d-d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-1/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-e/(e^2)^(1/2)*arctan((e^
2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 56, normalized size = 1.22

$$-\frac{e\left(\frac{d\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{d\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{e}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -e*(d*arcsin(e*x/d)/e + d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x
))/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)), x)
```

3.98 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=51

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 807, 266, 63, 208}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} - e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} - \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 1.04

$$-\frac{\sqrt{d^2-e^2x^2} - ex\log\left(\sqrt{d^2-e^2x^2}+d\right) + ex\log(x)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2] + e*x*Log[x] - e*x*Log[d + Sqrt[d^2 - e^2*x^2]])/(d*
x))
```

fricas [A] time = 0.91, size = 50, normalized size = 0.98

$$-\frac{ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2+d^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] -(e*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/(d*x)
```

giac [B] time = 0.21, size = 102, normalized size = 2.00

$$\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d} + \frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d + 1/2*x*e
^3/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
e^(-1)/(d*x)
```

maple [B] time = 0.01, size = 222, normalized size = 4.35

$$\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d} - \frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d} + \frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} - \frac{\sqrt{-e^2x^2+d^2}\,e^2x}{d^3} - \frac{\sqrt{-e^2x^2+d}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d^3/x*(-e^2*x^2+d^2)^(3/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(1/2)-1/d*e^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-e/d^2*(-e^2*x^2+d^2)^(1/
2)+e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e/d^2*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+e^2/d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2 - e^2\,x^2}}{x^2\,(d + e\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^2\,(d + ex)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)), x)
```

3.99 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=82

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

```
[Out] -1/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/2*(-e^2*x^2+d^2)^(1/2)/d/x^2
+e*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*d*x^2) + (e*Sqrt[d^2 - e^2*x^2])/(d^2*x) - (e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx &= \int \frac{d-ex}{x^3\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{\int \frac{2d^2e-de^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}+\frac{e^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}+\frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 70, normalized size = 0.85

$$-\frac{(d-2ex)\sqrt{d^2-e^2x^2}+e^2x^2\log\left(\sqrt{d^2-e^2x^2}+d\right)-e^2x^2\log(x)}{2d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*((d - 2*e*x)*Sqrt[d^2 - e^2*x^2] - e^2*x^2*Log[x] + e^2*x^2*Log[d + Sq
rt[d^2 - e^2*x^2]])/(d^2*x^2)
```

fricas [A] time = 0.94, size = 63, normalized size = 0.77

$$\frac{e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}\,(2\,ex-d)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)*(2*e
*x - d))/(d^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/8*(
exp(2)^3+2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/x/exp(2))/d
^2/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2/exp(1)^4+1
/16*(-2*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^4*exp(2)^5-4*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6
*exp(2)^4/x/exp(2))/d^4/exp(1)^6/exp(2)^3+1/2*(exp(2)^3-2*exp(1)^4*exp(2))*
ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^2/ex
p(1)^3/exp(1)+1/2*(4*exp(1)^3*exp(2)-4*exp(1)*exp(2)^2)*atan((-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^2
/sqrt(-exp(1)^4+exp(2)^2)/exp(1)
```

maple [B] time = 0.01, size = 254, normalized size = 3.10

$$-\frac{e^2 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d}-\frac{e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,d^2}+\frac{e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^2}+\frac{\sqrt{-e^2x^2+d^2}\,e^3x}{d^4}+\frac{\sqrt{-e^2x^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] e/d^4/x*(-e^2*x^2+d^2)^(3/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(1/2)+e^3/d^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/d^3/x^2*(-e^2*x^2+d^2)^
(3/2)+1/2/d^3*e^2*(-e^2*x^2+d^2)^(1/2)-1/2/d*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d
^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d^3*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)-1/d^2*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

3.100 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=114

$$\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

```
[Out] 1/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/3*(-e^2*x^2+d^2)^(1/2)/d/x^3+
1/2*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-2/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*d*x^3) + (e*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) - (2*e
^2*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) + (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx &= \int \frac{d-ex}{x^4\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{3d^2e-2de^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} + \frac{\int \frac{4d^3e^2-3d^2e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.74

$$\frac{\left(-2d^2+3dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}+3e^3x^3\log\left(\sqrt{d^2-e^2x^2}+d\right)-3e^3x^3\log(x)}{6d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] ((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2] - 3*e^3*x^3*Log[x] + 3*
e^3*x^3*Log[d + Sqrt[d^2 - e^2*x^2]])/(6*d^3*x^3)
```

fricas [A] time = 0.59, size = 75, normalized size = 0.66

$$-\frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(4\,e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (4*e^2*x^2 - 3*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/24*
((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(12*exp(1)^4
*exp(2)^2-3*exp(2)^4)+exp(2)^4+3/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))*exp(2)^4/x/exp(2))/d^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^3/exp(1)^5+1/512*(64*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^7-64/3*d^6*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^8+96*d^6*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^8/x/exp(2)-128*d^6*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^7/x/exp(2)+128*d^6*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^6/x/exp(2))/d^9/
exp(1)^15/exp(2)^3+1/2*(4*exp(2)^3-4*exp(1)^4*exp(2))*atan((-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^3/s
qrt(-exp(1)^4+exp(2)^2)/exp(1)+1/2*(-exp(2)^3+2*exp(1)^4*exp(2))*ln(1/2*abs
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)/exp(2
)
```

maple [B] time = 0.01, size = 280, normalized size = 2.46

$$\frac{e^3 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^2}+\frac{e^4 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,d^3}-\frac{e^4 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^3}-\frac{\sqrt{-e^2x^2+d^2}\,e^4x}{d^5}-\frac{\sqrt{-e^2x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] -1/d^5*e^2/x*(-e^2*x^2+d^2)^(3/2)-1/d^5*e^4*x*(-e^2*x^2+d^2)^(1/2)-1/d^3*e^
4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2*e/d^4/x^2*(-e^
2*x^2+d^2)^(3/2)-1/2*e^3/d^4*(-e^2*x^2+d^2)^(1/2)+1/2*e^3/d^2/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3*(-e^2*x^2+d^2)^
(3/2)+1/d^4*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+1/d^3*e^4/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^{4}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)), x)
```

3.101 $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}$$

```
[Out] -3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/4*(-e^2*x^2+d^2)^(1/2)/d/x^4
+1/3*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x^2+2/
3*e^3*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*d*x^4) + (e*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) - (3*e
^2*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) + (2*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^4*x)
 - (3*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
```

```
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \int \frac{d-ex}{x^5\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} - \frac{\int \frac{4d^2e-3de^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} + \frac{\int \frac{9d^3e^2-8d^2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{\int \frac{16d^4e^3-9d^3e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} + \frac{\left(3e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^3}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} + \frac{\left(3e^4\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.}{16d^3}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} - \frac{\left(3e^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\right.}{8d^3}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} - \frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

Mathematica [A] time = 0.13, size = 95, normalized size = 0.66

$$\frac{-9e^4x^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex-9de^2x^2+16e^3x^3\right)+9e^4x^4\log(x)}{24d^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3) + 9*e^
4*x^4*Log[x] - 9*e^4*x^4*Log[d + Sqrt[d^2 - e^2*x^2]])/(24*d^4*x^4)
```

fricas [A] time = 0.89, size = 86, normalized size = 0.60

$$\frac{9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^3x^3-9\,de^2x^2+8\,d^2ex-6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(9*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 - 9*d*e^2*
x^2 + 8*d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/192
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-96*exp(1)
^6*exp(2)^2+96*exp(1)^4*exp(2)^3-72*exp(2)^5)+24*exp(1)^4*exp(2)^3*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2+3*exp(2)^5+4*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2))/d^4/(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4/exp(1)^6+1/65536*(-8192*d^12*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^22*exp
(2)^7+8192/3*d^12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(1)^20*exp(2)^8-1024*d^12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^4*exp(1)^18*exp(2)^9+8192*d^12*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(2)^8-8192*d^12*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^18*exp(2)^9-122
88*d^12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^20*exp(2)^8/x/ex
p(2)+16384*d^12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^22*exp(2
)^7/x/exp(2)-16384*d^12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^
24*exp(2)^6/x/exp(2))/d^16/exp(1)^24/exp(2)^4+1/2*(-4*exp(1)^3*exp(2)^2+4*e
xp(1)^5*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+ex
p(2))/sqrt(-exp(1)^4+exp(2)^2))/d^4/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/8*(8*
exp(1)^6*exp(2)^2-4*exp(1)^4*exp(2)^3+exp(2)^5-8*exp(1)^8*exp(2))*ln(1/2*ab
s(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/exp(1)^5/ex
p(1)
```

maple [B] time = 0.01, size = 304, normalized size = 2.13

$$-\frac{3e^4 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\,d^3}-\frac{e^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^4}+\frac{e^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^4}+\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^6}+\frac{3\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] -1/4/d^3/x^4*(-e^2*x^2+d^2)^(3/2)-5/8/d^5*e^2/x^2*(-e^2*x^2+d^2)^(3/2)+3/8/
d^5*e^4*(-e^2*x^2+d^2)^(1/2)-3/8/d^3*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^6*e^3/x*(-e^2*x^2+d^2)^(3/2)+1/d^6*e^5*x*(-e
^2*x^2+d^2)^(1/2)+1/d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(
1/2)*x)+1/3*e/d^4/x^3*(-e^2*x^2+d^2)^(3/2)-1/d^5*e^4*(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(1/2)-1/d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^5\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**5*(d + e*x)), x)
```

3.102 $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

```
[Out] 1/12*d*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)/e^3-1/5*(-e^2*x^2+d^2)^(5/2)/e^3+1
/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2)/
e^2
```

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 12, 785, 780, 195, 217, 203}

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) + (d*(4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2
))/(12*e^3) - (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(8*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{\int \frac{5de^3x\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{5e^4} \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{d\int \frac{x\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{e} \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{\int x\left(d^2e-de^2x\right)\sqrt{d^2-e^2x^2}\,dx}{e^2} \\
&= \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{4e^2} \\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2} \\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^5\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x\right.}{8e^2} \\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.99

$$
\frac{\sqrt{d^2-e^2x^2}\left(15d^4\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(16d^4-15d^3ex+8d^2e^2x^2+30de^3x^3-24e^4x^4\right)\right)}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*
e^2*x^2 + 30*d*e^3*x^3 - 24*e^4*x^4) + 15*d^4*ArcSin[(e*x)/d]))/(120*e^3*Sq
rt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.88, size = 94, normalized size = 0.83

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(24\,e^4x^4-30\,de^3x^3-8\,d^2e^2x^2+15\,d^3ex-16\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 - 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 + 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
4*d^5*exp(2)^3-4*d^5*exp(1)^4*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)
^2)/exp(1)^6/exp(1)+1/8*d^5*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)+2
*((((-192*exp(1)^7*1/1920/exp(1)^6*x+240*exp(1)^6*d*1/1920/exp(1)^6)*x+64*e
xp(1)^5*d^2*1/1920/exp(1)^6)*x-120*exp(1)^4*d^3*1/1920/exp(1)^6)*x+128*exp(
1)^3*d^4*1/1920/exp(1)^6)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.02, size = 222, normalized size = 1.96

$$\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,e^2}-\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{3\sqrt{-e^2x^2+d^2}\,d^3x}{8e^2}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/e^3-1/4*(-e^2*x^2+d^2)^(3/2)*d/e^2*x-3/8*(-e^2*x^
2+d^2)^(1/2)*d^3/e^2*x-3/8/(e^2)^(1/2)*d^5/e^2*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)+1/3*d^2/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+1/2*d^3/e^2*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/2*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^
(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.01, size = 174, normalized size = 1.54

$$-\frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^3}-\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^3x}{2\,e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}\,d^3x}{8\,e^2}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*I*d^5*arcsin(e*x/d + 2)/e^3 - 3/8*d^5*arcsin(e*x/d)/e^3 + 1/2*sqrt(e^2
*x^2 + 4*d*e*x + 3*d^2)*d^3*x/e^2 - 3/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^2 + sq
rt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^2
+ 1/3*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3 - 1/5*(-e^2*x^2 + d^2)^(5/2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(d^2 - e^2 x^2\right)^{3/2}}{d + e x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x), x)
```

sympy [C] time = 7.85, size = 279, normalized size = 2.47

$$d\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - e\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4}{} \\ \frac{x^4\sqrt{d^2}}{4} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(3/2)/(e*x+d),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - e*Piece
wise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2
*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**
2)/4, True))
```

3.103 $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex} dx$

Optimal. Leaf size=201

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}+\frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-\right.}{64}$$

```
[Out] 1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-1
/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2+1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e+1/5040*d^3
*(-315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 833, 780, 195, 217, 203}

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= \int x^4(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx\\
&= \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{\int x^3\left(4d^2e-9de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2}\\
&= -\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}+\frac{\int x^2\left(27d^3e^2-32d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{72e^4}\\
&= \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{\int x\left(64d^4e^3-189d^3e^4x\right)}{504e^6}\\
&= \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}+\frac{d^3(128d-315ex)\left(d^2-\right.}{5040e^5}\\
&= \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}+\frac{d^3(}{}\\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4(}{}\\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4(}{}\\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4(}{}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 135, normalized size = 0.67

$$
\frac{945d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(1024d^8-945d^7ex+512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x^5-\right.}{40320e^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*e^
3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^7*
x^7 + 4480*e^8*x^8) + 945*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40320*e^5
)
```

fricas [A] time = 0.81, size = 139, normalized size = 0.69

$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(4480\,e^8x^8-5040\,de^7x^7-6400\,d^2e^6x^6+7560\,d^3e^5x^5+384\,d^4e^4x^4-630\,d\right.}{40320\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (4480*e^8*x^
8 - 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 + 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
- 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 - 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^9*exp(1)^4*exp(2)^2-8*d^9*exp(2)^4-4*d^9*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^10/exp(1)+3/128*d^9*sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)^5+2*((((((((322560*exp(1)^17*1/5806080/exp(1)^14*x-3628
80*exp(1)^16*d*1/5806080/exp(1)^14)*x-460800*exp(1)^15*d^2*1/5806080/exp(1)
^14)*x+544320*exp(1)^14*d^3*1/5806080/exp(1)^14)*x+27648*exp(1)^13*d^4*1/58
06080/exp(1)^14)*x-45360*exp(1)^12*d^5*1/5806080/exp(1)^14)*x+36864*exp(1)^
11*d^6*1/5806080/exp(1)^14)*x-68040*exp(1)^10*d^7*1/5806080/exp(1)^14)*x+73
728*exp(1)^9*d^8*1/5806080/exp(1)^14)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.02, size = 330, normalized size = 1.64

$$\frac{3d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,e^4}-\frac{45d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}\,e^4}-\frac{45\sqrt{-e^2x^2+d^2}\,d^7x}{128e^4}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e}}{8e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/9/e^3*x^2*(-e^2*x^2+d^2)^(7/2)-11/63*d^2/e^5*(-e^2*x^2+d^2)^(7/2)+1/8*d/
e^4*x*(-e^2*x^2+d^2)^(7/2)-3/16*(-e^2*x^2+d^2)^(5/2)*d^3/e^4*x-15/64*(-e^2*
x^2+d^2)^(3/2)*d^5/e^4*x-45/128*(-e^2*x^2+d^2)^(1/2)*d^7/e^4*x-45/128/(e^2)
^(1/2)*d^9/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^4/e^5*(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/4*d^5/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3
/2)*x+3/8*d^7/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d^9/e^4/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.04, size = 246, normalized size = 1.22

$$-\frac{3i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^5}-\frac{45\,d^9\arcsin\left(\frac{ex}{d}\right)}{128\,e^5}+\frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^7x}{8\,e^4}-\frac{45\sqrt{-e^2x^2+d^2}\,d^7x}{128\,e^4}+\frac{3\sqrt{e^2x^2+4\,dex}}{4\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^9*arcsin(e*x/d + 2)/e^5 - 45/128*d^9*arcsin(e*x/d)/e^5 + 3/8*sqrt(
e^2*x^2 + 4*d*e*x + 3*d^2)*d^7*x/e^4 - 45/128*sqrt(-e^2*x^2 + d^2)*d^7*x/e^
4 + 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^8/e^5 + 1/64*(-e^2*x^2 + d^2)^(3/
2)*d^5*x/e^4 - 3/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 1/9*(-e^2*x^2 + d^2)
^(7/2)*x^2/e^3 + 1/5*(-e^2*x^2 + d^2)^(5/2)*d^4/e^5 + 1/8*(-e^2*x^2 + d^2)^
(7/2)*d*x/e^4 - 11/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{d+e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [C] time = 25.15, size = 830, normalized size = 4.13

$$d^3 \left(\begin{cases} -\frac{i d^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{i d^5 x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{i d^3 x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5 i d x^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{i e^2 x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5 x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3 x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5 d x^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2 x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - d^2 e \left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} \\ \frac{x^6\sqrt{d^2}}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) - d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7
) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e*
*4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e*
*7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) +
 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8
) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 -
 e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*
sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.104 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=172

$$-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}+\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}$$

```
[Out] -1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2+1/8
*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e
^4-3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/128*d^6*x*(-e^2*x^2+d^2
)^(1/2)/e^3
```

Rubi [A] time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 833, 780, 195, 217, 203}

$$-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}+\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^(3/2))/(6
4*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) + (x^3*(d^2 - e^2*x^2)^(5/2)
)/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) - (3*d^8*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= \int x^3(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx\\
&= \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{\int x^2\left(3d^2e-8de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}\\
&= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} + \frac{\int x\left(16d^3e^2-21d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}\\
&= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{d^4\int\left(d^2-e^2\right.}{16e^3}\\
&= -\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x\right.}{560e^4}\\
&= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32}{}\\
&= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32}{}\\
&= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32}{}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(-256d^7+105d^6ex-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x^5-640de^6x^6+560e^7x^7\right)-105}{4480e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7) -
105*d^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4480*e^4)
```

fricas [A] time = 0.90, size = 127, normalized size = 0.74

$$\frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(560\,e^7x^7-640\,de^6x^6-840\,d^2e^5x^5+1024\,d^3e^4x^4+70\,d^4e^3x^3-128\,d^5e^2x^2+105\right.}{4480\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 -
640*d*e^6*x^6 - 840*d^2*e^5*x^5 + 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 - 128*d
^5*e^2*x^2 + 105*d^6*e*x - 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-12*d^8*exp(1)^4*exp(2)^2+8*d^8*exp(2)^4+4*d^8*exp(1)^6*exp(2))*atan((-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)
^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^9/exp(1)-3/128*d^8*sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)^4+2*((((((((40320*exp(1)^15*1/645120/exp(1)^12*x-46080*e
xp(1)^14*d*1/645120/exp(1)^12)*x-60480*exp(1)^13*d^2*1/645120/exp(1)^12)*x+
73728*exp(1)^12*d^3*1/645120/exp(1)^12)*x+5040*exp(1)^11*d^4*1/645120/exp(1
)^12)*x-9216*exp(1)^10*d^5*1/645120/exp(1)^12)*x+7560*exp(1)^9*d^6*1/645120
/exp(1)^12)*x-18432*exp(1)^8*d^7*1/645120/exp(1)^12)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 305, normalized size = 1.77

$$-\frac{3d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,e^3}+\frac{45d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}\,e^3}+\frac{45\sqrt{-e^2x^2+d^2}\,d^6x}{128e^3}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)}}{8e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/8/e^3*x*(-e^2*x^2+d^2)^(7/2)+3/16*(-e^2*x^2+d^2)^(5/2)*d^2/e^3*x+15/64*(
-e^2*x^2+d^2)^(3/2)*d^4/e^3*x+45/128*(-e^2*x^2+d^2)^(1/2)*d^6/e^3*x+45/128/
(e^2)^(1/2)*d^8/e^3*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/7*d/e^4*(-
e^2*x^2+d^2)^(7/2)-1/5*d^3/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4*d^4/
e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8*d^6/e^3*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)*x-3/8*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.03, size = 221, normalized size = 1.28

$$\frac{3i\,d^8\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^4}+\frac{45\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^4}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^6x}{8\,e^3}+\frac{45\,\sqrt{-e^2x^2+d^2}\,d^6x}{128\,e^3}-\frac{3\,\sqrt{e^2x^2+4\,dex+}}{4\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^8*arcsin(e*x/d + 2)/e^4 + 45/128*d^8*arcsin(e*x/d)/e^4 - 3/8*sqrt(e
^2*x^2 + 4*d*e*x + 3*d^2)*d^6*x/e^3 + 45/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^3
 - 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^7/e^4 - 1/64*(-e^2*x^2 + d^2)^(3/2
)*d^4*x/e^3 + 3/16*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^3 - 1/5*(-e^2*x^2 + d^2)^
(5/2)*d^3/e^4 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^3 + 1/7*(-e^2*x^2 + d^2)^(7/
2)*d/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [A] time = 23.01, size = 775, normalized size = 4.51

$$d^3\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d}{24\sqrt{1}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e
*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48
*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) -
e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-8*d**
6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e*
*2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + e**3*Piecewise((-5*I*d*
*8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e
**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5
*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2))
 + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sq
rt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.105 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex} dx$

Optimal. Leaf size=140

$$\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}$$

```
[Out] 1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/30*d*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2
)/e^3-1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 12, 785, 780, 195, 217, 203}

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) + (d*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^3) - (d^2 - e^2*x^2)^(7/
2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\int \frac{7de^3x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{7e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{d\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}\\
&= \frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^3\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{6e^2}\\
&= \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^5\int \sqrt{d^2-e^2x^2}\,dx}{8e^2}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7}{}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7}{}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7}{}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 113, normalized size = 0.81

$$\frac{105d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(96d^6-105d^5ex+48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-280de^5x^5+240e^6x\right.}{1680e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3*x
^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6) + 105*d^7*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(1680*e^3)
```

fricas [A] time = 0.86, size = 117, normalized size = 0.84

$$-\frac{210\,d^{7}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(240\,e^{6}x^{6}-280\,de^{5}x^{5}-384\,d^{2}e^{4}x^{4}+490\,d^{3}e^{3}x^{3}+48\,d^{4}e^{2}x^{2}-105\,d^{5}ex+96\right.}{1680\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (240*e^6*x^6 -
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 + 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 - 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^7*exp(1)^4*exp(2)^2-8*d^7*exp(2)^4-4*d^7*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^8/exp(1)+1/16*d^7*sign(d)*asin(x*exp(2)
/d/exp(1))/exp(1)/exp(2)+2*((((((5760*exp(1)^13*1/80640/exp(1)^10*x-6720*ex
p(1)^12*d*1/80640/exp(1)^10)*x-9216*exp(1)^11*d^2*1/80640/exp(1)^10)*x+1176
0*exp(1)^10*d^3*1/80640/exp(1)^10)*x+1152*exp(1)^9*d^4*1/80640/exp(1)^10)*x
-2520*exp(1)^8*d^5*1/80640/exp(1)^10)*x+2304*exp(1)^7*d^6*1/80640/exp(1)^10
)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 282, normalized size = 2.01

$$\frac{3d^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,e^{2}}-\frac{5d^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{16\sqrt{e^{2}}\,e^{2}}-\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{5}x}{16e^{2}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}{8e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/7*(-e^2*x^2+d^2)^(7/2)/e^3-1/6*(-e^2*x^2+d^2)^(5/2)*d/e^2*x-5/24*(-e^2*x
^2+d^2)^(3/2)*d^3/e^2*x-5/16*(-e^2*x^2+d^2)^(1/2)*d^5/e^2*x-5/16/(e^2)^(1/2
)*d^7/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^2/e^3*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(5/2)+1/4*d^3/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x
+3/8*d^5/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d^7/e^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.04, size = 198, normalized size = 1.41

$$-\frac{3i\,d^{7}\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^{3}}-\frac{5\,d^{7}\arcsin\left(\frac{ex}{d}\right)}{16\,e^{3}}+\frac{3\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{5}x}{8\,e^{2}}-\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{5}x}{16\,e^{2}}+\frac{3\sqrt{e^{2}x^{2}+4\,dex+3}}{4\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^7*arcsin(e*x/d + 2)/e^3 - 5/16*d^7*arcsin(e*x/d)/e^3 + 3/8*sqrt(e^
2*x^2 + 4*d*e*x + 3*d^2)*d^5*x/e^2 - 5/16*sqrt(-e^2*x^2 + d^2)*d^5*x/e^2 +
```

```
3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^6/e^3 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d
^3*x/e^2 - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)*
d^2/e^3 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(d^2 - e^2 x^2\right)^{5/2}}{d + e x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [C] time = 16.66, size = 653, normalized size = 4.66

$$d^3\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \\ \frac{x^4\sqrt{d^2}}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - d**2
*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**
2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*
sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I
*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x
/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e
**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e*
*2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**3*Piecewise((-8*d**6*sq
rt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x
**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.106 $\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=116

$$-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

```
[Out] -1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e
^2-1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/16*d^4*x*(-e^2*x^2+d^2)^
(1/2)/e
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {785, 780, 195, 217, 203}

$$-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -(d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) - (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e)
- ((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) - (d^6*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(16*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
```

```
&& EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{de}$$
$$= -\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}$$
$$= -\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e}$$
$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx\right.}{16e}$$
$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Mathematica [**A**] time = 0.09, size = 102, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(-48d^5+15d^4ex+96d^3e^2x^2-70d^2e^3x^3-48de^4x^4+40e^5x^5\right)-15d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 + 40*e^5*x^5) - 15*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(240*e^2)
```

fricas [**A**] time = 0.96, size = 105, normalized size = 0.91

$$\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5-48\,de^4x^4-70\,d^2e^3x^3+96\,d^3e^2x^2+15\,d^4ex-48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 - 48*
d*e^4*x^4 - 70*d^2*e^3*x^3 + 96*d^3*e^2*x^2 + 15*d^4*e*x - 48*d^5)*sqrt(-e^
2*x^2 + d^2))/e^2
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-12*d^6*exp(1)^4*exp(2)^2+8*d^6*exp(2)^4+4*d^6*exp(1)^6*exp(2))*atan((-1/2*
```

```
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)
^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^7/exp(1)-1/16*d^6*sign(d)*asin(x*exp(2
)/d/exp(1))/exp(1)^2+2*(((((960*exp(1)^11*1/11520/exp(1)^8*x-1152*exp(1)^10
*d*1/11520/exp(1)^8)*x-1680*exp(1)^9*d^2*1/11520/exp(1)^8)*x+2304*exp(1)^8*
d^3*1/11520/exp(1)^8)*x+360*exp(1)^7*d^4*1/11520/exp(1)^8)*x-1152*exp(1)^6*
d^5*1/11520/exp(1)^8)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 260, normalized size = 2.24

$$-\frac{3d^6 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\,e}+\frac{5d^6 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e}+\frac{5\sqrt{-e^2x^2+d^2}\,d^4x}{16e}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/6*(-e^2*x^2+d^2)^(5/2)/e*x+5/24*(-e^2*x^2+d^2)^(3/2)*d^2/e*x+5/16*(-e^2*x
^2+d^2)^(1/2)*d^4/e*x+5/16/(e^2)^(1/2)*d^6/e*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)-1/5*d/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4*d^2/e*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8*d^4/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/
2)*x-3/8*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x)
```

maxima [C] time = 1.02, size = 176, normalized size = 1.52

$$\frac{3i\,d^6 \arcsin\left(\frac{ex}{d}+2\right)}{8\,e^2}+\frac{5\,d^6 \arcsin\left(\frac{ex}{d}\right)}{16\,e^2}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^4x}{8\,e}+\frac{5\,\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d}}{4\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^6*arcsin(e*x/d + 2)/e^2 + 5/16*d^6*arcsin(e*x/d)/e^2 - 3/8*sqrt(e^2
*x^2 + 4*d*e*x + 3*d^2)*d^4*x/e + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e - 3/4*s
qrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5/e^2 - 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/
e + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2-e^2\,x^2\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [A] time = 16.12, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)-d^2e\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.107 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=100

$$\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}+\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}$$

```
[Out] 1/4*d*x*(-e^2*x^2+d^2)^(3/2)+1/5*(-e^2*x^2+d^2)^(5/2)/e+3/8*d^5*arctan(e*x/
(-e^2*x^2+d^2)^(1/2))/e+3/8*d^3*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {665, 195, 217, 203}

$$\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (d*x*(d^2 - e^2*x^2)^(3/2))/4 + (d^2 - e^
2*x^2)^(5/2)/(5*e) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + d\int \left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{4}\left(3d^3\right)\int \sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{8}\left(3d^5\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{8}\left(3d^5\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x,$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.91

$$\frac{15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \sqrt{d^2-e^2x^2}\left(8d^4+25d^3ex-16d^2e^2x^2-10de^3x^3+8e^4x^4\right)}{40e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
8*e^4*x^4) + 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40*e)
```

fricas [A] time = 0.95, size = 95, normalized size = 0.95

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(8\,e^4x^4-10\,de^3x^3-16\,d^2e^2x^2+25\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (8*e^4*x^4 - 10*d
*e^3*x^3 - 16*d^2*e^2*x^2 + 25*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/e
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^5*exp(1)^4*exp(2)^2-8*d^5*exp(2)^4-4*d^5*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^6/exp(1)+3/8*d^5*sign(d)*asin(x*exp(2)/
d/exp(1))/exp(1)+2*((((192*exp(1)^9*1/1920/exp(1)^6*x-240*exp(1)^8*d*1/1920
/exp(1)^6)*x-384*exp(1)^7*d^2*1/1920/exp(1)^6)*x+600*exp(1)^6*d^3*1/1920/ex
p(1)^6)*x+192*exp(1)^5*d^4*1/1920/exp(1)^6)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.01, size = 147, normalized size = 1.47

$$\frac{3d^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\; d^3 x}{8}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} dx}{4}+\frac{\left(2\left(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/4*d*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(3/2)*x+3/8*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d^5/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 0.99, size = 109, normalized size = 1.09

$$-\frac{3i\,d^5 \arcsin\left(\frac{ex}{d}+2\right)}{8\,e}+\frac{3}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^3x+\frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^4}{4\,e}+\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx+\frac{\left(-e^2x^2+\right.}{5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^5*arcsin(e*x/d + 2)/e + 3/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
+ 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*
x + 1/5*(-e^2*x^2 + d^2)^(5/2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x), x)
```

sympy [C] time = 10.46, size = 435, normalized size = 4.35

$$d^3\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right)-d^2e\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)-de^2\left(\begin{cases} -\frac{id^4\operatorname{aco}}{8} \\ \frac{d^4\operatorname{asin}(}{8e^3} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - d**2
*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/
(3*e**2), True)) - d*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3
*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) +
```

```
 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/
d**2)), True)) + e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) -
 d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5
, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

3.108 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/12*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)-3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(-3*e*x+8*d)*(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (d^2*(8*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 - (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$$
$$= \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2+3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4-3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{8}\left(3d^4e\right.$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x\right.$$
$$d^5\,\mathrm{S}$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - $$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\,\mathrm{ta}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.96

$$d^4\log(x) - d^4\log\left(\sqrt{d^2-e^2x^2}+d\right) - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3-15d^2ex-8de^2x^2+6e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 -
(3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + d^4*Log[x] - d^4*Log[d + Sqrt
[d^2 - e^2*x^2]]
```

fricas [A] time = 0.86, size = 107, normalized size = 0.95

$$\frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{24}\left(6\,e^3x^3-8\,de^2x^2-15\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + 1/24*(6*e^3*x^3 - 8*d*e^2*x^2 - 15*d^2*e*x + 32*d^3)*sqrt
(-e^2*x^2 + d^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -3/8*
d^4*sign(d)*asin(x*exp(2)/d/exp(1))+1/2*(-12*d^4*exp(1)^4*exp(2)^2+8*d^4*ex
p(2)^4+4*d^4*exp(1)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(
1)^5/exp(1)-d^4*exp(2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/abs(x)/exp(2))/exp(1)^2+2*(((24*exp(1)^7*1/192/exp(1)^4*x-32*exp(1)^6*d*1
/192/exp(1)^4)*x-60*exp(1)^5*d^2*1/192/exp(1)^4)*x+128*exp(1)^4*d^3*1/192/e
xp(1)^4)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 245, normalized size = 2.17

$$-\frac{d^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{3d^4e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2ex}{8}+\sqrt{-e^2x^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x)
```

```
[Out] 1/5/d*(-e^2*x^2+d^2)^(5/2)+1/3*(-e^2*x^2+d^2)^(3/2)*d+(-e^2*x^2+d^2)^(1/2)*
d^3-1/(d^2)^(1/2)*d^5*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/
d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(
3/2)*x-3/8*d^2*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-3/8*d^4*e/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 124, normalized size = 1.10

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{ex}{d}\right)-d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3}{8}\sqrt{-e^2x^2+d^2}\,d^2ex+\sqrt{-e^2x^2+d^2}\,d^3-\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(e*x/d) - d^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/ab
s(x)) - 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(-e^2*x^2 + d^2)*d^3 - 1/4*(
-e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2)^(3/2)*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)), x)
```

sympy [C] time = 25.65, size = 469, normalized size = 4.15

$$d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{fo} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{ot} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d
**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) + e**3*Piecewise((-I*d**4*acosh(e*x/d
)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
 e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.109 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))+d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*d*e*(3*e*x+2*d)*(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {850, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d + e*x)*(d^2 - e^2*x^2)^(
3/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + d^3*e*ArcTanh
[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$$
$$= -\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx$$
$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{\int \frac{-4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$
$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}($$
$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.$$
$$d^4$$
$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + -$$
$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^3$$

Mathematica [A] time = 0.13, size = 114, normalized size = 0.99

$$-d^3e\log(x)+d^3e\log\left(\sqrt{d^2-e^2x^2}+d\right)-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(-\frac{d^3}{x}-\frac{4d^2e}{3}-\frac{1}{2}de^2x+\frac{e^3x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-4*d^2*e)/3 - d^3/x - (d*e^2*x)/2 + (e^3*x^2)/3) - (3
*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*Log[x] + d^3*e*Log[d +
Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.67, size = 123, normalized size = 1.07

$$\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-8\,d^3ex+\left(2\,e^3x^3-3\,de^2x^2-8\,d^2ex-6\,d^3\right)\sqrt{-e^2x}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) - 8*d^3*e*x + (2*e^3*x^3 - 3*d*e^2*x^2 - 8*d^
2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^3*exp(1)^4*exp(2)^2-8*d^3*exp(2)^4-4*d^3*exp(1)^6*exp(2))*atan((-1/2*(
```

```
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)-d^3*x*exp(2)^3/(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/exp(1)/exp(2)+1/4*d^3*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(2)^4/exp(1)^4/x/exp(1)/exp(2)^2+d^3*exp(2)*ln(1/2*a
bs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)-3/2*d^3
*sign(d)*asin(x*exp(2)/d/exp(1))*exp(2)/exp(1)+2*((4*exp(1)^5*1/24/exp(1)^2
*x-6*exp(1)^4*d*1/24/exp(1)^2)*x-16*exp(1)^3*d^2*1/24/exp(1)^2)*sqrt(d^2-x^
2*exp(2))
```

maple [B] time = 0.01, size = 380, normalized size = 3.30

$$\frac{d^4 e \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{3d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{15d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}\,d\,e^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d^3/x*(-e^2*x^2+d^2)^(7/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4*(-e^2*x^
2+d^2)^(3/2)/d*e^2*x-15/8*(-e^2*x^2+d^2)^(1/2)*d*e^2*x-15/8/(e^2)^(1/2)*d^3
*e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/5*e/d^2*(-e^2*x^2+d^2)^(5
/2)-1/3*(-e^2*x^2+d^2)^(3/2)*e-(-e^2*x^2+d^2)^(1/2)*d^2*e+1/(d^2)^(1/2)*d^4
*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5*e/d^2*(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(5/2)+1/4*e^2/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+3/8*
e^2*d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*e^2*d^3/(e^2)^(1/2)*arctan(
(e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 131, normalized size = 1.14

$$-\frac{3}{2}\,d^3e\arcsin\left(\frac{ex}{d}\right)+d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{1}{2}\sqrt{-e^2x^2+d^2}\,de^2x-\sqrt{-e^2x^2+d^2}\,d^2e-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/2*d^3*e*arcsin(e*x/d) + d^3*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x - sqrt(-e^2*x^2 + d^2)*d^2*e -
 1/3*(-e^2*x^2 + d^2)^(3/2)*e - sqrt(-e^2*x^2 + d^2)*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

sympy [C] time = 10.20, size = 386, normalized size = 3.36

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\,\mathrm{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\,\mathrm{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\mathrm{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{fo}\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\mathrm{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{ot}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
- I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 -
e**2*x**2/d**2)/2, True)) + e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0))
, (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

3.110 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=121

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2+3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3/2*d*e*(-e*x+d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 813, 844, 217, 203, 266, 63, 208}

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d + e*x)*(d^2 - e^2*x^2)^(3
/2))/(2*x^2) + (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^2
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2+8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{2}\big(3$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\text{Subst}\bigg(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\big(3d^3$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2$$

Mathematica [A] time = 0.16, size = 119, normalized size = 0.98

$$\frac{1}{2}\Bigg(3d^2e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 3d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 3d^2e^2\log(x) + \frac{\sqrt{d^2-e^2x^2}\left(-d^3+2d^2ex-2de^2\right.}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/x^2 + 3*d
^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 3*d^2*e^2*Log[x] + 3*d^2*e^2*Log
[d + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.92, size = 135, normalized size = 1.12

$$-\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2\,d^2e^2x^2-\left(e^3x^3-2\,de^2x^2+2\,d^2ex-d^3\right)\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*d^2*e^2*x
^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2*d^2*e^2*x^2 - (e^3*x^3 - 2*d*e^2*
x^2 + 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/8*(
d^2*exp(2)^3+2*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/x/e
xp(2))/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2/exp(1)
^4+1/16*(-2*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^2*exp(1)^4*exp(2)^5-4*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^6*exp(2)^4/x/exp(2))/exp(1)^6/exp(2)^3+1/2*(5*d^2*exp(2)^3-2*d^2*exp(1)^
4*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(
2))/exp(1)^3/exp(1)+1/2*(-12*d^2*exp(1)^4*exp(2)^2+8*d^2*exp(2)^4+4*d^2*exp
(1)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(
2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^3/exp(1)+3/2*
d^2*sign(d)*asin(x*exp(2)/d/exp(1))*exp(1)^2+2*(2*exp(1)^3/8*x-4*exp(1)^2*d
/8)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 411, normalized size = 3.40

$$\frac{3d^3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{3d^2e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{15d^2e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{15\sqrt{-e^2x^2+d^2}\,e^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x)
```

```
[Out] e/d^4/x*(-e^2*x^2+d^2)^(7/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(5/2)+5/4*(-e^2*x^2+d
^2)^(3/2)/d^2*e^3*x+15/8*(-e^2*x^2+d^2)^(1/2)*e^3*x+15/8/(e^2)^(1/2)*d^2*e^
3*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/d^3/x^2*(-e^2*x^2+d^2)^(7/
2)-3/10/d^3*e^2*(-e^2*x^2+d^2)^(5/2)-1/2*(-e^2*x^2+d^2)^(3/2)/d*e^2-3/2*(-e
^2*x^2+d^2)^(1/2)*d*e^2+3/2/(d^2)^(1/2)*d^3*e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-1/5/d^3*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4/d
^2*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8*e^3*(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(1/2)*x-3/8*d^2*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 138, normalized size = 1.14

$$\frac{3}{2}\,d^2e^2\arcsin\left(\frac{ex}{d}\right)+\frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,e^3x-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,de^2+\frac{\sqrt{-e^2x^2+d^2}\,d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*e^2*arcsin(e*x/d) + 3/2*d^2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
+ d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^3*x - 3/2*sqrt(-e^2*x^2 + d^2
)*d*e^2 + sqrt(-e^2*x^2 + d^2)*d^2*e/x - 1/2*(-e^2*x^2 + d^2)^(3/2)*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

sympy [C] time = 13.35, size = 461, normalized size = 3.81

$$d^{3}\left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) - d^{2} e \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} \\ - \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) - d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d
/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
- d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
> 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*
x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**3*Piecewise((-I*d**2*acosh(e*x/d
)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*s
qrt(1 - e**2*x**2/d**2)/2, True))
```

3.111 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/6*(-3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(3*e*x+2*d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {850, 811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^2*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d - 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$$
$$= -\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{\int \frac{\left(4d^3e^2-6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{\int \frac{12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \left(de^4\right.$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{1}{4}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}\left(3d^2e\right)$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3\tan$$

Mathematica [A] time = 0.15, size = 116, normalized size = 0.97

$$-\frac{3}{2}de^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\left(-\frac{d^3}{3x^3}+\frac{d^2e}{2x^2}+\frac{4de^2}{3x}+e^3\right)\sqrt{d^2-e^2x^2}+\frac{3}{2}de^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^3 - d^3/(3*x^3) + (d^2*e)/(2*x^2) + (4*d*e^2)/(3*x))*Sqrt[d^2 - e^2*x^2]
 + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Log[x])/2 - (3*d*e^3*
Log[d + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.82, size = 130, normalized size = 1.08

$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - 6\,de^3x^3 - \left(6\,e^3x^3+8\,de^2x^2+3\,d^2ex-2\,d^3\right)\sqrt{-e^2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 6*d*e^3*x^3 - (6*e^3*x^3 + 8*d*e^2*x^2
 + 3*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/24*
((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(12*d*exp(1)
```

```
^4*exp(2)^2-27*d*exp(2)^4)+d*exp(2)^4+3/2*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(2)^4/x/exp(2))/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3/exp(1)^5+1/512*(64*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^7-64/3*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^8+96*d*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^8/x/exp(2)-384*d*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^7/x/exp(2)+128*d*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^6/x/exp(2))/exp(1)^15
/exp(2)^3+1/2*(-5*d*exp(2)^3+2*d*exp(1)^4*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)/exp(2)+1/2*(12*d*exp(1)^
4*exp(2)^2-8*d*exp(2)^4-4*d*exp(1)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4
+exp(2)^2)/exp(1)/exp(2)+d*sign(d)*asin(x*exp(2)/d/exp(1))*exp(1)^3+4*exp(1
)^3/4*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 439, normalized size = 3.66

$$-\frac{3d^2e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}+\frac{3d\,e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{5d\,e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{5\sqrt{-e^2x^2+d^2}\,e}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x)
```

```
[Out] 1/3/d^5*e^2/x*(-e^2*x^2+d^2)^(7/2)+1/3/d^5*e^4*x*(-e^2*x^2+d^2)^(5/2)+5/12*
(-e^2*x^2+d^2)^(3/2)/d^3*e^4*x+5/8*(-e^2*x^2+d^2)^(1/2)/d*e^4*x+5/8/(e^2)^(
1/2)*d*e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2*e/d^4/x^2*(-e^2*x
^2+d^2)^(7/2)+3/10*e^3/d^4*(-e^2*x^2+d^2)^(5/2)+1/2*(-e^2*x^2+d^2)^(3/2)/d^
2*e^3+3/2*(-e^2*x^2+d^2)^(1/2)*e^3-3/2/(d^2)^(1/2)*d^2*e^3*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3*(-e^2*x^2+d^2)^(7/2)+1/5/d^4*e
^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/4/d^3*e^4*(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(3/2)*x+3/8/d*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d*e^4/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 132, normalized size = 1.10

$$de^3\arcsin\left(\frac{ex}{d}\right)-\frac{3}{2}de^3\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{3}{2}\sqrt{-e^2x^2+d^2}\,e^3+\frac{\sqrt{-e^2x^2+d^2}\,de^2}{x}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{2x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] d*e^3*arcsin(e*x/d) - 3/2*d*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x)) + 3/2*sqrt(-e^2*x^2 + d^2)*e^3 + sqrt(-e^2*x^2 + d^2)*d*e^2/x + 1/
2*(-e^2*x^2 + d^2)^(3/2)*e/x^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

sympy [C] time = 11.68, size = 457, normalized size = 3.81

$$d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)-d^2e\left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 d**2*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d)
, True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(
e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
(-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*
x**2/d**2)), True)) + e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
 - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
 > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e
*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.112 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)} dx$

Optimal. Leaf size=119

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*(-4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4-e^4*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(-8*e*x+3*d)*(-e^2*x^
2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 811, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (e^2*(3*d - 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d - 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx$$

$$= -\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{\left(6d^3e^2-8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4-32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^5\int$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{8}\left(3de^2\right)\text{S}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\tanh^{-}$$

Mathematica [A] time = 0.19, size = 111, normalized size = 0.93

$$\frac{1}{24}\left(-9e^4\log\left(\sqrt{d^2-e^2x^2}+d\right) - 24e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex+15de^2x^2-32e^3x^3\right)}{x^4} + 9\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/x^4
 - 24*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + 9*e^4*Log[x] - 9*e^4*Log[d +
Sqrt[d^2 - e^2*x^2]])/24
```

fricas [A] time = 0.95, size = 119, normalized size = 1.00

$$\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3-15\,de^2x^2-8\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 - 15*d*e^2*x^2 - 8*d^2*e*x + 6
*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/192
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-96*exp(1)
^6*exp(2)^2+288*exp(1)^4*exp(2)^3-72*exp(2)^5)+(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*(24*exp(1)^4*exp(2)^3-48*exp(2)^5)+3*exp(
2)^5+4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2))/(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4/exp(1)^6+1/65536*(
-8192*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
22*exp(2)^7+8192/3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^3*exp(1)^20*exp(2)^8-1024*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^4*exp(1)^18*exp(2)^9+24576*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(2)^8-8192*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^18*exp(2)^9-12288*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^20*exp(2)^8/x/exp(2)+49152*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^22*exp(2)^7/x/exp(2)-16384*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^24*exp(2)^6/x/exp(2))/exp(1)
^24/exp(2)^4+1/2*(-12*exp(1)^4*exp(2)^2+8*exp(2)^4+4*exp(1)^6*exp(2))*atan(
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+
exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^2+1/8*(24*exp(1)^6*exp(2)^2-28*e
xp(1)^4*exp(2)^3+9*exp(2)^5-8*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^5/exp(1)-sign(d)*asin(x*exp
(2)/d/exp(1))*exp(1)^4
```

maple [B] time = 0.01, size = 463, normalized size = 3.89

$$-\frac{3d\,e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}-\frac{3e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{5e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x)
```

```
[Out] -5/8/(e^2)^(1/2)*e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/8*(-e^2*x
^2+d^2)^(3/2)/d^3*e^4+3/8*(-e^2*x^2+d^2)^(1/2)/d*e^4-1/4/d^4*e^5*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8/d^2*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

```
*x+1/3*e/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-1/3/d^6*e^3/x*(-e^2*x^2+d^2)^(7/2)-1/
3/d^6*e^5*x*(-e^2*x^2+d^2)^(5/2)-1/8/d^5*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/12*
(-e^2*x^2+d^2)^(3/2)/d^4*e^5*x-5/8*(-e^2*x^2+d^2)^(1/2)/d^2*e^5*x-3/8/(d^2)
^(1/2)*d*e^4*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/d^5*e^4*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-3/8*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/4/d^3/x^4*(-e^2*x^2+d^2)^(7/2)+3/40
/d^5*e^4*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 1.01, size = 159, normalized size = 1.34

$$-e^4 \arcsin\left(\frac{ex}{d}\right) - \frac{3}{8} e^4 \log\left(\frac{2 d^2}{|x|} + \frac{2 \sqrt{-e^2x^2 + d^2}\, d}{|x|}\right) + \frac{3 \sqrt{-e^2x^2 + d^2}\, e^4}{8 d} - \frac{\sqrt{-e^2x^2 + d^2}\, e^3}{x} + \frac{3 \left(-e^2x^2 + d^2\right)^{\frac{3}{2}} e^2}{8 dx^2} + \frac{(-e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] -e^4*arcsin(e*x/d) - 3/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/ab
s(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^2*x^2 + d^2)*e^3/x + 3/8*(
-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) + 1/3*(-e^2*x^2 + d^2)^(3/2)*e/x^3 - 1/4*
(-e^2*x^2 + d^2)^(3/2)*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\, x^2\right)^{5/2}}{x^5\, \left(d + e\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

sympy [C] time = 14.37, size = 541, normalized size = 4.55

$$d^3 \left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4 \operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4 \operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right) - d^2e \left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) -
1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d
**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asi
n(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)
) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.113 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

```
[Out] 1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5+3/8*e^5*arcta
nh((-e^2*x^2+d^2)^(1/2)/d)/d-3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 807, 266, 47, 63, 208}

$$-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (-3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) + (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) -
(d^2 - e^2*x^2)^(5/2)/(5*d*x^5) + (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8
*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - \frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right) \\
&= \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - \frac{1}{16}\left(3e^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, \right. \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2} \right. \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 106, normalized size = 0.98

$$\frac{15e^5x^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-8d^4+10d^3ex+16d^2e^2x^2-25de^3x^3-8e^4x^4\right)-15e^5x^5\log(x)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3 -
 8*e^4*x^4) - 15*e^5*x^5*Log[x] + 15*e^5*x^5*Log[d + Sqrt[d^2 - e^2*x^2]])/
(40*d*x^5)
```

fricas [A] time = 0.72, size = 97, normalized size = 0.90

$$-\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^4x^4+25\,de^3x^3-16\,d^2e^2x^2-10\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 + 25*d*e^
3*x^3 - 16*d^2*e^2*x^2 - 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/960
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(480*exp(1)
^8*exp(2)^2-1440*exp(1)^6*exp(2)^3+1800*exp(1)^4*exp(2)^4-780*exp(2)^6)+(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-120*exp(1)^6*e
xp(2)^3+360*exp(1)^4*exp(2)^4-120*exp(2)^6)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*(40*exp(1)^4*exp(2)^4-70*exp(2)^6)+6*exp(2)^
6+15/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^6/x/exp(2))/(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5/exp(1)^6/d/exp(1)+
1/33554432*(4194304*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^34*exp(2)^8-4194304/3*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^32*exp(2)^9+524288*d^4*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^30*exp(2)^10-1048576
/5*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)
^28*exp(2)^11-12582912*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^2*exp(1)^32*exp(2)^9+4194304*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^30*exp(2)^10+4194304*d^4*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^30*exp(2)^10-5242
880/3*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^28*exp(2)^11+5242880*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*ex
p(1)^28*exp(2)^11/x/exp(2)-18874368*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))*exp(1)^30*exp(2)^10/x/exp(2)+31457280*d^4*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^32*exp(2)^9/x/exp(2)-25165824*d^4*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^34*exp(2)^8/x/exp(2)+8388608*d^4*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^36*exp(2)^7/x/exp(2))/d^5/exp
(1)^35/exp(2)^5+1/2*(12*exp(1)^4*exp(2)^2-8*exp(2)^4-4*exp(1)^6*exp(2))*ata
n((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^
4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/d/exp(1)+1/8*(-24*exp(1)^6*exp(2)^2+2
8*exp(1)^4*exp(2)^3-9*exp(2)^5+8*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^4/d/exp(1)
```

maple [B] time = 0.02, size = 493, normalized size = 4.56

$$\frac{3e^6 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\, d}-\frac{3e^6 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\, d}+\frac{3e^5 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\right.}}{8d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x)
```

```
[Out] -1/8*(-e^2*x^2+d^2)^(3/2)/d^4*e^5-3/8*(-e^2*x^2+d^2)^(1/2)/d^2*e^5+3/8/(d^2
)^(1/2)*e^5*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/4/d^5*e^6*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+3/8/d^3*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(1/2)*x+3/8/d*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)*x)-1/5/d^5*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^4/x*(-e^2*x^
2+d^2)^(7/2)-1/5/d^7*e^6*x*(-e^2*x^2+d^2)^(5/2)-1/4/d^5*e^6*x*(-e^2*x^2+d^2
)^(3/2)-3/8/d^3*e^6*x*(-e^2*x^2+d^2)^(1/2)-3/8/d*e^6/(e^2)^(1/2)*arctan((e^
2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/4*e/d^4/x^4*(-e^2*x^2+d^2)^(7/2)+1/8*e^3
```

```
/d^6/x^2*(-e^2*x^2+d^2)^(7/2)+1/5/d^6*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/
2)-1/5/d^3/x^5*(-e^2*x^2+d^2)^(7/2)-3/40*e^5/d^6*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 1.00, size = 153, normalized size = 1.42

$$\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^5}{8\,d^2}-\frac{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^2}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}{5\,dx^3}+\frac{(-e^2x^2+d^2)^{\frac{3}{2}}e}{4\,x^4}-\frac{(-e^2x^2+}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 3/8*sqrt(-e
^2*x^2 + d^2)*e^5/d^2 - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^2) + 1/5*(-e^
2*x^2 + d^2)^(3/2)*e^2/(d*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e/x^4 - 1/5*(-e
^2*x^2 + d^2)^(3/2)*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

sympy [C] time = 13.94, size = 774, normalized size = 7.17

$$d^3\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+ \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**3*Piecewise((-d**2/(2*e*x**
3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*a
cosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**
2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

3.114 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=143

$$-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

```
[Out] -1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6+1/5*e*(
-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/
16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) + (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{\int \frac{\left(6d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)}{12d}\\
&= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)}{16d}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^6\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2}}\right.}{32d}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right.}{16}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.}{16d^2}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 117, normalized size = 0.82

$$\frac{-15e^6x^6\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-40d^5+48d^4ex+70d^3e^2x^2-96d^2e^3x^3-15de^4x^4+48e^5x^5\right)+15e^6}{240d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 + 48*e^5*x^5) + 15*e^6*x^6*Log[x] - 15*e^6*x^6*Log[d + Sqr
t[d^2 - e^2*x^2]])/(240*d^2*x^6)
```

fricas [A] time = 0.91, size = 108, normalized size = 0.76

$$\frac{15\,e^{6}x^{6}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(48\,e^{5}x^{5}-15\,de^{4}x^{4}-96\,d^{2}e^{3}x^{3}+70\,d^{3}e^{2}x^{2}+48\,d^{4}ex-40\,d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{240\,d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*x^5 - 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 + 70*d^3*e^2*x^2 + 48*d^4*e*x - 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/192
0*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-960*exp(
1)^10*exp(2)^2+2880*exp(1)^8*exp(2)^3-3600*exp(1)^6*exp(2)^4+2160*exp(1)^4*
exp(2)^5-600*exp(2)^7)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*(240*exp(1)^8*exp(2)^3-720*exp(1)^6*exp(2)^4+960*exp(1)^4*exp(2)^
5-495*exp(2)^7)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*(-80*exp(1)^6*exp(2)^4+240*exp(1)^4*exp(2)^5-100*exp(2)^7)+(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(30*exp(1)^4*exp(2)^5-45*e
xp(2)^7)+5*exp(2)^7+6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^7/
x/exp(2))/d^2/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6
/exp(1)^8+1/68719476736*(-8589934592*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^2*exp(1)^48*exp(2)^9+8589934592/3*d^10*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^46*exp(2)^10-10
73741824*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
*exp(1)^44*exp(2)^11+2147483648/5*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^42*exp(2)^12-536870912/3*d^10*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^40*exp(2)^13+25769
803776*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^46*exp(2)^10-8589934592*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^44*exp(2)^11+3221225472*d^10*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^42*exp(2)^12-3435973836
8*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^44*exp(2)^11+10737418240/3*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^42*exp(2)^12-1610612736*d^10*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^40*exp(2)^13+25769803776*
d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4
2*exp(2)^12-8053063680*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^40*exp(2)^13-10737418240*d^10*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(1)^42*exp(2)^12/x/exp(2)+38654705664*d^10*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^44*exp(2)^11/x/exp(2)-6442450944
0*d^10*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^46*exp(2)^10/x/ex
p(2)+51539607552*d^10*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^48
*exp(2)^9/x/exp(2)-17179869184*d^10*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))*exp(1)^50*exp(2)^8/x/exp(2))/d^12/exp(1)^48/exp(2)^6+1/2*(-12*exp(1)^5
```

```
*exp(2)^2+12*exp(1)^3*exp(2)^3+4*exp(1)^7*exp(2)-4*exp(1)*exp(2)^4)*atan((-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+ex
p(2)^2))/d^2/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/16*(48*exp(1)^10*exp(2)^2-56
*exp(1)^8*exp(2)^3+40*exp(1)^6*exp(2)^4-30*exp(1)^4*exp(2)^5+13*exp(2)^7-16
*exp(1)^12*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/ab
s(x)/exp(2))/d^2/exp(1)^7/exp(1)
```

maple [B] time = 0.01, size = 521, normalized size = 3.64

$$-\frac{e^{6}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}\,d}-\frac{3e^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,d^{2}}+\frac{3e^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}\,d^{2}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right.}}{8d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x)
```

```
[Out] 1/48*(-e^2*x^2+d^2)^(3/2)/d^5*e^6+1/16*(-e^2*x^2+d^2)^(1/2)/d^3*e^6+1/5*e/d
^4/x^5*(-e^2*x^2+d^2)^(7/2)-1/4/d^6*e^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
*x-3/8/d^4*e^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-3/8/d^2*e^7/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/5/d^6*e^3/x^3
*(-e^2*x^2+d^2)^(7/2)+1/5/d^8*e^5/x*(-e^2*x^2+d^2)^(7/2)+1/5/d^8*e^7*x*(-e^
2*x^2+d^2)^(5/2)+1/4/d^6*e^7*x*(-e^2*x^2+d^2)^(3/2)+3/8/d^4*e^7*x*(-e^2*x^2
+d^2)^(1/2)+3/8/d^2*e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)-5/24/d^5*e^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/16/d^7*e^4/x^2*(-e^2*x^2+d^2)^(
7/2)-1/16/(d^2)^(1/2)/d*e^6*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)-1/5/d^7*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/6/d^3/x^6*(-e^2*x^2+d^2
)^(7/2)+1/80/d^7*e^6*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 1.00, size = 178, normalized size = 1.24

$$-\frac{e^{6}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{2}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16\,d^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{16\,d^{3}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{5\,d^{2}x^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{8\,dx^{4}}+\frac{\left(-e^{2}x^{2}+d\right.}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^3 + 1/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^2) - 1/
5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^3) + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*
x^4) + 1/5*(-e^2*x^2 + d^2)^(3/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(3/2)*d/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^{2}-e^{2}\,x^{2}\right)^{5/2}}{x^{7}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

sympy [C] time = 18.69, size = 918, normalized size = 6.42

$$d^3\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise} \end{cases}\right)-d^2e\left(\begin{cases} \frac{3id^3\sqrt{\dots}}{-15d^2\dots} \\ \frac{3d^3\sqrt{\dots}}{-15d^2\dots} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) + e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

3.115 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=172

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] 1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7+1/6*e*
(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5+1/16*e^7
*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] -(e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) + (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d
^2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) + (e*(d^2 - e^2*x^2)^(5/2))/(6*d^
2*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) + (e^7*ArcTanh[Sqrt[d^2
 - e^2*x^2]/d])/(16*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(7d^2e-2de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} + \frac{\int \frac{\left(12d^3e^2-7d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^3\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right.}{12d^2}$$

$$= \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^5\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)}{35d^3x^5}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)}{35d^3x^5}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)}{35d^3x^5}$$

Mathematica [A] time = 0.19, size = 128, normalized size = 0.74

$$\frac{105e^7x^7\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-240d^6+280d^5ex+384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+105de^5x^5\right.}{1680d^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6) - 105*e^7*x^7*Log[x] +
 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(1680*d^3*x^7)
```

fricas [A] time = 0.89, size = 119, normalized size = 0.69

$$-\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(96\,e^6x^6-105\,de^5x^5+48\,d^2e^4x^4+490\,d^3e^3x^3-384\,d^4e^2x^2-280\,d^5ex+240\,d^6\right)}{1680\,d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (96*e^6*x^6 - 105
*d*e^5*x^5 + 48*d^2*e^4*x^4 + 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 - 280*d^5*e
*x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/134
40*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(6720*exp
(1)^12*exp(2)^2-20160*exp(1)^10*exp(2)^3+25200*exp(1)^8*exp(2)^4-21840*exp(
1)^6*exp(2)^5+19320*exp(1)^4*exp(2)^6-8925*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-1680*exp(1)^10*exp(2)^3+5040*exp(
1)^8*exp(2)^4-6720*exp(1)^6*exp(2)^5+5040*exp(1)^4*exp(2)^6-1575*exp(2)^8)+
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(560*exp(1)^8
*exp(2)^4-1680*exp(1)^6*exp(2)^5+2380*exp(1)^4*exp(2)^6-1365*exp(2)^8)+(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-210*exp(1)^6*ex
p(2)^5+630*exp(1)^4*exp(2)^6-315*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^2*(84*exp(1)^4*exp(2)^6-105*exp(2)^8)+15*exp(2)
^8+35/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2))/d^3/
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7/exp(1)^9+1/56
2949953421312*(70368744177664*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^2*exp(1)^64*exp(2)^10-70368744177664/3*d^18*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^62*exp(2)^11+8796
093022208*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
4*exp(1)^60*exp(2)^12-17592186044416/5*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^58*exp(2)^13+4398046511104/3*d^18*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^56*exp(2)
^14-4398046511104/7*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^7*exp(1)^54*exp(2)^15-211106232532992*d^18*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^62*exp(2)^11+70368744177664*
d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6
0*exp(2)^12-26388279066624*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^4*exp(1)^58*exp(2)^13+52776558133248/5*d^18*(-1/2*(-2*d*ex
```

```
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^56*exp(2)^14+2814749
76710656*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(1)^60*exp(2)^12-299067162755072/3*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^58*exp(2)^13+13194139533312*d^18*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^56*exp(2)^
14-30786325577728/5*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(1)^54*exp(2)^15-211106232532992*d^18*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^58*exp(2)^13+8796093022080*
d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5
6*exp(2)^14+65970697666560*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^2*exp(1)^56*exp(2)^14-30786325577728*d^18*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^54*exp(2)^15+769658139
44320*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^54*exp(2)^15/
x/exp(2)-263882790666240*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
xp(1)^56*exp(2)^14/x/exp(2)+404620279021568*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))*exp(1)^58*exp(2)^13/x/exp(2)-457396837154816*d^18*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^60*exp(2)^12/x/exp(2)+527765581
332480*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^62*exp(2)^11
/x/exp(2)-422212465065984*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^64*exp(2)^10/x/exp(2)+140737488355328*d^18*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))*exp(1)^66*exp(2)^9/x/exp(2))/d^21/exp(1)^63/exp(2)^7+1/2
*(12*exp(1)^6*exp(2)^2-12*exp(1)^4*exp(2)^3+4*exp(2)^5-4*exp(1)^8*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/d^3/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/16*(-48*exp(1)^10*exp(
2)^2+56*exp(1)^8*exp(2)^3-40*exp(1)^6*exp(2)^4+30*exp(1)^4*exp(2)^5-13*exp(
2)^7+16*exp(1)^12*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/abs(x)/exp(2))/d^3/exp(1)^6/exp(1)
```

maple [B] time = 0.02, size = 546, normalized size = 3.17

$$\frac{e^7 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}\,d^2}+\frac{3e^8 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,d^3}-\frac{3e^8 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,d^3}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\right.}}{8d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] 1/4/d^7*e^8*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+3/8/d^5*e^8*(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)*x+3/8/d^3*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/5/d^5*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+1/6*e/
d^4/x^6*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-1/5/d^9*e
^6/x*(-e^2*x^2+d^2)^(7/2)-1/5/d^9*e^8*x*(-e^2*x^2+d^2)^(5/2)-1/4/d^7*e^8*x*
(-e^2*x^2+d^2)^(3/2)-3/8/d^5*e^8*x*(-e^2*x^2+d^2)^(1/2)-3/8/d^3*e^8/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+5/24/d^6*e^3/x^4*(-e^2*x^2+
d^2)^(7/2)+3/16/d^8*e^5/x^2*(-e^2*x^2+d^2)^(7/2)+1/16/(d^2)^(1/2)/d^2*e^7*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^8*e^7*(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(5/2)-1/80/d^8*e^7*(-e^2*x^2+d^2)^(5/2)-1/7/d^3/x^7*(-e^2*x^
2+d^2)^(7/2)-1/48*(-e^2*x^2+d^2)^(3/2)/d^6*e^7-1/16*(-e^2*x^2+d^2)^(1/2)/d^
4*e^7
```

maxima [A] time = 0.99, size = 203, normalized size = 1.18

$$\frac{e^7 \log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{16\,d^3}-\frac{\sqrt{-e^2x^2+d^2}\,e^7}{16\,d^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{16\,d^4x^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{35\,d^3x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{8\,d^2x^4}+\frac{3\left(-e^2x^{\cdots}\right.}{35\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 1/16*sqr
t(-e^2*x^2 + d^2)*e^7/d^4 - 1/16*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^2) + 2/3
5*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^
2*x^4) + 3/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^5) + 1/6*(-e^2*x^2 + d^2)^(3/
2)*e/x^6 - 1/7*(-e^2*x^2 + d^2)^(3/2)*d/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^8\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

sympy [C] time = 18.79, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

3.116 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=201

$$-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}+\frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}+\frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5}-\frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6}+\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}$$

```
[Out] -1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8+1/7*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6+2/35*e^
3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^
4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4}+\frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5}-\frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6}+\frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}-3e^8\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) + (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) + (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(8d^2e-3de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{\left(21d^3e^2-16d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{\left(96d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{336d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{\left(d^2-e^2x^2\right)^{3/}}{x^5}}{16d^3}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\,\text{Subst}\left(\int \right.}{3}$$

$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)}{35d^4x^5}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} +$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} +$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} +$$

Mathematica [A] time = 0.21, size = 139, normalized size = 0.69

$$\frac{-105e^8x^8\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-560d^7+640d^6ex+840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128\right.}{4480d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7) +
105*e^8*x^8*Log[x] - 105*e^8*x^8*Log[d + Sqrt[d^2 - e^2*x^2]])/(4480*d^4*x^
8)
```

fricas [A] time = 1.05, size = 130, normalized size = 0.65

$$\frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(256\,e^7x^7-105\,de^6x^6+128\,d^2e^5x^5-70\,d^3e^4x^4-1024\,d^4e^3x^3+840\,d^5e^2x^2+6\right.}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (256*e^7*x^7 - 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 - 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 + 840*d^5*
e^2*x^2 + 640*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/215
040*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(-107520
*exp(1)^14*exp(2)^2+322560*exp(1)^12*exp(2)^3-403200*exp(1)^10*exp(2)^4+349
440*exp(1)^8*exp(2)^5-309120*exp(1)^6*exp(2)^6+201600*exp(1)^4*exp(2)^7-588
00*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*
(26880*exp(1)^12*exp(2)^3-80640*exp(1)^10*exp(2)^4+107520*exp(1)^8*exp(2)^5
-107520*exp(1)^6*exp(2)^6+105840*exp(1)^4*exp(2)^7-52080*exp(2)^9)+(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-8960*exp(1)^10*exp(
2)^4+26880*exp(1)^8*exp(2)^5-38080*exp(1)^6*exp(2)^6+33600*exp(1)^4*exp(2)^
7-11760*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^4*(3360*exp(1)^8*exp(2)^5-10080*exp(1)^6*exp(2)^6+15120*exp(1)^4*exp(2)^
7-9240*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^3*(-1344*exp(1)^6*exp(2)^6+4032*exp(1)^4*exp(2)^7-2352*exp(2)^9)+(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(560*exp(1)^4*exp(2)^
7-560*exp(2)^9)+105*exp(2)^9+120*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)*exp(2)^9/x/exp(2))/d^4/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^8/exp(1)^10+1/18446744073709551616*(-2305843009213693952*d^28*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^82*exp(2)
^11+2305843009213693952/3*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^80*exp(2)^12-288230376151711744*d^28*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^78*exp(2)^13+576460
752303423488/5*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^5*exp(1)^76*exp(2)^14-144115188075855872/3*d^28*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^74*exp(2)^15+144115188075855
872/7*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*ex
```

```
p(1)^72*exp(2)^16-9007199254740992*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^8*exp(1)^70*exp(2)^17+6917529027641081856*d^28*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^80*exp(2)
^12-2305843009213693952*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^3*exp(1)^78*exp(2)^13+864691128455135232*d^28*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^76*exp(2)^14-17293822
56910270464/5*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^5*exp(1)^74*exp(2)^15+144115188075855872*d^28*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^72*exp(2)^16-922337203685477580
8*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^78*exp(2)^13+9799832789158199296/3*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(1)^76*exp(2)^14-1297036692682702848*d^28*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^74*exp(2
)^15+1008806316530991104/5*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^72*exp(2)^16-288230376151711744/3*d^28*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^70*exp(2)^17+922
3372036854775808*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^76*exp(2)^14-2882303761517117440*d^28*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^74*exp(2)^15+12970366926827
02848*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*ex
p(1)^72*exp(2)^16-9079256848778919936*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^74*exp(2)^15+1008806316530991104*d^28*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^72*exp
(2)^16-504403158265495552*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^4*exp(1)^70*exp(2)^17+6485183463413514240*d^28*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^72*exp(2)^16-20176
1263306198208*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^2*exp(1)^70*exp(2)^17-2522015791327477760*d^28*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(1)^72*exp(2)^16/x/exp(2)+8646911284551352320*d^28*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^74*exp(2)^15/x/exp(2)-13
258597302978740224*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^
76*exp(2)^14/x/exp(2)+14987979559889010688*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))*exp(1)^78*exp(2)^13/x/exp(2)-17293822569102704640*d^28*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^80*exp(2)^12/x/exp(2)+13835
058055282163712*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^82*
exp(2)^11/x/exp(2)-4611686018427387904*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(1)^84*exp(2)^10/x/exp(2))/d^32/exp(1)^80/exp(2)^8+1/2*(-12
*exp(1)^7*exp(2)^2+12*exp(1)^5*exp(2)^3-4*exp(1)^3*exp(2)^4+4*exp(1)^9*exp(
2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-
exp(1)^4+exp(2)^2))/d^4/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/128*(384*exp(1)^1
4*exp(2)^2-448*exp(1)^12*exp(2)^3+320*exp(1)^10*exp(2)^4-240*exp(1)^8*exp(2
)^5+208*exp(1)^6*exp(2)^6-184*exp(1)^4*exp(2)^7+85*exp(2)^9-128*exp(1)^16*e
xp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))
/d^4/exp(1)^9/exp(1)
```

maple [B] time = 0.02, size = 571, normalized size = 2.84

$$-\frac{3e^{8}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{128\sqrt{d^{2}}\,d^{3}}-\frac{3e^{9}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,d^{4}}+\frac{3e^{9}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}\,d^{4}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\right.}}{8d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x)
```

```
[Out] -1/4/d^8*e^9*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8/d^6*e^9*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)*x-3/8/d^4*e^9/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/5/d^6*e^3/x^5*(-e^2*x^2+d^2)^(7/2)-3/16/
d^5*e^2/x^6*(-e^2*x^2+d^2)^(7/2)+1/5/d^8*e^5/x^3*(-e^2*x^2+d^2)^(7/2)+1/5/d
```

```
^10*e^7/x*(-e^2*x^2+d^2)^(7/2)+1/5/d^10*e^9*x*(-e^2*x^2+d^2)^(5/2)+1/4/d^8*
e^9*x*(-e^2*x^2+d^2)^(3/2)+3/8/d^6*e^9*x*(-e^2*x^2+d^2)^(1/2)+3/8/d^4*e^9/(
e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-13/64/d^7*e^4/x^4*(-e
^2*x^2+d^2)^(7/2)-25/128/d^9*e^6/x^2*(-e^2*x^2+d^2)^(7/2)+1/7*e/d^4/x^7*(-e
^2*x^2+d^2)^(7/2)-3/128/(d^2)^(1/2)/d^3*e^8*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x)-1/5/d^9*e^8*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/8/d^3/x
^8*(-e^2*x^2+d^2)^(7/2)+3/640/d^9*e^8*(-e^2*x^2+d^2)^(5/2)+1/128*(-e^2*x^2+
d^2)^(3/2)/d^7*e^8+3/128*(-e^2*x^2+d^2)^(1/2)/d^5*e^8
```

maxima [A] time = 1.00, size = 228, normalized size = 1.13

$$-\frac{3\,e^{8}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{128\,d^{4}}+\frac{3\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}}{128\,d^{5}}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}{128\,d^{5}x^{2}}-\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}{35\,d^{4}x^{3}}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{64\,d^{3}x^{4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 3/128*(-e^2*x^2 + d^2)^(3/2)*e^6/(d^5*x^2) -
 2/35*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^3) + 3/64*(-e^2*x^2 + d^2)^(3/2)*e^
4/(d^3*x^4) - 3/35*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^5) + 1/16*(-e^2*x^2 +
d^2)^(3/2)*e^2/(d*x^6) + 1/7*(-e^2*x^2 + d^2)^(3/2)*e/x^7 - 1/8*(-e^2*x^2 +
 d^2)^(3/2)*d/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^9\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

sympy [C] time = 27.71, size = 1159, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
```

```
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

3.117 $\int \frac{x\sqrt{1-x^2}}{1+x} \, dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {785, 780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(1 + x),x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1-x^2}}{1+x} \, dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}} \, dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.96

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(1 + x),x]
[Out] (-1 + x/2)*Sqrt[1 - x^2] - ArcSin[x]/2
```

fricas [A] time = 0.81, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.16, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

maple [A] time = 0.01, size = 34, normalized size = 1.26

$$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{2x-(x+1)^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(1+x),x)
[Out] 1/2*(-x^2+1)^(1/2)*x-1/2*arcsin(x)-(-(1+x)^2+2+2*x)^(1/2)
```

maxima [A] time = 0.98, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(x + 1),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

sympy [A] time = 3.35, size = 29, normalized size = 1.07

$$\begin{cases} \frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\mathrm{asin}\,(x)}{2} & \text{for } x>-1 \wedge x<1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(1+x),x)
[Out] Piecewise((x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2, (x > -1) & (x <
 1)))
```

3.118 $\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)}\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a\sin^{-1}(ax)$$

```
[Out] -a*arcsin(a*x)-a*arctanh((-a^2*x^2+1)^(1/2))-(-a*x+1)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {850, 813, 844, 216, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] -(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)}\,dx &= \int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx\\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - \frac{1}{2}\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - a^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}\\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.96

$$\frac{\sqrt{1-a^2x^2}\,(ax-1)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] ((-1 + a*x)*Sqrt[1 - a^2*x^2])/x - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2*x
^2]]
```

fricas [A] time = 0.89, size = 74, normalized size = 1.45

$$\frac{2\,ax\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + ax + \sqrt{-a^2x^2+1}\,(ax-1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*x*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) + a*x + sqrt(-a^2*x^2 + 1)*(a*x - 1))/x
```

giac [B] time = 0.19, size = 125, normalized size = 2.45

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\sqrt{-a^2x^2+1}\,a-\frac{\sqrt{-a^2x^2+1}\,|a|+}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*arcsin(a*x)*sgn(a)
/abs(a) - a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))
/abs(a) + sqrt(-a^2*x^2 + 1)*a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs
(a))
```

maple [B] time = 0.02, size = 238, normalized size = 4.67

$$\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}\;a^2x}{2}-\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2x-\frac{3\sqrt{-a^2x^2+1}\;a^2x}{2}+\frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}\right)}{2\sqrt{a^2}}-\frac{3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x)
```

```
[Out] -1/3*a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/2*a^2*(-(x-1/a)^2*a^2-2*(x-1/a)
*a)^(1/2)*x+1/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*(x-1
/a)*a)^(1/2))-1/x*(-a^2*x^2+1)^(5/2)-a^2*x*(-a^2*x^2+1)^(3/2)-3/2*a^2*x*(-a
^2*x^2+1)^(1/2)-3/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(1/2)*x
)+1/3*a*(-a^2*x^2+1)^(3/2)+a*(-a^2*x^2+1)^(1/2)-a*arctanh(1/(-a^2*x^2+1)^(1
/2))
```

maxima [A] time = 0.99, size = 68, normalized size = 1.33

$$-a\arcsin(ax)-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}\,a-\frac{\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -a*arcsin(a*x) - a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*
x^2 + 1)*a - sqrt(-a^2*x^2 + 1)/x
```

mupad [B] time = 0.05, size = 74, normalized size = 1.45

$$a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x}-\frac{a^2\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}+a\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - a^2*x^2)^(3/2)/(x^2*(a*x - 1)),x)
```

```
[Out] a*atan((1 - a^2*x^2)^(1/2)*1i)*1i + a*(1 - a^2*x^2)^(1/2) - (1 - a^2*x^2)^(
1/2)/x - (a^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [C] time = 6.53, size = 170, normalized size = 3.33

$$a\left(\begin{cases} i\sqrt{a^2x^2-1} - \log(ax) + \frac{\log(a^2x^2)}{2} + i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2| > 1 \\ \sqrt{-a^2x^2+1} + \frac{\log(a^2x^2)}{2} - \log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}} + ia\,\mathrm{acosh}(ax) + \frac{i}{x\sqrt{a^2x^2-1}} & \text{f} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}} - a\,\mathrm{asin}(ax) - \frac{1}{x\sqrt{-a^2x^2+1}} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/x**2/(-a*x+1),x)
```

```
[Out] a*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True)) + Piecewise((-I*a**2*x/sqrt(a**2*x**2 -
 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**
2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), True))
```

3.119 $\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] -3/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+x^3*(-e*x+d)/e^2/(-e^2*x^2+d^
2)^(1/2)-4/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/6*d*(-9*e*x+16*d)*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 819, 833, 780, 217, 203}

$$-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}+\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^3*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) - (4*x^2*Sqrt[d^2 - e^2*x^2])/(3*
e^3) - (d*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} + \frac{\int \frac{x\left(8d^4e-9d^3e^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^2e^4} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{\left(3d^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^4} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{2e^4} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 91, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^3-7d^2ex+de^2x^2-2e^3x^3\right)-9d^3(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3) - 9*d^3*
(d + e*x)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(6*e^5*(d + e*x))
```

fricas [A] time = 0.62, size = 112, normalized size = 0.95

$$-\frac{16\,d^3ex+16\,d^4-18\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^3x^3-de^2x^2+7\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^3*e*x + 16*d^4 - 18*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (2*e^3*x^3 - d*e^2*x^2 + 7*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^6*x + d*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -3/2*
d^3*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^5-2*d^3*exp(2)*atan((-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/s
qrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)+2*((-16*exp(1)^13*1/96/exp(1)^16*x+
24*exp(1)^12*d*1/96/exp(1)^16)*x-80*exp(1)^11*d^2*1/96/exp(1)^16)*sqrt(-exp
(2)*x^2+d^2)
```

maple [A] time = 0.01, size = 147, normalized size = 1.25

$$-\frac{3d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3e^3}+\frac{\sqrt{-e^2x^2+d^2}\,dx}{2e^4}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{\left(x+\frac{d}{e}\right)e^6}-\frac{5\sqrt{-e^2x^2+d^2}}{3e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-5/3/e^5*d^2*(-e^2*x^2+d^2)^(1/2)+1/2*d/e^
4*x*(-e^2*x^2+d^2)^(1/2)-3/2*d^3/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x
^2+d^2)^(1/2)*x)-d^3/e^6/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 1.01, size = 113, normalized size = 0.96

$$-\frac{\sqrt{-e^2x^2+d^2}\,d^3}{e^6x+de^5}-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3\,e^3}-\frac{3\,d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^5}+\frac{\sqrt{-e^2x^2+d^2}\,dx}{2\,e^4}-\frac{5\,\sqrt{-e^2x^2+d^2}\,d^2}{3\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*d^3/(e^6*x + d*e^5) - 1/3*sqrt(-e^2*x^2 + d^2)*x^2/e^
3 - 3/2*d^3*arcsin(e*x/d)/e^5 + 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^4 - 5/3*sqrt
(-e^2*x^2 + d^2)*d^2/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.120 $\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

```
[Out] 3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2
)^(1/2)+1/2*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^2*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) + ((4*d - 3*e*x)*Sqrt[d^2 - e^2*x
^2])/(2*e^4) + (3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-e^2x^2\right)+3d^2(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2) + 3*d^2*(d + e*x)*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4*(d + e*x))
```

fricas [A] time = 0.90, size = 101, normalized size = 1.11

$$\frac{4\,d^2ex+4\,d^3-6\left(d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(e^2x^2-dex-4\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*d^2*e*x + 4*d^3 - 6*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (e^2*x^2 - d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 3/2*d
^2*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^4+2*d^2*exp(2)*atan((-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sq
```

```
rt(-exp(1)^4+exp(2)^2)/exp(1)^3/exp(1)+2*(-4*exp(1)^7*1/16/exp(1)^10*x+8*ex
p(1)^6*d*1/16/exp(1)^10)*sqrt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.01, size = 120, normalized size = 1.32

$$\frac{3d^2 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^3} - \frac{\sqrt{-e^2x^2+d^2}\,x}{2e^3} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}\,d^2}{\left(x+\frac{d}{e}\right)e^5} + \frac{\sqrt{-e^2x^2+d^2}\,d}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2/e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/
(-e^2*x^2+d^2)^(1/2)*x)+d/e^4*(-e^2*x^2+d^2)^(1/2)+d^2/e^5/(x+d/e)*(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 1.00, size = 86, normalized size = 0.95

$$\frac{\sqrt{-e^2x^2+d^2}\,d^2}{e^5x+de^4} + \frac{3\,d^2 \arcsin\left(\frac{ex}{d}\right)}{2\,e^4} - \frac{\sqrt{-e^2x^2+d^2}\,x}{2\,e^3} + \frac{\sqrt{-e^2x^2+d^2}\,d}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-e^2*x^2 + d^2)*d^2/(e^5*x + d*e^4) + 3/2*d^2*arcsin(e*x/d)/e^4 - 1/2*
sqrt(-e^2*x^2 + d^2)*x/e^3 + sqrt(-e^2*x^2 + d^2)*d/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.121 $\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-(-e^2*x^2+d^2)^(1/2)/e^3-d*(-e^2*x^
2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 12, 793, 217, 203}

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e^3) - (d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
```

```
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{de^3x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.77

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(2d+ex)}{d+ex}+d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((((2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + d*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/e^3)
```

fricas [A] time = 0.84, size = 85, normalized size = 1.10

$$-\frac{2\,dex+2\,d^2-2\left(dex+d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}\,(ex+2\,d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*d*e*x + 2*d^2 - 2*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(e^4*x + d*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -d*si
gn(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)-2*d*exp(2)*atan((-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt
(-exp(1)^4+exp(2)^2)/exp(1)/exp(2)-4*exp(1)^2*1/4/exp(1)^5*sqrt(-exp(2)*x^2
+d^2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.26

$$-\frac{d \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,d}{\left(x+\frac{d}{e}\right)e^4}-\frac{\sqrt{-e^2x^2+d^2}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^3-1/(e^2)^(1/2)*d/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-d/e^4/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.98, size = 63, normalized size = 0.82

$$-\frac{\sqrt{-e^2x^2+d^2}\,d}{e^4x+de^3}-\frac{d\arcsin\left(\frac{ex}{d}\right)}{e^3}-\frac{\sqrt{-e^2x^2+d^2}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*d/(e^4*x + d*e^3) - d*arcsin(e*x/d)/e^3 - sqrt(-e^2*x
^2 + d^2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.122 $\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}\\
&= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}\\
&= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex}+\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

fricas [A] time = 0.80, size = 67, normalized size = 1.29

$$\frac{ex-2\,(ex+d)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d+\sqrt{-e^2x^2+d^2}}{e^3x+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x - 2*(e*x + d)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d + sqrt(-e^
2*x^2 + d^2))/(e^3*x + d*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: sign(
d)*asin(x*exp(2)/d/exp(1))/exp(1)^2+2*exp(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4
+exp(2)^2)/exp(1)^2
```

maple [A] time = 0.01, size = 74, normalized size = 1.42

$$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{\left(x+\frac{d}{e}\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/e^3/(x+d/e)*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.97, size = 40, normalized size = 0.77

$$\frac{\sqrt{-e^2x^2+d^2}}{e^3x+de^2}+\frac{\arcsin\left(\frac{ex}{d}\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-e^2*x^2 + d^2)/(e^3*x + d*e^2) + arcsin(e*x/d)/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.123 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.03

$$-\frac{\sqrt{d^2-e^2x^2}}{d^2e+de^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d^2*e + d*e^2*x))
```

fricas [A] time = 0.60, size = 35, normalized size = 1.13

$$-\frac{ex+d+\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -2*ex
p(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(
-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/d/exp(1)
```

maple [A] time = 0.01, size = 29, normalized size = 0.94

$$-\frac{-ex+d}{\sqrt{-e^2x^2+d^2}\ de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.98, size = 30, normalized size = 0.97

$$-\frac{\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)/(d*e^2*x + d^2*e)
```

mupad [B] time = 2.64, size = 29, normalized size = 0.94

$$-\frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] -(d^2 - e^2*x^2)^(1/2)/(d*e*(d + e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\ \left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.124 $\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+(-e^2*x^2+d^2)^(1/2)/d^2/(e*x+d)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {857, 12, 266, 63, 208}

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\int \frac{de^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{de^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.96

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex} - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

fricas [A] time = 1.12, size = 62, normalized size = 1.15

$$\frac{ex + (ex+d)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + d + \sqrt{-e^2x^2+d^2}}{d^2ex + d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + (e*x + d)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + d + sqrt(-e^2*x^2 + d
^2))/(d^2*e*x + d^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -exp(
2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^2
/exp(1)^2+2*exp(1)*exp(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^2/sqrt(-exp(1)^4+exp(2)^2)/exp(
1)
```

maple [A] time = 0.01, size = 88, normalized size = 1.63

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}}{\left(x+\frac{d}{e}\right)d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^2/e/(
x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}\,(ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{d^2 - e^2\,x^2}\,\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-(-d + ex)(d + ex)}\,\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.125 $\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-2*(-e^2*x^2+d^2)^(1/2)/d^3/x+(-e^2*x^
2+d^2)^(1/2)/d^2/x/(e*x+d)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {857, 807, 266, 63, 208}

$$-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(d^3*x) + Sqrt[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) +
(e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
```

```
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{\int \frac{-2de^2+e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.77

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{(d+2ex)\sqrt{d^2-e^2x^2}}{x(d+ex)}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-(((d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x))) + e*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/d^3
```

fricas [A] time = 0.85, size = 88, normalized size = 1.09

$$-\frac{e^2x^2 + dex + \left(e^2x^2 + dex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2+d^2}\,(2\,ex + d)}{d^3ex^2 + d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e^2*x^2 + d*e*x + (e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) +
sqrt(-e^2*x^2 + d^2)*(2*e*x + d))/(d^3*e*x^2 + d^4*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -x*ex
p(2)^3/d^3/(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/exp(1)/exp(2)-2*exp(
```

```
2)^2*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(
-exp(1)^4+exp(2)^2))/d^3/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/4*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/d^3/x/exp(1)/exp(2)^3+exp(2)*ln(1/2*
abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)
```

maple [A] time = 0.01, size = 108, normalized size = 1.33

$$\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{\left(x+\frac{d}{e}\right)d^3}-\frac{\sqrt{-e^2x^2+d^2}}{d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^3/x+e/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)-1/d^3/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.126 $\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

```
[Out] -3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-3/2*(-e^2*x^2+d^2)^(1/2)/d^3/x
^2+2*e*(-e^2*x^2+d^2)^(1/2)/d^4/x+(-e^2*x^2+d^2)^(1/2)/d^2/x^2/(e*x+d)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 835, 807, 266, 63, 208}

$$\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^4*x) +
Sqrt[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/(2*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{\int \frac{-3de^2+2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\int \frac{-4d^2e^3+3de^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{(3e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{(3e^2)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x\right.}{4d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3\operatorname{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-\right.}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Mathematica [A] time = 0.32, size = 127, normalized size = 1.12

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4} - \frac{d^3+de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)-2d^2ex-3de^2x^2+4e^3x^3}{2d^4x^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4) - (d^3 - 2*d^2*e*x - 3*d*e^2*x^
2 + 4*e^3*x^3 + d*e^2*x^2*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2
)/d^2]])/(2*d^4*x^2*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 0.82, size = 113, normalized size = 1.00

$$\frac{2e^3x^3+2de^2x^2+3\left(e^3x^3+de^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(4e^2x^2+dex-d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(d^4ex^3+d^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^3*x^3 + 2*d*e^2*x^2 + 3*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (4*e^2*x^2 + d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^3
 + d^5*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/8*(
exp(2)^3+2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/x/exp(2))/d
^4/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2/exp(1)^4+1
/16*(-2*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^4*exp(2)^5-4*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6
*exp(2)^4/x/exp(2))/d^8/exp(1)^6/exp(2)^3+1/2*(-exp(2)^3-2*exp(1)^4*exp(2))
*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/e
xp(1)^3/exp(1)+2*exp(1)^3*exp(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^4/sqrt(-exp(1)^4+exp(2)^
2)/exp(1)
```

maple [A] time = 0.01, size = 133, normalized size = 1.18

$$-\frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e}{\left(x+\frac{d}{e}\right)d^4}+\frac{\sqrt{-e^2x^2+d^2}\,e}{d^4x}-\frac{\sqrt{-e^2x^2+d^2}}{2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] e*(-e^2*x^2+d^2)^(1/2)/d^4/x-1/2*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-3/2/d^3*e^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^4*e/(x+d/e)
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.127 $\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-5/2*d^2*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))/e^6-1/3*x^2*(-5*e*x+4*d)/e^4/(-e^2*x^2+d^2)^(1/2)-1/6*(-15*e*x+16*d
)*(-e^2*x^2+d^2)^(1/2)/e^6
```

Rubi [A] time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*
Sqrt[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^6) - (5*d
^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{\left(5d^2\right)\int \frac{1}{\sqrt{d^2-e}}}{2e^5}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{\left(5d^2\right)\text{Subst}\left(\right.}{}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\tan^{-1}\left(\frac{}{\sqrt{d}}\right.}{2e^6}$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.83

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(16d^4+d^3ex-23d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{(ex-d)(d+ex)^2}-15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(16*d^4 + d^3*e*x - 23*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/((-d + e*x)*(d + e*x)^2) - 15*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(6*e^6)
```

fricas [A] time = 0.82, size = 190, normalized size = 1.48

$$-\frac{16\,d^2e^3x^3+16\,d^3e^2x^2-16\,d^4ex-16\,d^5-30\left(d^2e^3x^3+d^3e^2x^2-d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(3\,e^4x^4\right.}{6\left(e^9x^3+de^8x^2-d^2e^7x-d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 - 16*d^4*e*x - 16*d^5 - 30*(d^2*e^3*x
^3 + d^3*e^2*x^2 - d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
```

```
 - (3*e^4*x^4 - 3*d*e^3*x^3 - 23*d^2*e^2*x^2 + d^3*e*x + 16*d^4)*sqrt(-e^2*
x^2 + d^2))/(e^9*x^3 + d*e^8*x^2 - d^2*e^7*x - d^3*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 208, normalized size = 1.62

$$-\frac{x^3}{2\sqrt{-e^2x^2+d^2}\,e^3}+\frac{d\,x^2}{\sqrt{-e^2x^2+d^2}\,e^4}+\frac{7d^2x}{2\sqrt{-e^2x^2+d^2}\,e^5}-\frac{2d^2x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^5}-\frac{5d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/2/e^3*x^3/(-e^2*x^2+d^2)^(1/2)+7/2/(-e^2*x^2+d^2)^(1/2)*d^2/e^5*x-5/2/(e
^2)^(1/2)*d^2/e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+d/e^4*x^2/(-e^
2*x^2+d^2)^(1/2)-3*d^3/e^6/(-e^2*x^2+d^2)^(1/2)+1/3*d^4/e^7/(x+d/e)/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(1/2)-2/3*d^2/e^5/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2
)*x
```

maxima [A] time = 1.01, size = 151, normalized size = 1.18

$$\frac{d^4}{3\left(\sqrt{-e^2x^2+d^2}\,e^7x+\sqrt{-e^2x^2+d^2}\,de^6\right)}-\frac{x^3}{2\sqrt{-e^2x^2+d^2}\,e^3}+\frac{dx^2}{\sqrt{-e^2x^2+d^2}\,e^4}+\frac{17\,d^2x}{6\sqrt{-e^2x^2+d^2}\,e^5}-\frac{5\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*d^4/(sqrt(-e^2*x^2 + d^2)*e^7*x + sqrt(-e^2*x^2 + d^2)*d*e^6) - 1/2*x^3
/(sqrt(-e^2*x^2 + d^2)*e^3) + d*x^2/(sqrt(-e^2*x^2 + d^2)*e^4) + 17/6*d^2*x
/(sqrt(-e^2*x^2 + d^2)*e^5) - 5/2*d^2*arcsin(e*x/d)/e^6 - 3*d^3/(sqrt(-e^2*
x^2 + d^2)*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.128 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x^3*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)+d*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e^5-1/3*x*(-4*e*x+3*d)/e^4/(-e^2*x^2+d^2)^(1/2)+8/3*(-e^2*x^2+d^2)^(1/2)/
e^5
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 641, 217, 203}

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*Sq
rt[d^2 - e^2*x^2]) + (8*Sqrt[d^2 - e^2*x^2])/(3*e^5) + (d*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{3d^5-8d^4ex}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \cdot\right)}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] time = 0.14, size = 93, normalized size = 0.82

$$\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(8d^3+5d^2ex-7de^2x^2-3e^3x^3\right)}{(d-ex)(d+ex)^2}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/((d -
e*x)*(d + e*x)^2) + 3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^5)
```

fricas [A] time = 0.91, size = 175, normalized size = 1.55

$$\frac{8\,de^3x^3 + 8\,d^2e^2x^2 - 8\,d^3ex - 8\,d^4 - 6\left(de^3x^3 + d^2e^2x^2 - d^3ex - d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(3\,e^3x^3 + 7\,de^2x^2 \cdots\right.}{3\left(e^8x^3 + de^7x^2 - d^2e^6x - d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*d*e^3*x^3 + 8*d^2*e^2*x^2 - 8*d^3*e*x - 8*d^4 - 6*(d*e^3*x^3 + d^2*e
^2*x^2 - d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^3*
x^3 + 7*d*e^2*x^2 - 5*d^2*e*x - 8*d^3)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + d*e
^7*x^2 - d^2*e^6*x - d^3*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 179, normalized size = 1.58

$$-\frac{x^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{3}}-\frac{2 d x}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{4}}+\frac{2 d x}{3 \sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, e^{4}}+\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\, e^{4}}-\frac{}{3 \left(x+\frac{d}{e}\right) \sqrt{2 \left(x+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/e^3*x^2/(-e^2*x^2+d^2)^(1/2)+3*d^2/e^5/(-e^2*x^2+d^2)^(1/2)-2/(-e^2*x^2+
d^2)^(1/2)*d/e^4*x+1/(e^2)^(1/2)*d/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1
/2)*x)-1/3*d^3/e^6/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+2/3*d/e^4/(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [A] time = 1.01, size = 124, normalized size = 1.10

$$-\frac{d^{3}}{3 \left(\sqrt{-e^{2} x^{2}+d^{2}}\, e^{6} x+\sqrt{-e^{2} x^{2}+d^{2}}\, d e^{5}\right)}-\frac{x^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{3}}-\frac{4\, d x}{3 \sqrt{-e^{2} x^{2}+d^{2}}\, e^{4}}+\frac{d \arcsin\left(\frac{e x}{d}\right)}{e^{5}}+\frac{3\, d^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*d^3/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt(-e^2*x^2 + d^2)*d*e^5) - x^2/(s
qrt(-e^2*x^2 + d^2)*e^3) - 4/3*d*x/(sqrt(-e^2*x^2 + d^2)*e^4) + d*arcsin(e*
x/d)/e^5 + 3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4}}{\left(d^{2}-e^{2}\, x^{2}\right)^{3/2}\, \left(d+e\, x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(-\left(-d+e x\right) \left(d+e x\right)\right)^{\frac{3}{2}} \left(d+e x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.129 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/3*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/
e^4+1/3*(3*e*x-2*d)/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 217, 203}

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*Sqrt[d
^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\ &= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}\\ &= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}\\ &= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\\ &= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}\end{aligned}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.90

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-2d^2+dex+4e^2x^2\right)}{(d-ex)(d+ex)^2}-3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/((d - e*x)*(d + e*x)^2)
 - 3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^4)
```

fricas [A] time = 0.81, size = 157, normalized size = 1.76

$$-\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3-6\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^2x^2+dex-2\,d^2\right)\sqrt{-}}{3\left(e^7x^3+de^6x^2-d^2e^5x-d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 - 6*(e^3*x^3 + d*e^2*x^2
- d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^2*x^2 + d
*e*x - 2*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + d*e^6*x^2 - d^2*e^5*x - d^3*
e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 153, normalized size = 1.72

$$\frac{2x}{\sqrt{-e^2x^2+d^2}\,e^3}-\frac{2x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^3}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^3}+\frac{d^2}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 2/(-e^2*x^2+d^2)^(1/2)/e^3*x-1/(e^2)^(1/2)/e^3*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)-d/e^4/(-e^2*x^2+d^2)^(1/2)+1/3*d^2/e^5/(x+d/e)/(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)-2/3/e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [A] time = 1.00, size = 99, normalized size = 1.11

$$\frac{d^2}{3\left(\sqrt{-e^2x^2+d^2}\,e^5x+\sqrt{-e^2x^2+d^2}\,de^4\right)}+\frac{4\,x}{3\,\sqrt{-e^2x^2+d^2}\,e^3}-\frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}-\frac{d}{\sqrt{-e^2x^2+d^2}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2*x^2 + d^2)*d*e^4) + 4/3*x/(
sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e*x/d)/e^4 - d/(sqrt(-e^2*x^2 + d^2)*e^4
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.130 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/3/e^3/(-e^2*x^2+d^2)^(1/2)-1/3*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {855, 12, 261}

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] 2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{2dx}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3de}\\
&= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\
&= \frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.00

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^2+2dex-e^2x^2\right)}{3de^3(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d + e
*x)^2)
```

fricas [A] time = 0.67, size = 103, normalized size = 1.72

$$\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3+\left(e^2x^2-2\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(de^6x^3+d^2e^5x^2-d^3e^4x-d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 + (e^2*x^2 - 2*d*e*x - 2*d
^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + d^2*e^5*x^2 - d^3*e^4*x - d^4*e^3)
```

giac [A] time = 0.24, size = 1, normalized size = 0.02

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{(-ex+d)\left(-e^2x^2+2dex+2d^2\right)}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/3*(-e*x+d)*(-e^2*x^2+2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(3/2)
```

maxima [A] time = 0.46, size = 86, normalized size = 1.43

$$-\frac{d}{3\left(\sqrt{-e^2x^2+d^2}\,e^4x+\sqrt{-e^2x^2+d^2}\,de^3\right)}-\frac{x}{3\sqrt{-e^2x^2+d^2}\,de^2}+\frac{1}{\sqrt{-e^2x^2+d^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*d/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x^2 + d^2)*d*e^3) - 1/3*x/(s
qrt(-e^2*x^2 + d^2)*d*e^2) + 1/(sqrt(-e^2*x^2 + d^2)*e^3)
```

mupad [B] time = 2.71, size = 56, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+2\,d\,e\,x-e^2\,x^2\right)}{3\,d\,e^3\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 - e^2*x^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)^2*(d
 - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-(-d+ex)(d+ex))^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.131 $\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x/d^2/e/(-e^2*x^2+d^2)^(1/2)+1/3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {793, 191}

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] x/(3*d^2*e*Sqrt[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(d^2+dex+e^2x^2\right)}{3d^2e^2(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x
)^2)
```

fricas [B] time = 1.05, size = 101, normalized size = 1.74

$$\frac{e^3x^3 + de^2x^2 - d^2ex - d^3 - \left(e^2x^2 + dex + d^2\right)\sqrt{-e^2x^2 + d^2}}{3\left(d^2e^5x^3 + d^3e^4x^2 - d^4e^3x - d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 - (e^2*x^2 + d*e*x + d^2)*sqrt(-e^
2*x^2 + d^2))/(d^2*e^5*x^3 + d^3*e^4*x^2 - d^4*e^3*x - d^5*e^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.01, size = 44, normalized size = 0.76

$$\frac{(-ex + d)\left(e^2x^2 + dex + d^2\right)}{3\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/3*(-e*x+d)*(e^2*x^2+d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(3/2)
```

maxima [A] time = 0.46, size = 67, normalized size = 1.16

$$\frac{1}{3\left(\sqrt{-e^2x^2 + d^2}\, e^3x + \sqrt{-e^2x^2 + d^2}\, de^2\right)} + \frac{x}{3\sqrt{-e^2x^2 + d^2}\, d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3/(sqrt(-e^2*x^2 + d^2)*e^3*x + sqrt(-e^2*x^2 + d^2)*d*e^2) + 1/3*x/(sqrt
(-e^2*x^2 + d^2)*d^2*e)
```

mupad [B] time = 2.71, size = 52, normalized size = 0.90

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(d^2 + d\,e\,x + e^2\,x^2\right)}{3\,d^2\,e^2\,{\left(d + e\,x\right)}^2\,\left(d - e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + d*e*x))/(3*d^2*e^2*(d + e*x)^2*(d -
 e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.132 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

[Out] 2/3*x/d^3/(-e^2*x^2+d^2)^(1/2)-1/3/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 191}

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]

[Out] (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d}\\ &= \frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$-\frac{\left(d^2-2dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{3d^3e(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]

[Out] -1/3*((d^2 - 2*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(d^3*e*(d - e*x)*(d + e*x)^2)

fricas [B] time = 0.56, size = 102, normalized size = 1.76

$$-\frac{e^3x^3 + de^2x^2 - d^2ex - d^3 + \left(2\,e^2x^2 + 2\,dex - d^2\right)\sqrt{-e^2x^2 + d^2}}{3\left(d^3e^4x^3 + d^4e^3x^2 - d^5e^2x - d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 + (2*e^2*x^2 + 2*d*e*x - d^2)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + d^4*e^3*x^2 - d^5*e^2*x - d^6*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.01, size = 46, normalized size = 0.79

$$-\frac{(-ex + d)\left(-2e^2x^2 - 2dex + d^2\right)}{3\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)
```

maxima [A] time = 0.45, size = 65, normalized size = 1.12

$$-\frac{1}{3\left(\sqrt{-e^2x^2 + d^2}\,de^2x + \sqrt{-e^2x^2 + d^2}\,d^2e\right)} + \frac{2\,x}{3\,\sqrt{-e^2x^2 + d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e) + 2/3*x/(s
qrt(-e^2*x^2 + d^2)*d^3)
```

mupad [B] time = 2.71, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(-d^2 + 2\,d\,e\,x + 2\,e^2\,x^2\right)}{3\,d^3\,e\,(d + e\,x)^2\,(d - e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^2*x^2 - d^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)^2*(d
 - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.133 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] -arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/3*(-2*e*x+3*d)/d^4/(-e^2*x^2+d^2)^(1
/2)+1/3/d^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 12, 266, 63, 208}

$$\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (3*d - 2*e*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*(d + e*x)*Sqrt[d^2 - e
^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-3de^2+2e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{3d^3e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.94

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-2e^2x^2\right)}{(d-ex)(d+ex)^2} - 3\log\left(\sqrt{d^2-e^2x^2}+d\right) + 3\log(x)}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (((4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/((d - e*x)*(d + e*x)^2)
+ 3*Log[x] - 3*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^4)
```

fricas [A] time = 0.90, size = 155, normalized size = 1.76

$$\frac{4e^3x^3+4de^2x^2-4d^2ex-4d^3+3\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(2e^2x^2-dex-4d^2\right)\sqrt{-e^2x^2}}{3\left(d^4e^3x^3+d^5e^2x^2-d^6ex-d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*e^3*x^3 + 4*d*e^2*x^2 - 4*d^2*e*x - 4*d^3 + 3*(e^3*x^3 + d*e^2*x^2 -
 d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (2*e^2*x^2 - d*e*x - 4
*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + d^5*e^2*x^2 - d^6*e*x - d^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 142, normalized size = 1.61

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3}-\frac{2ex}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4}+\frac{1}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2e}+\frac{}{\sqrt{-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/(-e^2*x^2+d^2)^(1/2)/d^3-1/(d^2)^(1/2)/d^3*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+1/3/d^2/e/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-2/
3/d^4*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.134 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/3*(-3*e*x+4*d)/d^4/x/(-e^2*x^2+d^2)
^(1/2)+1/3/d^2/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-8/3*(-e^2*x^2+d^2)^(1/2)/d^5/
x
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 807, 266, 63, 208}

$$\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*d - 3*e*x)/(3*d^4*x*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*Sqrt[d^2
 - e^2*x^2]) - (8*Sqrt[d^2 - e^2*x^2])/(3*d^5*x) + (e*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/d^5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-4de^2+3e^3x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-8d^3e^4+3d^2e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d}{d^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-}\right.}{}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\tanh^{-1}\left(\sqrt{d^2}\right.}{d^5}$$

Mathematica [A] time = 0.12, size = 101, normalized size = 0.84

$$\frac{3e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(3d^3+7d^2ex-5de^2x^2-8e^3x^3\right)}{x(ex-d)(d+ex)^2}-3e\log(x)}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*(-d
 + e*x)*(d + e*x)^2) - 3*e*Log[x] + 3*e*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^
5)
```

fricas [A] time = 0.65, size = 181, normalized size = 1.51

$$-\frac{4\,e^4x^4+4\,de^3x^3-4\,d^2e^2x^2-4\,d^3ex+3\left(e^4x^4+de^3x^3-d^2e^2x^2-d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^3x^3+5\,de^2x^2-\right.}{3\left(d^5e^3x^4+d^6e^2x^3-d^7ex^2-d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*e^4*x^4 + 4*d*e^3*x^3 - 4*d^2*e^2*x^2 - 4*d^3*e*x + 3*(e^4*x^4 + d*
e^3*x^3 - d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^
3*x^3 + 5*d*e^2*x^2 - 7*d^2*e*x - 3*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^4
 + d^6*e^2*x^3 - d^7*e*x^2 - d^8*x)
```

giac [A] time = 0.25, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.02, size = 188, normalized size = 1.57

$$\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}+\frac{2e^2x}{\sqrt{-e^2x^2+d^2}\,d^5}+\frac{2e^2x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5}-\frac{1}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/d^3/x/(-e^2*x^2+d^2)^(1/2)+2/(-e^2*x^2+d^2)^(1/2)/d^5*e^2*x-1/(-e^2*x^2+
d^2)^(1/2)/d^4*e+1/(d^2)^(1/2)/d^4*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-1/3/d^3/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+2/3*e^2/d^5/
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.135 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}$$

```
[Out] -5/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/3*(-4*e*x+5*d)/d^4/x^2/(-e^2
*x^2+d^2)^(1/2)+1/3/d^2/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-5/2*(-e^2*x^2+d^2)
^(1/2)/d^5/x^2+8/3*e*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (5*d - 4*e*x)/(3*d^4*x^2*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*Sqrt
[d^2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*Sqrt[d^2 - e^
2*x^2])/(3*d^6*x) - (5*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-5de^2+4e^3x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-16d^4e^5+15\cdots}{x^2\sqrt{d^2-e\cdots}}}{6d^8e\cdots}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2\cdots}}{3d^6x}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2\cdots}}{3d^6x}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2\cdots}}{3d^6x}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2\cdots}}{3d^6x}$$

Mathematica [A] time = 0.10, size = 115, normalized size = 0.76

$$\frac{-15e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(3d^4-3d^3ex-23d^2e^2x^2+de^3x^3+16e^4x^4\right)}{x^2(ex-d)(d+ex)^2}+15e^2\log(x)}{6d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 + 16*
e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) + 15*e^2*Log[x] - 15*e^2*Log[d + Sqr
t[d^2 - e^2*x^2]])/(6*d^6)
```

fricas [A] time = 0.87, size = 201, normalized size = 1.32

$$\frac{14\,e^5x^5+14\,de^4x^4-14\,d^2e^3x^3-14\,d^3e^2x^2+15\left(e^5x^5+de^4x^4-d^2e^3x^3-d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^4x^4+\right.}{6\left(d^6e^3x^5+d^7e^2x^4-d^8ex^3-d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(14*e^5*x^5 + 14*d*e^4*x^4 - 14*d^2*e^3*x^3 - 14*d^3*e^2*x^2 + 15*(e^5*
x^5 + d*e^4*x^4 - d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (16*e^4*x^4 + d*e^3*x^3 - 23*d^2*e^2*x^2 - 3*d^3*e*x + 3*d^4)*sqrt(-
e^2*x^2 + d^2))/(d^6*e^3*x^5 + d^7*e^2*x^4 - d^8*e*x^3 - d^9*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 216, normalized size = 1.42

$$-\frac{5e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^5}-\frac{2e^3x}{\sqrt{-e^2x^2+d^2}\,d^6}-\frac{2e^3x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^6}+\frac{e}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] e/d^4/x/(-e^2*x^2+d^2)^(1/2)-2/(-e^2*x^2+d^2)^(1/2)/d^6*e^3*x-1/2/d^3/x^2/(
-e^2*x^2+d^2)^(1/2)+5/2/(-e^2*x^2+d^2)^(1/2)/d^5*e^2-5/2/(d^2)^(1/2)/d^5*e^
2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/3/d^4*e/(x+d/e)/(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-2/3/d^6*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/
2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(d^2-e^2x^2\right)^{3/2}(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.136 $\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^6*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(-7*e*x+6*d)/e^4/(-e^2*x
^2+d^2)^(3/2)+7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(-35*e*
x+24*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e
^8
```

Rubi [A] time = 0.16, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^6*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d - 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d - 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d - 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) + (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3-7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5-35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7-105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,d}{15d^6e^6}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2}}{10e^8}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2}}{10e^8}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2}}{10e^8}$$

Mathematica [A] time = 0.24, size = 128, normalized size = 0.79

$$\frac{105d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6\right)}{(d-ex)^2(d+ex)^3}}{30e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^3
 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3)
+ 105*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8)
```

fricas [A] time = 0.92, size = 274, normalized size = 1.69

$$\frac{96\,d^2e^5x^5+96\,d^3e^4x^4-192\,d^4e^3x^3-192\,d^5e^2x^2+96\,d^6ex+96\,d^7-210\left(d^2e^5x^5+d^3e^4x^4-2\,d^4e^3x^3-2\,d^5e^2x\right.}{30\left(e^{13}x^5+de^{12}x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 + 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 - 192*d^5*e^2*x^2 +
 96*d^6*e*x + 96*d^7 - 210*(d^2*e^5*x^5 + d^3*e^4*x^4 - 2*d^4*e^3*x^3 - 2*d
^5*e^2*x^2 + d^6*e*x + d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15
*e^6*x^6 - 15*d*e^5*x^5 - 176*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 + 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 + d*e^12*x^4 - 2*d^2
*e^11*x^3 - 2*d^3*e^10*x^2 + d^4*e^9*x + d^5*e^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.05, size = 318, normalized size = 1.96

$$-\frac{x^{5}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}+\frac{d\,x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}+\frac{7d^{2}x^{3}}{6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{5d^{3}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}+\frac{2d^{4}x}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}-\frac{}{15\left(2\left(x+\frac{d}{e}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/2/e^3*x^5/(-e^2*x^2+d^2)^(3/2)+7/6/(-e^2*x^2+d^2)^(3/2)*d^2/e^5*x^3-19/6
/(-e^2*x^2+d^2)^(1/2)*d^2/e^7*x+7/2/(e^2)^(1/2)*d^2/e^7*arctan((e^2)^(1/2)/
(-e^2*x^2+d^2)^(1/2)*x)+d/e^4*x^4/(-e^2*x^2+d^2)^(3/2)-5*d^3/e^6*x^2/(-e^2*
x^2+d^2)^(3/2)+3*d^5/e^8/(-e^2*x^2+d^2)^(3/2)+2/3*d^4/e^7*x/(-e^2*x^2+d^2)^
(3/2)+1/5*d^6/e^9/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-4/15*d^4/e^7/
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-8/15*d^2/e^7/(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(1/2)*x
```

maxima [B] time = 1.09, size = 289, normalized size = 1.78

$$\frac{d^{6}}{5\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{9}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}de^{8}\right)}-\frac{x^{5}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}+\frac{dx^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}+\frac{25\,d^{2}x^{3}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{65\,d^{3}x}{6\left(-e^{2}x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^6/((-e^2*x^2 + d^2)^(3/2)*e^9*x + (-e^2*x^2 + d^2)^(3/2)*d*e^8) - 1/2
*x^5/((-e^2*x^2 + d^2)^(3/2)*e^3) + d*x^4/((-e^2*x^2 + d^2)^(3/2)*e^4) + 25
/2*d^2*x^3/((-e^2*x^2 + d^2)^(3/2)*e^5) - 65/6*d^3*x^2/((-e^2*x^2 + d^2)^(3
/2)*e^6) - 164/15*d^4*x/((-e^2*x^2 + d^2)^(3/2)*e^7) - 7/6*d*x^2/(sqrt(-e^2
*x^2 + d^2)*e^6) + 53/6*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8) + 229/30*d^2*x/(sq
rt(-e^2*x^2 + d^2)*e^7) + 7/2*d^2*arcsin(e*x/d)/e^8 - 14/3*d^3/(sqrt(-e^2*x
^2 + d^2)*e^8) - 7/6*sqrt(-e^2*x^2 + d^2)*d/e^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(x**7/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.137 $\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^5*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(-6*e*x+5*d)/e^4/(-e^2*x
^2+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(-8*e*x+5*d)/e^6
/(-e^2*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 641, 217, 203}

$$\frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^5*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d - 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d - 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) - (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4\left(5d^3-6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(15d^5-24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7-48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d}{\ldots}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d}{\ldots}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d}{\ldots}$$

Mathematica [A] time = 0.19, size = 115, normalized size = 0.78

$$-\frac{15d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(48d^5+33d^4ex-87d^3e^2x^2-52d^2e^3x^3+38de^4x^4+15e^5x^5\right)}{(d-ex)^2(d+ex)^3}}{15e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(48*d^5 + 33*d^4*e*x - 87*d^3*e^2*x^2 - 52*d^2*
e^3*x^3 + 38*d*e^4*x^4 + 15*e^5*x^5))/((d - e*x)^2*(d + e*x)^3) + 15*d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

fricas [A] time = 0.88, size = 258, normalized size = 1.74

$$-\frac{48\,de^5x^5 + 48\,d^2e^4x^4 - 96\,d^3e^3x^3 - 96\,d^4e^2x^2 + 48\,d^5ex + 48\,d^6 - 30\left(de^5x^5 + d^2e^4x^4 - 2\,d^3e^3x^3 - 2\,d^4e^2x^2 + \ldots\right.}{15\left(e^{12}x^5 + de^{11}x^4 - 2\,d^2e^{1}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(48*d*e^5*x^5 + 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 - 96*d^4*e^2*x^2 + 48
*d^5*e*x + 48*d^6 - 30*(d*e^5*x^5 + d^2*e^4*x^4 - 2*d^3*e^3*x^3 - 2*d^4*e^2
*x^2 + d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x
^5 + 38*d*e^4*x^4 - 52*d^2*e^3*x^3 - 87*d^3*e^2*x^2 + 33*d^4*e*x + 48*d^5)*
sqrt(-e^2*x^2 + d^2))/(e^12*x^5 + d*e^11*x^4 - 2*d^2*e^10*x^3 - 2*d^3*e^9*x
^2 + d^4*e^8*x + d^5*e^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 288, normalized size = 1.95

$$-\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^3}-\frac{d\,x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^4}+\frac{5d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^5}-\frac{2d^3x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^6}+\frac{4d^3x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/e^3*x^4/(-e^2*x^2+d^2)^(3/2)+5/e^5*d^2*x^2/(-e^2*x^2+d^2)^(3/2)-3*d^4/e^
7/(-e^2*x^2+d^2)^(3/2)-1/3/(-e^2*x^2+d^2)^(3/2)*d/e^4*x^3+2/3/(-e^2*x^2+d^2
)^(1/2)*d/e^6*x-1/(e^2)^(1/2)*d/e^6*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)-2/3*d^3/e^6*x/(-e^2*x^2+d^2)^(3/2)-1/5*d^5/e^8/(x+d/e)/(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(3/2)+4/15*d^3/e^6/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+8/15
*d/e^6/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [A] time = 1.09, size = 259, normalized size = 1.75

$$-\frac{d^5}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^8 x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d e^7\right)}-\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^3}-\frac{5\,dx^3}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^4}+\frac{20\,d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^5}+\frac{64\,d^3}{15\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8*x + (-e^2*x^2 + d^2)^(3/2)*d*e^7) - x^
4/((-e^2*x^2 + d^2)^(3/2)*e^3) - 5*d*x^3/((-e^2*x^2 + d^2)^(3/2)*e^4) + 20/
3*d^2*x^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 64/15*d^3*x/((-e^2*x^2 + d^2)^(3/2
)*e^6) + 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*e^5) - 14/3*d^4/((-e^2*x^2 + d^2)^(3
/2)*e^7) - 52/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) - d*arcsin(e*x/d)/e^7 + 4/3
*d^2/(sqrt(-e^2*x^2 + d^2)*e^7) + 1/3*sqrt(-e^2*x^2 + d^2)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(x**6/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.138 $\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}-\frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(-5*e*x+4*d)/e^4/(-e^2*x
^2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(-15*e*x+8*d)/e^6/(
-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 217, 203}

$$\frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) + Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,d\right.}{e^5}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] time = 0.14, size = 103, normalized size = 0.84

$$\frac{15\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4\right)}{(d-ex)^2(d+ex)^3}}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 + 2
3*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 15*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(15*e^6)
```

fricas [B] time = 0.87, size = 241, normalized size = 1.98

$$\frac{8e^5x^5 + 8de^4x^4 - 16d^2e^3x^3 - 16d^3e^2x^2 + 8d^4ex + 8d^5 - 30\left(e^5x^5 + de^4x^4 - 2d^2e^3x^3 - 2d^3e^2x^2 + d^4ex + d^5\right)}{15\left(e^{11}x^5 + de^{10}x^4 - 2d^2e^9x^3 - 2d^3e^8x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*x
 + 8*d^5 - 30*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*
x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (23*e^4*x^4 + 8*d*e^3*
```

```
x^3 - 27*d^2*e^2*x^2 - 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 +
 d*e^10*x^4 - 2*d^2*e^9*x^3 - 2*d^3*e^8*x^2 + d^4*e^7*x + d^5*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 259, normalized size = 2.12

$$\frac{x^{3}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}-\frac{d\,x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}+\frac{2d^{2}x}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{4d^{2}x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{5}}+\frac{}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3/(-e^2*x^2+d^2)^(3/2)/e^3*x^3-2/3/(-e^2*x^2+d^2)^(1/2)/e^5*x+1/(e^2)^(1/
2)/e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-d/e^4*x^2/(-e^2*x^2+d^2)^
(3/2)+1/3*d^3/e^6/(-e^2*x^2+d^2)^(3/2)+2/3*d^2/e^5*x/(-e^2*x^2+d^2)^(3/2)+1
/5*d^4/e^7/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-4/15*d^2/e^5/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-8/15/e^5/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
*x
```

maxima [B] time = 1.06, size = 234, normalized size = 1.92

$$\frac{d^{4}}{5\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}de^{6}\right)}+\frac{x^{3}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}-\frac{8\,dx^{2}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}-\frac{4\,d^{2}x}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{x^{2}}{3\sqrt{-e^{2}x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^4/((-e^2*x^2 + d^2)^(3/2)*e^7*x + (-e^2*x^2 + d^2)^(3/2)*d*e^6) + x^3
/((-e^2*x^2 + d^2)^(3/2)*e^3) - 8/3*d*x^2/((-e^2*x^2 + d^2)^(3/2)*e^4) - 4/
15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*d*e^4
) + 2*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6) - 8/15*x/(sqrt(-e^2*x^2 + d^2)*e^5)
+ arcsin(e*x/d)/e^6 - 4/3*d/(sqrt(-e^2*x^2 + d^2)*e^6) - 1/3*sqrt(-e^2*x^2
+ d^2)/(d*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.139 $\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}} \, dx$

Optimal. Leaf size=85

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -1/5*x^4*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2
)-4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {850, 805, 266, 43}

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(x^4*(d - e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e^2
*x^2)^(3/2)) - 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{x^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)^{5/2}}\,dx, x, x^2\right)}{5e}$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \left(\frac{d^2}{e^2\left(d^2-e^2x\right)^{5/2}} - \frac{1}{e^2\left(d^2-e^2x\right)^{3/2}}\right)dx, x, x^2\right)}{5e}$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.09, size = 82, normalized size = 0.96

$$-\frac{\sqrt{d^2-e^2x^2}\left(8d^4+8d^3ex-12d^2e^2x^2-12de^3x^3+3e^4x^4\right)}{15de^5(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(8*d^4 + 8*d^3*e*x - 12*d^2*e^2*x^2 - 12*d*e^3*x
^3 + 3*e^4*x^4))/(d*e^5*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.73, size = 168, normalized size = 1.98

$$-\frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5+\left(3\,e^4x^4-12\,de^3x^3-12\,d^2e^2x^2+8\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(de^{10}x^5+d^2e^9x^4-2\,d^3e^8x^3-2\,d^4e^7x^2+d^5e^6x+d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*
x + 8*d^5 + (3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2 + 8*d^3*e*x + 8*d^4)
*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 + d^2*e^9*x^4 - 2*d^3*e^8*x^3 - 2*d^4*e^
7*x^2 + d^5*e^6*x + d^6*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.82

$$-\frac{(-ex+d)\left(3x^4e^4-12x^3d\,e^3-12d^2x^2e^2+8d^3xe+8d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(3*e^4*x^4-12*d*e^3*x^3-12*d^2*e^2*x^2+8*d^3*e*x+8*d^4)/d/e^
5/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.50, size = 134, normalized size = 1.58

$$-\frac{d^3}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^5\right)}+\frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{2\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}-\frac{d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}+\frac{x}{5\sqrt{-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6*x + (-e^2*x^2 + d^2)^(3/2)*d*e^5) + x^
2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

mupad [B] time = 2.95, size = 78, normalized size = 0.92

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4+8\,d^3\,e\,x-12\,d^2\,e^2\,x^2-12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2
+ 8*d^3*e*x))/(15*d*e^5*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.140 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(3/2)-1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {850, 819, 778, 191}

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$
$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$
$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}$$
$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^4-2d^3ex+3d^2e^2x^2+3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.88, size = 171, normalized size = 1.88

$$-\frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5-\left(3\,e^4x^4+3\,de^3x^3+3\,d^2e^2x^2-2\,d^3ex-2\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^9x^5+d^3e^8x^4-2\,d^4e^7x^3-2\,d^5e^6x^2+d^6e^5x+d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x
+ 2*d^5 - (3*e^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 + d^3*e^8*x^4 - 2*d^4*e^7*x^3 - 2*d^5*e^6*x
^2 + d^6*e^5*x + d^7*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.77

$$-\frac{(-ex+d)\left(-3x^4e^4-3x^3d\,e^3-3d^2x^2e^2+2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(-3*e^4*x^4-3*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^2/e
^4/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.47, size = 110, normalized size = 1.21

$$\frac{d^2}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4\right)}+\frac{2\,x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{d}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}-\frac{x}{5\sqrt{-e^2x^2+d^2}\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5*x + (-e^2*x^2 + d^2)^(3/2)*d*e^4) + 2/5
*x/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*d/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*
x/(sqrt(-e^2*x^2 + d^2)*d^2*e^3)
```

mupad [B] time = 2.84, size = 78, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4-2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.141 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/5*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)
^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {855, 778, 191}

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -x^2/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(15*d*e^3*(d^2
 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x(2d+2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}\\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de^2}\\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4\right)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*e
^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.81, size = 170, normalized size = 1.79

$$\frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5+\left(2\,e^4x^4+2\,de^3x^3-3\,d^2e^2x^2+2\,d^3ex+2\,d^4\right)\sqrt{-e^2x^2+}}{15\left(d^3e^8x^5+d^4e^7x^4-2\,d^5e^6x^3-2\,d^6e^5x^2+d^7e^4x+d^8e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x +
 2*d^5 + (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2*d^3*e*x + 2*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^3*e^8*x^5 + d^4*e^7*x^4 - 2*d^5*e^6*x^3 - 2*d^6*e^5*x^
2 + d^7*e^4*x + d^8*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.74

$$\frac{(-ex+d)\left(2x^4e^4+2x^3d\,e^3-3d^2x^2e^2+2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/15*(-e*x+d)*(2*e^4*x^4+2*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^3/e^3
/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.48, size = 110, normalized size = 1.16

$$-\frac{d}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^3\right)}-\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d/((-e^2*x^2 + d^2)^(3/2)*e^4*x + (-e^2*x^2 + d^2)^(3/2)*d*e^3) - 1/15
*x/((-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/15
*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

mupad [B] time = 2.79, size = 78, normalized size = 0.82

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(2\,d^4 + 2\,d^3\,e\,x - 3\,d^2\,e^2\,x^2 + 2\,d\,e^3\,x^3 + 2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d + e\,x\right)}^3\,{\left(d - e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4 + 2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{5}{2}}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.142 $\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)+1/5/e^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15
*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 192, 191}

$$\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}+\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$
$$= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e}$$
$$= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^4+3d^3ex+3d^2e^2x^2-2de^3x^3-2e^4x^4\right)}{15d^4e^2(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.69, size = 171, normalized size = 2.01

$$\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5-\left(2\,e^4x^4+2\,de^3x^3-3\,d^2e^2x^2-3\,d^3ex-3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^7x^5+d^5e^6x^4-2\,d^6e^5x^3-2\,d^7e^4x^2+d^8e^3x+d^9e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x +
 3*d^5 - (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 + d^5*e^6*x^4 - 2*d^6*e^5*x^3 - 2*d^7*e^4*x^
2 + d^8*e^3*x + d^9*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.82

$$\frac{(-ex+d)\left(-2x^4e^4-2x^3d\,e^3+3d^2x^2e^2+3d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/15*(-e*x+d)*(-2*e^4*x^4-2*d*e^3*x^3+3*d^2*e^2*x^2+3*d^3*e*x+3*d^4)/d^4/e^
2/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.49, size = 90, normalized size = 1.06

$$\frac{1}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2\right)}+\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e}+\frac{2x}{15\sqrt{-e^2x^2+d^2}\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5/((-e^2*x^2 + d^2)^(3/2)*e^3*x + (-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^2*e) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

mupad [B] time = 2.78, size = 78, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+3\,d^3\,e\,x+3\,d^2\,e^2\,x^2-2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 - 2*d*e^3*x^3 + 3*d^2*e^2*x^2 + 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.143 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)-1/5/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+8/15*x
/d^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}\\
&= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.00

$$-\frac{\sqrt{d^2-e^2x^2}\left(3d^4-12d^3ex-12d^2e^2x^2+8de^3x^3+8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(3*d^4 - 12*d^3*e*x - 12*d^2*e^2*x^2 + 8*d*e^3*x
^3 + 8*e^4*x^4))/(d^5*e*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.92, size = 168, normalized size = 2.05

$$-\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
+ 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*e^3
*x^2 + d^9*e^2*x + d^10*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.85

$$-\frac{(-ex+d)\left(8x^4e^4+8x^3d\,e^3-12d^2x^2e^2-12d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\,d^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5/
e/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.45, size = 85, normalized size = 1.04

$$-\frac{1}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e\right)}+\frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5/((-e^2*x^2 + d^2)^(3/2)*d*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^2*e) + 4/15
*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

mupad [B] time = 2.76, size = 78, normalized size = 0.95

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-12\,d^3\,e\,x-12\,d^2\,e^2\,x^2+8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 12*d^3*e*x))/(15*d^5*e*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.144 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/15*(-4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/(e*x+d)/(-e^2*x^2+d^2)^(
3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(-8*e*x+15*d)/d^6/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 12, 266, 63, 208}

$$\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (5*d - 4*e*x)/(15*d^4*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*(d + e*x)*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-5de^2+4e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^5e^6}{x\sqrt{d^2-e^2 \cdots}}}{15d^{10}e \cdots}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^{\cdots}}}}{d^5}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.}{d^6}$$

Mathematica [A] time = 0.09, size = 106, normalized size = 0.89

$$\frac{-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(23d^4+8d^3ex-27d^2e^2x^2-7de^3x^3+8e^4x^4\right)}{(d-ex)^2(d+ex)^3}+15\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 +
8*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 15*Log[x] - 15*Log[d + Sqrt[d^2 - e
^2*x^2]])/(15*d^6)
```

fricas [B] time = 0.88, size = 237, normalized size = 1.99

$$\frac{23\,e^{5}x^{5} + 23\,de^{4}x^{4} - 46\,d^{2}e^{3}x^{3} - 46\,d^{3}e^{2}x^{2} + 23\,d^{4}ex + 23\,d^{5} + 15\left(e^{5}x^{5} + de^{4}x^{4} - 2\,d^{2}e^{3}x^{3} - 2\,d^{3}e^{2}x^{2} + d^{4}ex\right.}{15\left(d^{6}e^{5}x^{5} + d^{7}e^{4}x^{4} - 2\,d^{8}e^{3}x^{3} - 2\,d^{9}e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 + 23*d*e^4*x^4 - 46*d^2*e^3*x^3 - 46*d^3*e^2*x^2 + 23*d^4*
e*x + 23*d^5 + 15*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^
4*e*x + d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 - 7*d*e^3*x^3
- 27*d^2*e^2*x^2 + 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 +
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 - 2*d^9*e^2*x^2 + d^10*e*x + d^11)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 196, normalized size = 1.65

$$-\frac{4ex}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{4}}+\frac{1}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{2}e}+\frac{1}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{3}}-\frac{\ln\left(\frac{2d^{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3/(-e^2*x^2+d^2)^(3/2)/d^3+1/(-e^2*x^2+d^2)^(1/2)/d^5-1/(d^2)^(1/2)/d^5*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^2/e/(x+d/e)/(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(3/2)-4/15/d^4*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x
-8/15/d^6*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(d^{2}-e^{2}\,x^{2}\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.145 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/15*(-5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x/(e*x+d)/(-e^2*x^2+d^
2)^(3/2)+e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(-5*e*x+8*d)/d^6/x/(-e^2
*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

Rubi [A] time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 807, 266, 63, 208}

$$\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (6*d - 5*e*x)/(15*d^4*x*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x*(d + e*x)*(d^2
- e^2*x^2)^(3/2)) + (8*d - 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[
d^2 - e^2*x^2])/(5*d^7*x) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-6de^2+5e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-24d^3e^4+15d^2e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-48d^5}{x^2}}{1}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2}}{5d}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2}}{5d}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2}}{5d}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2}}{5d}$$

Mathematica [A] time = 0.12, size = 122, normalized size = 0.79

$$-\frac{-15e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(15d^5+38d^4ex-52d^3e^2x^2-87d^2e^3x^3+33de^4x^4+48e^5x^5\right)}{x(d-ex)^2(d+ex)^3}+15e\log(x)}{15d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(15*d^5 + 38*d^4*e*x - 52*d^3*e^2*x^2 - 87*d^2*
e^3*x^3 + 33*d*e^4*x^4 + 48*e^5*x^5))/(x*(d - e*x)^2*(d + e*x)^3) + 15*e*Lo
g[x] - 15*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^7
```

fricas [A] time = 0.73, size = 265, normalized size = 1.72

$$-\frac{23\,e^{6}x^{6} + 23\,de^{5}x^{5} - 46\,d^{2}e^{4}x^{4} - 46\,d^{3}e^{3}x^{3} + 23\,d^{4}e^{2}x^{2} + 23\,d^{5}ex + 15\left(e^{6}x^{6} + de^{5}x^{5} - 2\,d^{2}e^{4}x^{4} - 2\,d^{3}e^{3}x^{3} + \right.}{15\left(d^{7}e^{5}x^{6} + d^{8}e^{4}x^{5} - 2\,d^{9}e^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(23*e^6*x^6 + 23*d*e^5*x^5 - 46*d^2*e^4*x^4 - 46*d^3*e^3*x^3 + 23*d^4
*e^2*x^2 + 23*d^5*e*x + 15*(e^6*x^6 + d*e^5*x^5 - 2*d^2*e^4*x^4 - 2*d^3*e^3
*x^3 + d^4*e^2*x^2 + d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*
x^5 + 33*d*e^4*x^4 - 87*d^2*e^3*x^3 - 52*d^3*e^2*x^2 + 38*d^4*e*x + 15*d^5)
*sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 + d^8*e^4*x^5 - 2*d^9*e^3*x^4 - 2*d^10*
e^2*x^3 + d^11*e*x^2 + d^12*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 268, normalized size = 1.74

$$\frac{4e^{2}x}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}}+\frac{4e^{2}x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{5}}-\frac{1}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{3}}-\frac{e}{3\left(-e^{2}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/d^3/x/(-e^2*x^2+d^2)^(3/2)+4/3/(-e^2*x^2+d^2)^(3/2)/d^5*e^2*x+8/3/(-e^2*
x^2+d^2)^(1/2)/d^7*e^2*x-1/3/(-e^2*x^2+d^2)^(3/2)/d^4*e-1/(-e^2*x^2+d^2)^(1
/2)/d^6*e+1/(d^2)^(1/2)/d^6*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))
/x)-1/5/d^3/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/15*e^2/d^5/(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+8/15*e^2/d^7/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}(ex+d)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(d^2 - e^2 x^2\right)^{5/2} (d + e x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{5}{2}} \left(d + ex\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.146 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{}{15d^4}$$

```
[Out] 1/15*(-6*e*x+7*d)/d^4/x^2/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^2/(e*x+d)/(-e^2*x^
2+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(-24*e*x+35*d
)/d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2+16/5*e*(-e^
2*x^2+d^2)^(1/2)/d^8/x
```

Rubi [A] time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{7e^2\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (7*d - 6*e*x)/(15*d^4*x^2*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^2*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (35*d - 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) -
(7*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) + (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x)
- (7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-7de^2+6e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-35d^3e^4+24d^2e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} -$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} -$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} -$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} -$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} -$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} -$$

Mathematica [A] time = 0.13, size = 137, normalized size = 0.74

$$\frac{-105e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(-15d^6+15d^5ex+176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4-9de^5x^5+96e^6x^6\right)}{x^2(d-ex)^2(d+ex)^3} + 105e^2\log(x)}{30d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^3*x
^3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(x^2*(d - e*x)^2*(d + e*x
)^3) + 105*e^2*Log[x] - 105*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(30*d^8)
```

fricas [A] time = 1.06, size = 286, normalized size = 1.54

$$\frac{116\,e^7x^7 + 116\,de^6x^6 - 232\,d^2e^5x^5 - 232\,d^3e^4x^4 + 116\,d^4e^3x^3 + 116\,d^5e^2x^2 + 105\left(e^7x^7 + de^6x^6 - 2\,d^2e^5x^5 - 2\,d\right.}{30\left(d^8e^5x^7 + d^9e^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 + 116*d*e^6*x^6 - 232*d^2*e^5*x^5 - 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 + 116*d^5*e^2*x^2 + 105*(e^7*x^7 + d*e^6*x^6 - 2*d^2*e^5*x^5 -
```

```
2*d^3*e^4*x^4 + d^4*e^3*x^3 + d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) + (96*e^6*x^6 - 9*d*e^5*x^5 - 249*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 + 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 + d^9*e^
4*x^6 - 2*d^10*e^3*x^5 - 2*d^11*e^2*x^4 + d^12*e*x^3 + d^13*x^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [**A**] time = 0.02, size = 298, normalized size = 1.60

$$-\frac{4e^3x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}-\frac{4e^3x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^6}+\frac{e}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^4}+\frac{7e^2}{6\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] e/d^4/x/(-e^2*x^2+d^2)^(3/2)-4/3/(-e^2*x^2+d^2)^(3/2)/d^6*e^3*x-8/3/(-e^2*x
^2+d^2)^(1/2)/d^8*e^3*x-1/2/d^3/x^2/(-e^2*x^2+d^2)^(3/2)+7/6/(-e^2*x^2+d^2)
^(3/2)/d^5*e^2+7/2/(-e^2*x^2+d^2)^(1/2)/d^7*e^2-7/2/(d^2)^(1/2)/d^7*e^2*ln(
(2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^4*e/(x+d/e)/(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(3/2)-4/15/d^6*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x
-8/15/d^8*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.147 $\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=215

$$\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{48d}{15d^6x^3}$$

```
[Out] 1/15*(-7*e*x+8*d)/d^4/x^3/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^3/(e*x+d)/(-e^2*x^
2+d^2)^(3/2)+7/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^9+1/15*(-35*e*x+48*d
)/d^6/x^3/(-e^2*x^2+d^2)^(1/2)-64/15*(-e^2*x^2+d^2)^(1/2)/d^7/x^3+7/2*e*(-e
^2*x^2+d^2)^(1/2)/d^8/x^2-128/15*e^2*(-e^2*x^2+d^2)^(1/2)/d^9/x
```

Rubi [A] time = 0.21, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}+\frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^3*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (48*d - 35*e*x)/(15*d^6*x^3*Sqrt[d^2 - e^2*x^2]) -
(64*Sqrt[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*Sqrt[d^2 - e^2*x^2])/(2*d^8*x^
2) - (128*e^2*Sqrt[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/(2*d^9)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-8de^2+7e^3x}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-48d^3e^4+35d^2e^5x}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{\int}{}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64}{}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64}{}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64}{}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64}{}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64}{}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64}{}$$

Mathematica [A] time = 0.16, size = 148, normalized size = 0.69

$$-\frac{-105e^3\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(10d^7-5d^6ex+75d^5e^2x^2+236d^4e^3x^3-244d^3e^4x^4-489d^2e^5x^5+151de^6x^6+256e^7x^7\right)}{x^3(d-ex)^2(d+ex)^3} + 105e^3\log}{30d^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/30*((Sqrt[d^2 - e^2*x^2]*(10*d^7 - 5*d^6*e*x + 75*d^5*e^2*x^2 + 236*d^4*
e^3*x^3 - 244*d^3*e^4*x^4 - 489*d^2*e^5*x^5 + 151*d*e^6*x^6 + 256*e^7*x^7))
/(x^3*(d - e*x)^2*(d + e*x)^3) + 105*e^3*Log[x] - 105*e^3*Log[d + Sqrt[d^2
- e^2*x^2]])/d^9
```

fricas [A] time = 1.45, size = 297, normalized size = 1.38

$$-\frac{116e^8x^8+116de^7x^7-232d^2e^6x^6-232d^3e^5x^5+116d^4e^4x^4+116d^5e^3x^3+105\left(e^8x^8+de^7x^7-2d^2e^6x^6-2d^3\right.}{30\left(d^9e^5x^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(116*e^8*x^8 + 116*d*e^7*x^7 - 232*d^2*e^6*x^6 - 232*d^3*e^5*x^5 + 11
6*d^4*e^4*x^4 + 116*d^5*e^3*x^3 + 105*(e^8*x^8 + d*e^7*x^7 - 2*d^2*e^6*x^6
- 2*d^3*e^5*x^5 + d^4*e^4*x^4 + d^5*e^3*x^3)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (256*e^7*x^7 + 151*d*e^6*x^6 - 489*d^2*e^5*x^5 - 244*d^3*e^4*x^4 + 2
36*d^4*e^3*x^3 + 75*d^5*e^2*x^2 - 5*d^6*e*x + 10*d^7)*sqrt(-e^2*x^2 + d^2))
/(d^9*e^5*x^8 + d^10*e^4*x^7 - 2*d^11*e^3*x^6 - 2*d^12*e^2*x^5 + d^13*e*x^4
 + d^14*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 326, normalized size = 1.52

$$\frac{4e^4x}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^7}+\frac{4e^4x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} d^7}-\frac{e^2}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} d^5}-\frac{7e^3}{6\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -3/d^5*e^2/x/(-e^2*x^2+d^2)^(3/2)+4/d^7*e^4*x/(-e^2*x^2+d^2)^(3/2)+8/d^9*e^
4*x/(-e^2*x^2+d^2)^(1/2)+1/2*e/d^4/x^2/(-e^2*x^2+d^2)^(3/2)-7/6*e^3/d^6/(-e
^2*x^2+d^2)^(3/2)-7/2*e^3/d^8/(-e^2*x^2+d^2)^(1/2)+7/2*e^3/d^8/(d^2)^(1/2)*
ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3/(-e^2*x^2+d^2)
^(3/2)-1/5/d^5*e^2/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/15/d^7*e^4
/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+8/15/d^9*e^4/(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(d^2-e^2\,x^2\right)^{5/2}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

[Out] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.148 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/7*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(7/2)+1/35*(3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(5/2)-1/35*x/d^2/e^3/(-e^2*x^2+d^2)^(3/2)-2/35*x/d^4/e^3/(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 192, 191}

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2
- e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^
3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$$
$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2}$$
$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35e^3}$$
$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,d}{35d^2e^3}$$
$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2}}$$

Mathematica [A] time = 0.12, size = 104, normalized size = 0.88

$$-\frac{\sqrt{d^2-e^2x^2}\left(2d^6+2d^5ex-5d^4e^2x^2-5d^3e^3x^3-5d^2e^4x^4+2de^5x^5+2e^6x^6\right)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -1/35*(Sqrt[d^2 - e^2*x^2]*(2*d^6 + 2*d^5*e*x - 5*d^4*e^2*x^2 - 5*d^3*e^3*x
^3 - 5*d^2*e^4*x^4 + 2*d*e^5*x^5 + 2*e^6*x^6))/(d^4*e^4*(d - e*x)^3*(d + e*
x)^4)
```

fricas [B] time = 1.02, size = 239, normalized size = 2.03

$$-\frac{2e^7x^7+2de^6x^6-6d^2e^5x^5-6d^3e^4x^4+6d^4e^3x^3+6d^5e^2x^2-2d^6ex-2d^7-\left(2e^6x^6+2de^5x^5-5d^2e^4x^4-5d^3\right.}{35\left(d^4e^{11}x^7+d^5e^{10}x^6-3d^6e^9x^5-3d^7e^8x^4+3d^8e^7x^3+3d^9e^6x^2-d^{10}e^5x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(2*e^7*x^7 + 2*d*e^6*x^6 - 6*d^2*e^5*x^5 - 6*d^3*e^4*x^4 + 6*d^4*e^3*
x^3 + 6*d^5*e^2*x^2 - 2*d^6*e*x - 2*d^7 - (2*e^6*x^6 + 2*d*e^5*x^5 - 5*d^2*
e^4*x^4 - 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 + 2*d^5*e*x + 2*d^6)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^11*x^7 + d^5*e^10*x^6 - 3*d^6*e^9*x^5 - 3*d^7*e^8*x^4 + 3*d^
8*e^7*x^3 + 3*d^9*e^6*x^2 - d^10*e^5*x - d^11*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 92, normalized size = 0.78

$$-\frac{(-ex+d)\left(2e^6x^6+2e^5x^5d-5e^4x^4d^2-5x^3d^3e^3-5x^2d^4e^2+2d^5xe+2d^6\right)}{35\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/35*(-e*x+d)*(2*e^6*x^6+2*d*e^5*x^5-5*d^2*e^4*x^4-5*d^3*e^3*x^3-5*d^4*e^2
*x^2+2*d^5*e*x+2*d^6)/d^4/e^4/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.51, size = 133, normalized size = 1.13

$$\frac{d^2}{7\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^4\right)}+\frac{8x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/7*d^2/((-e^2*x^2 + d^2)^(5/2)*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d*e^4) + 8/3
5*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/3
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^3) - 2/35*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^3
)
```

mupad [B] time = 2.95, size = 161, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{56\,d\,e^4\,{\left(d+e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d\,e^4}+\frac{x}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2\,d}{35\,e^4}-\frac{11\,x}{70\,e^3}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{2\,x\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,e^3\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d*e^4*(d + e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(1/(5
6*d*e^4) + x/(35*d^2*e^3)))/((d + e*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1
/2)*((2*d)/(35*e^4) - (11*x)/(70*e^3)))/((d + e*x)^3*(d - e*x)^3) - (2*x*(d
^2 - e^2*x^2)^(1/2))/(35*d^4*e^3*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.149 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -1/7*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+2/35*(2*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {855, 778, 192, 191}

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -x^2/(7*d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (2*(d + 2*e*x))/(35*d*e^3*(d
^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(1
05*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{x(2d+4ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7de}$$

$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$

$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \ldots$$

$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \ldots$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6x^6\right)}{105d^5e^3(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d + e
*x)^4)
```

fricas [B] time = 1.01, size = 238, normalized size = 1.93

$$\frac{6\,e^7x^7+6\,de^6x^6-18\,d^2e^5x^5-18\,d^3e^4x^4+18\,d^4e^3x^3+18\,d^5e^2x^2-6\,d^6ex-6\,d^7+\left(8\,e^6x^6+8\,de^5x^5-20\,d^2e^4x \ldots\right.}{105\left(d^5e^{10}x^7+d^6e^9x^6-3\,d^7e^8x^5-3\,d^8e^7x^4+3\,d^9e^6x^3+3\,d^{10}e^5x^2- \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*e^7*x^7 + 6*d*e^6*x^6 - 18*d^2*e^5*x^5 - 18*d^3*e^4*x^4 + 18*d^4*e
^3*x^3 + 18*d^5*e^2*x^2 - 6*d^6*e*x - 6*d^7 + (8*e^6*x^6 + 8*d*e^5*x^5 - 20
*d^2*e^4*x^4 - 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 - 6*d^5*e*x - 6*d^6)*sqrt(-e
^2*x^2 + d^2))/(d^5*e^10*x^7 + d^6*e^9*x^6 - 3*d^7*e^8*x^5 - 3*d^8*e^7*x^4
+ 3*d^9*e^6*x^3 + 3*d^10*e^5*x^2 - d^11*e^4*x - d^12*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 92, normalized size = 0.75

$$\frac{(-ex+d)\left(-8e^6x^6-8e^5x^5d+20e^4x^4d^2+20x^3d^3e^3-15x^2d^4e^2+6d^5xe+6d^6\right)}{105\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/105*(-e*x+d)*(-8*e^6*x^6-8*d*e^5*x^5+20*d^2*e^4*x^4+20*d^3*e^3*x^3-15*d^4
*e^2*x^2+6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.48, size = 133, normalized size = 1.08

$$-\frac{d}{7\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^3\right)}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{4x}{105\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}-\frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/7*d/((-e^2*x^2 + d^2)^(5/2)*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d*e^3) - 1/35
*x/((-e^2*x^2 + d^2)^(5/2)*d*e^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e^3) - 4/10
5*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d^2)*d^5*e^
2)
```

mupad [B] time = 2.88, size = 161, normalized size = 1.31

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}-\frac{4\,x}{105\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}+\frac{3\,x}{70\,d\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,{\left(d+e\,x\right)}^4}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(1/(56*d^2*e^3) - (4*x)/(105*d^3*e^2)))/((d + e*x)^2
*(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*(2/(35*e^3) + (3*x)/(70*d*e^2)))/((d
 + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d + e*x)^4) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.150 $\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=66

$$\frac{3\sin^{-1}(ax)}{2a^4}+\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}$$

```
[Out] 3/2*arcsin(a*x)/a^4+x^2*(-a*x+1)/a^2/(-a^2*x^2+1)^(1/2)+1/2*(-3*a*x+4)*(-a^
2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {850, 819, 780, 216}

$$\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}+\frac{3\sin^{-1}(ax)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (x^2*(1 - a*x))/(a^2*Sqrt[1 - a^2*x^2]) + ((4 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
2*a^4) + (3*ArcSin[a*x])/(2*a^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx$$
$$= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{x(2-3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3}$$
$$= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\sin^{-1}(ax)}{2a^4}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-a^2x^2+ax+4\right)+3(ax+1)\sin^{-1}(ax)}{2a^4(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2) + 3*(1 + a*x)*ArcSin[a*x])/(2*a^4*(1
 + a*x))
```

fricas [A] time = 0.67, size = 75, normalized size = 1.14

$$\frac{4\,ax - 6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(a^2x^2 - ax - 4\right)\sqrt{-a^2x^2+1} + 4}{2\left(a^5x + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^2
- a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(a^5*x + a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 100, normalized size = 1.52

$$-\frac{\sqrt{-a^2x^2+1}\,x}{2a^3} + \frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}\,a^3} + \frac{\sqrt{-a^2x^2+1}}{a^4} + \frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2 + 2\left(x+\frac{1}{a}\right)a}}{\left(x+\frac{1}{a}\right)a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/a^3*x*(-a^2*x^2+1)^(1/2)+3/2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*
x^2+1)^(1/2)*x)+1/a^4*(-a^2*x^2+1)^(1/2)+1/a^5/(x+1/a)*(-(x+1/a)^2*a^2+2*(x
+1/a)*a)^(1/2)
```

maxima [A] time = 0.98, size = 68, normalized size = 1.03

$$\frac{\sqrt{-a^2x^2+1}}{a^5x+a^4}-\frac{\sqrt{-a^2x^2+1}\,x}{2\,a^3}+\frac{3\,\arcsin\left(ax\right)}{2\,a^4}+\frac{\sqrt{-a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) - 1/2*sqrt(-a^2*x^2 + 1)*x/a^3 + 3/2*arcsi
n(a*x)/a^4 + sqrt(-a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.07, size = 116, normalized size = 1.76

$$\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^3\,\sqrt{-a^2}}-\frac{\left(\frac{1}{a^2\,\sqrt{-a^2}}+\frac{x\,\sqrt{-a^2}}{2\,a^3}\right)\,\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (3*asinh(x*(-a^2)^(1/2)))/(2*a^3*(-a^2)^(1/2)) - ((1/(a^2*(-a^2)^(1/2)) + (
x*(-a^2)^(1/2))/(2*a^3))*(1 - a^2*x^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^
(1/2)/(a^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.151 $\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=55

$$-\frac{\sin^{-1}(ax)}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

```
[Out] -arcsin(a*x)/a^3-(-a^2*x^2+1)^(1/2)/a^3-(-a^2*x^2+1)^(1/2)/a^3/(a*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1639, 12, 793, 216}

$$-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x)) - ArcSin[a*x]/
a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{a^3x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a^4}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\sin^{-1}(ax)}{a^3}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 0.67

$$-\frac{\frac{\sqrt{1-a^2x^2}(ax+2)}{ax+1} + \sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -((((2 + a*x)*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x])/a^3)
```

fricas [A] time = 0.87, size = 66, normalized size = 1.20

$$-\frac{2\,ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax+2) + 2}{a^4x + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

giac [A] time = 0.20, size = 70, normalized size = 1.27

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.01, size = 84, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,a^2} - \frac{\sqrt{-a^2x^2+1}}{a^3} - \frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2 + 2\left(x+\frac{1}{a}\right)a}}{\left(x+\frac{1}{a}\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a^3-1/(a^2)^(1/2)/a^2*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(
1/2)*x)-1/a^4/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

maxima [A] time = 0.98, size = 52, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3}-\frac{\arcsin(ax)}{a^3}-\frac{\sqrt{-a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) - arcsin(a*x)/a^3 - sqrt(-a^2*x^2 + 1)/a^
3
```

mupad [B] time = 0.07, size = 84, normalized size = 1.53

$$\frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-(ax-1)(ax+1)}\;(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.152 $\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=34

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sin^{-1}(ax)}{a^2}$$

```
[Out] arcsin(a*x)/a^2+(-a^2*x^2+1)^(1/2)/a^2/(a*x+1)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {793, 216}

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx&=\frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a}\\&=\frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\sin^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.91

$$\frac{\frac{\sqrt{1-a^2x^2}}{ax+1}+\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]/(1 + a*x) + ArcSin[a*x])/a^2
```

fricas [A] time = 0.88, size = 58, normalized size = 1.71

$$\frac{ax - 2\,(ax + 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2 + 1} + 1}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1) + 1)/(a^3*x + a^2)
```

giac [A] time = 0.22, size = 52, normalized size = 1.53

$$\frac{\arcsin\left(ax\right)\mathrm{sgn}(a)}{a|a|} - \frac{2}{a\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.01, size = 65, normalized size = 1.91

$$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,a} + \frac{\sqrt{-\left(x + \frac{1}{a}\right)^2 a^2 + 2\left(x + \frac{1}{a}\right)a}}{\left(x + \frac{1}{a}\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(1/2)*x)+1/a^3/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

maxima [A] time = 0.99, size = 33, normalized size = 0.97

$$\frac{\sqrt{-a^2x^2 + 1}}{a^3x + a^2} + \frac{\arcsin\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2
```

mupad [B] time = 2.60, size = 57, normalized size = 1.68

$$\frac{1}{a^2\,\sqrt{1 - a^2\,x^2}} - \frac{x}{a\,\sqrt{1 - a^2\,x^2}} - \frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] 1/(a^2*(1 - a^2*x^2)^(1/2)) - x/(a*(1 - a^2*x^2)^(1/2)) - (asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.153 $\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/a/(a*x+1)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {651}

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx=-\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$-\frac{\sqrt{1-a^2x^2}}{a^2x+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a + a^2*x))
```

fricas [A] time = 0.72, size = 28, normalized size = 1.08

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*x + a)
```

giac [A] time = 0.20, size = 34, normalized size = 1.31

$$\frac{2}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] 2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{ax-1}{\sqrt{-a^2x^2+1}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
[Out] (a*x-1)/a/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 23, normalized size = 0.88

$$-\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-a^2*x^2 + 1)/(a^2*x + a)
```

mupad [B] time = 2.59, size = 23, normalized size = 0.88

$$-\frac{\sqrt{1-a^2\,x^2}}{x\,a^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
[Out] -(1 - a^2*x^2)^(1/2)/(a + a^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\ (ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.154 $\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -arctanh((-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {857, 12, 266, 63, 208}

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{1-ax} + \frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{1-ax} + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}}{1-ax} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1-a^2x^2}}{1-ax} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{1-ax} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}}{1-ax} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.87, size = 52, normalized size = 1.27

$$\frac{ax + (ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1} - 1}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*x - 1)
```

giac [A] time = 0.21, size = 74, normalized size = 1.80

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{2a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 2
*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.01, size = 58, normalized size = 1.41

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}{\left(x-\frac{1}{a}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-a^2x^2+1}\,(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x)
```

mupad [B] time = 2.65, size = 58, normalized size = 1.41

$$\frac{a\,\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}\,\left(\frac{a}{\sqrt{-a^2}}+x\,\sqrt{-a^2}\right)}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/((-a^2)^(1/2)*(a/(-a^2)^(1/2) + x*(-a^2)^(1/2))) -
atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x)
```

3.155 $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-2*(-a^2*x^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x/
(-a*x+1)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = $\{857, 807, 266, 63, 208\}$

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x^2]/(x*(1 - a*x)) - a*ArcTanh[Sqrt
[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
```

```
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\int \frac{-2a^2-a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.78

$$\frac{(1-2ax)\sqrt{1-a^2x^2}}{x(ax-1)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x)) - a*ArcTanh[Sqrt[1 - a^2*x^2
]]
```

fricas [A] time = 0.89, size = 76, normalized size = 1.19

$$\frac{a^2x^2 - ax + \left(a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\,(2\,ax-1)}{ax^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^
2*x^2 + 1)*(2*a*x - 1))/(a*x^2 - x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 73, normalized size = 1.14

$$-a \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-a^2x^2+1}}{x}-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2-2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-(-a^2*x^2+1)^(1/2)/x-a*arctan
h(1/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-a^2x^2+1}\,(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)
```

mupad [B] time = 2.59, size = 81, normalized size = 1.27

$$\frac{a^2\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{x}-a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (1 - a^2*x^2)^(1/2)/x - a*atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)
```

3.156 $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -3/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-3/2*(-a^2*x^2+1)^(1/2)/x^2-2*a*(-a^2*x
^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x^2/(-a*x+1)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {857, 835, 807, 266, 63, 208}

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x
^2]/(x^2*(1 - a*x)) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{\int \frac{-3a^2-2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2} \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{\int \frac{4a^3+3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2} \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{2}\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{4}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2\right. \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.70

$$\frac{1}{2}\left(\frac{\left(-4a^2x^2+ax+1\right)\sqrt{1-a^2x^2}}{x^2(ax-1)} - 3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(x^2*(-1 + a*x)) - 3*a^2*ArcTanh
[Sqrt[1 - a^2*x^2]])/2
```

fricas [A] time = 0.90, size = 97, normalized size = 1.08

$$\frac{2\,a^3x^3 - 2\,a^2x^2 + 3\left(a^3x^3 - a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(4\,a^2x^2 - ax - 1\right)\sqrt{-a^2x^2+1}}{2\left(ax^3 - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

giac [B] time = 0.21, size = 213, normalized size = 2.37

$$-\frac{\left(a^{3}+\frac{3\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}}{ax^{2}}\right)a^{4}x^{2}}{8\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}\left(\frac{\sqrt{-a^{2}x^{2}+1}\,|a|+a}{a^{2}x}-1\right)|a|}-\frac{3\,a^{3}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}\,|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{2\,|a|}-\frac{\frac{4\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^{2}x^{2}+1}\,|a|\right.}{ax^{2}}}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2
))/a^2
```

maple [A] time = 0.01, size = 94, normalized size = 1.04

$$-\frac{3a^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^{2}a^{2}-2\left(x-\frac{1}{a}\right)a}\;a}{x-\frac{1}{a}}-\frac{\sqrt{-a^{2}x^{2}+1}\;a}{x}-\frac{\sqrt{-a^{2}x^{2}+1}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -a/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-a*(-a^2*x^2+1)^(1/2)/x-1/2*(-
a^2*x^2+1)^(1/2)/x^2-3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\sqrt{-a^{2}x^{2}+1}\,(ax-1)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^3), x)
```

mupad [B] time = 2.61, size = 105, normalized size = 1.17

$$\frac{a^{3}\sqrt{1-a^{2}\,x^{2}}}{\left(x\,\sqrt{-a^{2}}-\frac{\sqrt{-a^{2}}}{a}\right)\sqrt{-a^{2}}}-\frac{a\,\sqrt{1-a^{2}\,x^{2}}}{x}-\frac{\sqrt{1-a^{2}\,x^{2}}}{2\,x^{2}}+\frac{a^{2}\,\mathrm{atan}\left(\sqrt{1-a^{2}\,x^{2}}\,1\mathrm{i}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a*
(1 - a^2*x^2)^(1/2))/x + (a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2
)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{ax^{4}\sqrt{-a^{2}x^{2}+1}-x^{3}\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x)
```

3.157 $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}+\frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}-\frac{5d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-$$

```
[Out] -4/21*d^4*x^2*(-e^2*x^2+d^2)^(3/2)/e^4+5/24*d^3*x^3*(-e^2*x^2+d^2)^(3/2)/e^
3-5/21*d^2*x^4*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d*x^5*(-e^2*x^2+d^2)^(3/2)/e-1/
9*x^6*(-e^2*x^2+d^2)^(3/2)-1/2016*d^5*(-315*e*x+256*d)*(-e^2*x^2+d^2)^(3/2)
/e^6-5/64*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6-5/64*d^7*x*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A] time = 0.31, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right.}{21e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (-5*d^7*x*Sqrt[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^(3/2))
/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^(3/2))/(24*e^3) - (5*d^2*x^4*(d^2 -
e^2*x^2)^(3/2))/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e) - (x^6*(d^2
- e^2*x^2)^(3/2))/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^(3/2))/(2016*e
^6) - (5*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(64*e^6)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^5(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^5\left(-15d^2e^2+18de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{9e^2}$$

$$= \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^4\left(-90d^3e^3+120d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{72e^4}$$

$$= -\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3\left(-480d^4e^4+\cdots\right.}{\cdots}$$

$$= \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}\cdots$$

$$= -\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right.}{4e}\cdots$$

$$= -\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right.}{4e}\cdots$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)}{21e^2}\cdots$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)}{21e^2}\cdots$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)}{21e^2}\cdots$$

Mathematica [A] time = 0.21, size = 135, normalized size = 0.59

$$\frac{\sqrt{d^2-e^2x^2}\left(-512d^8+315d^7ex-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5x^5+512d^2e^6x^6-1008de^7x^7+4\right.}{4032e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5*e^
3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^7*x^
7 + 448*e^8*x^8) - 315*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4032*e^6)
```

fricas [A] time = 0.88, size = 138, normalized size = 0.60

$$\frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(448\,e^8x^8-1008\,de^7x^7+512\,d^2e^6x^6+168\,d^3e^5x^5-192\,d^4e^4x^4+210\,d^5e^3x^3-25\right.}{4032\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (448*e^8*x^8 -
1008*d*e^7*x^7 + 512*d^2*e^6*x^6 + 168*d^3*e^5*x^5 - 192*d^4*e^4*x^4 + 210*
d^5*e^3*x^3 - 256*d^6*e^2*x^2 + 315*d^7*e*x - 512*d^8)*sqrt(-e^2*x^2 + d^2)
)/e^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 375, normalized size = 1.64

$$\frac{5d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{4\sqrt{e^2}\,e^5}-\frac{85d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{64\sqrt{e^2}\,e^5}-\frac{85\sqrt{-e^2x^2+d^2}\,d^7x}{64e^5}+\frac{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,c}{4e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/9/e^4*x^2*(-e^2*x^2+d^2)^(7/2)-29/63*d^2/e^6*(-e^2*x^2+d^2)^(7/2)+1/4/e^
5*d*x*(-e^2*x^2+d^2)^(7/2)-17/24/e^5*d^3*x*(-e^2*x^2+d^2)^(5/2)-85/96/e^5*d
^5*x*(-e^2*x^2+d^2)^(3/2)-85/64*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^5-85/64/e^5*d^
9/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+2/3/e^6*d^4*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+5/6/e^5*d^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/
2)*x+5/4/e^5*d^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+5/4/e^5*d^9/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3*d^4/e^8/(
x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [C] time = 1.07, size = 299, normalized size = 1.31

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5}{4\left(e^7x+de^6\right)}-\frac{5i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^6}-\frac{85\,d^9\arcsin\left(\frac{ex}{d}\right)}{64\,e^6}+\frac{5\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^7x}{4\,e^5}-\frac{85\,\sqrt{-e^2x^2+d^2}\,d^7x}{64\,e^5}+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^7*x + d*e^6) - 5/4*I*d^9*arcsin(e*x/d +
2)/e^6 - 85/64*d^9*arcsin(e*x/d)/e^6 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*
d^7*x/e^5 - 85/64*sqrt(-e^2*x^2 + d^2)*d^7*x/e^5 + 5/2*sqrt(e^2*x^2 + 4*d*e
*x + 3*d^2)*d^8/e^6 + 35/96*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^5 - 5/12*(-e^2*x
^2 + d^2)^(3/2)*d^6/e^6 - 17/24*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^5 - 1/9*(-e^
2*x^2 + d^2)^(7/2)*x^2/e^4 + (-e^2*x^2 + d^2)^(5/2)*d^4/e^6 + 1/4*(-e^2*x^2
 + d^2)^(7/2)*d*x/e^5 - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [A] time = 17.48, size = 571, normalized size = 2.49

$$d^2\left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases} -\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7}+\frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7}-\frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 2
*d*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sq
rt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2
)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt
(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e
**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sq
rt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) +
 d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2
*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piec
ewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 -
 e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d*
*2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, N
e(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.158 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=200

$$-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}+\frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2}+\frac{13d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1\cdots}{}$$

```
[Out] 8/35*d^3*x^2*(-e^2*x^2+d^2)^(3/2)/e^3-13/48*d^2*x^3*(-e^2*x^2+d^2)^(3/2)/e^
2+2/7*d*x^4*(-e^2*x^2+d^2)^(3/2)/e-1/8*x^5*(-e^2*x^2+d^2)^(3/2)+1/6720*d^4*
(-1365*e*x+1024*d)*(-e^2*x^2+d^2)^(3/2)/e^5+13/128*d^8*arctan(e*x/(-e^2*x^2
+d^2)^(1/2))/e^5+13/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.27, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5}+\frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3}-\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2}+\frac{2dx^4\left(d^2-e^2\cdots\right.}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (13*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^(3/2)
)/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^(3/2))/(48*e^2) + (2*d*x^4*(d^2 -
e^2*x^2)^(3/2))/(7*e) - (x^5*(d^2 - e^2*x^2)^(3/2))/8 + (d^4*(1024*d - 1365
*e*x)*(d^2 - e^2*x^2)^(3/2))/(6720*e^5) + (13*d^8*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(128*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^4(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^4\left(-13d^2e^2+16de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{8e^2}$$

$$= \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^3\left(-64d^3e^3+91d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{56e^4}$$

$$= -\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2\left(-273d^4e^4\right.}{}$$

$$= \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)$$

$$= \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)}{7e}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)}{7e}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)}{7e}$$

Mathematica [A] time = 0.15, size = 124, normalized size = 0.62

$$\frac{1365d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(2048d^7-1365d^6ex+1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e\right.}{13440e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^4*
e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*x^
7) + 1365*d^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(13440*e^5)
```

fricas [A] time = 0.82, size = 128, normalized size = 0.64

$$-\frac{2730\,d^{8}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(1680\,e^{7}x^{7}-3840\,de^{6}x^{6}+1960\,d^{2}e^{5}x^{5}+768\,d^{3}e^{4}x^{4}-910\,d^{4}e^{3}x^{3}+1024\,d^{5}e^{2}\cdots}{13440\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/13440*(2730*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (1680*e^7*x^
7 - 3840*d*e^6*x^6 + 1960*d^2*e^5*x^5 + 768*d^3*e^4*x^4 - 910*d^4*e^3*x^3 +
 1024*d^5*e^2*x^2 - 1365*d^6*e*x + 2048*d^7)*sqrt(-e^2*x^2 + d^2))/e^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 350, normalized size = 1.75

$$-\frac{7d^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,e^{4}}+\frac{125d^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{128\sqrt{e^{2}}\,e^{4}}+\frac{125\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}x}{128e^{4}}-\frac{7\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}\cdots}}{8e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/8/e^4*x*(-e^2*x^2+d^2)^(7/2)+25/48*d^2/e^4*x*(-e^2*x^2+d^2)^(5/2)+125/19
2/e^4*d^4*x*(-e^2*x^2+d^2)^(3/2)+125/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4+125
/128/e^4*d^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+2/7*d/e
^5*(-e^2*x^2+d^2)^(7/2)-7/15/e^5*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-7/
12/e^4*d^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-7/8/e^4*d^6*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x-7/8/e^4*d^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/3*d^3/e^7/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(7/2)
```

maxima [C] time = 1.04, size = 275, normalized size = 1.38

$$\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}}{4\left(e^{6}x+de^{5}\right)}+\frac{7i\,d^{8}\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^{5}}+\frac{125\,d^{8}\arcsin\left(\frac{ex}{d}\right)}{128\,e^{5}}-\frac{7\,\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{6}x}{8\,e^{4}}+\frac{125\,\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}x}{128\,e^{4}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^6*x + d*e^5) + 7/8*I*d^8*arcsin(e*x/d + 2
)/e^5 + 125/128*d^8*arcsin(e*x/d)/e^5 - 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)
*d^6*x/e^4 + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^4 - 7/4*sqrt(e^2*x^2 + 4*
d*e*x + 3*d^2)*d^7/e^5 - 67/192*(-e^2*x^2 + d^2)^(3/2)*d^4*x/e^4 + 5/12*(-e
^2*x^2 + d^2)^(3/2)*d^5/e^5 + 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^4 - 4/5*
(-e^2*x^2 + d^2)^(5/2)*d^3/e^5 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^4 + 2/7*(-e
^2*x^2 + d^2)^(7/2)*d/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(d^2-e^2\,x^2\right)^{5/2}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [C] time = 21.40, size = 690, normalized size = 3.45

$$d^{2}\left(\begin{cases} -\frac{i d^{6} \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{16 e^{5}} + \frac{i d^{5} x}{16 e^{4} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{i d^{3} x^{3}}{48 e^{2} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{5 i d x^{5}}{24 \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + \frac{i e^{2} x^{7}}{6 d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\ \frac{d^{6} \operatorname{asin}{\left(\frac{e x}{d} \right)}}{16 e^{5}} - \frac{d^{5} x}{16 e^{4} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{d^{3} x^{3}}{48 e^{2} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{5 d x^{5}}{24 \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - \frac{e^{2} x^{7}}{6 d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) - 2 d e \left(\begin{cases} - \frac{8 d^{6} \sqrt{d^{2} - \ldots}}{105 \ldots} \\ \frac{x^{6} \sqrt{d^{2}}}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) - 2*d*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(10
5*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d*
*2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6
*sqrt(d**2)/6, True)) + e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) +
 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*
sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2
)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 +
 e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7)
 - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqr
t(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*
d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d*
*2)), True))
```

3.159 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal. Leaf size=171

$$-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-1\cdots}{\cdots}$$

```
[Out] -11/35*d^2*x^2*(-e^2*x^2+d^2)^(3/2)/e^2+1/3*d*x^3*(-e^2*x^2+d^2)^(3/2)/e-1/
7*x^4*(-e^2*x^2+d^2)^(3/2)-1/420*d^3*(-105*e*x+88*d)*(-e^2*x^2+d^2)^(3/2)/e
^4-1/8*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/8*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^3
```

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(d^5*x*Sqrt[d^2 - e^2*x^2])/(8*e^3) - (11*d^2*x^2*(d^2 - e^2*x^2)^(3/2))/(
35*e^2) + (d*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e) - (x^4*(d^2 - e^2*x^2)^(3/2))
/7 - (d^3*(88*d - 105*e*x)*(d^2 - e^2*x^2)^(3/2))/(420*e^4) - (d^7*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^3(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3\left(-11d^2e^2+14de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{7e^2}$$

$$= \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^2\left(-42d^3e^3+66d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{42e^4}$$

$$= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x\left(-132d^4e^4+\right.}{}$$

$$= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d-105ex)\left(\right.}{420e}$$

$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} -$$

$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} -$$

$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} -$$

Mathematica [A] time = 0.13, size = 113, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(-176d^6+105d^5ex-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4-280de^5x^5+120e^6x^6\right)-105d^7\tan^{-1}\left(\frac{}{\sqrt{d}}\right.}{840e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

[Out] (Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e^3*
x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6) - 105*d^7*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(840*e^4)

fricas [A] time = 0.88, size = 116, normalized size = 0.68

$$\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(120\,e^6x^6-280\,de^5x^5+144\,d^2e^4x^4+70\,d^3e^3x^3-88\,d^4e^2x^2+105\,d^5ex-176\,d^6\right)}{840\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")

[Out] 1/840*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (120*e^6*x^6 - 2
80*d*e^5*x^5 + 144*d^2*e^4*x^4 + 70*d^3*e^3*x^3 - 88*d^4*e^2*x^2 + 105*d^5*
e*x - 176*d^6)*sqrt(-e^2*x^2 + d^2))/e^4

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.02, size = 327, normalized size = 1.91

$$\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,e^3}-\frac{5d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^3}-\frac{5\sqrt{-e^2x^2+d^2}\,d^5x}{8e^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5x}{2e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)

[Out] -1/7/e^4*(-e^2*x^2+d^2)^(7/2)-1/3*d/e^3*x*(-e^2*x^2+d^2)^(5/2)-5/12/e^3*d^3
x(-e^2*x^2+d^2)^(3/2)-5/8*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^3-5/8/e^3*d^7/(e^2
)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+4/15/e^4*d^2*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(5/2)+1/3/e^3*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+
1/2/e^3*d^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/2/e^3*d^7/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3*d^2/e^6/(x+d/e
)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)

maxima [C] time = 1.04, size = 251, normalized size = 1.47

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{4\left(e^5x+de^4\right)}-\frac{i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^4}-\frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{8\,e^4}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^5x}{2\,e^3}-\frac{5\sqrt{-e^2x^2+d^2}\,d^5x}{8\,e^3}+\frac{\sqrt{e^2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")

[Out] -1/4*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^5*x + d*e^4) - 1/2*I*d^7*arcsin(e*x/d +
2)/e^4 - 5/8*d^7*arcsin(e*x/d)/e^4 + 1/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
5*x/e^3 - 5/8*sqrt(-e^2*x^2 + d^2)*d^5*x/e^3 + sqrt(e^2*x^2 + 4*d*e*x + 3*d
^2)*d^6/e^4 + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^3 - 5/12*(-e^2*x^2 + d^2)^

```
(3/2)*d^4/e^4 - 1/3*(-e^2*x^2 + d^2)^(5/2)*d*x/e^3 + 3/5*(-e^2*x^2 + d^2)^(
5/2)*d^2/e^4 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(d^2 - e^2 x^2\right)^{5/2}}{(d + e x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [A] time = 12.03, size = 450, normalized size = 2.63

$$d^2 \left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - 2*d*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*
x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*
e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e
**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecewise((-8*d**6*s
qrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*
e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*
x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.160 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=142

$$\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}$$

```
[Out] 2/5*d*x^2*(-e^2*x^2+d^2)^(3/2)/e-1/6*x^3*(-e^2*x^2+d^2)^(3/2)+1/120*d^2*(-4
5*e*x+32*d)*(-e^2*x^2+d^2)^(3/2)/e^3+3/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))/e^3+3/16*d^4*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.18, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (3*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (2*d*x^2*(d^2 - e^2*x^2)^(3/2))/(5
*e) - (x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^2*(32*d - 45*e*x)*(d^2 - e^2*x^2)^
(3/2))/(120*e^3) + (3*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int x^2(d-ex)^2\sqrt{d^2-e^2x^2}\,dx\\
&= -\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^2\left(-9d^2e^2+12de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{6e^2}\\
&= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{\int x\left(-24d^3e^3+45d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{30e^4}\\
&= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{\left(3d^4\right)\int \sqrt{}}{}\\
&= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e\right.}{120e^3}\\
&= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e\right.}{120e^3}\\
&= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e\right.}{120e^3}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 102, normalized size = 0.72

$$
\frac{45d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(64d^5-45d^4ex+32d^3e^2x^2+50d^2e^3x^3-96de^4x^4+40e^5x^5\right)}{240e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x^3
 - 96*d*e^4*x^4 + 40*e^5*x^5) + 45*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(
240*e^3)
```

fricas [A] time = 0.64, size = 106, normalized size = 0.75

$$-\frac{90\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,e^5x^5-96\,de^4x^4+50\,d^2e^3x^3+32\,d^3e^2x^2-45\,d^4ex+64\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(90*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (40*e^5*x^5 - 96
*d*e^4*x^4 + 50*d^2*e^3*x^3 + 32*d^3*e^2*x^2 - 45*d^4*e*x + 64*d^5)*sqrt(-e
^2*x^2 + d^2))/e^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 303, normalized size = 2.13

$$-\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,e^2}+\frac{5d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^2}+\frac{5\sqrt{-e^2x^2+d^2}\,d^4x}{16e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4x}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/6*x*(-e^2*x^2+d^2)^(5/2)/e^2+5/24/e^2*d^2*x*(-e^2*x^2+d^2)^(3/2)+5/16*d^4
*x*(-e^2*x^2+d^2)^(1/2)/e^2+5/16/e^2*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e
^2*x^2+d^2)^(1/2)*x)-1/15*d/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/12*d^
2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-1/8*d^4/e^2*(2*(x+d/e)*d*e-(x+d
/e)^2*e^2)^(1/2)*x-1/8*d^6/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)*x)+1/3*d/e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(7/2)
```

maxima [C] time = 1.03, size = 230, normalized size = 1.62

$$\frac{i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3}+\frac{5\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{4\left(e^4x+de^3\right)}-\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^4x}{8\,e^2}+\frac{5\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e^2}-\sqrt{e^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*I*d^6*arcsin(e*x/d + 2)/e^3 + 5/16*d^6*arcsin(e*x/d)/e^3 + 1/4*(-e^2*x^
2 + d^2)^(5/2)*d^2/(e^4*x + d*e^3) - 1/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
4*x/e^2 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x
+ 3*d^2)*d^5/e^3 - 7/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e^2 + 5/12*(-e^2*x^2 +
 d^2)^(3/2)*d^3/e^3 + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e^2 - 2/5*(-e^2*x^2 + d^
2)^(5/2)*d/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [C] time = 14.45, size = 541, normalized size = 3.81

$$d^2\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)-2de\left(\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} \\ \frac{x^4\sqrt{d^2}}{4} \end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 2*d*
e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2
 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*s
qrt(d**2)/4, True)) + e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d*
*5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(
6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)
/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2
*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.161 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}$$

```
[Out] -1/6*d*x*(-e^2*x^2+d^2)^(3/2)/e-2/15*(-e^2*x^2+d^2)^(5/2)/e^2-1/3*(-e^2*x^2
+d^2)^(7/2)/e^2/(e*x+d)^2-1/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/4*
d^3*x*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {793, 665, 195, 217, 203}

$$-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(d^3*x*Sqrt[d^2 - e^2*x^2])/(4*e) - (d*x*(d^2 - e^2*x^2)^(3/2))/(6*e) - (2
*(d^2 - e^2*x^2)^(5/2))/(15*e^2) - (d^2 - e^2*x^2)^(7/2)/(3*e^2*(d + e*x)^2
) - (d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{3e}$$
$$= -\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{(2d)\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{3e}$$
$$= -\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{2e}$$
$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\int \frac{1}{\sqrt{d^2-e^2}}}{4e}$$
$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\,\mathrm{Subst}\left(\right.}{}$$
$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\tan^{-1}\left(\right.}{4e^2}$$

Mathematica [A] time = 0.08, size = 91, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(-28d^4+15d^3ex+16d^2e^2x^2-30de^3x^3+12e^4x^4\right)-15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{60e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x^3
+ 12*e^4*x^4) - 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(60*e^2)
```

fricas [A] time = 0.81, size = 94, normalized size = 0.69

$$\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(12\,e^4x^4-30\,de^3x^3+16\,d^2e^2x^2+15\,d^3ex-28\,d^4\right)\sqrt{-e^2x^2+d^2}}{60\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (12*e^4*x^4 - 30*d
*e^3*x^3 + 16*d^2*e^2*x^2 + 15*d^3*e*x - 28*d^4)*sqrt(-e^2*x^2 + d^2))/e^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

[Out] sage0*x

maple [A] time = 0.01, size = 198, normalized size = 1.46

$$-\frac{d^5 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{4\sqrt{e^2}\, e}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\, d^3 x}{4e}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} dx}{6e}-\frac{2\left(2\left(x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -2/15/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/6/e*d*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(3/2)*x-1/4/e*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-1/4/e*d^5/
(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/e
^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [C] time = 1.00, size = 167, normalized size = 1.23

$$\frac{i\, d^5 \arcsin\left(\frac{ex}{d}+2\right)}{4\, e^2}-\frac{\sqrt{e^2x^2+4\, dex+3\, d^2}\, d^3 x}{4\, e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d}{4\left(e^3 x+de^2\right)}-\frac{\sqrt{e^2x^2+4\, dex+3\, d^2}\, d^4}{2\, e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} dx}{4\, e}-\frac{5\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*d^5*arcsin(e*x/d + 2)/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
/e - 1/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^3*x + d*e^2) - 1/2*sqrt(e^2*x^2 + 4*d*
e*x + 3*d^2)*d^4/e^2 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e - 5/12*(-e^2*x^2 +
d^2)^(3/2)*d^2/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2-e^2\, x^2\right)^{5/2}}{\left(d+e\, x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [A] time = 8.57, size = 321, normalized size = 2.36

$$d^2\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - 2*d*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**
```

```
3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/
d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
+ 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4)
- d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/
5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

3.162 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] 5/12*d*(-e^2*x^2+d^2)^(3/2)/e+1/4*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/e+5/8*d^4*a
rctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {655, 671, 641, 195, 217, 203}

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (5*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + ((d
- e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(8*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx \\
&= \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}(5d)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx \\
&= \frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}\left(5d^2\right)\int \sqrt{d^2-e^2x^2}\,dx \\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}} \\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\mathrm{Subst}\left(\int \frac{1}{1+} \right. \\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.74

$$
\frac{15d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(16d^3+9d^2ex-16de^2x^2+6e^3x^3\right)}{24e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3) + 15*d
^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e)
```

fricas [A] time = 0.58, size = 84, normalized size = 0.78

$$
-\frac{30\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,e^3x^3-16\,de^2x^2+9\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(30*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (6*e^3*x^3 - 16*d
*e^2*x^2 + 9*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
sage_0x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

[Out] sage0*x

maple [B] time = 0.01, size = 194, normalized size = 1.80

$$\frac{5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2x}{8}+\frac{5\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}x}{12}+\frac{\left(2\left(x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/3/e^3/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+1/3/e/d*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(5/2)+5/12*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+5/8*d^2
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+5/8*d^4/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 0.99, size = 119, normalized size = 1.10

$$-\frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e}+\frac{5}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^2x+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^3}{4\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\left(e^2x+de\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -5/8*I*d^4*arcsin(e*x/d + 2)/e + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^2*x
+ 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e + 1/4*(-e^2*x^2 + d^2)^(5/2)/(e
^2*x + d*e) + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2, x)
```

sympy [C] time = 9.35, size = 350, normalized size = 3.24

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{e}{}\right.}{8e^3}\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 2*d*
```

```
e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(
3*e**2), True)) + e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/
(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*
d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**
2)), True))
```

3.163 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^2}\,dx$

Optimal. Leaf size=96

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)-d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^3*arctanh(
(-e^2*x^2+d^2)^(1/2)/d)+d*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1809, 815, 844, 217, 203, 266, 63, 208}

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] d*(d - e*x)*Sqrt[d^2 - e^2*x^2] - (d^2 - e^2*x^2)^(3/2)/3 - d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= -\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-3d^2e^2+6de^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{3e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{6d^4e^4-6d^3e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{6e^4}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + d^4\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \left(d^3e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^4\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \left(d^3e\right)$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^4\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,d\right.}{e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2}}{d}\right.$$

Mathematica [A]   time = 0.10, size = 96, normalized size = 1.00

$$d^3\log(x)+\sqrt{d^2-e^2x^2}\left(\frac{2d^2}{3}-dex+\frac{e^2x^2}{3}\right)-d^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((2*d^2)/3 - d*e*x + (e^2*x^2)/3) - d^3*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]] + d^3*Log[x] - d^3*Log[d + Sqrt[d^2 - e^2*x^2]]
```

fricas [A]   time = 0.84, size = 95, normalized size = 0.99

$$2\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{3}\left(e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 2*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^3*log(-(d - sqrt(-e^2*x
^2 + d^2))/x) + 1/3*(e^2*x^2 - 3*d*e*x + 2*d^2)*sqrt(-e^2*x^2 + d^2)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B]   time = 0.01, size = 290, normalized size = 3.02

$$-\frac{d^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{d^3e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}-\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\;dex+\sqrt{-e^2x^2+d^2}\;d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x)
```

```
[Out] 1/5/d^2*(-e^2*x^2+d^2)^(5/2)+1/3*(-e^2*x^2+d^2)^(3/2)+d^2*(-e^2*x^2+d^2)^(1
/2)-d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-8/15/d
^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-2/3/d*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(3/2)*x-d*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-d^3*e/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/d^2/e^2/(x+d/e)^2*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [A]   time = 0.99, size = 103, normalized size = 1.07

$$-d^3\arcsin\left(\frac{ex}{d}\right)-d^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\sqrt{-e^2x^2+d^2}\,dex+\sqrt{-e^2x^2+d^2}\,d^2-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -d^3*arcsin(e*x/d) - d^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)
) - sqrt(-e^2*x^2 + d^2)*d*e*x + sqrt(-e^2*x^2 + d^2)*d^2 - 1/3*(-e^2*x^2 +
 d^2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2), x)
```

sympy [C] time = 14.83, size = 267, normalized size = 2.78

$$d^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d*
*2)/2, True)) + e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 -
e**2*x**2)**(3/2)/(3*e**2), True))
```

3.164 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(-e^2*x^2+d^2)^(3/2)/x-1/2*d^2*e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+2*d^2*e*
arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] -(e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/2 - (d^2 - e^2*x^2)^(3/2)/x - (d^2*e*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + 2*d^2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^2}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \frac{\int \frac{\left(2d^3e+d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{d^2}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} + \frac{\int \frac{-4d^5e^3-d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2e^2}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \left(2d^3e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(d^2e^2\right)\int \frac{1}{\sqrt{d^2 -}}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \left(d^3e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \frac{1}{2}\left(d^2\right.$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\left(2d^3\right)\text{Subst}\left(\int \right.}{}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.95

$$\left(-\frac{d^2}{x}-2de+\frac{e^2x}{2}\right)\sqrt{d^2-e^2x^2}+2d^2e\log\left(\sqrt{d^2-e^2x^2}+d\right)-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2d^2e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] (-2*d*e - d^2/x + (e^2*x)/2)*Sqrt[d^2 - e^2*x^2] - (d^2*e*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/2 - 2*d^2*e*Log[x] + 2*d^2*e*Log[d + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.62, size = 111, normalized size = 1.06

$$\frac{2\,d^2ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-4\,d^2ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-4\,d^2ex+\left(e^2x^2-4\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 4*d^2*e*x*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - 4*d^2*e*x + (e^2*x^2 - 4*d*e*x - 2*d^2)*sqrt
(-e^2*x^2 + d^2))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 425, normalized size = 4.05

$$\frac{2d^3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{11d^2e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{15d^2e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x)
```

```
[Out] -1/d^4/x*(-e^2*x^2+d^2)^(7/2)-1/d^4*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4/d^2*e^2*
x*(-e^2*x^2+d^2)^(3/2)-15/8*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/8*d^2*e^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-2/5/d^3*e*(-e^2*x^2+d^2)^(5
/2)-2/3/d*e*(-e^2*x^2+d^2)^(3/2)-2*d*e*(-e^2*x^2+d^2)^(1/2)+2*d^3*e/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+11/15/d^3*e*(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(5/2)+11/12/d^2*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2
)*x+11/8*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+11/8*d^2*e^2/(e^2)^(1/2)
*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/3/d^3/e/(x+d/e
)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [A] time = 1.12, size = 112, normalized size = 1.07

$$-\frac{1}{2}\,d^2e\arcsin\left(\frac{ex}{d}\right)+2\,d^2e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,e^2x-2\,\sqrt{-e^2x^2+d^2}\,de-\frac{\sqrt{-e^2x^2+d^2}\,d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*e*arcsin(e*x/d) + 2*d^2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2
)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2*x - 2*sqrt(-e^2*x^2 + d^2)*d*e -
 sqrt(-e^2*x^2 + d^2)*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^2\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2), x)
```

sympy [C] time = 9.85, size = 347, normalized size = 3.30

$$d^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\,\mathrm{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\,\mathrm{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\mathrm{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}}\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\mathrm{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
```

```
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - 2*d*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d
/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) + e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I
*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d
**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**
2*x**2/d**2)/2, True))
```

3.165 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=110

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(3/2)/x^2+2*d*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*
d*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/
x
```

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - (d^2 - e^2*x^2)^(3/2)/(2*x^2) +
 2*d*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (d*e^2*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^3}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{\int \frac{\left(4d^3e-d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{2d^2}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{\int \frac{2d^4e^2+8d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{1}{2}\left(d^2e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \left(2de^3\right)\int \frac{1}{\sqrt{d^2-e}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{1}{4}\left(d^2e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) + \left(2de$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^2\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-}}{d}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.93

$$\left(-\frac{d^2}{2x^2}+\frac{2de}{x}+e^2\right)\sqrt{d^2-e^2x^2}-\frac{1}{2}de^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}de^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e^2 - d^2/(2*x^2) + (2*d*e)/x)*Sqrt[d^2 - e^2*x^2] + 2*d*e^2*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]] + (d*e^2*Log[x])/2 - (d*e^2*Log[d + Sqrt[d^2 - e^2*x^2
]])/2
```

fricas [A] time = 0.98, size = 119, normalized size = 1.08

$$-\frac{8\,de^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-de^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,de^2x^2-\left(2\,e^2x^2+4\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*d*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - d*e^2*x^2*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d*e^2*x^2 - (2*e^2*x^2 + 4*d*e*x - d^2)
*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 456, normalized size = 4.15

$$-\frac{d^2e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{7d\,e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{4\sqrt{e^2}}+\frac{15d\,e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}}+\frac{15\sqrt{-e^2x^2+d^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x)
```

```
[Out] 2/d^5*e/x*(-e^2*x^2+d^2)^(7/2)+2/d^5*e^3*x*(-e^2*x^2+d^2)^(5/2)+5/2/d^3*e^3
*x*(-e^2*x^2+d^2)^(3/2)+15/4/d*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/4*d*e^3/(e^2)^
(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/d^4/x^2*(-e^2*x^2+d^2)
^(7/2)+1/10/d^4*e^2*(-e^2*x^2+d^2)^(5/2)+1/6/d^2*e^2*(-e^2*x^2+d^2)^(3/2)+1
/2*e^2*(-e^2*x^2+d^2)^(1/2)-1/2*d^2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x)-14/15/d^4*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
-7/6/d^3*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-7/4/d*e^3*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x-7/4*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/d^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(7/2)
```

maxima [A] time = 0.99, size = 111, normalized size = 1.01

$$2\,de^2\arcsin\left(\frac{ex}{d}\right)-\frac{1}{2}\,de^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,e^2+\frac{2\,\sqrt{-e^2x^2+d^2}\,de}{x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 2*d*e^2*arcsin(e*x/d) - 1/2*d*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)
*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e/x -
1/2*(-e^2*x^2 + d^2)^(3/2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}}{x^3\,\left(d + e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2), x)
```

sympy [C] time = 10.17, size = 347, normalized size = 3.15

$$d^2\left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{fo} \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{ot} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) - 2*d*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/
d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/
(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d
*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1
), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/s
qrt(-d**2/(e**2*x**2) + 1), True))
```

3.166 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=102

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/x^3-e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-e^3*arct
anh((-e^2*x^2+d^2)^(1/2)/d)+e*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 811, 844, 217, 203, 266, 63, 208}

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] (e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2 - e^2*x^2)^(3/2)/(3*x^3) - e^3
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
```

```
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^4}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}-\frac{\int \frac{\left(6d^3e-3d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{3d^2}\\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+\frac{\int \frac{12d^5e^3-12d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{12d^4}\\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+\left(de^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-e^4\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+\frac{1}{2}\left(de^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)-e^4\,\text{Subs}\\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}-e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-(de)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx\right.\\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}-e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 96, normalized size = 0.94

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^2-3dex+2e^2x^2\right)}{3x^3}-e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+e^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] -1/3*(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + 2*e^2*x^2))/x^3 - e^3*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + e^3*Log[x] - e^3*Log[d + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.93, size = 106, normalized size = 1.04

$$\frac{6\,e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(2\,e^2x^2-3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*e^3*x^3*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - (2*e^2*x^2 - 3*d*e*x + d^2)*sqrt(-e^2*x^2 +
d^2))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.71index.cc index_m i_lex_is_greater Error: Bad Ar
gument Value
```

maple [B] time = 0.02, size = 479, normalized size = 4.70

$$-\frac{d\,e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{17e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{25e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{17\sqrt{2\left(x+\frac{d}{e}\right)de-}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x)
```

```
[Out] 17/12/d^4*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+17/8/d^2*e^4*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/3/d^5*e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(7/2)+1/d^5*e/x^2*(-e^2*x^2+d^2)^(7/2)-d*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^
2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-5/3/d^6*e^2/x*(-e^2*x^2+d^2)^(7/2)-5/3/d^
6*e^4*x*(-e^2*x^2+d^2)^(5/2)-25/12/d^4*e^4*x*(-e^2*x^2+d^2)^(3/2)-25/8/d^2*
e^4*x*(-e^2*x^2+d^2)^(1/2)+17/15/d^5*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2
)+17/8*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/
2)*x)-1/3/d^4/x^3*(-e^2*x^2+d^2)^(7/2)+1/5/d^5*e^3*(-e^2*x^2+d^2)^(5/2)+1/3
/d^3*e^3*(-e^2*x^2+d^2)^(3/2)+1/d*e^3*(-e^2*x^2+d^2)^(1/2)-25/8*e^4/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 134, normalized size = 1.31

$$-e^3 \arcsin\left(\frac{ex}{d}\right) - e^3 \log\left(\frac{2 d^2}{|x|} + \frac{2 \sqrt{-e^2x^2 + d^2}\, d}{|x|}\right) + \frac{\sqrt{-e^2x^2 + d^2}\, e^3}{d} - \frac{\sqrt{-e^2x^2 + d^2}\, e^2}{x} + \frac{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} e}{dx^2} - \frac{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}}{3\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -e^3*arcsin(e*x/d) - e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)
) + sqrt(-e^2*x^2 + d^2)*e^3/d - sqrt(-e^2*x^2 + d^2)*e^2/x + (-e^2*x^2 + d
^2)^(3/2)*e/(d*x^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\, x^2\right)^{5/2}}{x^4\, (d + e\, x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2), x)
```

sympy [C] time = 9.73, size = 338, normalized size = 3.31

$$d^2 \left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right) - 2de \left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 2*d*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2))
> 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d),
 True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d
/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**
2/d**2)), True))
```

3.167 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2} \, dx$

Optimal. Leaf size=108

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] -1/4*(-e^2*x^2+d^2)^(3/2)/x^4+2/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^3+5/8*e^4*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d-5/8*e^2*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {852, 1807, 807, 266, 47, 63, 208}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] (-5*e^2*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d^2 - e^2*x^2)^(3/2)/(4*x^4) + (2*e
*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) + (5*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(8*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^5}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\int \frac{\left(8d^3e-5d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{4d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{1}{4}\left(5e^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{1}{8}\left(5e^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}-\frac{1}{16}\left(5e^4\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.\\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{1}{8}\left(5e^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{\right.\\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.88

$$
-\frac{-15e^4x^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(6d^3-16d^2ex+9de^2x^2+16e^3x^3\right)+15e^4x^4\log(x)}{24dx^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(6*d^3 - 16*d^2*e*x + 9*d*e^2*x^2 + 16*e^3*x^3)
+ 15*e^4*x^4*Log[x] - 15*e^4*x^4*Log[d + Sqrt[d^2 - e^2*x^2]])/(d*x^4)
```

fricas [A] time = 0.84, size = 86, normalized size = 0.80

$$-\frac{15\,e^{4}x^{4}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(16\,e^{3}x^{3}+9\,de^{2}x^{2}-16\,d^{2}ex+6\,d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24\,dx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(15*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 + 9*d*e^
2*x^2 - 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 0.69Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 513, normalized size = 4.75

$$-\frac{5e^{5}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{2\sqrt{e^{2}}\,d}+\frac{5e^{5}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}\,d}+\frac{5e^{4}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{8\sqrt{d^{2}}}-\frac{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x\right.}}{2d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x)
```

```
[Out] -1/3/d^6*e^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-5/3/d^5*e^5*(2*(
x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-5/2/d^3*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x-5/2/d*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*
e^2)^(1/2)*x)+2/3/d^5*e/x^3*(-e^2*x^2+d^2)^(7/2)+4/3/d^7*e^5*x*(-e^2*x^2+d^
2)^(5/2)+5/3/d^5*e^5*x*(-e^2*x^2+d^2)^(3/2)+5/2/d^3*e^5*x*(-e^2*x^2+d^2)^(1
/2)+5/2/d*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+4/3/d^
7*e^3/x*(-e^2*x^2+d^2)^(7/2)-9/8/d^6*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-4/3/d^6*e
^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-5/8/d^2*e^4*(-e^2*x^2+d^2)^(1/2)+5/8
*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4/d^4/x
^4*(-e^2*x^2+d^2)^(7/2)-1/8/d^6*e^4*(-e^2*x^2+d^2)^(5/2)-5/24/d^4*e^4*(-e^2
*x^2+d^2)^(3/2)
```

maxima [A] time = 1.01, size = 130, normalized size = 1.20

$$\frac{5\,e^{4}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{8\,d}-\frac{5\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}}{8\,d^{2}}-\frac{5\,\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{8\,d^{2}x^{2}}+\frac{2\,\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{3\,dx^{3}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 5/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 5/8*sqrt(-e
^2*x^2 + d^2)*e^4/d^2 - 5/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^2) + 2/3*(-e^
2*x^2 + d^2)^(3/2)*e/(d*x^3) - 1/4*(-e^2*x^2 + d^2)^(3/2)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2 x^2\right)^{5/2}}{x^5 \left(d + e x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2), x)
```

sympy [C] time = 12.36, size = 422, normalized size = 3.91

$$d^2 \left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4 \operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4 \operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right) - 2de \left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True)) + e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1))
 + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2
/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d
/(e*x))/(2*d), True))
```

3.168 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

Optimal. Leaf size=140

$$-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}+\frac{e(d^2-e^2x^2)^{3/2}}{2dx^4}-\frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}+\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(3/2)/x^5+1/2*e*(-e^2*x^2+d^2)^(3/2)/d/x^4-7/15*e^2*(-e
^2*x^2+d^2)^(3/2)/d^2/x^3-1/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/4*e
^3*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A] time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}-\frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3}+\frac{e(d^2-e^2x^2)^{3/2}}{2dx^4}-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (e^3*Sqrt[d^2 - e^2*x^2])/(4*d*x^2) - (d^2 - e^2*x^2)^(3/2)/(5*x^5) + (e*(d
^2 - e^2*x^2)^(3/2))/(2*d*x^4) - (7*e^2*(d^2 - e^2*x^2)^(3/2))/(15*d^2*x^3)
 - (e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(4*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^6}\,dx \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} - \frac{\int \frac{(10d^3e-7d^2e^2x)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{5d^2} \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} + \frac{\int \frac{(28d^4e^2-10d^3e^3x)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{20d^4} \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx}{2d} \\
&= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)}{4d} \\
&= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} + \frac{e^5\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right)}{8} \\
&= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}}\right)}{} \\
&= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-}}{}\right)}{4d^2}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.76

$$
\frac{-15e^5x^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-12d^4+30d^3ex-16d^2e^2x^2-15de^3x^3+28e^4x^4\right)+15e^5x^5\log(x)}{60d^2x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
+ 28*e^4*x^4) + 15*e^5*x^5*Log[x] - 15*e^5*x^5*Log[d + Sqrt[d^2 - e^2*x^2]]
)/(60*d^2*x^5)
```

fricas [A] time = 0.88, size = 97, normalized size = 0.69

$$
\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(28\,e^4x^4-15\,de^3x^3-16\,d^2e^2x^2+30\,d^3ex-12\,d^4\right)\sqrt{-e^2x^2+d^2}}{60\,d^2x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (28*e^4*x^4 - 15*d*e^
3*x^3 - 16*d^2*e^2*x^2 + 30*d^3*e*x - 12*d^4)*sqrt(-e^2*x^2 + d^2))/(d^2*x^
5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 0.69Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 541, normalized size = 3.86

$$-\frac{e^5 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4\sqrt{d^2}\,d}+\frac{23e^6 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\,d^2}-\frac{23e^6 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,d^2}+\frac{23\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x\right.}}{8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x)
```

```
[Out] 1/3/d^7*e^3/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+23/12/d^6*e^6*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+23/8/d^4*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(1/2)*x+23/8/d^2*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)*x)-13/15/d^6*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-23/15/d^8*e^4/x*(
-e^2*x^2+d^2)^(7/2)-23/15/d^8*e^6*x*(-e^2*x^2+d^2)^(5/2)-23/12/d^6*e^6*x*(-
e^2*x^2+d^2)^(3/2)-23/8/d^4*e^6*x*(-e^2*x^2+d^2)^(1/2)-23/8/d^2*e^6/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2/d^5*e/x^4*(-e^2*x^2+d^2
)^(7/2)+5/4/d^7*e^3/x^2*(-e^2*x^2+d^2)^(7/2)-1/4/d*e^5/(d^2)^(1/2)*ln((2*d^
2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+23/15/d^7*e^5*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(5/2)-1/5/d^4/x^5*(-e^2*x^2+d^2)^(7/2)+1/20/d^7*e^5*(-e^2*x^2+d^2
)^(5/2)+1/12/d^5*e^5*(-e^2*x^2+d^2)^(3/2)+1/4/d^3*e^5*(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.99, size = 155, normalized size = 1.11

$$-\frac{e^5 \log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{4\,d^2}+\frac{\sqrt{-e^2x^2+d^2}\,e^5}{4\,d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{4\,d^3x^2}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{15\,d^2x^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{2\,dx^4}-\frac{\left(-e^2x^2+\right.}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/4*sqrt
(-e^2*x^2 + d^2)*e^5/d^3 + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^2) - 7/15*
(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^3) + 1/2*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^4)
 - 1/5*(-e^2*x^2 + d^2)^(3/2)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2), x)
```

sympy [C] time = 13.42, size = 660, normalized size = 4.71

$$d^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)-2de\left(\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - 2*d*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*
x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2
)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqr
t(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(
e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(
-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

3.169 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}-\frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4}-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}+\frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3}$$

```
[Out] -1/6*(-e^2*x^2+d^2)^(3/2)/x^6+2/5*e*(-e^2*x^2+d^2)^(3/2)/d/x^5-3/8*e^2*(-e^
2*x^2+d^2)^(3/2)/d^2/x^4+4/15*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^3+3/16*e^6*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-3/16*e^4*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A] time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3}-\frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] (-3*e^4*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (d^2 - e^2*x^2)^(3/2)/(6*x^6) +
 (2*e*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (3*e^2*(d^2 - e^2*x^2)^(3/2))/(8*d
^2*x^4) + (4*e^3*(d^2 - e^2*x^2)^(3/2))/(15*d^3*x^3) + (3*e^6*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^7}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} - \frac{\int \frac{\left(12d^3e-9d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^6}dx}{6d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} + \frac{\int \frac{\left(45d^4e^2-24d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^5}dx}{30d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} - \frac{\int \frac{\left(96d^5e^3-45d^4e^4x\right)\sqrt{d^2-e^2x^2}}{x^4}dx}{120d^6}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{\left(3e^4\right)\int \frac{\sqrt{d^2}}{}}{8d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{\left(3e^4\right)\text{Subs}}{}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right.}{15d^3x^3}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right.}{15d^3x^3}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right.}{15d^3x^3}$$

Mathematica [A] time = 0.23, size = 117, normalized size = 0.69

$$-\frac{-45e^6x^6\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(40d^5-96d^4ex+50d^3e^2x^2+32d^2e^3x^3-45de^4x^4+64e^5x^5\right)+45e}{240d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2*
e^3*x^3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 45*e^6*x^6*Log[x] - 45*e^6*x^6*Log[d
 + Sqrt[d^2 - e^2*x^2]])/(d^3*x^6)
```

fricas [A] time = 0.92, size = 108, normalized size = 0.64

$$-\frac{45\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(64\,e^5x^5-45\,de^4x^4+32\,d^2e^3x^3+50\,d^3e^2x^2-96\,d^4ex+40\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(45*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (64*e^5*x^5 - 45*d*
e^4*x^4 + 32*d^2*e^3*x^3 + 50*d^3*e^2*x^2 - 96*d^4*e*x + 40*d^5)*sqrt(-e^2*
x^2 + d^2))/(d^3*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 1.04Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 566, normalized size = 3.35

$$\frac{3e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}\,d^2}-\frac{13e^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{4\sqrt{e^2}\,d^3}+\frac{13e^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,d^3}-\frac{13\sqrt{2\left(x+\frac{d}{e}\right)de-\left(\right.}}{4d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x)
```

```
[Out] -13/6/d^7*e^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-13/4/d^5*e^7*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(1/2)*x-13/4/d^3*e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/d^8*e^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x
+d/e)^2*e^2)^(7/2)+2/5/d^5*e/x^5*(-e^2*x^2+d^2)^(7/2)+16/15/d^7*e^3/x^3*(-e
^2*x^2+d^2)^(7/2)+13/4/d^5*e^7*x*(-e^2*x^2+d^2)^(1/2)+13/4/d^3*e^7/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+26/15/d^9*e^5/x*(-e^2*x^2+d^
2)^(7/2)+26/15/d^9*e^7*x*(-e^2*x^2+d^2)^(5/2)+13/6/d^7*e^7*x*(-e^2*x^2+d^2)
^(3/2)-17/24/d^6*e^2/x^4*(-e^2*x^2+d^2)^(7/2)-23/16/d^8*e^4/x^2*(-e^2*x^2+d
^2)^(7/2)+3/16/d^2*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)-26/15/d^8*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/6/d^4/x^6*(-e^
2*x^2+d^2)^(7/2)-3/80/d^8*e^6*(-e^2*x^2+d^2)^(5/2)-1/16/d^6*e^6*(-e^2*x^2+d
^2)^(3/2)-3/16/d^4*e^6*(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.01, size = 180, normalized size = 1.07

$$\frac{3\,e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{16\,d^3}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^6}{16\,d^4}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{16\,d^4x^2}+\frac{4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{15\,d^3x^3}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{8\,d^2x^4}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 3/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/16*sqr
t(-e^2*x^2 + d^2)*e^6/d^4 - 3/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^2) + 4/1
5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^3) - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^
2*x^4) + 2/5*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^5) - 1/6*(-e^2*x^2 + d^2)^(3/2)/
x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^7\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2), x)
```

sympy [C] time = 19.72, size = 808, normalized size = 4.78

$$d^2\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{3id^3\sqrt{-1}}{-15d^2x^5+} \\ \frac{3d^3\sqrt{1-}}{-15d^2x^5+}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 2*d*e*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e*
*2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqr
t(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**
4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sq
rt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I
*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3
), True))
```

3.170 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx$

Optimal. Leaf size=198

$$-\frac{(d^2-e^2x^2)^{3/2}}{7x^7}+\frac{e(d^2-e^2x^2)^{3/2}}{3dx^6}-\frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}-\frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3}+\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}$$

```
[Out] -1/7*(-e^2*x^2+d^2)^(3/2)/x^7+1/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^6-11/35*e^2*(-
e^2*x^2+d^2)^(3/2)/d^2/x^5+1/4*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-22/105*e^4*
(-e^2*x^2+d^2)^(3/2)/d^4/x^3-1/8*e^7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/
8*e^5*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A] time = 0.24, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3}+\frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4}-\frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5}+\frac{e(d^2-e^2x^2)^{3/2}}{3dx^6}-\frac{(d^2-e^2x^2)^{3/2}}{7x^7}-\frac{e^7 \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) - (d^2 - e^2*x^2)^(3/2)/(7*x^7) + (e*
(d^2 - e^2*x^2)^(3/2))/(3*d*x^6) - (11*e^2*(d^2 - e^2*x^2)^(3/2))/(35*d^2*x
^5) + (e^3*(d^2 - e^2*x^2)^(3/2))/(4*d^3*x^4) - (22*e^4*(d^2 - e^2*x^2)^(3/
2))/(105*d^4*x^3) - (e^7*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^8}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} - \frac{\int \frac{(14d^3e-11d^2e^2x)\sqrt{d^2-e^2x^2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} + \frac{\int \frac{(66d^4e^2-42d^3e^3x)\sqrt{d^2-e^2x^2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} - \frac{\int \frac{(210d^5e^3-132d^4e^4x)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{210d^6}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} + \int \frac{(528d^6e^4\ldots}{}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-\ldots}{105d\ldots}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-\ldots}{105d\ldots}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)\ldots}{4d^3x^4}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)\ldots}{4d^3x^4}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)\ldots}{4d^3x^4}$$

Mathematica [A] time = 0.23, size = 128, normalized size = 0.65

$$\frac{-105e^7x^7\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-120d^6+280d^5ex-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4-105de\ldots\right.}{840d^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3*e^3
*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6) + 105*e^7*x^7*Log[x] -
 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(840*d^4*x^7)
```

fricas [A] time = 0.90, size = 119, normalized size = 0.60

$$\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(176\,e^6x^6-105\,de^5x^5+88\,d^2e^4x^4-70\,d^3e^3x^3-144\,d^4e^2x^2+280\,d^5ex-120\,d^6\ldots\right.}{840\,d^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (176*e^6*x^6 - 105*
d*e^5*x^5 + 88*d^2*e^4*x^4 - 70*d^3*e^3*x^3 - 144*d^4*e^2*x^2 + 280*d^5*e*x
 - 120*d^6)*sqrt(-e^2*x^2 + d^2))/(d^4*x^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 1.08Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 591, normalized size = 2.98

$$-\frac{e^7 \ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\, d^3}+\frac{29e^8 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\, d^4}-\frac{29e^8 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\, d^4}+\frac{29\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x\right.}}{8d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x)
```

```
[Out] 1/3/d^9*e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+29/12/d^8*e^8*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+29/8/d^6*e^8*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(1/2)*x+29/8/d^4*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)*x)-3/5/d^6*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+1/3/d^5*e/x^6*(-e^2
*x^2+d^2)^(7/2)-19/15/d^8*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-29/15/d^10*e^6/x*(-e
^2*x^2+d^2)^(7/2)-29/15/d^10*e^8*x*(-e^2*x^2+d^2)^(5/2)-29/12/d^8*e^8*x*(-e
^2*x^2+d^2)^(3/2)-29/8/d^6*e^8*x*(-e^2*x^2+d^2)^(1/2)-29/8/d^4*e^8/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+11/12/d^7*e^3/x^4*(-e^2*x^2+
d^2)^(7/2)+13/8/d^9*e^5/x^2*(-e^2*x^2+d^2)^(7/2)-1/8/d^3*e^7/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+29/15/d^9*e^7*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(5/2)+1/40/d^9*e^7*(-e^2*x^2+d^2)^(5/2)+1/24/d^7*e^7*(-e^2*
x^2+d^2)^(3/2)+1/8/d^5*e^7*(-e^2*x^2+d^2)^(1/2)-1/7/d^4/x^7*(-e^2*x^2+d^2)^
(7/2)
```

maxima [A] time = 0.99, size = 205, normalized size = 1.04

$$-\frac{e^7 \log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d^4}+\frac{\sqrt{-e^2x^2+d^2}\,e^7}{8\,d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8\,d^5x^2}-\frac{22\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{105\,d^4x^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{4\,d^3x^4}-\frac{11\left(-e^2x\right.}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 1/8*sqrt
(-e^2*x^2 + d^2)*e^7/d^5 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^5*x^2) - 22/10
5*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^
3*x^4) - 11/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^5) + 1/3*(-e^2*x^2 + d^2)^
(3/2)*e/(d*x^6) - 1/7*(-e^2*x^2 + d^2)^(3/2)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^8\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2), x)
```

sympy [C] time = 18.18, size = 835, normalized size = 4.22

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{7x^6}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{35d^2x^4}+\frac{4e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^4x^2}+\frac{8e^7\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{7x^6}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{35d^2x^4}+\frac{4ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^4x^2}+\frac{8ie^7\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^6} & \text{otherwise} \end{cases}\right)-2de\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\cdots}} \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\cdots}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 2*d*e*Pi
ecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d*
*2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**
5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), A
bs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) -
 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d
**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*
e**6*asin(d/(e*x))/(16*d**5), True)) + e**2*Piecewise((3*I*d**3*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**
2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (
3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x
**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*s
qrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqr
t(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True))
```

3.171 $\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] -1/5*d^3*(-e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)+17/15*d^2*(-e*x+d)/e^5/(-e^2*x
^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-2/15*(-13*e*x+15*d)/e^5/
(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {852, 1635, 1814, 12, 217, 203}

$$-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -(d^3*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (17*d^2*(d - e*x))/(15*e
^5*(d^2 - e^2*x^2)^(3/2)) - (2*(15*d - 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2]
) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(\frac{2d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{e}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.}{e^5}$$

Mathematica [A] time = 0.16, size = 106, normalized size = 0.86

$$\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{10e^5(d+ex)^3}+\frac{31d}{60e^5(d+ex)^2}-\frac{1}{8e^5(ex-d)}-\frac{193}{120e^5(d+ex)}\right)-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(-1/8*1/(e^5*(-d + e*x)) - d^2/(10*e^5*(d + e*x)^3) + (
31*d)/(60*e^5*(d + e*x)^2) - 193/(120*e^5*(d + e*x))) - ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]]/e^5
```

fricas [A] time = 0.95, size = 171, normalized size = 1.39

$$-\frac{16\,e^{4}x^{4} + 32\,de^{3}x^{3} - 32\,d^{3}ex - 16\,d^{4} - 30\left(e^{4}x^{4} + 2\,de^{3}x^{3} - 2\,d^{3}ex - d^{4}\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right) + \left(26\,e^{3}x^{3} + 22\,\ldots\right.}{15\left(e^{9}x^{4} + 2\,de^{8}x^{3} - 2\,d^{3}e^{6}x - d^{4}e^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(16*e^4*x^4 + 32*d*e^3*x^3 - 32*d^3*e*x - 16*d^4 - 30*(e^4*x^4 + 2*d*
e^3*x^3 - 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (26*
e^3*x^3 + 22*d*e^2*x^2 - 17*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^
4 + 2*d*e^8*x^3 - 2*d^3*e^6*x - d^4*e^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 198, normalized size = 1.61

$$\frac{4x}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}} - \frac{34x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{4}} - \frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,e^{4}} - \frac{d^{3}}{5\left(x+\frac{d}{e}\right)^{2}\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 4/(-e^2*x^2+d^2)^(1/2)/e^4*x-1/(e^2)^(1/2)/e^4*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)-2*d/e^5/(-e^2*x^2+d^2)^(1/2)+17/15/e^6*d^2/(x+d/e)/(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(1/2)-34/15/e^4/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-
1/5*d^3/e^7/(x+d/e)^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 1.02, size = 170, normalized size = 1.38

$$-\frac{d^{3}}{5\left(\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}x^{2} + 2\sqrt{-e^{2}x^{2}+d^{2}}\,de^{6}x + \sqrt{-e^{2}x^{2}+d^{2}}\,d^{2}e^{5}\right)} + \frac{17\,d^{2}}{15\left(\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}x + \sqrt{-e^{2}x^{2}+d^{2}}\,de^{5}\right)} + \frac{\ldots}{15\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/(sqrt(-e^2*x^2 + d^2)*e^7*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^6*x + s
qrt(-e^2*x^2 + d^2)*d^2*e^5) + 17/15*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt
(-e^2*x^2 + d^2)*d*e^5) + 26/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e*x/d
)/e^5 - 2*d/(sqrt(-e^2*x^2 + d^2)*e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4}}{\left(d^{2}-e^{2}\,x^{2}\right)^{3/2}\,\left(d+e\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.172 $\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d^2*(-e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(-e*x+d)/e^4/(-e^2*x^2+d^
2)^(3/2)+1/5*(-2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {852, 1635, 637}

$$\frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d - e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(-\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{-\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}dx}{15d^2}$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^3+4d^2ex+de^2x^2-2e^3x^3\right)}{5de^4(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*
(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.86, size = 116, normalized size = 1.17

$$\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3-de^2x^2-4\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(de^8x^4+2\,d^2e^7x^3-2\,d^4e^5x-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 - d*e^2*x^2 -
 4*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 + 2*d^2*e^7*x^3 - 2*d^
4*e^5*x - d^5*e^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 65, normalized size = 0.66

$$\frac{(-ex+d)\left(-2e^3x^3+d\,e^2x^2+4d^2ex+2d^3\right)}{5\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/5*(-e*x+d)*(-2*e^3*x^3+d*e^2*x^2+4*d^2*e*x+2*d^3)/(e*x+d)/d/e^4/(-e^2*x^2
+d^2)^(3/2)
```

maxima [A] time = 0.47, size = 157, normalized size = 1.59

$$\frac{d^2}{5\left(\sqrt{-e^2x^2+d^2}\,e^6x^2+2\sqrt{-e^2x^2+d^2}\,de^5x+\sqrt{-e^2x^2+d^2}\,d^2e^4\right)}-\frac{4\,d}{5\left(\sqrt{-e^2x^2+d^2}\,e^5x+\sqrt{-e^2x^2+d^2}\,de^4\right)}-\frac{}{5\sqrt{-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^5*x + sq
rt(-e^2*x^2 + d^2)*d^2*e^4) - 4/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2
*x^2 + d^2)*d*e^4) - 2/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^3) + 1/(sqrt(-e^2*x^2
+ d^2)*e^4)
```

mupad [B] time = 2.97, size = 66, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+4\,d^2\,e\,x+d\,e^2\,x^2-2\,e^3\,x^3\right)}{5\,d\,e^4\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 - 2*e^3*x^3 + d*e^2*x^2 + 4*d^2*e*x))/(5*d*e^
4*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.173 $\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}-\frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)-1/5*d/e^3/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2
)+7/15/e^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {852, 1635, 778, 191}

$$-\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -(d*(d - e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) + (7*(d - e*x))/(15*e^3*(d^2
 - e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^2(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}-\frac{5dx}{e}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}$$

$$= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.79

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^3+8d^2ex+2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e
^3*(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.88, size = 118, normalized size = 1.33

$$\frac{4\,e^4x^4+8\,de^3x^3-8\,d^3ex-4\,d^4-\left(e^3x^3+2\,de^2x^2+8\,d^2ex+4\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^7x^4+2\,d^3e^6x^3-2\,d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*e^4*x^4 + 8*d*e^3*x^3 - 8*d^3*e*x - 4*d^4 - (e^3*x^3 + 2*d*e^2*x^2
+ 8*d^2*e*x + 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 + 2*d^3*e^6*x^3 - 2
*d^5*e^4*x - d^6*e^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 65, normalized size = 0.73

$$\frac{(-ex+d)\left(e^3x^3+2d\,e^2x^2+8d^2ex+4d^3\right)}{15\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e^3*x^3+2*d*e^2*x^2+8*d^2*e*x+4*d^3)/(e*x+d)/d^2/e^3/(-e^2*x
^2+d^2)^(3/2)
```

maxima [A] time = 0.47, size = 136, normalized size = 1.53

$$-\frac{d}{5\left(\sqrt{-e^2x^2+d^2}\,e^5x^2+2\sqrt{-e^2x^2+d^2}\,de^4x+\sqrt{-e^2x^2+d^2}\,d^2e^3\right)}+\frac{7}{15\left(\sqrt{-e^2x^2+d^2}\,e^4x+\sqrt{-e^2x^2+d^2}\,de^3\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^4*x + sqr
t(-e^2*x^2 + d^2)*d^2*e^3) + 7/15/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x
^2 + d^2)*d*e^3) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^2)
```

mupad [B] time = 2.90, size = 66, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3+8\,d^2\,e\,x+2\,d\,e^2\,x^2+e^3\,x^3\right)}{15\,d^2\,e^3\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(4*d^3 + e^3*x^3 + 2*d*e^2*x^2 + 8*d^2*e*x))/(15*d^2
*e^3*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.174 $\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)+1/5/e^2/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-2/
15/d/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 659, 191}

$$\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2]) - 2/(15*d*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e}\\
&= \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\
&= \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(d^3+2d^2ex+8de^2x^2+4e^3x^3\right)}{15d^3e^2(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.90, size = 116, normalized size = 1.27

$$\frac{e^4x^4+2\,de^3x^3-2\,d^3ex-d^4-\left(4\,e^3x^3+8\,de^2x^2+2\,d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^6x^4+2\,d^4e^5x^3-2\,d^6e^3x-d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4 - (4*e^3*x^3 + 8*d*e^2*x^2 +
2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 + 2*d^4*e^5*x^3 - 2*d^6
*e^3*x - d^7*e^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 64, normalized size = 0.70

$$\frac{(-ex+d)\left(4e^3x^3+8d\,e^2x^2+2d^2ex+d^3\right)}{15\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/15*(-e*x+d)*(4*e^3*x^3+8*d*e^2*x^2+2*d^2*e*x+d^3)/(e*x+d)/d^3/e^2/(-e^2*x
^2+d^2)^(3/2)
```

maxima [A] time = 0.46, size = 138, normalized size = 1.52

$$\frac{1}{5\left(\sqrt{-e^2x^2+d^2}\,e^4x^2+2\sqrt{-e^2x^2+d^2}\,de^3x+\sqrt{-e^2x^2+d^2}\,d^2e^2\right)}-\frac{2}{15\left(\sqrt{-e^2x^2+d^2}\,de^3x+\sqrt{-e^2x^2+d^2}\,d^2e^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5/(sqrt(-e^2*x^2 + d^2)*e^4*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-
e^2*x^2 + d^2)*d^2*e^2) - 2/15/(sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-e^2*x^
2 + d^2)*d^2*e^2) + 4/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

mupad [B] time = 2.88, size = 65, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3+2\,d^2\,e\,x+8\,d\,e^2\,x^2+4\,e^3\,x^3\right)}{15\,d^3\,e^2\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 + 4*e^3*x^3 + 8*d*e^2*x^2 + 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.175 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*x/d^4/(-e^2*x^2+d^2)^(1/2)-1/5/d/e/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-1/5/d
^2/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2]) - 1/(5*d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2
]) - 1/(5*d^2*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{3\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d}\\
&= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}\\
&= \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^3+d^2ex+4de^2x^2+2e^3x^3\right)}{5d^4e(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.72, size = 115, normalized size = 1.26

$$-\frac{2\,e^4x^4 + 4\,de^3x^3 - 4\,d^3ex - 2\,d^4 + \left(2\,e^3x^3 + 4\,de^2x^2 + d^2ex - 2\,d^3\right)\sqrt{-e^2x^2 + d^2}}{5\,\left(d^4e^5x^4 + 2\,d^5e^4x^3 - 2\,d^7e^2x - d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 + 4*d*e^2*x^
2 + d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 + 2*d^5*e^4*x^3 - 2
*d^7*e^2*x - d^8*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 66, normalized size = 0.73

$$-\frac{(-ex + d)\left(-2e^3x^3 - 4d\,e^2x^2 - d^2ex + 2d^3\right)}{5\,(ex + d)\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x^
2+d^2)^(3/2)
```

maxima [A] time = 0.44, size = 136, normalized size = 1.49

$$-\frac{1}{5\left(\sqrt{-e^2x^2 + d^2}\,de^3x^2 + 2\,\sqrt{-e^2x^2 + d^2}\,d^2e^2x + \sqrt{-e^2x^2 + d^2}\,d^3e\right)} - \frac{1}{5\left(\sqrt{-e^2x^2 + d^2}\,d^2e^2x + \sqrt{-e^2x^2 + d^2}\,d^3e\right)} + \frac{}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(-e^2*x^2 + d^2)*d*e^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + s
qrt(-e^2*x^2 + d^2)*d^3*e) - 1/5/(sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + sqrt(-e^
2*x^2 + d^2)*d^3*e) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

mupad [B] time = 2.85, size = 66, normalized size = 0.73

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(-2\,d^3 + d^2\,e\,x + 4\,d\,e^2\,x^2 + 2\,e^3\,x^3\right)}{5\,d^4\,e\,{\left(d + e\,x\right)}^3\,\left(d - e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^3*x^3 - 2*d^3 + 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.176 $\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 2/5*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(-16*e*x+15*d)/d^5/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2+16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-\cdots}}\cdots\right.}{2d^4}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\cdots\right.}{\cdots}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x\cdots}}{d}\right.}{d^5}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.81

$$\frac{-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(26d^3+22d^2ex-17de^2x^2-16e^3x^3\right)}{(d-ex)(d+ex)^3}+15\log(x)}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/((
d - e*x)*(d + e*x)^3) + 15*Log[x] - 15*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^
5)
```

fricas [A] time = 1.00, size = 168, normalized size = 1.42

$$\frac{26\,e^4x^4+52\,de^3x^3-52\,d^3ex-26\,d^4+15\left(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^3x^3+17\,de^2x^2\right.}{15\left(d^5e^4x^4+2\,d^6e^3x^3-2\,d^8ex-d^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 + 52*d*e^3*x^3 - 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 + 2*d*e
^3*x^3 - 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3
+ 17*d*e^2*x^2 - 22*d^2*e*x - 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 +
2*d^6*e^3*x^3 - 2*d^8*e*x - d^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 187, normalized size = 1.58

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}-\frac{16ex}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5}+\frac{1}{5\left(x+\frac{d}{e}\right)^2\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2e^2}+\frac{}{15\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/(-e^2*x^2+d^2)^(1/2)/d^4-1/(d^2)^(1/2)/d^4*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+8/15/d^3/e/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-1
6/15/d^5*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/5/d^2/e^2/(x+d/e)^2/(2*(
x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(d^2-e^2x^2\right)^{3/2}(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.177 $\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -2/5*e*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(-13*e*x+10*d)/d^4/(-e^2*x^
2+d^2)^(3/2)+2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/15*e*(-41*e*x+30*d)/
d^6/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] time = 0.30, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^
4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
 - Sqrt[d^2 - e^2*x^2]/(d^6*x) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2-30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30de\cdots}{x^2\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x}$$

Mathematica [A] time = 0.11, size = 112, normalized size = 0.77

$$\frac{30e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(15d^4+76d^3ex+32d^2e^2x^2-82de^3x^3-56e^4x^4\right)}{x(ex-d)(d+ex)^3}-30e\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^4 + 76*d^3*e*x + 32*d^2*e^2*x^2 - 82*d*e^3*x^3
- 56*e^4*x^4))/(x*(-d + e*x)*(d + e*x)^3) - 30*e*Log[x] + 30*e*Log[d + Sqrt
[d^2 - e^2*x^2]])/(15*d^6)
```

fricas [A] time = 1.17, size = 194, normalized size = 1.33

$$-\frac{46\,e^5x^5+92\,de^4x^4-92\,d^3e^2x^2-46\,d^4ex+30\left(e^5x^5+2\,de^4x^4-2\,d^3e^2x^2-d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(56\,e^4x^4\right.}{15\left(d^6e^4x^5+2\,d^7e^3x^4-2\,d^9ex^2-d^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(46*e^5*x^5 + 92*d*e^4*x^4 - 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^
5 + 2*d*e^4*x^4 - 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/
x) + (56*e^4*x^4 + 82*d*e^3*x^3 - 32*d^2*e^2*x^2 - 76*d^3*e*x - 15*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^6*e^4*x^5 + 2*d^7*e^3*x^4 - 2*d^9*e*x^2 - d^10*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 234, normalized size = 1.60

$$\frac{2e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5}+\frac{2e^2x}{\sqrt{-e^2x^2+d^2}\,d^6}+\frac{26e^2x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^6}-\frac{1}{5\left(x+\frac{d}{e}\right)^2\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/d^4/x/(-e^2*x^2+d^2)^(1/2)+2/(-e^2*x^2+d^2)^(1/2)/d^6*e^2*x-2/(-e^2*x^2+
d^2)^(1/2)/d^5*e+2/(d^2)^(1/2)/d^5*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-13/15/d^4/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+26/15/d^6*
e^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-1/5/d^3/e/(x+d/e)^2/(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{3}{2}} \left(d + ex\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.178 $\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] 2/5*e^2*(-e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(-6*e*x+5*d)/d^5/(-e^2*x^
2+d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(-11*e*x+1
0*d)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2+2*e*(-e^2*x^
2+d^2)^(1/2)/d^7/x
```

Rubi [A] time = 0.37, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d - 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d - 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+10dex-10e^2x^2+\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2-30dex+45e^2x^2-\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30dex-60e^2x}{x^3\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int}{}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e}{}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e}{}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e}{}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e}{}$$

Mathematica [A] time = 0.13, size = 127, normalized size = 0.69

$$\frac{-45e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(5d^5-10d^4ex-94d^3e^2x^2-58d^2e^3x^3+83de^4x^4+64e^5x^5\right)}{x^2(ex-d)(d+ex)^3} + 45e^2\log(x)}{10d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 10*d^4*e*x - 94*d^3*e^2*x^2 - 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 + 64*e^5*x^5))/(x^2*(-d + e*x)*(d + e*x)^3) + 45*e^2*Log[x]
 - 45*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(10*d^7)
```

fricas [A] time = 0.98, size = 215, normalized size = 1.17

$$\frac{54\,e^6x^6+108\,de^5x^5-108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6+2\,de^5x^5-2\,d^3e^3x^3-d^4e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(64\right.}{10\left(d^7e^4x^6+2\,d^8e^3x^5-2\,d^{10}ex^3-d^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 + 108*d*e^5*x^5 - 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 + 2*d*e^5*x^5 - 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) + (64*e^5*x^5 + 83*d*e^4*x^4 - 58*d^2*e^3*x^3 - 94*d^3*e^2*x^2
- 10*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 + 2*d^8*e^3*x^5 -
2*d^10*e*x^3 - d^11*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 259, normalized size = 1.42

$$-\frac{9e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^6}-\frac{4e^3x}{\sqrt{-e^2x^2+d^2}\,d^7}-\frac{12e^3x}{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^7}+\frac{1}{5\left(x+\frac{d}{e}\right)^2\sqrt{2\left(x+\frac{d}{e}\right)de-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 2/d^5*e/x/(-e^2*x^2+d^2)^(1/2)-4/(-e^2*x^2+d^2)^(1/2)/d^7*e^3*x-1/2/d^4/x^2
/(-e^2*x^2+d^2)^(1/2)+9/2/(-e^2*x^2+d^2)^(1/2)/d^6*e^2-9/2/(d^2)^(1/2)/d^6*
e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+6/5/d^5*e/(x+d/e)/(2*(
x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-12/5/d^7*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x+1/5/d^4/(x+d/e)^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(d^2-e^2x^2\right)^{3/2}(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.179 $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=177

$$\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*d^4*(-e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(-e*x+d)^2/e^6/(-e^2*
x^2+d^2)^(3/2)+13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(-e
*x+d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6-1/2*x*(-e^2*x^2
+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.44, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d - e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d - e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 - (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(-\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{\frac{195d^5}{e^3}\cdots}{\sqrt{d^2\cdots}}}{30\cdots}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2\cdots}}{\cdots}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2\cdots}}{\cdots}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2\cdots}}{\cdots}$$

Mathematica [A] time = 0.19, size = 98, normalized size = 0.55

$$\frac{195d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4\right)}{(d+ex)^3}}{30e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3*x
^3 - 15*e^4*x^4))/(d + e*x)^3 + 195*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(30*e^6)
```

fricas [A] time = 1.04, size = 190, normalized size = 1.07

$$\frac{304\,d^2e^3x^3+912\,d^3e^2x^2+912\,d^4ex+304\,d^5-390\left(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-}{30\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 + 912*d^3*e^2*x^2 + 912*d^4*e*x + 304*d^5 - 390*(d^2*
e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (15*e^4*x^4 - 45*d*e^3*x^3 - 479*d^2*e^2*x^2 - 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (3*d^
2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*ex
p(2)^3-18*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(1)^8*exp(2)-8*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^3*exp(1)^6*exp(2)^2+5*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^3*exp(2)^5-9*d^2*exp(1)^4*exp(2)^3+6*d^2*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^5+6*d^2*exp(2)^5-19/2
*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)+14*d^2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-exp(1)^12+2*exp(1)^8*exp(2
)^2-exp(1)^4*exp(2)^4)+1/2*(-58*d^2*exp(1)^4*exp(2)^3+24*d^2*exp(2)^5+40*d^
2*exp(1)^8*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^14-2*ex
p(1)^10*exp(2)^2+exp(1)^6*exp(2)^4)+13/2*d^2*sign(d)*asin(x*exp(2)/d/exp(1)
)/exp(1)^6+2*(-2*exp(1)^11*1/8/exp(1)^16*x+12*exp(1)^10*d*1/8/exp(1)^16)*sq
rt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.02, size = 212, normalized size = 1.20

$$\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^5}-\frac{\sqrt{-e^2x^2+d^2}\,x}{2e^5}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4}{5\left(x+\frac{d}{e}\right)^3e^9}-\frac{23\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{15\left(x+\frac{d}{e}\right)^2e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)/e^5+13/2/(e^2)^(1/2)*d^2/e^5*arctan((e^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2)*x)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+1/5*d^4/e^9/(x+d/e)^3
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-23/15*d^3/e^8/(x+d/e)^2*(2*(x+d/e)*d*e
```

```
-(x+d/e)^2*e^2)^(1/2)+127/15/e^7*d^2/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)
```

maxima [A] time = 1.00, size = 185, normalized size = 1.05

$$\frac{\sqrt{-e^2x^2+d^2}\,d^4}{5\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}-\frac{23\sqrt{-e^2x^2+d^2}\,d^3}{15\left(e^8x^2+2\,de^7x+d^2e^6\right)}+\frac{127\sqrt{-e^2x^2+d^2}\,d^2}{15\left(e^7x+de^6\right)}+\frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6}-\frac{\sqrt{-e^2x^2+\cdots}}{2\,e^5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)*d^4/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
) - 23/15*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 127/15
*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x + d*e^6) + 13/2*d^2*arcsin(e*x/d)/e^6 - 1/
2*sqrt(-e^2*x^2 + d^2)*x/e^5 + 3*sqrt(-e^2*x^2 + d^2)*d/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.180 $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] -1/5*d^3*(-e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)+6/5*d^2*(-e*x+d)^2/e^5/(-e^2*x
^2+d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-24/5*d*(-e*x+d)/e^5/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 641, 217, 203}

$$-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(d^3*(d - e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (6*d^2*(d - e*x)^2)/(5*e
^5*(d^2 - e^2*x^2)^(3/2)) - (24*d*(d - e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) -
Sqrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(\frac{27d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}-\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int \cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\cdots}{\cdots}$$

Mathematica [A] time = 0.15, size = 85, normalized size = 0.58

$$-\frac{15d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(24d^3+57d^2ex+39de^2x^2+5e^3x^3\right)}{(d+ex)^3}}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(24*d^3 + 57*d^2*e*x + 39*d*e^2*x^2 + 5*e^3*x^3)
)/(d + e*x)^3 + 15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

fricas [A] time = 0.86, size = 174, normalized size = 1.19

$$-\frac{24\,de^3x^3+72\,d^2e^2x^2+72\,d^3ex+24\,d^4-30\left(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^3x^3+\cdots\right.}{5\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*d*e^3*x^3 + 72*d^2*e^2*x^2 + 72*d^3*e*x + 24*d^4 - 30*(d*e^3*x^3 +
 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 + (5*e^3*x^3 + 39*d*e^2*x^2 + 57*d^2*e*x + 24*d^3)*sqrt(-e^2*x^2 + d^2))/(
e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-d*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2
)^3+14*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(
1)^8*exp(2)+6*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*exp(1)^6*exp(2)^2-3*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^3*exp(2)^5+7*d*exp(1)^4*exp(2)^3-4*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^5-4*d*exp(2)^5+13/2*d*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)-11*d*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x+exp(2))^2/(-exp(1)^11+2*exp(1)^7*exp(2)^2-exp(1)*exp(2)^5
)+1/2*(30*d*exp(1)^4*exp(2)^3-12*d*exp(2)^5-24*d*exp(1)^8*exp(2))*atan((-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(
2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^13-2*exp(1)^9*exp(2)^2+exp(1)^5*exp
(2)^4)-3*d*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^5-4*exp(1)^4*1/4/exp(1)^9
*sqrt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.02, size = 187, normalized size = 1.28

$$-\frac{3d\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,e^{4}}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{3}}{5\left(x+\frac{d}{e}\right)^{3}e^{8}}+\frac{6\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{2}}{5\left(x+\frac{d}{e}\right)^{2}e^{7}}-\frac{24\sqrt{2\left(x+\frac{d}{e}\right)de}}{5\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^5-3/(e^2)^(1/2)*d/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-1/5*d^3/e^8/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+6/5
*d^2/e^7/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-24/5/e^6*d/(x+d/e)*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.98, size = 160, normalized size = 1.10

$$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{3}}{5\left(e^{8}x^{3}+3\,de^{7}x^{2}+3\,d^{2}e^{6}x+d^{3}e^{5}\right)}+\frac{6\sqrt{-e^{2}x^{2}+d^{2}}\,d^{2}}{5\left(e^{7}x^{2}+2\,de^{6}x+d^{2}e^{5}\right)}-\frac{24\sqrt{-e^{2}x^{2}+d^{2}}\,d}{5\left(e^{6}x+de^{5}\right)}-\frac{3\,d\arcsin\left(\frac{ex}{d}\right)}{e^{5}}-\frac{\sqrt{-e^{2}x^{2}+}}{e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^
5) + 6/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 24/5*sq
rt(-e^2*x^2 + d^2)*d/(e^6*x + d*e^5) - 3*d*arcsin(e*x/d)/e^5 - sqrt(-e^2*x^
2 + d^2)/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{d^2 - e^2 x^2}\,(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(-d + ex)(d + ex)}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.181 $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(-e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(-e*x+d)^2/e^4/(-e^2*x^
2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(-e*x+d)/e^4/(-e^2*
x^2+d^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 778, 217, 203}

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d - e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d - e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) + Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
```

```
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(-\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \right.}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.61

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(22d^2+51dex+32e^2x^2\right)}{(d+ex)^3}+15\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(d + e*x)^3 + 15*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^4)
```

fricas [A] time = 0.82, size = 157, normalized size = 1.31

$$\frac{22\,e^3x^3+66\,de^2x^2+66\,d^2ex+22\,d^3-30\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(32\,e^2x^2+51\,d\right.}{15\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 + 66*d*e^2*x^2 + 66*d^2*e*x + 22*d^3 - 30*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (32*e
^2*x^2 + 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + 3*d*e^6*x^2 +
3*d^2*e^5*x + d^3*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^
3-10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8
*exp(2)-4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^6*exp(2)^2+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
3*exp(2)^5-5*exp(1)^4*exp(2)^3+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^2*exp(2)^5+2*exp(2)^5-7/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(2)^5/x/exp(2)+8*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)*exp(1)^6*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(
2))^2/(-exp(1)^10+2*exp(1)^6*exp(2)^2-exp(1)^2*exp(2)^4)+1/2*(-10*exp(1)^4*
exp(2)^3+4*exp(2)^5+12*exp(1)^8*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(
2)^2)/(exp(1)^12-2*exp(1)^8*exp(2)^2+exp(1)^4*exp(2)^4)+sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)^4
```

maple [A] time = 0.01, size = 163, normalized size = 1.36

$$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2}{5\left(x+\frac{d}{e}\right)^3e^7}-\frac{13\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d}{15\left(x+\frac{d}{e}\right)^2e^6}+\frac{32\sqrt{2\left(x+\frac{d}{e}\right)de-\cdots}}{15\left(x+\frac{d}{e}\right)e\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/(e^2)^(1/2)/e^3*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^2/e^7/(x
+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-13/15*d/e^6/(x+d/e)^2*(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(1/2)+32/15/e^5/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)
```

maxima [A] time = 0.99, size = 136, normalized size = 1.13

$$\frac{\sqrt{-e^2x^2+d^2}\,d^2}{5\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}-\frac{13\,\sqrt{-e^2x^2+d^2}\,d}{15\left(e^6x^2+2\,de^5x+d^2e^4\right)}+\frac{32\,\sqrt{-e^2x^2+d^2}}{15\left(e^5x+de^4\right)}+\frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4
) - 13/15*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 32/15*sq
rt(-e^2*x^2 + d^2)/(e^5*x + d*e^4) + arcsin(e*x/d)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.182 $\int \frac{x^2}{(d+ex)^3 \sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=95

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

```
[Out] -1/5*d*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)^3+8/15*(-e^2*x^2+d^2)^(1/2)/e^3/(e*
x+d)^2-7/15*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1639, 793, 659, 651}

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(d*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3) + (8*Sqrt[d^2 - e^2*x^2])/(15*
e^3*(d + e*x)^2) - (7*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{\int \frac{2d^2e^2+de^3x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$
$$= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3} + \frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{(7d)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e^2}$$
$$= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3} + \frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2} + \frac{7\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15e^2}$$
$$= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3} + \frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2} - \frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.55

$$-\frac{\sqrt{d^2-e^2x^2}\left(2d^2+6dex+7e^2x^2\right)}{15de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(2*d^2 + 6*d*e*x + 7*e^2*x^2))/(d*e^3*(d + e*x)^
3)
```

fricas [A] time = 0.98, size = 104, normalized size = 1.09

$$-\frac{2\,e^3x^3+6\,de^2x^2+6\,d^2ex+2\,d^3+\left(7\,e^2x^2+6\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x + 2*d^3 + (7*e^2*x^2 + 6*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^
4*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (3*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)
^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8
*exp(2)+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^6*exp(2)^2+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
3*exp(2)^5+3*exp(1)^4*exp(2)^3+1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))*exp(2)^5/x/exp(2)-5*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^
6*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d
*exp(1)^9+2*d*exp(1)^5*exp(2)^2-d*exp(1)*exp(2)^4)+1/2*(-2*exp(2)^4-4*exp(1
```

```
)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d*exp(1)^9-2*d*exp(1)
^5*exp(2)^2+d*exp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 55, normalized size = 0.58

$$-\frac{(-ex+d)\left(7e^2x^2+6dex+2d^2\right)}{15\,(ex+d)^2\,\sqrt{-e^2x^2+d^2}\;d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(7*e^2*x^2+6*d*e*x+2*d^2)/(e*x+d)^2/d/e^3/(-e^2*x^2+d^2)^(1/
2)
```

maxima [A] time = 0.98, size = 125, normalized size = 1.32

$$-\frac{\sqrt{-e^2x^2+d^2}\,d}{5\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}+\frac{8\,\sqrt{-e^2x^2+d^2}}{15\left(e^5x^2+2\,de^4x+d^2e^3\right)}-\frac{7\,\sqrt{-e^2x^2+d^2}}{15\left(de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
 + 8/15*sqrt(-e^2*x^2 + d^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 7/15*sqrt(-e
^2*x^2 + d^2)/(d*e^4*x + d^2*e^3)
```

mupad [B] time = 2.76, size = 48, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 + 6*d*e*x))/(15*d*e^3*(d + e*x)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.183 $\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)^3-1/5*(-e^2*x^2+d^2)^(1/2)/d/e^2/(e*x+
d)^2-1/5*(-e^2*x^2+d^2)^(1/2)/d^2/e^2/(e*x+d)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) - Sqrt[d^2 - e^2*x^2]/(5*d*e^2*(d +
 e*x)^2) - Sqrt[d^2 - e^2*x^2]/(5*d^2*e^2*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} + \frac{3\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2} + \frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{5de}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2} - \frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^2+3dex+e^2x^2\right)}{5d^2e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + e^2*x^2))/(d^2*e^2*(d + e*x)^3)
```

fricas [A] time = 0.96, size = 100, normalized size = 1.03

$$-\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3+\left(e^2x^2+3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/5*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 + (e^2*x^2 + 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (2*ex
p(1)*exp(2)^5+5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^2*exp(1)^5*exp(2)^3+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^9*exp(2)+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)*exp(2)^5+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^3*exp(2)^4+exp(1)^5*exp(2)^3-5/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^4/x/exp(2)-2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x+exp(2))^2/(d^2*exp(1)^9-2*d^2*exp(1)^5*exp(2)^2+d^2*ex
p(1)*exp(2)^4)+3*exp(1)^3*exp(2)^3*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)
/(d^2*exp(1)^9-2*d^2*exp(1)^5*exp(2)^2+d^2*exp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.54

$$-\frac{(-ex+d)\left(e^2x^2+3dex+d^2\right)}{5\,(ex+d)^2\,\sqrt{-e^2x^2+d^2}\,d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*(-e*x+d)*(e^2*x^2+3*d*e*x+d^2)/(e*x+d)^2/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.98, size = 129, normalized size = 1.33

$$\frac{\sqrt{-e^2x^2+d^2}}{5\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+d^2}}{5\left(de^4x^2+2\,d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+d^2}}{5\left(d^2e^3x+d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) -
1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/5*sqrt(-e^
2*x^2 + d^2)/(d^2*e^3*x + d^3*e^2)
```

mupad [B] time = 2.59, size = 45, normalized size = 0.46

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + 3*d*e*x))/(5*d^2*e^2*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.184 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)^3-2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(e*
x+d)^2-2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(e*x+d)
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*
e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx&=-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}+\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\&=-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}+\frac{2\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\&=-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.52

$$-\frac{\sqrt{d^2-e^2x^2}\left(7d^2+6dex+2e^2x^2\right)}{15d^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(7*d^2 + 6*d*e*x + 2*e^2*x^2))/(d^3*e*(d + e*x)^
3)
```

fricas [A] time = 0.91, size = 104, normalized size = 1.04

$$-\frac{7\,e^{3}x^{3}+21\,de^{2}x^{2}+21\,d^{2}ex+7\,d^{3}+\left(2\,e^{2}x^{2}+6\,dex+7\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\,\left(d^{3}e^{4}x^{3}+3\,d^{4}e^{3}x^{2}+3\,d^{5}e^{2}x+d^{6}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*x
+ d^6*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (7*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)
^3-2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^1
0*exp(2)-2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(1)^8*exp(2)^2+5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(1)^4*exp(2)^4-exp(1)^6*exp(2)^3+4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^6+4*exp(2)^6-11/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^4/x/exp(2)+(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x+exp(2))^2/(-d^3*exp(1)^9+2*d^3*exp(1)^5*exp(2)^2-d^3*exp(1)*e
xp(2)^4)+1/2*(-2*exp(1)^4*exp(2)^3-4*exp(2)^5)*atan((-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)
^4+exp(2)^2)/(d^3*exp(1)^9-2*d^3*exp(1)^5*exp(2)^2+d^3*exp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 55, normalized size = 0.55

$$-\frac{\left(-ex+d\right)\left(2e^{2}x^{2}+6dex+7d^{2}\right)}{15\left(ex+d\right)^{2}\sqrt{-e^{2}x^{2}+d^{2}}\,d^{3}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*e^2*x^2+6*d*e*x+7*d^2)/(e*x+d)^2/d^3/e/(-e^2*x^2+d^2)^(1/
2)
```

maxima [A] time = 0.98, size = 128, normalized size = 1.28

$$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{5\left(de^{4}x^{3}+3\,d^{2}e^{3}x^{2}+3\,d^{3}e^{2}x+d^{4}e\right)}-\frac{2\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(d^{2}e^{3}x^{2}+2\,d^{3}e^{2}x+d^{4}e\right)}-\frac{2\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(d^{3}e^{2}x+d^{4}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e)
 - 2/15*sqrt(-e^2*x^2 + d^2)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2/15*sqr
t(-e^2*x^2 + d^2)/(d^3*e^2*x + d^4*e)
```

mupad [B] time = 2.62, size = 48, normalized size = 0.48

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2+6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 + 6*d*e*x))/(15*d^3*e*(d + e*x)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.185 $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] 4/5*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(-11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/
2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(-22*e*x+15*d)/d^4/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d - 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d - 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2+22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.}{2d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\right.}{d^3e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] time = 0.12, size = 76, normalized size = 0.66

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(32d^2+51dex+22e^2x^2\right)}{(d+ex)^3}-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+15\log(x)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(d + e*x)^3 + 15*Lo
g[x] - 15*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^4)
```

fricas [A] time = 0.95, size = 153, normalized size = 1.33

$$\frac{32\,e^3x^3+96\,de^2x^2+96\,d^2ex+32\,d^3+15\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(22\,e^2x^2+51\,dex\cdots\right.}{15\left(d^4e^3x^3+3\,d^5e^2x^2+3\,d^6ex+d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 + 96*d*e^2*x^2 + 96*d^2*e*x + 32*d^3 + 15*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (22*e^2*x^2
+ 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + 3*d^5*e^2*x^2 + 3
*d^6*e*x + d^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-9*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2
)^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
11*exp(2)+4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(1)^9*exp(2)^2-7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^3*exp(1)^5*exp(2)^4+3*exp(1)^7*exp(2)^3-6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^5-6*exp(1)^3*exp(2)^5+17/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^4/x/exp(2)-4*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*exp(2)^2/x/exp(2))/((-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d^4*exp(1)^9+2*d^4*exp(1)^
5*exp(2)^2-d^4*exp(1)*exp(2)^4)+1/2*(-10*exp(1)^5*exp(2)^3+4*exp(1)^9*exp(2
)+12*exp(1)*exp(2)^5)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d^4*exp(1)^
9-2*d^4*exp(1)^5*exp(2)^2+d^4*exp(1)*exp(2)^4)-exp(2)*ln(1/2*abs(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/exp(1)^2
```

maple [A] time = 0.01, size = 179, normalized size = 1.56

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^3d^2e^3}+\frac{7\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{15\left(x+\frac{d}{e}\right)^2d^3e^2}+\frac{22\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\cdots\right.}}{15\left(x+\frac{d}{e}\right)d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/(d^2)^(1/2)/d^3*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^2
/e^3/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+7/15/d^3/e^2/(x+d/e)^2*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+22/15/d^4/e/(x+d/e)*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}\,(ex + d)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{d^2 - e^2\,x^2}\,\left(d + e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(-d + ex\right)\left(d + ex\right)}\,\left(d + ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.186 $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -4/5*e*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)-1/5*e*(-7*e*x+5*d)/d^3/(-e^2*x^2+d^2
)^(3/2)+3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5-1/5*e*(-19*e*x+15*d)/d^5/(-
e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A] time = 0.30, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^
2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqr
t[d^2 - e^2*x^2]/(d^5*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{(3e}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{(3e}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3\,S}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3e}{}$$

Mathematica [A] time = 0.18, size = 92, normalized size = 0.63

$$-\frac{-15e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(5d^3+39d^2ex+57de^2x^2+24e^3x^3\right)}{x(d+ex)^3}+15e\log(x)}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x^3)
)/(x*(d + e*x)^3) + 15*e*Log[x] - 15*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^5
```

fricas [A] time = 0.88, size = 181, normalized size = 1.24

$$-\frac{24\,e^4x^4+72\,de^3x^3+72\,d^2e^2x^2+24\,d^3ex+15\left(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(24\,e^3x^3\right.}{5\left(d^5e^3x^4+3\,d^6e^2x^3+3\,d^7ex^2+d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*e^4*x^4 + 72*d*e^3*x^3 + 72*d^2*e^2*x^2 + 24*d^3*e*x + 15*(e^4*x^4
 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) + (24*e^3*x^3 + 57*d*e^2*x^2 + 39*d^2*e*x + 5*d^3)*sqrt(-e^2*x^2 + d^2))/
(d^5*e^3*x^4 + 3*d^6*e^2*x^3 + 3*d^7*e*x^2 + d^8*x)
```

giac [A] time = 0.30, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.01, size = 199, normalized size = 1.36

$$\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^3d^3e^2}-\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^2d^4e}-\frac{19\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x\right.}}{5\left(x+\frac{d}{e}\right)d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^5/x+3/(d^2)^(1/2)/d^4*e*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-1/5/d^3/e^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(
1/2)-4/5/d^4/e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-19/5/d^5/(x+d/
e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)

3.187 $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=183

$$\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 4/5*e^2*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(-31*e*x+25*d)/d^4/(-e^2
*x^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(-107
*e*x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2+3*e*(-
e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] time = 0.38, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d - 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d - 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) + (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-20de^2x^2+16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+75de^2x^2-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex-90de^2}{x^3\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-9}{}}{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{}}{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{}}{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{}}{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{}}{}$$

Mathematica [A] time = 0.17, size = 107, normalized size = 0.58

$$\frac{-195e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4\right)}{x^2(d+ex)^3} + 195e^2\log(x)}{30d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^3*x
^3 + 304*e^4*x^4))/(x^2*(d + e*x)^3) + 195*e^2*Log[x] - 195*e^2*Log[d + Sqr
t[d^2 - e^2*x^2]])/(30*d^6)
```

fricas [A] time = 0.79, size = 202, normalized size = 1.10

$$\frac{254\,e^5x^5 + 762\,de^4x^4 + 762\,d^2e^3x^3 + 254\,d^3e^2x^2 + 195\left(e^5x^5 + 3\,de^4x^4 + 3\,d^2e^3x^3 + d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) +}{30\left(d^6e^3x^5 + 3\,d^7e^2x^4 + 3\,d^8ex^3 + d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 + 762*d*e^4*x^4 + 762*d^2*e^3*x^3 + 254*d^3*e^2*x^2 + 195
*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (304*e^4*x^4 + 717*d*e^3*x^3 + 479*d^2*e^2*x^2 + 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 + 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 + d^9*x^2)
```

giac [A] time = 0.29, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.01, size = 222, normalized size = 1.21

$$-\frac{13 e^{2} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 \sqrt{d^{2}}\, d^{5}}+\frac{\sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}{5 \left(x+\frac{d}{e}\right)^{3} d^{4} e}+\frac{17 \sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}{15 \left(x+\frac{d}{e}\right)^{2} d^{5}}+\frac{107 \sqrt{2 \left(x+\frac{d}{e}\right. \cdots}}{15 \left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 3*e*(-e^2*x^2+d^2)^(1/2)/d^6/x-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2-13/2/(d^2)^
(1/2)/d^5*e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^4/e/(x
+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+17/15/d^5/(x+d/e)^2*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(1/2)+107/15/d^6*e/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^{2} x^{2}+d^{2}}\, (ex+d)^{3} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3}\, \sqrt{d^{2}-e^{2}\, x^{2}}\, \left(d+e\, x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt{-\left(-d+ex\right)\left(d+ex\right)}\, \left(d+ex\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.188 $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{18d^3\,\mathrm{ta}}{}$$

```
[Out] 1/5*d^4*(-e*x+d)^4/e^6/(-e^2*x^2+d^2)^(5/2)-8/5*d^3*(-e*x+d)^3/e^6/(-e^2*x^
2+d^2)^(3/2)+18*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+10*d^2*(-e*x+d)^2/
e^6/(-e^2*x^2+d^2)^(1/2)+59/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^6-2*d*x*(-e^2*x^2+
d^2)^(1/2)/e^5+1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A] time = 0.59, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}+\frac{18d^3\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (8*d^3*(d - e*x)^3)/(5*e^
6*(d^2 - e^2*x^2)^(3/2)) + (10*d^2*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*x^2]) +
 (59*d^2*Sqrt[d^2 - e^2*x^2])/(3*e^6) - (2*d*x*Sqrt[d^2 - e^2*x^2])/e^5 + (
x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + (18*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(-\frac{60d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)\left(-\frac{240d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{\int \frac{\frac{720d^6}{e^3}-\frac{885d^5}{e^2}}{\sqrt{d^2-}}}{45d}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2}}{3e^4}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-}}{e^5}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-}}{e^5}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-}}{e^5}$$

Mathematica [A] time = 0.18, size = 109, normalized size = 0.53

$$\frac{270d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(424d^5+1002d^4ex+674d^3e^2x^2+70d^2e^3x^3-15de^4x^4+5e^5x^5\right)}{(d+ex)^3}}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^
3*x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(d + e*x)^3 + 270*d^3*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(15*e^6)
```

fricas [A] time = 0.96, size = 200, normalized size = 0.98

$$\frac{424\,d^3e^3x^3 + 1272\,d^4e^2x^2 + 1272\,d^5ex + 424\,d^6 - 540\left(d^3e^3x^3 + 3\,d^4e^2x^2 + 3\,d^5ex + d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) +}{15\left(e^9x^3 + 3\,de^8x^2 + 3\,d^2e^7x + d^3e^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(424*d^3*e^3*x^3 + 1272*d^4*e^2*x^2 + 1272*d^5*e*x + 424*d^6 - 540*(d^
3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + (5*e^5*x^5 - 15*d*e^4*x^4 + 70*d^2*e^3*x^3 + 674*d^3*e^2*x^2
+ 1002*d^4*e*x + 424*d^5)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*
d^2*e^7*x + d^3*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-162
*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^1
2*exp(2)^2-36*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^10*exp(2)^3+240*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2+228*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+54*d^3*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-402*d^3*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)
^2+158*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(1)^10*exp(2)^3+339*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(1)^8*exp(2)^4+87*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+492*d^3*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+192*d^3*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-96*d^3
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp
(2)^5-36*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*
exp(1)^4*exp(2)^6+840*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^8*exp(2)^4+420*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-102*d^3*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-48*d^3*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-228*d^3*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-25
2*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
4*exp(2)^6-47*d^3*exp(1)^8*exp(2)^4-102*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-288*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6+60*d^3*exp(1)^6*exp(2)
^5-360*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(2)^8+110*d^3*exp(1)^4*exp(2)^6-204*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-102*d^3*exp(2)^8-188*d^3*(-1/2*(-2*d*e
```

```
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+156*d^3*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+108*d^3*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-573/2*d^3*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-153*
d^3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+
123*d^3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/ex
p(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)
-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^16-6*exp
(1)^12*exp(2)^2-6*exp(1)^10*exp(2)^3+3*exp(1)^8*exp(2)^4+3*exp(1)^6*exp(2)^
5+3*exp(1)^14*exp(2))+1/2*(-100*d^3*exp(1)^10*exp(2)^2-170*d^3*exp(1)^8*exp
(2)^3+152*d^3*exp(1)^6*exp(2)^4+208*d^3*exp(1)^4*exp(2)^5-144*d^3*exp(2)^7+
40*d^3*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^
18+2*exp(1)^14*exp(2)^2+2*exp(1)^12*exp(2)^3-exp(1)^10*exp(2)^4-exp(1)^8*ex
p(2)^5-exp(1)^16*exp(2))+18*d^3*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^6+2*
((2*exp(1)^16*1/12/exp(1)^20*x-12*exp(1)^15*d*1/12/exp(1)^20)*x+58*exp(1)^1
4*d^2*1/12/exp(1)^20)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.02, size = 297, normalized size = 1.46

$$\frac{20d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e^5}-\frac{2d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^5}-\frac{2\sqrt{-e^2x^2+d^2}\,dx}{e^5}+\frac{20\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/3/e^6*(-e^2*x^2+d^2)^(3/2)-2*d*x*(-e^2*x^2+d^2)^(1/2)/e^5-2/e^5*d^3/(e^2
)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^4/e^10/(x+d/e)^4*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-8/5/e^9*d^3/(x+d/e)^3*(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(3/2)+20/e^6*d^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+20/e^5*d^3
/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+10/e
^8*d^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \sqrt{- \left(- d + e x\right) \left(d + e x\right)}}{\left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.189 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=160

$$\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] -1/5*d^3*(-e*x+d)^4/e^5/(-e^2*x^2+d^2)^(5/2)+19/15*d^2*(-e*x+d)^3/e^5/(-e^2
*x^2+d^2)^(3/2)-19/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-6*d*(-e*x+d)^
2/e^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e*x+20*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.42, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 780, 217, 203}

$$-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -(d^3*(d - e*x)^4)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (19*d^2*(d - e*x)^3)/(15
*e^5*(d^2 - e^2*x^2)^(3/2)) - (6*d*(d - e*x)^2)/(e^5*Sqrt[d^2 - e^2*x^2]) -
 ((20*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(\frac{45d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{135d^4}{e^4}-\frac{15d^3x}{e^3}\right)(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(19d\cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(19d\cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{19d^2\cdots}{\cdots}$$

Mathematica [A] time = 0.18, size = 98, normalized size = 0.61

$$-\frac{285d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(448d^4+1059d^3ex+713d^2e^2x^2+75de^3x^3-15e^4x^4\right)}{(d+ex)^3}}{30e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/30*((Sqrt[d^2 - e^2*x^2]*(448*d^4 + 1059*d^3*e*x + 713*d^2*e^2*x^2 + 75*
d*e^3*x^3 - 15*e^4*x^4))/(d + e*x)^3 + 285*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*
x^2]])/e^5
```

fricas [A] time = 0.98, size = 190, normalized size = 1.19

$$-\frac{448\,d^2e^3x^3+1344\,d^3e^2x^2+1344\,d^4ex+448\,d^5-570\left(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)\cdots}{30\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/30*(448*d^2*e^3*x^3 + 1344*d^3*e^2*x^2 + 1344*d^4*e*x + 448*d^5 - 570*(d
^2*e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*e^4*x^4 - 75*d*e^3*x^3 - 713*d^2*e^2*x^2 - 1059*d^3*e*x
- 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3
*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (84*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*
exp(2)^2+18*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^10*exp(2)^3-192*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^12*exp(2)^2-180*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-42*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+228*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2
-104*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(
1)^10*exp(2)^3-192*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*exp(1)^8*exp(2)^4-48*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-396*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-168*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+60*d^2*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2
)^5+24*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*ex
p(1)^4*exp(2)^6-510*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^8*exp(2)^4-246*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5+57*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+27*d^2*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+156*d^2*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+180*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*e
xp(2)^6+26*d^2*exp(1)^8*exp(2)^4+66*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^4*exp(2)^8+180*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-48*d^2*exp(1)^6*exp(2)^5+2
16*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)
^8-65*d^2*exp(1)^4*exp(2)^6+132*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(2)^8+66*d^2*exp(2)^8+104*d^2*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)-189/2*d^2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)-78*d^2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+171*d^2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+123*d^2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-69*d^2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^15-6*exp(1)^11*e
xp(2)^2-6*exp(1)^9*exp(2)^3+3*exp(1)^7*exp(2)^4+3*exp(1)^5*exp(2)^5+3*exp(1
)^13*exp(2))+1/2*(64*d^2*exp(1)^10*exp(2)^2+80*d^2*exp(1)^8*exp(2)^3-88*d^2
*exp(1)^6*exp(2)^4-102*d^2*exp(1)^4*exp(2)^5+76*d^2*exp(2)^7-16*d^2*exp(1)^
12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))
/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^17+2*exp(1)^13
*exp(2)^2+2*exp(1)^11*exp(2)^3-exp(1)^9*exp(2)^4-exp(1)^7*exp(2)^5-exp(1)^1
5*exp(2))-19/2*d^2*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^5+2*(2*exp(1)^9*1
/8/exp(1)^13*x-16*exp(1)^8*d*1/8/exp(1)^13)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.01, size = 273, normalized size = 1.71

$$-\frac{10d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^4}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^4}+\frac{\sqrt{-e^2x^2+d^2}\,x}{2e^4}-\frac{10\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d}{e^5}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/2/e^4*x*(-e^2*x^2+d^2)^(1/2)+1/2/e^4*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(
-e^2*x^2+d^2)^(1/2)*x)-1/5*d^3/e^9/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(3/2)+19/15/e^8*d^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-10/e^5*d*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-10/e^4*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-6/e^7*d/(x+d/e)^2*(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \sqrt{-\left(-d + ex\right)\left(d + ex\right)}}{\left(d + ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.190 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=148

$$\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^4-14/15*d*(-e^2*x^2+d^2)^(3/2)/e^4
/(e*x+d)^3-(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^2+4*d*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))/e^4+8*d*(-e^2*x^2+d^2)^(1/2)/e^4/(e*x+d)
```

Rubi [A] time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1639, 1637, 659, 651, 663, 217, 203}

$$\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (8*d*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*
e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2
 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
```

```
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(2d^3e^2+5d^2e^3x+4de^4x^2\right)}{(d+ex)^4}\,dx}{e^5}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{\int \left(\frac{d^3e^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}-\frac{3d^2e^2\sqrt{d^2-e^2x^2}}{(d+ex)^3}+\frac{4de^2\sqrt{d^2-e^2x^2}}{(d+ex)^2}\right)dx}{e^5}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{(4d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^3}+\frac{\left(3d^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^3}-\frac{d^3\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^3}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{(4d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{(4d)\,\mathrm{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e}}\right.}{e^4}$$

Mathematica [A] time = 0.13, size = 85, normalized size = 0.57

$$\frac{60d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(94d^3+222d^2ex+149de^2x^2+15e^3x^3\right)}{(d+ex)^3}}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/
(d + e*x)^3 + 60*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^4)
```

fricas [A] time = 1.08, size = 174, normalized size = 1.18

$$\frac{94\,de^3x^3 + 282\,d^2e^2x^2 + 282\,d^3ex + 94\,d^4 - 120\left(de^3x^3 + 3\,d^2e^2x^2 + 3\,d^3ex + d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (15}{15\left(e^7x^3 + 3\,de^6x^2 + 3\,d^2e^5x + d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*d*e^3*x^3 + 282*d^2*e^2*x^2 + 282*d^3*e*x + 94*d^4 - 120*(d*e^3*x^
3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*
x)) + (15*e^3*x^3 + 149*d*e^2*x^2 + 222*d^2*e*x + 94*d^3)*sqrt(-e^2*x^2 + d
^2))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-30*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*e
xp(2)^2-6*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*e
xp(1)^10*exp(2)^3+144*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^3*exp(1)^12*exp(2)^2+132*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+30*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-102*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2+62*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+
87*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8
*exp(2)^4+21*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^6*exp(2)^5+300*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^10*exp(2)^3+144*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-24*d*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-12*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6+264*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4+1
20*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6
*exp(2)^5-24*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
4*exp(1)^4*exp(2)^6-12*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(2)^8-84*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^6*exp(2)^5-108*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6-11*d*exp(1)^8*exp(2)^4-36*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-96*d*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6
+36*d*exp(1)^6*exp(2)^5-108*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(2)^8+32*d*exp(1)^4*exp(2)^6-72*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-36*d*exp(2)^8-44*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+48
*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+48*d*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-171/2*d*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-93*d*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+30*d
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/(
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^14-6*exp(1)^10*
exp(2)^2-6*exp(1)^8*exp(2)^3+3*exp(1)^6*exp(2)^4+3*exp(1)^4*exp(2)^5+3*exp(
```

```
1)^12*exp(2))+1/2*(-36*d*exp(1)^10*exp(2)^2-30*d*exp(1)^8*exp(2)^3+40*d*exp
(1)^6*exp(2)^4+40*d*exp(1)^4*exp(2)^5-32*d*exp(2)^7+4*d*exp(1)^12*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^16+2*exp(1)^12*exp(2)^2+2*
exp(1)^10*exp(2)^3-exp(1)^8*exp(2)^4-exp(1)^6*exp(2)^5-exp(1)^14*exp(2))+4*
d*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^4+2*exp(1)^3*1/2/exp(1)^7*sqrt(d^2
-x^2*exp(2))
```

maple [A] time = 0.01, size = 212, normalized size = 1.43

$$\frac{4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^3}+\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{e^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^2}{5\left(x+\frac{d}{e}\right)^4e^8}-\frac{14\left(2\left(x+\frac{d}{e}\right)d\right.}{15\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/5*d^2/e^8/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-14/15*d/e^7/(x+d/
e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(1/2)+4/e^3*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(1/2)*x)+3/e^6/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.191 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=115

$$\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -1/5*d*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x+d)^4+3/5*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x
+d)^3-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+
d)
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1637, 659, 651, 663, 217, 203}

$$\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3
*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]]/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
```

```
2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \left(\frac{d^2\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4} - \frac{2d\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3} + \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right) dx$$

$$= \frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2} - \frac{(2d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2} + \frac{d^2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4} + \frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e^3(d+ex)^3} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e^2}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4} + \frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4} + \frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Mathematica [A] time = 0.12, size = 73, normalized size = 0.63

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(8d^2+19dex+13e^2x^2\right)}{(d+ex)^3} + 5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(8*d^2 + 19*d*e*x + 13*e^2*x^2))/(d + e*x)^3 + 5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

fricas [A] time = 0.94, size = 157, normalized size = 1.37

$$-\frac{8\,e^3x^3 + 24\,de^2x^2 + 24\,d^2ex + 8\,d^3 - 10\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(13\,e^2x^2 + 19\,de\right.}{5\left(e^6x^3 + 3\,de^5x^2 + 3\,d^2e^4x + d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/5*(8*e^3*x^3 + 24*d*e^2*x^2 + 24*d^2*e*x + 8*d^3 - 10*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (13*e^2
*x^2 + 19*d*e*x + 8*d^2)*sqrt(-e^2*x^2 + d^2))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-96*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*exp
(2)^2-84*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^10*exp(2)^3-18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^8*exp(2)^4+24*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^12*exp(2)^2-32*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+8*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)-24*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-6*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-204*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-120
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp
(2)^4-12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^6*exp(2)^5-102*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^2*exp(1)^8*exp(2)^4-42*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^3*exp(1)^6*exp(2)^5+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^5*exp(2)^8+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*exp(1)^8*exp(2)^4+12*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+36*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2
)^6-24*exp(1)^6*exp(2)^5+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(2)^8-11*exp(1)^4*exp(2)^6+24*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+12*exp(2)^8-33/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)-18*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+30*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+63*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-6*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x+exp(2))^3/(3*exp(1)^13-6*exp(1)^9*exp(2)^2-6*exp(1)^7*exp(2)^3+3
*exp(1)^5*exp(2)^4+3*exp(1)^11*exp(2)+3*exp(1)*exp(2)^6)+1/2*(16*exp(1)^10*
exp(2)^2+8*exp(1)^8*exp(2)^3-8*exp(1)^6*exp(2)^4-10*exp(1)^4*exp(2)^5+8*exp
(2)^7)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqr
t(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^15+2*exp(1)^11*exp
(2)^2+2*exp(1)^9*exp(2)^3-exp(1)^7*exp(2)^4-exp(1)^5*exp(2)^5-exp(1)^13*exp
(2))-sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)
```

maple [B] time = 0.01, size = 214, normalized size = 1.86

$$-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{d\,e^3}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d}{5\left(x+\frac{d}{e}\right)^4e^7}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\cdots\right.}{\left(x+\frac{d}{e}\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/5*d/e^7/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+3/5/e^6/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-1/e^5/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(3/2)-1/e^3/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-1/e^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2 + d^2}\, x^2}{(ex + d)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate(sqrt(-e^2*x^2 + d^2)*x^2/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\, \sqrt{d^2 - e^2\, x^2}}{(d + e\, x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(-d + ex)(d + ex)}}{(d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.192 $\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=64

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(3/2)/e^2/(e*x+d)^4-4/15*(-e^2*x^2+d^2)^(3/2)/d/e^2/(e*x
+d)^3
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {793, 651}

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*d
*e^2*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}+\frac{4\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e}\\ &= \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.78

$$-\frac{\left(d^2+3dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/15*((d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(d*e^2*(d + e*x)^3)
```

fricas [A] time = 0.95, size = 102, normalized size = 1.59

$$-\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3-\left(4\,e^2x^2-3\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 - (4*e^2*x^2 - 3*d*e*x - d^2
)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (6*ex
p(1)*exp(2)^7+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)*exp(2)^7+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^11*exp(2)^2+48*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^11*exp(2)^2+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^4*exp(1)^9*exp(2)^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^7*exp(2)^4+6*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^11*exp(2)^2+14*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^3+4*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^13*exp(2)+3
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp
(2)^4+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1
)*exp(2)^7+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*
exp(1)^5*exp(2)^5+108*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^9*exp(2)^3+96*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^7*exp(2)^4+48*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^4*exp(1)^5*exp(2)^5+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^3*exp(2)^6+24*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^4+12*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)^5+6*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^3*exp(2)^6+6
0*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*ex
p(2)^5+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^3*exp(2)^6+exp(1)^7*exp(2)^4+12*exp(1)^5*exp(2)^5+2*exp(1)^3*exp(2)^6-1
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^6/x/exp(2)-9/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^5/x/exp(2)-33
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^4/x/exp(2)-3*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*exp(2)^3/x/exp(2))/((-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*d*exp(1)^11-6*d*exp(1)^7*
exp(2)^2-6*d*exp(1)^5*exp(2)^3+3*d*exp(1)^9*exp(2)+6*d*exp(1)*exp(2)^5)+1/2
*(4*exp(1)^7*exp(2)^2+2*exp(1)^5*exp(2)^3+8*exp(1)^3*exp(2)^4)*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/(d*exp(1)^11-2*d*exp(1)^7*exp(2)^2-2*d*exp(1)^
5*exp(2)^3+d*exp(1)^9*exp(2)+2*d*exp(1)*exp(2)^5)
```

maple [A] time = 0.01, size = 42, normalized size = 0.66

$$-\frac{(-ex+d)(4ex+d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3 d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/15*(-e*x+d)*(4*e*x+d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/e^2/d
```

maxima [B] time = 0.45, size = 125, normalized size = 1.95

$$\frac{2\sqrt{-e^2x^2+d^2}\,d}{5\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}-\frac{11\sqrt{-e^2x^2+d^2}}{15\left(e^4x^2+2\,de^3x+d^2e^2\right)}+\frac{4\sqrt{-e^2x^2+d^2}}{15\left(de^3x+d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(-e^2*x^2 + d^2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)
- 11/15*sqrt(-e^2*x^2 + d^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 4/15*sqrt(-e
^2*x^2 + d^2)/(d*e^3*x + d^2*e^2)
```

mupad [B] time = 2.90, size = 46, normalized size = 0.72

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x-4\,e^2\,x^2\right)}{15\,d\,e^2\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 - 4*e^2*x^2 + 3*d*e*x))/(15*d*e^2*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.193 $\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal. Leaf size=67

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(3/2)/d/e/(e*x+d)^4-1/15*(-e^2*x^2+d^2)^(3/2)/d^2/e/(e*
x+d)^3
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {659, 651}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -(d^2 - e^2*x^2)^(3/2)/(5*d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^2*
e*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}+\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3} dx}{5d}\\ &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-4d^2+3dex+e^2x^2\right)}{15d^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)

fricas [A] time = 1.03, size = 104, normalized size = 1.55

$$-\frac{4\,e^{3}x^{3}+12\,de^{2}x^{2}+12\,d^{2}ex+4\,d^{3}-\left(e^{2}x^{2}+3\,dex-4\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(d^{2}e^{4}x^{3}+3\,d^{3}e^{3}x^{2}+3\,d^{4}e^{2}x+d^{5}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")

[Out] -1/15*(4*e^3*x^3 + 12*d*e^2*x^2 + 12*d^2*e*x + 4*d^3 - (e^2*x^2 + 3*d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp(2)^2+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^10*exp(2)^3+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-8*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-24*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-72*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-84*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-24*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6-30*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4-30*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-132*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-108*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*exp(1)^8*exp(2)^4-18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-5*exp(1)^4*exp(2)^6-36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-18*exp(2)^8+8*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+3/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+42*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*d^2*exp(1)^11-6*d^2*exp(1)^7*exp(2)^2-6*d^2*exp(1)^5*exp(2)^3+3*d^2*exp(1)^9*exp(2)+6*d^2*exp(1)*exp(2)^5)+1/2*(8*exp(1)^4*exp(2)^4+6*exp(2)^6)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^2*exp(1)^11+2*d^2*exp(1)^7*exp(2)^2+2*d^2*exp(1)^5*exp(2)^3-d^2*exp(1)^9*exp(2)-2*d^2*exp(1)*exp(2)^5)

maple [A] time = 0.01, size = 43, normalized size = 0.64

$$-\frac{(-ex+d)(ex+4d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3 d^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
[Out] -1/15*(-e*x+d)*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d^2/e
```

maxima [B] time = 0.44, size = 123, normalized size = 1.84

$$-\frac{2\sqrt{-e^2x^2+d^2}}{5\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}+\frac{\sqrt{-e^2x^2+d^2}}{15\left(de^3x^2+2\,d^2e^2x+d^3e\right)}+\frac{\sqrt{-e^2x^2+d^2}}{15\left(d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
[Out] -2/5*sqrt(-e^2*x^2 + d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 1
/15*sqrt(-e^2*x^2 + d^2)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) + 1/15*sqrt(-e^2
*x^2 + d^2)/(d^2*e^2*x + d^3*e)
```

mupad [B] time = 2.78, size = 47, normalized size = 0.70

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-4\,d^2+3\,d\,e\,x+e^2\,x^2\right)}{15\,d^2\,e\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x)^4,x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 4*d^2 + 3*d*e*x))/(15*d^2*e*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.194 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4} dx$

Optimal. Leaf size=110

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] 8/5*d*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/5*e*x/d/(-e^2*x^2+d^2)^(3/2)-arctanh(
(-e^2*x^2+d^2)^(1/2)/d)/d^3+1/5*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^(3
/2)) + (5*d - 8*e*x)/(5*d^3*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+12d^3ex+5d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-24d^3ex}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^6e^2}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^8e^2}\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^2}\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^2e^2}\\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 76, normalized size = 0.69

$$\frac{\frac{\sqrt{d^2-e^2x^2}(13d^2+19dex+8e^2x^2)}{(d+ex)^3} - 5\log\left(\sqrt{d^2-e^2x^2}+d\right) + 5\log(x)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 5*Log[
x] - 5*Log[d + Sqrt[d^2 - e^2*x^2]])/(5*d^3)
```

fricas [A] time = 0.83, size = 153, normalized size = 1.39

$$\frac{13\,e^3x^3 + 39\,de^2x^2 + 39\,d^2ex + 13\,d^3 + 5\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \left(8\,e^2x^2 + 19\,dex\right.}{5\left(d^3e^3x^3 + 3\,d^4e^2x^2 + 3\,d^5ex + d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(13*e^3*x^3 + 39*d*e^2*x^2 + 39*d^2*e*x + 13*d^3 + 5*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^2*x^2 + 1
9*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^
5*e*x + d^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (6*ex
p(1)*exp(2)^8+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)*exp(2)^8+54*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*exp(1)^13*exp(2)^2+18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^11*exp(2)^3+48*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(1)^13*exp(2)^2+44*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^15*exp(2)+60*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^11*exp(2)^3+6*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)*exp(2)^8+18*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^9*exp(2)^4
+78*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^13
*exp(2)^2-14*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*
exp(1)^11*exp(2)^3-87*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^9*exp(2)^4-33*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^5*exp(1)^7*exp(2)^5+84*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^2*exp(1)^11*exp(2)^3-48*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^4-120*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^5-36*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^5*exp(2)^6-120*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^9*exp(2)
^4-96*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
7*exp(2)^5+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
*exp(1)^5*exp(2)^6+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^5*exp(1)^3*exp(2)^7-204*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^7*exp(2)^5-180*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
```

```
xp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)^6-30*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^3*exp(2)^7+11*exp(1)^9*exp(2)^4+3
6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^3*ex
p(2)^7+12*exp(1)^7*exp(2)^5-60*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^3*exp(2)^7-20*exp(1)^5*exp(2)^6-30*exp(1)^3*exp(2)
^7-12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^7/x/exp(2
)+72*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^6/x/exp(2)
+87/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^5/x/exp(2
)-27*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*exp(2)^4/x/exp(2)
-24*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^11*exp(2)^3/x/exp(2)
)/(-(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)+(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x-exp(2))^3/(3*d^3*exp(1)^11-6*d^
3*exp(1)^7*exp(2)^2-6*d^3*exp(1)^5*exp(2)^3+3*d^3*exp(1)^9*exp(2)+6*d^3*exp
(1)*exp(2)^5)+1/2*(-4*exp(1)^9*exp(2)^2+10*exp(1)^7*exp(2)^3+8*exp(1)^5*exp
(2)^4-8*exp(1)^3*exp(2)^5-4*exp(1)^11*exp(2)-16*exp(1)*exp(2)^6)*atan((-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2
)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^3*exp(1)^11+2*d^3*exp(1)^7*exp(2)^2+2*d^
3*exp(1)^5*exp(2)^3-d^3*exp(1)^9*exp(2)-2*d^3*exp(1)*exp(2)^5)-exp(2)*ln(1/
2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)^
2
```

maple [B] time = 0.01, size = 196, normalized size = 1.78

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^2}+\frac{\sqrt{-e^2x^2+d^2}}{d^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^4d^2e^4}+\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^3d^3e^3}+\frac{\left(2\left(x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x)
```

```
[Out] 1/5/d^2/e^4/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+2/5/e^3/d^3/(x+d/
e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+1/d^4*(-e^2*x^2+d^2)^(1/2)-1/d^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^4/e^2/(x+d/
e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

3.195 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4} \, dx$

Optimal. Leaf size=143

$$-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] -8/5*e*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/15*e*(-8*e*x+5*d)/d^2/(-e^2*x^2+d^2)
^(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/15*e*(-79*e*x+60*d)/d^4/(-
e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] time = 0.31, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d
^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*Sqrt[d^2 - e^2*x^2]) - S
qrt[d^2 - e^2*x^2]/(d^4*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-27d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+64d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(4e)\int \frac{1}{x\sqrt{d^2-e^2}}}{d^3}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(2e)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4\,\mathrm{Subst}\left(\int \frac{}{\frac{d^2}{e^2}}\right.}{}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e\tanh^{-1}\left(\sqrt{\right.}{d^4}$$

Mathematica [A] time = 0.21, size = 92, normalized size = 0.64

$$-\frac{-60e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(15d^3+149d^2ex+222de^2x^2+94e^3x^3\right)}{x(d+ex)^3}+60e\log(x)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e^3*
x^3))/(x*(d + e*x)^3) + 60*e*Log[x] - 60*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^
4
```

fricas [A] time = 0.94, size = 181, normalized size = 1.27

$$-\frac{104\,e^4x^4 + 312\,de^3x^3 + 312\,d^2e^2x^2 + 104\,d^3ex + 60\left(e^4x^4 + 3\,de^3x^3 + 3\,d^2e^2x^2 + d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + (94}{15\left(d^4e^3x^4 + 3\,d^5e^2x^3 + 3\,d^6ex^2 + d^7x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(104*e^4*x^4 + 312*d*e^3*x^3 + 312*d^2*e^2*x^2 + 104*d^3*e*x + 60*(e^
4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) + (94*e^3*x^3 + 222*d*e^2*x^2 + 149*d^2*e*x + 15*d^3)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^3*x^4 + 3*d^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x)
```

giac [A] time = 0.28, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [B] time = 0.01, size = 361, normalized size = 2.52

$$\frac{4e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3}+\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^4}-\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^4}-\frac{\sqrt{-e^2x^2+d^2}\,e^2x}{d^6}-\frac{4\sqrt{-e^2x^2}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x)
```

```
[Out] -1/5/d^3/e^3/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-11/15/d^4/e^2/(x
+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-1/d^6/x*(-e^2*x^2+d^2)^(3/2)-1/
d^6*e^2*x*(-e^2*x^2+d^2)^(1/2)-1/d^4*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e
^2*x^2+d^2)^(1/2)*x)-4/d^5*e*(-e^2*x^2+d^2)^(1/2)+4/d^3*e/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^5*e*(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)+1/d^4*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)*x)-3/d^5/e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{d^2-e^2\,x^2}}{x^2\,(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^2(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)**4), x)
```

3.196 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 8/5*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+4/15*e^2*(-13*e*x+10*d)/d^3/(-e^2*x
^2+d^2)^(3/2)-19/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*e^2*(-164*e
*x+135*d)/d^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^4/x^2+4*e*(-e
^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A] time = 0.39, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (4*e^2*(10*d - 13*e*x))/(15
*d^3*(d^2 - e^2*x^2)^(3/2)) + (e^2*(135*d - 164*e*x))/(15*d^5*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*Sqrt[d^2 - e^2*x^2])/(d^5*
x) - (19*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^5)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+32de^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-104de^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{15d^4}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}dx}{15d^6}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{\int \frac{-120d^5e+285d^4e^2}{x^2\sqrt{d^2-e^2x^2}}}{30d^8}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} +$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} +$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} -$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} -$$

Mathematica [A] time = 0.23, size = 107, normalized size = 0.58

$$\frac{-285e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(-15d^4+75d^3ex+713d^2e^2x^2+1059de^3x^3+448e^4x^4\right)}{x^2(d+ex)^3} + 285e^2\log(x)}{30d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3*
x^3 + 448*e^4*x^4))/(x^2*(d + e*x)^3) + 285*e^2*Log[x] - 285*e^2*Log[d + Sq
rt[d^2 - e^2*x^2]])/(30*d^5)
```

fricas [A] time = 0.96, size = 202, normalized size = 1.10

$$\frac{398\,e^5x^5 + 1194\,de^4x^4 + 1194\,d^2e^3x^3 + 398\,d^3e^2x^2 + 285\left(e^5x^5 + 3\,de^4x^4 + 3\,d^2e^3x^3 + d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)}{30\left(d^5e^3x^5 + 3\,d^6e^2x^4 + 3\,d^7ex^3 + d^8x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/30*(398*e^5*x^5 + 1194*d*e^4*x^4 + 1194*d^2*e^3*x^3 + 398*d^3*e^2*x^2 + 2
85*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) + (448*e^4*x^4 + 1059*d*e^3*x^3 + 713*d^2*e^2*x^2 + 75*d^3
*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^5 + 3*d^6*e^2*x^4 + 3*d^7*e
*x^3 + d^8*x^2)
```

giac [A] time = 0.30, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [B] time = 0.01, size = 389, normalized size = 2.13

$$-\frac{19e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^4}-\frac{4e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^5}+\frac{4e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^5}+\frac{4\sqrt{-e^2x^2+d^2}\,e^3x}{d^7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x)
```

```
[Out] 1/5/d^4/e^2/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+16/15/d^5/e/(x+d/
e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/d^7*e/x*(-e^2*x^2+d^2)^(3/2)+4/d
^7*e^3*x*(-e^2*x^2+d^2)^(1/2)+4/d^5*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^
2*x^2+d^2)^(1/2)*x)-1/2/d^6/x^2*(-e^2*x^2+d^2)^(3/2)+19/2/d^6*e^2*(-e^2*x^2
+d^2)^(1/2)-19/2/d^4*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-4/d^6*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-4/d^5*e^3/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+6/d^6/(x+d/e)
^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,{(d+e\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^3\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)**4), x)
```

3.197 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}+\frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}-$$

```
[Out] -8/5*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-4/5*e^3*(-6*e*x+5*d)/d^4/(-e^2*x
^2+d^2)^(3/2)+18*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/5*e^3*(-93*e*x+8
0*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^4/x^3+2*e*(-e^2*x^
2+d^2)^(1/2)/d^5/x^2-29/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A] time = 0.49, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{18e^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (4*e^3*(5*d - 6*e*x))/(5
*d^4*(d^2 - e^2*x^2)^(3/2)) - (e^3*(80*d - 93*e*x))/(5*d^6*Sqrt[d^2 - e^2*x
^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^5*x^2
) - (29*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+40de^3x^3-32e^4x^4}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-180de^3x^3+144e^4x^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2+240de}{x^4\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{\int \frac{-180d^5e+435d}{x^3\sqrt{d}}}{4}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

Mathematica [A] time = 0.27, size = 118, normalized size = 0.56

$$-\frac{-270e^3\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(5d^5-15d^4ex+70d^3e^2x^2+674d^2e^3x^3+1002de^4x^4+424e^5x^5\right)}{x^3(d+ex)^3} + 270e^3\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 15*d^4*e*x + 70*d^3*e^2*x^2 + 674*d^2*
e^3*x^3 + 1002*d*e^4*x^4 + 424*e^5*x^5))/(x^3*(d + e*x)^3) + 270*e^3*Log[x]
 - 270*e^3*Log[d + Sqrt[d^2 - e^2*x^2]])/d^6
```

fricas [A] time = 0.82, size = 213, normalized size = 1.01

$$-\frac{324\,e^6x^6 + 972\,de^5x^5 + 972\,d^2e^4x^4 + 324\,d^3e^3x^3 + 270\left(e^6x^6 + 3\,de^5x^5 + 3\,d^2e^4x^4 + d^3e^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d}}{x}\right.}{15\left(d^6e^3x^6 + 3\,d^7e^2x^5 + 3\,d^8ex^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(324*e^6*x^6 + 972*d*e^5*x^5 + 972*d^2*e^4*x^4 + 324*d^3*e^3*x^3 + 27
0*(e^6*x^6 + 3*d*e^5*x^5 + 3*d^2*e^4*x^4 + d^3*e^3*x^3)*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + (424*e^5*x^5 + 1002*d*e^4*x^4 + 674*d^2*e^3*x^3 + 70*d^3*
e^2*x^2 - 15*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^6 + 3*d^7*e^
2*x^5 + 3*d^8*e*x^4 + d^9*x^3)
```

giac [A] time = 0.37, size = 1, normalized size = 0.00

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [B] time = 0.02, size = 412, normalized size = 1.96

$$\frac{18e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5}+\frac{10e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^6}-\frac{10e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^6}-\frac{10\sqrt{-e^2x^2+d^2}\,e^4x}{d^8}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x)
```

```
[Out] -1/5/d^5/e/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-7/5/d^6/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-10/d^8*e^2/x*(-e^2*x^2+d^2)^(3/2)-10/d^
8*e^4*x*(-e^2*x^2+d^2)^(1/2)-10/d^6*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^
2*x^2+d^2)^(1/2)*x)+2/d^7*e/x^2*(-e^2*x^2+d^2)^(3/2)-18/d^7*e^3*(-e^2*x^2+d
^2)^(1/2)+18/d^5*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x)-1/3/d^6/x^3*(-e^2*x^2+d^2)^(3/2)+10/d^7*e^3*(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(1/2)+10/d^6*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)*x)-10/d^7*e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^4\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)**4), x)
```

3.198 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal. Leaf size=252

$$\frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} + \frac{65d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e}}{4e^5}$$

```
[Out] 65/4*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+d^4*(-e*x+d)^4/e^6/(-e^2*x^2+
d^2)^(1/2)+515/21*d^6*(-e^2*x^2+d^2)^(1/2)/e^6-49/4*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^5+121/21*d^4*x^2*(-e^2*x^2+d^2)^(1/2)/e^4-17/6*d^3*x^3*(-e^2*x^2+d^2)
^(1/2)/e^3+11/7*d^2*x^4*(-e^2*x^2+d^2)^(1/2)/e^2-2/3*d*x^5*(-e^2*x^2+d^2)^(
1/2)/e+1/7*x^6*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} + \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x}}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(e^6*Sqrt[d^2 - e^2*x^2]) + (515*d^6*Sqrt[d^2 - e^2*x^2])
/(21*e^6) - (49*d^5*x*Sqrt[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*Sqrt[d^2
- e^2*x^2])/(21*e^4) - (17*d^3*x^3*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x
^4*Sqrt[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*Sqrt[d^2 - e^2*x^2])/(3*e) + (x^
6*Sqrt[d^2 - e^2*x^2])/7 + (65*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^
6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=\int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{d^4x}{e^4}-\frac{d^3x^2}{e^3}+\frac{d^2x^3}{e^2}-\frac{dx^4}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}+\frac{\int \frac{\frac{28d^8}{e^3}-\frac{91d^7x}{e^2}+\frac{112d^6x^2}{e}-77d^5x^3+56d^4ex^4-55d^3e^2x^5+28d^2e^3x^6}{\sqrt{d^2-e^2x^2}}\,d}{7de^2}$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}-\frac{\int \frac{-\frac{168d^8}{e}+546d^7x-672d^6ex^2+462d^5e^2}{\sqrt{d^2-e^2x^2}}}{42de^4}$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}+\frac{\int \frac{840d^8e-}{}}{}$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{1}{7}x^6$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}-2d$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+11d$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-17d$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-17d$$

$$=\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-17d$$

Mathematica [A] time = 0.23, size = 131, normalized size = 0.52

$$\frac{1365d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7\right)}{d+ex}}{84e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^4*e
^3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))/(d
+ e*x) + 1365*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(84*e^6)
```

fricas [A] time = 0.81, size = 156, normalized size = 0.62

$$\frac{2144\,d^7ex+2144\,d^8-2730\left(d^7ex+d^8\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(12\,e^7x^7-44\,de^6x^6+76\,d^2e^5x^5-106\,d^3e^4x^4+\cdots\right.}{84\left(e^7x+de^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/84*(2144*d^7*e*x + 2144*d^8 - 2730*(d^7*e*x + d^8)*arctan(-(d - sqrt(-e^2
*x^2 + d^2))/(e*x)) + (12*e^7*x^7 - 44*d*e^6*x^6 + 76*d^2*e^5*x^5 - 106*d^3
*e^4*x^4 + 162*d^4*e^3*x^3 - 293*d^5*e^2*x^2 + 779*d^6*e*x + 2144*d^7)*sqrt
(-e^2*x^2 + d^2))/(e^7*x + d*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-162
*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^1
2*exp(2)^2-36*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^10*exp(2)^3+720*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2+684*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+162*d^7*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-402*d^7*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2
)^2+350*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(1)^10*exp(2)^3+507*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^4*exp(1)^8*exp(2)^4+123*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+1476*d^7*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-864*d^7*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-252
*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4
*exp(2)^6+1248*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*exp(1)^8*exp(2)^4+84*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-654*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-192*d^7*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-1836*d^7*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-1620*d^7
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp
(2)^6-47*d^7*exp(1)^8*exp(2)^4-486*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^4*exp(2)^8-1464*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
```

```
^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6+180*d^7*exp(1)^6*exp(2)^5-
1296*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(
2)^8+158*d^7*exp(1)^4*exp(2)^6-972*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^2*exp(2)^8-486*d^7*exp(2)^8-188*d^7*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+552*d^7*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+684*d^7*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-825/2*d^7*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-459*d^
7*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+12
3*d^7*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(
2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^16+9*exp(1
)^12*exp(2)^2+3*exp(1)^10*exp(2)^3+9*exp(1)^14*exp(2))+1/2*(-300*d^7*exp(1)
^10*exp(2)^2-82*d^7*exp(1)^8*exp(2)^3+1000*d^7*exp(1)^6*exp(2)^4+464*d^7*ex
p(1)^4*exp(2)^5-1248*d^7*exp(2)^7+40*d^7*exp(1)^12*exp(2))*atan((-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/
sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^18-3*exp(1)^14*exp(2)^2-exp(1)^12*exp(2)^
3-3*exp(1)^16*exp(2))+65/4*d^7*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^6+2*(
(((((720*exp(1)^26*1/10080/exp(1)^26*x-3360*exp(1)^25*d*1/10080/exp(1)^26)*
x+7920*exp(1)^24*d^2*1/10080/exp(1)^26)*x-14280*exp(1)^23*d^3*1/10080/exp(1
)^26)*x+24000*exp(1)^22*d^4*1/10080/exp(1)^26)*x-41580*exp(1)^21*d^5*1/1008
0/exp(1)^26)*x+88320*exp(1)^20*d^6*1/10080/exp(1)^26)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.03, size = 416, normalized size = 1.65

$$\frac{35d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,e^5}-\frac{5d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,e^5}+\frac{35\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5x}{2e^5}-\frac{5\sqrt{-e^2x^2+d}}{4e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] d^4/e^10/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+8*d^3/e^9/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+22/3*d^2/e^8/(x+d/e)^2*(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(7/2)+35/3*d^3/e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+35/2
*d^5/e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+35/2*d^7/e^5/(e^2)^(1/2)*arc
tan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-2/3/e^5*d*x*(-e^2*x^
2+d^2)^(5/2)-5/6/e^5*d^3*x*(-e^2*x^2+d^2)^(3/2)-5/4*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^5-5/4/e^5*d^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-
1/7/e^6*(-e^2*x^2+d^2)^(7/2)+28/3*d^2/e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/
2)
```

maxima [C] time = 1.06, size = 478, normalized size = 1.90

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5}{2\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}{2\left(e^8x^2+2\,de^7x+d^2e^6\right)}+\frac{15\sqrt{-e^2x^2+d^2}\,d^7}{e^7x+de^6}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}{3\left(e^8x^2+2\,de^7x+d^2e^6\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*
e^6) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^6/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 15*
sqrt(-e^2*x^2 + d^2)*d^7/(e^7*x + d*e^6) + 5/3*(-e^2*x^2 + d^2)^(5/2)*d^4/(
e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 25/6*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x + d
*e^6) - 5/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x + d*e^6) + 5/2*I*d^7*arcsin(e
*x/d + 2)/e^6 + 75/4*d^7*arcsin(e*x/d)/e^6 - 5/2*sqrt(e^2*x^2 + 4*d*e*x + 3
```

```
*d^2)*d^5*x/e^5 - 5/4*sqrt(-e^2*x^2 + d^2)*d^5*x/e^5 - 5*sqrt(e^2*x^2 + 4*d
*e*x + 3*d^2)*d^6/e^6 + 25/2*sqrt(-e^2*x^2 + d^2)*d^6/e^6 + 5/3*(-e^2*x^2 +
 d^2)^(3/2)*d^3*x/e^5 - 25/6*(-e^2*x^2 + d^2)^(3/2)*d^4/e^6 - 2/3*(-e^2*x^2
 + d^2)^(5/2)*d*x/e^5 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2/e^6 - 1/7*(-e^2*x^2 +
d^2)^(7/2)/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(d^2 - e^2 x^2\right)^{5/2}}{(d + e x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (-(-d + ex)(d + ex))^{\frac{5}{2}}}{(d + ex)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.199 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=224

$$\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{239d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{}}{16}$$

```
[Out] -239/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-d^3*(-e*x+d)^4/e^5/(-e^2*x
^2+d^2)^(1/2)-337/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5+175/16*d^4*x*(-e^2*x^2+d^
2)^(1/2)/e^4-71/15*d^3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3+47/24*d^2*x^3*(-e^2*x^2
+d^2)^(1/2)/e^2-4/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e+1/6*x^5*(-e^2*x^2+d^2)^(1/
2)
```

Rubi [A] time = 0.53, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{4dx^4\sqrt{d^2-}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d^3*(d - e*x)^4)/(e^5*Sqrt[d^2 - e^2*x^2])) - (337*d^5*Sqrt[d^2 - e^2*x^
2])/(15*e^5) + (175*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^4) - (71*d^3*x^2*Sqrt[
d^2 - e^2*x^2])/(15*e^3) + (47*d^2*x^3*Sqrt[d^2 - e^2*x^2])/(24*e^2) - (4*d
*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (x^5*Sqrt[d^2 - e^2*x^2])/6 - (239*d^6*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{d^3x}{e^3}+\frac{d^2x^2}{e^2}-\frac{dx^3}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-\frac{24d^7}{e^2}+\frac{78d^6x}{e}-96d^5x^2+66d^4ex^3-47d^3e^2x^4+24d^2e^3x^5}{\sqrt{d^2-e^2x^2}}\,dx}{6de^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{120d^7-390d^6ex+480d^5e^2x^2-426d^4e^3x^3 \cdots}{\sqrt{d^2-e^2x^2}}}{30de^4}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-480d^7e^2 \cdots}{\cdots}}{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4 \cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2 \cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2 \cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2 \cdots}{\cdots}$$

Mathematica [A] time = 0.16, size = 125, normalized size = 0.56

$$\frac{\sqrt{d^2-e^2x^2}\left(-5632d^6-2047d^5ex+769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-152de^5x^5+40e^6x^6\right)-3585d^6(d+ex) \cdots}{240e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^3*
e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6) - 3585*d^6*(d + e*x
)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(240*e^5*(d + e*x))
```

fricas [A] time = 0.97, size = 146, normalized size = 0.65

$$-\frac{5632\,d^6ex + 5632\,d^7 - 7170\left(d^6ex + d^7\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(40\,e^6x^6 - 152\,de^5x^5 + 278\,d^2e^4x^4 - 426\,d^3\right.}{240\left(e^6x + de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(5632*d^6*e*x + 5632*d^7 - 7170*(d^6*e*x + d^7)*arctan(-(d - sqrt(-e
^2*x^2 + d^2))/(e*x)) - (40*e^6*x^6 - 152*d*e^5*x^5 + 278*d^2*e^4*x^4 - 426
*d^3*e^3*x^3 + 769*d^4*e^2*x^2 - 2047*d^5*e*x - 5632*d^6)*sqrt(-e^2*x^2 + d
^2))/(e^6*x + d*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (84*d
^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*
exp(2)^2+18*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^10*exp(2)^3-576*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^12*exp(2)^2-540*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-126*d^6*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+228*d^6*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^
2-200*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^10*exp(2)^3-264*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^4*exp(1)^8*exp(2)^4-60*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-1188*d^6*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+72*d^6*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+756*d^6
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp
(2)^5+216*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5
*exp(1)^4*exp(2)^6-726*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^2*exp(1)^8*exp(2)^4+138*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5+537*d^6*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+147*d^6*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+1620*d^6*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5
+1404*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^4*exp(2)^6+26*d^6*exp(1)^8*exp(2)^4+402*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+1212*d^6*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-144*d^6*exp(1)^6*
exp(2)^5+1008*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(2)^8-89*d^6*exp(1)^4*exp(2)^6+804*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+402*d^6*exp(2)^8+104*d^6*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)-861
```

```
/2*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)-594*d^
6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+23
7*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2
)+369*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/e
xp(2)-69*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3
/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^15+
9*exp(1)^11*exp(2)^2+3*exp(1)^9*exp(2)^3+9*exp(1)^13*exp(2))+1/2*(-192*d^6*
exp(1)^10*exp(2)^2+32*d^6*exp(1)^8*exp(2)^3+712*d^6*exp(1)^6*exp(2)^4+230*d
^6*exp(1)^4*exp(2)^5-924*d^6*exp(2)^7+16*d^6*exp(1)^12*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^17+3*exp(1)^13*exp(2)^2+exp(1)^11*exp(
2)^3+3*exp(1)^15*exp(2))-239/16*d^6*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^
5+2*(((((240*exp(1)^19*1/2880/exp(1)^19*x-1152*exp(1)^18*d*1/2880/exp(1)^19
)*x+2820*exp(1)^17*d^2*1/2880/exp(1)^19)*x-5376*exp(1)^16*d^3*1/2880/exp(1)
^19)*x+9990*exp(1)^15*d^4*1/2880/exp(1)^19)*x-22272*exp(1)^14*d^5*1/2880/ex
p(1)^19)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.02, size = 393, normalized size = 1.75

$$-\frac{61 d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{4 \sqrt{e^{2}}\, e^{4}}+\frac{5 d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 \sqrt{e^{2}}\, e^{4}}-\frac{61 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, d^{4} x}{4 e^{4}}+\frac{5 \sqrt{-e^{2} x^{2}+d^{2}}}{16 e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -d^3/e^9/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-7*d^2/e^8/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-22/3*d/e^7/(x+d/e)^2*(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(7/2)-61/6*d^2/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-61/4*d
^4/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-61/4*d^6/e^4/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+5/24/e^4*d^2*x*(-e^2*x
^2+d^2)^(3/2)+5/16/(e^2)^(1/2)*d^6/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1
/2)*x)+5/16*(-e^2*x^2+d^2)^(1/2)*d^4/e^4*x-122/15*d/e^5*(2*(x+d/e)*d*e-(x+d
/e)^2*e^2)^(5/2)+1/6/e^4*x*(-e^2*x^2+d^2)^(5/2)
```

maxima [C] time = 1.04, size = 456, normalized size = 2.04

$$\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{4}}{2\left(e^{8} x^{3}+3 d e^{7} x^{2}+3 d^{2} e^{6} x+d^{3} e^{5}\right)}+\frac{5\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{5}}{2\left(e^{7} x^{2}+2 d e^{6} x+d^{2} e^{5}\right)}-\frac{15 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{6}}{e^{6} x+d e^{5}}-\frac{4\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{3}}{3\left(e^{7} x^{2}+2 d e^{6} x+d^{2} e^{5}\right)}-\frac{10\,(}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e
^5) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 15*s
qrt(-e^2*x^2 + d^2)*d^6/(e^6*x + d*e^5) - 4/3*(-e^2*x^2 + d^2)^(5/2)*d^3/(e
^7*x^2 + 2*d*e^6*x + d^2*e^5) - 10/3*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x + d*
e^5) + 3/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x + d*e^5) - 9/4*I*d^6*arcsin(e*
x/d + 2)/e^5 - 275/16*d^6*arcsin(e*x/d)/e^5 + 9/4*sqrt(e^2*x^2 + 4*d*e*x +
3*d^2)*d^4*x/e^4 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^4 + 9/2*sqrt(e^2*x^2 +
 4*d*e*x + 3*d^2)*d^5/e^5 - 10*sqrt(-e^2*x^2 + d^2)*d^5/e^5 - 19/24*(-e^2*x
^2 + d^2)^(3/2)*d^2*x/e^4 + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/e^5 + 1/6*(-e^2*
x^2 + d^2)^(5/2)*x/e^4 - 4/5*(-e^2*x^2 + d^2)^(5/2)*d/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(- \left(- d + e x\right) \left(d + e x\right)\right)^{\frac{5}{2}}}{\left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.200 $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=192

$$\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}+\frac{27d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}+\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-$$

```
[Out] 27/2*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+d^2*(-e*x+d)^4/e^4/(-e^2*x^2+
d^2)^(1/2)+101/5*d^4*(-e^2*x^2+d^2)^(1/2)/e^4-19/2*d^3*x*(-e^2*x^2+d^2)^(1/
2)/e^3+18/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-d*x^3*(-e^2*x^2+d^2)^(1/2)/e+1
/5*x^4*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-\frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}+\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{27d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^2*(d - e*x)^4)/(e^4*Sqrt[d^2 - e^2*x^2]) + (101*d^4*Sqrt[d^2 - e^2*x^2])
/(5*e^4) - (19*d^3*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (18*d^2*x^2*Sqrt[d^2 -
e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/e + (x^4*Sqrt[d^2 - e^2*x^2
])/5 + (27*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
```

```
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx &= \int \frac{x^3(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^3}{e^3}+\frac{d^2x}{e^2}-\frac{dx^2}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{20d^6}{e}-65d^5x+80d^4ex^2-54d^3e^2x^3+20d^2e^3x^4}{\sqrt{d^2-e^2x^2}}\,dx}{5de^2} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-80d^6e+260d^5e^2x-380d^4e^3x^2+216d^3e}{\sqrt{d^2-e^2x^2}}}{20de^4} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int 240d^6e^3-}{} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{}}{} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{}}{} \\
&= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{}}{}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.57

$$
\frac{135d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(212d^5+77d^4ex-29d^3e^2x^2+16d^2e^3x^3-8de^4x^4+2e^5x^5\right)}{d+ex}}{10e^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^3*x
^3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(d + e*x) + 135*d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(10*e^4)
```

fricas [A] time = 0.96, size = 134, normalized size = 0.70

$$\frac{212\,d^5ex + 212\,d^6 - 270\left(d^5ex + d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(2\,e^5x^5 - 8\,de^4x^4 + 16\,d^2e^3x^3 - 29\,d^3e^2x^2 + 77\,d^4ex}{10\left(e^5x + de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/10*(212*d^5*e*x + 212*d^6 - 270*(d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (2*e^5*x^5 - 8*d*e^4*x^4 + 16*d^2*e^3*x^3 - 29*d^3*e^2*x
^2 + 77*d^4*e*x + 212*d^5)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-30*
d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12
*exp(2)^2-6*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^10*exp(2)^3+432*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^12*exp(2)^2+396*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+90*d^5*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-102*d^5*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2
+62*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1
)^10*exp(2)^3+63*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^4*exp(1)^8*exp(2)^4+9*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+900*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-144*d^5*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-648*d^5*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^
5-180*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp
(1)^4*exp(2)^6+288*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^8*exp(2)^4-312*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-432*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-108*d^5*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-1404*d^5*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-118
8*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
4*exp(2)^6-11*d^5*exp(1)^8*exp(2)^4-324*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-984*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6+108*d^5*exp(1)^6*exp(2
)^5-756*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(2)^8+32*d^5*exp(1)^4*exp(2)^6-648*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-324*d^5*exp(2)^8-44*d^5*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+324*d^5*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+504*d^5*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-183/2*d^5*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-279*d
^5*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+3
0*d^5*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(
2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^14+9*exp(1
)^10*exp(2)^2+3*exp(1)^8*exp(2)^3+9*exp(1)^12*exp(2))+1/2*(-108*d^5*exp(1)^
```

```
10*exp(2)^2+90*d^5*exp(1)^8*exp(2)^3+472*d^5*exp(1)^6*exp(2)^4+72*d^5*exp(1
)^4*exp(2)^5-656*d^5*exp(2)^7+4*d^5*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(
-exp(1)^4+exp(2)^2)/(-exp(1)^16-3*exp(1)^12*exp(2)^2-exp(1)^10*exp(2)^3-3*e
xp(1)^14*exp(2))+27/2*d^5*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^4+2*((((24
*exp(1)^13*1/240/exp(1)^13*x-120*exp(1)^12*d*1/240/exp(1)^13)*x+312*exp(1)^
11*d^2*1/240/exp(1)^13)*x-660*exp(1)^10*d^3*1/240/exp(1)^13)*x+1584*exp(1)^
9*d^4*1/240/exp(1)^13)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.02, size = 285, normalized size = 1.48

$$\frac{27d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,e^3}+\frac{27\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3x}{2e^3}+\frac{9\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}dx}{e^3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] d^2/e^8/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+6*d/e^7/(x+d/e)^3*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+7/e^6/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e
^2)^(7/2)+36/5/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+9*d/e^3*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(3/2)*x+27/2*d^3/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*
x+27/2*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x)
```

maxima [C] time = 1.03, size = 407, normalized size = 2.12

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{2\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}{2\left(e^6x^2+2\,de^5x+d^2e^4\right)}+\frac{15\sqrt{-e^2x^2+d^2}\,d^5}{e^5x+de^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{e^6x^2+2\,de^5x+d^2e^4}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*
e^4) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 15*
sqrt(-e^2*x^2 + d^2)*d^5/(e^5*x + d*e^4) + (-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*
x^2 + 2*d*e^5*x + d^2*e^4) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x + d*e^4)
 - 3/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x + d*e^4) + 3/2*I*d^5*arcsin(e*x/d +
2)/e^4 + 15*d^5*arcsin(e*x/d)/e^4 - 3/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3
*x/e^3 - 3*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^4 + 15/2*sqrt(-e^2*x^2 + d
^2)*d^4/e^4 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^3 - 5/4*(-e^2*x^2 + d^2)^(3/
2)*d^2/e^4 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.201 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=182

$$-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] -95/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d*(-e*x+d)^4/e^3/(-e^2*x^2+d
^2)^(1/2)-95/8*d^3*(-e^2*x^2+d^2)^(1/2)/e^3-95/24*d^2*(-e*x+d)*(-e^2*x^2+d^
2)^(1/2)/e^3-19/12*d*(-e*x+d)^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/4*(-e*x+d)^3*(-e
^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1635, 795, 671, 641, 217, 203}

$$-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d*(d - e*x)^4)/(e^3*Sqrt[d^2 - e^2*x^2])) - (95*d^3*Sqrt[d^2 - e^2*x^2])
/(8*e^3) - (95*d^2*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) - (19*d*(d - e*x
)^2*Sqrt[d^2 - e^2*x^2])/(12*e^3) - ((d - e*x)^3*Sqrt[d^2 - e^2*x^2])/(4*e^
3) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
```

```
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{4d^2}{e^2}-\frac{dx}{e}\right)(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(19d)\int \frac{(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{\left(95d^2\right)\int \frac{(d-ex)^2}{\sqrt{d^2-e^2x^2}}\,d}{12e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-}}{4e^3}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x}}{12e^3}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x}}{12e^3}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x}}{12e^3}$$

Mathematica [A] time = 0.12, size = 103, normalized size = 0.57

$$\sqrt{d^2-e^2x^2}\left(-\frac{8d^4}{e^3(d+ex)}-\frac{32d^3}{3e^3}+\frac{31d^2x}{8e^2}-\frac{4dx^2}{3e}+\frac{x^3}{4}\right)-\frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-32*d^3)/(3*e^3) + (31*d^2*x)/(8*e^2) - (4*d*x^2)/(3*
e) + x^3/4 - (8*d^4)/(e^3*(d + e*x))) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(8*e^3)
```

fricas [A] time = 0.80, size = 124, normalized size = 0.68

$$-\frac{448\,d^4ex + 448\,d^5 - 570\left(d^4ex + d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(6\,e^4x^4 - 26\,de^3x^3 + 61\,d^2e^2x^2 - 163\,d^3ex - 44\right.}{24\left(e^4x + de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(448*d^4*e*x + 448*d^5 - 570*(d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) - (6*e^4*x^4 - 26*d*e^3*x^3 + 61*d^2*e^2*x^2 - 163*d^3*e*
x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-288
*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
2*exp(2)^2-252*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^4*exp(1)^10*exp(2)^3-54*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+24*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2+64*d^4*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+8*d^4*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+96
*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8
*exp(2)^4+30*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^5*exp(1)^6*exp(2)^5-612*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+216*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+540*d^4*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5+144*d^4*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6+
66*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^8*exp(2)^4+438*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^6*exp(2)^5+339*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+75*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+1188*d^4*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+972*d^4*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*d^4*e
xp(1)^8*exp(2)^4+252*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(2)^8+780*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(1)^4*exp(2)^6-72*d^4*exp(1)^6*exp(2)^5+540*d^4*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8+13*d^4*exp(1
)^4*exp(2)^6+504*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^2*exp(2)^8+252*d^4*exp(2)^8-465/2*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(2)^8/x/exp(2)-414*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-24*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+189*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-6*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^
```

```
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x+exp(2))^3/(3*exp(1)^13+9*exp(1)^9*exp(2)^2+3*exp(1)^7*exp(2)^3+
9*exp(1)^11*exp(2))+1/2*(48*d^4*exp(1)^10*exp(2)^2-104*d^4*exp(1)^8*exp(2)^
3-280*d^4*exp(1)^6*exp(2)^4+22*d^4*exp(1)^4*exp(2)^5+440*d^4*exp(2)^7)*atan
((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4
+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^15-3*exp(1)^11*exp(2)^2-exp(1
)^9*exp(2)^3-3*exp(1)^13*exp(2))-95/8*d^4*sign(d)*asin(x*exp(2)/d/exp(1))/e
xp(1)/exp(2)+2*(((12*exp(1)^8*1/96/exp(1)^8*x-64*exp(1)^7*d*1/96/exp(1)^8)*
x+186*exp(1)^6*d^2*1/96/exp(1)^8)*x-512*exp(1)^5*d^3*1/96/exp(1)^8)*sqrt(d^
2-x^2*exp(2))
```

maple [A] time = 0.01, size = 288, normalized size = 1.58

$$-\frac{95 d^4 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\, e^2}-\frac{95\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2}\, d^2 x}{8 e^2}-\frac{95\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} x}{12 e^2}-\frac{19\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -d/e^7/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-5/e^6/(x+d/e)^3*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-19/3/d/e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(7/2)-19/3/d/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-95/12/e^2*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-95/8*d^2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x-95/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)*x)
```

maxima [C] time = 1.02, size = 363, normalized size = 1.99

$$\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^2}{2\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^3}{2\left(e^5x^2+2\,de^4x+d^2e^3\right)}-\frac{15\,\sqrt{-e^2x^2+d^2}\,d^4}{e^4x+de^3}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d}{3\left(e^5x^2+2\,de^4x+d^2e^3\right)}-\frac{5\left(-\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e
^3) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 15*s
qrt(-e^2*x^2 + d^2)*d^4/(e^4*x + d*e^3) - 2/3*(-e^2*x^2 + d^2)^(5/2)*d/(e^5
*x^2 + 2*d*e^4*x + d^2*e^3) - 5/3*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x + d*e^3
) - 5/8*I*d^4*arcsin(e*x/d + 2)/e^3 - 25/2*d^4*arcsin(e*x/d)/e^3 + 1/4*(-e^
2*x^2 + d^2)^(5/2)/(e^4*x + d*e^3) + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
2*x/e^2 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e^3 - 5*sqrt(-e^2*x^2 + d
^2)*d^3/e^3 + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.202 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=130

$$\frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4}+\frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2}+\frac{20(d^2-e^2x^2)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] 20/3*(-e^2*x^2+d^2)^(3/2)/e^2+8*(-e^2*x^2+d^2)^(5/2)/e^2/(e*x+d)^2+(-e^2*x^
2+d^2)^(7/2)/e^2/(e*x+d)^4+10*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+10*d
*x*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {793, 663, 665, 195, 217, 203}

$$\frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4}+\frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2}+\frac{20(d^2-e^2x^2)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (10*d*x*Sqrt[d^2 - e^2*x^2])/e + (20*(d^2 - e^2*x^2)^(3/2))/(3*e^2) + (8*(d
^2 - e^2*x^2)^(5/2))/(e^2*(d + e*x)^2) + (d^2 - e^2*x^2)^(7/2)/(e^2*(d + e*
x)^4) + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx &= \frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{4\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx}{e}\\ &= \frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{20\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{e}\\ &= \frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{(20d)\int \sqrt{d^2-e^2x^2}\,dx}{e}\\ &= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{\left(10d^3\right)\int \cdots}{e}\\ &= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{\left(10d^3\right)\text{Sub}\cdots}{\cdots}\\ &= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{10d^3\tan^{-1}\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.64

$$\frac{1}{3}\sqrt{d^2-e^2x^2}\left(\frac{24d^3}{e^2(d+ex)}+\frac{23d^2}{e^2}-\frac{6dx}{e}+x^2\right)+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((23*d^2)/e^2 - (6*d*x)/e + x^2 + (24*d^3)/(e^2*(d + e
*x))))/3 + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

fricas [A] time = 0.99, size = 111, normalized size = 0.85

$$\frac{47\,d^3ex+47\,d^4-60\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(e^3x^3-5\,de^2x^2+17\,d^2ex+47\,d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(e^3x+de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(47*d^3*e*x + 47*d^4 - 60*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (e^3*x^3 - 5*d*e^2*x^2 + 17*d^2*e*x + 47*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^3*x + d*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (6*d^
3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*e
xp(2)^2+144*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*exp(1)^12*exp(2)^2+108*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+18*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+6*d^3*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-178*d^3*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-2
13*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^8*exp(2)^4-57*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^5*exp(1)^6*exp(2)^5+324*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-288*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-432*d^3*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-108*d^3*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^
6-336*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^8*exp(2)^4-516*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^3*exp(1)^6*exp(2)^5-258*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-48*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-972*d^3*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-756*d^3*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+d^3*e
xp(1)^8*exp(2)^4-186*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(2)^8-600*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(1)^4*exp(2)^6+36*d^3*exp(1)^6*exp(2)^5-360*d^3*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8-46*d^3*exp(1
)^4*exp(2)^6-372*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^2*exp(2)^8-186*d^3*exp(2)^8+4*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+156*d^3*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+324*d^3*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+219/2*d^3*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-99*d^3*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-3*d^3*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^12+9*exp(1)^8*exp(2)^2+3*exp(1)^6
*exp(2)^3+9*exp(1)^10*exp(2))+1/2*(12*d^3*exp(1)^10*exp(2)^2-86*d^3*exp(1)^
8*exp(2)^3-136*d^3*exp(1)^6*exp(2)^4+64*d^3*exp(1)^4*exp(2)^5+272*d^3*exp(2
)^7)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(
-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^14+3*exp(1)^10*exp(2)
^2+exp(1)^8*exp(2)^3+3*exp(1)^12*exp(2))+10*d^3*sign(d)*asin(x*exp(2)/d/exp
(1))/exp(1)^2+2*((2*exp(1)^4*1/12/exp(1)^4*x-12*exp(1)^3*d*1/12/exp(1)^4)*x
+46*exp(1)^2*d^2*1/12/exp(1)^4)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 290, normalized size = 2.23

$$\frac{10d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e}+\frac{10\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,dx}{e}+\frac{20\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} x}{3de}+\frac{16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 1/e^6/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+4/d/e^5/(x+d/e)^3*(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+16/3/d^2/e^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(7/2)+16/3/d^2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+20/3/d/e*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+10*d/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1
/2)*x+10*d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x)
```

maxima [A] time = 0.99, size = 235, normalized size = 1.81

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{2\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{2\left(e^4x^2+2\,de^3x+d^2e^2\right)}+\frac{15\sqrt{-e^2x^2+d^2}\,d^3}{e^3x+de^2}+\frac{10\,d^3\arcsin\left(\frac{ex}{d}\right)}{e^2}+\frac{\left(\right.}{3\left(e^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^
2) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 15*sq
rt(-e^2*x^2 + d^2)*d^3/(e^3*x + d*e^2) + 10*d^3*arcsin(e*x/d)/e^2 + 1/3*(-e
^2*x^2 + d^2)^(5/2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 5/6*(-e^2*x^2 + d^2)^
(3/2)*d/(e^3*x + d*e^2) + 5/2*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.203 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=113

$$-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] -5/2*(-e^2*x^2+d^2)^(3/2)/e/(e*x+d)-2*(-e^2*x^2+d^2)^(5/2)/e/(e*x+d)^3-15/2
*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-15/2*d*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {663, 665, 217, 203}

$$-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (-15*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x
)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(2*e)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = -\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3} - 5\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{(d+ex)^2}\,dx$$
$$= -\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3} - \frac{1}{2}\left(15d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3} - \frac{1}{2}\left(15d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx\right.$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3} - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [**A**] time = 0.06, size = 75, normalized size = 0.66

$$\sqrt{d^2-e^2x^2}\left(-\frac{8d^2}{e(d+ex)} - \frac{4d}{e} + \frac{x}{2}\right) - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-4*d)/e + x/2 - (8*d^2)/(e*(d + e*x))) - (15*d^2*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

fricas [**A**] time = 0.93, size = 99, normalized size = 0.88

$$-\frac{24\,d^2ex + 24\,d^3 - 30\left(d^2ex + d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(e^2x^2 - 7\,dex - 24\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(e^2x + de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^2*e*x + 24*d^3 - 30*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) - (e^2*x^2 - 7*d*e*x - 24*d^2)*sqrt(-e^2*x^2 + d^2))/(e^2*x +
 d*e)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (12*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*
exp(2)^2+6*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^10*exp(2)^3+36*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^4*exp(1)^10*exp(2)^3+18*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+12*d^2*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2+280*d^2*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+28
```

```
8*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^
8*exp(2)^4+72*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^6*exp(2)^5-36*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+360*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+324*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5+72*d^2*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6+5
22*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^8*exp(2)^4+546*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^6*exp(2)^5+189*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+27*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+756*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+540*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*d^2*ex
p(1)^8*exp(2)^4+126*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^4*exp(2)^8+444*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^4*exp(2)^6+216*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(2)^8+67*d^2*exp(1)^4*exp(2)^6+252*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+126*d^2*e
xp(2)^8+8*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3
*exp(1)^14*exp(2)-189/2*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp
(2)^8/x/exp(2)-234*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4
*exp(2)^6/x/exp(2)-165*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^6*exp(2)^5/x/exp(2)+9*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*ex
p(1)^8*exp(2)^4/x/exp(2)-3*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2
))^3/(3*exp(1)^11+9*exp(1)^7*exp(2)^2+3*exp(1)^5*exp(2)^3+9*exp(1)^9*exp(2)
)+1/2*(48*d^2*exp(1)^8*exp(2)^3+40*d^2*exp(1)^6*exp(2)^4-66*d^2*exp(1)^4*ex
p(2)^5-148*d^2*exp(2)^7)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^1
3+3*exp(1)^9*exp(2)^2+exp(1)^7*exp(2)^3+3*exp(1)^11*exp(2))-15/2*d^2*sign(d
)*asin(x*exp(2)/d/exp(1))/exp(1)+2*(2*exp(1)*1/8/exp(1)*x-16*d*1/8/exp(1))*
sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 284, normalized size = 2.51

$$-\frac{15d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}}-\frac{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,x}{2}-\frac{5\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}x}{d^2}-\frac{4\left(2\left(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -1/e^5/d/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-3/e^4/d^2/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-4/e^3/d^3/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d
/e)^2*e^2)^(7/2)-4/e/d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-5/d^2*(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-15/2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-15
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
)
```

maxima [A] time = 0.98, size = 134, normalized size = 1.19

$$-\frac{15\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{2\left(e^3x^2+2\,de^2x+d^2e\right)}-\frac{15\sqrt{-e^2x^2+d^2}\,d^2}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -15/2*d^2*arcsin(e*x/d)/e + 1/2*(-e^2*x^2 + d^2)^(5/2)/(e^4*x^3 + 3*d*e^3*x
^2 + 3*d^2*e^2*x + d^3*e) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d/(e^3*x^2 + 2*d*e^2
*x + d^2*e) - 15*sqrt(-e^2*x^2 + d^2)*d^2/(e^2*x + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.204 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=89

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 4*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)+8*d*
(-e*x+d)/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1805, 1809, 844, 217, 203, 266, 63, 208}

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/Sqrt[d^2 - e^2*x^2] + Sqrt[d^2 - e^2*x^2] + 4*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4-4d^3ex+d^2e^2x^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{\int \frac{d^4e^2+4d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (4de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{1}{2}d^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + (4de)\,\mathrm{Subst}\left(\int \frac{}{1-}\right.\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e}\right.}{e^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 79, normalized size = 0.89

$$\sqrt{d^2-e^2x^2}\left(\frac{8d}{d+ex}+1\right)-d\log\left(\sqrt{d^2-e^2x^2}+d\right)+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(1 + (8*d)/(d + e*x)) + 4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]] + d*Log[x] - d*Log[d + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.93, size = 111, normalized size = 1.25

$$\frac{9\,dex+9\,d^2-8\left(dex+d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(dex+d^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}\,(ex+9\,d)}{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] (9*d*e*x + 9*d^2 - 8*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (d*e*x + d^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)
*(e*x + 9*d))/(e*x + d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-54*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*e
xp(2)^2-18*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*
exp(1)^10*exp(2)^3-144*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^3*exp(1)^12*exp(2)^2-180*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-54*d*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-78*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-370*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3
-321*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^8*exp(2)^4-75*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^5*exp(1)^6*exp(2)^5-252*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^10*exp(2)^3-432*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-216*d*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-36*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6-624*d*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^
4-528*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1
)^6*exp(2)^5-132*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^4*exp(1)^4*exp(2)^6-12*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^5*exp(2)^8-540*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-324*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6-11*d*exp(1)^8*exp(2)^4-72*d*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-312*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*ex
p(2)^6-36*d*exp(1)^6*exp(2)^5-108*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(2)^8-76*d*exp(1)^4*exp(2)^6-144*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-72*d*exp(2)^8-44*d
```

```
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*ex
p(2)+48*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+144
*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+3
81/2*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(
2)+81*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp
(2)+24*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/e
xp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2
)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^10+9*ex
p(1)^6*exp(2)^2+3*exp(1)^4*exp(2)^3+9*exp(1)^8*exp(2))+4*d*sign(d)*asin(x*e
xp(2)/d/exp(1))+1/2*(-12*d*exp(1)^10*exp(2)^2+2*d*exp(1)^8*exp(2)^3-8*d*exp
(1)^6*exp(2)^4-40*d*exp(1)^4*exp(2)^5-64*d*exp(2)^7-4*d*exp(1)^12*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^12-3*exp(1)^8*exp(2)^2-exp
(1)^6*exp(2)^3-3*exp(1)^10*exp(2))-d*exp(2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^2+sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 378, normalized size = 4.25

$$-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{4de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}+\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\;ex}{d}+\frac{8\left(2\left(x+\frac{d}{e}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x)
```

```
[Out] -1/(d^2)^(1/2)*d^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^2/e
^4/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+2/d^3/e^3/(x+d/e)^3*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+7/3/d^4/e^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(7/2)+8/3/d^3*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+4/d*e*(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+(-e^2*x^2+d^2)^(1/2)+4*d*e/(e^2)^(1/2)*arctan
((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/5/d^4*(-e^2*x^2+d^2)^
(5/2)+1/3/d^2*(-e^2*x^2+d^2)^(3/2)+32/15/d^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{5}{2}}}{x\left(d + ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)
```

3.205 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=94

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-8*e
*(-e*x+d)/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A] time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1805, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/Sqrt[d^2 - e^2*x^2] - Sqrt[d^2 - e^2*x^2]/x - e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + 4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4+4d^3ex+d^2e^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}+\frac{\int \frac{-4d^5e-d^4e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-(4de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-(2de)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)-e^2\,\text{Subst}\left(\int \frac{1}{1+e^2
\end{aligned}$$

$$\begin{aligned}
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{(4d)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2}\right.}{e}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.19, size = 84, normalized size = 0.89

$$\sqrt{d^2-e^2x^2}\left(-\frac{8e}{d+ex}-\frac{1}{x}\right)+4e\log\left(\sqrt{d^2-e^2x^2}+d\right)-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-4e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(-x^(-1) - (8*e)/(d + e*x)) - e*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]] - 4*e*Log[x] + 4*e*Log[d + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.90, size = 127, normalized size = 1.35

$$-\frac{8\,e^2x^2+8\,dex-2\left(e^2x^2+dex\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+4\left(e^2x^2+dex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}\,(9}{ex^2+dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -(8*e^2*x^2 + 8*d*e*x - 2*(e^2*x^2 + d*e*x)*arctan(-(d - sqrt(-e^2*x^2 + d^
2))/(e*x)) + 4*(e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(
-e^2*x^2 + d^2)*(9*e*x + d))/(e*x^2 + d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/4*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/exp(1)^4/x/exp(1)/exp(2
)+1/2*(-48*exp(1)^10*exp(2)^2-40*exp(1)^8*exp(2)^3-8*exp(1)^6*exp(2)^4+2*ex
p(1)^4*exp(2)^5-16*exp(2)^7-16*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(
1)^4+exp(2)^2)/(exp(1)^11+3*exp(1)^7*exp(2)^2+exp(1)^5*exp(2)^3+3*exp(1)^9*
exp(2))+4*exp(2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(
x)/exp(2))/exp(1)-1/3*x*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^3*(408*exp(1)^12*exp(2)^2+1152*exp(1)^10*exp(2)^3+1392*exp(1)^8*e
xp(2)^4+780*exp(1)^6*exp(2)^5+516*exp(1)^4*exp(2)^6+132*exp(2)^8)+(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(576*exp(1)^12*exp(2)^
2+932*exp(1)^10*exp(2)^3+1116*exp(1)^8*exp(2)^4+1041*exp(1)^6*exp(2)^5+279*
exp(1)^4*exp(2)^6+108*exp(2)^8+208*exp(1)^14*exp(2))+(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(240*exp(1)^12*exp(2)^2+648*exp(1)^
10*exp(2)^3+642*exp(1)^8*exp(2)^4+270*exp(1)^6*exp(2)^5+228*exp(1)^4*exp(2)
^6+66*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^6*(72*exp(1)^10*exp(2)^3+180*exp(1)^8*exp(2)^4+135*exp(1)^6*exp(2)^5+9*exp
(1)^4*exp(2)^6+18*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*(276*exp(1)^10*exp(2)^3+792*exp(1)^8*exp(2)^4+861*exp(1)^6*e
xp(2)^5+279*exp(1)^4*exp(2)^6+102*exp(2)^8)+3*exp(1)^6*exp(2)^5+9*exp(1)^4*
exp(2)^6+12*exp(2)^8-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(70*ex
p(1)^8*exp(2)^4+198*exp(1)^6*exp(2)^5+200*exp(1)^4*exp(2)^6+66*exp(2)^8)/x/
exp(2))*exp(2)/(2*exp(2))^3/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)^3/(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/exp(1)/exp(2)-sign(d)*asin(
x*exp(2)/d/exp(1))*exp(2)/exp(1)
```

maple [B] time = 0.01, size = 515, normalized size = 5.48

$$\frac{4de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{7e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{15e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}\,e^2x}{8d^2}+\frac{7\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x)
```

```
[Out] -15/8/(e^2)^(1/2)*e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/d^6*e^2*
x*(-e^2*x^2+d^2)^(5/2)-5/4/d^4*e^2*x*(-e^2*x^2+d^2)^(3/2)-15/8/d^2*e^2*x*(-
e^2*x^2+d^2)^(1/2)-1/d^3/e^3/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-
1/d^4/e^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-1/3/d^5/e/(x+d/e)^2
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+7/12/d^4*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*
e^2)^(3/2)*x+7/8/d^2*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+4/(d^2)^(1/2
)*d*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-4/d*e*(-e^2*x^2+d^2)
^(1/2)-4/5/d^5*e*(-e^2*x^2+d^2)^(5/2)-4/3/d^3*e*(-e^2*x^2+d^2)^(3/2)-1/d^6/
x*(-e^2*x^2+d^2)^(7/2)+7/15/d^5*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+7/8*e
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^2\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^2\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)
```

3.206 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=110

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -15/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d+8*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)
^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/x^2+4*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A] time = 0.22, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(d*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*x^2) + (
4*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (15*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{\int \frac{-8d^5e+15d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4}\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{2}\left(15e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{4}\left(15e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 85, normalized size = 0.77

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(-d^2+7dex+24e^2x^2\right)}{x^2(d+ex)} - 15e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15e^2\log(x)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(x^2*(d + e*x)) + 15*e
^2*Log[x] - 15*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(2*d)
```

fricas [A] time = 0.66, size = 112, normalized size = 1.02

$$\frac{16\,e^{3}x^{3}+16\,de^{2}x^{2}+15\left(e^{3}x^{3}+de^{2}x^{2}\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(24\,e^{2}x^{2}+7\,dex-d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{2\left(dex^{3}+d^{2}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(16*e^3*x^3 + 16*d*e^2*x^2 + 15*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + (24*e^2*x^2 + 7*d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d*e
*x^3 + d^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/16*
(-2*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
4*exp(2)^11-16*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2
)^10/x/exp(2))/d^2/exp(1)^6/exp(2)^9+1/24*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^4*(-3216*exp(1)^14*exp(2)^2-7776*exp(1)^12*exp(
2)^3-6300*exp(1)^10*exp(2)^4-2868*exp(1)^8*exp(2)^5-2571*exp(1)^6*exp(2)^6-
225*exp(1)^4*exp(2)^7+36*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^5*(-3456*exp(1)^14*exp(2)^2-4688*exp(1)^12*exp(2)^3-633
6*exp(1)^10*exp(2)^4-4638*exp(1)^8*exp(2)^5-90*exp(1)^6*exp(2)^6-378*exp(1)
^4*exp(2)^7-126*exp(2)^9-1504*exp(1)^16*exp(2))+(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(-1680*exp(1)^14*exp(2)^2-3744*exp(1)^12
*exp(2)^3-2376*exp(1)^10*exp(2)^4-864*exp(1)^8*exp(2)^5-1293*exp(1)^6*exp(2
)^6-135*exp(1)^4*exp(2)^7+12*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^7*(-480*exp(1)^12*exp(2)^3-1008*exp(1)^10*exp(2)^4-
408*exp(1)^8*exp(2)^5+144*exp(1)^6*exp(2)^6-144*exp(1)^4*exp(2)^7-48*exp(2)
^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-2328*ex
p(1)^12*exp(2)^3-5832*exp(1)^10*exp(2)^4-4188*exp(1)^8*exp(2)^5-300*exp(1)^
6*exp(2)^6-324*exp(1)^4*exp(2)^7-108*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*(-628*exp(1)^10*exp(2)^4-1620*exp(1)^8*ex
p(2)^5-1211*exp(1)^6*exp(2)^6-81*exp(1)^4*exp(2)^7+36*exp(2)^9)+3*exp(1)^6*
exp(2)^6+9*exp(1)^4*exp(2)^7+12*exp(2)^9-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))*(-30*exp(1)^8*exp(2)^5-90*exp(1)^6*exp(2)^6-90*exp(1)^4*exp(2
)^7-30*exp(2)^9)/x/exp(2))/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^2/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))
^3/exp(1)^3/d/exp(1)+1/2*(5*exp(2)^3-20*exp(1)^4*exp(2))*ln(1/2*abs(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^3/d/exp(1)+1/2*(-
108*exp(1)^7*exp(2)^2-66*exp(1)^5*exp(2)^3+40*exp(1)^3*exp(2)^4-40*exp(1)^9
*exp(2)+48*exp(1)*exp(2)^5)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d*ex
p(1)^7-3*d*exp(1)^5*exp(2)-4*d*exp(1)*exp(2)^3)
```

maple [B] time = 0.01, size = 504, normalized size = 4.58

$$-\frac{15e^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{2\sqrt{e^{2}}\,d}+\frac{15e^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}\,d}-\frac{15e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}}+\frac{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}x}{2d^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x)
```

```
[Out] -15/2/(d^2)^(1/2)*e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+4/d^
7*e/x*(-e^2*x^2+d^2)^(7/2)+4/d^7*e^3*x*(-e^2*x^2+d^2)^(5/2)+5/d^5*e^3*x*(-e
^2*x^2+d^2)^(3/2)+15/2/d^3*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/2/d*e^3/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/d^4/e^2/(x+d/e)^4*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(7/2)-5/d^5*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-1
5/2/d^3*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-15/2/d*e^3/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/2/d^6/x^2*(-e^2*x
^2+d^2)^(7/2)+3/2/d^6*e^2*(-e^2*x^2+d^2)^(5/2)+5/2/d^4*e^2*(-e^2*x^2+d^2)^(
3/2)+15/2/d^2*e^2*(-e^2*x^2+d^2)^(1/2)-2/d^6/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(7/2)-4/d^6*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{(ex+d)^{4}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^{2}-e^{2}\,x^{2}\right)^{5/2}}{x^{3}\,\left(d+e\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^{3}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**3*(d + e*x)**4), x)
```

3.207 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^4} \, dx$

Optimal. Leaf size=137

$$-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] 10*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-8*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2
)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/x^3+2*e*(-e^2*x^2+d^2)^(1/2)/d/x^2-23/3*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A] time = 0.30, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(d^2*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*x^3)
+ (2*e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^2*x
) + (10*e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{\int \frac{-12d^5e+23d^4e^2x-24d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^4}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{\int \frac{-46d^6e^2+60d^5e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^6}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{\left(10e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{\left(5e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-\ldots}}\right.}{d}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{(10e)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\ldots\right.}{d}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\ldots\right.}{d^2}$$

Mathematica [A] time = 0.25, size = 94, normalized size = 0.69

$$-\frac{-30e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(d^3-5d^2ex+17de^2x^2+47e^3x^3\right)}{x^3(d+ex)}+30e^3\log(x)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/3*((Sqrt[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3))/(
x^3*(d + e*x)) + 30*e^3*Log[x] - 30*e^3*Log[d + Sqrt[d^2 - e^2*x^2]])/d^2
```

fricas [A] time = 0.56, size = 123, normalized size = 0.90

$$-\frac{24\,e^4x^4+24\,de^3x^3+30\left(e^4x^4+de^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(47\,e^3x^3+17\,de^2x^2-5\,d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(d^2ex^4+d^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*e^4*x^4 + 24*d*e^3*x^3 + 30*(e^4*x^4 + d*e^3*x^3)*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + (47*e^3*x^3 + 17*d*e^2*x^2 - 5*d^2*e*x + d^3)*sqrt(-e^
2*x^2 + d^2))/(d^2*e*x^4 + d^3*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/512
*(256*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^10*exp(2)^16-64/3*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^3*exp(1)^8*exp(2)^17+96*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))*exp(1)^8*exp(2)^17/x/exp(2)-384*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(1)^10*exp(2)^16/x/exp(2)+1280*d^4*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^15/x/exp(2))/d^6/exp(1)^15/exp(2)^12+1
/72*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(16416*e
xp(1)^16*exp(2)^2+34560*exp(1)^14*exp(2)^3+20376*exp(1)^12*exp(2)^4+8712*ex
p(1)^10*exp(2)^5+9234*exp(1)^8*exp(2)^6-1674*exp(1)^6*exp(2)^7-594*exp(1)^4
*exp(2)^8+234*exp(2)^10)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^6*(13824*exp(1)^16*exp(2)^2+17952*exp(1)^14*exp(2)^3+27216*exp(1)
^12*exp(2)^4+14724*exp(1)^10*exp(2)^5-4104*exp(1)^8*exp(2)^6+1380*exp(1)^6*
exp(2)^7+468*exp(1)^4*exp(2)^8+12*exp(2)^10+7104*exp(1)^18*exp(2))+(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(7776*exp(1)^16*exp(2
)^2+14688*exp(1)^14*exp(2)^3+6192*exp(1)^12*exp(2)^4+3240*exp(1)^10*exp(2)^
5+5454*exp(1)^8*exp(2)^6-702*exp(1)^6*exp(2)^7-270*exp(1)^4*exp(2)^8+126*ex
p(2)^10)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*(216
0*exp(1)^14*exp(2)^3+3888*exp(1)^12*exp(2)^4+648*exp(1)^10*exp(2)^5-756*exp
(1)^8*exp(2)^6+855*exp(1)^6*exp(2)^7+117*exp(1)^4*exp(2)^8)+(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(12528*exp(1)^14*exp(2)^3+27
648*exp(1)^12*exp(2)^4+13392*exp(1)^10*exp(2)^5-3204*exp(1)^8*exp(2)^6+774*
exp(1)^6*exp(2)^7+306*exp(1)^4*exp(2)^8+36*exp(2)^10)+(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(3528*exp(1)^12*exp(2)^4+8064*exp(
1)^10*exp(2)^5+4146*exp(1)^8*exp(2)^6-1218*exp(1)^6*exp(2)^7-378*exp(1)^4*e
xp(2)^8+90*exp(2)^10)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*(180*exp(1)^10*exp(2)^5+432*exp(1)^8*exp(2)^6+252*exp(1)^6*exp(2)^
7-36*exp(1)^4*exp(2)^8+36*exp(2)^10)+3*exp(1)^6*exp(2)^7+9*exp(1)^4*exp(2)^
8+12*exp(2)^10-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(-18*exp(1)^
8*exp(2)^6-54*exp(1)^6*exp(2)^7-54*exp(1)^4*exp(2)^8-18*exp(2)^10)/x/exp(2)
)/d^2/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^3/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)
```

```
-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/exp(1)^5-(10*exp(2
)^3-20*exp(1)^4*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/abs(x)/exp(2))/d^2/exp(1)/exp(2)+1/2*(-192*exp(1)^8*exp(2)^2-64*exp(1)^6
*exp(2)^3+136*exp(1)^4*exp(2)^4+74*exp(2)^6-80*exp(1)^10*exp(2))*atan((-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2
)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d^2*exp(1)^7+3*d^2*exp(1)^5*exp(2)+4*d^2*ex
p(1)*exp(2)^3)
```

maple [B] time = 0.02, size = 575, normalized size = 4.20

$$\frac{10e^3 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d}+\frac{65e^4 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{4\sqrt{e^2}\,d^2}-\frac{65e^4 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,d^2}-\frac{65\sqrt{-e^2x^2+d^2}\,e^4x}{4d^4}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x)
```

```
[Out] -26/3/d^8*e^4*x*(-e^2*x^2+d^2)^(5/2)-65/6/d^6*e^4*x*(-e^2*x^2+d^2)^(3/2)-65
/4/d^4*e^4*x*(-e^2*x^2+d^2)^(1/2)-65/4/d^2*e^4/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(-e^2*x^2+d^2)^(1/2)*x)+2/d^7*e/x^2*(-e^2*x^2+d^2)^(7/2)-1/d^5/e/(x+d/e)
^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+14/3/d^7*e/(x+d/e)^2*(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(7/2)+65/6/d^6*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+65/
4/d^4*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+65/4/d^2*e^4/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-26/3/d^8*e^2/x*(-e^
2*x^2+d^2)^(7/2)+10/(d^2)^(1/2)/d*e^3*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2
)^(1/2))/x)-1/3/d^6/x^3*(-e^2*x^2+d^2)^(7/2)-2/d^7*e^3*(-e^2*x^2+d^2)^(5/2)
-10/3/d^5*e^3*(-e^2*x^2+d^2)^(3/2)-10/d^3*e^3*(-e^2*x^2+d^2)^(1/2)+26/3/d^7
*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/d^6/(x+d/e)^3*(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^4\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^4\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**4*(d + e*x)**4), x)
```

3.208 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

```
[Out] -95/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+8*e^4*(-e*x+d)/d^3/(-e^2*x^2+
d^2)^(1/2)-1/4*(-e^2*x^2+d^2)^(1/2)/x^4+4/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-31
/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2+32/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A] time = 0.39, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] (8*e^4*(d - e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(4*x^4) +
 (4*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (31*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^2*
x^2) + (32*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) - (95*e^4*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/(8*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^5\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{\int \frac{-16d^5e+31d^4e^2x-32d^3e^3x^2+32d^2e^4x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^4}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{-93d^6e^2+128d^5e^3x-96d^4e^4x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^6}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{\int \frac{-256d^7e^3+285d^6e^4x}{x^2\sqrt{d^2-e^2x^2}}\,d}{24d^8}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} +$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} +$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} -$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} -$$

Mathematica [A] time = 0.27, size = 107, normalized size = 0.63

$$\frac{-285e^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\,(-6d^4+26d^3ex-61d^2e^2x^2+163de^3x^3+448e^4x^4)}{x^4(d+ex)}+285e^4\log(x)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^3
 + 448*e^4*x^4))/(x^4*(d + e*x)) + 285*e^4*Log[x] - 285*e^4*Log[d + Sqrt[d^
2 - e^2*x^2]])/(24*d^3)
```

fricas [A] time = 0.91, size = 136, normalized size = 0.80

$$\frac{192\,e^5x^5+192\,de^4x^4+285\,(e^5x^5+de^4x^4)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+(448\,e^4x^4+163\,de^3x^3-61\,d^2e^2x^2+26\,d^3ex-6\,}{24\,(d^3ex^5+d^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(192*e^5*x^5 + 192*d*e^4*x^4 + 285*(e^5*x^5 + d*e^4*x^4)*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + (448*e^4*x^4 + 163*d*e^3*x^3 - 61*d^2*e^2*x^2 + 26*
d^3*e*x - 6*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e*x^5 + d^4*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/655
36*(-81920*d^9*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
2*exp(1)^22*exp(2)^19+32768/3*d^9*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^3*exp(1)^20*exp(2)^20-1024*d^9*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^18*exp(2)^21+24576*d^9*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(2)^20-8
192*d^9*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1
)^18*exp(2)^21-49152*d^9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^20*exp(2)^20/x/exp(2)+196608*d^9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))*exp(1)^22*exp(2)^19/x/exp(2)-327680*d^9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))*exp(1)^24*exp(2)^18/x/exp(2))/d^12/exp(1)^24/exp(2)^16+1/192*((
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(-67968*exp(1)
^18*exp(2)^2-126720*exp(1)^16*exp(2)^3-53184*exp(1)^14*exp(2)^4-21408*exp(1
)^12*exp(2)^5-23472*exp(1)^10*exp(2)^6+19800*exp(1)^8*exp(2)^7+3699*exp(1)^
6*exp(2)^8-3063*exp(1)^4*exp(2)^9+84*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(-46080*exp(1)^18*exp(2)^2-62080*exp(1)^
16*exp(2)^3-101376*exp(1)^14*exp(2)^4-33888*exp(1)^12*exp(2)^5+32688*exp(1)
^10*exp(2)^6-3488*exp(1)^8*exp(2)^7-960*exp(1)^6*exp(2)^8+768*exp(1)^4*exp(
2)^9-752*exp(2)^11-27392*exp(1)^20*exp(2))+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^8*(-29568*exp(1)^18*exp(2)^2-48384*exp(1)^16*ex
p(2)^3-13632*exp(1)^14*exp(2)^4-13824*exp(1)^12*exp(2)^5-16848*exp(1)^10*ex
p(2)^6+10440*exp(1)^8*exp(2)^7+1872*exp(1)^6*exp(2)^8-1632*exp(1)^4*exp(2)^
9+24*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^9*(-8064*exp(1)^16*exp(2)^3-12672*exp(1)^14*exp(2)^4+192*exp(1)^12*exp(2)^
5+2304*exp(1)^10*exp(2)^6-3360*exp(1)^8*exp(2)^7+672*exp(1)^6*exp(2)^8+288*
```

```
exp(1)^4*exp(2)^9-288*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^5*(-54144*exp(1)^16*exp(2)^3-106560*exp(1)^14*exp(2)^4-29
184*exp(1)^12*exp(2)^5+29280*exp(1)^10*exp(2)^6-750*exp(1)^8*exp(2)^7-714*e
xp(1)^6*exp(2)^8+630*exp(1)^4*exp(2)^9-654*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(-15744*exp(1)^14*exp(2)^4-32160*e
xp(1)^12*exp(2)^5-9100*exp(1)^10*exp(2)^6+11588*exp(1)^8*exp(2)^7+1893*exp(
1)^6*exp(2)^8-1473*exp(1)^4*exp(2)^9+108*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-840*exp(1)^12*exp(2)^5-1800*exp(1)
^10*exp(2)^6-564*exp(1)^8*exp(2)^7+708*exp(1)^6*exp(2)^8+108*exp(1)^4*exp(2
)^9-204*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*(84*exp(1)^10*exp(2)^6+180*exp(1)^8*exp(2)^7+69*exp(1)^6*exp(2)^8-33*
exp(1)^4*exp(2)^9+60*exp(2)^11)+3*exp(1)^6*exp(2)^8+9*exp(1)^4*exp(2)^9+12*
exp(2)^11-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(-14*exp(1)^8*exp
(2)^7-42*exp(1)^6*exp(2)^8-42*exp(1)^4*exp(2)^9-14*exp(2)^11)/x/exp(2))/d^3
/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/exp(1)^6+1/8*(240*exp(1
)^6*exp(2)^2-64*exp(1)^4*exp(2)^3+9*exp(2)^5-280*exp(1)^8*exp(2))*ln(1/2*ab
s(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)^5/ex
p(1)+1/2*(-300*exp(1)^9*exp(2)^2-22*exp(1)^7*exp(2)^3+280*exp(1)^5*exp(2)^4
+104*exp(1)^3*exp(2)^5-140*exp(1)^11*exp(2)-48*exp(1)*exp(2)^6)*atan((-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)
^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^3*exp(1)^7-3*d^3*exp(1)^5*exp(2)-4*d^3*ex
p(1)*exp(2)^3)
```

maple [B] time = 0.02, size = 600, normalized size = 3.53

$$-\frac{95e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\,d^2}-\frac{55e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,d^3}+\frac{55e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,d^3}+\frac{55\sqrt{-e^2x^2+d^2}\,e}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x)
```

```
[Out] 4/3/d^7*e/x^3*(-e^2*x^2+d^2)^(7/2)-37/8/d^8*e^2/x^2*(-e^2*x^2+d^2)^(7/2)+44
/3/d^9*e^3/x*(-e^2*x^2+d^2)^(7/2)+44/3/d^9*e^5*x*(-e^2*x^2+d^2)^(5/2)+55/3/
d^7*e^5*x*(-e^2*x^2+d^2)^(3/2)+55/2/d^5*e^5*x*(-e^2*x^2+d^2)^(1/2)+55/2/d^3
*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-2/d^7*e/(x+d/e)
^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-23/3/d^8*e^2/(x+d/e)^2*(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(7/2)-55/3/d^7*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-5
5/2/d^5*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-55/2/d^3*e^5/(e^2)^(1/2)*
arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-95/8/(d^2)^(1/2)/
d^2*e^4*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4/d^6/x^4*(-e^2*
x^2+d^2)^(7/2)+19/8/d^8*e^4*(-e^2*x^2+d^2)^(5/2)+95/24/d^6*e^4*(-e^2*x^2+d^
2)^(3/2)+95/8/d^4*e^4*(-e^2*x^2+d^2)^(1/2)+1/d^6/(x+d/e)^4*(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(7/2)-44/3/d^8*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^5\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**5*(d + e*x)**4), x)
```

3.209 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}+\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+11e$$

```
[Out] 27/2*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-8*e^5*(-e*x+d)/d^4/(-e^2*x^2+d
^2)^(1/2)-1/5*(-e^2*x^2+d^2)^(1/2)/x^5+e*(-e^2*x^2+d^2)^(1/2)/d/x^4-13/5*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3+11/2*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-66/5*e
^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A] time = 0.52, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{27e^5\tanh^{-1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] (-8*e^5*(d - e*x))/(d^4*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(5*x^5)
+ (e*Sqrt[d^2 - e^2*x^2])/(d*x^4) - (13*e^2*Sqrt[d^2 - e^2*x^2])/(5*d^2*x^3
) + (11*e^3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) - (66*e^4*Sqrt[d^2 - e^2*x^2])
/(5*d^4*x) + (27*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^6\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4+\frac{8e^5x^5}{d}}{x^6\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{\int \frac{-20d^5e+39d^4e^2x-40d^3e^3x^2+40d^2e^4x^3-40de^5x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^4}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{\int \frac{-156d^6e^2+220d^5e^3x-160d^4e^4x^2+160d^3e^5x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^6}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{\int \frac{-660d^7e^3+792d^6e^4x-}{x^3\sqrt{d^2-e^2x^2}}}{60d^8}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} -$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} -$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} -$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} -$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} -$$

Mathematica [A] time = 0.33, size = 118, normalized size = 0.60

$$-\frac{-135e^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(2d^5-8d^4ex+16d^3e^2x^2-29d^2e^3x^3+77de^4x^4+212e^5x^5\right)}{x^5(d+ex)}+135e^5\log(x)}{10d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] -1/10*((Sqrt[d^2 - e^2*x^2]*(2*d^5 - 8*d^4*e*x + 16*d^3*e^2*x^2 - 29*d^2*e^
3*x^3 + 77*d*e^4*x^4 + 212*e^5*x^5))/(x^5*(d + e*x)) + 135*e^5*Log[x] - 135
*e^5*Log[d + Sqrt[d^2 - e^2*x^2]])/d^4
```

fricas [A] time = 1.10, size = 147, normalized size = 0.75

$$-\frac{80\,e^6x^6+80\,de^5x^5+135\left(e^6x^6+de^5x^5\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(212\,e^5x^5+77\,de^4x^4-29\,d^2e^3x^3+16\,d^3e^2x^2-\right.}{10\left(d^4ex^6+d^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/10*(80*e^6*x^6 + 80*d*e^5*x^5 + 135*(e^6*x^6 + d*e^5*x^5)*log(-(d - sqrt
(-e^2*x^2 + d^2))/x) + (212*e^5*x^5 + 77*d*e^4*x^4 - 29*d^2*e^3*x^3 + 16*d^
3*e^2*x^2 - 8*d^4*e*x + 2*d^5)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^6 + d^5*x^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/480
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(247680*exp
(1)^20*exp(2)^2+414720*exp(1)^18*exp(2)^3+115200*exp(1)^16*exp(2)^4+45600*e
xp(1)^14*exp(2)^5+43920*exp(1)^12*exp(2)^6-106200*exp(1)^10*exp(2)^7-10680*
exp(1)^8*exp(2)^8+15720*exp(1)^6*exp(2)^9-1080*exp(1)^4*exp(2)^10+1200*exp(
2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*(13824
0*exp(1)^20*exp(2)^2+203200*exp(1)^18*exp(2)^3+342720*exp(1)^16*exp(2)^4+46
320*exp(1)^14*exp(2)^5-158400*exp(1)^12*exp(2)^6+12440*exp(1)^10*exp(2)^7-3
60*exp(1)^8*exp(2)^8-3965*exp(1)^6*exp(2)^9+4505*exp(1)^4*exp(2)^10+220*exp
(2)^12+93440*exp(1)^22*exp(2))+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^9*(99840*exp(1)^20*exp(2)^2+144000*exp(1)^18*exp(2)^3+27360
*exp(1)^16*exp(2)^4+56160*exp(1)^14*exp(2)^5+39840*exp(1)^12*exp(2)^6-60120
*exp(1)^10*exp(2)^7-5460*exp(1)^8*exp(2)^8+8340*exp(1)^6*exp(2)^9-540*exp(1
)^4*exp(2)^10+660*exp(2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^10*(26880*exp(1)^18*exp(2)^3+37440*exp(1)^16*exp(2)^4-6000*ex
p(1)^14*exp(2)^5-5040*exp(1)^12*exp(2)^6+10440*exp(1)^10*exp(2)^7-6180*exp(
1)^8*exp(2)^8-1110*exp(1)^6*exp(2)^9+1350*exp(1)^4*exp(2)^10+60*exp(2)^12)+
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(204480*exp(1
)^18*exp(2)^3+362880*exp(1)^16*exp(2)^4+30960*exp(1)^14*exp(2)^5-144240*exp
(1)^12*exp(2)^6+3600*exp(1)^10*exp(2)^7+840*exp(1)^8*exp(2)^8-3432*exp(1)^6
*exp(2)^9+3384*exp(1)^4*exp(2)^10+312*exp(2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(60960*exp(1)^16*exp(2)^4+112320*exp(1)
^14*exp(2)^5+9120*exp(1)^12*exp(2)^6-56840*exp(1)^10*exp(2)^7-4512*exp(1)^8
*exp(2)^8+6704*exp(1)^6*exp(2)^9-576*exp(1)^4*exp(2)^10+408*exp(2)^12)+(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(3360*exp(1)^14*e
xp(2)^5+6480*exp(1)^12*exp(2)^6+936*exp(1)^10*exp(2)^7-3192*exp(1)^8*exp(2)
^8-558*exp(1)^6*exp(2)^9+198*exp(1)^4*exp(2)^10+216*exp(2)^12)+(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-336*exp(1)^12*exp(2)^6-
648*exp(1)^10*exp(2)^7-72*exp(1)^8*exp(2)^8+312*exp(1)^6*exp(2)^9-72*exp(1)
^4*exp(2)^10-144*exp(2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*(56*exp(1)^10*exp(2)^7+108*exp(1)^8*exp(2)^8+22*exp(1)^6*exp
(2)^9-22*exp(1)^4*exp(2)^10+76*exp(2)^12)+3*exp(1)^6*exp(2)^9+9*exp(1)^4*ex
p(2)^10+12*exp(2)^12-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(-12*e
xp(1)^8*exp(2)^8-36*exp(1)^6*exp(2)^9-36*exp(1)^4*exp(2)^10-12*exp(2)^12)/x
/exp(2))/d^4/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^5/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/exp(1)^7+1/
33554432*(83886080*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^34*exp(2)^23-41943040/3*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^32*exp(2)^24+2097152*d^16*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^30*exp(2)^25-1
048576/5*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5
*exp(1)^28*exp(2)^26-50331648*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^2*exp(1)^32*exp(2)^24+4194304*d^16*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^30*exp(2)^25+16777216*d^16
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^30*ex
p(2)^25-5242880/3*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^3*exp(1)^28*exp(2)^26+5242880*d^16*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))*exp(1)^28*exp(2)^26/x/exp(2)-18874368*d^16*(-2*d*exp(1)-2*sqrt(
```

```
d^2-x^2*exp(2))*exp(1))*exp(1)^30*exp(2)^25/x/exp(2)+88080384*d^16*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^32*exp(2)^24/x/exp(2)-251658240*
d^16*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^34*exp(2)^23/x/exp(
2)+293601280*d^16*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^36*exp
(2)^22/x/exp(2))/d^20/exp(1)^35/exp(2)^20+1/2*(-120*exp(1)^6*exp(2)^2+44*ex
p(1)^4*exp(2)^3-9*exp(2)^5+112*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/exp(1)^4/exp(1)+1/2*(-432*exp
(1)^10*exp(2)^2+72*exp(1)^8*exp(2)^3+472*exp(1)^6*exp(2)^4+90*exp(1)^4*exp(
2)^5-104*exp(2)^7-224*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(
2)^2)/(d^4*exp(1)^7+3*d^4*exp(1)^5*exp(2)+4*d^4*exp(1)*exp(2)^3)
```

maple [B] time = 0.02, size = 628, normalized size = 3.20

$$\frac{27e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^3}+\frac{333e^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,d^4}-\frac{333e^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,d^4}-\frac{333\sqrt{-e^2x^2+d^2}}{8d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x)
```

```
[Out] 1/d^7*e/x^4*(-e^2*x^2+d^2)^(7/2)-16/5/d^8*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-111/
5/d^10*e^4/x*(-e^2*x^2+d^2)^(7/2)-111/5/d^10*e^6*x*(-e^2*x^2+d^2)^(5/2)-111
/4/d^8*e^6*x*(-e^2*x^2+d^2)^(3/2)-333/8/d^6*e^6*x*(-e^2*x^2+d^2)^(1/2)-333/
8/d^4*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+17/2/d^9*e
^3/x^2*(-e^2*x^2+d^2)^(7/2)+333/8/d^4*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/d^7*e/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(7/2)+3/d^8*e^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+11/
d^9*e^3/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+111/4/d^8*e^6*(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+333/8/d^6*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x+27/2/(d^2)^(1/2)/d^3*e^5*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x)-1/5/d^6/x^5*(-e^2*x^2+d^2)^(7/2)-27/10/d^9*e^5*(-e^2*x^2+d^2)^(5/2)-
9/2/d^7*e^5*(-e^2*x^2+d^2)^(3/2)-27/2/d^5*e^5*(-e^2*x^2+d^2)^(1/2)+111/5/d^
9*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^4x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^{6}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**6*(d + e*x)**4), x)
```

3.210 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$

Optimal. Leaf size=95

$$-\frac{\sin^{-1}(ax)}{a^3}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^4-3/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
-arcsin(a*x)/a^3+2*(-a^2*x^2+1)^(1/2)/a^3/(-a*x+1)
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1637, 659, 651, 663, 216}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*x
)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \int \left(\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^2}\right)dx$$
$$=\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2}$$
$$=\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}-\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$=\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Mathematica [A] time = 0.11, size = 50, normalized size = 0.53

$$\frac{\frac{\left(-13a^2x^2+19ax-8\right)\sqrt{1-a^2x^2}}{(ax-1)^3}-5\sin^{-1}(ax)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(-1 + a*x)^3 - 5*ArcSin[a*x
])/(5*a^3)
```

fricas [A] time = 0.96, size = 126, normalized size = 1.33

$$\frac{8\,a^3x^3-24\,a^2x^2+24\,ax+10\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(13\,a^2x^2-19\,ax+8\right)\sqrt{-a^2x^2+1}}{5\left(a^6x^3-3\,a^5x^2+3\,a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 200, normalized size = 2.11

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}\right)}{\sqrt{a^2}\,a^2}+\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}{a^3}+\frac{\left(-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a\right)^{\frac{3}{2}}}{\left(x-\frac{1}{a}\right)^2a^5}+\frac{3\left(-\left(x-\frac{1}{a}\right)^2a^2-\right.}{5\left(x-\frac{1}{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x)
```

```
[Out] 1/a^5/(x-1/a)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/a^3*(-(x-1/a)^2*a^2-2*
(x-1/a)*a)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-(x-1/a)^2*a^2-2*(x-
1/a)*a)^(1/2)*x)+1/5/a^7/(x-1/a)^4*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+3/5/a
^6/(x-1/a)^3*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,x^2}{(ax-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)
```

mupad [B] time = 2.70, size = 220, normalized size = 2.32

$$\frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,x^2-2\,a^6\,x+a^5\right)}-\frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a\,\sqrt{-a^2}-3\,a^2\,x\,\sqrt{-a^2}+3\,a^3\,x^2\,\sqrt{-a^2}-a^4\,x^3\,\sqrt{-a^2}\right)}-\frac{}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^4,x)
```

```
[Out] (4*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^5 - 2*a^6*x + a^7*x^2)) - asinh(x*(-a^2)
^(1/2))/(a^2*(-a^2)^(1/2)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a*(-a
^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2) - a^4*x^3*(-a^2)^
(1/2))) - (13*(1 - a^2*x^2)^(1/2))/(5*(a*(-a^2)^(1/2) - a^2*x*(-a^2)^(1/2))
*(-a^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{(ax-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)
```

3.211 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$

Optimal. Leaf size=88

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^5-12/35*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)
^4+23/105*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1639, 793, 659, 651}

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*a^3*
(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
```

```
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}+\frac{\int \frac{\left(4a^2-3a^3x\right)\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{a^4}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{7a^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{35a^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.57

$$\frac{\sqrt{1-a^2x^2}\left(23a^3x^3+13a^2x^2-8ax+2\right)}{105a^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a*
x)^4)
```

fricas [A] time = 0.88, size = 102, normalized size = 1.16

$$\frac{2\,a^4x^4-8\,a^3x^3+12\,a^2x^2-8\,ax+\left(23\,a^3x^3+13\,a^2x^2-8\,ax+2\right)\sqrt{-a^2x^2+1}+2}{105\left(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4
*a^4*x + a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 44, normalized size = 0.50

$$\frac{\sqrt{-a^2x^2+1}\,\left(23a^2x^2-10ax+2\right)(ax+1)}{105\,(ax-1)^4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x)
```

```
[Out] 1/105*(-a^2*x^2+1)^(1/2)*(23*a^2*x^2-10*a*x+2)*(a*x+1)/(a*x-1)^4/a^3
```

maxima [B] time = 0.44, size = 153, normalized size = 1.74

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3\right)}+\frac{29\sqrt{-a^2x^2+1}}{35\left(a^6x^3-3\,a^5x^2+3\,a^4x-a^3\right)}+\frac{82\sqrt{-a^2x^2+1}}{105\left(a^5x^2-2\,a^4x+a^3\right)}+\frac{23\sqrt{-a^2x^2+}}{105\left(a^4x-a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4*a^4*x + a^3) +
29/35*sqrt(-a^2*x^2 + 1)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3) + 82/105*sqr
t(-a^2*x^2 + 1)/(a^5*x^2 - 2*a^4*x + a^3) + 23/105*sqrt(-a^2*x^2 + 1)/(a^4*
x - a^3)
```

mupad [B] time = 0.06, size = 287, normalized size = 3.26

$$\frac{2\sqrt{1-a^2\,x^2}}{7\left(a^7\,x^4-4\,a^6\,x^3+6\,a^5\,x^2-4\,a^4\,x+a^3\right)}+\frac{4\sqrt{1-a^2\,x^2}}{3\left(a^5\,x^2-2\,a^4\,x+a^3\right)}+\frac{4\,a\sqrt{1-a^2\,x^2}}{35\left(a^6\,x^2-2\,a^5\,x+a^4\right)}+\frac{}{35\sqrt{-a^2}\left(a\sqrt{-a^2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^5,x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^3 - 4*a^4*x + 6*a^5*x^2 - 4*a^6*x^3 + a^7*x^4
)) + (4*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2)) + (4*a*(1 - a^2*
x^2)^(1/2))/(35*(a^4 - 2*a^5*x + a^6*x^2)) + (29*(1 - a^2*x^2)^(1/2))/(35*(
-a^2)^(1/2)*(a*(-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2)
 - a^4*x^3*(-a^2)^(1/2))) + (23*(1 - a^2*x^2)^(1/2))/(105*(a*(-a^2)^(1/2) -
 a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(a^5 -
2*a^6*x + a^7*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)
```

```
[Out] -Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**3
 - 10*a**2*x**2 + 5*a*x - 1), x)
```

3.212 $\int \frac{x^3}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=209

$$\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{1001de^4(d+ex)\left(d^2\right.}$$

```
[Out] -24/5005*x/d^3/e^3/(-e^2*x^2+d^2)^(5/2)+1/13*d^2/e^4/(e*x+d)^4/(-e^2*x^2+d^
2)^(5/2)-30/143*d/e^4/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)+21/143/e^4/(e*x+d)^2/(
-e^2*x^2+d^2)^(5/2)+4/1001/d/e^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2)-32/5005*x/d^5
/e^3/(-e^2*x^2+d^2)^(3/2)-64/5005*x/d^7/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 793, 659, 192, 191}

$$\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{1001de^4(d+ex)\left(d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^(5/2)) + d^2/(13*e^4*(d + e*x)^4*(d^2
 - e^2*x^2)^(5/2)) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + 2
1/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + 4/(1001*d*e^4*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^(3/2)) - (64*x)/
(5005*d^7*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{2d^3e^2-3d^2e^3x-12de^4x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7e^5}$$

$$= -\frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{-20d^3e^6+36d^2e^7x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}}{56e^9}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-e\right.}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-\right.}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{21}{143e^4(d+ex)^2\left(d^2\right.}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{21}{143e^4(d+ex)^2\left(d^2\right.}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e\right.}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e\right.}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e\right.}$$

Mathematica [A] time = 0.18, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(90d^9+360d^8ex+315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d^4e^5x^5+384d^3e^6x^6-224d^2e^7x^7-256\right.}{5005d^7e^4(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3*
x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x^7
 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

fricas [A] time = 1.92, size = 316, normalized size = 1.51

$$\frac{90\,e^{10}x^{10}+360\,de^{9}x^{9}+270\,d^{2}e^{8}x^{8}-720\,d^{3}e^{7}x^{7}-1260\,d^{4}e^{6}x^{6}+1260\,d^{6}e^{4}x^{4}+720\,d^{7}e^{3}x^{3}-270\,d^{8}e^{2}x^{2}-36\cdots}{5005\,(d^{7}e^{14}x^{10}+4\,d^{8}e^{13}x^{9}+3\,d^{9}e^{12}x^{8}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5005*(90*e^10*x^10 + 360*d*e^9*x^9 + 270*d^2*e^8*x^8 - 720*d^3*e^7*x^7 -
1260*d^4*e^6*x^6 + 1260*d^6*e^4*x^4 + 720*d^7*e^3*x^3 - 270*d^8*e^2*x^2 - 3
60*d^9*e*x - 90*d^10 + (64*e^9*x^9 + 256*d*e^8*x^8 + 224*d^2*e^7*x^7 - 384*
d^3*e^6*x^6 - 776*d^4*e^5*x^5 - 160*d^5*e^4*x^4 + 540*d^6*e^3*x^3 - 315*d^7
*e^2*x^2 - 360*d^8*e*x - 90*d^9)*sqrt(-e^2*x^2 + d^2))/(d^7*e^14*x^10 + 4*d
^8*e^13*x^9 + 3*d^9*e^12*x^8 - 8*d^10*e^11*x^7 - 14*d^11*e^10*x^6 + 14*d^13
*e^8*x^4 + 8*d^14*e^7*x^3 - 3*d^15*e^6*x^2 - 4*d^16*e^5*x - d^17*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.63

$$\frac{(-ex+d)\,(-64e^{9}x^{9}-256e^{8}x^{8}d-224e^{7}x^{7}d^{2}+384e^{6}x^{6}d^{3}+776e^{5}x^{5}d^{4}+160x^{4}d^{5}e^{4}-540x^{3}d^{6}e^{3}+315x^{2}d^{7}e^{2}\cdots}{5005\,(ex+d)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{7}e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5005*(-e*x+d)*(-64*e^9*x^9-256*d*e^8*x^8-224*d^2*e^7*x^7+384*d^3*e^6*x^6+
776*d^4*e^5*x^5+160*d^5*e^4*x^4-540*d^6*e^3*x^3+315*d^7*e^2*x^2+360*d^8*e*x
+90*d^9)/(e*x+d)^3/d^7/e^4/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.50, size = 399, normalized size = 1.91

$$\frac{d^{2}}{13\,(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{8}x^{4}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{7}x^{3}+6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{6}x^{2}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{5}x+(-e^{2}x^{2}+d^{2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13*d^2/((-e^2*x^2 + d^2)^(5/2)*e^8*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^7*x
^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^6*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^
5*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^4) - 30/143*d/((-e^2*x^2 + d^2)^(5/2)*e^
```

```
7*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e
^5*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^4) + 21/143/((-e^2*x^2 + d^2)^(5/2)*e^6
*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e^4) +
 4/1001/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e^4) -
 24/5005*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 32/5005*x/((-e^2*x^2 + d^2)^(
3/2)*d^5*e^3) - 64/5005*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^3)
```

mupad [B] time = 3.22, size = 252, normalized size = 1.21

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{107}{4004\,d^2\,e^4}-\frac{1139\,x}{80080\,d^3\,e^3}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{23}{32032\,d^4\,e^4}+\frac{32\,x}{5005\,d^5\,e^3}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}}{104\,d\,e^4\,{\left(d+e\,x\right)}^7}-\frac{27\,\sqrt{d^2-e^2\,x^2}}{2288\,d^2\,e^4\,{\left(d+e\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(107/(4004*d^2*e^4) - (1139*x)/(80080*d^3*e^3)))/((d
 + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(23/(32032*d^4*e^4) + (32*x
)/(5005*d^5*e^3)))/((d + e*x)^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(104*d
*e^4*(d + e*x)^7) - (27*(d^2 - e^2*x^2)^(1/2))/(2288*d^2*e^4*(d + e*x)^6) -
 (15*(d^2 - e^2*x^2)^(1/2))/(2288*d^3*e^4*(d + e*x)^5) + (23*(d^2 - e^2*x^2
)^(1/2))/(32032*d^4*e^4*(d + e*x)^4) - (64*x*(d^2 - e^2*x^2)^(1/2))/(5005*d
^7*e^3*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.213 $\int \frac{x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+e}$$

```
[Out] 14/2145*x/d^4/e^2/(-e^2*x^2+d^2)^(5/2)-1/13*d/e^3/(e*x+d)^4/(-e^2*x^2+d^2)^
(5/2)+17/143/e^3/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-7/1287/d/e^3/(e*x+d)^2/(-e^
2*x^2+d^2)^(5/2)-7/1287/d^2/e^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+56/6435*x/d^6/
e^2/(-e^2*x^2+d^2)^(3/2)+112/6435*x/d^8/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 793, 659, 192, 191}

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+e}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (14*x)/(2145*d^4*e^2*(d^2 - e^2*x^2)^(5/2)) - d/(13*e^3*(d + e*x)^4*(d^2 -
e^2*x^2)^(5/2)) + 17/(143*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*
d*e^3*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*d^2*e^3*(d + e*x)*(d^2 -
 e^2*x^2)^(5/2)) + (56*x)/(6435*d^6*e^2*(d^2 - e^2*x^2)^(3/2)) + (112*x)/(6
435*d^8*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{3d^2e^2-5de^3x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{8e^4}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{(7d)\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right.}}{104e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right.}}{143e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x\right.}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x\right.}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x\right.}$$

Mathematica [A] time = 0.10, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(200d^9+800d^8ex+700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-1358d^4e^5x^5-672d^3e^6x^6+392d^2e^7x^7+4\right.}{6435d^8e^3(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^3
*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7*x
^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7)
```

fricas [A] time = 2.06, size = 317, normalized size = 1.52

$$\frac{200\,e^{10}x^{10}+800\,de^{9}x^{9}+600\,d^{2}e^{8}x^{8}-1600\,d^{3}e^{7}x^{7}-2800\,d^{4}e^{6}x^{6}+2800\,d^{6}e^{4}x^{4}+1600\,d^{7}e^{3}x^{3}-600\,d^{8}e^{2}x^{2}-}{6435\left(d^{8}e^{13}x^{10}+4\,d^{9}e^{12}x^{9}+3\,d^{10}e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/6435*(200*e^10*x^10 + 800*d*e^9*x^9 + 600*d^2*e^8*x^8 - 1600*d^3*e^7*x^7
- 2800*d^4*e^6*x^6 + 2800*d^6*e^4*x^4 + 1600*d^7*e^3*x^3 - 600*d^8*e^2*x^2
- 800*d^9*e*x - 200*d^10 - (112*e^9*x^9 + 448*d*e^8*x^8 + 392*d^2*e^7*x^7 -
 672*d^3*e^6*x^6 - 1358*d^4*e^5*x^5 - 280*d^5*e^4*x^4 + 945*d^6*e^3*x^3 + 7
00*d^7*e^2*x^2 + 800*d^8*e*x + 200*d^9)*sqrt(-e^2*x^2 + d^2))/(d^8*e^13*x^1
0 + 4*d^9*e^12*x^9 + 3*d^10*e^11*x^8 - 8*d^11*e^10*x^7 - 14*d^12*e^9*x^6 +
14*d^14*e^7*x^4 + 8*d^15*e^6*x^3 - 3*d^16*e^5*x^2 - 4*d^17*e^4*x - d^18*e^3
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.63

$$\frac{(-ex+d)\left(112e^{9}x^{9}+448e^{8}x^{8}d+392e^{7}x^{7}d^{2}-672e^{6}x^{6}d^{3}-1358e^{5}x^{5}d^{4}-280x^{4}d^{5}e^{4}+945x^{3}d^{6}e^{3}+700x^{2}d^{7}e^{2}\right.}{6435\,(ex+d)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{8}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/6435*(-e*x+d)*(112*e^9*x^9+448*d*e^8*x^8+392*d^2*e^7*x^7-672*d^3*e^6*x^6-
1358*d^4*e^5*x^5-280*d^5*e^4*x^4+945*d^6*e^3*x^3+700*d^7*e^2*x^2+800*d^8*e*
x+200*d^9)/(e*x+d)^3/d^8/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.49, size = 401, normalized size = 1.92

$$-\frac{d}{13\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}x^{4}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{6}x^{3}+6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{5}x^{2}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{4}x+\left(-e^{2}x^{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13*d/((-e^2*x^2 + d^2)^(5/2)*e^7*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^
3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^5*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^4
*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^3) + 17/143/((-e^2*x^2 + d^2)^(5/2)*e^6*x
^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*
x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 7/1287/((-e^2*x^2 + d^2)^(5/2)*d*e^5*
x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3)
- 7/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3
```

```
) + 14/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2) + 56/6435*x/((-e^2*x^2 + d^2
)^(3/2)*d^6*e^2) + 112/6435*x/(sqrt(-e^2*x^2 + d^2)*d^8*e^2)
```

mupad [B] time = 3.19, size = 252, normalized size = 1.21

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{227}{6864\,d^3\,e^3}-\frac{353\,x}{17160\,d^4\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{353}{41184\,d^5\,e^3}-\frac{56\,x}{6435\,d^6\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^2\,e^3\,{\left(d+e\,x\right)}^7}+\frac{\sqrt{d^2-e^2\,x^2}}{2288\,d^3\,e^3\,{\left(d+e\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(227/(6864*d^3*e^3) - (353*x)/(17160*d^4*e^2)))/((d
+ e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(353/(41184*d^5*e^3) - (56*x
)/(6435*d^6*e^2)))/((d + e*x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(104*d
^2*e^3*(d + e*x)^7) + (d^2 - e^2*x^2)^(1/2)/(2288*d^3*e^3*(d + e*x)^6) + (3
7*(d^2 - e^2*x^2)^(1/2))/(5148*d^4*e^3*(d + e*x)^5) + (353*(d^2 - e^2*x^2)^
(1/2))/(41184*d^5*e^3*(d + e*x)^4) + (112*x*(d^2 - e^2*x^2)^(1/2))/(6435*d^
8*e^2*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.214 $\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{512x}{6435d^9e\sqrt{d^2}}$$

```
[Out] 64/2145*x/d^5/e/(-e^2*x^2+d^2)^(5/2)+1/13/e^2/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2
)-4/143/d/e^2/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-32/1287/d^2/e^2/(e*x+d)^2/(-e^
2*x^2+d^2)^(5/2)-32/1287/d^3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+256/6435*x/d^
7/e/(-e^2*x^2+d^2)^(3/2)+512/6435*x/d^9/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {793, 659, 192, 191}

$$\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{}{1287d}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (64*x)/(2145*d^5*e*(d^2 - e^2*x^2)^(5/2)) + 1/(13*e^2*(d + e*x)^4*(d^2 - e^
2*x^2)^(5/2)) - 4/(143*d*e^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*
d^2*e^2*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*d^3*e^2*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) + (256*x)/(6435*d^7*e*(d^2 - e^2*x^2)^(3/2)) + (512*x)/
(6435*d^9*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13e}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{32\int \frac{1}{(d+ex)^2\left(d^2-e^2x\right.}}{143de}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x\right.}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x\right.}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x\right.}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.65

$$\frac{\sqrt{d^2-e^2x^2}\left(-5d^9-20d^8ex+3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-6208d^4e^5x^5-3072d^3e^6x^6+1792d^2e^7x^7\right.}{6435d^9e^2(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^3
*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*e^
7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*x)^
7)
```

fricas [A] time = 2.34, size = 316, normalized size = 1.50

$$-\frac{5e^{10}x^{10}+20de^9x^9+15d^2e^8x^8-40d^3e^7x^7-70d^4e^6x^6+70d^6e^4x^4+40d^7e^3x^3-15d^8e^2x^2-20d^9ex-5d^{10}+}{6435\left(d^9e^{12}x^{10}+4d^{10}e^{11}x^9+3d^{11}e^{10}x^8-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6435*(5*e^10*x^10 + 20*d*e^9*x^9 + 15*d^2*e^8*x^8 - 40*d^3*e^7*x^7 - 70*
d^4*e^6*x^6 + 70*d^6*e^4*x^4 + 40*d^7*e^3*x^3 - 15*d^8*e^2*x^2 - 20*d^9*e*x
 - 5*d^10 + (512*e^9*x^9 + 2048*d*e^8*x^8 + 1792*d^2*e^7*x^7 - 3072*d^3*e^6
*x^6 - 6208*d^4*e^5*x^5 - 1280*d^5*e^4*x^4 + 4320*d^6*e^3*x^3 + 3200*d^7*e^
2*x^2 - 20*d^8*e*x - 5*d^9)*sqrt(-e^2*x^2 + d^2))/(d^9*e^12*x^10 + 4*d^10*e
```

```
^11*x^9 + 3*d^11*e^10*x^8 - 8*d^12*e^9*x^7 - 14*d^13*e^8*x^6 + 14*d^15*e^6*
x^4 + 8*d^16*e^5*x^3 - 3*d^17*e^4*x^2 - 4*d^18*e^3*x - d^19*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.63

$$-\frac{(-ex+d)\left(-512e^9x^9-2048e^8x^8d-1792e^7x^7d^2+3072e^6x^6d^3+6208e^5x^5d^4+1280x^4d^5e^4-4320x^3d^6e^3-3\right.}{6435\,(ex+d)^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^9e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/6435*(-e*x+d)*(-512*e^9*x^9-2048*d*e^8*x^8-1792*d^2*e^7*x^7+3072*d^3*e^6
*x^6+6208*d^4*e^5*x^5+1280*d^5*e^4*x^4-4320*d^6*e^3*x^3-3200*d^7*e^2*x^2+20
*d^8*e*x+5*d^9)/(e*x+d)^3/d^9/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.49, size = 405, normalized size = 1.92

$$\frac{1}{13\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6x^4+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^5x^3+6\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4x^2+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3x+\left(-e^2x^2+d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13/((-e^2*x^2 + d^2)^(5/2)*e^6*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3 +
 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x
+ (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 4/143/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3
 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*
x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e
^4*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^
2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4
*e^2) + 64/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^5*e) + 256/6435*x/((-e^2*x^2 +
d^2)^(3/2)*d^7*e) + 512/6435*x/(sqrt(-e^2*x^2 + d^2)*d^9*e)
```

mupad [B] time = 3.19, size = 252, normalized size = 1.19

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{41}{41184\,d^6\,e^2}+\frac{256\,x}{6435\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{47}{1716\,d^4\,e^2}-\frac{1369\,x}{34320\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}+\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^3\,e^2\,{\left(d+e\,x\right)}^7}+\frac{25\,\sqrt{d^2-e^2\,x^2}}{2288\,d^4\,e^2\,(d+e\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(41/(41184*d^6*e^2) + (256*x)/(6435*d^7*e)))/((d + e
*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(47/(1716*d^4*e^2) - (1369*x)/(
34320*d^5*e)))/((d + e*x)^3*(d - e*x)^3) + (d^2 - e^2*x^2)^(1/2)/(104*d^3*e
^2*(d + e*x)^7) + (25*(d^2 - e^2*x^2)^(1/2))/(2288*d^4*e^2*(d + e*x)^6) + (
```

```
125*(d^2 - e^2*x^2)^(1/2))/(20592*d^5*e^2*(d + e*x)^5) - (41*(d^2 - e^2*x^2
)^(1/2))/(41184*d^6*e^2*(d + e*x)^4) + (512*x*(d^2 - e^2*x^2)^(1/2))/(6435*
d^9*e*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.215 $\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{}{715d}$$

```
[Out] 48/715*x/d^6/(-e^2*x^2+d^2)^(5/2)-1/13/d/e/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2)-9
/143/d^2/e/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-8/143/d^3/e/(e*x+d)^2/(-e^2*x^2+d
^2)^(5/2)-8/143/d^4/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+64/715*x/d^8/(-e^2*x^2+d
^2)^(3/2)+128/715*x/d^10/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{9\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13d}$$

$$= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{72\int \frac{1}{(d+ex)^2\left(d^2-e\right.}}{143d^2}$$

$$= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2}$$

$$= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2}$$

$$= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)}$$

$$= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)}$$

$$= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)}$$

Mathematica [A] time = 0.07, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(-180d^9-5d^8ex+800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552d^4e^5x^5-768d^3e^6x^6+448d^2e^7x^7+5\right.}{715d^{10}e(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

fricas [A] time = 2.33, size = 314, normalized size = 1.53

$$-\frac{180\,e^{10}x^{10}+720\,de^{9}x^{9}+540\,d^2e^8x^8-1440\,d^3e^7x^7-2520\,d^4e^6x^6+2520\,d^6e^4x^4+1440\,d^7e^3x^3-540\,d^8e^2x^2-7}{715\left(d^{10}e^{11}x^{10}+4\,d^{11}e^{10}x^9+3\,d^{12}e^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/715*(180*e^10*x^10 + 720*d*e^9*x^9 + 540*d^2*e^8*x^8 - 1440*d^3*e^7*x^7
- 2520*d^4*e^6*x^6 + 2520*d^6*e^4*x^4 + 1440*d^7*e^3*x^3 - 540*d^8*e^2*x^2
- 720*d^9*e*x - 180*d^10 + (128*e^9*x^9 + 512*d*e^8*x^8 + 448*d^2*e^7*x^7 -
 768*d^3*e^6*x^6 - 1552*d^4*e^5*x^5 - 320*d^5*e^4*x^4 + 1080*d^6*e^3*x^3 +
800*d^7*e^2*x^2 - 5*d^8*e*x - 180*d^9)*sqrt(-e^2*x^2 + d^2))/(d^10*e^11*x^1
0 + 4*d^11*e^10*x^9 + 3*d^12*e^9*x^8 - 8*d^13*e^8*x^7 - 14*d^14*e^7*x^6 + 1
4*d^16*e^5*x^4 + 8*d^17*e^4*x^3 - 3*d^18*e^3*x^2 - 4*d^19*e^2*x - d^20*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.64

$$-\frac{(-ex+d)\left(-128e^9x^9-512e^8x^8d-448e^7x^7d^2+768e^6x^6d^3+1552e^5x^5d^4+320x^4d^5e^4-1080x^3d^6e^3-800x^2d\right.}{715\,(ex+d)^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^{10}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/715*(-e*x+d)*(-128*e^9*x^9-512*d*e^8*x^8-448*d^2*e^7*x^7+768*d^3*e^6*x^6
+1552*d^4*e^5*x^5+320*d^5*e^4*x^4-1080*d^6*e^3*x^3-800*d^7*e^2*x^2+5*d^8*e*
x+180*d^9)/(e*x+d)^3/d^10/e/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.50, size = 393, normalized size = 1.92

$$-\frac{1}{13\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^5x^4+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4x^3+6\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3x^2+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4e^2x+\left(-e^2x^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*
x^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^4*e
^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 9/143/((-e^2*x^2 + d^2)^(5/2)*d^2*e^
4*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^4
*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^3*
e^3*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e
) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e)
 + 48/715*x/((-e^2*x^2 + d^2)^(5/2)*d^6) + 64/715*x/((-e^2*x^2 + d^2)^(3/2)
*d^8) + 128/715*x/(sqrt(-e^2*x^2 + d^2)*d^10)
```

mupad [B] time = 3.12, size = 242, normalized size = 1.18

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{64\,x}{715\,d^8}+\frac{189}{4576\,d^7\,e}\right)}{(d+e\,x)^2\,(d-e\,x)^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1139\,x}{5720\,d^6}-\frac{427}{2288\,d^5\,e}\right)}{(d+e\,x)^3\,(d-e\,x)^3}-\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^4\,e\,(d+e\,x)^7}-\frac{51\,\sqrt{d^2-e^2\,x^2}}{2288\,d^5\,e\,(d+e\,x)^6}-\frac{19}{572\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*((64*x)/(715*d^8) + 189/(4576*d^7*e)))/((d + e*x)^2*
(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*((1139*x)/(5720*d^6) - 427/(2288*d^5*
e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(104*d^4*e*(d + e*x)
^7) - (51*(d^2 - e^2*x^2)^(1/2))/(2288*d^5*e*(d + e*x)^6) - (19*(d^2 - e^2*
x^2)^(1/2))/(572*d^6*e*(d + e*x)^5) - (189*(d^2 - e^2*x^2)^(1/2))/(4576*d^7
```

```
*e*(d + e*x)^4) + (128*x*(d^2 - e^2*x^2)^(1/2))/(715*d^10*(d + e*x)*(d - e*
x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.216 $\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}+\frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}+\frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{27}{1365d}$$

```
[Out] 8/13*d*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/13*e*x/d/(-e^2*x^2+d^2)^(11/2)+1/11
7*(-40*e*x+13*d)/d^3/(-e^2*x^2+d^2)^(9/2)+1/819*(-320*e*x+117*d)/d^5/(-e^2*
x^2+d^2)^(7/2)+1/1365*(-640*e*x+273*d)/d^7/(-e^2*x^2+d^2)^(5/2)+1/819*(-512
*e*x+273*d)/d^9/(-e^2*x^2+d^2)^(3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^11+1
/819*(-1024*e*x+819*d)/d^11/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}}+\frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}}+\frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{273d-640ex}{1365d^7\left(d^2-e^2x^2\right)^{5/2}}+\frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)
^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e
*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^
2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d
- 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
/d^11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+44d^3ex+13d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-440d^3ex}{x\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{\int \frac{1287d^6e^2-3\cdots}{x\left(d^2-e^2\cdots\right.}}{1287\cdots}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-3\cdots}{819d^5\left(d^2-\cdots\right.}$$

Mathematica [A] time = 0.17, size = 161, normalized size = 0.69

$$\frac{-4095\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(9839d^9+22976d^8ex-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e^6x^6-15\cdots\right.}{(d-ex)^3(d+ex)^7}}{4095d^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304*
d^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 - 1
540*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/((d - e*x)^3*(d + e*x)^7
) + 4095*Log[x] - 4095*Log[d + Sqrt[d^2 - e^2*x^2]])/(4095*d^11)
```

fricas [B] time = 2.40, size = 432, normalized size = 1.85

$$\frac{9839\,e^{10}x^{10} + 39356\,de^{9}x^{9} + 29517\,d^{2}e^{8}x^{8} - 78712\,d^{3}e^{7}x^{7} - 137746\,d^{4}e^{6}x^{6} + 137746\,d^{6}e^{4}x^{4} + 78712\,d^{7}e^{3}x^{3} - 29}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4095*(9839*e^10*x^10 + 39356*d*e^9*x^9 + 29517*d^2*e^8*x^8 - 78712*d^3*e^
7*x^7 - 137746*d^4*e^6*x^6 + 137746*d^6*e^4*x^4 + 78712*d^7*e^3*x^3 - 29517
*d^8*e^2*x^2 - 39356*d^9*e*x - 9839*d^10 + 4095*(e^10*x^10 + 4*d*e^9*x^9 +
3*d^2*e^8*x^8 - 8*d^3*e^7*x^7 - 14*d^4*e^6*x^6 + 14*d^6*e^4*x^4 + 8*d^7*e^3
*x^3 - 3*d^8*e^2*x^2 - 4*d^9*e*x - d^10)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 + (5120*e^9*x^9 + 16385*d*e^8*x^8 + 1540*d^2*e^7*x^7 - 45735*d^3*e^6*x^6 -
 40240*d^4*e^5*x^5 + 34156*d^5*e^4*x^4 + 56304*d^6*e^3*x^3 + 4466*d^7*e^2*x
^2 - 22976*d^8*e*x - 9839*d^9)*sqrt(-e^2*x^2 + d^2))/(d^11*e^10*x^10 + 4*d^
12*e^9*x^9 + 3*d^13*e^8*x^8 - 8*d^14*e^7*x^7 - 14*d^15*e^6*x^6 + 14*d^17*e^
4*x^4 + 8*d^18*e^3*x^3 - 3*d^19*e^2*x^2 - 4*d^20*e*x - d^21)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.03, size = 385, normalized size = 1.65

$$-\frac{128ex}{273\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}d^{7}}+\frac{1}{13\left(x+\frac{d}{e}\right)^{4}\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}d^{2}e^{4}}+\frac{2}{13\left(x+\frac{d}{e}\right)^{3}\left(2\left(x+\frac{d}{e}\right)de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/13/d^2/e^4/(x+d/e)^4/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+2/13/d^3/e^3/(x+
d/e)^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+29/117/d^4/e^2/(x+d/e)^2/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(5/2)+320/819/d^5/e/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(5/2)-128/273/d^7*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)*x-512/819/d^9
*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-1024/819/d^11*e/(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(1/2)*x+1/5/d^6/(-e^2*x^2+d^2)^(5/2)+1/3/d^8/(-e^2*x^2+d^2)^(3/
2)+1/d^10/(-e^2*x^2+d^2)^(1/2)-1/d^10/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}(ex+d)^{4}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(d^2-e^2\,x^2\right)^{7/2}\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.217 $\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}-\frac{e(12012d}{9009d^{10}\left(\right.}$$

```
[Out] -8/13*e*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/143*e*(-24*e*x+13*d)/d^2/(-e^2*x^2
+d^2)^(11/2)-1/1287*e*(-1103*e*x+572*d)/d^4/(-e^2*x^2+d^2)^(9/2)-1/9009*e*(
-10111*e*x+5148*d)/d^6/(-e^2*x^2+d^2)^(7/2)-1/15015*e*(-23225*e*x+12012*d)/
d^8/(-e^2*x^2+d^2)^(5/2)-1/9009*e*(-21583*e*x+12012*d)/d^10/(-e^2*x^2+d^2)^
(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^12-1/9009*e*(-52175*e*x+36036*d
)/d^12/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^12/x
```

Rubi [A] time = 0.68, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}-\frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\frac{e}{128}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*(13*d - 24*e*x))/(143*d
^2*(d^2 - e^2*x^2)^(11/2)) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^
2)^(9/2)) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^(7/2)) - (e*
(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^(5/2)) - (e*(12012*d - 21
583*e*x))/(9009*d^10*(d^2 - e^2*x^2)^(3/2)) - (e*(36036*d - 52175*e*x))/(90
09*d^12*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^12*x) + (4*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^12
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+52d^3ex-83d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-572d^3ex+960d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{-1287d}{}}{}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148}{9009d^6}$$

Mathematica [A] time = 0.21, size = 183, normalized size = 0.68

$$-\frac{4e\log(x)}{d^{12}}+\frac{4e\log\left(\sqrt{d^2-e^2x^2}+d\right)}{d^{12}}+\frac{\sqrt{d^2-e^2x^2}\left(45045d^{10}+546316d^9ex+1014094d^8e^2x^2-700504d^7e^3x^3-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(45045*d^10 + 546316*d^9*e*x + 1014094*d^8*e^2*x^2 - 7
00504*d^7*e^3*x^3 - 3157776*d^6*e^4*x^4 - 1301264*d^5*e^5*x^5 + 2748320*d^4
*e^6*x^6 + 2496180*d^3*e^7*x^7 - 350000*d^2*e^8*x^8 - 1043500*d*e^9*x^9 - 3
05920*e^10*x^10))/(45045*d^12*x*(-d + e*x)^3*(d + e*x)^7) - (4*e*Log[x])/d^
12 + (4*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^12
```

fricas [A] time = 3.67, size = 458, normalized size = 1.69

$$-\frac{366136\,e^{11}x^{11} + 1464544\,de^{10}x^{10} + 1098408\,d^2e^9x^9 - 2929088\,d^3e^8x^8 - 5125904\,d^4e^7x^7 + 5125904\,d^6e^5x^5 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/45045*(366136*e^11*x^11 + 1464544*d*e^10*x^10 + 1098408*d^2*e^9*x^9 - 29
29088*d^3*e^8*x^8 - 5125904*d^4*e^7*x^7 + 5125904*d^6*e^5*x^5 + 2929088*d^7
*e^4*x^4 - 1098408*d^8*e^3*x^3 - 1464544*d^9*e^2*x^2 - 366136*d^10*e*x + 18
0180*(e^11*x^11 + 4*d*e^10*x^10 + 3*d^2*e^9*x^9 - 8*d^3*e^8*x^8 - 14*d^4*e^
7*x^7 + 14*d^6*e^5*x^5 + 8*d^7*e^4*x^4 - 3*d^8*e^3*x^3 - 4*d^9*e^2*x^2 - d^
10*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (305920*e^10*x^10 + 1043500*d*
e^9*x^9 + 350000*d^2*e^8*x^8 - 2496180*d^3*e^7*x^7 - 2748320*d^4*e^6*x^6 +
1301264*d^5*e^5*x^5 + 3157776*d^6*e^4*x^4 + 700504*d^7*e^3*x^3 - 1014094*d^
8*e^2*x^2 - 546316*d^9*e*x - 45045*d^10)*sqrt(-e^2*x^2 + d^2))/(d^12*e^10*x
^11 + 4*d^13*e^9*x^10 + 3*d^14*e^8*x^9 - 8*d^15*e^7*x^8 - 14*d^16*e^6*x^7 +
 14*d^18*e^4*x^5 + 8*d^19*e^3*x^4 - 3*d^20*e^2*x^3 - 4*d^21*e*x^2 - d^22*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.02, size = 484, normalized size = 1.79

$$\frac{6e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^8}+\frac{20222e^2x}{15015\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^8}-\frac{1}{13\left(x+\frac{d}{e}\right)^4\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^3e^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/d^11*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+6/5/d
^8*e^2*x/(-e^2*x^2+d^2)^(5/2)+8/5/d^10*e^2*x/(-e^2*x^2+d^2)^(3/2)+16/5/d^12
*e^2*x/(-e^2*x^2+d^2)^(1/2)-35/143/d^4/e^2/(x+d/e)^3/(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(5/2)-709/1287/d^5/e/(x+d/e)^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+
20222/15015/d^8*e^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)*x+80888/45045/d^10*
e^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+161776/45045/d^12*e^2/(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(1/2)*x-1/13/d^3/e^3/(x+d/e)^4/(2*(x+d/e)*d*e-(x+d/e)^2*
e^2)^(5/2)-4/5/d^7*e/(-e^2*x^2+d^2)^(5/2)-4/3/d^9*e/(-e^2*x^2+d^2)^(3/2)-4/
d^11*e/(-e^2*x^2+d^2)^(1/2)-1/d^6/x/(-e^2*x^2+d^2)^(5/2)-10111/9009/d^6/(x+
d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(d^2-e^2\,x^2\right)^{7/2}\,(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.218 $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx$

Optimal. Leaf size=102

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -c^2*(-a^2*x^2+1)^(3/2)/x/(-a*c*x+c)^(3/2)+a*arctanh(c^(1/2)*(-a^2*x^2+1)^(
1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-a*c*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {879, 865, 875, 208}

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] -((a*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]) - (c^2*(1 - a^2*x^2)^(3/2))/(x*(
c - a*c*x)^(3/2)) + a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c -
a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx = -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}(ac)\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}\,dx$$
$$= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}$$
$$= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Mathematica [A] time = 0.10, size = 93, normalized size = 0.91

$$\frac{\sqrt{1-a^2x^2}\left(a\sqrt{c}\,x\tanh^{-1}\left(\sqrt{c}\sqrt{\frac{ax+1}{c}}\right)-c(2ax+1)\sqrt{\frac{ax+1}{c}}\right)}{x\sqrt{\frac{ax+1}{c}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(c*Sqrt[(1 + a*x)/c]*(1 + 2*a*x)) + a*Sqrt[c]*x*ArcTan
h[Sqrt[c]*Sqrt[(1 + a*x)/c]]))/(x*Sqrt[(1 + a*x)/c]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.90, size = 217, normalized size = 2.13

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(2ax+1)}{2\left(ax^2-x\right)}, \frac{\left(a^2x^2-ax\right)\sqrt{-c}\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x), ((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt
(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 95, normalized size = 0.93

$$\frac{\left(-acx\,\mathrm{arctanh}\left(\frac{\sqrt{(ax+1)c}}{\sqrt{c}}\right)+2\sqrt{(ax+1)c}\,a\sqrt{c}\,x+\sqrt{(ax+1)c}\,\sqrt{c}\right)\sqrt{-(ax-1)c}\,\sqrt{-a^2x^2+1}}{(ax-1)\sqrt{(ax+1)c}\,\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+
(c*(a*x+1))^(1/2)*c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c
*(a*x+1))^(1/2)/x/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2,x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\sqrt{-(ax-1)\,(ax+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/x**2, x)
```

3.219 $\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=39

$$-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {875, 208}

$$-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx &= \left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= -2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.72

$$-\frac{2\sqrt{c}\sqrt{\frac{ax}{c}+\frac{1}{c}}\sqrt{c-acx}\tanh^{-1}\left(\sqrt{c}\sqrt{\frac{ax}{c}+\frac{1}{c}}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[c^(-1) + (a*x)/c]*Sqrt[c - a*c*x]*ArcTanh[Sqrt[c]*Sqrt[c^(
-1) + (a*x)/c]])/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.91, size = 110, normalized size = 2.82

$$\left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right), -2\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sq
rt(c) - 2*c)/(a*x^2 - x)), -2*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c))]
```

giac [A] time = 0.17, size = 57, normalized size = 1.46

$$-\frac{2\,c^{3}\left(\frac{\arctan\left(\frac{\sqrt{2}\,\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c^3*(arctan(sqrt(2)*sqrt(c)/sqrt(-c))/(sqrt(-c)*c) - arctan(sqrt(a*c*x +
 c)/sqrt(-c))/(sqrt(-c)*c))/abs(c)
```

maple [A] time = 0.02, size = 58, normalized size = 1.49

$$\frac{2\sqrt{-\left(ax-1\right)c}\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{\left(ax+1\right)c}}{\sqrt{c}}\right)}{\left(ax-1\right)\sqrt{\left(ax+1\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2*(-(a*x-1)*c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/((a*x+1)*c)^(1/2)*c^(1/2)*a
rctanh(((a*x+1)*c)^(1/2)/c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}}{\sqrt{-a^{2}x^{2}+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.220 $\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

[Out] arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - a*x]/Sqrt[x],x]

[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{1-ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx\\
&= \sqrt{x}\sqrt{1-ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

fricas [A] time = 0.70, size = 92, normalized size = 2.63

$$\left[\frac{2\sqrt{-ax+1}\,a\sqrt{x}-\sqrt{-a}\,\log\left(-2\,ax+2\,\sqrt{-ax+1}\,\sqrt{-a}\,\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{-ax+1}\,a\sqrt{x}-\sqrt{a}\,\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\,\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*x + 1)*a*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt(-a*x + 1)*s
qrt(-a)*sqrt(x) + 1))/a, (sqrt(-a*x + 1)*a*sqrt(x) - sqrt(a)*arctan(sqrt(-a
*x + 1)/(sqrt(a)*sqrt(x))))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-15.6438432182,61.793747834
9]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-29.292030761,78.649
3344628]1/abs(a)*a^2/a*(1/a*sqrt(-a*x+1)*sqrt(-a*(-a*x+1)+a)+1/sqrt(-a)*ln(
abs(sqrt(-a*(-a*x+1)+a)-sqrt(-a)*sqrt(-a*x+1))))
```

maple [B] time = 0.01, size = 62, normalized size = 1.77

$$\sqrt{-ax+1}\,\sqrt{x}+\frac{\sqrt{(-ax+1)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{2a}\right)\sqrt{a}}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{a}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-a*x+1)^(1/2)+1/2*((-a*x+1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)/a^(1/2
)*arctan(a^(1/2)*(x-1/2/a)/(-a*x^2+x)^(1/2))
```

maxima [A] time = 0.96, size = 48, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{a}}+\frac{\sqrt{-ax+1}}{\left(a-\frac{ax-1}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))/sqrt(a) + sqrt(-a*x + 1)/((a - (a
*x - 1)/x)*sqrt(x))
```

mupad [B] time = 2.99, size = 38, normalized size = 1.09

$$\sqrt{x}\,\sqrt{1-a\,x}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{1-a\,x}-1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(1 - a*x)^(1/2) + (2*atan((a^(1/2)*x^(1/2))/((1 - a*x)^(1/2) - 1)))
/a^(1/2)
```

sympy [A] time = 1.90, size = 83, normalized size = 2.37

$$\begin{cases} \frac{iax^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{i\sqrt{x}}{\sqrt{ax-1}}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{for } |ax|>1 \\ \sqrt{x}\sqrt{-ax+1}+\frac{\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(3/2)/sqrt(a*x - 1) - I*sqrt(x)/sqrt(a*x - 1) - I*acosh(s
qrt(a)*sqrt(x))/sqrt(a), Abs(a*x) > 1), (sqrt(x)*sqrt(-a*x + 1) + asin(sqrt
(a)*sqrt(x))/sqrt(a), True))
```

3.221 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {848, 50, 54, 216}

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx = \int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$$
$$= \sqrt{x}\sqrt{1-ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx$$
$$= \sqrt{x}\sqrt{1-ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

fricas [B] time = 0.78, size = 199, normalized size = 5.69

$$\left[\frac{4\sqrt{-a^2x^2+1}\sqrt{ax+1}\,a\sqrt{x} - (ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{4\left(a^2x+a\right)}, \frac{2\sqrt{-a^2x^2+1}\sqrt{ax}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(-a)*log
(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*sqrt(-a)*sqrt
(x) - 7*a*x + 1)/(a*x + 1)))/(a^2*x + a), 1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(a*
x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(a*x +
 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^2*x + a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
```

```
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-15.6438432182,61.793747834
9]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-29.292030761,78.649
3344628]2/abs(a)*a^2/a*(1/2*ln(abs(-sqrt(-a)*sqrt(-a*x+1)+sqrt(-a*(-a*x+1)+
a)))/sqrt(-a)+1/2*sqrt(-a*x+1)*sqrt(-a*(-a*x+1)+a)/a+(sqrt(2)-ln(abs(-sqrt(
-a)*sqrt(2)+sqrt(-a))))/2/sqrt(-a))
```

maple [B] time = 0.02, size = 76, normalized size = 2.17

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\left(\arctan\left(\frac{2ax-1}{2\sqrt{-(ax-1)x}\,\sqrt{a}}\right)+2\sqrt{-\left(ax-1\right)x}\,\sqrt{a}\right)\sqrt{x}}{2\sqrt{ax+1}\,\sqrt{-\left(ax-1\right)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)/(a*x+1)^(1/2)*(2*a^(1/2)*(-x*(a*x-1))^(1/2)+
arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/(-x*(a*x-1))^(1/2)/a^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\sqrt{ax+1}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-a^{2}\,x^{2}}}{\sqrt{x}\,\sqrt{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\sqrt{x}\,\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)
```

3.222 $\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{1+ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx\\
&= \sqrt{x}\sqrt{1+ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

fricas [A] time = 0.89, size = 90, normalized size = 2.65

$$\left[\frac{2\,\sqrt{ax+1}\,a\sqrt{x}\,+\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{ax+1}\,\sqrt{a}\,\sqrt{x}\,+1\right)}{2\,a},\,\frac{\sqrt{ax+1}\,a\sqrt{x}\,-\sqrt{-a}\,\arctan\left(\frac{\sqrt{ax+1}\,\sqrt{-a}}{a\,\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a*x + 1)*a*sqrt(x) + sqrt(a)*log(2*a*x + 2*sqrt(a*x + 1)*sqrt(
a)*sqrt(x) + 1))/a, (sqrt(a*x + 1)*a*sqrt(x) - sqrt(-a)*arctan(sqrt(a*x + 1
)*sqrt(-a)/(a*sqrt(x))))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}+
%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[
3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%}
+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,
[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]%%%
}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,
[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]%%%}+
%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[0
,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [85.3561567818,61.7937478349]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{4,[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]
%%%}+%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%
{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0
]%%%}+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0
]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]
%%%}+%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{
-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [71.707969239,78.6493344
628]1/abs(a)*a^2/a*(1/a*sqrt(a*x+1)*sqrt(a*(a*x+1)-a)-1/sqrt(a)*ln(abs(sqrt
(a*(a*x+1)-a)-sqrt(a)*sqrt(a*x+1))))
```

maple [B] time = 0.01, size = 57, normalized size = 1.68

$$\sqrt{ax+1}\,\sqrt{x}+\frac{\sqrt{(ax+1)\,x}\,\ln\left(\frac{ax+\frac{1}{2}}{\sqrt{a}}+\sqrt{a\,x^{2}+x}\right)}{2\sqrt{ax+1}\,\sqrt{a}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x+1)^(1/2)+1/2*((a*x+1)*x)^(1/2)/(a*x+1)^(1/2)/x^(1/2)*ln((1/2+a
*x)/a^(1/2)+(a*x^2+x)^(1/2))/a^(1/2)
```

maxima [B] time = 0.96, size = 68, normalized size = 2.00

$$-\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax+1}}{\sqrt{x}}}{\sqrt{a}+\frac{\sqrt{ax+1}}{\sqrt{x}}}\right)}{2\sqrt{a}}-\frac{\sqrt{ax+1}}{\left(a-\frac{ax+1}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(-(sqrt(a) - sqrt(a*x + 1)/sqrt(x))/(sqrt(a) + sqrt(a*x + 1)/sqrt(x
)))/sqrt(a) - sqrt(a*x + 1)/((a - (a*x + 1)/x)*sqrt(x))
```

mupad [B] time = 3.00, size = 36, normalized size = 1.06

$$\sqrt{x}\,\sqrt{a\,x+1}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{a\,x+1}-1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x + 1)^(1/2) + (2*atanh((a^(1/2)*x^(1/2))/((a*x + 1)^(1/2) - 1))
)/a^(1/2)
```

sympy [A] time = 1.97, size = 29, normalized size = 0.85

$$\sqrt{x}\,\sqrt{ax+1}+\frac{\operatorname{asinh}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a*x + 1) + asinh(sqrt(a)*sqrt(x))/sqrt(a)
```

3.223 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {848, 50, 54, 215}

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx &= \int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

fricas [B] time = 1.03, size = 208, normalized size = 6.12

$$\left[-\frac{4\sqrt{-a^2x^2+1}\sqrt{-ax+1}\,a\sqrt{x}-(ax-1)\sqrt{a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}\,(2ax+1)\sqrt{-ax+1}\sqrt{a}\sqrt{x}-7ax-1}{ax-1}\right)}{4\left(a^2x-a\right)},-\frac{2\sqrt{-a^2x^2+1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(a)*lo
g(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 1)*sqrt(-a*x + 1)*sqrt(a)*sqr
t(x) - 7*a*x - 1)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(-a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(-
a*x + 1)*sqrt(-a)*sqrt(x)/(2*a^2*x^2 - a*x - 1)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}+
%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[
3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%}
+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,
[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]%%%
}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,
[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6,[
```

```
2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]%%%}+
%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[0
,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [85.3561567818,61.7937478349]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{4,[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]
%%%}+%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%
{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0
]%%%}+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0
]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]
%%%}+%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{
-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [71.707969239,78.6493344
628]-2/abs(a)*a^2/a*(1/2*ln(abs(-sqrt(a)*sqrt(a*x+1)+sqrt(a*(a*x+1)-a)))/sq
rt(a)-1/2*sqrt(a*(a*x+1)-a)*sqrt(a*x+1)/a+(sqrt(2)-ln(abs(-sqrt(2)*sqrt(a)+
sqrt(a))))/2/sqrt(a))
```

maple [B] time = 0.01, size = 86, normalized size = 2.53

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-ax+1}\,\left(\ln\left(\frac{2ax+2\sqrt{(ax+1)x}\,\sqrt{a}+1}{2\sqrt{a}}\right)+2\sqrt{(ax+1)\,x}\,\sqrt{a}\right)\sqrt{x}}{2\,(ax-1)\,\sqrt{(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)*(-a*x+1)^(1/2)*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)
^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}}{\sqrt{-ax+1}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(ax-1)\,(ax+1)}}{\sqrt{x}\,\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)
```

3.224 $\int \sqrt{x}\sqrt{1-ax}\,dx$

Optimal. Leaf size=63

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

```
[Out] 1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*
(-a*x+1)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{1-ax}\,dx &= \frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)}{4a}\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(2ax-1)+\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

fricas [A] time = 0.75, size = 111, normalized size = 1.76

$$\left[\frac{2\left(2a^2x-a\right)\sqrt{-ax+1}\sqrt{x}-\sqrt{-a}\log\left(-2ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{8a^2}, \frac{\left(2a^2x-a\right)\sqrt{-ax+1}\sqrt{x}-\sqrt{a}\,\mathrm{arc}}{4a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x - a)*sqrt(-a*x + 1)*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt
(-a*x + 1)*sqrt(-a)*sqrt(x) + 1))/a^2, 1/4*((2*a^2*x - a)*sqrt(-a*x + 1)*sq
rt(x) - sqrt(a)*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x))))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-41.1343540126,25.838873679
7]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-67.0714422017,15.45
```

```
1549686]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{
-4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]
%%%}+%%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%
%{4,[0,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,
1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%
%%{4,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[
1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},
0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[
4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+
%%%{4,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2
,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%
%{-8,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]
%%%}+%%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-46.2420096635
,81.9516051291]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%
%}+%%%{-4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{
6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]
%%%}+%%%{4,[0,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%
{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1
]%%%}+%%%{4,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%
%%{-4,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,
0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+
%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,
1]%%%}+%%%{4,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%
%{-8,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]
%%%}+%%%{-8,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{
6,[0,2]%%%}+%%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-82.594
7937798,51.6443148847]1/a*(-2*a*abs(a)/a^2/a*(2*(1/8*sqrt(-a*x+1)*sqrt(-a*x
+1)-5/16)*sqrt(-a*x+1)*sqrt(-a*(-a*x+1)+a)+6*a/16/sqrt(-a)*ln(abs(sqrt(-a*(
-a*x+1)+a)-sqrt(-a)*sqrt(-a*x+1))))-2*abs(a)/a^2*(1/2*sqrt(-a*x+1)*sqrt(-a*
(-a*x+1)+a)-2*a/4/sqrt(-a)*ln(abs(sqrt(-a*(-a*x+1)+a)-sqrt(-a)*sqrt(-a*x+1)
))))
```

maple [A] time = 0.00, size = 79, normalized size = 1.25

$$\frac{\sqrt{-ax+1}\,x^{\frac{3}{2}}}{2}-\frac{\sqrt{-ax+1}\,\sqrt{x}}{4a}+\frac{\sqrt{(-ax+1)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{2a}\right)\sqrt{a}}{\sqrt{-a\,x^2+x}}\right)}{8\sqrt{-ax+1}\,a^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+1)^(1/2)*x^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*(-a*x+1)^(1/2)/a+1/8/a^(3/2)*((-a*x+
1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)*arctan((x-1/2/a)/(-a*x^2+x)^(1/2)*a^(1/2
))
```

maxima [A] time = 0.96, size = 82, normalized size = 1.30

$$\frac{\frac{\sqrt{-ax+1}\,a}{\sqrt{x}}-\frac{(-ax+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{4\left(a^3-\frac{2\,(ax-1)a^2}{x}+\frac{(ax-1)^2a}{x^2}\right)}-\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\,\sqrt{x}}\right)}{4\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a*x + 1)*a/sqrt(x) - (-a*x + 1)^(3/2)/x^(3/2))/(a^3 - 2*(a*x - 1
)*a^2/x + (a*x - 1)^2*a/x^2) - 1/4*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))
/a^(3/2)
```

mupad [B] time = 2.60, size = 54, normalized size = 0.86

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{1}{4\,a}\right)\,\sqrt{1-a\,x}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{x}\,\sqrt{1-a\,x}-2\,a\,x+1\right)}{8\,{\left(-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1 - a*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(4*a))*(1 - a*x)^(1/2) - log(2*(-a)^(1/2)*x^(1/2)*(1 - a*x
)^(1/2) - 2*a*x + 1)/(8*(-a)^(3/2))
```

sympy [A] time = 3.39, size = 148, normalized size = 2.35

$$\begin{cases} \frac{iax^{\frac{5}{2}}}{2\sqrt{ax-1}} - \frac{3ix^{\frac{3}{2}}}{4\sqrt{ax-1}} + \frac{i\sqrt{x}}{4a\sqrt{ax-1}} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{for}\ |ax| > 1 \\ -\frac{ax^{\frac{5}{2}}}{2\sqrt{-ax+1}} + \frac{3x^{\frac{3}{2}}}{4\sqrt{-ax+1}} - \frac{\sqrt{x}}{4a\sqrt{-ax+1}} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1))
+ I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)), Ab
s(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x + 1)
) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)), True
))
```

3.225 $\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$

Optimal. Leaf size=63

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

```
[Out] 1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*
(-a*x+1)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {848, 50, 54, 216}

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \sqrt{x}\sqrt{1-ax}\,dx$$
$$= \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}\,(2ax-1)+\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

fricas [B] time = 0.71, size = 221, normalized size = 3.51

$$\left[\frac{4\sqrt{-a^2x^2+1}\left(2a^2x-a\right)\sqrt{ax+1}\sqrt{x}-(ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}\,(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{16\left(a^3x+a^2\right)}, \frac{2\sqrt{-a\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a^2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)
*sqrt(-a)*log(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*
sqrt(-a)*sqrt(x) - 7*a*x + 1)/(a*x + 1)))/(a^3*x + a^2), 1/8*(2*sqrt(-a^2*x
^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sq
rt(-a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^3
*x + a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 92, normalized size = 1.46

$$\frac{\sqrt{-a^2x^2+1}\,\left(4\sqrt{-(ax-1)\,x}\;a^{\frac{3}{2}}x+\arctan\left(\frac{2ax-1}{2\sqrt{-(ax-1)x}\;\sqrt{a}}\right)-2\sqrt{-(ax-1)\,x}\;\sqrt{a}\right)\sqrt{x}}{8\sqrt{ax+1}\;\sqrt{-(ax-1)\,x}\;a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x)
```

```
[Out] 1/8*x^(1/2)*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*x*a^(3/2)*(-(a*x-1)*x)^(1/2)-2*(-
(a*x-1)*x)^(1/2)*a^(1/2)+arctan(1/2*(2*a*x-1)/(-(a*x-1)*x)^(1/2)/a^(1/2)))/
(a*x+1)^(1/2)/(-(a*x-1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{x}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}\,\sqrt{1-a^2\,x^2}}{\sqrt{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2),x)
```

```
[Out] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\,\sqrt{-(ax-1)(ax+1)}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a**2*x**2+1)**(1/2)/(a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-(a*x - 1)*(a*x + 1))/sqrt(a*x + 1), x)
```

3.226 $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=250

$$-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}-\frac{3d\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}+\frac{d^7(4m+11)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^
(7/2)/g^2/(9+m)+d^7*(11+4*m)*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1
/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8+m)/(1-e^2*x^2/d^2)^(1/2)
+d^6*e*(29+4*m)*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2
)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(9+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1809, 808, 365, 364}

$$\frac{d^6e(4m+29)\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+9)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}+\frac{d^7(4m+11)\sqrt{d^2-e^2x^2}\,(g\ldots}{g(m+1)(m\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m)) - (e*(g*x)^(2 + m)*(
d^2 - e^2*x^2)^(7/2))/(g^2*(9 + m)) + (d^7*(11 + 4*m)*(g*x)^(1 + m)*Sqrt[d^
2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/
(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^6*e*(29 + 4*m)*(g*x)^(2 +
m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x
^2)/d^2])/(g^2*(2 + m)*(9 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
```

```
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} - \frac{\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\left(-d^3e^2(9+m)-d^2e^3(29+\right.}{e^2(9+m)}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{\int (gx)^m\left(d^3e^4(9+m\right)}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{\left(d^3(11+4m)\right)\int (gx)}{8+}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{\left(d^7(11+4m)\sqrt{d^2-}\right.}{(8+}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{d^7(11+4m)(gx)^{1+m}}{g(1+}$$

Mathematica [A] time = 0.20, size = 199, normalized size = 0.80

$$\frac{d^4x\sqrt{d^2-e^2x^2}\,(gx)^m\left(ex\left(\frac{3d^2\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(-\frac{5}{2},\frac{m+3}{2};\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}+\frac{ex\,{}_2F_1\left(-\frac{5}{2},\frac{m+4}{2};\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)\right)+\frac{d^3\,{}_2F_1\left(-\frac{5}{2},\right.}{}\right.}{\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*((d^3*Hypergeometric2F1[-5/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[-5/2, (
2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[
-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F
1[-5/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/Sqrt[1 - (e^2*x^2
)/d^2]
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^7x^7+3\,de^6x^6+d^2e^5x^5-5\,d^3e^4x^4-5\,d^4e^3x^3+d^5e^2x^2+3\,d^6ex+d^7\right)\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*x
^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^3\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^{\frac{5}{2}} \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^3 \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d^2-e^2\,x^2\right)^{5/2}\,\left(g\,x\right)^m\,\left(d+e\,x\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3, x)
```

sympy [C] time = 45.91, size = 513, normalized size = 2.05

$$\frac{d^8g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{3d^7eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{d^6e^2g^mx^3x^m\Gamma\left(\frac{m}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**8*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**7*e*g**m*x**2*x
**m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2
*I*pi)/d**2)/(2*gamma(m/2 + 2)) + d**6*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)
*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(
2*gamma(m/2 + 5/2)) - 5*d**5*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2
, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3)
) - 5*d**4*e**4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m
/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**3*e
**5*g**m*x**6*x**m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x
**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4)) + 3*d**2*e**6*g**m*x**7*x**m
*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/(2*gamma(m/2 + 9/2)) + d*e**7*g**m*x**8*x**m*gamma(m/2 + 4)
*hyper((-1/2, m/2 + 4), (m/2 + 5,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*ga
mma(m/2 + 5))
```

3.227 $\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=206

$$-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}+\frac{d^6(2m+9)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{2d^5e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\right.}{g^2(m+2)\sqrt{1-}}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)+d^6*(9+2*m)*(g*x)^(1+m)*hypergeom
([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8
+m)/(1-e^2*x^2/d^2)^(1/2)+2*d^5*e*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+
1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1809, 808, 365, 364}

$$\frac{2d^5e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^6(2m+9)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{\left(d^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m))) + (d^6*(9 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (2*d^5*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{\int (gx)^m\left(-d^2e^2(9+2m)-2de^3(8+m)x\right)\left(d^2-\right.}{e^2(8+m)}\\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g}+\frac{\left(d^2(9+2\right.}{}\\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{\left(2d^5e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}+\\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\right.}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 174, normalized size = 0.84

$$\frac{d^4x\sqrt{d^2-e^2x^2}\,(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2}{d}\right.\right.\right.}{(m+1)(m+2)(m+3)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-
5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperg
eometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hyperg
eometric2F1[-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*
(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^6x^6+2\,de^5x^5-d^2e^4x^4-4\,d^3e^3x^3-d^4e^2x^2+2\,d^5ex+d^6\right)\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^6*x^6 + 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3 - d^4*e^2*x^2
 + 2*d^5*e*x + d^6)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^2\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^{\frac{5}{2}} \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}} (ex+d)^2 \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2, x)
```

sympy [C] time = 33.19, size = 442, normalized size = 2.15

$$\frac{d^7g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^6eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^5e^2g^mx^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)\cdots}{2\Gamma\left(\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**6*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/gamma(m/2 + 2) - d**5*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper
((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamm
a(m/2 + 5/2)) - 2*d**4*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2
+ 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) - d**3*e
**4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**2*e**5*g**m*x**
6*x**m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/gamma(m/2 + 4) + d*e**6*g**m*x**7*x**m*gamma(m/2 + 7/2)*hyp
er((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*ga
mma(m/2 + 9/2))
```

3.228 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{d^5\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^5*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)+d^4*e*(g*x)^(2+m)*hypergeom([-
5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*
x^2/d^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {808, 365, 364}

$$\frac{d^4e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^5\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^5*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^4*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx = d\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\,dx + \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g}$$
$$= \frac{\left(d^5\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\left(d^4e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2}{}\right.}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$
$$= \frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^4e(gx)^{2+m}\sqrt{d^2-e^2x^2}\ {}_2F_1\left(\right.}{g^2(2+m)\sqrt{1}}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.75

$$\frac{d^4x\sqrt{d^2-e^2x^2}\,(gx)^m\left(d(m+2)\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d*(2 + m)*Hypergeometric2F1[-5/2, (1 +
m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[-5/2, (2 +
m)/2, (4 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^5x^5+de^4x^4-2\,d^2e^3x^3-2\,d^3e^2x^2+d^4ex+d^5\right)\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*x + d
^5)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x), x)
```

sympy [C] time = 24.58, size = 374, normalized size = 2.31

$$\frac{d^6g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^5eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^4e^2g^mx^3x^m\Gamma\left(\frac{m}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**6*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**5*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 2)) - d**4*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*h
yper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gam
ma(m/2 + 5/2) - d**3*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 +
2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) + d**2*e**
4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d*e**5*g**m*x**6*x**
m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 4))
```

3.229 $\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=80

$$\frac{d^4\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^4*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {365, 364}

$$\frac{d^4\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx &= \frac{\left(d^4\sqrt{d^2-e^2x^2}\right)\int (gx)^m \left(1-\frac{e^2x^2}{d^2}\right)^{5/2} \, dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}\\ &= \frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.98

$$\frac{d^4x\sqrt{d^2-e^2x^2}\,(gx)^m\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+1}{2}+1;\frac{e^2x^2}{d^2}\right)}{(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, 1 + (
1 + m)/2, (e^2*x^2)/d^2])/((1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^4x^4 - 2\,d^2e^2x^2 + d^4\right)\sqrt{-e^2x^2 + d^2}\,\left(gx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}}\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}}\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}}\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2 - e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m, x)
```

sympy [C] time = 10.07, size = 61, normalized size = 0.76

$$\frac{d^5g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**5*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

3.230 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=163

$$\frac{d^3\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^3*(g*x)^(1+m)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)-d^2*e*(g*x)^(2+m)*hypergeom([-
3/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*
x^2/d^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {892, 82, 126, 365, 364}

$$\frac{d^3\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^3*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (d^2*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}\int (gx)^m (d-ex)^{5/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= \frac{\left(d\sqrt{d^2-e^2x^2}\right)\int (gx)^m (d-ex)^{3/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left(e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}(d-ex)^{3/2}}{g\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= d\int (gx)^m \left(d^2-e^2x^2\right)^{3/2}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}\,dx}{g}$$

$$= \frac{\left(d^3\sqrt{d^2-e^2x^2}\right)\int (gx)^m \left(1-\frac{e^2x^2}{d^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{\left(d^2e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}\,d}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$= \frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{3}{2},\frac{2+}{2}\right.}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 0.75

$$\frac{d^2x\sqrt{d^2-e^2x^2}\,(gx)^m\left(d(m+2)\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\,{}_2F_1\left(-\frac{3}{2},\frac{m}{2}+1;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^2*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(-(e*(1 + m)*x*Hypergeometric2F1[-3/2, 1
 + m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[-3/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^3x^3-de^2x^2-d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2)*(g*x)^m
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(g\,x\right)^m}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x),x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x), x)
```

sympy [C] time = 25.46, size = 248, normalized size = 1.52

$$\frac{d^4g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{d^3eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^2e^2g^mx^3x^m\Gamma\left(\frac{m}{2}+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**4*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**3*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 2)) - d**2*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*h
yper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*
gamma(m/2 + 5/2)) + d*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 +
 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3))
```

3.231 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{2de\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^2(2m+5)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{(d^2-$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(3/2)/g/(4+m)+d^2*(5+2*m)*(g*x)^(1+m)*hypergeom
([-1/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(4
+m)/(1-e^2*x^2/d^2)^(1/2)-2*d*e*(g*x)^(2+m)*hypergeom([-1/2, 1+1/2*m],[2+1/
2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{2de\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^2(2m+5)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{(d^2-$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(3/2))/(g*(4 + m))) + (d^2*(5 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(4 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (2*d*e*(g*x)^
(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int (gx)^m (d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$
$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\int (gx)^m\left(-d^2e^2(5+2m)+2de^3(4+m)x\right)\sqrt{d^2-e^2x^2}\,d}{e^2(4+m)}$$
$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{(2de)\int (gx)^{1+m}\sqrt{d^2-e^2x^2}\,dx}{g} + \frac{\left(d^2(5+2m)\right)\int (gx)^{m}}{4+m}$$
$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\left(2de\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\left(d^2(5+2m\right.}{}$$
$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} + \frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right.}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] time = 0.11, size = 173, normalized size = 0.85

$$\frac{x\sqrt{d^2-e^2x^2}\,(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(2d(m+3)\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right.\right.\right.}{(m+1)(m+2)(m+3)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-1/2,
 (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeome
tric2F1[-1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome
tric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 +
 m)*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^2-2\,dex+d^2\right)\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(g\,x\right)^m}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2, x)
```

sympy [C] time = 91.47, size = 185, normalized size = 0.91

$$\frac{d^3g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{d^2eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{de^2g^mx^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)\cdots}{2\Gamma\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
[Out] d**3*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**2*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/gamma(m/2 + 2) + d*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper((-
1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m
/2 + 5/2))
```

3.232 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{d^2e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+3)\sqrt{d^2-e^2x^2}}+\frac{e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}}{g^2(m+3)}-\frac{3d\sqrt{d^2-e^2x^2}\,(gx)^{m+1}}{g(m+2)}+\frac{d^3(4m$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(1/2)/g/(2+m)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^
(1/2)/g^2/(3+m)+d^3*(5+4*m)*(g*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2
*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/g/(1+m)/(2+m)/(-e^2*x^2+d^2)^(1/2)-d
^2*e*(11+4*m)*(g*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(
1-e^2*x^2/d^2)^(1/2)/g^2/(2+m)/(3+m)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{d^2e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+3)\sqrt{d^2-e^2x^2}}+\frac{e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}}{g^2(m+3)}+\frac{d^3(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}}{g(m+1)(m+2)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2])/(g*(2 + m)) + (e*(g*x)^(2 + m)*Sqr
t[d^2 - e^2*x^2])/(g^2*(3 + m)) + (d^3*(5 + 4*m)*(g*x)^(1 + m)*Sqrt[1 - (e^
2*x^2)/d^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(g
*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2]) - (d^2*e*(11 + 4*m)*(g*x)^(2 + m)*Sqr
t[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)
/d^2])/(g^2*(2 + m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \int \frac{(gx)^m(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} - \frac{\int \frac{(gx)^m\left(-d^3e^2(3+m)+d^2e^3(11+4m)x-3de^4(3+m)x^2\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2(3+m)}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\int \frac{(gx)^m\left(d^3e^4(3+m)(5+4m)-d^2e^5(2+m)(11+\cdots\right.}{\sqrt{d^2-e^2x^2}}}{e^4(2+m)(3+m)}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\left(d^3(5+4m)\right)\int \frac{(gx)^m}{\sqrt{d^2-e^2x^2}}\,dx}{2+m} - \frac{\left(d^2\cdots\right.}{\cdots}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\left(d^3(5+4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\sqrt{1-\frac{e^2x^2}{d^2}}}\cdots}{(2+m)\sqrt{d^2-e^2x^2}}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\cdots\right.}{g(1+m)(2+m)\sqrt{d^2-\cdots}}$$

Mathematica [A] time = 0.18, size = 245, normalized size = 0.98

$$\frac{x\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d^3\left(m^3+9m^2+26m+24\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(3d^2\left(m^2+7m+12\cdots\right.\right.\right.}{(m+1)(m+2)(m+3)(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*(d^3*(24 + 26*m + 9*
m^2 + m^3)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*
(1 + m)*x*(3*d^2*(12 + 7*m + m^2)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2] + e*(2 + m)*x*(-3*d*(4 + m)*Hypergeometric2F1[1/2, (3 +
m)/2, (5 + m)/2, (e^2*x^2)/d^2] + e*(3 + m)*x*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, (e^2*x^2)/d^2]))))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(d - e*
x)*(d + e*x))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2-2\,dex+d^2\right)\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e*x + d),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^{\frac{5}{2}} \left(gx\right)^m}{\left(ex + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^{\frac{5}{2}} \left(gx\right)^m}{\left(ex + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^{\frac{5}{2}} \left(gx\right)^m}{\left(ex + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}\,\left(g\,x\right)^m}{\left(d + e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.233 $\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=213

$$\frac{4(d+ex)(gx)^{m+1}}{5g\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\ldots\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/5*(g*x)^(1+m)*(e*x+d)/g/(-e^2*x^2+d^2)^(5/2)+1/5*(1-4*m)*(g*x)^(1+m)*hype
rgeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^3/g
/(1+m)/(-e^2*x^2+d^2)^(1/2)+1/5*e*(7-4*m)*(g*x)^(2+m)*hypergeom([5/2, 1+1/2
*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4/g^2/(2+m)/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1806, 808, 365, 364}

$$\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}+\frac{4(d+ex)\ldots}{5g\left(d^2-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d + e*x))/(5*g*(d^2 - e^2*x^2)^(5/2)) + ((1 - 4*m)*(g*x)^
(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2
, (e^2*x^2)/d^2])/(5*d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(7 - 4*m)*(g*x
)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^3(1-4m)-d^2e(7-4m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{5}(d(1-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx + \frac{(e(7-4m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5g}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left((1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\left(e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int}{5d^4g\sqrt{d^2-e^2x^2}}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(7-4m)(gx}{}$$

Mathematica [A] time = 0.19, size = 199, normalized size = 0.93

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\left(ex\left(\frac{3d^2\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(\frac{7}{2},\frac{m+3}{2};\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}+\frac{ex\,{}_2F_1\left(\frac{7}{2},\frac{m+4}{2};\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)\right)+\frac{d^3\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+}{2}\right.}{m+1}\right.}{d^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[7/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[7/2, (2
+ m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[7/
2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[7
/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*Sqrt[d^2 - e^2*x
^2])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^5x^5-3\,de^4x^4+2\,d^2e^3x^3+2\,d^3e^2x^2-3\,d^4ex+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^5*x^5 - 3*d*e^4*x^4 + 2*d^2*e^3*x^
3 + 2*d^3*e^2*x^2 - 3*d^4*e*x + d^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\,(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((g*x)**m*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.234 $\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=216

$$\frac{2(d+ex)(gx)^{m+1}}{5dg\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+}{2}\right.}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*(g*x)^(1+m)*(e*x+d)/d/g/(-e^2*x^2+d^2)^(5/2)+1/5*(3-2*m)*(g*x)^(1+m)*hy
pergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4
/g/(1+m)/(-e^2*x^2+d^2)^(1/2)+2/5*e*(3-m)*(g*x)^(2+m)*hypergeom([5/2, 1+1/2
*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^5/g^2/(2+m)/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1806, 808, 365, 364}

$$\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}+\frac{2(d+}{5dg\,(}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d + e*x))/(5*d*g*(d^2 - e^2*x^2)^(5/2)) + ((3 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (2*e*(3 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^2(3-2m)-2de(3-m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2e(3-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5dg} - \frac{1}{5}(-3+2m)\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left(2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^5g\sqrt{d^2-e^2x^2}} - \frac{\left((-3+2m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{}{\left(1\right.}}{5d^4\sqrt{d^2-e^2x^2}}$$

$$= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}} + \frac{2e(3-m)(gx)^{2}}{5d}$$

Mathematica [A] time = 0.11, size = 174, normalized size = 0.81

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)+e(m\right.\right.}{d^6(m+1)(m+2)(m+3)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[7
/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperge
ometric2F1[7/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeo
metric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^6*(1 + m)*(2 + m)
*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m}{e^6x^6-2\,de^5x^5-d^2e^4x^4+4\,d^3e^3x^3-d^4e^2x^2-2\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^6*x^6 - 2*d*e^5*x^5 - d^2*e^4*x^4
+ 4*d^3*e^3*x^3 - d^4*e^2*x^2 - 2*d^5*e*x + d^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(gx\right)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((g*x)**m*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.235 $\int \frac{(gx)^m(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=124

$$\frac{e(gx)^{m+2}\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^2g^2(m+2)\left(d^2-e^2x^2\right)^{5/2}}+\frac{(gx)^{m+1}\,{}_2F_1\left(1,\frac{m-4}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([1, -2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d/g/(1+m)/(-e^
2*x^2+d^2)^(5/2)+e*(g*x)^(2+m)*hypergeom([1, -3/2+1/2*m],[2+1/2*m],e^2*x^2/
d^2)/d^2/g^2/(2+m)/(-e^2*x^2+d^2)^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {808, 365, 364}

$$\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^6g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^5*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^6*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = d\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx + \frac{e\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{g}$$
$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^5\sqrt{d^2-e^2x^2}} + \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6g\sqrt{d^2-e^2x^2}}$$
$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\left(d(m+2)\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)\right)}{d^6(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d*(2 + m)*Hypergeometric2F1[7/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[7/2, (2 + m)
/2, (4 + m)/2, (e^2*x^2)/d^2]))/(d^6*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m}{e^7x^7-de^6x^6-3\,d^2e^5x^5+3\,d^3e^4x^4+3\,d^4e^3x^3-3\,d^5e^2x^2-d^6ex+d^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^7*x^7 - d*e^6*x^6 - 3*d^2*e^5*x^5
+ 3*d^3*e^4*x^4 + 3*d^4*e^3*x^3 - 3*d^5*e^2*x^2 - d^6*e*x + d^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(gx\right)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(gx\right)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,(d+e\,x)}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [C] time = 63.25, size = 117, normalized size = 0.94

$$\frac{g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{7}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^6\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{eg^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}\frac{7}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
[Out] g**m*x*x**m*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**6*gamma(m/2 + 3/2)) + e*g**m*x**2*x**m*gamm
a(m/2 + 1)*hyper((7/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/(2*d**7*gamma(m/2 + 2))
```

3.236 $\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([7/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/
d^2)^(1/2)/d^6/g/(1+m)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {365, 364}

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^6*g*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6\sqrt{d^2-e^2x^2}}\\ &= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+1}{2}+1;\frac{e^2x^2}{d^2}\right)}{d^6(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (1
 + m)/2, (e^2*x^2)/d^2])/(d^6*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^8x^8-4\,d^2e^6x^6+6\,d^4e^4x^4-4\,d^6e^2x^2+d^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^8*x^8 - 4*d^2*e^6*x^6 + 6*d^4*e^4*
x^4 - 4*d^6*e^2*x^2 + d^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [C] time = 11.40, size = 60, normalized size = 0.75

$$\frac{g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix}\frac{7}{2}, \frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] g**m*x*x**m*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**7*gamma(m/2 + 3/2))
```

3.237 $\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/
d^2)^(1/2)/d^7/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-e*(g*x)^(2+m)*hypergeom([9/2, 1
+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^8/g^2/(2+m)/(-e^2*x^
2+d^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {892, 82, 126, 365, 364}

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\left(\sqrt{d-ex}\,\sqrt{d+ex}\right)\int \frac{(gx)^m}{(d-ex)^{7/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}}$$

$$= \frac{\left(d\sqrt{d-ex}\,\sqrt{d+ex}\right)\int \frac{(gx)^m}{(d-ex)^{9/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}} - \frac{\left(e\sqrt{d-ex}\,\sqrt{d+ex}\right)\int \frac{(gx)^{1+m}}{(d-ex)^{9/2}(d+ex)^{9}}}{g\sqrt{d^2-e^2x^2}}$$

$$= d\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}\,dx - \frac{e\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{g}$$

$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^7\sqrt{d^2-e^2x^2}} - \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^8g\sqrt{d^2-e^2x^2}}$$

$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{9}{2},\frac{2+m}{2};\frac{4+}{2}\right.}{d^8g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 0.75

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\left(d(m+2)\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\,{}_2F_1\left(\frac{9}{2},\frac{m}{2}+1;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right)\right)}{d^8(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(-(e*(1 + m)*x*Hypergeometric2F1[9/2, 1
+ m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[9/2, (1 + m)/
2, (3 + m)/2, (e^2*x^2)/d^2]))/(d^8*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,\left(gx\right)^m}{e^9x^9+de^8x^8-4\,d^2e^7x^7-4\,d^3e^6x^6+6\,d^4e^5x^5+6\,d^5e^4x^4-4\,d^6e^3x^3-4\,d^7e^2x^2+d^8ex+d^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^9*x^9 + d*e^8*x^8 - 4*d^2*e^7*x^7
- 4*d^3*e^6*x^6 + 6*d^4*e^5*x^5 + 6*d^5*e^4*x^4 - 4*d^6*e^3*x^3 - 4*d^7*e^2
*x^2 + d^8*e*x + d^9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(ex+d)\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m}{\left(d^2-e^2\,x^2\right)^{7/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((g*x)**m/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.238 $\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=217

$$\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+}{2}\right.}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/9*(g*x)^(1+m)*(-e*x+d)/d/g/(-e^2*x^2+d^2)^(9/2)+1/9*(7-2*m)*(g*x)^(1+m)*h
ypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^
8/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-2/9*e*(7-m)*(g*x)^(2+m)*hypergeom([9/2, 1+1/
2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^9/g^2/(2+m)/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {852, 1806, 808, 365, 364}

$$-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}+\frac{2(d}{9dg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d - e*x))/(9*d*g*(d^2 - e^2*x^2)^(9/2)) + ((7 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(9*d^8*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (2*e*(7 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(9*d^9*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m(d-ex)^2}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{(gx)^m\left(-d^2(7-2m)+2de(7-m)x\right)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9d^2}$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{(2e(7-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9dg} - \frac{1}{9}(-7+2m)\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\left(2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{9d^9g\sqrt{d^2-e^2x^2}} - \frac{\left((-7+2m)\sqrt{1-\frac{e^2}{d}}\right.}{9d^8\sqrt{d^2}}$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} + \frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}} - \frac{2e(7-}{}$$

Mathematica [A] time = 0.12, size = 176, normalized size = 0.81

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{11}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)-e(\right.\right.}{d^{10}(m+1)(m+2)(m+3)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[1
1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hyperg
eometric2F1[11/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyperg
eometric2F1[11/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^10*(1 + m)*(2
+ m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^{10}x^{10}+2\,de^9x^9-3\,d^2e^8x^8-8\,d^3e^7x^7+2\,d^4e^6x^6+12\,d^5e^5x^5+2\,d^6e^4x^4-8\,d^7e^3x^3-3\,d^8e^2x^2+2\,d^9ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^10*x^10 + 2*d*e^9*x^9 - 3*d^2*e^8*
x^8 - 8*d^3*e^7*x^7 + 2*d^4*e^6*x^6 + 12*d^5*e^5*x^5 + 2*d^6*e^4*x^4 - 8*d^
7*e^3*x^3 - 3*d^8*e^2*x^2 + 2*d^9*e*x + d^10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(ex+d)^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m}{\left(d^2-e^2\,x^2\right)^{7/2}\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2),x)
```

```
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.239 $\int \frac{(gx)^m}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=214

$$\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{11}{2},\frac{m+}{2}\right.}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/11*(g*x)^(1+m)*(-e*x+d)/g/(-e^2*x^2+d^2)^(11/2)+1/11*(7-4*m)*(g*x)^(1+m)*
hypergeom([11/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/
d^9/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-1/11*e*(25-4*m)*(g*x)^(2+m)*hypergeom([11/
2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^10/g^2/(2+m)/(-e
^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1806, 808, 365, 364}

$$-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{11}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}+\frac{4(d}{11g}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d - e*x))/(11*g*(d^2 - e^2*x^2)^(11/2)) + ((7 - 4*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1 + m)/2, (3 + m
)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(25 - 4*m)
*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2 + m)/2, (
4 + m)/2, (e^2*x^2)/d^2])/(11*d^10*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m(d-ex)^3}{\left(d^2-e^2x^2\right)^{13/2}}\,dx$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} - \frac{\int \frac{(gx)^m\left(-d^3(7-4m)+d^2e(25-4m)x\right)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx}{11d^2}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{1}{11}(d(7-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{11/2}}\,dx - \frac{(e(25-4m))\int \frac{}{(d^2}}{11g}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{\left((7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}\,dx}{11d^9\sqrt{d^2-e^2x^2}} - \frac{\left(e(25-4m)\sqrt{1}\right.}{11d^{10}}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{11}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(2}{}$$

Mathematica [A] time = 0.21, size = 200, normalized size = 0.93

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\left(ex\left(ex\left(\frac{3d\,{}_2F_1\left(\frac{13}{2},\frac{m+3}{2};\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}-\frac{ex\,{}_2F_1\left(\frac{13}{2},\frac{m+4}{2};\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)-\frac{3d^2\,{}_2F_1\left(\frac{13}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}\right)+\frac{d^3\,{}_2F_1\left(\frac{13}{2},\frac{m+1}{2}\right.}{m+1}\right.}{d^{12}\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[13/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[13/2,
(2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1
[13/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2
F1[13/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^12*Sqrt[d^2 -
 e^2*x^2])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^{11}x^{11}+3\,de^{10}x^{10}-d^2e^9x^9-11\,d^3e^8x^8-6\,d^4e^7x^7+14\,d^5e^6x^6+14\,d^6e^5x^5-6\,d^7e^4x^4-11\,d^8e^3x^3-d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^11*x^11 + 3*d*e^10*x^10 - d^2*e^9*
x^9 - 11*d^3*e^8*x^8 - 6*d^4*e^7*x^7 + 14*d^5*e^6*x^6 + 14*d^6*e^5*x^5 - 6*
d^7*e^4*x^4 - 11*d^8*e^3*x^3 - d^9*e^2*x^2 + 3*d^10*e*x + d^11), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(ex + d)^3 (-e^2x^2 + d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m}{(d^2 - e^2\,x^2)^{7/2}\,(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3),x)
```

```
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.240 $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=148

$$\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+3}}{2e^6(p+3)}-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{2e^6(p+1)}+\frac{d^3\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}$$

```
[Out] -1/2*d^5*(-e^2*x^2+d^2)^(1+p)/e^6/(1+p)+d^3*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-
1/2*d*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/7*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([
7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 43, 365, 364}

$$\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{2e^6(p+1)}+\frac{d^3\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}-\frac{d\left(d^2-e^2x^2\right)^{p+3}}{2e^6(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^5*(d^2 - e^2*x^2)^(1 + p))/(2*e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2 +
p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^
2*x^2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^5\left(d^2-e^2x^2\right)^p\,dx + e\int x^6\left(d^2-e^2x^2\right)^p\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^6\left(1-\frac{e}{\cdot}\right.$$

$$= \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}d\,\text{Subst}\left(\int\left(\frac{d^4\left(d^2-e^2x\right.}{e^4}\right.\right.$$

$$= -\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{2e^6(1+p)} + \frac{d^3\left(d^2-e^2x^2\right)^{2+p}}{e^6(2+p)} - \frac{d\left(d^2-e^2x^2\right)^{3+p}}{2e^6(3+p)} + \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1\right.$$

Mathematica [A] time = 0.09, size = 132, normalized size = 0.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{7d\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(p+1)x^2+e^4\left(p^2+3p+2\right)x^4\right)}{(p+1)(p+2)(p+3)}\right)}{14e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-7*d*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 +
e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (2*e^7*x^7*Hypergeome
tric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^6+dx^5\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^6 + d*x^5)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)\left(-e^2x^2+d^2\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^5, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int(ex+d)\,x^5\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int x^{6}e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx+\frac{\left(\left(p^{2}+3\,p+2\right)e^{6}x^{6}-\left(p^{2}+p\right)d^{2}e^{4}x^{4}-2\,d^{4}e^{2}px^{2}-2\,d^{6}\right)\left(-e^{2}x^{2}+d^{2}\right)^{p}d}{2\left(p^{3}+6\,p^{2}+11\,p+6\right)e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^6*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*((p^2 + 3*p
+ 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*d^6)*(-e^2*x^2 +
 d^2)^p*d/((p^3 + 6*p^2 + 11*p + 6)*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5}\left(d^{2}-e^{2}x^{2}\right)^{p}\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

sympy [B] time = 7.46, size = 972, normalized size = 6.57

$$d\left(\begin{cases}\frac{x^{6}\left(d^{2}\right)^{p}}{6} \\ -\frac{2d^{4}\log\left(-\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}-\frac{2d^{4}\log\left(\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}-\frac{3d^{4}}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}+\frac{4d^{2}e^{2}x^{2}\log\left(-\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}+\frac{4d^{2}e^{2}x^{2}\log\left(\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}+ \\ -\frac{2d^{4}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}-\frac{2d^{4}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}-\frac{2d^{4}}{-2d^{2}e^{6}+2e^{8}x^{2}}+\frac{2d^{2}e^{2}x^{2}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}+\frac{2d^{2}e^{2}x^{2}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}+\frac{e^{4}x^{4}}{-2d^{2}e^{6}+2e^{8}x^{2}} \\ -\frac{d^{4}\log\left(-\frac{d}{e}+x\right)}{2e^{6}}-\frac{d^{4}\log\left(\frac{d}{e}+x\right)}{2e^{6}}-\frac{d^{2}x^{2}}{2e^{4}}-\frac{x^{4}}{4e^{2}} \\ -\frac{2d^{6}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}-\frac{2d^{4}e^{2}px^{2}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}-\frac{d^{2}e^{4}p^{2}x^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}-\frac{d^{2}e^{4}px^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}+\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 +
 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e*
*8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d*
*2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e
**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e*
*4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e
**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(
p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)
```

```
**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(
d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e
**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
 + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p*
*2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e*x**7*hyper((7/2,
 -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

3.241 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=147

$$\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{d^2\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^5(p+3)}-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{2e^5(p+1)}$$

```
[Out] -1/2*d^4*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+d^2*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-
1/2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+1/5*d*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/
2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 365, 364, 266, 43}

$$\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{2e^5(p+1)}+\frac{d^2\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^5(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^4*(d^2 - e^2*x^2)^(1 + p))/(2*e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2 +
p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^
2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^4\left(d^2-e^2x^2\right)^p\,dx + e\int x^5\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e}{}\right.$$
$$= \frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{d^4\left(d^2-e^2x\right.}{e^4}\right.\right.$$
$$= -\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{2e^5(1+p)} + \frac{d^2\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)} - \frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^5(3+p)} + \frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\right.$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.88

$$\frac{1}{10}\left(d^2-e^2x^2\right)^p\left(2dx^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) - \frac{5\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(p+1)x^2+e^4\left(p^2+3p+2\right)\right.}{e^5(p+1)(p+2)(p+3)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 + e^
4*(2 + 3*p + p^2)*x^4))/(e^5*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeome
tric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/10
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^5+dx^4\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)\,x^4\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)\left(-e^2x^2 + d^2\right)^p x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(d^2 - e^2\, x^2\right)^p\, (d + e\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

sympy [B] time = 7.10, size = 972, normalized size = 6.61

$$\frac{dd^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}+e\left(\begin{cases}\frac{x^6\left(d^2\right)^p}{6} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}\cdots} \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\cdots \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2} \\ -\frac{2d^6\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^{\cdots}}{2e^6p^3+1\cdots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
5 + e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**
4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e
**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**
8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**
8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-
2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)
+ e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/
(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
 Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 2
2*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x
**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x*
*4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
```

```
 + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

3.242 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=120

$$\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

```
[Out] -1/2*d^3*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)+1/2*d*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p
)+1/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2
*x^2/d^2)^p)
```

Rubi [A] time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 43, 365, 364}

$$\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^3*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 + p)
)/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^3\left(d^2-e^2x^2\right)^p\,dx + e\int x^4\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2}{d}\right.$$
$$= \frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}d\,\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right.}{e^2}\right.\right.$$
$$= -\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)} + \frac{d\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)} + \frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},\right.$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.88

$$\frac{\left(d^2-e^2x^2\right)^p\left(2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{5d\left(d^2-e^2x^2\right)\left(d^2+e^2(p+1)x^2\right)}{(p+1)(p+2)}\right)}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/((1 + p)
*(2 + p)) + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/(10*e^4)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^4+dx^3\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)\left(-e^2x^2+d^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int(ex+d)\,x^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int x^4 e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx + \frac{\left(e^4(p+1)x^4 - d^2e^2px^2 - d^4\right)\left(-e^2x^2+d^2\right)^p d}{2\left(p^2+3p+2\right)e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^4*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*(e^4*(p + 1)
*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d/((p^2 + 3*p + 2)*e^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(d^2-e^2x^2\right)^p\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

sympy [B] time = 5.46, size = 382, normalized size = 3.18

$$d\left(\begin{cases}\frac{x^4\left(d^2\right)^p}{4} & \text{for}\ e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for}\ p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for}\ p=-1\\ -\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}ex^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\ \frac{e}{}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-
2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (
-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2),
Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
- d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
+ e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**
4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) + d*
*(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

3.243 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=119

$$\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^3(p+2)}$$

```
[Out] -1/2*d^2*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)+1/2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)+
1/3*d*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x
^2/d^2)^p)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 365, 364, 266, 43}

$$\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^3(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e
^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^2\left(d^2-e^2x^2\right)^p\,dx + e\int x^3\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}e\,\mathrm{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\right.$$
$$= \frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}e\,\mathrm{Subst}\left(\int\left(\frac{d^2\left(d^2-\right.}{e^2}\right.\right.$$
$$= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)} + \frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)} + \frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2}\right.$$

Mathematica [A] time = 0.07, size = 103, normalized size = 0.87

$$\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(2dx^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{3\left(d^2-e^2x^2\right)\left(d^2+e^2(p+1)x^2\right)}{e^3(p+1)(p+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/(e^3*(1 +
p)*(2 + p)) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/6
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^3+dx^2\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex+d)\,x^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(d^2-e^2x^2\right)^p(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

sympy [B] time = 5.07, size = 382, normalized size = 3.21

$$\frac{dd^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+e\left(\begin{cases}\frac{x^4\left(d^2\right)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
3 + e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**
2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*
e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)
), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**
2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
 e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True))
```

3.244 $\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=89

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/e^2/(1+p)+1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom(
[3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {764, 261, 365, 364}

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x\left(d^2-e^2x^2\right)^p\,dx + e\int x^2\left(d^2-e^2x^2\right)^p\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)} + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)} + \frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.00

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right) - \frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+dx\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x^2 + d*x)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex+d)\,x\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int x^2e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx - \frac{\left(-e^2x^2+d^2\right)^{p+1}d}{2\,e^2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^2*e^(p*log(e*x + d) + p*log(-e*x + d)), x) - 1/2*(-e^2*x^2 +
d^2)^(p + 1)*d/(e^2*(p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

sympy [A] time = 3.52, size = 85, normalized size = 0.96

$$d\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2
)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tr
ue)) + d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)
/d**2)/3
```

3.245 $\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=83

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(1+p)/e/(1+p)+d*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],
[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {641, 246, 245}

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+d\int \left(d^2-e^2x^2\right)^p\,dx\\ &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+\left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\ &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 1.00

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left((ex+d)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

mupad [B] time = 4.35, size = 78, normalized size = 0.94

$$\frac{d\,x\,\left(d^2-e^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^p}-\frac{\left(d^2-e^2\,x^2\right)^{p+1}}{2\,e\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] (d*x*(d^2 - e^2*x^2)^p*hypergeom([1/2, -p], 3/2, (e^2*x^2)/d^2))/(1 - (e^2*
x^2)/d^2)^p - (d^2 - e^2*x^2)^(p + 1)/(2*e*(p + 1))
```

sympy [A] time = 4.18, size = 82, normalized size = 0.99

$$dd^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right) + e\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1\\ \log\left(d^2 - e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e
*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)
**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tru
e))
```

3.246 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx$

Optimal. Leaf size=104

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

```
[Out] e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2
)^p)-1/2*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/d/(1+
p)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 65, 246, 245}

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx + e\int \left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,d$$
$$= ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+\right.}{2d(1+p)}$$

Mathematica [A] time = 0.04, size = 104, normalized size = 1.00

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x, x)
```

sympy [C] time = 9.37, size = 78, normalized size = 0.75

$$-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+d^{2p}ex{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p)) + d**(2*p)*e*x*hyper((1/2, -p), (3/2,), e**2*x
**2*exp_polar(2*I*pi)/d**2)
```

3.247 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

```
[Out] -d*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d
^2)^p)-1/2*e*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/d
^2/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 365, 364, 266, 65}

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx + e\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx$$
$$= \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{p}}{x^2}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1,1+p\right.}{2d^2(1+p)}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 1.00

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2, x)
```

sympy [C] time = 5.53, size = 82, normalized size = 0.76

$$-\frac{dd^{2p}{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{ee^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x -
 e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p))
```

3.248 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=110

$$-\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

```
[Out] -e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d
^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^(1+p)*hypergeom([2, 1+p],[2+p],1-e^2*x^2/d^2)
/d^3/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 65, 365, 364}

$$-\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -((e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e^2*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x^3}\,dx + e\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}\,d$$
$$= -\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2\left(d^2-e^2x^2\right)^{1+p} {}_2F_1\left(2,1+p;2\right.}{2d^3(1+p)}$$

Mathematica [A] time = 0.05, size = 106, normalized size = 0.96

$$\frac{1}{2}e\left(d^2-e^2x^2\right)^p\left(\frac{e\left(e^2x^2-d^2\right) {}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d^3(p+1)} - \frac{2\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/
(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*Hypergeometric2F1[2, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2])/(d^3*(1 + p))))/2
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3, x)
```

sympy [C] time = 5.07, size = 85, normalized size = 0.77

$$-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma\left(2-p\right)}-\frac{d^{2p}e_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,),
d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - d**(2*p)*e*hyper((-1/2, -p), (1/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x
```

3.249 $\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=178

$$\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{2d^2\left(d^2-e^2x^2\right)^{p+3}}{e^6(p+3)}+\frac{\left(d^2-e^2x^2\right)^{p+4}}{2e^6(p+4)}-\frac{d^6\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{5d}{}$$

```
[Out] -d^6*(-e^2*x^2+d^2)^(1+p)/e^6/(1+p)+5/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-
2*d^2*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/2*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7
*d*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^
2/d^2)^p)
```

Rubi [A] time = 0.15, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1652, 446, 77, 12, 365, 364}

$$\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^6\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{5d^4\left(d^2-e^2x^2\right)^{p+2}}{2e^6(p+2)}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+3}}{e^6(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^6*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (5*d^4*(d^2 - e^2*x^2)^(2
+ p))/(2*e^6*(2 + p)) - (2*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d^
2 - e^2*x^2)^(4 + p)/(2*e^6*(4 + p)) + (2*d*e*x^7*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \int 2dex^6\left(d^2-e^2x^2\right)^p\,dx + \int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)dx, x, x^2\right) + (2de)\int x^6\left(d^2-e^2x^2\right)^p\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{2d^6\left(d^2-e^2x\right)^p}{e^4} - \frac{5d^4\left(d^2-e^2x\right)^{1+p}}{e^4} + \frac{4d^2\left(d^2-e^2x\right)^{2+p}}{e^4} - \frac{(d^2 \cdots}{\cdots}\right.\right.$$

$$= -\frac{d^6\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)} + \frac{5d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)} - \frac{2d^2\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)} + \frac{\left(d^2-e^2x^2\right)^{4 \cdots}}{2e^6(4+p)\cdots}$$

Mathematica [A] time = 0.13, size = 159, normalized size = 0.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{7\left(d^2-e^2x^2\right)^4}{p+4} - \frac{28d^2\left(d^2-e^2x^2\right)^3}{p+3} + 4de^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right) - \frac{14d^6\left(d^2-e^2x^2\right)}{p+1} + \frac{35d^4\left(d^2-e^2x^2\right)^2}{p+2}\right)}{14e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-14*d^6*(d^2 - e^2*x^2))/(1 + p) + (35*d^4*(d^2 - e^2*
x^2)^2)/(2 + p) - (28*d^2*(d^2 - e^2*x^2)^3)/(3 + p) + (7*(d^2 - e^2*x^2)^4
)/(4 + p) + (4*d*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1
 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^7 + 2\,dex^6 + d^2x^5\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^5, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^5 \left(-e^2x^2+d^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(p^2+3\,p+2\right)e^6x^6-\left(p^2+p\right)d^2e^4x^4-2\,d^4e^2px^2-2\,d^6\right)\left(-e^2x^2+d^2\right)^p d^2}{2\left(p^3+6\,p^2+11\,p+6\right)e^6}+\int \left(e^2x^7+2\,dex^6\right)e^{\left(p\log(ex+d)+p\log(-ex+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*
d^6)*(-e^2*x^2 + d^2)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e^2
*x^7 + 2*d*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(d^2-e^2\,x^2\right)^p (d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

sympy [B] time = 14.93, size = 2924, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2
*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2
*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) +
 e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(
2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x*
*2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**
```

```
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
+ e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**
6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6
*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p*
*3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 2*d*d**(2*p)*e*x**7*hype
r((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e**2*Piecewise((
x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-
12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 1
1*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*
x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2
 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-1
2*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 27
*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 +
12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e
**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e
 + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x
**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**1
2*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e +
x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6
), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*
e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**1
2*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4
*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) +
 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*
x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
- 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12
*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
, Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6
*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**1
0*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*
d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**
4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2),
Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d*
*4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8
*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**
8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20
*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(
d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*
p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e
**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2
 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p +
48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e*
*8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 -
 e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48
*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 +
 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)
**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11
*e**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p*
*2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4
 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True))
```

3.250 $\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=185

$$\frac{2d^2(p+6)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{d\left(d^2-e^2x^2\right)^{p+3}}{e^5(p+3)}-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}$$

```
[Out] -d^5*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)-x^5*(-e^2*x^2+d^2)^(1+p)/(7+2*p)+2*d^3*
(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-d*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+2/5*d^2*(6+
p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-
e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$\frac{2d^2(p+6)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{2d^3\left(d^2-e^2x^2\right)^p}{e^5(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p))) - (x^5*(d^2 - e^2*x^2)^(1 +
p))/(7 + 2*p) + (2*d^3*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d*(d^2 - e
^2*x^2)^(3 + p))/(e^5*(3 + p)) + (2*d^2*(6 + p)*x^5*(d^2 - e^2*x^2)^p*Hyper
geometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)
^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int 2dex^5\left(d^2-e^2x^2\right)^p\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$=-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+(2de)\int x^5\left(d^2-e^2x^2\right)^p\,dx+\frac{\left(2d^2(6+p)\right)\int x^4\left(d^2-e^2\right.}{7+2p}$$

$$=-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+(de)\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\frac{\left(2d^2(6+p)\left(\right.\right.}{}$$

$$=-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^2(6+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right.}{5(7+2p)}$$

$$=-\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^3\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)}-\frac{d\left(d^2-e^2x^2\right)^3}{e^5(3+p)}$$

Mathematica [A] time = 0.12, size = 186, normalized size = 1.01

$$\frac{1}{35}\left(d^2-e^2x^2\right)^p\left(5e^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+7d^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{35d\left(d^2\right.}{e^5(p}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d^5*(d^2 - e^2*x^2))/(e^5*(1 + p)) + (70*d^3*(d^2
- e^2*x^2)^2)/(e^5*(2 + p)) - (35*d*(d^2 - e^2*x^2)^3)/(e^5*(3 + p)) + (7*d
^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^
p + (5*e^2*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^
2)/d^2)^p))/35
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^6+2\,dex^5+d^2x^4\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^4\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
[Out] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(d^2-e^2x^2\right)^p(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

sympy [B] time = 8.90, size = 1015, normalized size = 5.49

$$\frac{d^2d^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}+2de\left(\begin{cases}\frac{x^6\left(d^2\right)^p}{6}\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}\\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2}\\ -\frac{2d^6\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e}{2e^6p^3+}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 2*d*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)
/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*
e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*
d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8
*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/
e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4
*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p
**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e*
*6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 -
 e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e*
*4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 1
2*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 +
 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2
*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e**2*x*
*7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

3.251 $\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=149

$$\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{3d^2\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^4(p+3)}-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}$$

```
[Out] -d^4*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)+3/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-
1/2*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2/5*d*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([
5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1652, 446, 77, 12, 365, 364}

$$\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{3d^2\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^4(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (3*d^2*(d^2 - e^2*x^2)^(2
+ p))/(2*e^4*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^4*(3 + p)) + (2*d*e*x^
5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 -
 (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^4\left(d^2-e^2x^2\right)^p\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^4\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{2d^4\left(d^2-e^2x\right)^p}{e^2}-\frac{3d^2\left(d^2-e^2x\right)^{1+p}}{e^2}+\frac{\left(d^2-e^2x\right)^{2+p}}{e^2}\right)dx,x,x^2\right.\\ &= -\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}+\frac{3d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^4(3+p)}+\frac{2}{5}dex^5\left(d^2-e^2x^2\right.\end{aligned}$$

Mathematica [A] time = 0.12, size = 138, normalized size = 0.93

$$\frac{\left(d^2-e^2x^2\right)^p\left(4de^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{5\left(d^2-e^2x^2\right)\left(d^4(p+5)+d^2e^2\left(p^2+6p+5\right)x^2+e^4\left(p^2+3p+2\right)x^4\right)}{(p+1)(p+2)(p+3)}\right)}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(d^4*(5 + p) + d^2*e^2*(5 + 6*p + p
^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (4*d*e^5*x^
5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(
10*e^4)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^5+2\,dex^4+d^2x^3\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^3 \left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^4(p+1)x^4 - d^2e^2px^2 - d^4\right)\left(-e^2x^2+d^2\right)^p d^2}{2\left(p^2+3p+2\right)e^4} + \int \left(e^2x^5 + 2\,dex^4\right)e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^2/((p^2 +
3*p + 2)*e^4) + integrate((e^2*x^5 + 2*d*e*x^4)*e^(p*log(e*x + d) + p*log(-
e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

sympy [B] time = 9.14, size = 1328, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 2*d*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d
**2)/5 + e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x
)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2
```

```
*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2
*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**
8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d
/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(
4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*
p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e
**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2
- e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e
**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(
2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

3.252 $\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=155

$$\frac{2d^2(p+4)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{e^3(p+2)}-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}$$

```
[Out] -d^3*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)-x^3*(-e^2*x^2+d^2)^(1+p)/(5+2*p)+d*(-e^
2*x^2+d^2)^(2+p)/e^3/(2+p)+2/3*d^2*(4+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/
2, -p],[5/2],e^2*x^2/d^2)/(5+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$\frac{2d^2(p+4)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{e^3(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p))) - (x^3*(d^2 - e^2*x^2)^(1 +
p))/(5 + 2*p) + (d*(d^2 - e^2*x^2)^(2 + p))/(e^3*(2 + p)) + (2*d^2*(4 + p)*
x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5
 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int 2dex^3\left(d^2-e^2x^2\right)^p\,dx+\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx$$

$$=-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+(2de)\int x^3\left(d^2-e^2x^2\right)^p\,dx+\frac{\left(2d^2(4+p)\right)\int x^2\left(d^2-e^2\right.}{5+2p}$$

$$=-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+(de)\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\frac{\left(2d^2(4+p)\left(d^2\right.\right.}{}$$

$$=-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{2d^2(4+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right.}{3(5+2p)}$$

$$=-\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)}-\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{d\left(d^2-e^2x^2\right)^{2+p}}{e^3(2+p)}+\frac{2d^2(4+p)x^3\left(d^2\right.}{}$$

Mathematica [A] time = 0.11, size = 168, normalized size = 1.08

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-15d\left(d^2-e^2x^2\right)\left(d^2+e^2(p+1)x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p+3e^5\left(p^2+3p+2\right)x^5{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e}{}\right.\right.}{15e^3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-15*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(d^2 + e^2*
(1 + p)*x^2) + 5*d^2*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2] + 3*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2]))/(15*e^3*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^4+2\,dex^3+d^2x^2\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^2 \left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
[Out] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(d^2-e^2\,x^2\right)^p (d+e\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

sympy [B] time = 6.43, size = 425, normalized size = 2.74

$$\frac{d^2 d^{2p} x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + 2de \begin{cases} \frac{x^4\left(d^2\right)^p}{4} & \text{for } e \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p \\ -\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
[Out] d**2*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 2*d*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2
```

```
) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e*
*4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq
(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2
/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
True)) + d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*
I*pi)/d**2)/5
```

3.253 $\int x(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=118

$$\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^2(p+2)}$$

```
[Out] -d^2*(-e^2*x^2+d^2)^(1+p)/e^2/(1+p)+1/2*(-e^2*x^2+d^2)^(2+p)/e^2/(2+p)+2/3*
d*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2
/d^2)^p)
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1652, 444, 43, 12, 365, 364}

$$\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p))) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^2*(2 + p)) + (2*d*e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^2\left(d^2-e^2x^2\right)^p\,dx+\int x\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^2\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(2d^2\left(d^2-e^2x\right)^p-\left(d^2-e^2x\right)^{1+p}\right)dx,x,x^2\right)+\left(2de\left(d^2-e^2x^2\right)^p\left(1\right.\right.\\ &= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^2(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^2(2+p)}+\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2}\right.\end{aligned}$$

Mathematica [A] time = 0.07, size = 110, normalized size = 0.93

$$\frac{\left(d^2-e^2x^2\right)^p\left(4de^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{3\left(d^2-e^2x^2\right)\left(d^2(p+3)+e^2(p+1)x^2\right)}{(p+1)(p+2)}\right)}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2*(3 + p) + e^2*(1 + p)*x^2))/((
1 + p)*(2 + p)) + (4*d*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^
2])/(1 - (e^2*x^2)/d^2)^p))/(6*e^2)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(e^2x^3+2\,dex^2+d^2x\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^2\left(-e^2x^2+d^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int(ex+d)^2\,x\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-e^2x^2+d^2\right)^{p+1}d^2}{2\,e^2\left(p+1\right)}+\int\left(e^2x^3+2\,dex^2\right)e^{\left(p\log\left(ex+d\right)+p\log\left(-ex+d\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(p + 1)*d^2/(e^2*(p + 1)) + integrate((e^2*x^3 + 2*d*
e*x^2)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

sympy [A] time = 5.92, size = 440, normalized size = 3.73

$$d^2\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for}\ e^2=0\\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for}\ p\neq-1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{2dd^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+e^2\left(\begin{cases}\frac{x^4\left(d^2\right)^p}{4}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6 \cdots}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}\cdots\\ -\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2\left(d \cdots\right.}{2e^4p^2+6e \cdots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + 2*d*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)/3 + e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-
d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e*
*4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e
**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4), True))
```

3.254 $\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=71

$$-\frac{d2^{p+2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -2^(2+p)*d*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, -2-p],[2+p
],1/2*(-e*x+d)/d)/e/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{d2^{p+2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*d*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \left(d(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{2+p}\,dx\\ &= -\frac{2^{2+p}d\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(-2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 134, normalized size = 1.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(3d^2e(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-3d\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p+e^3(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\right.\right.}{3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-3*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p + 3*d^2*e*(1
 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + e^3*(1 + p)*x^3*Hy
pergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2]))/(3*e*(1 + p)*(1 - (e^2*x^2)/
d^2)^p)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^2 + 2\,dex + d^2\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^2\left(-e^2x^2 + d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex + d)^2\left(-e^2x^2 + d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^2\left(-e^2x^2 + d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d^2 - e^2\,x^2\right)^p\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

sympy [A] time = 4.95, size = 124, normalized size = 1.75

$$d^2d^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+2de\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}e^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 2*d*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**
2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**
2), True)) + d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/3
```

3.255 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x} dx$

Optimal. Leaf size=128

$$2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2(p+1)}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(1+p)/(1+p)+2*d*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p
],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^(1+p)*hypergeom
([1, 1+p],[2+p],1-e^2*x^2/d^2)/(1+p)
```

Rubi [A] time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1652, 446, 80, 65, 12, 246, 245}

$$2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*(1 + p)) + (2*d*e*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
 p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x}\,dx &= \int 2de\left(d^2-e^2x^2\right)^p\,dx + \int \frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)}{x}\,dx, x, x^2\right) + (2de)\int \left(d^2-e^2x^2\right)^p\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} + \frac{1}{2}d^2\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(2de\left(d^2-e^2x^2\right)^p\left(1-\right.\right. \\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} + 2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 0.80

$$\frac{1}{2}\left(d^2-e^2x^2\right)^p\left(4dex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)\left({}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)+1\right)}{p+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)*(1 + Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2]))/(1 + p)))/2
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(-e^2x^2+d^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x, x)
```

sympy [A] time = 9.05, size = 136, normalized size = 1.06

$$-\frac{d^2e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+2dd^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e^2\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for } e \\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{othe}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 2*d*d**(2*p)*e*x*hyper((1/2, -p), (3/2,),
 e**2*x**2*exp_polar(2*I*pi)/d**2) + e**2*Piecewise((x**2*(d**2)**p/2, Eq(e
**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (lo
g(d**2 - e**2*x**2), True))/(2*e**2), True))
```

3.256 $\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=128

$$-2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{\left(d^2-e^2\right.}{x}$$

```
[Out] -(-e^2*x^2+d^2)^(1+p)/x-2*e^2*p*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2
],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p
],[2+p],1-e^2*x^2/d^2)/d/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1807, 764, 266, 65, 246, 245}

$$-2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{\left(d^2-e^2\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d^2 - e^2*x^2)^(1 + p)/x) - (2*e^2*p*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e*(d^2 - e^2*x^2)
^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x}-\frac{\int \frac{\left(-2d^3e+2d^2e^2px\right)\left(d^2-e^2x^2\right)^p}{x}\,dx}{d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x}+(2de)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx-\left(2e^2p\right)\int \left(d^2-e^2x^2\right)^p\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x}+(de)\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)-\left(2e^2p\left(d^2-e^2x^2\right)^p\left(1-\frac{e}{}\right.\right.$$
$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x}-2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e\left(d^2-e\right.}{}$$

Mathematica [A] time = 0.07, size = 153, normalized size = 1.20

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(de(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)\right.\right.}{d(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-(d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2
)/d^2]) + e*x*(d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
 - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 - (e^2*x^2)/d^2])))/(d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(-e^2x^2+d^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2 \left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2, x)
```

sympy [C] time = 6.89, size = 116, normalized size = 0.91

$$-\frac{d^2d^{2p}{}_2F_1\left(\begin{matrix} -\frac{1}{2},-p \\ \frac{1}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{dee^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix} -p,-p \\ 1-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{\Gamma\left(1-p\right)}+d^{2p}e^2x{}_2F_1\left(\begin{matrix} \frac{1}{2},-p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**2,x)
[Out] -d**2*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
x - d*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d
**2/(e**2*x**2))/gamma(1 - p) + d**(2*p)*e**2*x*hyper((1/2, -p), (3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)
```

3.257 $\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=139

$$-\frac{2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}-\frac{\left(d^2-\right.}{2}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(1+p)/x^2-2*d*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[
1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d^2)^p)-1/2*e^2*(1-p)*(-e^2*x^2+d^2)^(1+p)*
hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/d^2/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1807, 764, 365, 364, 266, 65}

$$-\frac{2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}-\frac{\left(d^2-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*x^2) - (2*d*e*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^
2])/(2*d^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^3}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{\int \frac{\left(-4d^3e-2d^2e^2(1-p)x\right)\left(d^2-e^2x^2\right)^p}{x^2}\,dx}{2d^2}\\ &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}+(2de)\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx+\left(e^2(1-p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx\\ &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}+\frac{1}{2}\left(e^2(1-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)+\left(2de\left(d^2-e^2\right.\right.\\ &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 131, normalized size = 0.94

$$\frac{e\left(d^2-e^2x^2\right)^p\left(\frac{e\left(e^2x^2-d^2\right)\left({}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)+{}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)\right)}{p+1}-\frac{4d^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-4*d^3*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*(Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e
^2*x^2)/d^2]))/(1 + p)))/(2*d^2)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(-e^2x^2+d^2\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2 \left(-e^2x^2+d^2\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3, x)
```

sympy [C] time = 6.92, size = 139, normalized size = 1.00

$$-\frac{d^2e^{2p}x^{2p}e^{i\pi p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma\left(2-p\right)}-\frac{2dd^{2p}e_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{e^2e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,
), d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - 2*d*d**(2*p)*e*hyper((-1/2, -p
), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - e**2*e**(2*p)*x**(2*p)*exp
(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gamma(1 -
 p))
```

3.258 $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=222

$$\frac{2d^2e(3p+17)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}+\frac{3d\left(d^2-e^2x^2\right)^{p+4}}{2e^6(p+4)}-\frac{2d^7\left(d^2-\right.}{e^6(p}$$

```
[Out] -2*d^7*(-e^2*x^2+d^2)^(1+p)/e^6/(1+p)-e*x^7*(-e^2*x^2+d^2)^(1+p)/(9+2*p)+11
/2*d^5*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-5*d^3*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+
3/2*d*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7*d^2*e*(17+3*p)*x^7*(-e^2*x^2+d^2)^
p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/(9+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.19, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1652, 446, 77, 459, 365, 364}

$$\frac{2d^2e(3p+17)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}-\frac{2d^7\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{11d^5\left(d^2\right.}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1
+ p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3
*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*
e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^5\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^6\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\frac{(2d^2\ldots}{\ldots}$$

$$=-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^7\left(d^2-e^2x\right)^p}{e^4}-\frac{11d^5\left(d^2-e^2x\right)^{1+p}}{e^4}+\frac{10d\ldots}{\ldots}\right.\right.$$

$$=-\frac{2d^7\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{11d^5\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{5d^3\left(d^2-e^2x\ldots\right)}{e^6(3+p)\ldots}$$

Mathematica [A] time = 0.24, size = 205, normalized size = 0.92

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{630\left(d^3-de^2x^2\right)^3}{p+3}+\frac{189d\left(d^2-e^2x^2\right)^4}{p+4}+14e^9x^9\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{9}{2},-p;\frac{11}{2};\frac{e^2x^2}{d^2}\right)+54d^2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\ldots\right.}{126e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-252*d^7*(d^2 - e^2*x^2))/(1 + p) + (693*d^5*(d^2 - e^
2*x^2)^2)/(2 + p) + (189*d*(d^2 - e^2*x^2)^4)/(4 + p) - (630*(d^3 - d*e^2*x
^2)^3)/(3 + p) + (54*d^2*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/
d^2])/(1 - (e^2*x^2)/d^2)^p + (14*e^9*x^9*Hypergeometric2F1[9/2, -p, 11/2,
(e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(126*e^6)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^8+3\,de^2x^7+3\,d^2ex^6+d^3x^5\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(-e^2*x^2 + d^2)^p
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^5, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^5 \left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(p^2+3\,p+2\right)e^6x^6-\left(p^2+p\right)d^2e^4x^4-2\,d^4e^2px^2-2\,d^6\right)\left(-e^2x^2+d^2\right)^p d^3}{2\,\left(p^3+6\,p^2+11\,p+6\right)e^6}+\int \left(e^3x^8+3\,de^2x^7+3\,d^2ex^6\right)e^{\left(p\log(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*
d^6)*(-e^2*x^2 + d^2)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e^3
*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

sympy [B] time = 16.25, size = 2966, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2
*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2
```

*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d**2*d**(2*p)*e*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + 3*d*e**2*Piecewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4), Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*

```
e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e*
*8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True)) + d**
(2*p)*e**3*x**9*hyper((9/2, -p), (11/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
/9
```

3.259 $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=218

$$-\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{3d^2\left(d^2-e^2x^2\right)^{p+3}}{e^5(p+3)}+\frac{\left(d^2-e^2x^2\right)^{p+4}}{2e^5(p+4)}-\frac{2d^6\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{9d^4\left(d^2-e^2x^2\right)^{p+2}}{2e^5(p+2)}+\frac{2d^3(p+11)x}{}$$

```
[Out] -2*d^6*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)-3*d*x^5*(-e^2*x^2+d^2)^(1+p)/(7+2*p)+
9/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-3*d^2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)
+1/2*(-e^2*x^2+d^2)^(4+p)/e^5/(4+p)+2/5*d^3*(11+p)*x^5*(-e^2*x^2+d^2)^p*hyp
ergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.18, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1652, 459, 365, 364, 446, 77}

$$\frac{2d^3(p+11)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{2d^6\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{9d^4\left(d^2-e^2\right.}{2e^5(p+}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(
1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^
2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(
4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p,
7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^5\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+$$

$$=-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^6\left(d^2-e^2x\right)^p}{e^3}-\frac{9d^4\left(d^2-e^2x\right)^{1+p}}{e^3}+\right.\right.$$

$$=-\frac{2d^6\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)}-\frac{3d^2\left(d^2-\right.}{e^5(3}$$

Mathematica [A] time = 0.24, size = 219, normalized size = 1.00

$$\frac{1}{70}\left(d^2-e^2x^2\right)^p\left(30de^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+14d^3x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{35(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*(d^2 - e^2*x^2)*(6*d^6*(5 + p) + 6*d^4*e^2*(5 + 6*
p + p^2)*x^2 + 3*d^2*e^4*(10 + 17*p + 8*p^2 + p^3)*x^4 + e^6*(6 + 11*p + 6*
p^2 + p^3)*x^6))/(e^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyperg
eometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (30*d*e^2
*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p)
)/70
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^7+3\,de^2x^6+3\,d^2ex^5+d^3x^4\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(-e^2*x^2 + d^2)^p
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^4 \left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
[Out] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(d^2-e^2x^2\right)^p (d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

sympy [B] time = 16.05, size = 2966, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
[Out] d**3*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 3*d**2*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6
- 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d*
*2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) -
 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d
**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e
```

```
**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(
-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4
/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2
*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**
2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2
*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
+ 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p*
*2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**
3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d*d**(2*p)*
e**2*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e*
*3*Piecewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*l
og(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e
**14*x**6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x*
*4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d
**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*lo
g(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e*
*14*x**6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2
*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e*
*8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**
4*x**4*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**
4 + 12*e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2
- 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x
**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 +
 12*e**14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e
**10*x**2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10
*x**2 + 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x
**4) + 12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x
**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
4*e**12*x**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*
x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*
e**10*x**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
 4*e**12*x**4), Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*
x**2) - 6*d**6*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2
*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e*
*10*x**2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3
*d**2*e**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4
*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)
/(2*e**8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p,
-1)), (-6*d**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*
p**2 + 100*e**8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*
e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e*
*4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p*
*2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e*
*8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p
**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 +
 100*e**8*p + 48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**
8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*
p*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 1
00*e**8*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 +
20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**
2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p +
 48*e**8) + 11*e**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**
3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**
p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True)
```

)

3.260 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=193

$$\frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{3d\left(d^2-e^2x^2\right)^{p+3}}{2e^4(p+3)}-\frac{2d^5\left(d^2-\right.}{e^4(p}$$

```
[Out] -2*d^5*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)-e*x^5*(-e^2*x^2+d^2)^(1+p)/(7+2*p)+7/
2*d^3*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-3/2*d*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2
/5*d^2*e*(13+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^
2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.18, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1652, 446, 77, 459, 365, 364}

$$\frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{2d^5\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{7d^3\left(d^2\right.}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1
+ p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d
^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (
e^2*x^2)/d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^3\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^4\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\frac{\left(2d^2e\right.}{}$$

$$=-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^5\left(d^2-e^2x\right)^p}{e^2}-\frac{7d^3\left(d^2-e^2x\right)^{1+p}}{e^2}+\frac{3d\left(d\right.}{}\right.\right.$$

$$=-\frac{2d^5\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{7d^3\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{3d\left(d^2-e^2x^2\right)}{2e^4(3+p)}$$

Mathematica [A] time = 0.18, size = 187, normalized size = 0.97

$$\frac{\left(d^2-e^2x^2\right)^p\left(10e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+42d^2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{35d\left(d^2-e^2x^2\right)\left(d^4(p\right.}{}\right.}{70e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d*(d^2 - e^2*x^2)*(d^4*(9 + p) + d^2*e^2*(9 + 10*p
 + p^2)*x^2 + 3*e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (42*d
^2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d
^2)^p + (10*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e
^2*x^2)/d^2)^p))/(70*e^4)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^6+3\,de^2x^5+3\,d^2ex^4+d^3x^3\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(-e^2*x^2 + d^2)^p
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^3\, x^3 \left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^4\left(p+1\right)x^4-d^2e^2px^2-d^4\right)\left(-e^2x^2+d^2\right)^p d^3}{2\left(p^2+3\,p+2\right)e^4}+\int \left(e^3x^6+3\,de^2x^5+3\,d^2ex^4\right)e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^3/((p^2 +
3*p + 2)*e^4) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*e^(p*log(e*
x + d) + p*log(-e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

sympy [B] time = 11.26, size = 1370, normalized size = 7.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 3*d**2*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi
```

```
)/d**2)/5 + 3*d*e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-
d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 -
 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(
-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x*
*2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/
(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6
+ 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4
*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) -
 x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4
*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) -
 d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**
6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**
2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)
*e**3*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

3.261 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=189

$$-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}+\frac{5d^2\left(d^2-e^2x^2\right)^{p+2}}{2e^3(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^3(p+3)}-\frac{2d^4\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{2d^3(p+7)x^3\left(d^2-e^2x^2\right)^p\left(1-\right.}{3(2p+}$$

```
[Out] -2*d^4*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)-3*d*x^3*(-e^2*x^2+d^2)^(1+p)/(5+2*p)+
5/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)-1/2*(-e^2*x^2+d^2)^(3+p)/e^3/(3+p)+2
/3*d^3*(7+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/(5
+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.17, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1652, 459, 365, 364, 446, 77}

$$\frac{2d^3(p+7)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}-\frac{2d^4\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{5d^2\left(d^2-\right.}{2e^3(p}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(
1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2
- e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/
d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= \int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^3\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx\\ &= -\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\frac{(2d^3\cdots}{\cdots}\\ &= -\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^4\left(d^2-e^2x\right)^p}{e}-\frac{5d^2\left(d^2-e^2x\right)^{1+p}}{e}+\frac{(d^2\cdots}{\cdots}\right.\right.\\ &= -\frac{2d^4\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{5d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+\cdots}}{2e^3(3+p)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 187, normalized size = 0.99

$$\frac{1}{30}\left(d^2-e^2x^2\right)^p\left(18de^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{15\left(d^2-e^2x^2\right)\left(d^4(3p+11)+d^2e^2\left(3p^2+14p+11\right)\cdots}{e^3(p+1)(p+2)(p+3)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-15*(d^2 - e^2*x^2)*(d^4*(11 + 3*p) + d^2*e^2*(11 + 14
*p + 3*p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/(e^3*(1 + p)*(2 + p)*(3 + p)) +
 (10*d^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)
/d^2)^p + (18*d*e^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p))/30
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(-e^2*x^2 + d^2)^p
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^2 \left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
[Out] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

sympy [B] time = 10.46, size = 1370, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
[Out] d**3*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 3*d**2*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x
**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2),
 Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x
**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4
), True)) + 3*d*d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/5 + e**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**
4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*lo
```

```
g(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e
 + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*
e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/
e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)),
 (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*
e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2
*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*
*2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*
e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e
**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

3.262 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=116

$$-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}-\frac{3d^3 2^{p+3} \left(d^2-e^2x^2\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}$$

```
[Out] -(e*x+d)^3*(-e^2*x^2+d^2)^(1+p)/e^2/(5+2*p)-3*2^(3+p)*d^3*(1+e*x/d)^(-1-p)*
(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, -3-p],[2+p],1/2*(-e*x+d)/d)/e^2/(2*p^2
+7*p+5)
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {795, 678, 69}

$$-\frac{3d^3 2^{p+3} \left(d^2-e^2x^2\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((d + e*x)^3*(d^2 - e^2*x^2)^(1 + p))/(e^2*(5 + 2*p))) - (3*2^(3 + p)*d^3
*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(5 + 2*p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} + \frac{(3d)\int (d+ex)^3\left(d^2-e^2x^2\right)^p\,dx}{e(5+2p)}$$
$$= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} + \frac{\left(3d^3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-e}{e(5+2p)}$$
$$= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} - \frac{3\,2^{3+p}d^3\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(-3-p,1+\right.}{e^2(1+p)(5+2p)}$$

Mathematica [A] time = 0.26, size = 159, normalized size = 1.37

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{5d\left(d^2-e^2x^2\right)\left(d^2(p+5)+3e^2(p+1)x^2\right)}{(p+1)(p+2)}+2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+10d^2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2}\right.\right.}{10e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2*(5 + p) + 3*e^2*(1 + p)*x^2)
)/((1 + p)*(2 + p)) + (10*d^2*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*
x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/
2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(10*e^2)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(-e^2*x^2 + d^2)^p,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^3\,x\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-e^2x^2+d^2\right)^{p+1}d^3}{2\,e^2\left(p+1\right)} + \int \left(e^3x^4+3\,de^2x^3+3\,d^2ex^2\right)e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(p + 1)*d^3/(e^2*(p + 1)) + integrate((e^3*x^4 + 3*d*
e^2*x^3 + 3*d^2*e*x^2)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2x^2\right)^p(d+ex)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

sympy [A] time = 7.58, size = 479, normalized size = 4.13

$$d^3\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+d^2d^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+3de^2\left(\begin{cases}\frac{x^4(d^2)^p}{4}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\,\mathrm{l}}{-2d}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\,\mathrm{l}}{}\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2}{2}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + d**2*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar
(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*lo
g(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4
+ 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2
*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 +
6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*
p + 4*e**4), True)) + d**(2*p)*e**3*x**5*hyper((5/2, -p), (7/2,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/5
```

3.263 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=73

$$-\frac{d^2 2^{p+3} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -2^(3+p)*d^2*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, -3-p],[2
+p],1/2*(-e*x+d)/d)/e/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{d^2 2^{p+3} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(3 + p)*d^2*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeomet
ric2F1[-3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= \left(d^2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{3+p}\,dx\\ &= -\frac{2^{3+p}d^2\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [B] time = 0.16, size = 155, normalized size = 2.12

$$\frac{1}{2}\left(d^2-e^2x^2\right)^p\left(2de^2x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+\frac{\left(e^2x^2-d^2\right)\left(d^2(3p+7)+e^2(p+1)x^2\right)}{e(p+1)(p+2)}+2d^3x\left(1-\frac{e^2}{d}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(((-d^2 + e^2*x^2)*(d^2*(7 + 3*p) + e^2*(1 + p)*x^2))/(e
*(1 + p)*(2 + p)) + (2*d^3*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p + (2*d*e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2
*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/2
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2 + d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2 + d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2 + d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

sympy [B] time = 6.58, size = 476, normalized size = 6.52

$$d^3d^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+3d^2e\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+dd^{2p}e^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 3*d**2*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 -
e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*
e**2), True)) + d*d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp
_polar(2*I*pi)/d**2) + e**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*
log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e +
 x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/
(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4), True))
```

3.264 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$

Optimal. Leaf size=171

$$\frac{2d^2e(3p+5)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{ex\,(}{}$$

```
[Out] -3/2*d*(-e^2*x^2+d^2)^(1+p)/(1+p)-e*x*(-e^2*x^2+d^2)^(1+p)/(3+2*p)+2*d^2*e*
(5+3*p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/(3+2*p)/(
(1-e^2*x^2/d^2)^p)-1/2*d*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^
2*x^2/d^2)/(1+p)
```

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1652, 446, 80, 65, 388, 246, 245}

$$\frac{2d^2e(3p+5)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{ex\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (-3*d*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (e*x*(d^2 - e^2*x^2)^(1 + p))/
(3 + 2*p) + (2*d^2*e*(5 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, (e^2*x^2)/d^2])/((3 + 2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x}\,dx = \int \frac{\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)}{x}\,dx + \int \left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx$$

$$= -\frac{ex\left(d^2-e^2x^2\right)^{1+p}}{3+2p} + \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)}{x}\,dx, x, x^2\right) + \frac{\left(2d^2e(5+\right.}{}$$

$$= -\frac{3d\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{ex\left(d^2-e^2x^2\right)^{1+p}}{3+2p} + \frac{1}{2}d^3\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + $$

$$= -\frac{3d\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{ex\left(d^2-e^2x^2\right)^{1+p}}{3+2p} + \frac{2d^2e(5+3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}}{3+2p}$$

Mathematica [A] time = 0.14, size = 169, normalized size = 0.99

$$\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(18d^2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{3d\left(d^2-e^2x^2\right){}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{p+1} - \frac{9d\left(d^2\right.}{p+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-9*d*(d^2 - e^2*x^2))/(1 + p) + (18*d^2*e*x*Hypergeome
tric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*d*(d^2 - e
^2*x^2)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 + p) + (2
*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2
)^p))/6
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x, x)
```

sympy [A] time = 12.29, size = 178, normalized size = 1.04

$$-\frac{d^3e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+3d^2d^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+3de^2\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2}\\-\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\\log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d**2*d**(2*p)*e*x*hyper((1/2, -p), (3/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**2*(d**2)**p/
2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**(2*p)*e**3*x**3*hy
per((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3
```

3.265 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=159

$$2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{3e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-e(\ldots$$

```
[Out] -1/2*e*(-e^2*x^2+d^2)^(1+p)/(1+p)-d*(-e^2*x^2+d^2)^(1+p)/x+2*d*e^2*(1-p)*x*
(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
-3/2*e*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/(1+p)
```

Rubi [A] time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1807, 1652, 446, 80, 65, 12, 246, 245}

$$2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{3e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-e(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -(e*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (d*(d^2 - e^2*x^2)^(1 + p))/x +
(2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x
^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-3d^4e-2d^3e^2(1-p)x-d^2e^3x^2\right)}{x}\,dx}{d^2}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int -2d^3e^2(1-p)\left(d^2-e^2x^2\right)^p\,dx}{d^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-3d^4e-d^2e^3x^2\right)}{x}\,dx}{d^2}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(-3d^4e-d^2e^3x\right)}{x}\,dx,x,x^2\right)}{2d^2} + \left(2de^2(1-p)\right)\int \left(d^2\right.$$
$$= -\frac{e\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right) +$$
$$= -\frac{e\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} + 2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1$$

Mathematica [A] time = 0.09, size = 158, normalized size = 0.99

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(6de(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(3\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2}{d}\right.\right.\right.\right.}{2(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-2*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^
2)/d^2] + e*x*(6*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^
```

```
2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(1 + 3*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 - (e^2*x^2)/d^2]))))/(2*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}\right)\left(-e^{2}x^{2}+d^{2}\right)^{p}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^2,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^{2}-e^{2}\,x^{2}\right)^{p}\,(d+e\,x)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2, x)
```

sympy [A] time = 7.45, size = 177, normalized size = 1.11

$$-\frac{d^3 d^{2p}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{3d^2ee^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+3dd^{2p}e^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e^3\left\{\left(\left(\frac{x^2\left(d^2\right)^p}{2}\right.\right.\right.\ \left\{\frac{\left(d^2-e^2\ldots\right.}{p\ldots}\right.\ \log\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d**3*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
x - 3*d**2*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p
,), d**2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d*d**(2*p)*e**2*x*hyper((1/2, -p
), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**3*Piecewise((x**2*(d**2)*
*p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p,
 -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True))
```

3.266 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}-\frac{3e\left(d^2-e^2x^2\right)^{p+1}}{x}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2x^2}-2e^3(3p+1)x\left(d^2-\right.$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/x^2-3*e*(-e^2*x^2+d^2)^(1+p)/x-2*e^3*(1+3*p)*x*
(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
-1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)
/d/(1+p)
```

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1807, 764, 266, 65, 246, 245}

$$-2e^3(3p+1)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\right.}{2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2
*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2
)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^3}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-6d^4e-2d^3e^2(3-p)x-2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\frac{\int \frac{\left(2d^5e^2(3-p)-4d^4e^3(1+3p)x\right)\left(d^2-e^2x^2\right)^p}{x}\,dx}{2d^4}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\left(de^2(3-p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx-\left(2e^3(1+\right.\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\frac{1}{2}\left(de^2(3-p)\right)\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x\right.\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}-2e^3(1+3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F
\end{aligned}$$

Mathematica [A] time = 0.10, size = 182, normalized size = 1.10

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(2de(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(3\,{}_2F_1\left(1,p+1;p+2;1-\right.\right.\right.\right.}{2d(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*(-6*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*
x^2)/d^2] + e*x*(2*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/
d^2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(3*Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e^2*x^
2)/d^2]))))/(2*d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^3,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,(d+e\,x)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3, x)
```

sympy [C] time = 8.37, size = 177, normalized size = 1.07

$$-\frac{d^3e^{2p}x^{2p}e^{i\pi p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma\left(2-p\right)}-\frac{3d^2d^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{3de^2e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**3,x)
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,
), d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - 3*d**2*d**(2*p)*e*hyper((-1/2,
 -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - 3*d*e**2*e**(2*p)*x**(2
*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*ga
mma(1 - p)) + d**(2*p)*e**3*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)
```

3.267 $\int \frac{x^4\left(d^2-e^2x^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=148

$$\frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^5(p+2)}+\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^5p}$$

```
[Out] 1/2*d^4*(-e^2*x^2+d^2)^p/e^5/p-d^2*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+1/2*(-e^2
*x^2+d^2)^(2+p)/e^5/(2+p)+1/5*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/
2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {850, 764, 365, 364, 266, 43}

$$\frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}+\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^5p}-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^5(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^p)/(2*e^5*p) - (d^2*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 +
 p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^5*(2 + p)) + (x^5*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^p}{d+ex}\,dx = \int x^4(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx$$
$$= d\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^5\left(d^2-e^2x^2\right)^{-1+p}\,dx$$
$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2}{d}\right.}{d}$$
$$= \frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{d^4\left(d^2-e^2x\right)^{-1+p}}{e^4} - \right.\right.$$
$$= \frac{d^4\left(d^2-e^2x^2\right)^p}{2e^5p} - \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)} + \frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)} + \frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F}{5d}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.45

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,1-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 1 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^4}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(-e^2 x^2 + d^2\right)^p}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2 x^2 + d^2\right)^p x^4}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \left(d^2 - e^2 x^2\right)^p}{d + e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

sympy [C] time = 16.54, size = 4442, normalized size = 30.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((-6*0**p*d**4*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p
 - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3)) + 6*0**p*d**4*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*
gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*
gamma(p + 3)) + 12*0**p*d**4*d**(2*p)*p*acoth(d/(e*x))*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(
p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(
p + 3)) - 6*0**p*d**4*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p - 1
/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
+ 3)) + 6*0**p*d**4*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(
p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(
p + 3)) + 12*0**p*d**4*d**(2*p)*acoth(d/(e*x))*gamma(-p)*gamma(-p - 1/2)*ga
mma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3))
```

```
- 12*0**p*d**3*d**(2*p)*e*p*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(
p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*
e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d*
*(2*p)*e*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**2*d**(2*p)*e**2*p*x**2*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gam
ma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*
gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**2*d**(2*p)*e**2*x**2*gamma(-p)*gamma
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3)) - 4*0**p*d*d**(2*p)*e**3*p*x**3*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gam
ma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) -
4*0**p*d*d**(2*p)*e**3*x**3*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
+ 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e*
*5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 3*0**p*d**(2*p)*e
**4*p*x**4*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 3*0**p*d**(2*p)*e**4*x**4*gamma(-p
)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
 + 1)*gamma(p + 3)) + 12*d**4*e**(2*p)*x**(2*p)*(d**2/(e**2*x**2) - 1)**p*g
amma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-
p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3)) + 12*d**2*e**2*e**(2*p)*p*x**2*x**(2*p)*(d**2/(e**2*
x**2) - 1)**p*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*ga
mma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p**2*x**3*x**(2*p
)*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p,
 -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p -
1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p
)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p, -p - 3/2), (-p - 1/2,
), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6
*e**4*e**(2*p)*p**2*x**4*x**(2*p)*(d**2/(e**2*x**2) - 1)**p*gamma(-p)*gamma
(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma
(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma
(p + 3)) + 6*e**4*e**(2*p)*p*x**4*x**(2*p)*(d**2/(e**2*x**2) - 1)**p*gamma(
-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1
/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p +
 1)*gamma(p + 3)), Abs(d**2/(e**2*x**2)) > 1), (-6*0**p*d**4*d**(2*p)*p*log
(d**2/(e**2*x**2))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*
e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(
-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**4*d**(2*p)*p*log
(-d**2/(e**2*x**2) + 1)*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)
/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 12*0**p*d**4*d**(2*p)
*p*atanh(d/(e*x))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e
**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-
p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 6*0**p*d**4*d**(2*p)*log(d*
*2/(e**2*x**2))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**
5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)
*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**4*d**(2*p)*log(-d**
2/(e**2*x**2) + 1)*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*
e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(
-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 12*0**p*d**4*d**(2*p)*atan
h(d/(e*x))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*g
```

```
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d**(2*p)*e*p*x*gamma(
-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p
 - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma
(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d**(2*p)*e*x*gamma(-p)*gamma(-p - 1/2)
*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1
)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3
)) + 6*0**p*d**2*d**(2*p)*e**2*p*x**2*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1
)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p +
3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*
d**2*d**(2*p)*e**2*x**2*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)
/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 4*0**p*d*d**(2*p)*e**
3*p*x**3*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gam
ma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(
-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 4*0**p*d*d**(2*p)*e**3*x**3*gamma(-p
)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
 + 1)*gamma(p + 3)) + 3*0**p*d**(2*p)*e**4*p*x**4*gamma(-p)*gamma(-p - 1/2)
*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1
)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3
)) + 3*0**p*d**(2*p)*e**4*x**4*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma
(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12
*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 12*d**4*e**(2*
p)*x**(2*p)*(-d**2/(e**2*x**2) + 1)**p*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma
(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3))
+ 12*d**2*e**2*e**(2*p)*p*x**2*x**(2*p)*(-d**2/(e**2*x**2) + 1)**p*exp(I*pi
*p)*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*ga
mma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)
*gamma(p + 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi
*p)*gamma(-p)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p, -p - 3/2)
, (-p - 1/2,), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma
(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma
(p + 3)) + 6*d*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p)
*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e
**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) +
 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*e**4*e**(
2*p)*p**2*x**4*x**(2*p)*(-d**2/(e**2*x**2) + 1)**p*exp(I*pi*p)*gamma(-p)*ga
mma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*ga
mma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*ga
mma(p + 3)) + 6*e**4*e**(2*p)*p*x**4*x**(2*p)*(-d**2/(e**2*x**2) + 1)**p*ex
p(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(
-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p
- 1/2)*gamma(p + 1)*gamma(p + 3)), True))
```

3.268 $\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=121

$$-\frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}-\frac{d^3\left(d^2-e^2x^2\right)^p}{2e^4p}$$

```
[Out] -1/2*d^3*(-e^2*x^2+d^2)^p/e^4/p+1/2*d*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)-1/5*e*
x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/2],e^2*x^2/d^2)/d^2/((1-e^2*x^
2/d^2)^p)
```

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {850, 764, 266, 43, 365, 364}

$$-\frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}-\frac{d^3\left(d^2-e^2x^2\right)^p}{2e^4p}+\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -(d^3*(d^2 - e^2*x^2)^p)/(2*e^4*p) + (d*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1
+ p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^
2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(d^2-e^2x^2\right)^p}{d+ex}\,dx &= \int x^3(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= d\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= \frac{1}{2}d\,\mathrm{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx, x, x^2\right) - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-1+}}{d^2}\\ &= -\frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{1}{2}d\,\mathrm{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^{-1+p}}{e^2} -\right.\right.\\ &= -\frac{d^3\left(d^2-e^2x^2\right)^p}{2e^4p} + \frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)} - \frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}\end{aligned}$$

Mathematica [B] time = 0.29, size = 245, normalized size = 2.02

$$\frac{\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(6d^2e(p+1)x\left(\frac{ex}{d}+1\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+3d\left(d(d-ex)\left(2-\frac{2e^2x^2}{d^2}\right)^p{}_2F_1\left(1\right.\right.\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(6*d^2*e*(1 + p)*x*(1 + (e*x)/d)^p*Hypergeometric2F1[1/2
, -p, 3/2, (e^2*x^2)/d^2] + 2*e^3*(1 + p)*x^3*(1 + (e*x)/d)^p*Hypergeometri
c2F1[3/2, -p, 5/2, (e^2*x^2)/d^2] + 3*d*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e
^2*x^2)/d^2)^p) + d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + d*(d - e*x)*(2 - (2*e
^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(
6*e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^3}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(d^2-e^2\,x^2\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

sympy [C] time = 10.52, size = 5090, normalized size = 42.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] Piecewise((3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gam
ma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*g
amma(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p -
1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
- 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*acoth(d/(e*x))*gamma(-p - 1/2
)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1
/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1
/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(
-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamm
a(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*acoth(d/(e*x))*gamma(-p -
1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
- 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p +
 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*ga
mma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*
```

```
d*d**(2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*
e**2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p +
1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gam
ma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*g
amma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*ga
mma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*
gamma(p + 1)) - 3*d**3*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(
p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) - 3*d*d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*e
xp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
+ 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)
*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,),
 d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p
)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(
6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)
), (Abs(e**2*x**2/d**2) > 1) & (Abs(d**2/(e**2*x**2)) > 1)), (3*0**p*d**3*d
**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamm
a(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*
*3*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e*
*4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) -
6*0**p*d**3*d**(2*p)*p*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*
p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 3*0
**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**
4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3
*0**p*d**3*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)
/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p +
1)) - 6*0**p*d**3*d**(2*p)*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e
**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) +
 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p
 - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**2*d
**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p
+ 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*p*x**2
*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e*
*4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*x**2*gamma(-p - 1
/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p +
 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*g
amma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3
*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(6*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1))
- 3*d*d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/
2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*
gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1))
- 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hype
r((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)), Abs(e**2*x**2/d**2)
 > 1), (3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(
p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamm
a(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2
)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1
/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*acoth(d/(e*x))*gamma(-p - 1/2)*g
amma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)
*gamma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)
*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/
```

```
2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(-p
- 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-
p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*acoth(d/(e*x))*gamma(-p - 1/2
)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1
/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p + 1)
/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p +
1)) + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma
(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d
**(2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*
gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**
2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
+ 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gamma(
-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamm
a(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*gamma
(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gam
ma(p + 1)) - 3*d**3*d**(2*p)*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(-p - 1/
2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
+ 1)) - 3*d*d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(-p
- 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamm
a(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*
gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**
3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 -
p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamm
a(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)), Abs(d**2/(e**2*x**2)) > 1)
, (3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1
)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p +
 1)) - 3*0**p*d**3*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*ga
mma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*
gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*atanh(d/(e*x))*gamma(-p - 1/2)*gamma
(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gam
ma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gam
ma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*g
amma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1
/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*atanh(d/(e*x))*gamma(-p - 1/2)*g
amma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)
*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p + 1)/(6
*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1))
 + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p
 - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(
2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gam
ma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*x
**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6
*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gamma(-p
- 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-
p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*gamma(p
+ 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(
p + 1)) - 3*d**3*d**(2*p)*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(-p - 1/2)/
(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1
)) - 3*d*d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(-p - 1
/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p -
 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*gam
ma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e
**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p,
-p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)), True))
```

3.269 $\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=119

$$\frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}+\frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}$$

```
[Out] 1/2*d^2*(-e^2*x^2+d^2)^p/e^3/p-1/2*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)+1/3*x^3*(
-e^2*x^2+d^2)^p*hypergeom([3/2, 1-p],[5/2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^
p)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {850, 764, 365, 364, 266, 43}

$$\frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}+\frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^p)/(2*e^3*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^3*(1 + p))
 + (x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2]
)/(3*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^2 (d^2-e^2x^2)^p}{d+ex}\,dx = \int x^2(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx$$
$$= d\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx$$
$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx, x, x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x}{d^2}\right.}{d}$$
$$= \frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d} - \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^{-1+p}}{e^2} - \right.\right.$$
$$= \frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)} + \frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}$$

Mathematica [A] time = 0.25, size = 198, normalized size = 1.66

$$-\frac{\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2de(p+1)x\left(\frac{ex}{d}+1\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+d(d-ex)\left(2-\frac{2e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1\right.\right.}{2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*((d^2 - e^2*x^2)^p*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e^2*x^2)/d^2)^p)
+ d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + 2*d*e*(1 + p)*x*(1 + (e*x)/d)^p*Hyper
geometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + d*(d - e*x)*(2 - (2*e^2*x^2)/d^
2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^3*(1 + p)
*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(d^2-e^2\,x^2\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

sympy [C] time = 8.39, size = 4124, normalized size = 34.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma
(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma
(p + 1)) + 0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*ga
mma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*ga
mma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p
+ 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p
+ 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1
)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1
)) + 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p +
1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p +
1)) + 2*0**p*d**2*d**(2*p)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*
0**p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*
gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*
gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*
gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gam
```

```
ma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gam
ma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*
gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d*
*(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(2*e**3
*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*
e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p,
 -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p
+ 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(
I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/
(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*
gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*g
amma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma
(1/2 - p)*gamma(p + 1)), (Abs(e**2*x**2/d**2) > 1) & (Abs(d**2/(e**2*x**2))
 > 1)), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p
 + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p
 + 1)) + 0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamm
a(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamm
a(p + 1)) + 2*0**p*d**2*d**(2*p)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p +
1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p +
1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/
(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1))
 + 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)
/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)
) + 2*0**p*d**2*d**(2*p)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3
*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0*
*p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*ga
mma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*ga
mma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*ga
mma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gamma
(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma
(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*ga
mma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d**(
2*p)*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 -
p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p**2*x
*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/
2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*g
amma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p
)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2
*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) +
 d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)), Abs
(d**2/(e**2*x**2)) > 1), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma
(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma
(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*g
amma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*g
amma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*atanh(d/(e*x))*gamma(1
/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1
/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2
- p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2
- p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(1/
2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/
2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*atanh(d/(e*x))*gamma(1/2 - p)*g
amma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*g
amma(p + 1)) - 2*0**p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*
p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**
p*d*d**(2*p)*e*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma
(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*g
amma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*g
amma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(
```

```
p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(
p + 1)) + d**2*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma
(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gam
ma(p + 1)) + d*e*e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1
/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(
1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*
p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2),
(1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**
3*gamma(1/2 - p)*gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)
**p*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p +
1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)), Abs(e**2*x**2/d**2) > 1), (-0**p*
d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p
*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d
**2*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 2*
0**p*d**2*d**(2*p)*p*atanh(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*g
amma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**
2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamm
a(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d
**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*ga
mma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d*
*2*d**(2*p)*atanh(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2
- p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)
*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) +
2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*gamma(1/2 - p)*
gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*
gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0*
*p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*
gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d**(2*p)*(1 - e**
2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p +
1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p**2*x*x**(2*p)*exp
(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2
/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)
*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p -
1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma
(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**(2*p)*e**
2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/
2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)), True))
```

3.270 $\int \frac{x\left(d^2-e^2x^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=90

$$-\frac{ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^p/e^2/p-1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 1-
p],[5/2],e^2*x^2/d^2)/d^2/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {785, 764, 261, 365, 364}

$$-\frac{ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^p)/(2*e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d^2-e^2x^2\right)^p}{d+ex}\,dx &= \frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{-1+p}\,dx}{de}\\
&= d\int x\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx\\
&= -\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p} - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p} - \frac{ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 147, normalized size = 1.63

$$\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2e(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1-p\right.\right.}{e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(2*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^2*(1 + p)*(1 + (
e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2-e^2\,x^2\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

sympy [C] time = 7.00, size = 427, normalized size = 4.74

$$\begin{cases} \frac{0^p d d^{2p}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d d^{2p}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2e^2} - \frac{0^p d d^{2p}\operatorname{acoth}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p}x}{e} - \frac{e^{2p}pxx^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,-p-\frac{1}{2}\\ \frac{1}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} - \frac{d^{2p}x^2 \cdots}{\cdots} \\ \frac{0^p d d^{2p}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d d^{2p}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2e^2} - \frac{0^p d d^{2p}\operatorname{atanh}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p}x}{e} - \frac{e^{2p}pxx^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,-p-\frac{1}{2}\\ \frac{1}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} - \frac{d^{2p}x \cdots}{\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] Piecewise((0**p*d*d**(2*p)*log(d**2/(e**2*x**2))/(2*e**2) - 0**p*d*d**(2*p)
*log(d**2/(e**2*x**2) - 1)/(2*e**2) - 0**p*d*d**(2*p)*acoth(d/(e*x))/e**2 +
 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p -
1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e*gamma(1/2
- p)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(-p)*gamma(p + 1)),
Abs(d**2/(e**2*x**2)) > 1), (0**p*d*d**(2*p)*log(d**2/(e**2*x**2))/(2*e**2)
 - 0**p*d*d**(2*p)*log(-d**2/(e**2*x**2) + 1)/(2*e**2) - 0**p*d*d**(2*p)*at
anh(d/(e*x))/e**2 + 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*g
amma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**
2))/(2*e*gamma(1/2 - p)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*gamma(1 - p)
*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(
-p)*gamma(p + 1)), True))
```

3.271 $\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

```
[Out] -2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1-p, 1+p],[2+p],
1/2*(-e*x+d)/d)/d^2/e/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{d+ex}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}\,dx}{d^2}\\ &= -\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.03

$$-\frac{2^{p-1}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^{p}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(1 + (e*x)/d)^p))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x), x)
```

sympy [C] time = 7.32, size = 321, normalized size = 4.40

$$\begin{cases} \frac{0^p \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{acoth}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix} 1-p,\frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{for} \\ \frac{0^p \log\left(1-\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{atanh}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix} 1-p,\frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*log(-1 + e**2*x**2/d**2)/(2*e) + 0**p*acoth(e*x/d)/e + d*e*
*(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p + 1)) + d
**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2
*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), Abs(e**2*x**2/d**
2) > 1), (0**p*log(1 - e**2*x**2/d**2)/(2*e) + 0**p*atanh(e*x/d)/e + d*e**(
2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p),
 (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p + 1)) + d**
(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*e
xp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

3.272 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$

Optimal. Leaf size=104

$$-\frac{ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

```
[Out] -e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 1-p],[3/2],e^2*x^2/d^2)/d^2/((1-e^2*x
^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1, p],[1+p],1-e^2*x^2/d^2)/d/p
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {850, 764, 266, 65, 246, 245}

$$-\frac{ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2*x^2)/d^2])
/(d^2*(1 - (e^2*x^2)/d^2)^p)) - ((d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p,
1 + p, 1 - (e^2*x^2)/d^2])/(2*d*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx\\ &= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx - e\int \left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx,x,x^2\right) - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}dx}{d^2}\\ &= -\frac{ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2} - \frac{\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2dp}\end{aligned}$$

Mathematica [A] time = 0.11, size = 151, normalized size = 1.45

$$\frac{2^{p-1}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+d(p+1)\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2\right.}{d^2p(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeome
tric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*(1 + p)*(1/2 + (e*x)/(2*d
))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^2*p*(1 + p)*(1 -
d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)), x)
```

sympy [C] time = 6.90, size = 355, normalized size = 3.41

$$\begin{cases} -\frac{0^p d^{2p}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2d} - \frac{0^p d^{2p}\operatorname{acoth}\left(\frac{d}{ex}\right)}{d} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^2\Gamma(2-p)\Gamma(p+1)} - \frac{e^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\\frac{3}{2}-p\end{matrix}\middle| \right.}{2ex\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} \\ -\frac{0^p d^{2p}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2d} - \frac{0^p d^{2p}\operatorname{atanh}\left(\frac{d}{ex}\right)}{d} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^2\Gamma(2-p)\Gamma(p+1)} - \frac{e^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\\frac{3}{2}-p\end{matrix}\middle| \right.}{2ex\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d),x)
[Out] Piecewise((-0**p*d**(2*p)*log(d**2/(e**2*x**2) - 1)/(2*d) - 0**p*d**(2*p)*a
coth(d/(e*x))/d + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*h
yper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*e**2*x**2*gamma(2 - p)*
gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyp
er((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*e*x*gamma(3/2 - p)*ga
mma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (-0**p*d**(2*p)*log(-d**2/(e**2*x*
*2) + 1)/(2*d) - 0**p*d**(2*p)*atanh(d/(e*x))/d + d*e**(2*p)*p*x**(2*p)*exp
```

```
(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x
**2))/(2*e**2*x**2*gamma(2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*p
i*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2
*x**2))/(2*e*x*gamma(3/2 - p)*gamma(p + 1)), True))
```

3.273 $\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)}\,dx$

Optimal. Leaf size=106

$$\frac{e\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}-\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}$$

```
[Out] -(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d/x/((1-e^2*x^2/
d^2)^p)+1/2*e*(-e^2*x^2+d^2)^p*hypergeom([1, p],[1+p],1-e^2*x^2/d^2)/d^2/p
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {850, 764, 365, 364, 266, 65}

$$\frac{e\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}-\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -(((d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
*x*(1 - (e^2*x^2)/d^2)^p)) + (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p, 1
 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx\\ &= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx - e\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx\\ &= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx,x,x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}\,dx}{d}\\ &= -\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx} + \frac{e\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}\end{aligned}$$

Mathematica [A] time = 0.18, size = 167, normalized size = 1.58

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}-\frac{2d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{e2^p(ex-d)\left(\frac{ex}{d}+1\right)^{-p}{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{p+1}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-2*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[1 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (d*e*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(2*d^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{{\left(e x + d\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)), x)
```

sympy [C] time = 7.86, size = 450, normalized size = 4.25

$$\begin{cases} - \frac{0^{p} d^{2 p}}{d x} - \frac{0^{p} d^{2 p} e \log{\left(\frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \log{\left(-1 + \frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \operatorname{acoth}{\left(\frac{e x}{d} \right)}}{d^{2}} + \frac{d e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{3}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{3}{2} - p \\ \frac{5}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{3} \Gamma\left(\frac{5}{2} - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2 p} p x^{2 p} \cdots}{\cdots} \\ - \frac{0^{p} d^{2 p}}{d x} - \frac{0^{p} d^{2 p} e \log{\left(\frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \log{\left(1 - \frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \operatorname{atanh}{\left(\frac{e x}{d} \right)}}{d^{2}} + \frac{d e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{3}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{3}{2} - p \\ \frac{5}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{3} \Gamma\left(\frac{5}{2} - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2 p} p x^{2 p} e^{i \cdots}}{\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d),x)
[Out] Piecewise((-0**p*d**(2*p)/(d*x) - 0**p*d**(2*p)*e*log(e**2*x**2/d**2)/(2*d*
*2) + 0**p*d**(2*p)*e*log(-1 + e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*a
coth(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p
)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/(e**2*x**2))/(2*e**2*x**3*gamma(
5/2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 -
 p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e*x**2))/(2*e*x**2*gamma(2 - p
```

```
)*gamma(p + 1)), Abs(e**2*x**2/d**2) > 1), (-0**p*d**(2*p)/(d*x) - 0**p*d**
(2*p)*e*log(e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*log(1 - e**2*x**2/d*
*2)/(2*d**2) + 0**p*d**(2*p)*e*atanh(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*p)*ex
p(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/
(e**2*x**2))/(2*e**2*x**3*gamma(5/2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p
)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e
**2*x**2))/(2*e*x**2*gamma(2 - p)*gamma(p + 1)), True))
```

3.274 $\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)}\,dx$

Optimal. Leaf size=108

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(2,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

```
[Out] e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d^2/x/((1-e^2*x
^2/d^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^p*hypergeom([2, p],[1+p],1-e^2*x^2/d^2)/d
^3/p
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {850, 764, 266, 65, 365, 364}

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(2,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
^2*x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2, p
, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx &= \int \frac{(d-ex)(d^2-e^2x^2)^{-1+p}}{x^3}\,dx\\ &= d\int \frac{(d^2-e^2x^2)^{-1+p}}{x^3}\,dx - e\int \frac{(d^2-e^2x^2)^{-1+p}}{x^2}\,dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{-1+p}}{x^2}\,dx, x, x^2\right) - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}\,dx}{d^2}\\ &= \frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2(d^2-e^2x^2)^p\,{}_2F_1\left(2,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}\end{aligned}$$

Mathematica [B] time = 0.57, size = 219, normalized size = 2.03

$$\frac{(d^2-e^2x^2)^p\left(\frac{2d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(e^2\left(\frac{(d-ex)\left(2-\frac{2d^2}{e^2x^2}\right)^p\left(\frac{ex}{d}+1\right)^{-p}{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{p+1}+\frac{d\,{}_2F_1\left(-p,-p;1\right.}{p}\right.\right.\right.}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((2*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + ((d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d^
2/(e^2*x^2)])/((-1 + p)*x^2) + e^2*(((2 - (2*d^2)/(e^2*x^2))^p*(d - e*x)*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (d*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/p))/(1 - d^2/(e
^2*x^2))^p))/(2*d^4)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-e^2x^2+d^2)^p}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)), x)
```

sympy [C] time = 9.48, size = 498, normalized size = 4.61

$$\begin{cases} -\frac{0^p d^{2p}}{2dx^2}+\frac{0^p d^{2p} e}{d^2 x}+\frac{0^p d^{2p} e^2\log\left(\frac{e^2x^2}{d^2}\right)}{2d^3}-\frac{0^p d^{2p} e^2\log\left(-1+\frac{e^2x^2}{d^2}\right)}{2d^3}-\frac{0^p d^{2p} e^2\operatorname{acoth}\left(\frac{ex}{d}\right)}{d^3}+\frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right.}{2e^2x^4\Gamma(3-p)\Gamma(p+1)} \\ -\frac{0^p d^{2p}}{2dx^2}+\frac{0^p d^{2p} e}{d^2 x}+\frac{0^p d^{2p} e^2\log\left(\frac{e^2x^2}{d^2}\right)}{2d^3}-\frac{0^p d^{2p} e^2\log\left(1-\frac{e^2x^2}{d^2}\right)}{2d^3}-\frac{0^p d^{2p} e^2\operatorname{atanh}\left(\frac{ex}{d}\right)}{d^3}+\frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^4\Gamma(3-p)\Gamma(p+1)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) + 0**p*d**(
2*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*log(-1 + e**2*x
**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*acoth(e*x/d)/d**3 + d*e**(2*p)*p*x*
*(2*p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), d*
```

```
*2/(e**2*x**2))/(2*e**2*x**4*gamma(3 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,),
d**2/(e**2*x**2))/(2*e*x**3*gamma(5/2 - p)*gamma(p + 1)), Abs(e**2*x**2/d**
2) > 1), (-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) + 0**p*d**(2
*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*log(1 - e**2*x**
2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*atanh(e*x/d)/d**3 + d*e**(2*p)*p*x**(
2*p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), d**2
/(e**2*x**2))/(2*e**2*x**4*gamma(3 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)
*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d*
*2/(e**2*x**2))/(2*e*x**3*gamma(5/2 - p)*gamma(p + 1)), True))
```

3.275 $\int \frac{x^5\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=179

$$-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^6(p+2)}+\frac{d^6\left(d^2-e^2x^2\right)^{p-1}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^p}{2e^6p}-\frac{2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\right.}{7d^3}$$

```
[Out] d^6*(-e^2*x^2+d^2)^(-1+p)/e^6/(1-p)+5/2*d^4*(-e^2*x^2+d^2)^p/e^6/p-2*d^2*(-
e^2*x^2+d^2)^(1+p)/e^6/(1+p)+1/2*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-2/7*e*x^7*(
-e^2*x^2+d^2)^p*hypergeom([7/2, 2-p],[9/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2
)^p)
```

Rubi [A] time = 0.19, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1652, 446, 77, 12, 365, 364}

$$-\frac{2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{7}{2},2-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7d^3}+\frac{d^6\left(d^2-e^2x^2\right)^{p-1}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^p}{2e^6p}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^6*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (5*d^4*(d^2 - e^2*x^2)^p)/(2
*e^6*p) - (2*d^2*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^2 - e^2*x^2)^(
2 + p)/(2*e^6*(2 + p)) - (2*e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2,
2 - p, 9/2, (e^2*x^2)/d^2])/(7*d^3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \int x^5(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^6\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^5\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)dx\\
&= \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)dx,x,x^2\right)-(2de)\int x^6\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^6\left(d^2-e^2x\right)^{-2+p}}{e^4}-\frac{5d^4\left(d^2-e^2x\right)^{-1+p}}{e^4}+\frac{4d^2\left(d^2-e^2x\right)^p}{e^4}-\frac{\left(d^2-e^2x\right)^{1+p}}{e^4}\right.\right.\\
&= \frac{d^6\left(d^2-e^2x^2\right)^{-1+p}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^p}{2e^6p}-\frac{2d^2\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{2ex^7\left(d^2\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 66, normalized size = 0.37

$$\frac{x^6(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(6;-p,2-p;7;\frac{ex}{d},-\frac{ex}{d}\right)}{6d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x^6*(d - e*x)^p*(d + e*x)^p*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)]
)/(6*d^2*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^5}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^5/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^5}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^5}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\left(d^2-e^2\,x^2\right)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.276 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=184

$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}+\frac{d\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}-\frac{d^5\left(d^2-e^2x^2\right)^{p-1}}{e^5(1-p)}$$

```
[Out] -d^5*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-x^5*(-e^2*x^2+d^2)^(-1+p)/(3+2*p)-2*d^
3*(-e^2*x^2+d^2)^p/e^5/p+d*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+2/5*(4+p)*x^5*(-e
^2*x^2+d^2)^p*hypergeom([5/2, 2-p],[7/2],e^2*x^2/d^2)/d^2/(3+2*p)/((1-e^2*x
^2/d^2)^p)
```

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {852, 1652, 459, 365, 364, 12, 266, 43}

$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}-\frac{d^5\left(d^2-e^2x^2\right)^{p-1}}{e^5(1-p)}-\frac{2d^3\left(d^2-e^2x^2\right)^p}{e^5p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(-1 + p))/(e^5*(1 - p))) - (x^5*(d^2 - e^2*x^2)^(-1
+ p))/(3 + 2*p) - (2*d^3*(d^2 - e^2*x^2)^p)/(e^5*p) + (d*(d^2 - e^2*x^2)^(1
 + p))/(e^5*(1 + p)) + (2*(4 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5
/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(3 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \int x^4(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$

$$= \int -2dex^5\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^4\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - (2de)\int x^5\left(d^2-e^2x^2\right)^{-2+p}\,dx + \frac{\left(2d^2(4+p)\right)\int x^4\left(d^2-e^2x^2\right)^{-}}{3+2p}$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - (de)\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\,dx, x, x^2\right) + \frac{\left(2(4+p)\left(d^2-e^2\right.\right.}{}$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} + \frac{2(4+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)} - (de$$

$$= -\frac{d^5\left(d^2-e^2x^2\right)^{-1+p}}{e^5(1-p)} - \frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - \frac{2d^3\left(d^2-e^2x^2\right)^p}{e^5p} + \frac{d\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)} + \frac{2(4}{}$$

Mathematica [C] time = 0.12, size = 66, normalized size = 0.36

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,2-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 2 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^4}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\left(d^2-e^2\,x^2\right)^p}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.277 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=150

$$\frac{3d^2\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}+\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4(1-p)}-\frac{2ex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}$$

```
[Out] d^4*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)+3/2*d^2*(-e^2*x^2+d^2)^p/e^4/p-1/2*(-e^
2*x^2+d^2)^(1+p)/e^4/(1+p)-2/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 2-p],
[7/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1652, 446, 77, 12, 365, 364}

$$-\frac{2ex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}+\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(2
*e^4*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*(1 + p)) - (2*e*x^5*(d^2 - e^2*x^2
)^p*Hypergeometric2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 - (e^2*x^2
)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \int x^3(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^4\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^3\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)\,dx,x,x^2\right)-(2de)\int x^4\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{2d^4\left(d^2-e^2x\right)^{-2+p}}{e^2}-\frac{3d^2\left(d^2-e^2x\right)^{-1+p}}{e^2}+\frac{\left(d^2-e^2x\right)^p}{e^2}\right)dx,x,x^2\right)-\frac{(2e
\end{aligned}$$

$$= \frac{d^4\left(d^2-e^2x^2\right)^{-1+p}}{e^4(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)}-\frac{2ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)}{5d^3}$$

Mathematica [B] time = 0.29, size = 332, normalized size = 2.21

$$2^{p-2}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-8de(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-6d(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*d^2*(1/2 + (e*x)/(2*d))^p - 2*d^2*(1/2 + (
e*x)/(2*d))^p*(1 - (e^2*x^2)/d^2)^p + 2*e^2*x^2*(1/2 + (e*x)/(2*d))^p*(1 -
(e^2*x^2)/d^2)^p - 8*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[
1/2, -p, 3/2, (e^2*x^2)/d^2] - 6*d*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d^2*(1 - (e^2*x^2)/d^2)^p
*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - d*e*x*(1 - (e^2*
x^2)/d^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^4*
(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}x^{3}}{e^{2}x^{2}+2\,dex+d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}x^{3}}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}x^{3}}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3}\left(d^{2}-e^{2}\,x^{2}\right)^{p}}{\left(d+e\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.278 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=156

$$\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}-\frac{d\left(d^2-e^2x^2\right)^p}{e^3p}-\frac{d^3\left(d^2-e^2x^2\right)^{p-1}}{e^3(1-p)}$$

```
[Out] -d^3*(-e^2*x^2+d^2)^(-1+p)/e^3/(1-p)-x^3*(-e^2*x^2+d^2)^(-1+p)/(1+2*p)-d*(-
e^2*x^2+d^2)^p/e^3/p+2/3*(2+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 2-p],[5
/2],e^2*x^2/d^2)/d^2/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {852, 1652, 459, 365, 364, 12, 266, 43}

$$\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}-\frac{d^3\left(d^2-e^2x^2\right)^{p-1}}{e^3(1-p)}-\frac{d\left(d^2-e^2x^2\right)^p}{e^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(-1 + p))/(e^3*(1 - p))) - (x^3*(d^2 - e^2*x^2)^(-1
+ p))/(1 + 2*p) - (d*(d^2 - e^2*x^2)^p)/(e^3*p) + (2*(2 + p)*x^3*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[3/2, 2 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 + 2*p)
*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2 \left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \int x^2(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \int -2dex^3\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^2\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$
$$= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} - (2de)\int x^3\left(d^2-e^2x^2\right)^{-2+p}\,dx + \frac{\left(2d^2(2+p)\right)\int x^2\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}$$
$$= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} - (de)\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\,dx, x, x^2\right) + \frac{\left(2(2+p)\left(d^2-e^2x^2\right)^p\right.}{}$$
$$= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} + \frac{2(2+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)} - (de)\,\text{Su}$$
$$= -\frac{d^3\left(d^2-e^2x^2\right)^{-1+p}}{e^3(1-p)} - \frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} - \frac{d\left(d^2-e^2x^2\right)^p}{e^3p} + \frac{2(2+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\right.}{3d^2(1}$$

Mathematica [A] time = 0.16, size = 177, normalized size = 1.13

$$\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(4e(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(4\,{}_2F_1\left(1-\right.\right.\right.}{e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(4*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(4
*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F
1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(e^3*(1 + p)*(1 + (e*x)/d)^p*(1
- (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^2}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(d^2-e^2\,x^2\right)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-(-d+ex)(d+ex)\right)^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.279 $\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=115

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(1-p)(d+ex)^2}-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^2\left(1-p^2\right)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(1+p)/e^2/(1-p)/(e*x+d)^2-2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^
2*x^2+d^2)^(1+p)*hypergeom([1-p, 1+p],[2+p],1/2*(-e*x+d)/d)/d^2/e^2/(-p^2+1
)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {793, 678, 69}

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(1-p)(d+ex)^2}-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^2\left(1-p^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(1 - p)*(d + e*x)^2) - (2^(-1 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^2*e^2*(1 - p^2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1-p)(d+ex)^2} + \frac{\int \frac{\left(d^2-e^2x^2\right)^p}{d+ex}\,dx}{e(1-p)}$$

$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1-p)(d+ex)^2} + \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}\,d}{d^2e(1-p)}$$

$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1-p)(d+ex)^2} - \frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^2\left(1-p^2\right)}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.89

$$\frac{2^{p-2}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left({}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)-2\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{de^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
[Out] (2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2 - e^2\, x^2\right)^p}{(d + e\, x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(-\,(-d + ex)\,(d + ex))^p}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.280 $\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

```
[Out] -2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 2-p],[2+p],
1/2*(-e*x+d)/d)/d^3/e/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}\,dx}{d^3}\\ &= -\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.03

$$-\frac{2^{p-2}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(1 + (e*x)/d)^p))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.281 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{(d^2-e^2x^2)^p\ {}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}+\frac{(d^2-e^2x^2)^{p-1}}{1-p}-\frac{2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\ {}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}$$

```
[Out] (-e^2*x^2+d^2)^(-1+p)/(1-p)-2*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p],[3/
2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1,
p],[1+p],1-e^2*x^2/d^2)/d^2/p
```

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {852, 1652, 446, 79, 65, 12, 246, 245}

$$-\frac{2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\ {}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{(d^2-e^2x^2)^p\ {}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}+\frac{(d^2-e^2x^2)^{p-1}}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (d^2 - e^2*x^2)^(-1 + p)/(1 - p) - (2*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^3*(1 - (e^2*x^2)/d^2)^p) - ((d^2 - e
^2*x^2)^p*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx\\
&= \int -2de\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int \frac{\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)}{x}\,dx\\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)}{x}\,dx, x, x^2\right) - (2de)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p} + \frac{1}{2}\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx, x, x^2\right) - \frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (
\end{aligned}$$

$$= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p} - \frac{2ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,\right.}{2d^2}$$

Mathematica [A] time = 0.16, size = 201, normalized size = 1.57

$$\frac{2^{p-2}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(2p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right.\right.}{d^3p(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + p*(1 - d^2/(e^2*x^2))^p*(
```

```
d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*(1 +
 p)*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))
/(d^3*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^3+2\,dex^2+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{x\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**2), x)
```

3.282 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=137

$$-\frac{e\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{x}+\frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{1}{2},2-p\ldots\right.}{d^4}$$

```
[Out] -(-e^2*x^2+d^2)^(-1+p)/x+2*e^2*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p
],[3/2],e^2*x^2/d^2)/d^4/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(-1+p)*hyperg
eom([1, -1+p],[p],1-e^2*x^2/d^2)/d/(1-p)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 266, 65, 246, 245}

$$-\frac{e\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}+\frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{\left(d^2-\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] -((d^2 - e^2*x^2)^(-1 + p)/x) + (2*e^2*(2 - p)*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(1 - (e^2*x^2)/d^2)^p) - (e
*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2
])/(d*(1 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^2}\,dx = \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - \frac{\int \frac{\left(2d^3e-2d^2e^2(2-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx}{d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx + \left(2e^2(2-p)\right)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - (de)\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx, x, x^2\right) + \frac{\left(2e^2(2-p)\left(d^2-e^2x^2\right)^p\left(1-\right.\right.}{d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} + \frac{2e^2(2-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4} - \frac{e\left(d^2-e^2x\right.}{}$$

Mathematica [A] time = 0.33, size = 223, normalized size = 1.63

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{4de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{4d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{e2^{p+2}(ex-d)\left(\frac{ex}{d}+1\right)^{-p}{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{p+1} + \right.}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1 -
 p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d
+ e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) - (4*d*e*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p
*(1 - d^2/(e^2*x^2))^p)))/(4*d^4)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^4+2\,dex^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{x^2\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-(-d+ex)\,(d+ex)\right)^p}{x^2\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**2), x)
```

3.283 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=143

$$\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{2x^2}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(-1+p)/x^2+2*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 2-p],[
1/2],e^2*x^2/d^2)/d^3/x/((1-e^2*x^2/d^2)^p)+1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(-
1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/(1-p)
```

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(2*x^2) + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(d^3*x*(1 - (e^2*x^2)/d^2)^p) + (e^2*(3
 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2
)/d^2])/(2*d^2*(1 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}-\frac{\int \frac{\left(4d^3e-2d^2e^2(3-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx}{2d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}-(2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx+\left(e^2(3-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}+\frac{1}{2}\left(e^2(3-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx,x,x^2\right)-\frac{\left(2e\left(d^2-e^2x^2\right)^p\left(\right.\right.}{} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}+\frac{2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}+\frac{e^2(3-p)\left(d^2-\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 283, normalized size = 1.98

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{6de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}+\frac{8d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{2d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2}+\frac{3e}{}\right.}{4d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (2*d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d
^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d*e^2*Hypergeometric2F1[-p,
 -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(4*d^5)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^5+2\,dex^4+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^5 + 2*d*e*x^4 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{x^3\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-(-d+ex)\,(d+ex))^p}{x^3\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**2), x)
```

3.284 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{(d^2-e^2x^2)^{p-1}}{3x^3}-\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{e^3(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(2,p-1;p;\right.}{d^3(1-p)}$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(-1+p)/x^3-2/3*e^2*(4-p)*(-e^2*x^2+d^2)^p*hypergeom([-1
/2, 2-p],[1/2],e^2*x^2/d^2)/d^4/x/((1-e^2*x^2/d^2)^p)-e^3*(-e^2*x^2+d^2)^(-
1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^3/(1-p)
```

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 266, 65, 365, 364}

$$-\frac{e^3(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}-\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{(d^2-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(3*x^3) - (2*e^2*(4 - p)*(d^2 - e^2*x^2)^p*Hyperg
eometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(3*d^4*x*(1 - (e^2*x^2)/d^2)^
p) - (e^3*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^2
*x^2)/d^2])/(d^3*(1 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^2}\,dx = \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - \frac{\int \frac{\left(6d^3e-2d^2e^2(4-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx}{3d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx + \frac{1}{3}\left(2e^2(4-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - (de)\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x^2}\,dx, x, x^2\right) + \frac{\left(2e^2(4-p)\left(d^2-e^2x^2\right)^p\left(1-\right.\right.}{3d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - \frac{2e^2(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x} - \frac{e^3\left(d^2-e^2\right.}{}$$

Mathematica [B] time = 0.41, size = 334, normalized size = 2.30

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{36d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{24de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{4d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3} - \frac{12\ldots}{}\right.}{12d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (36*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1/
2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (12*d^3*e*Hypergeometric2F1[
1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3
*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1
[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (24*d*e
```

```
^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^
p)))/(12*d^6)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^6+2\,dex^5+d^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^6 + 2*d*e*x^5 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^4\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^4\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**2), x)
```

3.285 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{(d^2-e^2x^2)^{p-1}}{4x^4}+\frac{e^4(5-p)(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{3}{2},2-p\right.}{3d^3x^3}$$

```
[Out] -1/4*(-e^2*x^2+d^2)^(-1+p)/x^4+2/3*e*(-e^2*x^2+d^2)^p*hypergeom([-3/2, 2-p]
,[-1/2],e^2*x^2/d^2)/d^3/x^3/((1-e^2*x^2/d^2)^p)+1/4*e^4*(5-p)*(-e^2*x^2+d^
2)^(-1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^4/(1-p)
```

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$\frac{e^4(5-p)(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3}-\frac{(d^2-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(4*x^4) + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-3/2, 2 - p, -1/2, (e^2*x^2)/d^2])/(3*d^3*x^3*(1 - (e^2*x^2)/d^2)^p) + (e
^4*(5 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^
2*x^2)/d^2])/(4*d^4*(1 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^2}\,dx=\int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^5}\,dx$$
$$=-\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4}-\frac{\int \frac{\left(8d^3e-2d^2e^2(5-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx}{4d^2}$$
$$=-\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4}-(2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx+\frac{1}{2}\left(e^2(5-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx$$
$$=-\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4}+\frac{1}{4}\left(e^2(5-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x^2}\,dx,x,x^2\right)-\frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\right.\right.}{}$$
$$=-\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3}+\frac{e^4(5-p)\left(d^2-e\right.}{}$$

Mathematica [B] time = 0.50, size = 389, normalized size = 2.68

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{30de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}+\frac{48d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{6d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(p-2)x^4}+\frac{8d}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (48*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (18*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (3*2^p*e^4*(d
```

```
- e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(
1 + (e*x)/d)^p) + (30*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]
)/(p*(1 - d^2/(e^2*x^2))^p)))/(12*d^7)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^7+2\,dex^6+d^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^7 + 2*d*e*x^6 + d^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(d^2-e^2\,x^2\right)^p}{x^5\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-(-d+ex)\,(d+ex)\right)^p}{x^5\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**2), x)
```

3.286 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=220

$$-\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{e^5p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^5(p+1)}-\frac{2d^6\left(d^2-e^2x^2\right)^{p-2}}{e^5(2-p)}+\frac{9d^4\left(d^2-e^2x^2\right)^{p-1}}{2e^5(1-p)}+\frac{2(p+8)x^5\left(\right.}{}$$

```
[Out] -2*d^6*(-e^2*x^2+d^2)^(-2+p)/e^5/(2-p)-3*d*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p
)+9/2*d^4*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)+3*d^2*(-e^2*x^2+d^2)^p/e^5/p-1/2*
(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+2/5*(8+p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/
2, 3-p],[7/2],e^2*x^2/d^2)/d^3/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.23, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1652, 459, 365, 364, 446, 77}

$$\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}-\frac{2d^6\left(d^2-e^2x^2\right)^{p-2}}{e^5(2-p)}+\frac{9d^4\left(d^2-\right.}{2e^5(1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^
(-2 + p))/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(-1 + p))/(2*e^5*(1 - p)) + (3
*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^5*(1 + p)) +
 (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^3*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \int x^4(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx\\
&= \int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)dx+\int x^5\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)dx\\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-3+p}\left(-3d^2e-e^3x\right)dx,x,x^2\right)+\frac{(2d^3(8\cdots}{\cdots}\\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{4d^6\left(d^2-e^2x\right)^{-3+p}}{e^3}+\frac{9d^4\left(d^2-e^2x\right)^{-2+p}}{e^3}-\frac{6d^2(\cdots}{\cdots}\right.\right.\\
&= -\frac{2d^6\left(d^2-e^2x^2\right)^{-2+p}}{e^5(2-p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{-1+p}}{2e^5(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{e^5p}-\cdots\end{aligned}$$

Mathematica [A] time = 0.31, size = 245, normalized size = 1.11

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(24de(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(24d\,{}_2\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d^2 - e^2*x^2)^p*(24*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyp
ergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)
^p*(4*d*(1/2 + (e*x)/(2*d))^p + 4*e*x*(1/2 + (e*x)/(2*d))^p + 24*d*Hypergeo
```

```
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 8*d*Hypergeometric2F1[2 -
 p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*Hypergeometric2F1[3 - p, 1 + p, 2 +
p, (d - e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^4}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\left(d^2-e^2\,x^2\right)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.287 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=194

$$\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}-\frac{3d(d^2-e^2x^2)^p}{2e^4p}+\frac{2d^5(d^2-e^2x^2)^{p-2}}{e^4(2-p)}-\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2}\ldots\right.}{5d^4(2p+1)}$$

```
[Out] 2*d^5*(-e^2*x^2+d^2)^(-2+p)/e^4/(2-p)+e*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p)-7
/2*d^3*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)-3/2*d*(-e^2*x^2+d^2)^p/e^4/p-2/5*e*(
4+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 3-p],[7/2],e^2*x^2/d^2)/d^4/(1+
2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1652, 446, 77, 459, 365, 364}

$$-\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}+\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}+\frac{2d^5(d^2-e^2x^2)^{p-2}}{e^4(2-p)}-\frac{7d^3(d^2\ldots}{2e^4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^(-2
 + p))/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^(-1 + p))/(2*e^4*(1 - p)) - (3*d*
(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \int x^3(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx\\
&= \int x^3\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)dx+\int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)dx\\
&= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\operatorname{Subst}\left(\int x\left(d^2-e^2x\right)^{-3+p}\left(d^3+3de^2x\right)dx,x,x^2\right)-\frac{\left(2d^2e(4+3\right.}{}\\
&= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{4d^5\left(d^2-e^2x\right)^{-3+p}}{e^2}-\frac{7d^3\left(d^2-e^2x\right)^{-2+p}}{e^2}+\frac{3d\left(d^2-e\right.}{e^2}\right.\right.\\
&= \frac{2d^5\left(d^2-e^2x^2\right)^{-2+p}}{e^4(2-p)}+\frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}-\frac{7d^3\left(d^2-e^2x^2\right)^{-1+p}}{2e^4(1-p)}-\frac{3d\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{2e(}{}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 202, normalized size = 1.04

$$\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(8e(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(12\,{}_2F_1\left(1\right.\right.\right.}{e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2^(-3 + p)*(d^2 - e^2*x^2)^p*(8*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(1
2*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 6*Hypergeometri
```

```
c2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)])))/(e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^
2)^p)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^3}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(d^2-e^2\,x^2\right)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.288 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=157

$$\frac{2^{p-3}(p+4)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3p(d+ex)^2}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/e^3/(2-p)/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(1+p)/e^
3/p/(e*x+d)^2+2^(-3+p)*(4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeo
m([1+p, 2-p],[2+p],1/2*(-e*x+d)/d)/d^2/e^3/p/(-p^2+p+2)
```

Rubi [A] time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1639, 793, 678, 69}

$$\frac{2^{p-3}(p+4)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3p(d+ex)^2}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(2 - p)*(d + e*x)^3) - (d^2 - e^2*x^2)^
(1 + p)/(2*e^3*p*(d + e*x)^2) + (2^(-3 + p)*(4 + p)*(1 + (e*x)/d)^(-1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2
*d)])/(d^2*e^3*(2 - p)*p*(1 + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}-\frac{\int \frac{\left(2d^2e^2+2de^3(1+p)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{2e^4p}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}-\frac{(d(4+p))\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx}{2e^2(2-p)p}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}-\frac{\left((4+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right.}{2d^2e^2(2-p)p}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}+\frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2\right.}{d^2e^3(2-p)p(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 130, normalized size = 0.83

$$-\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(4\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)-4\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)+{}_2F_1\left(3-\right.\right.}{de^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[1 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e
^3*(1 + p)*(1 + (e*x)/d)^p))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^px^2}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^px^2}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p} x^{2}}{\left(e x+d\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{\left(d + e x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.289 $\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=118

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(2-p)(d+ex)^3}-\frac{3\,2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(1+p)/e^2/(2-p)/(e*x+d)^3-3*2^(-3+p)*(1+e*x/d)^(-1-p)*(-
e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 2-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^2/(-p^2
+p+2)
```

Rubi [A] time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {793, 678, 69}

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(2-p)(d+ex)^3}-\frac{3\,2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(2 - p)*(d + e*x)^3) - (3*2^(-3 + p)*(1 + (e
*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^3*e^2*(2 - p)*(1 + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx = \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(2-p)(d+ex)^3} + \frac{3\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx}{2e(2-p)}$$
$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(2-p)(d+ex)^3} + \frac{\left(3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-2-}}{2d^3e(2-p)}$$
$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(2-p)(d+ex)^3} - \frac{3\,2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(1+p)}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.86

$$\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left({}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)-2\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{d^2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^2*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2 - e^2\,x^2\right)^p}{(d + e\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.290 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

```
[Out] -2^(-3+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 3-p],[2+p],
1/2*(-e*x+d)/d)/d^4/e/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^4}\\ &= -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.03

$$-\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(1 + (e*x)/d)^p))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{p}}{\left(d + ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.291 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$

Optimal. Leaf size=175

$$\frac{(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}+\frac{2d(d^2-e^2x^2)^{p-2}}{2-p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^{p}}{d^4(3-2p)}$$

```
[Out] 2*d*(-e^2*x^2+d^2)^(-2+p)/(2-p)-e*x*(-e^2*x^2+d^2)^(-2+p)/(3-2*p)-2*e*(4-3*
p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^4/(3-2*p)/(
(1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*
x^2/d^2)/d/(1-p)
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {852, 1652, 446, 79, 65, 388, 246, 245}

$$\frac{(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^{p}\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2*d*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (e*x*(d^2 - e^2*x^2)^(-2 + p))/(3
- 2*p) - (2*e*(4 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3
/2, (e^2*x^2)/d^2])/(d^4*(3 - 2*p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2
)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d*(1 - p)
)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)^3}\,dx &= \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx\\
&= \int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)}{x}\,dx+\int \left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)dx\\
&= -\frac{ex\left(d^2-e^2x^2\right)^{-2+p}}{3-2p}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}\left(d^3+3de^2x\right)}{x}\,dx,x,x^2\right)-\frac{\left(2d^2e(4-3p\right.}{}\\
&= \frac{2d\left(d^2-e^2x^2\right)^{-2+p}}{2-p}-\frac{ex\left(d^2-e^2x^2\right)^{-2+p}}{3-2p}+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx,x,x^2\right)-\frac{\left(2e(4\right.}{}\\
&= \frac{2d\left(d^2-e^2x^2\right)^{-2+p}}{2-p}-\frac{ex\left(d^2-e^2x^2\right)^{-2+p}}{3-2p}-\frac{2e(4-3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3\right.}{d^4(3-2p)}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 328, normalized size = 1.87

$$2^{p-3}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(4p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+2p(d-ex)\left(1-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2^(-3 + p)*(d^2 - e^2*x^2)^p*(4*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*p*(1 - d^2/(e^2*x^2))^p
*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*p*(1
 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
 - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)] + 4*d*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^
2/(e^2*x^2)] + 4*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p,
 d^2/(e^2*x^2)]))/(d^4*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**3), x)
```

3.292 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{3e\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}-\frac{2e\left(d^2-e^2x^2\right)^{p-2}}{2-p}-\frac{d\left(d^2-e^2x^2\right)^{p-2}}{x}+\frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right.}{d^5}$$

```
[Out] -2*e*(-e^2*x^2+d^2)^(-2+p)/(2-p)-d*(-e^2*x^2+d^2)^(-2+p)/x+2*e^2*(4-p)*x*(-
e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^5/((1-e^2*x^2/d^2)
^p)-3/2*e*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/
(1-p)
```

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {852, 1807, 1652, 446, 79, 65, 12, 246, 245}

$$-\frac{3e\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}+\frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{2e\left(d^2-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] (-2*e*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (d*(d^2 - e^2*x^2)^(-2 + p))/x +
(2*e^2*(4 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*
x^2)/d^2])/(d^5*(1 - (e^2*x^2)/d^2)^p) - (3*e*(d^2 - e^2*x^2)^(-1 + p)*Hype
rgeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 - p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(d-ex)^3(d^2-e^2x^2)^{-3+p}}{x^2}\,dx$$
$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(3d^4e-2d^3e^2(4-p)x+d^2e^3x^2)}{x}\,dx}{d^2}$$
$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\int -2d^3e^2(4-p)(d^2-e^2x^2)^{-3+p}\,dx}{d^2} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(3d^4e+d^2e^3x^2)}{x}\,dx}{d^2}$$
$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}(3d^4e+d^2e^3x)}{x}\,dx, x, x^2\right)}{2d^2} + \left(2de^2(4-p)\right)\int \left(d^2-e^2x^2\right.$$
$$= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{1}{2}(3e)\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x}\,dx, x, x^2\right) + \frac{\left(2e^2(\right.}{}$$
$$= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{d(d^2-e^2x^2)^{-2+p}}{x} + \frac{2e^2(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},3-\right.}{d^5}$$

Mathematica [A] time = 0.43, size = 280, normalized size = 1.69

$$\frac{(d^2-e^2x^2)^p\left(-\frac{12de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\,{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{8d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{3e2^{p+2}(ex-d)\left(\frac{ex}{d}+1\right)^{-p}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{p+1}\right.}{8d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (3*2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1
 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^(2 + p
)*e*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (12*d*e*Hypergeometric
2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^5)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-e^2x^2+d^2)^p}{e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{3} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{{\left(e x + d\right)}^{3} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{x^2\,{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{x^{2} \left(d + e x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**3), x)
```

3.293 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$

Optimal. Leaf size=173

$$\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}+\frac{3e\left(d^2-e^2x^2\right)^{p-2}}{x}-\frac{d\left(d^2-e^2x^2\right)^{p-2}}{2x^2}-\frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right.}{}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(-2+p)/x^2+3*e*(-e^2*x^2+d^2)^(-2+p)/x-2*e^3*(8-3*p)*
x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^6/((1-e^2*x^2/
d^2)^p)+1/2*e^2*(6-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^
2*x^2/d^2)/d/(2-p)
```

Rubi [A] time = 0.27, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 266, 65, 246, 245}

$$\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}-\frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(2*x^2) + (3*e*(d^2 - e^2*x^2)^(-2 + p))/x -
(2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^
2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p)
)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(6d^4e-2d^3e^2(6-p)x+2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} + \frac{\int \frac{\left(2d^5e^2(6-p)-4d^4e^3(8-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx}{2d^4}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} - \left(2e^3(8-3p)\right)\int \left(d^2-e^2x^2\right)^{-3+p}\,dx + \left(de^2(6\right.$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} + \frac{1}{2}\left(de^2(6-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,dx, x\right.$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} - \frac{2e^3(8-3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},\right.}{d^6}$$

Mathematica [A] time = 0.65, size = 341, normalized size = 1.97

$$\left(d^2-e^2x^2\right)^p\left(\frac{24de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{24d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{4d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((24*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (4*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(3 + p)*e^2*(d
```

```
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hy
pergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (24*d*e^2*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 -
d^2/(e^2*x^2))^p)))/(8*d^6)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^3x^6+3\,de^2x^5+3\,d^2ex^4+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^3\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^3(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**3), x)
```

3.294 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

Optimal. Leaf size=179

$$\frac{3e\left(d^2-e^2x^2\right)^{p-2}}{2x^2}-\frac{d\left(d^2-e^2x^2\right)^{p-2}}{3x^3}-\frac{e^3(10-3p)\left(d^2-e^2x^2\right)^{p-2}\ {}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right.}{}$$

```
[Out] -1/3*d*(-e^2*x^2+d^2)^(-2+p)/x^3+3/2*e*(-e^2*x^2+d^2)^(-2+p)/x^2-2/3*e^2*(8
-p)*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^5/x/((1-e^2
*x^2/d^2)^p)-1/2*e^3*(10-3*p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1
+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

Rubi [A] time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$-\frac{e^3(10-3p)\left(d^2-e^2x^2\right)^{p-2}\ {}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\ {}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2}{d}\right.}{3d^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(3*x^3) + (3*e*(d^2 - e^2*x^2)^(-2 + p))/(2*x
^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2,
(e^2*x^2)/d^2])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^
2*x^2)^(-2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2
*d^2*(2 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^4}\,dx$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(9d^4e-2d^3e^2(8-p)x+3d^2e^3x^2\right)}{x^3}\,dx}{3d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{\int \frac{\left(4d^5e^2(8-p)-6d^4e^3(10-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx}{6d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \left(e^3(10-3p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx + \frac{1}{3}\left(2de^2\right.$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \frac{1}{2}\left(e^3(10-3p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,dx,\right.$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \frac{2e^2(8-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},\right.}{3d^5x}$$

Mathematica [B] time = 0.50, size = 393, normalized size = 2.20

$$\left(d^2-e^2x^2\right)^p\left(-\frac{144d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{120de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{8d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (144*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1
```

```
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (36*d^3*e*Hypergeometric2F1
[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (
15*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x
)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeom
etric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) +
 (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*
d)])/((1 + p)*(1 + (e*x)/d)^p) - (120*d*e^3*Hypergeometric2F1[-p, -p, 1 - p
, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(24*d^7)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{e^{3}x^{7}+3\,de^{2}x^{6}+3\,d^{2}ex^{5}+d^{3}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{3}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{3}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{3}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{x^4\,{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^4(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**3), x)
```

3.295 $\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^3}\,dx$

Optimal. Leaf size=174

$$-\frac{d\left(d^2-e^2x^2\right)^{p-2}}{4x^4}+\frac{e\left(d^2-e^2x^2\right)^{p-2}}{x^3}+\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}+\frac{e^4(10-p)\left(d^2-\right.}{}$$

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(-2+p)/x^4+e*(-e^2*x^2+d^2)^(-2+p)/x^3+2*e^3*(4-p)*(-
e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^6/x/((1-e^2*x^2/d
^2)^p)+1/4*e^4*(10-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([2, -2+p],[-1+p],1-e^
2*x^2/d^2)/d^3/(2-p)
```

Rubi [A] time = 0.28, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 266, 65, 365, 364}

$$\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2} {}_2F_1\left(2,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}+\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(4*x^4) + (e*(d^2 - e^2*x^2)^(-2 + p))/x^3 +
(2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x
^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 -
p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^5}\,dx$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(12d^4e-2d^3e^2(10-p)x+4d^2e^3x^2\right)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{\int \frac{\left(6d^5e^2(10-p)-24d^4e^3(4-p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx}{12d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} - \left(2e^3(4-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx + \frac{1}{2}\left(de^2(10$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{1}{4}\left(de^2(10-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x^2}\,dx, x,\right.$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{2e^3(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},3\right.}{d^6x}$$

Mathematica [B] time = 0.58, size = 446, normalized size = 2.56

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{60de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{80d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{4d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(p-2)x^4} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (80*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
```

```
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (24*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(2 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (4*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (5*2^(1 + p)*
e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (2^p*e^4*(d - e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (60*d*e^4*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^8)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2 + d^2\right)^p}{e^3x^8 + 3\,de^2x^7 + 3\,d^2ex^6 + d^3x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex + d)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex + d)^3\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex + d)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^p}{x^5\,\left(d + e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^5(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**3), x)
```

3.296 $\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=265

$$\frac{d^2(12p+13)x^5\left(d^2-e^2x^2\right)^{p-3}}{1-4p^2}-\frac{e^2x^7\left(d^2-e^2x^2\right)^{p-3}}{2p+1}-\frac{2d\left(d^2-e^2x^2\right)^p}{e^5p}-\frac{4d^7\left(d^2-e^2x^2\right)^{p-3}}{e^5(3-p)}+\frac{10d^5\left(d^2-e^2x^2\right)^{p-2}}{e^5(2-p)}-\frac{4}{}$$

```
[Out] -4*d^7*(-e^2*x^2+d^2)^(-3+p)/e^5/(3-p)+d^2*(13+12*p)*x^5*(-e^2*x^2+d^2)^(-3
+p)/(-4*p^2+1)-e^2*x^7*(-e^2*x^2+d^2)^(-3+p)/(1+2*p)+10*d^5*(-e^2*x^2+d^2)^
(-2+p)/e^5/(2-p)-8*d^3*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-2*d*(-e^2*x^2+d^2)^p
/e^5/p-4/5*(p^2+15*p+16)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 4-p],[7/2],e^
2*x^2/d^2)/d^4/(-4*p^2+1)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.31, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {852, 1652, 1267, 459, 365, 364, 446, 77}

$$-\frac{4\left(p^2+15p+16\right)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},4-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4\left(1-4p^2\right)}+\frac{d^2(12p+13)x^5\left(d^2-e^2x^2\right)^{p-3}}{1-4p^2}-\frac{e^2x^7\left(d^2-e^2x^2\right)^{p-3}}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (-4*d^7*(d^2 - e^2*x^2)^(-3 + p))/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2
 - e^2*x^2)^(-3 + p))/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^(-3 + p))/(1 +
 2*p) + (10*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (8*d^3*(d^2 - e^2
*x^2)^(-1 + p))/(e^5*(1 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 +
15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = \int x^4(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}\,dx$$

$$= \int x^5\left(d^2-e^2x^2\right)^{-4+p}\left(-4d^3e-4de^3x^2\right)dx + \int x^4\left(d^2-e^2x^2\right)^{-4+p}\left(d^4+6d^2e^2x^2+e^4x\right.$$

$$= -\frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{1}{2}\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-4+p}\left(-4d^3e-4de^3x\right)dx, x, x^2\right) - \int$$

$$= \frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2} - \frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{1}{2}\mathrm{Subst}\left(\int\left(-\frac{8d^7\left(d^2-e^2x\right.}{e^3}\right.\right.$$

$$= -\frac{4d^7\left(d^2-e^2x^2\right)^{-3+p}}{e^5(3-p)} + \frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2} - \frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{10d^5}{}$$

$$= -\frac{4d^7\left(d^2-e^2x^2\right)^{-3+p}}{e^5(3-p)} + \frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2} - \frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{10d^5}{}$$

Mathematica [A] time = 0.29, size = 231, normalized size = 0.87

$$2^{p-4}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(16e(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(32\,{}_2F_1\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d^2 - e^2*x^2)^p*(16*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperge
ometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(
32*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 24*Hypergeomet
ric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*Hypergeometric2F1[3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^4}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\frac{x^4\left(-e^2x^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(-\left(-d+ex\right) \left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.297 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=211

$$\frac{3\,2^{p-2}(p+2)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}-\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e^4(1-2p)p(d+ex)^3}$$

```
[Out] 1/2*d^2*(-e^2*x^2+d^2)^(1+p)/e^4/(3-p)/(e*x+d)^4-d*(1+2*p)*(-e^2*x^2+d^2)^(
1+p)/e^4/(1-2*p)/p/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(1+p)/e^4/p/(e*x+d)^2+3*2^(
-2+p)*(2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 3-p],[2+p
],1/2*(-e*x+d)/d)/d^2/e^4/p/(2*p^3-5*p^2-4*p+3)
```

Rubi [A] time = 0.38, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1639, 793, 678, 69}

$$\frac{3\,2^{p-2}(p+2)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}-\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e^4(1-2p)p(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d
^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 +
 p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d
^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(2d^3e^2+2d^2e^3(2+p)x+2de^4(1+2p)x^2\right)}{(d+ex)^4}\,dx}{2e^5p}$$

$$= -\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\int \frac{\left(8d^3e^6(1+p)+2d^2e^7\left(4+3p+2p^2\right)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx}{2e^9(1-2p)p}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\left(6d^2(2+p)\right)\int \frac{(d^2-}{(d+}}{e^3(1-2p)(3-}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\left(6(2+p)(d-ex)^{-1}\right.}{}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} + \frac{3\,2^{-2+p}(2+p)\left(1+\right.}{}$$

Mathematica [A] time = 0.20, size = 156, normalized size = 0.74

$$\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(-8\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+12\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)-6\,{}_2F\right.}{de^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-8*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + 12*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] - 6*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hy
pergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e^4*(1 + p)*(1 +
 (e*x)/d)^p)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^3}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p} x^{3}}{{\left(e x + d\right)}^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{\left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.298 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=163

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^3(1-2p)(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(3-p)(d+ex)^4}-\frac{2^{p-3}(p+7)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/e^3/(3-p)/(e*x+d)^4+(-e^2*x^2+d^2)^(1+p)/e^3/(1
-2*p)/(e*x+d)^3-2^(-3+p)*(7+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hyperg
eom([1+p, 3-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^3/(2*p^3-5*p^2-4*p+3)
```

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1639, 793, 678, 69}

$$-\frac{2^{p-3}(p+7)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^3(1-2p)(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(3-p)(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2)^
(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(-1
 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = \frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{\int \frac{\left(3d^2e^2+2de^3(1+p)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx}{e^4(1-2p)}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{(d(7+p))\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{e^2(1-2p)(3-p)}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{\left((7+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right.\right.}{d^3e^2(1-2p)(3}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3} - \frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F}{d^3e^3(1-2p)(3-p)(1}$$

Mathematica [A] time = 0.11, size = 130, normalized size = 0.80

$$-\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(4\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)-4\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)+\,{}_2F_1\left(4-\right.\right.}{d^2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[2 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d^2
*e^3*(1 + p)*(1 + (e*x)/d)^p))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^2}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p} x^{2}}{\left(e x+d\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{\left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.299 $\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=118

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(3-p)(d+ex)^4}-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(1+p)/e^2/(3-p)/(e*x+d)^4-2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^
2*x^2+d^2)^(1+p)*hypergeom([1+p, 3-p],[2+p],1/2*(-e*x+d)/d)/d^4/e^2/(-p^2+2
*p+3)
```

Rubi [A] time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {793, 678, 69}

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(3-p)(d+ex)^4}-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(3 - p)*(d + e*x)^4) - (2^(-2 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^4*e^2*(3 - p)*(1 + p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(3-p)(d+ex)^4} + \frac{2\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{e(3-p)}$$

$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(3-p)(d+ex)^4} + \frac{\left(2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3-}}{d^4e(3-p)}$$

$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(3-p)(d+ex)^4} - \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\ {}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(1+p)}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.86

$$\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left({}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)-2\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{d^3e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[3 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^3*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^
3*e*x + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x\left(-e^2x^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x}{(ex + d)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2 - e^2\,x^2\right)^p}{(d + e\,x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{(d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.300 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

```
[Out] -2^(-4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 4-p],[2+p],
1/2*(-e*x+d)/d)/d^5/e/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-4+p}\,dx}{d^5}\\ &= -\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^5e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.03

$$-\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^{p}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[4 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)*(1 + (e*x)/d)^p))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

3.301 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{(d^2-e^2x^2)^{p-2}\ {}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}-\frac{4dex\,(d^2-e^2x^2)^{p-3}}{5-2p}+\frac{4d^2\,(d^2-e^2x^2)^{p-3}}{3-p}-\frac{(d^2-e^2x^2)^{p-2}}{2(2-p)}-\frac{8e(2-p)x}{}$$

```
[Out] 4*d^2*(-e^2*x^2+d^2)^(-3+p)/(3-p)-4*d*e*x*(-e^2*x^2+d^2)^(-3+p)/(5-2*p)-1/2
*(-e^2*x^2+d^2)^(-2+p)/(2-p)-8*e*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 4
-p],[3/2],e^2*x^2/d^2)/d^5/(5-2*p)/((1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(
-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/(2-p)
```

Rubi [A] time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {852, 1652, 1251, 951, 79, 65, 388, 246, 245}

$$\frac{(d^2-e^2x^2)^{p-2}\ {}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\ {}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}-\frac{4dex\,(d^2}{5-}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

```
[Out] (4*d^2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (4*d*e*x*(d^2 - e^2*x^2)^(-3 + p
))/(5 - 2*p) - (d^2 - e^2*x^2)^(-2 + p)/(2*(2 - p)) - (8*e*(2 - p)*x*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(5 - 2*
p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2)^(-2 + p)*Hypergeometric2F1[1,
-2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*(2 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x}\,dx$$
$$= \int \left(d^2-e^2x^2\right)^{-4+p}\left(-4d^3e-4de^3x^2\right)dx + \int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(d^4+6d^2e^2x^2+e^4x^4\right)}{x}\,dx$$
$$= -\frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} + \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-4+p}\left(d^4+6d^2e^2x+e^4x^2\right)}{x}\,dx,x,x^2\right) - \frac{\left(8d^3e\right.}{}$$
$$= -\frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} - \frac{\left(d^2-e^2x^2\right)^{-2+p}}{2(2-p)} - \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-4+p}\left(-d^4e^4(2-p)-7d^2e^6(2-p)x\right)}{x}\,dx,x,x\right.}{2e^4(2-p)}$$
$$= \frac{4d^2\left(d^2-e^2x^2\right)^{-3+p}}{3-p} - \frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} - \frac{\left(d^2-e^2x^2\right)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x\left(d^2-e^2x^2\right)^p\left(1-\right.}{d^5(5}$$
$$= \frac{4d^2\left(d^2-e^2x^2\right)^{-3+p}}{3-p} - \frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} - \frac{\left(d^2-e^2x^2\right)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x\left(d^2-e^2x^2\right)^p\left(1-\right.}{d^5(5}$$

Mathematica [B] time = 0.28, size = 417, normalized size = 2.04

$$\frac{2^{p-4}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(8p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p {}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+4p(d-ex)\left(1-\frac{d^2}{e^2x}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

```
[Out] (2^(-4 + p)*(d^2 - e^2*x^2)^p*(8*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 4*p*(1 - d^2/(e^2*x^2))^p
*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*p*
(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)] - 2*e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)] + d*p*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[4 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[4
- p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*d*(1/2 + (e*x)/(2*d))^p*Hypergeomet
ric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)] + 8*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^5*p*(1 + p)*(1 - d^2/(e^2*x^2)
)^p*(1 + (e*x)/d)^p)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^5+4\,de^3x^4+6\,d^2e^2x^3+4\,d^3ex^2+d^4x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*
e*x^2 + d^4*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{4} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{4} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**4), x)
```

3.302 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4} dx$

Optimal. Leaf size=207

$$-\frac{2e\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}+\frac{e^2x\left(d^2-e^2x^2\right)^{p-3}}{5-2p}-\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3-p}-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{x}+\frac{4e^2\left(\right.}{}$$

```
[Out] -4*d*e*(-e^2*x^2+d^2)^(-3+p)/(3-p)-d^2*(-e^2*x^2+d^2)^(-3+p)/x+e^2*x*(-e^2*
x^2+d^2)^(-3+p)/(5-2*p)+4*e^2*(p^2-9*p+16)*x*(-e^2*x^2+d^2)^p*hypergeom([1/
2, 4-p],[3/2],e^2*x^2/d^2)/d^6/(5-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(-e^2*x^2+d^
2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d/(2-p)
```

Rubi [A] time = 0.28, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {852, 1807, 1652, 446, 79, 65, 388, 246, 245}

$$\frac{4e^2\left(p^2-9p+16\right)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}-\frac{2e\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

```
[Out] (-4*d*e*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (d^2*(d^2 - e^2*x^2)^(-3 + p))/
x + (e^2*x*(d^2 - e^2*x^2)^(-3 + p))/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(
5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^(-2 + p)*Hypergeomet
ric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx = \int \frac{(d-ex)^4(d^2-e^2x^2)^{-4+p}}{x^2}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(4d^5e-d^4e^2(13-2p)x+4d^3e^3x^2-d^2e^4x^3)}{x}dx}{d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(4d^5e+4d^3e^3x^2)}{x}dx}{d^2} - \frac{\int (d^2-e^2x^2)^{-4+p}\left(-d^4e^2(13-2p)-\right.}{d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}(4d^5e+4d^3e^3x)}{x}dx, x, x^2\right)}{2d^2} + \frac{(4}{}$$

$$= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} - (2de)\,\text{Subst}\left(\int \frac{(d^2-e^2}{x}\right.$$

$$= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} + \frac{4e^2(16-9p+p^2)x(d^2}{}$$

Mathematica [A] time = 0.44, size = 337, normalized size = 1.63

$$\frac{(d^2-e^2x^2)^p\left(-32de(p+1)x\left(1-\frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right) - 16d^2p(p+1)\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-16*d^2*p*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^
2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2^(5 + p)*e*p*x*(-d + e*x)*Hypergeome
tric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (3*2^(2 +
p)*e*p*x*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)/(1 + (e*x)/d)^p + (2^(2 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[3 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (2^p*e*p*x*(-d + e*x)*Hyper
geometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p - (32*d
*e*(1 + p)*x*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(1 - d^2/(e^2
*x^2))^p))/(16*d^6*p*(1 + p)*x)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-e^2x^2+d^2)^p}{e^4x^6+4\,de^3x^5+6\,d^2e^2x^4+4\,d^3ex^3+d^4x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3*
e*x^3 + d^4*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{4} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{4} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^{2}-e^{2}\, x^{2}\right)^{p}}{x^{2}\, \left(d+e\, x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{2}\left(d+ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**4), x)
```

3.303 $\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=211

$$\frac{e^2(10-p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}+\frac{e^2(11-p)\left(d^2-e^2x^2\right)^{p-3}}{2(3-p)}+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{x}-\frac{d^2\left(d^2-e^2\right.}{2x^2}$$

```
[Out] 1/2*e^2*(11-p)*(-e^2*x^2+d^2)^(-3+p)/(3-p)-1/2*d^2*(-e^2*x^2+d^2)^(-3+p)/x^
2+4*d*e*(-e^2*x^2+d^2)^(-3+p)/x-8*e^3*(4-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1
/2, 4-p],[3/2],e^2*x^2/d^2)/d^7/((1-e^2*x^2/d^2)^p)+1/2*e^2*(10-p)*(-e^2*x^
2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

Rubi [A] time = 0.37, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {852, 1807, 1652, 446, 79, 65, 12, 246, 245}

$$\frac{e^2(10-p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] (e^2*(11 - p)*(d^2 - e^2*x^2)^(-3 + p))/(2*(3 - p)) - (d^2*(d^2 - e^2*x^2)^
(-3 + p))/(2*x^2) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/x - (8*e^3*(4 - p)*x*(
d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^7*(1
 - (e^2*x^2)/d^2)^p) + (e^2*(10 - p)*(d^2 - e^2*x^2)^(-2 + p)*Hypergeometri
c2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x^3}\,dx$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(8d^5e-2d^4e^2(10-p)x+8d^3e^3x^2-2d^2e^4x^3\right)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{2x^2} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{x} + \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(2d^6e^2(10-p)-16d^5e^3(4-p)x+2d^4e^4x^2\right)}{x}\,dx}{2d^4}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{2x^2} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{x} + \frac{\int -16d^5e^3(4-p)\left(d^2-e^2x^2\right)^{-4+p}\,dx}{2d^4} + \frac{\int \frac{\left(d^2-e\right.}{}}{}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{2x^2} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{x} + \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-4+p}\left(2d^6e^2(10-p)+2d^4e^4x\right)}{x}\,dx,x,x^2\right.}{4d^4}$$

$$= \frac{e^2(11-p)\left(d^2-e^2x^2\right)^{-3+p}}{2(3-p)} - \frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{2x^2} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{x} + \frac{1}{2}\left(e^2(10-p)\right)\text{Subs}$$

$$= \frac{e^2(11-p)\left(d^2-e^2x^2\right)^{-3+p}}{2(3-p)} - \frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{2x^2} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{x} - \frac{8e^3(4-p)x\left(d^2-e^2\right.}{}$$

Mathematica [A] time = 0.77, size = 399, normalized size = 1.89

$$\left(d^2-e^2x^2\right)^p\left(\frac{80de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{64d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{8d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \frac{5e^2\ldots}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (8*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (5*2^(4 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(3 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (80*d*e^2*Hypergeometric2F1[-p
, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(16*d^7)
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^7+4\,de^3x^6+6\,d^2e^2x^5+4\,d^3ex^4+d^4x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^7 + 4*d*e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3*
e*x^4 + d^4*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex+d)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex+d)^4\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex+d)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2 - e^2\,x^2\right)^p}{x^3\,\left(d + e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{x^3\left(d + ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**4), x)
```

3.304 $\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$-\frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{p-3}}{3x}+\frac{2de\left(d^2-e^2x^2\right)^{p-3}}{x^2}-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{3x^3}-\frac{2e^3(5-p)\left(d^2-e^2x^2\right)^{p-3}\,{}_2F_1\left(1,p-3;p-2;\right.}{d(3-p)}$$

```
[Out] -1/3*d^2*(-e^2*x^2+d^2)^(-3+p)/x^3+2*d*e*(-e^2*x^2+d^2)^(-3+p)/x^2-1/3*e^2*
(27-2*p)*(-e^2*x^2+d^2)^(-3+p)/x+4/3*e^4*(p^2-17*p+48)*x*(-e^2*x^2+d^2)^p*h
ypergeom([1/2, 4-p],[3/2],e^2*x^2/d^2)/d^8/((1-e^2*x^2/d^2)^p)-2*e^3*(5-p)*
(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d/(3-p)
```

Rubi [A] time = 0.39, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {852, 1807, 764, 266, 65, 246, 245}

$$\frac{4e^4\left(p^2-17p+48\right)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3d^8}-\frac{2e^3(5-p)\left(d^2-e^2x^2\right)^{p-3}\,{}_2F_1\left(1,p-3;p-\right.}{d(3-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] -(d^2*(d^2 - e^2*x^2)^(-3 + p))/(3*x^3) + (2*d*e*(d^2 - e^2*x^2)^(-3 + p))/
x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^(-3 + p))/(3*x) + (4*e^4*(48 - 17*p +
 p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2]
)/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^(-3 + p)*H
ypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^4}\,dx=\int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x^4}\,dx$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(12d^5e-d^4e^2(27-2p)x+12d^3e^3x^2-3d^2e^4x^3\right)}{x^3}\,dx}{3d^2}$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3}+\frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2}+\frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(2d^6e^2(27-2p)-24d^5e^3(5-p)x+6d^4e^4x^2\right.}{x^2}}{6d^4}$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3}+\frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2}-\frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x}-\int \frac{\left(24d^7e^3(5-p)\right.}{}$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3}+\frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2}-\frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x}-\left(4de^3(5-p)\right)$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3}+\frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2}-\frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x}-\left(2de^3(5-p)\right)$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3}+\frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2}-\frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x}+\frac{4e^4\left(48-17p\right.}{}$$

Mathematica [B] time = 0.62, size = 452, normalized size = 2.15

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{480d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{480de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}-\frac{16d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right.}{x^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-16*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (480*d^2*e^2*Hypergeometric2F1[-1/2, -p,
1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (96*d^3*e*Hypergeometric2F
1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
(15*2^(5 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(3 + p)*e^3*(-d + e*x)*Hyperge
ometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p)
 + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometr
ic2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (4
80*d*e^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*
x^2))^p)))/(48*d^8)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^8+4\,de^3x^7+6\,d^2e^2x^6+4\,d^3ex^5+d^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^8 + 4*d*e^3*x^7 + 6*d^2*e^2*x^6 + 4*d^3*
e*x^5 + d^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^4\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^4(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**4), x)
```

3.305 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4} dx$

Optimal. Leaf size=216

$$-\frac{e^2(17-p)\left(d^2-e^2x^2\right)^{p-3}}{4x^2}-\frac{d^2\left(d^2-e^2x^2\right)^{p-3}}{4x^4}+\frac{4de\left(d^2-e^2x^2\right)^{p-3}}{3x^3}+\frac{e^4\left(p^2-21p+70\right)\left(d^2-e^2x^2\right)^{p-3}\,{}_2F_1\left(1,p-3\right.}{4d^2(3-p)}$$

```
[Out] -1/4*d^2*(-e^2*x^2+d^2)^(-3+p)/x^4+4/3*d*e*(-e^2*x^2+d^2)^(-3+p)/x^3-1/4*e^
2*(17-p)*(-e^2*x^2+d^2)^(-3+p)/x^2+8/3*e^3*(6-p)*(-e^2*x^2+d^2)^p*hypergeom
([-1/2, 4-p],[1/2],e^2*x^2/d^2)/d^7/x/((1-e^2*x^2/d^2)^p)+1/4*e^4*(p^2-21*p
+70)*(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d^2/(3
-p)
```

Rubi [A] time = 0.41, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$\frac{e^4\left(p^2-21p+70\right)\left(d^2-e^2x^2\right)^{p-3}\,{}_2F_1\left(1,p-3;p-2;1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(-\frac{1}{2},4-p\right.}{3d^7x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] -(d^2*(d^2 - e^2*x^2)^(-3 + p))/(4*x^4) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/
(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^(-3 + p))/(4*x^2) + (8*e^3*(6 - p)*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^
7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^(-3 + p
)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x^5}\,dx$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(16d^5e-2d^4e^2(17-p)x+16d^3e^3x^2-4d^2e^4x^3\right)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4x^4} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(6d^6e^2(17-p)-32d^5e^3(6-p)x+12d^4e^4x^2\right.}{x^3}}{12d^4}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4x^4} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} - \frac{e^2(17-p)\left(d^2-e^2x^2\right)^{-3+p}}{4x^2} - \int \frac{(64d^7e^3(6-p)-1}{}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4x^4} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} - \frac{e^2(17-p)\left(d^2-e^2x^2\right)^{-3+p}}{4x^2} - \frac{1}{3}\left(8de^3(6-p)\right.$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4x^4} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} - \frac{e^2(17-p)\left(d^2-e^2x^2\right)^{-3+p}}{4x^2} + \frac{1}{4}\left(e^4\left(70-21\right.\right.$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{4x^4} + \frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} - \frac{e^2(17-p)\left(d^2-e^2x^2\right)^{-3+p}}{4x^2} + \frac{8e^3(6-p)\left(d^2\right.}{}$$

Mathematica [B] time = 0.74, size = 505, normalized size = 2.34

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{840de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{960d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{24d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right.}{(p-2)x^4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (960*d^2*e^3*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (240*d^3*e^2*Hypergeometr
ic2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2
) + (105*2^(3 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
- e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (24*d^5*Hypergeometric2F1[2 - p,
 -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (45*2^(2
 + p)*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1
[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*
e^4*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (840*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^
2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(48*d^9)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^9+4\,de^3x^8+6\,d^2e^2x^7+4\,d^3ex^6+d^4x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^9 + 4*d*e^3*x^8 + 6*d^2*e^2*x^7 + 4*d^3*
e*x^6 + d^4*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^5\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^5\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**4), x)
```

3.306 $\int (gx)^m (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=264

$$\frac{2d^2e(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}-\frac{3d(gx)^{m+}}{g(m}$$

[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(1+p)/g/(3+m+2*p)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^(1+p)/g^2/(4+m+2*p)+2*d^3*(3+2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3+m+2*p)/((1-e^2*x^2/d^2)^p)+2*d^2*e*(7+2*m+3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/(4+m+2*p)/((1-e^2*x^2/d^2)^p)

Rubi [A] time = 0.37, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1809, 808, 365, 364}

$$\frac{2d^2e(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}+\frac{2d^3(2m+}{}$$

Antiderivative was successfully verified.

[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]

[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p)) - (e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^(1 + p))/(g^2*(4 + m + 2*p)) + (2*d^3*(3 + 2*m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e*(7 + 2*m + 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(4 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p)

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m] && !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q - 1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m + q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
```

```
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} - \frac{\int (gx)^m\left(d^2-e^2x^2\right)^p\left(-d^3e^2(4+m+2p)-2d\right.}{e^2(4+m}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\int (gx)^m\left(2d^3e^4(3\right.}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\left(2d^3(3+2m+p)\right.}{3}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\left(2d^3(3+2m+p)\right.}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{2d^3(3+2m+p)(}{}$$

Mathematica [A] time = 0.17, size = 194, normalized size = 0.73

$$x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(\frac{3d^2\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(\frac{m+3}{2},-p;\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}+\frac{ex\,{}_2F_1\left(\right.}{}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p,
(4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2,
 -p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2,
 -p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(1 - (e^2*x^2)/d^2)^p
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p*(g*x)
^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(gx\right)^m \left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d^2-e^2\,x^2\right)^p\,\left(g\,x\right)^m\,\left(d+e\,x\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3, x)
```

sympy [C] time = 25.29, size = 262, normalized size = 0.99

$$\frac{d^3d^{2p}g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3d^2d^{2p}eg^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{3dd^{2p}e^2g^m x^3 x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
[Out] d**3*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**2*d**(2*
p)*e*g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x*
*2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + 3*d*d**(2*p)*e**2*g**m*x**3
*x**m*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/(2*gamma(m/2 + 5/2)) + d**(2*p)*e**3*g**m*x**4*x**m*gamm
a(m/2 + 2)*hyper((-p, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/(2*gamma(m/2 + 3))
```

3.307 $\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=206

$$\frac{2de(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{2d^2(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}}{g(m+1)(m+2p+3)}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(1+p)/g/(3+m+2*p)+2*d^2*(2+m+p)*(g*x)^(1+m)*(-e
^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3
+m+2*p)/((1-e^2*x^2/d^2)^p)+2*d*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-
p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.17, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1809, 808, 365, 364}

$$\frac{2de(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{2d^2(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}}{g(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))) + (2*d^2*(2 +
m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3
+ m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (
2*d*e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{\int (gx)^m\left(-2d^2e^2(2+m+p)-2de^3(3+m+2p)x\right)}{e^2(3+m+2p)}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g} + \frac{\left(2d^2(2+m+p\right.}{}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right.}{g}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{2d^2(2+m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,2}{g(1+m)(3+m+2p)}$$

Mathematica [A] time = 0.09, size = 169, normalized size = 0.82

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{m+2}{2},-p\right.\right.\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, -p, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric
2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeometric2F
1[(3 + m)/2, -p, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*(1 -
 (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(-e^2x^2+d^2\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(gx\right)^m\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p (gx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d^2-e^2\,x^2\right)^p\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2, x)
```

sympy [C] time = 16.86, size = 192, normalized size = 0.93

$$\frac{d^2d^{2p}g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{dd^{2p}eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{d^{2p}e^2g^mx^3x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d*d**(2*p)*e*
g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*ex
p_polar(2*I*pi)/d**2)/gamma(m/2 + 2) + d**(2*p)*e**2*g**m*x**3*x**m*gamma(m
/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/
d**2)/(2*gamma(m/2 + 5/2))
```

3.308 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=153

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2}\right.}{g(m+1)}$$

```
[Out] d*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^
2/d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom
([-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {808, 365, 364}

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2}\right.}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^(2 + m)*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2])
/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int (gx)^m\left(d^2-e^2x^2\right)^p\,dx + \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g}$$

$$= \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx + \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\right.\right.}{}$$

$$= \frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(d^2\right.}{}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.76

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(m+2)\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3
 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 +
m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)\,(gx)^m\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d^2-e^2\,x^2\right)^p\,\left(g\,x\right)^m\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x), x)
```

sympy [C] time = 9.94, size = 122, normalized size = 0.80

$$\frac{dd^{2p}g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^{2p}eg^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**(2*p)*e*g**m*
x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/(2*gamma(m/2 + 2))
```

3.309 $\int (gx)^m \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=75

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/
d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {365, 364}

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2
, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m\left(d^2-e^2x^2\right)^p\,dx &= \left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\ &= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 0.97

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;\frac{e^2x^2}{d^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2
, (e^2*x^2)/d^2])/((1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-e^2x^2+d^2\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-e^2x^2+d^2\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\left(gx\right)^m\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-e^2x^2+d^2\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m, x)
```

sympy [C] time = 3.59, size = 61, normalized size = 0.81

$$\frac{d^{2p}g^m x x^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

3.310 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=163

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m}{\ }\right.}{d^2g^2(m+2)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2
/d^2)/d/g/(1+m)/((1-e^2*x^2/d^2)^p)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeo
m([1-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^2/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {892, 82, 126, 365, 364}

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m}{\ }\right.}{d^2g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m
)/2, (e^2*x^2)/d^2])/(d*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) - (e*(g*x)^(2 + m)
*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2, (e^2*x^2)
/d^2])/(d^2*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{d+ex}\,dx = \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{-1+p}\,dx$$

$$= \left(d(d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^{-1+p}(d+ex)^{-1+p}\,dx - \frac{\left(e(d-e\right.}{}$$

$$= d\int (gx)^m\left(d^2-e^2x^2\right)^{-1+p}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}\,dx}{g}$$

$$= \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d} - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right.}{d}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{dg(1+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right.}{}$$

Mathematica [A] time = 0.05, size = 124, normalized size = 0.76

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(m+2)\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\,{}_2F_1\left(\frac{m}{2}+1,1-p;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right.\right.}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(-(e*(1 + m)*x*Hypergeometric2F1[1 + m/2, 1 -
p, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p,
 (3 + m)/2, (e^2*x^2)/d^2]))/(d^2*(1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (-e^2x^2 + d^2)^p}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p\,(g\,x)^m}{d + e\,x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x),x)
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x), x)
```

sympy [C] time = 13.15, size = 337, normalized size = 2.07

$$-\frac{0^p d d^{2p} g^m m x^m \Phi\left(\frac{d^2}{e^2x^2}, 1, \frac{1}{2} - \frac{m}{2}\right) \Gamma\left(\frac{1}{2} - \frac{m}{2}\right)}{4e^2 x \Gamma\left(\frac{3}{2} - \frac{m}{2}\right)} + \frac{0^p d d^{2p} g^m x^m \Phi\left(\frac{d^2}{e^2x^2}, 1, \frac{1}{2} - \frac{m}{2}\right) \Gamma\left(\frac{1}{2} - \frac{m}{2}\right)}{4e^2 x \Gamma\left(\frac{3}{2} - \frac{m}{2}\right)} + \frac{0^p d^{2p} g^m m x^m \Phi\left(\frac{d^2}{e^2x^2}, 1, \frac{m}{2}\right.}{4e\Gamma\left(1 - \frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] -0**p*d*d**(2*p)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma
(1/2 - m/2)/(4*e**2*x*gamma(3/2 - m/2)) + 0**p*d*d**(2*p)*g**m*x**m*lerchph
i(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*e**2*x*gamma(3/2 - m/
2)) + 0**p*d**(2*p)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2), 1, m*exp_polar(I
*pi)/2)*gamma(-m/2)/(4*e*gamma(1 - m/2)) + d*e**(2*p)*g**m*p*x**m*x**(2*p)*
exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)*hyper((1 - p, -m/2 - p + 1/2), (
-m/2 - p + 3/2,), d**2/(e**2*x**2))/(2*e**2*x*gamma(p + 1)*gamma(-m/2 - p +
 3/2)) - e**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p)
*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,), d**2/(e**2*x**2))/(2*e*gamma(p +
 1)*gamma(-m/2 - p + 1))
```

3.311 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=214

$$-\frac{2(m+p)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},2-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^2g(m+1)(-m-2p+1)}+\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-1}}{g(-m-2p+1)}-\frac{2e(gx)^{m+2}\left(d^2\right.}{}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^(-1+p)/g/(1-m-2*p)-2*(m+p)*(g*x)^(1+m)*(-e^2*x^2
+d^2)^p*hypergeom([2-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^2/g/(1+m)/(1-
m-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([2-p,
 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^3/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.22, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{2e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},2-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^3g^2(m+2)}-\frac{2(m+p)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F}{d^2g(m+1)(-m-2p+}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^(-1 + p))/(g*(1 - m - 2*p)) - (2*(m + p)*(g*
x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 + m)/2,
 (e^2*x^2)/d^2])/(d^2*g*(1 + m)*(1 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e
*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 2 - p, (4 + m
)/2, (e^2*x^2)/d^2])/(d^3*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \int (gx)^m(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} + \frac{\int (gx)^m\left(-2d^2e^2(m+p)-2de^3(1-m-2p)x\right)\left(d^2-e^2x^2\right)^{-2}}{e^2(1-m-2p)}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}\,dx}{g} - \frac{\left(2d^2(m+p)\right)\int (gx)^m}{1-m}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^3g} -$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{2(m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},2-p;\right.}{d^2g(1+m)(1-m-2p)}$$

Mathematica [A] time = 0.11, size = 180, normalized size = 0.84

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{m+1}{2},2-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{m+2}{2},2\right.\right.\right.}{d^4(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, 2 - p, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeomet
ric2F1[(2 + m)/2, 2 - p, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome
tric2F1[(3 + m)/2, 2 - p, (5 + m)/2, (e^2*x^2)/d^2])))/(d^4*(1 + m)*(2 + m)
*(3 + m)*(1 - (e^2*x^2)/d^2)^p)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^{m} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p} \left(gx\right)^{m}}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(g\,x\right)^m}{\left(d+e\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^{m} \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

3.312 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=275

$$-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}+\frac{3d(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-2}}{g(-m-2p+3)}-\frac{2e(-2m-3p+2)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+}{2}\right.}{d^4g^2(m+2)(-m-2p+2)}$$

```
[Out] 3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(-2+p)/g/(3-m-2*p)-e*(g*x)^(2+m)*(-e^2*x^2+d
^2)^(-2+p)/g^2/(2-m-2*p)-2*(2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([
3-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^3/g/(1+m)/(3-m-2*p)/((1-e^2*x^2/
d^2)^p)-2*e*(2-2*m-3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([3-p, 1+1/2*
m],[2+1/2*m],e^2*x^2/d^2)/d^4/g^2/(2+m)/(2-m-2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A] time = 0.44, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{2e(-2m-3p+2)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},3-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)(-m-2p+2)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}-\frac{2(2m+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g*(3 - m - 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g^2*(2 - m - 2*p)) - (2*(2*m + p)*(g*x)^(
1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 + m)/2, (e^
2*x^2)/d^2])/(d^3*g*(1 + m)*(3 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(2
 - 2*m - 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3
 - p, (4 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*(2 - m - 2*p)*(1 - (e^2*x
^2)/d^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx = \int (gx)^m(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx$$
$$= -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} + \frac{\int (gx)^m\left(d^2-e^2x^2\right)^{-3+p}\left(d^3e^2(2-m-2p)-2d^2e^3(2\right.}{e^2(2-m-2p)}$$
$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} + \frac{\int (gx)^m\left(-2d^3e^4(2-m-\right.}{}$$
$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{\left(2d^2e(2-2m-3p)\right)\int (g}{g(2-m}$$
$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{\left(2e(2-2m-3p)\left(d^2-e^2\right.\right.}{}$$
$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{2(2m+p)(gx)^{1+m}\left(d^2-e\right.}{d^3}$$

Mathematica [A] time = 0.19, size = 206, normalized size = 0.75

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(ex\left(\frac{3d\,{}_2F_1\left(\frac{m+3}{2},3-p;\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}-\frac{ex\,{}_2F_1\left(\frac{m+4}{2},3-p;\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)-\frac{3d^2\,{}_2F_1\left(\frac{m+2}{2},3-p;\frac{m+4}{2};\frac{e^2x}{d^2}\right.}{m+2}\right.\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 +
 m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[(2 + m)/2,
3 - p, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3
+ m)/2, 3 - p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2F1[
(4 + m)/2, 3 - p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*(1 - (e^2*x^2
)/d^2)^p)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p \left(gx\right)^m}{\left(ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^m \left(-e^2x^2 + d^2\right)^p}{\left(ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2 + d^2\right)^p \left(gx\right)^m}{\left(ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2 - e^2\,x^2\right)^p\,\left(g\,x\right)^m}{\left(d + e\,x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^m \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{\left(d + ex\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

3.313 $\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax} dx$

Optimal. Leaf size=89

$$\frac{(gx)^{m+1}\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{g(m+1)}-\frac{a(gx)^{m+2}\,{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};a^2x^2\right)}{g^2(m+2)}$$

```
[Out] (g*x)^(1+m)*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/g/(1+m)-a*(g*x)
^(2+m)*hypergeom([1-p, 1+1/2*m],[2+1/2*m],a^2*x^2)/g^2/(2+m)
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {890, 82, 125, 364}

$$\frac{(gx)^{m+1}\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{g(m+1)}-\frac{a(gx)^{m+2}\,{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};a^2x^2\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

```
[Out] ((g*x)^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2])/(g*
(1 + m)) - (a*(g*x)^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2,
a^2*x^2])/(g^2*(2 + m))
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] && !RationalQ[
p] && !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 890

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] && !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] && !IGtQ[m, 0] && !
IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m\left(1-a^2x^2\right)^p}{1+ax}\,dx = \int (gx)^m(1-ax)^p(1+ax)^{-1+p}\,dx$$
$$= -\frac{a\int (gx)^{1+m}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx}{g} + \int (gx)^m(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx$$
$$= -\frac{a\int (gx)^{1+m}\left(1-a^2x^2\right)^{-1+p}\,dx}{g} + \int (gx)^m\left(1-a^2x^2\right)^{-1+p}\,dx$$
$$= \frac{(gx)^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};a^2x^2\right)}{g(1+m)} - \frac{a(gx)^{2+m}\,{}_2F_1\left(\frac{2+m}{2},1-p;\frac{4+m}{2};a^2x^2\right)}{g^2(2+m)}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.87

$$x(gx)^m\left(\frac{{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{m+1} - \frac{ax\,{}_2F_1\left(\frac{m}{2}+1,1-p;\frac{m}{2}+2;a^2x^2\right)}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
[Out] x*(g*x)^m*(-((a*x*Hypergeometric2F1[1 + m/2, 1 - p, 2 + m/2, a^2*x^2])/(2 +
 m)) + Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2]/(1 + m))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-a^2x^2+1\right)^p\left(gx\right)^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="fricas")
[Out] integral((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^p\left(gx\right)^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="giac")
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^m\left(-a^2x^2+1\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
[Out] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^p (gx)^m}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,\left(1-a^2\,x^2\right)^p}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1),x)
[Out] int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1), x)
```

sympy [C] time = 10.81, size = 308, normalized size = 3.46

$$\frac{0^p g^m m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{me^{i\pi}}{2}\right)\Gamma\left(-\frac{m}{2}\right)}{4a\Gamma\left(1-\frac{m}{2}\right)} - \frac{0^p g^m m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4a^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p g^m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4a^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-a**2*x**2+1)**p/(a*x+1),x)
[Out] 0**p*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, m*exp_polar(I*pi)/2)*gamma(-m/2
)/(4*a*gamma(1 - m/2)) - 0**p*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, 1/2 -
m/2)*gamma(1/2 - m/2)/(4*a**2*x*gamma(3/2 - m/2)) + 0**p*g**m*x**m*lerchphi
(1/(a**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*a**2*x*gamma(3/2 - m/2))
- a**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p)*hyper(
(1 - p, -m/2 - p), (-m/2 - p + 1,), 1/(a**2*x**2))/(2*a*gamma(p + 1)*gamma(
-m/2 - p + 1)) + a**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
m/2 - p + 1/2)*hyper((1 - p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), 1/(a**2*x*
*2))/(2*a**2*x*gamma(p + 1)*gamma(-m/2 - p + 3/2))
```

3.314 $\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=96

$$\frac{(gx)^{m+1}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\left(\frac{ex}{d}+1\right)^{-n-p}F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

```
[Out] (g*x)^(1+m)*(e*x+d)^n*(1+e*x/d)^(-n-p)*(-e^2*x^2+d^2)^p*AppellF1(1+m,-p,-n-
p,2+m,e*x/d,-e*x/d)/g/(1+m)/((1-e*x/d)^p)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {892, 135, 133}

$$\frac{(gx)^{m+1}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\left(\frac{ex}{d}+1\right)^{-n-p}F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*(1 + (e*x)/d)^(-n - p)*(d^2 - e^2*x^2)^p*AppellF
1[1 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/(g*(1 + m)*(1 - (e*x)/d)^
p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx &= \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{n+p}\,dx\\
&= \left((d+ex)^{-p}\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d+ex)^{n+p}\left(1-\frac{ex}{d}\right)^p\,dx\\
&= \left((d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m\left(1-\frac{ex}{d}\right)^p\left(1+\frac{ex}{d}\right)^{n}\\
&= \frac{(gx)^{1+m}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^pF_1\left(1+m;-p,-n-p;2\right.}{g(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.94

$$\frac{x(gx)^m(d-ex)^p\left(\frac{d-ex}{d}\right)^{-p}(d+ex)^{n+p}\left(\frac{d+ex}{d}\right)^{-n-p}F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d - e*x)^p*(d + e*x)^(n + p)*((d + e*x)/d)^(-n - p)*AppellF1[1
+ m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/((1 + m)*((d - e*x)/d)^p)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-e^2x^2+d^2\right)^p(ex+d)^n\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-e^2x^2+d^2\right)^p(ex+d)^n\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int\left(gx\right)^m(ex+d)^n\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-e^2x^2+d^2\right)^p(ex+d)^n\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (gx)^m \left(-(-d+ex)(d+ex)\right)^p (d+ex)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(-e**2*x**2+d**2)**p,x)
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p*(d + e*x)**n, x)
```

3.315 $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

Optimal. Leaf size=214

$$2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\right.$$

```
[Out] 2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2)
)^(1/2)+arctan((-2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))/(
2+2*2^(1/2))^(1/2)-arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2)
)^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {825, 827, 1169, 634, 618, 204, 628}

$$2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 +
 Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[
1 + x])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])
/2]*Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2 - (Sqrt[(1
+ Sqrt[2])/2]*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{1+x}+\int \frac{-1+x}{\sqrt{1+x}\,\left(1+x^2\right)}\,dx$$

$$= 2\sqrt{1+x}+2\,\text{Subst}\left(\int \frac{-2+x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\sqrt{1+x}+\frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}-\left(-2-\sqrt{2}\right)x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}+\left(-2-\sqrt{2}\right)x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx,\right.}{2\sqrt{1+\sqrt{2}}}$$

$$= 2\sqrt{1+x}-\frac{1}{2}\sqrt{3-2\sqrt{2}}\;\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+x}\right)-\frac{1}{2}\sqrt{3-2\sqrt{2}}\;\text{Su}$$

$$= 2\sqrt{1+x}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{1+x}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(1\right.$$

$$= 2\sqrt{1+x}+\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(1\right.}}{\sqrt{}}\right.$$

Mathematica [C] time = 0.04, size = 60, normalized size = 0.28

$$2\sqrt{x+1}-\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)-\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] - Sqrt[1 - I]*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] - Sqrt[1 + I]*
ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]
```

fricas [A] time = 1.00, size = 307, normalized size = 1.43

$$-\frac{1}{8}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\log\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+x+\sqrt{2}+1\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*2^(1/4)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)*log(1/2*2^(1/4)*sqrt(x + 1)
*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/8*2^(1/4)*(sqrt(
2) + 2)*sqrt(-2*sqrt(2) + 4)*log(-1/2*2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqr
t(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/2*2^(3/4)*sqrt(-2*sqrt(2) + 4)*arc
tan(1/4*2^(3/4)*sqrt(2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)
 + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) - 1/2*2^(3/4)*sq
rt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) - sqrt(2) - 1) + 1/2*2^(3/4)*s
qrt(-2*sqrt(2) + 4)*arctan(1/4*2^(3/4)*sqrt(-2^(1/4)*sqrt(x + 1)*(sqrt(2) +
 2)*sqrt(-2*sqrt(2) + 4) + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(
2) + 4) - 1/2*2^(3/4)*sqrt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + sqrt
(2) + 1) + 2*sqrt(x + 1)
```

giac [A] time = 0.90, size = 167, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/2*sqrt(2*sqrt(2) - 2)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/4*s
qrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2)
+ 1) + 1/4*sqrt(2*sqrt(2) + 2)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) +
 x + sqrt(2) + 1) + 2*sqrt(x + 1)
```

maple [A] time = 0.12, size = 240, normalized size = 1.12

$$-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+}}{\sqrt{-2+2\sqrt{}}}\right.}{\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+1)^(1/2)/(x^2+1),x)
```

```
[Out] 2*(x+1)^(1/2)+1/4*ln(1+x+2^(1/2)-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2)
)/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2
)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*ln(1+x+2^(1/2)+(x+1)^(1/2)
*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/(-2+2*2^(1/2))^(1/2)*arctan((2*
(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/(-2+2*2^(1/2))^(1/
2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+1}\,x}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

mupad [B] time = 0.11, size = 201, normalized size = 0.94

$$2\sqrt{x+1}+\operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}-2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)-\text{ata}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] 2*(x + 1)^(1/2) + atanh((x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (x + 1)
^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1
/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/
8 - 2^(1/2)/8)^(1/2) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((x + 1)^(1/2)/(4*
(1/8 - 2^(1/2)/8)^(1/2)) + (x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(
1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(
8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8
)^(1/2))
```

sympy [A] time = 11.16, size = 68, normalized size = 0.32

$$2\sqrt{x+1}-4\operatorname{RootSum}\left(512t^4+32t^2+1,\left(t\mapsto t\log\left(-128t^3+\sqrt{x+1}\right)\right)\right)+2\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x + 1) - 4*RootSum(512*_t**4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*
_t**3 + sqrt(x + 1)))) + 2*RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*
log(64*_t**3 + 4*_t + sqrt(x + 1))))
```

3.316 $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=255

$$-\frac{d\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3}-\frac{d^4\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cd}{\sqrt{a+cx^2}\sqrt{a}}\right.}{e^6}$$

```
[Out] 1/60*(-8*a*e^2+47*c*d^2)*(c*x^2+a)^(3/2)/c^2/e^3-13/20*d*(e*x+d)*(c*x^2+a)^
(3/2)/c/e^3+1/5*(e*x+d)^2*(c*x^2+a)^(3/2)/c/e^3-1/8*d*(-a^2*e^4+4*a*c*d^2*e
^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^6-d^4*arctanh((-
c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^6+1/8
*d*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^5
```

Rubi [A] time = 0.63, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 815, 844, 217, 206, 725}

$$-\frac{d\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3}+\frac{d\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-a\right.\right.}{8ce^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^5) + ((47*c*d^
2 - 8*a*e^2)*(a + c*x^2)^(3/2))/(60*c^2*e^3) - (13*d*(d + e*x)*(a + c*x^2)^
(3/2))/(20*c*e^3) + ((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - (d*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3
/2)*e^6) - (d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\left(-2ad^2e^2-de\left(3cd^2+4ae^2\right)x-e^2\left(11cd^2+2ae^2\right)x^2-13cde^3x^3\right)}{d+ex}\,dx}{5ce^4}$$

$$= -\frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\left(5acd^2e^5+3cde^4\left(9cd^2-ae^2\right)x+ce^5\left(47cd^2\right.\right.}{d+ex}}{20c^2e^7}$$

$$= \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \int \frac{\left(15ac^2d^2e\right.}{}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3}}{20ce^3}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3}}{20ce^3}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3}}{20ce^3}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3}}{20ce^3}$$

Mathematica [A] time = 0.61, size = 259, normalized size = 1.02

$$\frac{e\sqrt{a+cx^2}\left(-16a^2e^4+ace^2\left(40d^2-15dex+8e^2x^2\right)+2c^2\left(60d^4-30d^3ex+20d^2e^2x^2-15de^3x^3+12e^4x^4\right)\right)-120}{120c^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2) +
 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))
+ (15*Sqrt[a]*Sqrt[c]*d*e^2*(-4*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*ArcSinh[(Sqr
t[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a] - 120*c^(5/2)*d^5*ArcTanh[(Sqrt[c]*x)
/Sqrt[a + c*x^2]] - 120*c^2*d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(
Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(120*c^2*e^6)
```

fricas [A] time = 13.72, size = 1104, normalized size = 4.33

$$\left[\frac{120 \sqrt{cd^2 + ae^2}\, c^2 d^4 \log\left(\frac{2\, acdex - acd^2 - 2\, a^2e^2 - \left(2\, c^2d^2 + ace^2\right)x^2 - 2\, \sqrt{cd^2 + ae^2}\, (cdx - ae)\sqrt{cx^2 + a}}{e^2x^2 + 2\, dex + d^2}\right) - 15\left(8\, c^2d^5 + 4\, acd^3e^2 - a^2de\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/240*(120*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^
2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(24*c^
2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5
+ 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(
c*x^2 + a))/(c^2*e^6), -1/240*(240*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log
(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(24*c^2*e^5*x^4 - 30*c^2*d
*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 +
 a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6
), 1/120*(60*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (24*c^2*e^5*x^4 - 3
0*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^
2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(
c^2*e^6), -1/120*(120*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*
x^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) - (24*c^2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e +
40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d
^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6)]
```

giac [A] time = 0.21, size = 252, normalized size = 0.99

$$\frac{2\left(cd^6 + ad^4e^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)e + \sqrt{c}\,d}{\sqrt{-cd^2 - ae^2}}\right)e^{(-6)}}{\sqrt{-cd^2 - ae^2}} + \frac{1}{120}\sqrt{cx^2 + a}\left(\left(\left(2\left(3\left(4\, xe^{(-1)} - 5\, de^{(-2)}\right)x + \frac{4\left(5\, c^3d^2e^{18} + \right.}{c^3}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^6 + a*d^4*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^2 - a*e^2) + 1/120*sqrt(c*x^2 + a)*
((2*(3*(4*x*e^(-1) - 5*d*e^(-2))*x + 4*(5*c^3*d^2*e^18 + a*c^2*e^20)*e^(-21
)/c^3)*x - 15*(4*c^3*d^3*e^17 + a*c^2*d*e^19)*e^(-21)/c^3)*x + 8*(15*c^3*d^
4*e^16 + 5*a*c^2*d^2*e^18 - 2*a^2*c*e^20)*e^(-21)/c^3) + 1/8*(8*c^(5/2)*d^5
 + 4*a*c^(3/2)*d^3*e^2 - a^2*sqrt(c)*d*e^4)*e^(-6)*log(abs(-sqrt(c)*x + sqr
t(c*x^2 + a)))/c^2
```

maple [B] time = 0.02, size = 560, normalized size = 2.20

$$-\frac{a\,d^4\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^5}-\frac{c\,d^6\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\cdots}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/5/e*x^2*(c*x^2+a)^(3/2)/c-2/15/e*a/c^2*(c*x^2+a)^(3/2)-1/4*d/e^2*x*(c*x^2
+a)^(3/2)/c+1/8*d/e^2*a/c*x*(c*x^2+a)^(1/2)+1/8*d/e^2*a^2/c^(3/2)*ln(x*c^(1
/2)+(c*x^2+a)^(1/2))+1/3*d^2/e^3*(c*x^2+a)^(3/2)/c-1/2*d^3/e^4*x*(c*x^2+a)^
(1/2)-1/2*d^3/e^4*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+d^4/e^5*((x+d/e)^
2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^6*c^(1/2)*ln((-c*d/e+c*(
x+d/e))/c^(1/2)+((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-d^4/
e^5/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a
*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2
))/(x+d/e))*a-d^6/e^7/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c
*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^
2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.68, size = 249, normalized size = 0.98

$$\frac{\left(cx^2+a\right)^{\frac{3}{2}}x^2}{5\,ce}-\frac{\sqrt{cx^2+a}\,d^3x}{2\,e^4}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}dx}{4\,ce^2}+\frac{\sqrt{cx^2+a}\,adx}{8\,ce^2}-\frac{\sqrt{c}\,d^5\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^6}-\frac{ad^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}\,e^4}+\frac{a^2d\operatorname{arsinh}\cdots}{8\,c^{\frac{3}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2 + a)^(3/2)*x^2/(c*e) - 1/2*sqrt(c*x^2 + a)*d^3*x/e^4 - 1/4*(c*x^
2 + a)^(3/2)*d*x/(c*e^2) + 1/8*sqrt(c*x^2 + a)*a*d*x/(c*e^2) - sqrt(c)*d^5*
arcsinh(c*x/sqrt(a*c))/e^6 - 1/2*a*d^3*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^4)
 + 1/8*a^2*d*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e^2) + sqrt(a + c*d^2/e^2)*d^4
*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e^5
 + sqrt(c*x^2 + a)*d^4/e^5 + 1/3*(c*x^2 + a)^(3/2)*d^2/(c*e^3) - 2/15*(c*x^
2 + a)^(3/2)*a/(c^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(a + c*x**2)/(d + e*x), x)
```

3.317 $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=211

$$\frac{\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}+\frac{d^3\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^5}-\frac{\sqrt{a+cx^2}\,\left(8cd^3-ex\left(4c\right.\right.}{8ce^4}$$

```
[Out] -7/12*d*(c*x^2+a)^(3/2)/c/e^2+1/4*(e*x+d)*(c*x^2+a)^(3/2)/c/e^2+1/8*(-a^2*e
^4+4*a*c*d^2*e^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5+
d^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)
^(1/2)/e^5-1/8*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^4
```

Rubi [A] time = 0.39, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}-\frac{\sqrt{a+cx^2}\,\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^4}+\frac{d^3\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{a\ldots}{\sqrt{a+cx\ldots}}\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^4) - (7*d*(a +
c*x^2)^(3/2))/(12*c*e^2) + ((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) + ((8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c
^(3/2)*e^5) + (d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 +
a*e^2]*Sqrt[a + c*x^2])])/e^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+cx^2}\left(-ade^2-e\left(3cd^2+ae^2\right)x-7cde^2x^2\right)}{d+ex}\,dx}{4ce^3} \\
&= -\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\left(-3acde^4+3ce^3\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{12c^2e^5} \\
&= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \int \frac{-3ac^2de^4\left(4\cdots\right.}{\cdots} \\
&= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} - \frac{\left(d^3\left(cd^2+\cdots\right.\right.}{\cdots} \\
&= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(d^3\left(cd^2+\cdots\right.\right.}{\cdots} \\
&= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(8c^2d^4+4\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.40, size = 225, normalized size = 1.07

$$\frac{24c^{3/2}d^4\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) + 24cd^3\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + e\sqrt{a+cx^2}\,\left(ae^2(3ex-8d)+c\left(-24d^3+\cdots\right.\right.}{24ce^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -1/8*(Sqrt[a]*(-4*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a
]])/(c^(3/2)*e^3*Sqrt[1 + (c*x^2)/a]) + (e*Sqrt[a + c*x^2]*(a*e^2*(-8*d + 3
*e*x) + c*(-24*d^3 + 12*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3)) + 24*c^(3/2)*d^
4*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 24*c*d^3*Sqrt[c*d^2 + a*e^2]*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(24*c*e^5)
```

fricas [A] time = 13.97, size = 963, normalized size = 4.56

$$\left[\frac{24\sqrt{cd^2+ae^2}\,c^2d^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(8\,c^2d^4+4\,acd^2e^2-a^2e^4\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^
2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e
^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*
x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c
*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/48*(48*sqrt(-c*d^2 - a*e^2)*c^2*d^3*
arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^
2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqr
t(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*x^3 - 8
*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*x
)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(12*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a
*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 +
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (6*c^2*e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^
2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(24*sqrt(-c*d^2 -
a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*e^4*x^3 -
8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^2*e^5)]
```

giac [A] time = 0.22, size = 201, normalized size = 0.95

$$-\frac{2\left(cd^5+ad^3e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-5)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(3\,xe^{(-1)}-4\,de^{(-2)}\right)x+\frac{3\left(4\,c^2d^2e^{12}+ac\right.}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^5 + a*d^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))*e^(-5)/sqrt(-c*d^2 - a*e^2) + 1/24*sqrt(c*x^2 + a)*
((2*(3*x*e^(-1) - 4*d*e^(-2))*x + 3*(4*c^2*d^2*e^12 + a*c*e^14)*e^(-15)/c^2
)*x - 8*(3*c^2*d^3*e^11 + a*c*d*e^13)*e^(-15)/c^2) - 1/8*(8*c^2*d^4 + 4*a*c
*d^2*e^2 - a^2*e^4)*e^(-5)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [B] time = 0.01, size = 515, normalized size = 2.44

$$\frac{a\,d^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^4}+\frac{c\,d^5\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c}{e}}}{x+\frac{d}{e}}\right.}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/4/e*x*(c*x^2+a)^(3/2)/c-1/8/e*a/c*x*(c*x^2+a)^(1/2)-1/8/e*a^2/c^(3/2)*ln(
c^(1/2)*x+(c*x^2+a)^(1/2))-1/3*d*(c*x^2+a)^(3/2)/c/e^2+1/2*d^2/e^3*x*(c*x^2
+a)^(1/2)+1/2*d^2/e^3*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d^3/e^4*(-2*(
x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^4/e^5*c^(1/2)*ln((-c*d/
e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)
)+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^
2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e
^2)^(1/2))/(x+d/e))*a+d^5/e^6/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/
e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)
^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.58, size = 207, normalized size = 0.98

$$\frac{\sqrt{cx^2+a}\,d^2x}{2\,e^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}x}{4\,ce}-\frac{\sqrt{cx^2+a}\,ax}{8\,ce}+\frac{\sqrt{c}\,d^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^5}+\frac{ad^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}\,e^3}-\frac{a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}e}-\frac{\sqrt{a+\frac{cd^2}{e^2}}\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*d^2*x/e^3 + 1/4*(c*x^2 + a)^(3/2)*x/(c*e) - 1/8*sqrt(c*
x^2 + a)*a*x/(c*e) + sqrt(c)*d^4*arcsinh(c*x/sqrt(a*c))/e^5 + 1/2*a*d^2*arc
sinh(c*x/sqrt(a*c))/(sqrt(c)*e^3) - 1/8*a^2*arcsinh(c*x/sqrt(a*c))/(c^(3/2)
*e) - sqrt(a + c*d^2/e^2)*d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/
(sqrt(a*c)*abs(e*x + d)))/e^4 - sqrt(c*x^2 + a)*d^3/e^4 - 1/3*(c*x^2 + a)^(
3/2)*d/(c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(a + c*x**2)/(d + e*x), x)
```

3.318 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=153

$$-\frac{d^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4}+\frac{d\sqrt{a+cx^2}\,(2d-ex)}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-1/2*d*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/e^4/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1
/2))*(a*e^2+c*d^2)^(1/2)/e^4+1/2*d*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^3
```

Rubi [A] time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1654, 12, 815, 844, 217, 206, 725}

$$-\frac{d^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4}+\frac{d\sqrt{a+cx^2}\,(2d-ex)}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)/(3*c*e) - (d*(2
*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^4) - (d^
2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
```

```
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{\left(a+cx^2\right)^{3/2}}{3ce}+\frac{\int -\frac{3cdex\sqrt{a+cx^2}}{d+ex}\,dx}{3ce^2}\\ &= \frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx}{e}\\ &= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}\\ &= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}+\frac{\left(d^2\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4}-\frac{\left(d\left(2cd^2+ae^2\right)\right)}{2e^4}\\ &= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{\left(d^2\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}-\\ &= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4}-\frac{d^2\sqrt{cd^2+ae^2}\,\tanh}{e}\end{aligned}$$

Mathematica [A] time = 0.32, size = 193, normalized size = 1.26

$$\frac{-6c^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+e\sqrt{a+cx^2}\left(2ae^2+c\left(6d^2-3dex+2e^2x^2\right)\right)-6cd^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right.}{6ce^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(2*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - (3*Sqrt[a]
*Sqrt[c]*d*e^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^
```

```
2)/a] - 6*c^(3/2)*d^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - 6*c*d^2*Sqrt[c
*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(6*c*e^4)
```

fricas [A] time = 1.50, size = 776, normalized size = 5.07

$$\left[\frac{6\,\sqrt{cd^2+ae^2}\,cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+3\left(2\,cd^3+ade^2\right)\sqrt{c}\,\log\left(-2\,cx^2\right.}{12\,ce^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c
*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -1/12*(12*sqrt(-c*d^2 - a*e^2)*
c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6
*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), 1/6*(3*sqrt(c*d^2 + a*e^2)*c*
d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2
)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2
*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -
1/6*(6*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^
3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c*e^3*x^2 - 3
*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4)]
```

giac [A] time = 0.21, size = 157, normalized size = 1.03

$$\frac{2\left(cd^4+ad^2e^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}}+\frac{\left(2\,cd^3+ade^2\right)e^{(-4)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,\sqrt{c}}+\frac{1}{6}\,\sqrt{cx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^4 + a*d^2*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/2*(2*c*d^3 + a*d*e^2
)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/6*sqrt(c*x^2 +
a)*((2*x*e^(-1) - 3*d*e^(-2))*x + 2*(3*c*d^2*e^7 + a*e^9)*e^(-10)/c)
```

maple [B] time = 0.01, size = 448, normalized size = 2.93

$$-\frac{a\,d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^3}-\frac{c\,d^4\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right.}{e}}}{x+\frac{d}{e}}\right.}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-1/2*d/e^2*x*(c*x^2+a)^(1/2)-1/2/e^2*d*a/c^(1/2)*ln(
c^(1/2)*x+(c*x^2+a)^(1/2))+d^2/e^3*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2)-d^3/e^4*c^(1/2)*ln((-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*
d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))-d^2/e^3/((a*e^2+c*d^2)/e^2)^(1/2)
*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(
x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*a-d^4/e^5/((a*e
^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d
^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d
/e))*c
```

maxima [A] time = 0.52, size = 144, normalized size = 0.94

$$-\frac{\sqrt{cx^2+a}\,dx}{2\,e^2}-\frac{\sqrt{c}\,d^3\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^4}-\frac{ad\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}\,e^2}+\frac{\sqrt{a+\frac{cd^2}{e^2}}\,d^2\,\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e^3}+\frac{\sqrt{cx^2+a}\,d^2}{e^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*d*x/e^2 - sqrt(c)*d^3*arcsinh(c*x/sqrt(a*c))/e^4 - 1/2
*a*d*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^2) + sqrt(a + c*d^2/e^2)*d^2*arcsinh
(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e^3 + sqrt(
c*x^2 + a)*d^2/e^3 + 1/3*(c*x^2 + a)^(3/2)/(c*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x), x)
```

3.319 $\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=127

$$\frac{\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3}+\frac{d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}\,(2d-ex)}{2e^2}$$

```
[Out] 1/2*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^(1/2)+d*arctan
h((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^3
-1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^2
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {815, 844, 217, 206, 725}

$$\frac{\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3}+\frac{d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}\,(2d-ex)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -((2*d - e*x)*Sqrt[a + c*x^2])/(2*e^2) + ((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3) + (d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*
e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^2}$$

$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} - \frac{\left(d\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(2cd^2+ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2e^3}$$

$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\left(d\left(cd^2+ae^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{\left(2cd^2+ae^2\right)\mathrm{Sub}}{}$$

$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3} + \frac{d\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right.}{e^3}$$

Mathematica [A] time = 0.28, size = 175, normalized size = 1.38

$$\frac{\frac{a^{3/2}e^2\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}\,\sqrt{a+cx^2}} + 2d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + 2\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) - 2de\sqrt{a+cx^2} + e^2x}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (-2*d*e*Sqrt[a + c*x^2] + e^2*x*Sqrt[a + c*x^2] + (a^(3/2)*e^2*Sqrt[1 + (c*
x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[a + c*x^2]) + 2*Sqrt[c]
*d^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 2*d*Sqrt[c*d^2 + a*e^2]*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^3)
```

fricas [A] time = 1.39, size = 684, normalized size = 5.39

$$\left[\frac{2\sqrt{cd^2+ae^2}\,cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right) + \left(2\,cd^2+ae^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\right.}{4\,ce^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c
*e^3), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (2*c
*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(sqrt(c*d^2 + a*e^2)*c*d*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) -
(2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*e^2*x -
2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d*arctan(s
qrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2
```

```
*d^2 + a*c*e^2)*x^2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3)]
```

giac [A] time = 0.20, size = 135, normalized size = 1.06

$$-\frac{2\left(cd^3+ade^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-3)}}{\sqrt{-cd^2-ae^2}}-\frac{\left(2c^{\frac{3}{2}}d^2+a\sqrt{c}e^2\right)e^{(-3)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2c}+\frac{1}{2}\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^3 + a*d*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) - 1/2*(2*c^(3/2)*d^2 + a*
sqrt(c)*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c + 1/2*sqrt(c*x
^2 + a)*(x*e^(-1) - 2*d*e^(-2))
```

maple [B] time = 0.01, size = 423, normalized size = 3.33

$$\frac{ad\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e^2}+\frac{cd^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*x*(c*x^2+a)^(1/2)+1/2/e*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d/e^2
*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^2/e^3*c^(1/2)*ln(
(-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)
^(1/2))+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2
)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^
2)/e^2)^(1/2))/(x+d/e))*a+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*
c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+
d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.51, size = 122, normalized size = 0.96

$$\frac{\sqrt{cx^2+a}\,x}{2e}+\frac{\sqrt{c}d^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^3}+\frac{a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}e}-\frac{\sqrt{a+\frac{cd^2}{e^2}}\,d\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}|ex+d|}-\frac{ae}{\sqrt{ac}|ex+d|}\right)}{e^2}-\frac{\sqrt{cx^2+a}\,d}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x/e + sqrt(c)*d^2*arcsinh(c*x/sqrt(a*c))/e^3 + 1/2*a*ar
csinh(c*x/sqrt(a*c))/(sqrt(c)*e) - sqrt(a + c*d^2/e^2)*d*arcsinh(c*d*x/(sqr
t(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e^2 - sqrt(c*x^2 + a)*
d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{cx^2+a}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x), x)
```

3.320 $\int \frac{\sqrt{a+cx^2}}{d+ex} \, dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/e^2-arctanh((-c*d*x+a*e)/(a*e
^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^2+(c*x^2+a)^(1/2)/e
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {735, 844, 217, 206, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+cx^2}}{e} + \frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}$$
$$= \frac{\sqrt{a+cx^2}}{e} + \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx - \frac{(cd)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}$$
$$= \frac{\sqrt{a+cx^2}}{e} + \left(-a-\frac{cd^2}{e^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right) - \frac{(cd)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x$$
$$= \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2}$$

Mathematica [A] time = 0.02, size = 99, normalized size = 0.96

$$\frac{-\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right) - \sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) + e\sqrt{a+cx^2}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[
c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]
)/e^2
```

fricas [A] time = 1.15, size = 574, normalized size = 5.57

$$\left[\frac{\sqrt{c}\,d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,\sqrt{cx^2+a}\,e+\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+a}}{e^2x^2+2\,dex+d^2}\right.}{2\,e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*
x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(sqrt(
c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2 + a)*e
- 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2, (sqrt(-c)*d*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(c*x^2 + a)*e - sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2]
```

giac [A] time = 0.19, size = 109, normalized size = 1.06

$$\sqrt{c}\,de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\sqrt{cx^2+a}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] sqrt(c)*d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + 2*(c*d^2 + a*e^2)
*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2)
)*e^(-2)/sqrt(-c*d^2 - a*e^2) + sqrt(c*x^2 + a)*e^(-1)
```

maple [B] time = 0.01, size = 381, normalized size = 3.70

$$-\frac{a \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, e}-\frac{c d^2 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/e^2*c^(1/2)*d*
ln((-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e
^2)^(1/2))-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^
2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e))*a-1/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c
*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d
/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2
```

maxima [A] time = 0.48, size = 84, normalized size = 0.82

$$-\frac{\sqrt{c}\, d \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^2}+\frac{\sqrt{a+\frac{cd^2}{e^2}}\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e}+\frac{\sqrt{cx^2+a}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/e^2 + sqrt(a + c*d^2/e^2)*arcsinh(c*d*x/(
sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e + sqrt(c*x^2 + a)
/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

3.321 $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/e+arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
e^2+c*d^2)^(1/2)/d/e
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {896, 266, 63, 208, 844, 217, 206, 725}

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e + (Sqrt[c*d^2 + a*e^2]*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*e) - (Sqrt[a]
*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx &= -\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d}+\frac{a\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d}\\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}+\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e}-\left(\frac{cd}{e}+\frac{ae}{d}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx\\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e}-\left(-\frac{cd}{e}-\frac{ae}{d}\right)\mathrm{Subst}\Bigg(\\
&= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e}+\frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{de}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 113, normalized size = 0.97

$$
\frac{\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)+\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)-\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{de}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])] - Sqrt[a]*e*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/(d*e)
```

fricas [A] time = 2.49, size = 1316, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a)*e
*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(c*d^2 + a*e^2)*
log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt
(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/
(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*lo
g(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - sqrt(c*d^2 + a*e^2)*log
((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*
d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*
e), 1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a
)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a)
+ 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e),
1/2*(2*sqrt(-a)*e*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2
 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sq
rt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 2*sqrt(-a)*e*arctan(sqrt(-a)/
sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-a)*e*arctan(sqrt
(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) + 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -(sqr
t(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(-a)*e*arctan(sqrt(-a)/sqr
t(c*x^2 + a)) - sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 420, normalized size = 3.62

$$\frac{a \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\; d}+\frac{cd \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)}}{x+\frac{d}{e}}\right.}{\sqrt{\frac{a e^2+c d^2}{e^2}}\; e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(e*x+d),x)
```

```
[Out] -1/d*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d*(c*x^2+a)^(1/2)-1/d*
(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+c^(1/2)/e*ln((-c*d/e
+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))
+1/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))*a+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*
e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.50, size = 103, normalized size = 0.89

$$\frac{e\left(\frac{\sqrt{c}\,d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^2}-\frac{\sqrt{a+\frac{cd^2}{e^2}}\,\operatorname{arsinh}\left(\frac{2\,cdx}{\sqrt{ac}\,|2\,ex+2\,d|}-\frac{2\,ae}{\sqrt{ac}\,|2\,ex+2\,d|}\right)}{e}-\frac{\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ac}\,|x|}\right)}{e}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] e*(sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/e^2 - sqrt(a + c*d^2/e^2)*arcsinh(2*c*d
*x/(sqrt(a*c)*abs(2*e*x + 2*d)) - 2*a*e/(sqrt(a*c)*abs(2*e*x + 2*d)))/e - s
qrt(a)*arcsinh(a/(sqrt(a*c)*abs(x)))/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x)), x)
```

3.322 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 277, 217, 206, 266, 50, 63, 208, 735, 844, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]

[Out] -(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e^2\sqrt{a+cx^2}}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} + \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right.}{2d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,d\right.}{cd^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

Mathematica [A] time = 0.24, size = 178, normalized size = 1.70

$$\frac{-\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right) - \frac{d\sqrt{a+cx^2}}{x} + \frac{\sqrt{a}\sqrt{c}\,d\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a+cx^2}} - \sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) + \sqrt{a}\,e\tanh}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2])/x) + (Sqrt[a]*Sqrt[c]*d*Sqrt[1 + (c*x^2)/a]*ArcSinh[
(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[
a + c*x^2]] - Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])] + Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

fricas [A] time = 1.12, size = 599, normalized size = 5.70

$$\left[\frac{\sqrt{a}\,ex\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right) + \sqrt{cd^2+ae^2}\,x\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right) - 2\,\sqrt{}}{2\,d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*e*x*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqr
t(c*d^2 + a*e^2)*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), 1/2*(sqrt(a)*e*x*log(-(c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*x*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), -1/2*(2*sqrt(-a)*
e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*x*log((2*a*c*d*e
*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2
)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*x^2
+ a)*d)/(d^2*x), -(sqrt(-a)*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(-c*
```

```
d^2 - a*e^2)*x*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + sqrt(c*x^2 + a)*d)/(d^2*x)]
```

giac [A] time = 0.22, size = 145, normalized size = 1.38

$$-\frac{2\,a\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,d^2}+\frac{2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)d}+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*a*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*d^2) + 2*a
*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*d) + 2*(c*d^2 + a*e^2)*arct
an(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/(sq
rt(-c*d^2 - a*e^2)*d^2)
```

maple [B] time = 0.01, size = 486, normalized size = 4.63

$$-\frac{ae\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^2}-\frac{c\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d/a/x*(c*x^2+a)^(3/2)+1/d*c/a*x*(c*x^2+a)^(1/2)+1/d*c^(1/2)*ln(c^(1/2)*x
+(c*x^2+a)^(1/2))+e/d^2*a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-e/d^2
*(c*x^2+a)^(1/2)+e/d^2*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/
2)-1/d*c^(1/2)*ln((-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+
(a*e^2+c*d^2)/e^2)^(1/2))-e/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*
d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/
e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*a-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+
d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^2 (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x)), x)
```

3.323 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{a}\,e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

```
[Out] -1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-e^2*arctanh((c*x^2+a)^(1/
2)/a^(1/2))*a^(1/2)/d^3+e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a
)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(1/2
)/d^2/x
```

Rubi [A] time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {961, 266, 47, 63, 208, 277, 217, 206, 50, 735, 844, 725}

$$-\frac{\sqrt{a}\,e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2 +
a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^3 -
(c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{e^3\sqrt{a+cx^2}}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d} + \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(ce)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d} + \frac{(c}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{d^3} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d} - $$

Mathematica [A] time = 0.40, size = 283, normalized size = 1.77

$$-\frac{-2ex^2\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + cd^2x^2\sqrt{\frac{cx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right) + 2\sqrt{a}\,\sqrt{c}\,dex^2\sqrt{\frac{c}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(a*d^2 - 2*a*d*e*x + c*d^2*x^2 - 2*c*d*e*x^3 + 2*Sqrt[a]*Sqrt[c]*d*e*x
^2*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] - 2*Sqrt[c]*d*e*x^2*Sqr
t[a + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - 2*e*Sqrt[c*d^2 + a*e^2]
*x^2*Sqrt[a + c*x^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])] + 2*Sqrt[a]*e^2*x^2*Sqrt[a + c*x^2]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]
 + c*d^2*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]])/(d^3*x^2*Sqr
t[a + c*x^2])
```

fricas [A] time = 0.98, size = 726, normalized size = 4.54

$$\left[\frac{2\sqrt{cd^2+ae^2}\,aex^2\log\left(\frac{2\,acdex - acd^2 - 2\,a^2e^2 - \left(2\,c^2d^2 + ace^2\right)x^2 + 2\,\sqrt{cd^2+ae^2}\,(cdx - ae)\sqrt{cx^2+a}}{e^2x^2 + 2\,dex + d^2}\right) + \left(cd^2 + 2\,ae^2\right)\sqrt{a}\,x^2\log\left(-\frac{cx^2-}{}\right.}{4\,ad^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*
x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*
x^2 + a))/(a*d^3*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
```

```
(2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(s
qrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^
2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a
*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)]
```

giac [A] time = 0.22, size = 230, normalized size = 1.44

$$-\frac{2\left(cd^2e+ae^3\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^3}+\frac{\left(cd^2+2ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^3}+\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^3) + (c*d^2 + 2*a*e^2)*arctan
(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^3) + ((sqrt(c)*x - sq
rt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e + (sqr
t(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^
2 + a))^2 - a)^2*d^2)
```

maple [B] time = 0.01, size = 567, normalized size = 3.54

$$\frac{a e^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^3}+\frac{c\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\right.}}{x+\frac{d}{e}}\right.}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] e/d^2/a/x*(c*x^2+a)^(3/2)-e/d^2*c/a*x*(c*x^2+a)^(1/2)-e/d^2*c^(1/2)*ln(c^(1
/2)*x+(c*x^2+a)^(1/2))-1/2/d/a/x^2*(c*x^2+a)^(3/2)-1/2/d*c/a^(1/2)*ln((2*a+
2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/2/d*c/a*(c*x^2+a)^(1/2)-1/d^3*e^2*a^(1/2)*l
n((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/d^3*e^2*(c*x^2+a)^(1/2)-1/d^3*e^2*(-
2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+1/d^2*e*c^(1/2)*ln((-c
*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1
/2))+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^
2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e))*a+1/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d
/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e
)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x)), x)
```

3.324 $\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{a}\,e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^2}-\frac{e^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^4}-\ldots$$

```
[Out] -1/3*(c*x^2+a)^(3/2)/a/d/x^3+1/2*c*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a
^(1/2)+e^3*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^4-e^2*arctanh((-c*d*x
+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^4+1/2*e*(c
*x^2+a)^(1/2)/d^2/x^2-e^2*(c*x^2+a)^(1/2)/d^3/x
```

Rubi [A] time = 0.23, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {961, 264, 266, 47, 63, 208, 277, 217, 206, 50, 735, 844, 725}

$$-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{\sqrt{a}\,e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^4}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^2}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x^
2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[
a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^4} - \frac{e\sqrt{a+cx^2}}{d^2x^3} + \frac{e^2\sqrt{a+cx^2}}{d^3x^2} - \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e^4\sqrt{a+cx^2}}{d^4(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^4}$$

$$= \frac{e^3\sqrt{a+cx^2}}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{e\,\mathrm{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d^2} + \frac{\left(ce^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^3} -$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{(ce)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d^2} - \frac{\left(ce^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,d}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} + \frac{\sqrt{c}\,e^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \right.}{2d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{e^2\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{d^4} + \frac{ce\tanh^{-}}{2}$$

Mathematica [A] time = 1.02, size = 301, normalized size = 1.58

$$-\frac{\frac{2d^3\left(a+cx^2\right)^{3/2}}{ax^3} + 6e^2\left(\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + \sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\right) - \frac{3d^2e\left(cx^2\sqrt{\frac{cx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)\right.}{x^2\sqrt{a+cx^2}}}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/6*(-6*e^3*Sqrt[a + c*x^2] + (2*d^3*(a + c*x^2)^(3/2))/(a*x^3) + (6*d*e^2
*(a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqr
t[a]]))/(x*Sqrt[a + c*x^2]) + 6*e^2*(Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*S
qrt[a + c*x^2])]) + 6*e^3*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2
]/Sqrt[a]]) - (3*d^2*e*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[
1 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/d^4
```

fricas [A] time = 1.00, size = 824, normalized size = 4.31

$$\left[\frac{6\sqrt{cd^2+ae^2}\,ae^2x^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right) + 3\left(cd^2e+2\,ae^3\right)\sqrt{a}\,x^3\log\left(-\frac{cx^2+}{}\right.}{12\,ad^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(a)*x^3*l
og(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*d^2*e*x - 2*a*d
^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/12*(12*sqr
t(-c*d^2 - a*e^2)*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c*d^2*e + 2*
a*e^3)*sqrt(a)*x^3*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*
(3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4
*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)
*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 +
 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/6*(6*sqrt(-c*d^2 - a*e^2)
*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d
^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)*x
^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3
*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3)]
```

giac [A] time = 0.21, size = 309, normalized size = 1.62

$$\frac{2\left(cd^2e^2+ae^4\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^4}-\frac{\left(cd^2e+2\,ae^3\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^4}-\frac{3\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^5cde\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^4) - (c*d^2*e + 2*a*e^3)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^4) - 1/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(3/2)
*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^2*c*d*e - 2*a^2*c^(3/2)*d^2 + 12*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^2*sqrt(c)*e^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2 + a))^
2 - a)^3*d^3)
```

maple [B] time = 0.02, size = 600, normalized size = 3.14

$$-\frac{a\,e^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;d^4}-\frac{ce\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}\cdots}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;d^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] -1/d^3*e^2/a/x*(c*x^2+a)^(3/2)+1/d^3*e^2*c/a*x*(c*x^2+a)^(1/2)+1/d^3*e^2*c^
(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/2*e/d^2/a/x^2*(c*x^2+a)^(3/2)+1/2*e/d
^2*c/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-1/2*e/d^2*c/a*(c*x^2+a)^
(1/2)-1/3*(c*x^2+a)^(3/2)/a/d/x^3+1/d^4*e^3*a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/
2)*a^(1/2))/x)-1/d^4*e^3*(c*x^2+a)^(1/2)+1/d^4*e^3*(-2*(x+d/e)*c*d/e+(x+d/e
)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/d^3*e^2*c^(1/2)*ln((-c*d/e+(x+d/e)*c)/c^(1
/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))-1/d^4*e^3/((a*e
^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d
```

```
^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d
/e))*a-1/d^2*e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^
2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2 + a}}{x^4\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^4\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

3.325 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=274

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,e^4 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{ce^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^3}+\frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3}+$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3+1/8*c^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^
(3/2)/d-1/2*c*e^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-e^4*arctanh(
(c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^5+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)
^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^5-1/4*(c*x^2+a)^(1/2)/d/x^4-1
/8*c*(c*x^2+a)^(1/2)/a/d/x^2-1/2*e^2*(c*x^2+a)^(1/2)/d^3/x^2+e^3*(c*x^2+a)^
(1/2)/d^4/x
```

Rubi [A] time = 0.30, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {961, 266, 47, 51, 63, 208, 264, 277, 217, 206, 50, 735, 844, 725}

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{\sqrt{a}\,e^4 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(4*d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt[a
+ c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^(3/2
))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d
^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])
/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d^3) -
 (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
```

```
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^5} - \frac{e\sqrt{a+cx^2}}{d^2x^4} + \frac{e^2\sqrt{a+cx^2}}{d^3x^3} - \frac{e^3\sqrt{a+cx^2}}{d^4x^2} + \frac{e^4\sqrt{a+cx^2}}{d^5x} - \frac{e^5\sqrt{a+cx^2}}{d^5(d+ex)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^5}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^5} - \frac{e^5\int \sqrt{}}{}$$

$$= -\frac{e^4\sqrt{a+cx^2}}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^3}\,dx, x, x^2\right)}{2d} + \frac{e^2\,\text{Subst}\left(\int \sqrt{}\right)}{2d}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{c\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{8d}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} - \frac{\sqrt{c}\,e^3\tanh^{-1}\left(\right)}{d^4}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\,\tanh}{}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\,\tanh}{}$$

Mathematica [C] time = 1.10, size = 344, normalized size = 1.26

$$-\frac{2c^2d^4\left(a+cx^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{a}+1\right)}{a^3} + \frac{2d^3e\left(a+cx^2\right)^{3/2}}{ax^3} - \frac{3d^2e^2\left(cx^2\sqrt{\frac{cx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}} + 6e^3\left(\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (-6*e^4*Sqrt[a + c*x^2] + (2*d^3*e*(a + c*x^2)^(3/2))/(a*x^3) + (6*d*e^3*(a
 + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a
```

```
]]))/(x*Sqrt[a + c*x^2]) + 6*e^3*(Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*
x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt
[a + c*x^2])]) + 6*e^4*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/S
qrt[a]]) - (3*d^2*e^2*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1
 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]) - (2*c^2*d^4*(a + c*x^2)^(3/2)*Hyper
geometric2F1[3/2, 3, 5/2, 1 + (c*x^2)/a])/a^3)/(6*d^5)
```

fricas [A] time = 1.19, size = 1007, normalized size = 3.68

$$\left[\frac{24\sqrt{cd^2+ae^2}\,a^2e^3x^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(c^2d^4-4\,acd^2e^2-8\,a^2e^4\right)\sqrt{a}}{48\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a
^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2
*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4
+ 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/48*(48*sqrt(-c*d^2
- a*e^2)*a^2*e^3*x^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2
*e^2 - 8*a^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3
*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/24*(12*sq
rt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt
(-a)*x^4*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*
(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2
 + a))/(a^2*d^5*x^4), 1/24*(24*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt(-a)*x^4*a
rctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e
 + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^
2*d^5*x^4)]
```

giac [B] time = 0.26, size = 596, normalized size = 2.18

$$-\frac{2\left(cd^2e^3+ae^5\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^5}-\frac{\left(c^2d^4-4\,acd^2e^2-8\,a^2e^4\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,ad^5}+\frac{3\left(\sqrt{c}x-\sqrt{cx\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^5) - 1/4*(c^2*d^4 - 4*a*c*d
^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-
a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^2*d^3 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a*c^2*d^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c*d*e^2 + 24*(sqrt(c)*x -
sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a
^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*d*e^2 - 24*(sqrt(c)*x
```

```
 - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a
^3*c^(3/2)*d^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d^3 - 12*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a^3*c*d*e^2 + 72*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a
^3*sqrt(c)*e^3 + 8*a^4*c^(3/2)*d^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4
*c*d*e^2 - 72*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*e^3 + 24*a^5*sqrt
(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

maple [B] time = 0.02, size = 703, normalized size = 2.57

$$\frac{a e^4 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, d^5}+\frac{c e^2 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] 1/d^4*e^3/a/x*(c*x^2+a)^(3/2)-1/d^4*e^3*c/a*x*(c*x^2+a)^(1/2)-1/d^4*e^3*c^(
1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-1/2/d^3*e^2/a/x^2*(c*x^2+a)^(3/2)-1/2/d^
3*e^2*c/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/2/d^3*e^2*c/a*(c*x^
2+a)^(1/2)+1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3-1/d^5*e^4*a^(1/2)*ln((2*a+2*(c*x
^2+a)^(1/2)*a^(1/2))/x)+1/d^5*e^4*(c*x^2+a)^(1/2)-1/d^5*e^4*(-2*(x+d/e)*c*d
/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+1/d^4*e^3*c^(1/2)*ln((-c*d/e+(x+d/e
)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))+1/d^5*
e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1
/2))/(x+d/e))*a+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*
(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c
+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c-1/4/d/a/x^4*(c*x^2+a)^(3/2)+1/8/d*c/a
^2/x^2*(c*x^2+a)^(3/2)+1/8/d*c^2/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))
/x)-1/8/d*c^2/a^2*(c*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^5\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**5*(d + e*x)), x)
```

3.326 $\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6c^2e^3}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}\,(d+ex}{6ce^3}$$

```
[Out] -1/2*d*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4-d^4*
arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^4/(a*e^2+c*d^2)
^(1/2)+1/6*(-4*a*e^2+11*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^3-7/6*d*(e*x+d)*(c*x^2
+a)^(1/2)/c/e^3+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^3
```

Rubi [A] time = 0.48, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1654, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6c^2e^3}-\frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}\,(d+ex)}{6ce^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*Sqrt[a
+ c*x^2])/(6*c*e^3) + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2
 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*Sqrt[c*d^2 +
 a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{-2ad^2e^2-de\left(cd^2+4ae^2\right)x-e^2\left(5cd^2+2ae^2\right)x^2-7cde^3x^3}{(d+ex)\sqrt{a+cx^2}}\,dx}{3ce^4}$$

$$= -\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3acd^2e^5+cde^4\left(5cd^2-ae^2\right)x+ce^5\left(11cd^2-4ae^2\right)x^2}{(d+ex)\sqrt{a+cx^2}}\,d}{6c^2e^7}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3ac^2d^2e^7-3c^2a}{(d+ex)}}{6c}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+}}}{e^4}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d^4\,\mathrm{Subst}\left(\int \frac{}{c}\right.}{}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d\left(2cd^2-ae^2\right.}{2}$$

Mathematica [A] time = 0.23, size = 149, normalized size = 0.76

$$\frac{-\frac{3d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}} + \frac{e\sqrt{a+cx^2}\left(-4ae^2+6cd^2-3cdex+2ce^2x^2\right)}{c^2} - \frac{6d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(6*c*d^2 - 4*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2))/c^2 - (3
*d*(2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(3/2) - (6*d^4
*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 +
 a*e^2])/(6*e^4)
```

fricas [A] time = 5.99, size = 1060, normalized size = 5.44

$$\left[\frac{6\sqrt{cd^2+ae^2}\,c^2d^4\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-3\left(2\,c^2d^5+acd^3e^2-a^2de^4\right)\sqrt{c}\,\log}{12\left(c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x
```

```
^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^2 - a^2*d*e^
4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*d^4*e
 + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e
^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/12*(12*sq
rt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(2*c^2*d^5 + a
*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
- a) - 2*(6*c^2*d^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^
5)*x^2 - 3*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c
^2*e^6), 1/6*(3*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c^2*d^5 + a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*d^4*e + 2*a*
c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2 + a*
c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/6*(6*sqrt(-c*d^2
 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^
2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (6*c^2*d^4*e +
 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2
 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6)]
```

giac [A] time = 0.21, size = 163, normalized size = 0.84

$$\frac{2\,d^4\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2\,xe^{(-1)}}{c}-\frac{3\,de^{(-2)}}{c}\right)+\frac{2\left(3\,c^2d^2e^7-2\,ace^9\right)e^{(-10)}}{c^3}\right)+\frac{(2\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/6*sqrt(c*x^2 + a)*(x*(2*x*e^(-1)/c -
 3*d*e^(-2)/c) + 2*(3*c^2*d^2*e^7 - 2*a*c*e^9)*e^(-10)/c^3) + 1/2*(2*c^(3/2
)*d^3 - a*sqrt(c)*d*e^2)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^2
```

maple [A] time = 0.02, size = 260, normalized size = 1.33

$$\frac{\sqrt{c\,x^2+a}\;x^2}{3ce}-\frac{d^4\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^5}+\frac{ad\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}e^2}-\frac{d^3\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3/e*x^2/c*(c*x^2+a)^(1/2)-2/3/e*a/c^2*(c*x^2+a)^(1/2)-1/2*d/e^2*x/c*(c*x^
2+a)^(1/2)+1/2*d/e^2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+d^2/e^3/c*(c*x
^2+a)^(1/2)-d^3/e^4*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-d^4/e^5/((a*e^2+c
*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/
e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.55, size = 171, normalized size = 0.88

$$\frac{\sqrt{cx^2+a}\,x^2}{3\,ce}-\frac{\sqrt{cx^2+a}\,dx}{2\,ce^2}-\frac{d^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^4}+\frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}e^2}+\frac{d^4\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^5}+\frac{\sqrt{cx^2+a}\,d^2}{ce^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2 + a)*x^2/(c*e) - 1/2*sqrt(c*x^2 + a)*d*x/(c*e^2) - d^3*arcsi
nh(c*x/sqrt(a*c))/(sqrt(c)*e^4) + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e
^2) + d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x +
 d)))/(sqrt(a + c*d^2/e^2)*e^5) + sqrt(c*x^2 + a)*d^2/(c*e^3) - 2/3*sqrt(c*
x^2 + a)*a/(c^2*e)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.327 $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=152

$$\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^2}$$

```
[Out] 1/2*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^3+d^3*arc
tanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^3/(a*e^2+c*d^2)^(1
/2)-3/2*d*(c*x^2+a)^(1/2)/c/e^2+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^2
```

Rubi [A] time = 0.27, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1654, 844, 217, 206, 725}

$$\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-3*d*Sqrt[a + c*x^2])/(2*c*e^2) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) +
((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3) +
(d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^3*Sqr
t[c*d^2 + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
```

```
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-ade^2-e\left(cd^2+ae^2\right)x-3cde^2x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-acde^4+ce^3\left(2cd^2-ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2c^2e^5}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(2cd^2-ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{\left(2cd^2-a\right.}{}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3} + \frac{d^3\tanh^{-1}\left(\frac{}{\sqrt{cd^2}}\right.}{e^3\sqrt{cd^2+}}$$

Mathematica [A] time = 0.22, size = 131, normalized size = 0.86

$$\frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+\sqrt{c}\left(\frac{2cd^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}+e\sqrt{a+cx^2}\,(ex-2d)\right)}{2c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[c]*(e*(-2*d
+ e*x)*Sqrt[a + c*x^2] + (2*c*d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2]])])/Sqrt[c*d^2 + a*e^2]))/(2*c^(3/2)*e^3)
```

fricas [A] time = 5.57, size = 924, normalized size = 6.08

$$\left[\frac{2\sqrt{cd^2+ae^2}\,c^2d^3\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-\left(2\,c^2d^4+acd^2e^2-a^2e^4\right)\sqrt{c}\,\log\left(\right.}{4\left(c^3d^2e^3+ac^2e^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sq
rt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*
a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^
2*e^5), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
```

```
(2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(sqrt(c*d^2 + a*e^2)*c^2
*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c
*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c^2*d^3*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)
*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2
*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5)]
```

giac [A] time = 0.22, size = 129, normalized size = 0.85

$$-\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-3)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{2}\,\sqrt{cx^2+a}\left(\frac{xe^{(-1)}}{c}-\frac{2\,de^{(-2)}}{c}\right)-\frac{\left(2\,cd^2-ae^2\right)e^{(-3)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx}\right.\right.}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) + 1/2*sqrt(c*x^2 + a)*(x*e^(-1)/c - 2*d
*e^(-2)/c) - 1/2*(2*c*d^2 - a*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/c^(3/2)
```

maple [A] time = 0.01, size = 217, normalized size = 1.43

$$\frac{d^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^4}-\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}e}+\frac{d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2}\right.}{\sqrt{c}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*x/c*(c*x^2+a)^(1/2)-1/2/e*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d*(
c*x^2+a)^(1/2)/c/e^2+d^2/e^3*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/
((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^
2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))
/(x+d/e))
```

maxima [A] time = 0.52, size = 130, normalized size = 0.86

$$\frac{\sqrt{cx^2+a}\,x}{2\,ce}+\frac{d^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^3}-\frac{a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}e}-\frac{d^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^4}-\frac{\sqrt{cx^2+a}\,d}{ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x/(c*e) + d^2*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^3) - 1/
2*a*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e) - d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e
*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^4) - sqrt(c
*x^2 + a)*d/(c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.328 $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=109

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(1/2)+(c*x^2+a)^(1/2
)/c/e
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 12, 844, 217, 206, 725}

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e
^2) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e
^2*Sqrt[c*d^2 + a*e^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{\sqrt{a+cx^2}}{ce}-\frac{\int \frac{cdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}+\frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^2}-\frac{d^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.96

$$\frac{-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}+\frac{e\sqrt{a+cx^2}}{c}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2])/c - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] -
 (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[c*
d^2 + a*e^2])/e^2
```

fricas [A] time = 1.21, size = 745, normalized size = 6.83

$$\left[\frac{\sqrt{cd^2+ae^2}\,cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^3+ade^2\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\right.}{2\left(c^2d^2e^2+ace^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2
```

```
*d^2*e^2 + a*c*e^4), -1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2
- a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*
e^2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqr
t(c)*x - a) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4),
 1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-
c)*x/sqrt(c*x^2 + a)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 +
 a*c*e^4), -(sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*
d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*d^2*e + a*e
^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)]
```

giac [A] time = 0.23, size = 105, normalized size = 0.96

$$\frac{2\,d^2\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,e^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-2)/sqrt(-c*d^2 - a*e^2) + d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x
^2 + a)))/sqrt(c) + sqrt(c*x^2 + a)*e^(-1)/c
```

maple [A] time = 0.01, size = 172, normalized size = 1.58

$$-\frac{d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^3}-\frac{d\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\;e^2}+\frac{\sqrt{c\,x^2+a}}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/e-1/e^2*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2
+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/
(x+d/e))
```

maxima [A] time = 0.49, size = 90, normalized size = 0.83

$$-\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^2}+\frac{d^2\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^3}+\frac{\sqrt{cx^2+a}}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -d*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^2) + d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(
e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^3) + sqrt(
c*x^2 + a)/(c*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\;\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.329 $\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e/c^(1/2)+d*arctanh((-c*d*x+a*e)/(a*e^2+
c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {844, 217, 206, 725}

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \frac{d\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e} + \frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}$$

Mathematica [**A**] time = 0.03, size = 86, normalized size = 1.00

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

fricas [**A**] time = 1.15, size = 631, normalized size = 7.34

$$\left[\frac{\sqrt{cd^2+ae^2}\,cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+ae^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2}\right.}{2\left(c^2d^2e+ace^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(2*sqrt(-c*d^2 - a*
e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2
 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x
^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(sqrt(c*d
^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e
^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2
*d*e*x + d^2)) - 2*(c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
a)))/(c^2*d^2*e + a*c*e^3), (sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/(c^
2*d^2*e + a*c*e^3)]
```

giac [**A**] time = 0.20, size = 88, normalized size = 1.02

$$-\frac{2\,d\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-1)}}{\sqrt{-cd^2-ae^2}} - \frac{e^{(-1)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^(-1)/sqrt(-c*d^2 - a*e^2) - e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2
+ a)))/sqrt(c)
```

maple [B] time = 0.01, size = 151, normalized size = 1.76

$$\frac{d \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^2}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+
d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.49, size = 71, normalized size = 0.83

$$\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e}-\frac{d\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e) - d*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d
)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.330 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

```
[Out] -arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)^(1
/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx &= -\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

fricas [B] time = 0.89, size = 211, normalized size = 3.91

$$\left[\frac{\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2+ae^2}},\ -\frac{\sqrt{-cd^2-ae^2}\,\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)}{cd^2+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2))/sqrt(c*d^2 + a*e^2), -sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
)/(c*d^2 + a*e^2)]
```

giac [A] time = 0.19, size = 59, normalized size = 1.09

$$\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))/sqrt(-c*d^2 - a*e^2)
```

maple [B] time = 0.00, size = 127, normalized size = 2.35

$$-\frac{\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))
```

maxima [A] time = 0.46, size = 52, normalized size = 0.96

$$\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqr
t(a + c*d^2/e^2)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2+a}\,\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.331 $\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+e*arctanh((-c*d*x+a*e)/(a*e^2+c
*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {961, 266, 63, 208, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+cx^2}}\right)dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 1.00

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

fricas [A] time = 0.89, size = 634, normalized size = 7.37

$$\left[\frac{\sqrt{cd^2+ae^2}\,ae\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+ae^2\right)\sqrt{a}\,\log\left(-\frac{cx^2-2\,\sqrt{cx^2+a}\,\sqrt{a}+\cdots}{x^2}\right.}{2\left(acd^3+a^2de^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(2*sqrt(-c*d^2
- a*e^2)*a*e*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c
*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(
sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
```

```
 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2
*x^2 + 2*d*e*x + d^2)) + 2*(c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*
x^2 + a)))/(a*c*d^3 + a^2*d*e^2), (sqrt(-c*d^2 - a*e^2)*a*e*arctan(sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))
/(a*c*d^3 + a^2*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 158, normalized size = 1.84

$$\frac{\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, d}-\frac{\ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/d/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/d/((a*e^2+c*d^2)/e^2)^
(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*
(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + cx^2}\,(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.332 $\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=111

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-e^2*arctanh((-c*d*x+a*e)/(a*
e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)-(c*x^2+a)^(1/2)/a
/d/x
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {961, 264, 266, 63, 208, 725, 206}

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)\sqrt{a+cx^2}}\right)dx \\
&= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} \\
&= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.96

$$\frac{-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}} - \frac{d\sqrt{a+cx^2}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2])/(a*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*
e^2]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 + a*e^2] + (e*ArcTanh[Sqrt[a + c*x^2]/Sq
rt[a]])/Sqrt[a])/d^2
```

fricas [A] time = 1.05, size = 767, normalized size = 6.91

$$\left[\frac{\sqrt{cd^2+ae^2}\,ae^2x\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2e+ae^3\right)\sqrt{a}\,x\log\left(-\frac{cx^2+2\,\sqrt{c}}{}\right.}{2\left(acd^4+a^2d^2e^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
(2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2
+ 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 +
a))/((a*c*d^4 + a^2*d^2*e^2)*x), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e^2*x*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2 + 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*
c*d^4 + a^2*d^2*e^2)*x), 1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e
^3)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(c*d^3 + a*d*e^2)*sqrt(
c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^2)*x), -(sqrt(-c*d^2 - a*e^2)*a*e^2*x*arc
tan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(-a)*x*arctan(sqrt(-a)/s
qrt(c*x^2 + a)) + (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^
2)*x)]
```

giac [A] time = 0.22, size = 142, normalized size = 1.28

$$2\,c\left(\frac{\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\sqrt{-cd^2-ae^2}\,cd^2}-\frac{\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,cd^2}+\frac{1}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{c}\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*(arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^2/(sqrt(-c*d^2 - a*e^2)*c*d^2) - arctan(-(sqrt(c)*x - sqrt(c*x^2 +
a))/sqrt(-a))*e/(sqrt(-a)*c*d^2) + 1/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)
*sqrt(c)*d))
```

maple [A] time = 0.01, size = 180, normalized size = 1.62

$$-\frac{e\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;d^2}+\frac{e\ln\left(\frac{2a+2\sqrt{c\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\;d^2}-\frac{\sqrt{c\,x^2+a}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d/x+e/d^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-
1/d^2*e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+
2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \sqrt{a + cx^{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.333 $\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=168

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

```
[Out] 1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-e^2*arctanh((c*x^2+a)^(1/2
)/a^(1/2))/d^3/a^(1/2)+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+
a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(
1/2)/a/d^2/x
```

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {961, 266, 51, 63, 208, 264, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*
e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2}{d^3x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)\sqrt{a+cx^2}}\right)dx$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^3} + \frac{e^3\,\text{Su}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,\right.}{4ad}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3} -$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} -$$

Mathematica [A] time = 0.64, size = 163, normalized size = 0.97

$$\frac{\frac{2e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}} + \frac{d\left(cdx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)-\left(a+cx^2\right)(d-2ex)\right)}{ax^2\sqrt{a+cx^2}} - \frac{2e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[
c*d^2 + a*e^2] - (2*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (d*(-((
d - 2*e*x)*(a + c*x^2)) + c*d*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x
^2)/a]]))/(a*x^2*Sqrt[a + c*x^2]))/(2*d^3)
```

fricas [A] time = 1.04, size = 956, normalized size = 5.69

$$\left[\frac{2\,\sqrt{cd^2+ae^2}\,a^2e^3x^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-\left(c^2d^4-acd^2e^2-2\,a^2e^4\right)\sqrt{a}\,x^2\,l}{4\left(a^2cd^5+a^3d^3e^2\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4
)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*
d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d
^5 + a^3*d^3*e^2)*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(a)*x^2*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*d^4 + a^2*d^2*e^2 -
2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^
2), 1/2*(sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^
4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 -
 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x
^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x
^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2
 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^2)]
```

giac [A] time = 0.22, size = 239, normalized size = 1.42

$$-c^{\frac{3}{2}}\left(\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^3}{\sqrt{-cd^2-ae^2}\,c^{\frac{3}{2}}d^3}+\frac{\left(cd^2-2\,ae^2\right)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ac^{\frac{3}{2}}d^3}-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^3\sqrt{c}\,d-2\left(\sqrt{c}\right.}{\left(\left(\sqrt{c}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -c^(3/2)*(2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 - a*e^2))*e^3/(sqrt(-c*d^2 - a*e^2)*c^(3/2)*d^3) + (c*d^2 - 2*a*e^2)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^(3/2)*d^3) - ((s
qrt(c)*x - sqrt(c*x^2 + a))^3*sqrt(c)*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*sqrt(c)*d + 2*a^2*e)/(((sqrt(c)*x -
sqrt(c*x^2 + a))^2 - a)^2*a*c*d^2))
```

maple [A] time = 0.01, size = 236, normalized size = 1.40

$$\frac{e^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,d^3}-\frac{e^2\ln\left(\frac{2a+2\sqrt{c\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d^3}+\frac{c\ln\left(\frac{2a+2\sqrt{c\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] e*(c*x^2+a)^(1/2)/a/d^2/x-1/2*(c*x^2+a)^(1/2)/a/d/x^2+1/2/d*c/a^(3/2)*ln((2
*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-1/d^3*e^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2
)*a^(1/2))/x)+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a
*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(
a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.334 $\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^2-d^4*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(3/2)+a*(c*d*x+a*e)/
c^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)+(c*x^2+a)^(1/2)/c^2/e
```

Rubi [A] time = 0.31, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1647, 1654, 844, 217, 206, 725}

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + Sqrt[a + c*x^2]/(
c^2*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^2) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e
^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{\frac{a^2d^2}{cd^2+ae^2}-ax^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{\int \frac{\frac{a^2cd^2e^2}{cd^2+ae^2}+acdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac^2e^2} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce^2} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2\left(cd^2+ae^2\right)} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce^2} - \frac{d^4\,\text{Sub}}{} \\
&= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\tanh^{-1}\left(\frac{ae-c}{\sqrt{cd^2+ae^2}}\right)}{e^2\left(cd^2+ae^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 179, normalized size = 1.23

$$\frac{\frac{e\left(2a^2e^2+ac\left(d^2+dex+e^2x^2\right)+c^2d^2x^2\right)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{\sqrt{a}\,d\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}\sqrt{a+cx^2}} - \frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] ((e*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(c^2*(c*d^2 +
a*e^2)*Sqrt[a + c*x^2]) - (Sqrt[a]*d*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x
)/Sqrt[a]])/(c^(3/2)*Sqrt[a + c*x^2]) - (d^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*
d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2))/e^2
```

fricas [B] time = 9.36, size = 1525, normalized size = 10.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2
 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^4*e + 3*a^2*c
*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c
^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^
2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 + a^2*c^3*e^6)*x^2), -
1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^
3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c
^4*d^2*e^4 + a^2*c^3*e^6)*x^2), 1/2*(2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d
*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log(
(2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d
^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(
a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 +
a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d
^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 +
 a^2*c^3*e^6)*x^2), -((c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan
(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c
^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d
^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c
^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
 a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*
c^4*d^2*e^4 + a^2*c^3*e^6)*x^2)]
```

giac [B] time = 0.27, size = 299, normalized size = 2.05

$$\frac{2\,d^4\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e^2+ae^4\right)\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}+\frac{\left(\frac{\left(c^4d^4e^5+2\,ac^3d^2e^7+a^2c^2e^9\right)x}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}+\frac{ac^3d^3e^6+a^2c^2de^8}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3\cdots}\right.}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2*e^2 + a*e^4)*sqrt(-c*d^2 - a*e^2)) + d*e^(-2)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2) + (((c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c
^2*e^9)*x/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10) + (a*c^3*d^3*e^6 +
 a^2*c^2*d*e^8)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10))*x + (a*c^3*
d^4*e^5 + 3*a^2*c^2*d^2*e^7 + 2*a^3*c*e^9)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 +
 a^2*c^3*e^10))/sqrt(c*x^2 + a)
```

maple [B] time = 0.02, size = 396, normalized size = 2.71

$$\frac{c\,d^5 x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}\;a\,e^4}-\frac{d^4\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2}{}}}{x+\frac{d}{e}}\right.}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/e*x^2/c/(c*x^2+a)^(1/2)+2/e*a/c^2/(c*x^2+a)^(1/2)+d/e^2*x/c/(c*x^2+a)^(1/
2)-d/e^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d^2/e^3/c/(c*x^2+a)^(1/2)-d^
3/e^4*x/a/(c*x^2+a)^(1/2)+d^4/e^3/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^2
*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^5/e^4/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/
e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-d^4/e^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/
2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.62, size = 251, normalized size = 1.72

$$\frac{cd^5x}{\sqrt{cx^2+a}\,acd^2e^4+\sqrt{cx^2+a}\,a^2e^6}+\frac{d^4}{\sqrt{cx^2+a}\,cd^2e^3+\sqrt{cx^2+a}\,ae^5}+\frac{x^2}{\sqrt{cx^2+a}\,ce}-\frac{d^3x}{\sqrt{cx^2+a}\,ae^4}+\frac{dx}{\sqrt{cx^2+a}\,ce^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^5*x/(sqrt(c*x^2 + a)*a*c*d^2*e^4 + sqrt(c*x^2 + a)*a^2*e^6) + d^4/(sqrt
(c*x^2 + a)*c*d^2*e^3 + sqrt(c*x^2 + a)*a*e^5) + x^2/(sqrt(c*x^2 + a)*c*e)
- d^3*x/(sqrt(c*x^2 + a)*a*e^4) + d*x/(sqrt(c*x^2 + a)*c*e^2) - d*arcsinh(c
*x/sqrt(a*c))/(c^(3/2)*e^2) + d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) -
a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^5) - d^2/(sqrt(c*x^2
 + a)*c*e^3) + 2*a/(sqrt(c*x^2 + a)*c^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.335 $\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e+d^3*arctanh((-c*d*x+a*e)/(a*e^
2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(3/2)+a*(-e*x+d)/c/(a*e^2+c
*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1647, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]]/(c^(3/2)*e) + (d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2]])])/(e*(c*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{a^2de}{cd^2+ae^2}-ax}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac} \\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e\left(cd^2+ae^2\right)} \\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\right.}{e\left(cd^2+ae\right.} \\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 153, normalized size = 1.24

$$\frac{\frac{\sqrt{c}\left(ae(d-ex)\sqrt{ae^2+cd^2}+cd^3\sqrt{a+cx^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)\right)}{\left(ae^2+cd^2\right)^{3/2}}+\sqrt{a}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}e\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + (Sqrt[c]*(a*e*S
qrt[c*d^2 + a*e^2]*(d - e*x) + c*d^3*Sqrt[a + c*x^2]*ArcTanh[(a*e - c*d*x)/
(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]))/(c*d^2 + a*e^2)^(3/2))/(c^(3/2)*e*
Sqrt[a + c*x^2])
```

fricas [B] time = 9.78, size = 1323, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^3*e + a^2*c*d*e^3
 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3
*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2),
1/2*(2*(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*
e^2 + a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x -
```

```
a) + 2*(a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x
^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*
c^4*d^2*e^3 + a^2*c^3*e^5)*x^2), -1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2
*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2
 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*
c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(
a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^
3 + a^2*c^3*e^5)*x^2), ((c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)) + (a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(
c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2
*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2)]
```

giac [A] time = 0.24, size = 219, normalized size = 1.78

$$-\frac{2\, d^3 \arctan\left(-\frac{\left(\sqrt{c}\, x-\sqrt{cx^2+a}\right)e+\sqrt{c}\, d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e+ae^3\right)\sqrt{-cd^2-ae^2}}-\frac{\frac{\left(ac^2d^2e^3+a^2ce^5\right)x}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}-\frac{ac^2d^3e^2+a^2cde^4}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}}{\sqrt{cx^2+a}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}\, x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))/((c*d^2*e + a*e^3)*sqrt(-c*d^2 - a*e^2)) - ((a*c^2*d^2*e^3 + a^2*c*
e^5)*x/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6) - (a*c^2*d^3*e^2 + a^2
*c*d*e^4)/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6))/sqrt(c*x^2 + a) -
e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [B] time = 0.01, size = 354, normalized size = 2.88

$$-\frac{c\, d^4 x}{\left(a\, e^2+c\, d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\, e^2+c\, d^2}{e^2}}\; a\, e^3}+\frac{d^3 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\, e^2+2c\, d^2}{e^2}+2\sqrt{\frac{a\, e^2+c\, d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\, e^2+c\, d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\, e^2+c\, d^2\right)\sqrt{\frac{a\, e^2+c\, d^2}{e^2}}\; e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/e*x/c/(c*x^2+a)^(1/2)+1/e/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+d/e^2/c/
(c*x^2+a)^(1/2)+d^2/e^3*x/a/(c*x^2+a)^(1/2)-d^3/e^2/(a*e^2+c*d^2)/(-2*(x+d/
e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-d^4/e^3/(a*e^2+c*d^2)/a/(-2*(
x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x+d^3/e^2/(a*e^2+c*d^2)
/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e
^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)
)/(x+d/e))
```

maxima [A] time = 0.59, size = 211, normalized size = 1.72

$$-\frac{cd^4x}{\sqrt{cx^2+a}\, acd^2e^3+\sqrt{cx^2+a}\, a^2e^5}-\frac{d^3}{\sqrt{cx^2+a}\, cd^2e^2+\sqrt{cx^2+a}\, ae^4}+\frac{d^2x}{\sqrt{cx^2+a}\, ae^3}-\frac{x}{\sqrt{cx^2+a}\, ce}+\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -c*d^4*x/(sqrt(c*x^2 + a)*a*c*d^2*e^3 + sqrt(c*x^2 + a)*a^2*e^5) - d^3/(sqr
t(c*x^2 + a)*c*d^2*e^2 + sqrt(c*x^2 + a)*a*e^4) + d^2*x/(sqrt(c*x^2 + a)*a*
e^3) - x/(sqrt(c*x^2 + a)*c*e) + arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e) - d^3*a
rcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a +
 c*d^2/e^2)^(3/2)*e^4) + d/(sqrt(c*x^2 + a)*c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.336 $\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=95

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+(-c*d*x-a*e)/c/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1647, 12, 725, 206}

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{acd^2}{\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 1.00

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

fricas [B] time = 1.11, size = 455, normalized size = 4.79

$$\left[\frac{\left(c^2d^2x^2+acd^2\right)\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(acd^2e+a^2e^3+\left(c\right.\right.}{2\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^2*x^2 + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 +
 a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2), -((c^2*d^2*x^2
 + a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2)]
```

giac [A] time = 0.22, size = 174, normalized size = 1.83

$$-\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}} - \frac{\frac{\left(c^2d^3+acde^2\right)x}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}+\frac{acd^2e+a^2e^3}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) - ((c^2*d^3 + a*c*d*e^2)*x/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4) + (a*c*d^2*e + a^2*e^3)/(c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4))/sqrt(c*x^2 + a)
```

maple [B] time = 0.01, size = 311, normalized size = 3.27

$$\frac{c\,d^3 x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}\;a\,e^2}-\frac{d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/e/c/(c*x^2+a)^(1/2)-1/e^2*d*x/a/(c*x^2+a)^(1/2)+d^2/e/(a*e^2+c*d^2)/(-2*
(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^3/e^2/(a*e^2+c*d^2)/a/
(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-d^2/e/(a*e^2+c*d
^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1
/2))/(x+d/e))
```

maxima [A] time = 0.55, size = 171, normalized size = 1.80

$$\frac{cd^3x}{\sqrt{cx^2+a}\,acd^2e^2+\sqrt{cx^2+a}\,a^2e^4}+\frac{d^2}{\sqrt{cx^2+a}\,cd^2e+\sqrt{cx^2+a}\,ae^3}-\frac{dx}{\sqrt{cx^2+a}\,ae^2}+\frac{d^2\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^3*x/(sqrt(c*x^2 + a)*a*c*d^2*e^2 + sqrt(c*x^2 + a)*a^2*e^4) + d^2/(sqrt
(c*x^2 + a)*c*d^2*e + sqrt(c*x^2 + a)*a*e^3) - d*x/(sqrt(c*x^2 + a)*a*e^2)
+ d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d))
)/((a + c*d^2/e^2)^(3/2)*e^3) - 1/(sqrt(c*x^2 + a)*c*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.337 $\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=88

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

```
[Out] d*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)
^(3/2)+(e*x-d)/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {823, 12, 725, 206}

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{acde}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac\left(cd^2+ae^2\right)}$$
$$= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{(de)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.00

$$\frac{ex-d}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (d*e*ArcTanh[(a*e - c*d*x)/(
Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

fricas [B] time = 1.04, size = 425, normalized size = 4.83

$$\left[\frac{\left(cdex^2+ade\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^3+ade^2-\left(cd^2e+a\right.\right.}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^3 + a*d*e^2 - (c*d^2*
e + a*e^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^
3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2), ((c*d*e*x^2 + a*d*e)*sqrt(-c*d^2
 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^
2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2 - (c*d^2*e + a*e
^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)]
```

giac [A] time = 0.21, size = 162, normalized size = 1.84

$$\frac{2\,d\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}} + \frac{\frac{\left(cd^2e+ae^3\right)x}{c^2d^4+2\,acd^2e^2+a^2e^4} - \frac{cd^3+ade^2}{c^2d^4+2\,acd^2e^2+a^2e^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^
2))*e/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c*d^2*e + a*e^3)*x/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4) - (c*d^3 + a*d*e^2)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))/sqrt(c*x^2 + a)
```

maple [B] time = 0.01, size = 283, normalized size = 3.22

$$-\frac{c\,d^{2}x}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\;ae}+\frac{d\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/e*x/a/(c*x^2+a)^(1/2)-d/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^
2+c*d^2)/e^2)^(1/2)-d^2/e/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*
e^2+c*d^2)/e^2)^(1/2)*c*x+d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*
(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c
*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.54, size = 148, normalized size = 1.68

$$-\frac{cd^{2}x}{\sqrt{cx^{2}+a}\,acd^{2}e+\sqrt{cx^{2}+a}\,a^{2}e^{3}}-\frac{d}{\sqrt{cx^{2}+a}\,cd^{2}+\sqrt{cx^{2}+a}\,ae^{2}}+\frac{x}{\sqrt{cx^{2}+a}\,ae}-\frac{d\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^{2}}{e^{2}}\right)^{\frac{3}{2}}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -c*d^2*x/(sqrt(c*x^2 + a)*a*c*d^2*e + sqrt(c*x^2 + a)*a^2*e^3) - d/(sqrt(c*
x^2 + a)*c*d^2 + sqrt(c*x^2 + a)*a*e^2) + x/(sqrt(c*x^2 + a)*a*e) - d*arcsi
nh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d
^2/e^2)^(3/2)*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.338 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=94

$$\frac{ae + cdx}{a\sqrt{a + cx^2}\left(ae^2 + cd^2\right)} - \frac{e^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2 + cd^2\right)^{3/2}}$$

```
[Out] -e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {741, 12, 725, 206}

$$\frac{ae + cdx}{a\sqrt{a + cx^2}\left(ae^2 + cd^2\right)} - \frac{e^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2 + cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$
$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 1.00

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

fricas [B] time = 1.07, size = 456, normalized size = 4.85

$$\left[\frac{\left(ace^2x^2+a^2e^2\right)\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\left(acd^2e+a^2e^3+\left(c^2\right.\right.}{2\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), -((a*c*e^2*x^2
 + a^2*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

giac [A] time = 0.23, size = 172, normalized size = 1.83

$$\frac{\frac{\left(c^2d^3+acde^2\right)x}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}+\frac{acd^2e+a^2e^3}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}} - \frac{2\,\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^2*d^3 + a*c*d*e^2)*x/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2
*e + a^2*e^3)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a) - 2*
arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*
e^2/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2))
```

maple [B] time = 0.01, size = 260, normalized size = 2.77

$$\frac{cdx}{\left(a e^{2}+c d^{2}\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}\; a}-\frac{e\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^{2}+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a e^{2}+c d^{2}\right)\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d/(a
*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-e/
(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2
)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^
2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.50, size = 123, normalized size = 1.31

$$\frac{cdx}{\sqrt{cx^{2}+a}\,acd^{2}+\sqrt{cx^{2}+a}\,a^{2}e^{2}}+\frac{1}{\frac{\sqrt{cx^{2}+a}\,cd^{2}}{e}+\sqrt{cx^{2}+a}\,ae}+\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^{2}}{e^{2}}\right)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d*x/(sqrt(c*x^2 + a)*a*c*d^2 + sqrt(c*x^2 + a)*a^2*e^2) + 1/(sqrt(c*x^2 +
 a)*c*d^2/e + sqrt(c*x^2 + a)*a*e) + arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d))
 - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^{2}\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.339 $\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\,(ae^2+cd^2)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

```
[Out] e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^
2)^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)-e
*(c*d*x+a*e)/a/d/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {961, 266, 51, 63, 208, 741, 12, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\,(ae^2+cd^2)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a + c
*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}}-\frac{e}{d(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx\\
&= \frac{\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d}-\frac{e\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx}{d}\\
&= -\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx,x,x^2\right)}{2d}-\frac{e\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ad\left(cd^2+ae^2\right)}\\
&= \frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2ad}-\frac{e^3\int \frac{\cdots}{(d+ex)\cdots}}{d\left(cd^2\cdots\right.}\\
&= \frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{acd}+\frac{e^3\,\text{S}\cdots}{\cdots}\\
&= \frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\cdots\right.}{a^{3/2}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 132, normalized size = 0.90

$$\frac{-\frac{e(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((e*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (e^3*ArcTanh[(a
*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) +
 Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*Sqrt[a + c*x^2]))/d
```

fricas [B] time = 1.55, size = 1325, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*
d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log
(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2
*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4
*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^
2), 1/2*(2*(a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*
sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2), 1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(
sqrt(-a)/sqrt(c*x^2 + a)) + (a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 +
a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d
^3*e^2 + a^4*c*d*e^4)*x^2), ((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c
*x^2 + a)) + (a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sq
rt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 +
2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 318, normalized size = 2.16

$$-\frac{cex}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\;a}+\frac{e^{2}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/a/d/(c*x^2+a)^(1/2)-1/d/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-1/d
/(a*e^2+c*d^2)*e^2/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-e
/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x
+1/d/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*
e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2}+a\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c\,x^{2}+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+cx^{2}\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.340 $\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{\cdots}{adx\cdots}$$

```
[Out] -e^4*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c
*d^2)^(3/2)+e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a
)^(1/2)-1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+e^2*(c*d*x+a*e)
/a/d^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 271, 191, 266, 51, 63, 208, 741, 12, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{\cdots}{adx\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^2(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx$$

$$= \frac{\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}dx}{d} - \frac{e\int \frac{1}{x\left(a+cx^2\right)^{3/2}}dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}dx}{d^2}$$

$$= -\frac{1}{adx\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{(2c)\int \frac{1}{\left(a+cx^2\right)^{3/2}}dx}{ad} - \frac{e\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} -$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} -$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} -$$

Mathematica [C] time = 0.40, size = 163, normalized size = 0.84

$$-\frac{\frac{d\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} - \frac{e^2(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((-((e^2*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*(a + 2*c
*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c*x^2]))/d^2)
```

fricas [B] time = 1.47, size = 1556, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + ((c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*s
qrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*d^5 +
 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*
x^2 + (a*c^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a
^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*
d^2*e^4)*x), -1/2*(2*(a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 +
```

```
(a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(
c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4
 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d
^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e
^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -1/2*(2*((c^3*d
^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 +
a^3*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c*e^4*x^3 + a^
3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^
2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))
/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 +
(2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d^2*
e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)
*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -((a^2*c*e^4*x^3 +
 a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + ((c^3*d^4*
e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3
*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c^2*d^5 + 2*a^2*c*d
^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c
^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4
*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x
)]
```

giac [A] time = 0.25, size = 266, normalized size = 1.37

$$-\frac{\frac{\left(ac^3d^3+a^2c^2de^2\right)x}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}+\frac{a^2c^2d^2e+a^3ce^3}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}}{\sqrt{cx^2+a}}-\frac{2\,\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^4}{\left(cd^4+ad^2e^2\right)\sqrt{-cd^2-ae^2}}-\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,ad^2}+\frac{\cdots}{\left(\left(\sqrt{c}\,x-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a*c^3*d^3 + a^2*c^2*d*e^2)*x/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) +
 (a^2*c^2*d^2*e + a^3*c*e^3)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))/sqr
t(c*x^2 + a) - 2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 - a*e^2))*e^4/((c*d^4 + a*d^2*e^2)*sqrt(-c*d^2 - a*e^2)) - 2*arctan(
-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*a*d^2) + 2*sqrt(c)/(((
sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*a*d)
```

maple [B] time = 0.01, size = 363, normalized size = 1.87

$$\frac{c\,e^2x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;ad}-\frac{e^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)-e/a/d^2/(c*x^2+a)^(1/2
)+e/d^2/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+e^3/d^2/(a*e^2+c*d^2)
/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+e^2/d/(a*e^2+c*d^2)
/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-e^3/d^2/(a*e^
2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2
+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^
2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(c\,x^2+a\right)^{3/2}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.341 $\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{e^5\text{ta}}{}$$

```
[Out] e^5*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*
d^2)^(3/2)+3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(5/2)/d-e^2*arctanh((c*
x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^3-3/2*c/a^2/d/(c*x^2+a)^(1/2)+e^2/a/d^3/(c*
x^2+a)^(1/2)-1/2/a/d/x^2/(c*x^2+a)^(1/2)+e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*e*x/
a^2/d^2/(c*x^2+a)^(1/2)-e^3*(c*d*x+a*e)/a/d^3/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 266, 51, 63, 208, 271, 191, 741, 12, 725, 206}

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3\sqrt{a+cx^2}}{2a^2dx^2}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^2}{ad^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] e^2/(a*d^3*Sqrt[a + c*x^2]) + 1/(a*d*x^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt
[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sqrt[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a
*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (3*Sqrt[a + c*x^2])/(2*a^2*d*x^2) +
 (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*(c
*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5/2)*d)
 - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \left(\frac{1}{dx^3\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x^2\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^3x\left(a+cx^2\right)^{3/2}} - \frac{e^3}{d^3(d+ex)\left(a+cx^2\right)}\right.$$

$$= \frac{\int \frac{1}{x^3\left(a+cx^2\right)^{3/2}}dx}{d} - \frac{e\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}dx}{d^2} + \frac{e^2\int \frac{1}{x\left(a+cx^2\right)^{3/2}}dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}dx}{d^3}$$

$$= \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x^2(a+cx)^{3/2}}dx, x, x^2\right)}{2d} + \frac{(2ce}{}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

Mathematica [C] time = 0.34, size = 203, normalized size = 0.74

$$\frac{-\frac{cd^2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a^2\sqrt{a+cx^2}} + \frac{de\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} + \frac{e^5\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{e^3(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{e^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((e^3*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*(a + 2*c
*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e^2*Hypergeometric2F1[-1/
2, 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c*x^2]) - (c*d^2*Hypergeometric2F1[-
1/2, 2, 1/2, 1 + (c*x^2)/a])/(a^2*Sqrt[a + c*x^2]))/d^3
```

fricas [A] time = 2.13, size = 1943, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - ((3*c^4*d^6 + 4*
a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c
^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*
```

```
e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3*d^
6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*d^3
*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2 + a
^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2), 1/4
*(4*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e
^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*
x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sq
rt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6
+ 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^
3*c*d*e^5)*x^3 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*
(a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3
*d^7 + 2*a^4*c^2*d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*
e^2 + a^6*d^3*e^4)*x^2), -1/2*(((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*
e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 -
 2*a^4*e^6)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^3*c*e^5*x^4
 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d
^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3
*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*
d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2
+ a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2),
1/2*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^
6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2
 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 +
(3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e +
2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*
d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4
)*x^2)]
```

giac [A] time = 0.29, size = 358, normalized size = 1.30

$$\frac{\frac{(a^2c^3d^2e+a^3c^2e^3)x}{a^4c^2d^4+2a^5cd^2e^2+a^6e^4}-\frac{a^2c^3d^3+a^3c^2de^2}{a^4c^2d^4+2a^5cd^2e^2+a^6e^4}}{\sqrt{cx^2+a}}-\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^5}{\left(cd^5+ad^3e^2\right)\sqrt{-cd^2-ae^2}}-\frac{\left(3cd^2-2ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^2*c^3*d^2*e + a^3*c^2*e^3)*x/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4)
- (a^2*c^3*d^3 + a^3*c^2*d*e^2)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4))/
sqrt(c*x^2 + a) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/s
qrt(-c*d^2 - a*e^2))*e^5/((c*d^5 + a*d^3*e^2)*sqrt(-c*d^2 - a*e^2)) - (3*c*
d^2 - 2*a*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^
2*d^3) + ((sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)
/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^2*a^2*d^2)
```

maple [A] time = 0.01, size = 439, normalized size = 1.59

$$-\frac{c\,e^{3}x}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\;a\,d^{2}}+\frac{e^{4}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c}{e^{2}}}}{x+\frac{d}{e}}\right.}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\;d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*e*x/a^2/d^2/(c*x^2+a)^(1/2)-1/2/a/d/x^2/(c*x^
2+a)^(1/2)-3/2*c/a^2/d/(c*x^2+a)^(1/2)+3/2/d*c/a^(5/2)*ln((2*a+2*(c*x^2+a)^
(1/2)*a^(1/2))/x)+e^2/a/d^3/(c*x^2+a)^(1/2)-1/d^3*e^2/a^(3/2)*ln((2*a+2*(c*
x^2+a)^(1/2)*a^(1/2))/x)-1/d^3*e^4/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^
2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/d^2*e^3/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x
+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x+1/d^3*e^4/(a*e^2+c*d^2)/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2
)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2}+a\right)^{\frac{3}{2}}(ex+d)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\,\left(c\,x^{2}+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+cx^{2}\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.342 $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=244

$$-\frac{d\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)}{3c^2e^4}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{d^4\left(5ae^2+4cd^2\right)\tanh^{-1}(}{e^5\left(ae^2+cd^2\right.}$$

```
[Out] -d*(-a*e^2+4*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5-d^4*(5*a
*e^2+4*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^5
/(a*e^2+c*d^2)^(3/2)+1/3*(-2*a*e^2+13*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^4+d^5*(c
*x^2+a)^(1/2)/e^4/(a*e^2+c*d^2)/(e*x+d)-5/3*d*(e*x+d)*(c*x^2+a)^(1/2)/c/e^4
+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^4
```

Rubi [A] time = 0.89, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)}{3c^2e^4}-\frac{d\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{d^4\left(5ae^2+4cd^2\right)\tanh^{-1}(}{e^5\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((13*c*d^2 - 2*a*e^2)*Sqrt[a + c*x^2])/(3*c^2*e^4) + (d^5*Sqrt[a + c*x^2])/
(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*Sqrt[a + c*x^2])/(3*c*e^4)
 + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^5*(c*d^2 + a*e^
2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^4}{e^3}+\frac{d^3\left(cd^2+ae^2\right)x}{e^4}-\frac{d^2\left(cd^2+ae^2\right)x^2}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^3-\frac{\left(cd^2+ae^2\right)x^4}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-ad^2e\left(cd^2-2ae^2\right)+4d\left(cd^2+ae^2\right)^2x+2e\left(cd^2+ae^2\right)}{(d+ex)\sqrt{a+cx^2}}}{3ce^4\left(cd^2+ae^2\right)}$$

$$= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \int \frac{-6acd^2e^4\left(2cd^2+\right.}{}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2}{3}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2}{3}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2}{3}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2}{3}$$

Mathematica [A] time = 0.51, size = 230, normalized size = 0.94

$$\frac{-\frac{3d\left(4cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}} + e\sqrt{a+cx^2}\left(-\frac{2ae^2}{c^2}+\frac{3d^5}{(d+ex)\left(ae^2+cd^2\right)}+\frac{9d^2-3dex+e^2x^2}{c}\right) - \frac{3d^4\left(5ae^2+4cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}\right.}{\left(ae^2+cd^2\right)^{3/2}}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*((-2*a*e^2)/c^2 + (3*d^5)/((c*d^2 + a*e^2)*(d + e*x)) +
(9*d^2 - 3*d*e*x + e^2*x^2)/c) + (3*d^4*(4*c*d^2 + 5*a*e^2)*Log[d + e*x])/(
c*d^2 + a*e^2)^(3/2) - (3*d*(4*c*d^2 - a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*
x^2]])/c^(3/2) - (3*d^4*(4*c*d^2 + 5*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 +
a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(3*e^5)
```

fricas [B] time = 91.20, size = 2025, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e^6 + (4*c
^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(c)*log(-2
*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 3*(4*c^3*d^7 + 5*a*c^2*d^5*e^2
+ (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
 a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(12*c^3*d^7*e +
 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*
d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*
x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqrt
(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6
+ 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/6*(6*(4*c^3*d^7 + 5*a*c^2*d^5*e^2
+ (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e
^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(
c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(12*c^3*d^7*e + 19*a
*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e
^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*x^2 +
 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqrt(c*x^
2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6 + 2*a
*c^3*d^2*e^8 + a^2*c^2*e^10)*x), 1/6*(6*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^
2*c*d^4*e^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^
5 - a^3*d*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 3*(4*c^3*d^
7 + 5*a*c^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2
)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sq
rt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))
 + 2*(12*c^3*d^7*e + 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^
3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3
*e^5 + a^2*c*d*e^7)*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^
6 - a^3*e^8)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d
*e^9 + (c^4*d^4*e^6 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/3*(3*(4*c^3*d^
7 + 5*a*c^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^
2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2
*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c
*d^4*e^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 -
 a^3*d*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (12*c^3*d^7*e
+ 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2
*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)
*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqr
t(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6
 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 474, normalized size = 1.94

$$\frac{c\,d^6 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^6}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}\;d^5}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)e^5}+\frac{\sqrt{c\,x^2+}}{3c\,e^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3/e^2*x^2/c*(c*x^2+a)^(1/2)-2/3/e^2*a/c^2*(c*x^2+a)^(1/2)-d/e^3*x/c*(c*x^
2+a)^(1/2)+d/e^3*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+3*d^2/e^4/c*(c*x^2
+a)^(1/2)-4*d^3/e^5*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-5/e^6*d^4/((a*e^2
+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2
)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e
))+d^5/e^5/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2
)/e^2)^(1/2)+d^6/e^6*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/
e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+
(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.60, size = 274, normalized size = 1.12

$$\frac{\sqrt{cx^2+a}\,d^5}{cd^2e^5x+ae^7x+cd^3e^4+ade^6}+\frac{\sqrt{cx^2+a}\,x^2}{3\,ce^2}-\frac{\sqrt{cx^2+a}\,dx}{ce^3}-\frac{4\,d^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^5}+\frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}e^3}-\frac{cd^6\operatorname{arsinh}\left(\frac{c}{\sqrt{ac}}\right)}{\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + a)*d^5/(c*d^2*e^5*x + a*e^7*x + c*d^3*e^4 + a*d*e^6) + 1/3*sqr
t(c*x^2 + a)*x^2/(c*e^2) - sqrt(c*x^2 + a)*d*x/(c*e^3) - 4*d^3*arcsinh(c*x/
sqrt(a*c))/(sqrt(c)*e^5) + a*d*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e^3) - c*d^6
*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a
 + c*d^2/e^2)^(3/2)*e^8) + 5*d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a
*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^6) + 3*sqrt(c*x^2 + a)*
d^2/(c*e^4) - 2/3*sqrt(c*x^2 + a)*a/(c^2*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+cx^2}\,\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.343 $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{(6cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)}+\frac{d^3(4ae^2+3cd^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4(ae^2+cd^2)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\cdots$$

```
[Out] 1/2*(-a*e^2+6*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4+d^3*(4*
a*e^2+3*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^
4/(a*e^2+c*d^2)^(3/2)-5/2*d*(c*x^2+a)^(1/2)/c/e^3-d^4*(c*x^2+a)^(1/2)/e^3/(
a*e^2+c*d^2)/(e*x+d)+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^3
```

Rubi [A] time = 0.52, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{(6cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)}+\frac{d^3(4ae^2+3cd^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4(ae^2+cd^2)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-5*d*Sqrt[a + c*x^2])/(2*c*e^3) - (d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^
2)*(d + e*x)) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a
*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*(c
*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\frac{ad^3}{e^2}-\frac{d^2\left(cd^2+ae^2\right)x}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^2-\frac{\left(cd^2+ae^2\right)x^3}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{ade\left(3cd^2+ae^2\right)-\left(c^2d^4-a^2e^4\right)x+5cde\left(cd^2+a\right.}{(d+ex)\sqrt{a+cx^2}}}{2ce^3\left(cd^2+ae^2\right)}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{acde^3\left(3cd^2+ae^2\right)-ce^2\left(\right.}{(d+ex)\sqrt{}}}{2c^2e^5\left(cd\right.}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\int \frac{1}{\sqrt{a+c}}}{2ce^4}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\text{Subst}}{2}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\tanh^{-1}}{2c^{3/2}e^4}$$

Mathematica [A] time = 0.37, size = 208, normalized size = 1.02

$$\frac{\frac{\left(6cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}} + e\sqrt{a+cx^2}\left(\frac{ex-4d}{c}-\frac{2d^4}{(d+ex)\left(ae^2+cd^2\right)}\right) + \frac{2d^3\left(4ae^2+3cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{2d^3\left(4a\right.}{}}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*((-4*d + e*x)/c - (2*d^4)/((c*d^2 + a*e^2)*(d + e*x))) -
 (2*d^3*(3*c*d^2 + 4*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + ((6*c*d^2
 - a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/c^(3/2) + (2*d^3*(3*c*d^2 + 4
```

```
*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*
e^2)^(3/2))/(2*e^4)
```

fricas [B] time = 152.35, size = 1786, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3
*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (
3*c^3*d^5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*c^3*d^6*e + 10*
a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^
7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a)
)/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d
^2*e^7 + a^2*c^2*e^9)*x), 1/4*(4*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^5*
e + 4*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3*d^6*
e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^3*d^6*e + 10*a*c^2*d^4*e^3 + 4*a
^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3*(c^3*d^5
*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^4 + 2*
a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^
9)*x), -1/2*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 +
(6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^
5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (6*c^3*d^6*e + 10*a*c^2*d^4*
e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3
*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5
*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a
^2*c^2*e^9)*x), 1/2*(2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^5*e + 4*a*c^
2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e
)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (6*c^3*d
^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3*d^6*e + 11*a*c
^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(
c*x^2 + a)) - (6*c^3*d^6*e + 10*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*
e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 +
 a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*
d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 435, normalized size = 2.13

$$-\frac{c\,d^5 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^5}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}\,d^4}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)e^4}+\frac{4d^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e^2*x/c*(c*x^2+a)^(1/2)-1/2/e^2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
-2*d*(c*x^2+a)^(1/2)/c/e^3+3*d^2/e^4*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+
4/e^5*d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^
2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e
^2)^(1/2))/(x+d/e))-d^4/e^4/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)
^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^5*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(
1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.57, size = 233, normalized size = 1.14

$$-\frac{\sqrt{cx^2+a}\,d^4}{cd^2e^4x+ae^6x+cd^3e^3+ade^5}+\frac{\sqrt{cx^2+a}\,x}{2\,ce^2}+\frac{3\,d^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^4}-\frac{a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}e^2}+\frac{cd^5\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{a \cdots}{\sqrt{ac}\,| \cdots}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*d^4/(c*d^2*e^4*x + a*e^6*x + c*d^3*e^3 + a*d*e^5) + 1/2*sq
rt(c*x^2 + a)*x/(c*e^2) + 3*d^2*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^4) - 1/2*
a*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e^2) + c*d^5*arcsinh(c*d*x/(sqrt(a*c)*abs
(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^7) - 4*
d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/
(sqrt(a + c*d^2/e^2)*e^5) - 2*sqrt(c*x^2 + a)*d/(c*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.344 $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=160

$$-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

```
[Out] -d^2*(3*a*e^2+2*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(
1/2))/e^3/(a*e^2+c*d^2)^(3/2)-2*d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^
(1/2)+(c*x^2+a)^(1/2)/c/e^2+d^3*(c*x^2+a)^(1/2)/e^2/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.33, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e^2) + (d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e
*x)) - (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e^3) - (d^2*(2*c
*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(e^3*(c*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
```

```
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^2}{e}+d\left(a+\frac{cd^2}{e^2}\right)x-\frac{\left(cd^2+ae^2\right)x^2}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-acd^2e+2cd\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2\left(cd^2+ae^2\right)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{(2d)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(d^2\left(2cd^2+3ae^2\right)\right)\int \frac{}{(d+}}{e^3\left(cd^2+ae^2\right.}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^3} - \frac{\left(d^2\left(2c\right.\right.}{}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3} - \frac{d^2\left(2cd^2+3ae^2\right)\tanh}{e^3\left(cd^2+\right.}$$

Mathematica [A] time = 0.27, size = 184, normalized size = 1.15

$$\frac{-\frac{d^2\left(3ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{d^2\left(3ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} + e\sqrt{a+cx^2}\left(\frac{d^3}{(d+ex)\left(ae^2+cd^2\right)}+\frac{1}{c}\right) - \frac{2d\log\left(\sqrt{c}\,\sqrt{a+cx}\right.}{\sqrt{c}}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(c^(-1) + d^3/((c*d^2 + a*e^2)*(d + e*x))) + (d^2*(2*c*d
^2 + 3*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - (2*d*Log[c*x + Sqrt[c]*
Sqrt[a + c*x^2]])/Sqrt[c] - (d^2*(2*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/e^3
```

fricas [B] time = 16.10, size = 1449, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3
 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 -
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + 2*(2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d
^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*
c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*
a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(
sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^
2*d^2 + a*c*e^2)*x^2)) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*
e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - (2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 +
2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5
 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), 1/2*(4*(c^
2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^
5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c^2*d^5 + 3*a*c*d^3*
e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c^2*d^5*e
+ 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sq
rt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4
+ 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^
4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a
^2*d*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^5*e + 3
*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(
c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2
*a*c^2*d^2*e^6 + a^2*c*e^8)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0,0]%%
%},[4,1]%%%}+%%%{%%%{-2,[0,1,1,0]%%%},[2,1]%%%}+%%%{%%%{1,[0,1,2,0]%%%},[0,
1]%%%} / %%%{%%%{1,[1,2,0,0]%%%}+%%%{1,[0,0,1,2]%%%},[4,0]%%%}+%%%{%%%{-2,[
1,2,1,0]%%%}+%%%{-2,[0,0,2,2]%%%},[2,0]%%%}+%%%{%%%{1,[1,2,2,0]%%%}+%%%{1,[
0,0,3,2]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.01, size = 386, normalized size = 2.41

$$\frac{c\,d^4 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a e^2+c d^2\right)\sqrt{\frac{a e^2+c d^2}{e^2}}\; e^4}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}\; d^3}{\left(a e^2+c d^2\right)\left(x+\frac{d}{e}\right)e^3}-\frac{3d^2 \ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/e^2-2/e^3*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-3/e^4*d
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a
*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e))+d^3/e^3/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2)+d^4/e^4*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d
/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.54, size = 193, normalized size = 1.21

$$\frac{\sqrt{cx^2+a}\,d^3}{cd^2e^3x+ae^5x+cd^3e^2+ade^4}-\frac{2\,d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^3}-\frac{cd^4\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^6}+\frac{3\,d^2\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\cdots\right.}{\sqrt{a+\frac{cd^2}{e^2}}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + a)*d^3/(c*d^2*e^3*x + a*e^5*x + c*d^3*e^2 + a*d*e^4) - 2*d*arc
sinh(c*x/sqrt(a*c))/(sqrt(c)*e^3) - c*d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x
+ d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^6) + 3*d^2*a
rcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt
(a + c*d^2/e^2)*e^4) + sqrt(c*x^2 + a)/(c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.345 $\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}$$

```
[Out] d*(2*a*e^2+c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))
/e^2/(a*e^2+c*d^2)^(3/2)+arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2
*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1651, 844, 217, 206, 725}

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) + ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]]/(Sqrt[c]*e^2) + (d*(c*d^2 + 2*a*e^2)*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
```

```
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{ad-\frac{\left(cd^2+ae^2\right)x}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} - \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \right.}{cd^2+}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2} + \frac{d\left(2a+\frac{cd^2}{e^2}\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right.}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.32, size = 172, normalized size = 1.26

$$\frac{d\left(\frac{\left(2ae^2+cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{de\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}\right)-\frac{\left(2ade^2+cd^3\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(((c*d^3 + 2*a*d*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2)) + Log[c*x + Sq
rt[c]*Sqrt[a + c*x^2]]/Sqrt[c] + d*(-((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2
)*(d + e*x))) + ((c*d^2 + 2*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sq
rt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2)))/e^2
```

fricas [B] time = 13.03, size = 1260, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a
^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c^2*d
^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c
^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a
^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), 1/2*(2*(c^2*d^
4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*
e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 +
2*a*c^2*d^3*e^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)
*x), -1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*
```

```
e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4 +
2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c
*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a
*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^
4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*
d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), ((c^2*d^4 + 2*a*c*d
^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) - (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d
^2*e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4
*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d
*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.66Error: Bad Argument Type
```

maple [B] time = 0.01, size = 368, normalized size = 2.69

$$-\frac{c\,d^{3}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,e^{3}}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,d^{2}}{\left(a\,e^{2}+c\,d^{2}\right)\left(x+\frac{d}{e}\right)e^{2}}+\frac{2d\ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/e^2*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+2*d/e^3/((a*e^2+c*d^2)/e^2)^(1/
2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2
*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-d^2/e^2/(a*e^
2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-d^3
/e^3*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^
2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e
^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.53, size = 171, normalized size = 1.25

$$-\frac{\sqrt{cx^{2}+a}\,d^{2}}{cd^{2}e^{2}x+ae^{4}x+cd^{3}e+ade^{3}}+\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^{2}}+\frac{cd^{3}\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^{2}}{e^{2}}\right)^{\frac{3}{2}}e^{5}}-\frac{2\,d\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right.}{\sqrt{a+\frac{cd^{2}}{e^{2}}}\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*d^2/(c*d^2*e^2*x + a*e^4*x + c*d^3*e + a*d*e^3) + arcsinh(
c*x/sqrt(a*c))/(sqrt(c)*e^2) + c*d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d))
 - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^5) - 2*d*arcsinh(
c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*
d^2/e^2)*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a + cx^{2}} \left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.346 $\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -a*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+d*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {807, 725, 206}

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= \frac{d\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}+\frac{(ae)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\ &= \frac{d\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{(ae)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\ &= \frac{d\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.00

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

fricas [B] time = 1.06, size = 382, normalized size = 4.24

$$\left[\frac{\left(ae^2x+ade\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\left(cd^3+ade^2\right)\sqrt{cx^2+a}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*e^2*x + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((a*e^2*x + a*d*e)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 340, normalized size = 3.78

$$\frac{c\,d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^2}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;d}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)e}-\frac{\ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2
*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)
^(1/2))/(x+d/e))+d/e/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2)+d^2/e^2*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d
/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.52, size = 148, normalized size = 1.64

$$\frac{\sqrt{cx^2+a}\,d}{cd^2ex+ae^3x+cd^3+ade^2}-\frac{cd^2\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^4}+\frac{\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + a)*d/(c*d^2*e*x + a*e^3*x + c*d^3 + a*d*e^2) - c*d^2*arcsinh(c
*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e
^2)^(3/2)*e^4) + arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*ab
s(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.347 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -c*d*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)-e*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {731, 725, 206}

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}+\frac{(cd)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{(cd)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 115, normalized size = 1.26

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{cd\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (c*d*Log[d + e*x])/(c*
d^2 + a*e^2)^(3/2) - (c*d*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2]])/(c*d^2 + a*e^2)^(3/2)
```

fricas [B] time = 1.27, size = 381, normalized size = 4.19

$$\left[\frac{\left(cdex+cd^2\right)\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 210, normalized size = 2.31

$$-\frac{cd\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(
1/2)-1/e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2
*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*
c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.49, size = 93, normalized size = 1.02

$$-\frac{\sqrt{cx^2+a}}{cd^2x+ae^2x+\frac{cd^3}{e}+ade}+\frac{cd\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)/(c*d^2*x + a*e^2*x + c*d^3/e + a*d*e) + c*d*arcsinh(c*d*x/
(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(
3/2)*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.348 $\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=179

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}$$

```
[Out] c*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)
^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)+e*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)+e^2*(c*x^2+a)^
(1/2)/d/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.14, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {961, 266, 63, 208, 731, 725, 206}

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Ar
cTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2
 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{d^2x\sqrt{a+cx^2}} - \frac{e}{d(d+ex)^2\sqrt{a+cx^2}} - \frac{e}{d^2(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{e\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x,\right.}{d^2} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\right.}{cd^2} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 178, normalized size = 0.99

$$
\frac{\frac{de^2\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)} + \frac{e\left(ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{e\left(ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{\sqrt{a}} + \frac{\log(x)}{\sqrt{a}}}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((d*e^2*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + Log[x]/Sqrt[a] - (e*
(2*c*d^2 + a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - Log[a + Sqrt[a]*Sqr
t[a + c*x^2]]/Sqrt[a] + (e*(2*c*d^2 + a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 +
 a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/d^2
```

fricas [A] time = 1.81, size = 1261, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 +
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
 a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 +
 a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), 1/2*(2*(2*
a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*
d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*
d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*
d^2*e^5)*x), 1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*c
*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a
*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3
*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), ((2*a*c*d^3*e
+ a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*
a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c*
d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^
3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)]
```

giac [A] time = 0.57, size = 126, normalized size = 0.70

$$\left(\frac{\sqrt{c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}}\,d^2e^2\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{cd^5\mathrm{sgn}\left(\frac{1}{xe+d}\right)^2+ad^3e^2\mathrm{sgn}\left(\frac{1}{xe+d}\right)^2}-\frac{\sqrt{c}\,e^2\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{cd^3+ade^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)*d^2*e^2*
sgn(1/(x*e + d))/(c*d^5*sgn(1/(x*e + d))^2 + a*d^3*e^2*sgn(1/(x*e + d))^2)
- sqrt(c)*e^2*sgn(1/(x*e + d))/(c*d^3 + a*d*e^2))*e^(-1)
```

maple [B] time = 0.01, size = 364, normalized size = 2.03

$$\frac{c\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;e}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/d^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/d^2/((a*e^2+c*d^2)/e
^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1
/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/d*e/
(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2
```

```
)+c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c
*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+
c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2 + a}\,\left(d + e\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + cx^2}\,(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.349 $\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=212

$$\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{\sqrt{a+cx^2}}{ad^2x}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}$$

```
[Out] -c*e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c
*d^2)^(3/2)+2*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-2*e^2*arctanh(
(-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-(
c*x^2+a)^(1/2)/a/d^2/x-e^3*(c*x^2+a)^(1/2)/d^2/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.17, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {961, 264, 266, 63, 208, 731, 725, 206}

$$-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}-\frac{\sqrt{a+cx^2}}{ad^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(
d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^
2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{d^2x^2\sqrt{a+cx^2}} - \frac{2e}{d^3x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)^2\sqrt{a+cx^2}} + \frac{2e^2}{d^3(d+ex)\sqrt{a+cx}}\right.$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} + \frac{\left(2e^2\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3} + \frac{e^2\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{d^3} - \frac{\left(2e^2\right)\text{Sub}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{(2e)\text{Subs}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}} - \frac{2e^2\tanh}{d^3}$$

Mathematica [A] time = 0.33, size = 197, normalized size = 0.93

$$\frac{-\frac{e^2\left(2ae^2+3cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e^2\left(2ae^2+3cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} - d\sqrt{a+cx^2}\left(\frac{e^3}{(d+ex)\left(ae^2+cd^2\right)} + \frac{1}{ax}\right) + \frac{2e\log\left(\sqrt{a}\sqrt{a+c}\right.}{\sqrt{a}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(d*Sqrt[a + c*x^2]*(1/(a*x) + e^3/((c*d^2 + a*e^2)*(d + e*x)))) - (2*e*Lo
g[x])/Sqrt[a] + (e^2*(3*c*d^2 + 2*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2
```

```
) + (2*e*Log[a + Sqrt[a]*Sqrt[a + c*x^2]])/Sqrt[a] - (e^2*(3*c*d^2 + 2*a*e^
2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^
(3/2))/d^3
```

fricas [A] time = 1.63, size = 1512, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2
 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) + 2*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d
^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5
*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c^2*d^7*e + 2*a^2
*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^2 + a^3*d^4*e^4)
*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e
^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^2*d^4*e^2 + 2*a*c*
d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)
*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x), -1/2*(4*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a
^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*
x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^
2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c
^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^
2 + a^3*d^4*e^4)*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^
2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 2*((c^
2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%{[%%%{1,[0,0,0,
5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[4,1]%%%}+%%%{
%%%{-4,[1,2,0,4]%%%}+%%%{-4,[0,0,1,6]%%%},[3,1]%%%}+%%%{%%{[%%%{4,[1,2,0,3]
%%%}+%%%{6,[0,0,1,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]
%%},[2,1]%%%}+%%%{%%%{-4,[1,2,1,4]%%%}+%%%{-4,[0,0,2,6]%%%},[1,1]%%%}+%%%{%
%{[%%%{1,[0,0,2,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%
},[0,1]%%%} / %%%{%%%{1,[1,2,0,2]%%%}+%%%{1,[0,0,1,4]%%%},[4,0]%%%}+%%%{%%{
```

```
poly1[%%%{-4,[1,2,0,1]%%%}+%%%{-4,[0,0,1,3]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%
}+%%%{-1,[0,0,1,2]%%%}]%%},[3,0]%%%}+%%%{%%%{4,[2,4,0,0]%%%}+%%%{10,[1,2,1,
2]%%%}+%%%{6,[0,0,2,4]%%%},[2,0]%%%}+%%%{%%{poly1[%%%{-4,[1,2,1,1]%%%}+%%%{
-4,[0,0,2,3]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[1,0
]%%%}+%%%{%%%{1,[1,2,2,2]%%%}+%%%{1,[0,0,3,4]%%%},[0,0]%%%} Error: Bad Argu
ment Value
```

maple [B] time = 0.01, size = 395, normalized size = 1.86

$$-\frac{c e \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\frac{2 a e^{2}+2 c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a e^{2}+c d^{2}\right)\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, d}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}\, e^{2}}{\left(a e^{2}+c d^{2}\right)\left(x+\frac{d}{e}\right) d^{2}}-\frac{2 e \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d^2/x+2/d^3*e/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))
/x)-2/d^3*e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/
e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)
/e^2)^(1/2))/(x+d/e))-1/d^2/(a*e^2+c*d^2)*e^2/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+
d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/d*c*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)
*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{a+cx^{2}}\,\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.350 $\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=268

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^4}+\frac{2e\sqrt{a+cx^2}}{ad^3x}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{3e^3\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{d^4\sqrt{ae^2}}$$

```
[Out] c*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+
c*d^2)^(3/2)+1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-3*e^2*arcta
nh((c*x^2+a)^(1/2)/a^(1/2))/d^4/a^(1/2)+3*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c
*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^4/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/
d^2/x^2+2*e*(c*x^2+a)^(1/2)/a/d^3/x+e^4*(c*x^2+a)^(1/2)/d^3/(a*e^2+c*d^2)/(
e*x+d)
```

Rubi [A] time = 0.22, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {961, 266, 51, 63, 208, 264, 731, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}+\frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}+\frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{3e^2\tanh}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*Sqr
t[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (3*
e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^4*Sqrt
[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d^2) - (
3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{d^2x^3\sqrt{a+cx^2}} - \frac{2e}{d^3x^2\sqrt{a+cx^2}} + \frac{3e^2}{d^4x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)^2\sqrt{a+cx^2}} - \frac{\ldots}{d^4(\ldots}\right.$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^3} + \frac{\left(3e^2\right)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^4} - \frac{\left(3e^3\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^4}$$

$$= \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^2} + \frac{\left(3e^2\right)\text{Subst}\ldots}{\ldots}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}}\ldots$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}\ldots$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}\ldots$$

Mathematica [A] time = 0.45, size = 229, normalized size = 0.85

$$\frac{\frac{\left(cd^2-6ae^2\right)\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{a^{3/2}} + \frac{\log(x)\left(6ae^2-cd^2\right)}{a^{3/2}} + d\sqrt{a+cx^2}\left(\frac{2e^4}{(d+ex)\left(ae^2+cd^2\right)} - \frac{d-4ex}{ax^2}\right) + \frac{2e^3\left(3ae^2+4cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+a\ldots\right)}{\left(ae^2+cd^2\right)^{3/2}}\ldots}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2]*(-((d - 4*e*x)/(a*x^2)) + (2*e^4)/((c*d^2 + a*e^2)*(d +
e*x))) + ((-(c*d^2) + 6*a*e^2)*Log[x])/a^(3/2) - (2*e^3*(4*c*d^2 + 3*a*e^2)
*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + ((c*d^2 - 6*a*e^2)*Log[a + Sqrt[a]*S
qrt[a + c*x^2]])/a^(3/2) + (2*e^3*(4*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqr
t[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(2*d^4)
```

fricas [A] time = 2.79, size = 1867, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5
)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*
d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(
e^2*x^2 + 2*d*e*x + d^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^
5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*
d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*
d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5
)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3
+ (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), 1/4*(4*((4*a^2*c*d^2*
e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a
```

```
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^
2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^
4 - 6*a^3*d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2
+ 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a
^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4
*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), -1/2*(((c^3
*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4
*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*arctan(sqrt(
-a)/sqrt(c*x^2 + a)) - ((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^
3 + 3*a^3*d*e^5)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a
^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a
*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d
^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2
+ a^4*d^5*e^4)*x^2), 1/2*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d
^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2
)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c
^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4
 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e
 + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a
^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*
e^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.53Unable to divide, perhaps due to rounding error
%%%{%%%{1,[0,0,0,7]%%%},[6,1]%%%}+%%%{%%{[%%%{-6,[0,0,0,6]%%%},0]:[1,0,%%%{
-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[5,1]%%%}+%%%{%%%{12,[1,2,0,5]%%%
}+%%%{15,[0,0,1,7]%%%},[4,1]%%%}+%%%{%%{[%%%{-8,[1,2,0,4]%%%}+%%%{-20,[0,0,
1,6]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[3,1]%%%}+%%
%{%%%{12,[1,2,1,5]%%%}+%%%{15,[0,0,2,7]%%%},[2,1]%%%}+%%%{%%{[%%%{-6,[0,0,2
,6]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[1,1]%%%}+%%%
{%%%{1,[0,0,3,7]%%%},[0,1]%%%} / %%%{%%{[%%%{-1,[1,2,0,3]%%%}+%%%{-1,[0,0,1
,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[6,0]%%%}+%%%
{%%%{6,[2,4,0,2]%%%}+%%%{12,[1,2,1,4]%%%}+%%%{6,[0,0,2,6]%%%},[5,0]%%%}+%%%
{%%{[%%%{-12,[2,4,0,1]%%%}+%%%{-27,[1,2,1,3]%%%}+%%%{-15,[0,0,2,5]%%%},0]:[
1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[4,0]%%%}+%%%{%%%{8,[3,6,
0,0]%%%}+%%%{36,[2,4,1,2]%%%}+%%%{48,[1,2,2,4]%%%}+%%%{20,[0,0,3,6]%%%},[3,
0]%%%}+%%%{%%{[%%%{-12,[2,4,1,1]%%%}+%%%{-27,[1,2,2,3]%%%}+%%%{-15,[0,0,3,5
]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[2,0]%%%}+%%%{%
%%{6,[2,4,2,2]%%%}+%%%{12,[1,2,3,4]%%%}+%%%{6,[0,0,4,6]%%%},[1,0]%%%}+%%%{%
%{[%%%{-1,[1,2,3,3]%%%}+%%%{-1,[0,0,4,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%
%%{-1,[0,0,1,2]%%%}]%%},[0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.01, size = 452, normalized size = 1.69

$$\frac{c\,e^2 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,d^2}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;e^3}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)d^3}+\frac{3e^2\ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*e*(c*x^2+a)^(1/2)/a/d^3/x-1/2*(c*x^2+a)^(1/2)/a/d^2/x^2+1/2/d^2*c/a^(3/2)
*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-3/d^4*e^2/a^(1/2)*ln((2*a+2*(c*x^2+a
)^(1/2)*a^(1/2))/x)+3/d^4*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/
e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)
^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/d^3*e^3/(a*e^2+c*d^2)/(x+d/e)*(-2
*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+1/d^2*e^2*c/(a*e^2+c*d^
2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a
*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.351 $\int x^2(a+bx)^n\left(c+dx^2\right)\,dx$

Optimal. Leaf size=135

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)}{b^5(n+}$$

```
[Out] a^2*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^5/(1+n)-2*a*(2*a^2*d+b^2*c)*(b*x+a)^(2+n)
/b^5/(2+n)+(6*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/
(4+n)+d*(b*x+a)^(5+n)/b^5/(5+n)
```

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {948}

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)}{b^5(n+}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] (a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^2
*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 + n)
)/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5
 + n))/(b^5*(5 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)\,dx&=\int\left(\frac{\left(a^2b^2c+a^4d\right)(a+bx)^n}{b^4}-\frac{2\left(ab^2c+2a^3d\right)(a+bx)^{1+n}}{b^4}+\frac{\left(b^2c+6a^2d\right)(a+b}{b^4}\right.\\&=\frac{a^2\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a\left(b^2c+2a^2d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{\left(b^2c+6a^2d\right)(a+bx}{b^5(3+n)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 0.84

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(6a^2d+b^2c\right)}{n+3}-\frac{2a(a+bx)\left(2a^2d+b^2c\right)}{n+2}+\frac{a^4d+a^2b^2c}{n+1}+\frac{d(a+bx)^4}{n+5}-\frac{4ad(a+bx)^3}{n+4}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^2*c + a^4*d)/(1 + n) - (2*a*(b^2*c + 2*a^2*d)*(a
 + b*x))/(2 + n) + ((b^2*c + 6*a^2*d)*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*
x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

fricas [B] time = 0.90, size = 368, normalized size = 2.73

$$\frac{\left(2\,a^3b^2cn^2+18\,a^3b^2cn+40\,a^3b^2c+24\,a^5d+\left(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b^5d\right)x^5+\left(ab^4dn^4+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 +
10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 + (a*b^4*d*n^4 + 6
*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + (b^5*c*n^4 + 40*b^5*c +
4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5*c -
 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 12*a^3
*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 + 9*a^2
*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^
5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

giac [B] time = 0.19, size = 624, normalized size = 4.62

$$\frac{(bx+a)^n b^5dn^4x^5+(bx+a)^n ab^4dn^4x^4+10\,(bx+a)^n b^5dn^3x^5+(bx+a)^n b^5cn^4x^3+6\,(bx+a)^n ab^4dn^3x^4+35\,(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + (b*x + a)^n*b^5*c*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 +
35*(b*x + a)^n*b^5*d*n^2*x^5 + (b*x + a)^n*a*b^4*c*n^4*x^2 + 12*(b*x + a)^n
*b^5*c*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x + a)^n*a*b^4*d*n
^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + 10*(b*x + a)^n*a*b^4*c*n^3*x^2 + 49*(
b*x + a)^n*b^5*c*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b*x + a)^n
*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 - 2*(b*x + a)^n*a^2*b^3*c*n^3*x +
 29*(b*x + a)^n*a*b^4*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 + 78*(b*
x + a)^n*b^5*c*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - 18*(b*x + a)^n*a^2*b
^3*c*n^2*x + 20*(b*x + a)^n*a*b^4*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^2
+ 40*(b*x + a)^n*b^5*c*x^3 + 2*(b*x + a)^n*a^3*b^2*c*n^2 - 40*(b*x + a)^n*a
^2*b^3*c*n*x - 24*(b*x + a)^n*a^4*b*d*n*x + 18*(b*x + a)^n*a^3*b^2*c*n + 40
*(b*x + a)^n*a^3*b^2*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*b
^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

maple [B] time = 0.01, size = 328, normalized size = 2.43

$$\frac{\left(b^4d\,n^4x^4+10b^4d\,n^3x^4-4a\,b^3d\,n^3x^3+b^4c\,n^4x^2+35b^4d\,n^2x^4-24a\,b^3d\,n^2x^3+12b^4c\,n^3x^2+50b^4dn\,x^4+12a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c),x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+b^4*c*n^4*x
^2+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+12*b^4*c*n^3*x^2+50*b^4*d*n*x^4+12*a
^2*b^2*d*n^2*x^2-2*a*b^3*c*n^3*x-44*a*b^3*d*n*x^3+49*b^4*c*n^2*x^2+24*b^4*d
*x^4+36*a^2*b^2*d*n*x^2-20*a*b^3*c*n^2*x-24*a*b^3*d*x^3+78*b^4*c*n*x^2-24*a
^3*b*d*n*x+2*a^2*b^2*c*n^2+24*a^2*b^2*d*x^2-58*a*b^3*c*n*x+40*b^4*c*x^2-24*
a^3*b*d*x+18*a^2*b^2*c*n-40*a*b^3*c*x+24*a^4*d+40*a^2*b^2*c)/b^5/(n^5+15*n^
4+85*n^3+225*n^2+274*n+120)
```

maxima [A] time = 0.47, size = 210, normalized size = 1.56

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^n c}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n +
24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n
)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)
^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

mupad [B] time = 2.82, size = 363, normalized size = 2.69

$$(a+b\,x)^n\left(\frac{2\,a^3\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{d\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{x^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((2*a^3*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^5*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d*x^5*(50*n + 35*n^2 + 10*
n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^3*(3*
n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 4*a^2*d*n + 9*b^2*c*n))/(b^2*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (2*a^2*n*x*(12*a^2*d + 20*b^2*c +
 b^2*c*n^2 + 9*b^2*c*n))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (a*n*x^2*(n + 1)*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2
 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

sympy [A] time = 6.70, size = 4134, normalized size = 30.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) - a**2*b**2*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**
7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 108*a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b*
*7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*a*b**3*c*x/(12*a**4*b**5 + 48*
a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**
3*d*x**3*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b
**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 6*b**4*c*x**2/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d*lo
g(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 22
*a**4*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a*
*3*b*d*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 54*a**3*b*d*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**
8*x**3) - a**2*b**2*c/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 36*a**2*b**2*d*x**2*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*
```

```
a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d*x**2/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*a*b**3*c*x/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d*x**3*log(a/b + x)/(3*a**3*b
**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*b**4*c*x**2/(3*a**3*
b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + 3*b**4*d*x**4/(3*a**3
*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3), Eq(n, -4)), (12*a**4*
d*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2
*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5
 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*
b**7*x**2) + 2*a**2*b**2*c*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*
x**2) + 3*a**2*b**2*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b*
*2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*
c*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*c*x/(2
*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 + 4*a
*b**6*x + 2*b**7*x**2) + 2*b**4*c*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6
*x + 2*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), E
q(n, -3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*
b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**
2*b**2*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**2*b**2*c/(3*a*b**5 + 3*b
**6*x) + 6*a**2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) - 6*a*b**3*c*x*log(a/b +
x)/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c
*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2))
, (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*c*log(a/b + x)/b**3 + a*
*2*d*x**2/(2*b**3) - a*c*x/b**2 - a*d*x**3/(3*b**2) + c*x**2/(2*b) + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 2*a**3*b**2*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 18*a**3*b**2*c*n*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) + 40*a**3*b**2*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 2
*a**2*b**3*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 18*a**2*b**3*c*n**2*x*(a + b*x)**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) - 40*a**2*b**3*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**
2*b**3*d*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c*n**4*x**2*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 10*a*b**4*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 29*a*b**4*c*n**2*x**2*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 20*a*b**4*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**
4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5) + 6*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**
4*d*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b
**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n*
*5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
 b**5*c*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**5*c*n**3*x**3*(a + b*x)**n/(b
```

```
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) + 49*b**5*c*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5
*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 78*b**5*c*n*x**3*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 40*b**5*c*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**
5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 10*b**5*d*n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) + 50*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**
5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5), True))
```

3.352 $\int x(a+bx)^n\left(c+dx^2\right)dx$

Optimal. Leaf size=102

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[Out] -a*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+(3*a^2*d+b^2*c)*(b*x+a)^(2+n)/b^4/
(2+n)-3*a*d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A] time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {772}

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] -((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d)
*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n))
 + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^2\right)dx=\int\left(\frac{a\left(-b^2c-a^2d\right)(a+bx)^n}{b^3}+\frac{\left(b^2c+3a^2d\right)(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+
b\right.$$

$$=-\frac{a\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)}+\frac{\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx}{b^4(4+}$$

Mathematica [A] time = 0.10, size = 109, normalized size = 1.07

$$\frac{(a+bx)^{n+1}\left(-6a^3d+6a^2bd(n+1)x-ab^2\left(c\left(n^2+7n+12\right)+3d\left(n^2+3n+2\right)x^2\right)+b^3\left(n^2+4n+3\right)x\left(c(n+4\right)\right.}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x + b^3*(3 + 4*n + n^2)*x*
(c*(4 + n) + d*(2 + n)*x^2) - a*b^2*(c*(12 + 7*n + n^2) + 3*d*(2 + 3*n + n^
2)*x^2)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

fricas [B] time = 0.89, size = 250, normalized size = 2.45

$$-\frac{\left(a^2b^2cn^2+7\,a^2b^2cn+12\,a^2b^2c+6\,a^4d-\left(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d\right)x^4-\left(ab^3dn^3+3\,ab^3dn^2+2\,ab^3d\right.\right.}{b^4n^4+10\,b^4n^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*c*n^2 + 7*a^2*b^2*c*n + 12*a^2*b^2*c + 6*a^4*d - (b^4*d*n^3 + 6*b
^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 - (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b
^3*d*n)*x^3 - (b^4*c*n^3 + 12*b^4*c + (8*b^4*c - 3*a^2*b^2*d)*n^2 + (19*b^4
*c - 3*a^2*b^2*d)*n)*x^2 - (a*b^3*c*n^3 + 7*a*b^3*c*n^2 + 6*(2*a*b^3*c + a^
3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24
*b^4)
```

giac [B] time = 0.19, size = 410, normalized size = 4.02

$$\frac{(bx+a)^n b^4 d n^3 x^4 + (bx+a)^n a b^3 d n^3 x^3 + 6(bx+a)^n b^4 d n^2 x^4 + (bx+a)^n b^4 c n^3 x^2 + 3(bx+a)^n a b^3 d n^2 x^3 + 11(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + (b*x + a)^n*b^4*c*n^3*x^2 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 1
1*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*a*b^3*c*n^3*x + 8*(b*x + a)^n*b^4*c
*n^2*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^3 +
6*(b*x + a)^n*b^4*d*x^4 + 7*(b*x + a)^n*a*b^3*c*n^2*x + 19*(b*x + a)^n*b^4*
c*n*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 - (b*x + a)^n*a^2*b^2*c*n^2 + 12*(b
*x + a)^n*a*b^3*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 12*(b*x + a)^n*b^4*c*x^
2 - 7*(b*x + a)^n*a^2*b^2*c*n - 12*(b*x + a)^n*a^2*b^2*c - 6*(b*x + a)^n*a^
4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

maple [A] time = 0.00, size = 195, normalized size = 1.91

$$-\frac{\left(-b^3 d n^3 x^3 - 6b^3 d n^2 x^3 + 3a b^2 d n^2 x^2 - b^3 c n^3 x - 11b^3 dn x^3 + 9a b^2 dn x^2 - 8b^3 c n^2 x - 6d x^3 b^3 - 6a^2 bdnx + a}{\left(n^4 + 10n^3 + 35n^2 + 50n +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c),x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*
x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*c*n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*
b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6*a^2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a
^3*d+12*a*b^2*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 0.47, size = 146, normalized size = 1.43

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^n c}{\left(n^2+3n+2\right)b^2} + \frac{\left(\left(n^3+6n^2+11n+6\right)b^4x^4 + \left(n^3+3n^2+2n\right)ab^3x^3 - 3\left(n^2+n\right)a^2b^2x}{\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n
)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2
+ 50*n + 24)*b^4)
```

mupad [B] time = 2.70, size = 255, normalized size = 2.50

$$(a+bx)^n \left(\frac{dx^4\left(n^3+6n^2+11n+6\right)}{n^4+10n^3+35n^2+50n+24} - \frac{a^2\left(6da^2+cb^2n^2+7cb^2n+12cb^2\right)}{b^4\left(n^4+10n^3+35n^2+50n+24\right)} + \frac{x^2(n+1)\left(-3da^2n+\right.}{b^2\left(n^4+10n^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
 + 24) - (a^2*(6*a^2*d + 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^4*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24)) + (x^2*(n + 1)*(12*b^2*c + b^2*c*n^2 - 3*a^2*d*n
 + 7*b^2*c*n))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x*(6*a^2*d
+ 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

sympy [A] time = 3.53, size = 2181, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6
*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*
log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
+ 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) - a*b**2*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*
x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b +
 x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*
b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) -
 a*b**2*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*x**2*log(a/
b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*x/(2*a**2*b**4 +
 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3
*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 2*a*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c/(2*a*b**4 + 2*
b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**
3*d*log(a/b + x)/b**4 + a**2*d*x/b**3 - a*c*log(a/b + x)/b**2 - a*d*x**2/(2
*b**2) + c*x/b + d*x**3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n*
*4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a
 + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
- a**2*b**2*c*n**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 7*a**2*b**2*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 12*a**2*b**2*c*(a + b*x)**n/(b**
4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d
*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*
n + 24*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
+ 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*x*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 7*a*b**3*c*n**2
*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b
**4) + 12*a*b**3*c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**
2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*c*n**3*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 8*b**4*c*n**2*x**2*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 19*b**4*c
*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
```

```
 24*b**4) + 12*b**4*c*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 1
0*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
11*b**4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

3.353 $\int (a+bx)^n \left(c+dx^2\right) dx$

Optimal. Leaf size=70

$$\frac{\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

```
[Out] (a^2*d+b^2*c)*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+
a)^(3+n)/b^3/(3+n)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n \left(c+dx^2\right) dx &= \int \left(\frac{\left(b^2c+a^2d\right)(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right) dx\\ &= \frac{\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.93

$$\frac{(a+bx)^{n+1}\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) +
d*(1 + n)*x^2)))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

fricas [B] time = 0.59, size = 148, normalized size = 2.11

$$\frac{\left(ab^2cn^2+5\,ab^2cn+6\,ab^2c+2\,a^3d+\left(b^3dn^2+3\,b^3dn+2\,b^3d\right)x^3+\left(ab^2dn^2+ab^2dn\right)x^2+\left(b^3cn^2+6\,b^3c+\left(5\,b^3c\right.\right.\right.}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n +
 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5*b
^3*c - 2*a^2*b*d)*n)*x)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3
)
```

giac [B] time = 0.16, size = 237, normalized size = 3.39

$$\frac{(bx+a)^n b^3 dn^2 x^3 + (bx+a)^n ab^2 dn^2 x^2 + 3\,(bx+a)^n b^3 dnx^3 + (bx+a)^n b^3 cn^2 x + (bx+a)^n ab^2 dnx^2 + 2\,(bx+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*d*n^2*x^3 + (b*x + a)^n*a*b^2*d*n^2*x^2 + 3*(b*x + a)^n*b^
3*d*n*x^3 + (b*x + a)^n*b^3*c*n^2*x + (b*x + a)^n*a*b^2*d*n*x^2 + 2*(b*x +
a)^n*b^3*d*x^3 + (b*x + a)^n*a*b^2*c*n^2 + 5*(b*x + a)^n*b^3*c*n*x - 2*(b*x
 + a)^n*a^2*b*d*n*x + 5*(b*x + a)^n*a*b^2*c*n + 6*(b*x + a)^n*b^3*c*x + 6*(
b*x + a)^n*a*b^2*c + 2*(b*x + a)^n*a^3*d)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n +
 6*b^3)
```

maple [A] time = 0.00, size = 100, normalized size = 1.43

$$\frac{\left(b^2 d\, n^2 x^2 + 3b^2 dn\, x^2 - 2abdnx + b^2 c\, n^2 + 2d\, x^2 b^2 - 2adxb + 5b^2 cn + 2a^2 d + 6b^2 c\right)(bx+a)^{n+1}}{\left(n^3 + 6n^2 + 11n + 6\right) b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*d*n*x+b^2*c*n^2+2*b^2*d*x^
2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 0.46, size = 89, normalized size = 1.27

$$\frac{(bx+a)^{n+1} c}{b(n+1)} + \frac{\left(\left(n^2 + 3\,n + 2\right)b^3 x^3 + \left(n^2 + n\right)ab^2 x^2 - 2\,a^2 bnx + 2\,a^3\right)(bx+a)^n d}{\left(n^3 + 6\,n^2 + 11\,n + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^
2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*d/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 2.63, size = 163, normalized size = 2.33

$$(a+b\,x)^n \left(\frac{d\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6} + \frac{x\,\left(-2\,d\,a^2\,b\,n + c\,b^3\,n^2 + 5\,c\,b^3\,n + 6\,c\,b^3\right)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,\left(2\,d\,a^2 + c\,b^2\,n^2 + 5\,c\,b^2\,n +\right.}{b^3\,\left(n^3+6\,n^2+11\,n+6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*b^3*c
 + b^3*c*n^2 + 5*b^3*c*n - 2*a^2*b*d*n))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (
a*(2*a^2*d + 6*b^2*c + b^2*c*n^2 + 5*b^2*c*n))/(b^3*(11*n + 6*n^2 + n^3 + 6
)) + (a*d*n*x^2*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [A] time = 2.07, size = 952, normalized size = 13.60

$$\begin{cases} a^n\left(cx+\frac{dx^3}{3}\right) \\ \frac{2a^2d\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{3a^2d}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx}{2a^2b^3+4ab^4x+2b^5x^2}-\frac{b^2c}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2dx^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2d\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2d}{ab^3+b^4x}-\frac{2abdx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{b^2c}{ab^3+b^4x}+\frac{b^2dx^2}{ab^3+b^4x} \\ \frac{a^2d\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{adx}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx^2}{2b} \\ \frac{2a^3d(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bdnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2cn^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5ab^2cn(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6ab^2c(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{}{b^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**3/3), Eq(b, 0)), (2*a**2*d*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d/(2*a**2*b**3 + 4*a*b**4*x + 2*
b**5*x**2) + 4*a*b*d*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
) + 4*a*b*d*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*d*log(a/b + x)/(a*b**3 + b**
4*x) - 2*a**2*d/(a*b**3 + b**4*x) - 2*a*b*d*x*log(a/b + x)/(a*b**3 + b**4*x
) - b**2*c/(a*b**3 + b**4*x) + b**2*d*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (
a**2*d*log(a/b + x)/b**3 - a*d*x/b**2 + c*log(a/b + x)/b + d*x**2/(2*b), Eq
(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6
*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*
b**3) + a*b**2*d*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*
n + 6*b**3) + a*b**2*d*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b*
*3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**
3*n + 6*b**3) + b**3*d*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11
*b**3*n + 6*b**3) + 3*b**3*d*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
 11*b**3*n + 6*b**3) + 2*b**3*d*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
+ 11*b**3*n + 6*b**3), True))
```

3.354 $\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)}+\frac{d(a+bx)^{n+2}}{b^2(n+2)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] -a*d*(b*x+a)^(1+n)/b^2/(1+n)+d*(b*x+a)^(2+n)/b^2/(2+n)-c*(b*x+a)^(1+n)*hype
rgeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {952, 80, 65}

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)}+\frac{d(a+bx)^{n+2}}{b^2(n+2)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2))/x,x]
```

```
[Out] -((a*d*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (d*(a + b*x)^(2 + n))/(b^2*(2 +
n)) - (c*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
/(a*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+bx)^n\left(c+dx^2\right)}{x}\,dx = \frac{d(a+bx)^{2+n}}{b^2(2+n)} + \frac{\int \frac{(a+bx)^n\left(b^2c(2+n)-abd(2+n)x\right)}{x}\,dx}{b^2(2+n)}$$
$$= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)} + \frac{d(a+bx)^{2+n}}{b^2(2+n)} + c\int \frac{(a+bx)^n}{x}\,dx$$
$$= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)} + \frac{d(a+bx)^{2+n}}{b^2(2+n)} - \frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.83

$$-\frac{(a+bx)^{n+1}\left(b^2c(n+2)\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)+ad(a-b(n+1)x)\right)}{ab^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2))/x,x]
[Out] -(((a + b*x)^(1 + n)*(a*d*(a - b*(1 + n)*x) + b^2*c*(2 + n)*Hypergeometric2
F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/(a*b^2*(1 + n)*(2 + n)))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^2+c\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="fricas")
[Out] integral((d*x^2 + c)*(b*x + a)^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^2+c\right)(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)/x,x)
[Out] int((b*x+a)^n*(d*x^2+c)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x^2+c)\,(a+b\,x)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)*(a + b*x)^n)/x,x)
```

```
[Out] int(((c + d*x^2)*(a + b*x)^n)/x, x)
```

sympy [B] time = 7.22, size = 345, normalized size = 4.48

$$-\frac{b^n cn\left(\frac{a}{b}+x\right)^n\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}-\frac{b^n c\left(\frac{a}{b}+x\right)^n\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}+d\left(\begin{cases}\frac{a^n x^2}{2}\\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b}\\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b}\\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)/x,x)
```

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(n +
 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) + d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**
3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2))
, (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 +
 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2)
+ b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a +
 b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) - b*b**n*c*n*x*(a/b + x)**
n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*x*
(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

3.355 $\int x^2(a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=232

$$\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad\left(5a^2d+2b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{d\left(15a^2d+\right.}{b^7}$$

```
[Out] a^2*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^7/(1+n)-2*a*(a^2*d+b^2*c)*(3*a^2*d+b^2*
c)*(b*x+a)^(2+n)/b^7/(2+n)+(15*a^4*d^2+12*a^2*b^2*c*d+b^4*c^2)*(b*x+a)^(3+n
)/b^7/(3+n)-4*a*d*(5*a^2*d+2*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+d*(15*a^2*d+2*b
^2*c)*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(
7+n)/b^7/(7+n)
```

Rubi [A] time = 0.14, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {948}

$$\frac{\left(12a^2b^2cd+15a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^2
*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a^2
*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*c +
 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a + b
*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d
^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx &= \int\left(\frac{\left(ab^2c+a^3d\right)^2(a+bx)^n}{b^6}+\frac{2a\left(-b^2c-3a^2d\right)\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^6}+\frac{\left(b^4c^2+\right.}{}\right.\\ &= \frac{a^2\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^7(1+n)}-\frac{2a\left(b^2c+a^2d\right)\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^7(2+n)}+\frac{\left(b^4c^2+1\right.}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 199, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{\left(a^3d+ab^2c\right)^2}{n+1}+\frac{d(a+bx)^4\left(15a^2d+2b^2c\right)}{n+5}-\frac{4ad(a+bx)^3\left(5a^2d+2b^2c\right)}{n+4}-\frac{2a(a+bx)\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)}{n+2}+\frac{(a+bx)^2\left(15a^4d^2+12a^2b^2c\right.}{n+3}\right.}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^2*c + a^3*d)^2/(1 + n) - (2*a*(b^2*c + a^2*d)*(b^2
*c + 3*a^2*d)*(a + b*x))/(2 + n) + ((b^4*c^2 + 12*a^2*b^2*c*d + 15*a^4*d^2)
*(a + b*x)^2)/(3 + n) - (4*a*d*(2*b^2*c + 5*a^2*d)*(a + b*x)^3)/(4 + n) + (
d*(2*b^2*c + 15*a^2*d)*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6 + n)
 + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

fricas [B] time = 0.96, size = 1027, normalized size = 4.43

$$\frac{\left(2\,a^3b^4c^2n^4 + 44\,a^3b^4c^2n^3 + 1680\,a^3b^4c^2 + 2016\,a^5b^2cd + 720\,a^7d^2 + \left(b^7d^2n^6 + 21\,b^7d^2n^5 + 175\,b^7d^2n^4 + 735\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^4*c^2*n^4 + 44*a^3*b^4*c^2*n^3 + 1680*a^3*b^4*c^2 + 2016*a^5*b^2*c
*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*n^4 + 735*b^
7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^6*d
^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*a*b^
6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 + 2*(b^7*c*d*n^6 + 1008*b^7*c*d + (23*b^7*
c*d - 3*a^2*b^5*d^2)*n^5 + 3*(69*b^7*c*d - 10*a^2*b^5*d^2)*n^4 + 5*(185*b^7
*c*d - 21*a^2*b^5*d^2)*n^3 + 2*(1072*b^7*c*d - 75*a^2*b^5*d^2)*n^2 + 36*(67
*b^7*c*d - 2*a^2*b^5*d^2)*n)*x^5 + 2*(a*b^6*c*d*n^6 + 19*a*b^6*c*d*n^5 + (1
31*a*b^6*c*d + 15*a^3*b^4*d^2)*n^4 + (401*a*b^6*c*d + 90*a^3*b^4*d^2)*n^3 +
 15*(36*a*b^6*c*d + 11*a^3*b^4*d^2)*n^2 + 18*(14*a*b^6*c*d + 5*a^3*b^4*d^2)
*n)*x^4 + (b^7*c^2*n^6 + 1680*b^7*c^2 + (25*b^7*c^2 - 8*a^2*b^5*c*d)*n^5 +
(247*b^7*c^2 - 128*a^2*b^5*c*d)*n^4 + (1219*b^7*c^2 - 664*a^2*b^5*c*d - 120
*a^4*b^3*d^2)*n^3 + 8*(389*b^7*c^2 - 152*a^2*b^5*c*d - 45*a^4*b^3*d^2)*n^2
+ 4*(949*b^7*c^2 - 168*a^2*b^5*c*d - 60*a^4*b^3*d^2)*n)*x^3 + 2*(179*a^3*b^
4*c^2 + 24*a^5*b^2*c*d)*n^2 + (a*b^6*c^2*n^6 + 23*a*b^6*c^2*n^5 + 3*(67*a*b
^6*c^2 + 8*a^3*b^4*c*d)*n^4 + (817*a*b^6*c^2 + 336*a^3*b^4*c*d)*n^3 + 2*(73
9*a*b^6*c^2 + 660*a^3*b^4*c*d + 180*a^5*b^2*d^2)*n^2 + 24*(35*a*b^6*c^2 + 4
2*a^3*b^4*c*d + 15*a^5*b^2*d^2)*n)*x^2 + 4*(319*a^3*b^4*c^2 + 156*a^5*b^2*c
*d)*n - 2*(a^2*b^5*c^2*n^5 + 22*a^2*b^5*c^2*n^4 + (179*a^2*b^5*c^2 + 24*a^4
*b^3*c*d)*n^3 + 2*(319*a^2*b^5*c^2 + 156*a^4*b^3*c*d)*n^2 + 24*(35*a^2*b^5*
c^2 + 42*a^4*b^3*c*d + 15*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^
6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n
 + 5040*b^7)
```

giac [B] time = 0.22, size = 1750, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^5 + 15*(b*x + a)^n*a*b^6*d
^2*n^5*x^6 + 175*(b*x + a)^n*b^7*d^2*n^4*x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*
x^4 + 46*(b*x + a)^n*b^7*c*d*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^2*n^5*x^5 +
85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7 + (b*x +
 a)^n*b^7*c^2*n^6*x^3 + 38*(b*x + a)^n*a*b^6*c*d*n^5*x^4 + 414*(b*x + a)^n*
b^7*c*d*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^
6*d^2*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^2*n^2*x^7 + (b*x + a)^n*a*b^6*c^2*n^
6*x^2 + 25*(b*x + a)^n*b^7*c^2*n^5*x^3 - 8*(b*x + a)^n*a^2*b^5*c*d*n^5*x^3
+ 262*(b*x + a)^n*a*b^6*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 +
1850*(b*x + a)^n*b^7*c*d*n^3*x^5 - 210*(b*x + a)^n*a^2*b^5*d^2*n^3*x^5 + 27
4*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b^7*d^2*n*x^7 + 23*(b*x
+ a)^n*a*b^6*c^2*n^5*x^2 + 247*(b*x + a)^n*b^7*c^2*n^4*x^3 - 128*(b*x + a)^
```

```
n*a^2*b^5*c*d*n^4*x^3 + 802*(b*x + a)^n*a*b^6*c*d*n^3*x^4 + 180*(b*x + a)^n
*a^3*b^4*d^2*n^3*x^4 + 4288*(b*x + a)^n*b^7*c*d*n^2*x^5 - 300*(b*x + a)^n*a
^2*b^5*d^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*
d^2*x^7 - 2*(b*x + a)^n*a^2*b^5*c^2*n^5*x + 201*(b*x + a)^n*a*b^6*c^2*n^4*x
^2 + 24*(b*x + a)^n*a^3*b^4*c*d*n^4*x^2 + 1219*(b*x + a)^n*b^7*c^2*n^3*x^3
- 664*(b*x + a)^n*a^2*b^5*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^2*n^3*x^3
 + 1080*(b*x + a)^n*a*b^6*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*n^2*x^4
 + 4824*(b*x + a)^n*b^7*c*d*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 - 44*
(b*x + a)^n*a^2*b^5*c^2*n^4*x + 817*(b*x + a)^n*a*b^6*c^2*n^3*x^2 + 336*(b*
x + a)^n*a^3*b^4*c*d*n^3*x^2 + 3112*(b*x + a)^n*b^7*c^2*n^2*x^3 - 1216*(b*x
 + a)^n*a^2*b^5*c*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 504*(b*
x + a)^n*a*b^6*c*d*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n*x^4 + 2016*(b*x +
a)^n*b^7*c*d*x^5 + 2*(b*x + a)^n*a^3*b^4*c^2*n^4 - 358*(b*x + a)^n*a^2*b^5*
c^2*n^3*x - 48*(b*x + a)^n*a^4*b^3*c*d*n^3*x + 1478*(b*x + a)^n*a*b^6*c^2*n
^2*x^2 + 1320*(b*x + a)^n*a^3*b^4*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2
*n^2*x^2 + 3796*(b*x + a)^n*b^7*c^2*n*x^3 - 672*(b*x + a)^n*a^2*b^5*c*d*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 44*(b*x + a)^n*a^3*b^4*c^2*n^3 - 1
276*(b*x + a)^n*a^2*b^5*c^2*n^2*x - 624*(b*x + a)^n*a^4*b^3*c*d*n^2*x + 840
*(b*x + a)^n*a*b^6*c^2*n*x^2 + 1008*(b*x + a)^n*a^3*b^4*c*d*n*x^2 + 360*(b*
x + a)^n*a^5*b^2*d^2*n*x^2 + 1680*(b*x + a)^n*b^7*c^2*x^3 + 358*(b*x + a)^n
*a^3*b^4*c^2*n^2 + 48*(b*x + a)^n*a^5*b^2*c*d*n^2 - 1680*(b*x + a)^n*a^2*b^
5*c^2*n*x - 2016*(b*x + a)^n*a^4*b^3*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 1276*(b*x + a)^n*a^3*b^4*c^2*n + 624*(b*x + a)^n*a^5*b^2*c*d*n + 1680*(
b*x + a)^n*a^3*b^4*c^2 + 2016*(b*x + a)^n*a^5*b^2*c*d + 720*(b*x + a)^n*a^7
*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 1
3132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

maple [B] time = 0.02, size = 1000, normalized size = 4.31

$$\frac{\left(b^6 d^2 n^6 x^6 + 21 b^6 d^2 n^5 x^6 - 6 a\, b^5 d^2 n^5 x^5 + 2 b^6 c d\, n^6 x^4 + 175 b^6 d^2 n^4 x^6 - 90 a\, b^5 d^2 n^4 x^5 + 46 b^6 c d\, n^5 x^4 + 735 b^6 d^2 n^3 x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+2*b^6
*c*d*n^6*x^4+175*b^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+46*b^6*c*d*n^5*x^4+73
5*b^6*d^2*n^3*x^6+30*a^2*b^4*d^2*n^4*x^4-8*a*b^5*c*d*n^5*x^3-510*a*b^5*d^2*
n^3*x^5+b^6*c^2*n^6*x^2+414*b^6*c*d*n^4*x^4+1624*b^6*d^2*n^2*x^6+300*a^2*b^
4*d^2*n^3*x^4-152*a*b^5*c*d*n^4*x^3-1350*a*b^5*d^2*n^2*x^5+25*b^6*c^2*n^5*x
^2+1850*b^6*c*d*n^3*x^4+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+24*a^2*b
^4*c*d*n^4*x^2+1050*a^2*b^4*d^2*n^2*x^4-2*a*b^5*c^2*n^5*x-1048*a*b^5*c*d*n^
3*x^3-1644*a*b^5*d^2*n*x^5+247*b^6*c^2*n^4*x^2+4288*b^6*c*d*n^2*x^4+720*b^6
*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+384*a^2*b^4*c*d*n^3*x^2+1500*a^2*b^4*d^2*n
*x^4-46*a*b^5*c^2*n^4*x-3208*a*b^5*c*d*n^2*x^3-720*a*b^5*d^2*x^5+1219*b^6*c
^2*n^3*x^2+4824*b^6*c*d*n*x^4+360*a^4*b^2*d^2*n^2*x^2-48*a^3*b^3*c*d*n^3*x-
1320*a^3*b^3*d^2*n*x^3+2*a^2*b^4*c^2*n^4+1992*a^2*b^4*c*d*n^2*x^2+720*a^2*b
^4*d^2*x^4-402*a*b^5*c^2*n^3*x-4320*a*b^5*c*d*n*x^3+3112*b^6*c^2*n^2*x^2+20
16*b^6*c*d*x^4+1080*a^4*b^2*d^2*n*x^2-672*a^3*b^3*c*d*n^2*x-720*a^3*b^3*d^2
*x^3+44*a^2*b^4*c^2*n^3+3648*a^2*b^4*c*d*n*x^2-1634*a*b^5*c^2*n^2*x-2016*a*
b^5*c*d*x^3+3796*b^6*c^2*n*x^2-720*a^5*b*d^2*n*x+48*a^4*b^2*c*d*n^2+720*a^4
*b^2*d^2*x^2-2640*a^3*b^3*c*d*n*x+358*a^2*b^4*c^2*n^2+2016*a^2*b^4*c*d*x^2-
2956*a*b^5*c^2*n*x+1680*b^6*c^2*x^2-720*a^5*b*d^2*x+624*a^4*b^2*c*d*n-2016*
a^3*b^3*c*d*x+1276*a^2*b^4*c^2*n-1680*a*b^5*c^2*x+720*a^6*d^2+2016*a^4*b^2*
c*d+1680*a^2*b^4*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+1
3068*n+5040)
```

maxima [A] time = 0.50, size = 447, normalized size = 1.93

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc^2}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{2\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 +
6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

mupad [B] time = 3.12, size = 932, normalized size = 4.02

$$\frac{2\,a^3\,{\left(a+b\,x\right)}^n\,\left(360\,a^4\,d^2+24\,a^2\,b^2\,c\,d\,n^2+312\,a^2\,b^2\,c\,d\,n+1008\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+22\,b^4\,c^2\,n^3+179\,b^4\,c^2\,n\right.}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2
*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*d + 312*a^2*b^2*c*d*n
+ 24*a^2*b^2*c*d*n^2))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 32
2*n^5 + 28*n^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 7
35*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1
960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)
*(840*b^4*c^2 - 120*a^4*d^2*n + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^
2*n^3 + b^4*c^2*n^4 - 336*a^2*b^2*c*d*n - 104*a^2*b^2*c*d*n^2 - 8*a^2*b^2*c
*d*n^3))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6
 + n^7 + 5040)) - (2*a^2*n*x*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b
^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*
d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^6*(13068*n + 13132*n^2 + 67
69*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^5*(a + b*x)^n*
(42*b^2*c + b^2*c*n^2 - 3*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^
6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(360*a^4*d^2 + 840*b^
4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 10
08*a^2*b^2*c*d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n
*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(15*a^2*d + 42*b^2*c + b^2*c*n^2
+ 13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
28*n^6 + n^7 + 5040))
```

sympy [A] time = 21.59, size = 14317, normalized size = 61.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**3/3 + 2*c*d*x**5/5 + d**2*x**7/7), Eq(b, 0)), (60*
a**6*d**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x
 + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*
b**12*x**5 + 60*b**13*x**6) - 4*a**4*b**2*c*d/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a*
*6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900
*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*
x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10
*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 24*a**3*b
**3*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*
b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200
*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**
4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a
*b**12*x**5 + 60*b**13*x**6) - a**2*b**4*c**2/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) - 60*a**2*b**4*c*d*x**2/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x
)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b*
*4*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a*
*3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6
*a*b**5*c**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*
a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) -
 80*a*b**5*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x*
*6) + 360*a*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**
8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360
*a*b**12*x**5 + 60*b**13*x**6) - 15*b**6*c**2*x**2/(60*a**6*b**7 + 360*a**5
*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 +
 360*a*b**12*x**5 + 60*b**13*x**6) - 60*b**6*c*d*x**4/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-180*a
**6*d**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2
+ 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 411*a**6*d**2/(
30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 +
 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**5*b*d**2*x*log(a/b + x)/(30*a**
5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a
*b**11*x**4 + 30*b**12*x**5) - 1875*a**5*b*d**2*x/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
**12*x**5) - 12*a**4*b**2*c*d/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b*
*9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a*
*4*b**2*d**2*x**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b
**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 3300*a
**4*b**2*d**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 3
```

```
00*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*a**3*b**3*c*d*x
/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3
 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a**3*b**3*d**2*x**3*log(a/b + x
)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**
3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 2700*a**3*b**3*d**2*x**3/(30*a**5*b
**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b*
*11*x**4 + 30*b**12*x**5) - a**2*b**4*c**2/(30*a**5*b**7 + 150*a**4*b**8*x
+ 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x*
*5) - 120*a**2*b**4*c*d*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**
9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**2
*b**4*d**2*x**4*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**
9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**2
*b**4*d**2*x**4/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*
a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 5*a*b**5*c**2*x/(30*a
**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150
*a*b**11*x**4 + 30*b**12*x**5) - 120*a*b**5*c*d*x**3/(30*a**5*b**7 + 150*a*
*4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 3
0*b**12*x**5) - 180*a*b**5*d**2*x**5*log(a/b + x)/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
**12*x**5) - 10*b**6*c**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b
**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*b**
6*c*d*x**4/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*
b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) + 30*b**6*d**2*x**6/(30*a**5
*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*
b**11*x**4 + 30*b**12*x**5), Eq(n, -6)), (180*a**6*d**2*log(a/b + x)/(12*a*
*4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x
**4) + 375*a**6*d**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 4
8*a*b**10*x**3 + 12*b**11*x**4) + 720*a**5*b*d**2*x*log(a/b + x)/(12*a**4*b
**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4)
 + 1320*a**5*b*d**2*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 +
48*a*b**10*x**3 + 12*b**11*x**4) + 24*a**4*b**2*c*d*log(a/b + x)/(12*a**4*b
**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4)
 + 50*a**4*b**2*c*d/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48
*a*b**10*x**3 + 12*b**11*x**4) + 1080*a**4*b**2*d**2*x**2*log(a/b + x)/(12*
a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11
*x**4) + 1620*a**4*b**2*d**2*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*
b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 96*a**3*b**3*c*d*x*log(a/b +
 x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 +
12*b**11*x**4) + 176*a**3*b**3*c*d*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a*
*2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 720*a**3*b**3*d**2*x**3*l
og(a/b + x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10
*x**3 + 12*b**11*x**4) + 720*a**3*b**3*d**2*x**3/(12*a**4*b**7 + 48*a**3*b*
*8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - a**2*b**4*c**
2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12
*b**11*x**4) + 144*a**2*b**4*c*d*x**2*log(a/b + x)/(12*a**4*b**7 + 48*a**3*
b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 216*a**2*b*
*4*c*d*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10
*x**3 + 12*b**11*x**4) + 180*a**2*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**7
 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) -
4*a*b**5*c**2*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b
**10*x**3 + 12*b**11*x**4) + 96*a*b**5*c*d*x**3*log(a/b + x)/(12*a**4*b**7
+ 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 9
6*a*b**5*c*d*x**3/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a
*b**10*x**3 + 12*b**11*x**4) - 36*a*b**5*d**2*x**5/(12*a**4*b**7 + 48*a**3*
b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - 6*b**6*c**2
*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3
+ 12*b**11*x**4) + 24*b**6*c*d*x**4*log(a/b + x)/(12*a**4*b**7 + 48*a**3*b*
*8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 6*b**6*d**2*x
**6/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 +
```

```
12*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b + x)/(3*a**3*b**7 + 9*a*
*2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*d**2/(3*a**3*b**7 + 9*
a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**5*b*d**2*x*log(a/b + x
)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 270*a**5*b
*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 24*a
**4*b**2*c*d*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*
b**10*x**3) - 44*a**4*b**2*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a*
*2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a**3
*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**3*b**3*c*d*x*
log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) -
 108*a**3*b**3*c*d*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) - 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x +
 9*a*b**9*x**2 + 3*b**10*x**3) - a**2*b**4*c**2/(3*a**3*b**7 + 9*a**2*b**8*
x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**2*b**4*c*d*x**2*log(a/b + x)/(3*a
**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**2*b**4*c*d
*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**
2*b**4*d**2*x**4/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**
3) - 3*a*b**5*c**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) - 24*a*b**5*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a
*b**9*x**2 + 3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*
x + 9*a*b**9*x**2 + 3*b**10*x**3) - 3*b**6*c**2*x**2/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*b**6*c*d*x**4/(3*a**3*b**7 + 9*a
**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 +
9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*lo
g(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*
b**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b
**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8
*x + 4*b**9*x**2) + 48*a**4*b**2*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x
 + 4*b**9*x**2) + 72*a**4*b**2*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2)
 + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + 96*a**3*b**3*c*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + 96*a**3*b**3*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 20*a**
3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*a**2*b**4*c**
2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 6*a**2*b**4*c**2/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 48*a**2*b**4*c*d*x**2*log(a/b +
x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x**4/(4*a**2
*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 8*a*b**5*c**2*x*log(a/b + x)/(4*a**2*b*
*7 + 8*a*b**8*x + 4*b**9*x**2) + 8*a*b**5*c**2*x/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) - 16*a*b**5*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2
) - 2*a*b**5*d**2*x**5/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*b**6*c*
*2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*b**6*c*d*
x**4/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d**2*x**6/(4*a**2*b**7
 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-180*a**6*d**2*log(a/b + x)/(30*
a*b**7 + 30*b**8*x) - 180*a**6*d**2/(30*a*b**7 + 30*b**8*x) - 180*a**5*b*d*
*2*x*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 240*a**4*b**2*c*d*log(a/b + x)/
(30*a*b**7 + 30*b**8*x) - 240*a**4*b**2*c*d/(30*a*b**7 + 30*b**8*x) + 90*a*
*4*b**2*d**2*x**2/(30*a*b**7 + 30*b**8*x) - 240*a**3*b**3*c*d*x*log(a/b + x
)/(30*a*b**7 + 30*b**8*x) - 30*a**3*b**3*d**2*x**3/(30*a*b**7 + 30*b**8*x)
- 60*a**2*b**4*c**2*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 60*a**2*b**4*c**
2/(30*a*b**7 + 30*b**8*x) + 120*a**2*b**4*c*d*x**2/(30*a*b**7 + 30*b**8*x)
+ 15*a**2*b**4*d**2*x**4/(30*a*b**7 + 30*b**8*x) - 60*a*b**5*c**2*x*log(a/b
 + x)/(30*a*b**7 + 30*b**8*x) - 40*a*b**5*c*d*x**3/(30*a*b**7 + 30*b**8*x)
- 9*a*b**5*d**2*x**5/(30*a*b**7 + 30*b**8*x) + 30*b**6*c**2*x**2/(30*a*b**7
 + 30*b**8*x) + 20*b**6*c*d*x**4/(30*a*b**7 + 30*b**8*x) + 6*b**6*d**2*x**6
/(30*a*b**7 + 30*b**8*x), Eq(n, -2)), (a**6*d**2*log(a/b + x)/b**7 - a**5*d
**2*x/b**6 + 2*a**4*c*d*log(a/b + x)/b**5 + a**4*d**2*x**2/(2*b**5) - 2*a**
3*c*d*x/b**4 - a**3*d**2*x**3/(3*b**4) + a**2*c**2*log(a/b + x)/b**3 + a**2
*c*d*x**2/b**3 + a**2*d**2*x**4/(4*b**3) - a*c**2*x/b**2 - 2*a*c*d*x**3/(3*
```

b**2) - a*d**2*x**5/(5*b**2) + c**2*x**2/(2*b) + c*d*x**4/(2*b) + d**2*x**6
/(6*b), Eq(n, -1)), (720*a**7*d**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) - 720*a**6*b*d**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 48*a**5*b**2*c*d*n**2*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) + 624*a**5*b**2*c*d*n*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2016*a**5*b**2*c*d*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b**2*d**2*n**2*
x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5
*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) - 48*a**4*b**3*c*d*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
7*n5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) - 624*a**4*b**3*c*d*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) - 2016*a**4*b**3*c*d*n*x*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3*x**3*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**4*b**3*d**2*n*
*2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 240*a
4*b3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 2*a**3*b**4*c**2*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 44*a**3*b**4*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
6 + 322*b7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 358*a**3*b**4*c**2*n**2*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) + 1276*a**3*b**4*c**2*n*(a + b*x)**n/(b
7*n7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*a**3*b**4*c**2*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 24*a**3*b**4*c*d*n**4
*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 336*a**
3*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 1320*a**3*b**4*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 1008*a**3*b**4*c*d*n*x**2*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3*b**4*d**2*n**4*x**4*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
3 + 13132*b7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n**3
*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 330*a**
3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) - 2*a**2*b**5*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*

*2 + 13068*b**7*n + 5040*b**7) - 44*a**2*b**5*c**2*n**4*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 358*a**2*b**5*c**2*n**3*x*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1276*a**2*b**5*c*
*2*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 168
0*a**2*b**5*c**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) - 8*a**2*b**5*c*d*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
7*n + 5040*b7) - 128*a**2*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) - 664*a**2*b**5*c*d*n**3*x**3*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1216*a**2*b**5*c*d*n**2*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 672*a**2
*b**5*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) - 6*a**2*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) - 60*a**2*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x*
5(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b
5*d2*n*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 19
60*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + a*b**6*c**2*n**6*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 23*a*b**6*c**2*n**5*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 201*a*b**6*c**2*n**4*x**2*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 817*a*b**6*c**2*n**3*x**2*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1478*a*b**6*c**2*n**2*x**
2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 840*a*b**6*
c**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b
7*n4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2
*a*b**6*c*d*n**6*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 38*a*b**6*c*d*n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) + 262*a*b**6*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b
7*n6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 802*a*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13
132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1080*a*b**6*c*d*n**2*x**4*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b
7*n3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 504*a*b**6*c*d*n*x
4*(a + b*x)n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*d*
*2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*
b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +

```
15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**2*n**2*x**6*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 120*a*b**6*d**2*n*x**6*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*c**2*n**6*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 25*b**7*
c**2*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
+ 247*b**7*c**2*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) + 1219*b**7*c**2*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + 3112*b**7*c**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 3796*b**7*c**2*n*x**3*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*b**7*c**2*x**3*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*x**5*(
a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 67
69*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 46*b**7*c*d*n*
*5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 414*b
**7*c*d*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 1850*b**7*c*d*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) + 4288*b**7*c*d*n**2*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 4824*b**7*c*d*n*x**5*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 2016*b**7*c*d*x**5*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n/(
b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
+ 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n**
4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*b*
*7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
*6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13
068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7), True))
```

3.356 $\int x(a+bx)^n \left(c+dx^2\right)^2 dx$

Optimal. Leaf size=185

$$-\frac{a\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d\left(5a^2d+b^2\right.}{b^6(n}$$

```
[Out] -a*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^6/(1+n)+(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b
*x+a)^(2+n)/b^6/(2+n)-2*a*d*(5*a^2*d+3*b^2*c)*(b*x+a)^(3+n)/b^6/(3+n)+2*d*(
5*a^2*d+b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*
(b*x+a)^(6+n)/b^6/(6+n)
```

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {772}

$$-\frac{a\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d\left(5a^2d+b^2\right.}{b^6(n}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d)
*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5*a
^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x)^(
4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a
 + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^2\right)^2 dx = \int \left(-\frac{a\left(b^2c+a^2d\right)^2(a+bx)^n}{b^5}+\frac{\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{1+n}}{b^5}-\frac{2ad\left(3b^2c\right.}{}\right.$$

$$= -\frac{a\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6(1+n)}+\frac{\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{2+n}}{b^6(2+n)}-\frac{2ad\left(3b^2c+\right.}{b^6}$$

Mathematica [A] time = 0.50, size = 323, normalized size = 1.75

$$\frac{(a+bx)^{n+1}\left(4(n+1)(a+bx)\left((n+5)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)-a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(a + b*x)*(
c + d*x^2)^2 - a*(6 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2
 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*
```

```
(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 +
 n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2))) + 4*(1 + n)*(a + b*x)*((b
^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n)
 + d*(2 + n)*x^2)) - a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b
^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(5 + n)*(6 + n))
```

fricas [B] time = 0.96, size = 757, normalized size = 4.09

$$-\frac{\left(a^2b^4c^2n^4 + 18\,a^2b^4c^2n^3 + 360\,a^2b^4c^2 + 360\,a^4b^2cd + 120\,a^6d^2 - \left(b^6d^2n^5 + 15\,b^6d^2n^4 + 85\,b^6d^2n^3 + 225\,b^6d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d
+ 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*d^
2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2*n^
4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*c*d*
n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d - 15*
a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*d - 5*
a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*b^5*c*d
 + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*(3*a*b^5
*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^2 - (b^6*
c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b^6*c^2 - 7
2*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2*d^2)*n^2 +
 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(57*a^2*b^4*c
^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (119*a*b^5*c^2
 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^2 + 120*(3*a*b
^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5
 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

giac [B] time = 0.22, size = 1266, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d^2*n^5*x^6 + (b*x + a)^n*a*b^5*d^2*n^5*x^5 + 15*(b*x + a)
^n*b^6*d^2*n^4*x^6 + 2*(b*x + a)^n*b^6*c*d*n^5*x^4 + 10*(b*x + a)^n*a*b^5*d
^2*n^4*x^5 + 85*(b*x + a)^n*b^6*d^2*n^3*x^6 + 2*(b*x + a)^n*a*b^5*c*d*n^5*x
^3 + 34*(b*x + a)^n*b^6*c*d*n^4*x^4 - 5*(b*x + a)^n*a^2*b^4*d^2*n^4*x^4 + 3
5*(b*x + a)^n*a*b^5*d^2*n^3*x^5 + 225*(b*x + a)^n*b^6*d^2*n^2*x^6 + (b*x +
a)^n*b^6*c^2*n^5*x^2 + 28*(b*x + a)^n*a*b^5*c*d*n^4*x^3 + 214*(b*x + a)^n*b
^6*c*d*n^3*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n^3*x^4 + 50*(b*x + a)^n*a*b^5*
d^2*n^2*x^5 + 274*(b*x + a)^n*b^6*d^2*n*x^6 + (b*x + a)^n*a*b^5*c^2*n^5*x +
 19*(b*x + a)^n*b^6*c^2*n^4*x^2 - 6*(b*x + a)^n*a^2*b^4*c*d*n^4*x^2 + 130*(
b*x + a)^n*a*b^5*c*d*n^3*x^3 + 20*(b*x + a)^n*a^3*b^3*d^2*n^3*x^3 + 614*(b*
x + a)^n*b^6*c*d*n^2*x^4 - 55*(b*x + a)^n*a^2*b^4*d^2*n^2*x^4 + 24*(b*x + a
)^n*a*b^5*d^2*n*x^5 + 120*(b*x + a)^n*b^6*d^2*x^6 + 18*(b*x + a)^n*a*b^5*c^
2*n^4*x + 137*(b*x + a)^n*b^6*c^2*n^3*x^2 - 72*(b*x + a)^n*a^2*b^4*c*d*n^3*
x^2 + 224*(b*x + a)^n*a*b^5*c*d*n^2*x^3 + 60*(b*x + a)^n*a^3*b^3*d^2*n^2*x^
3 + 792*(b*x + a)^n*b^6*c*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n*x^4 - (b*x
 + a)^n*a^2*b^4*c^2*n^4 + 119*(b*x + a)^n*a*b^5*c^2*n^3*x + 12*(b*x + a)^n*
a^3*b^3*c*d*n^3*x + 461*(b*x + a)^n*b^6*c^2*n^2*x^2 - 246*(b*x + a)^n*a^2*b
^4*c*d*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n^2*x^2 + 120*(b*x + a)^n*a*b^5
*c*d*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d^2*n*x^3 + 360*(b*x + a)^n*b^6*c*d*x^4
 - 18*(b*x + a)^n*a^2*b^4*c^2*n^3 + 342*(b*x + a)^n*a*b^5*c^2*n^2*x + 132*(
```

```
b*x + a)^n*a^3*b^3*c*d*n^2*x + 702*(b*x + a)^n*b^6*c^2*n*x^2 - 180*(b*x + a
)^n*a^2*b^4*c*d*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n*x^2 - 119*(b*x + a)^n*
a^2*b^4*c^2*n^2 - 12*(b*x + a)^n*a^4*b^2*c*d*n^2 + 360*(b*x + a)^n*a*b^5*c^
2*n*x + 360*(b*x + a)^n*a^3*b^3*c*d*n*x + 120*(b*x + a)^n*a^5*b*d^2*n*x + 3
60*(b*x + a)^n*b^6*c^2*x^2 - 342*(b*x + a)^n*a^2*b^4*c^2*n - 132*(b*x + a)^
n*a^4*b^2*c*d*n - 360*(b*x + a)^n*a^2*b^4*c^2 - 360*(b*x + a)^n*a^4*b^2*c*d
 - 120*(b*x + a)^n*a^6*d^2)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n
^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

maple [B] time = 0.01, size = 677, normalized size = 3.66

$$-\frac{\left(-b^5d^2n^5x^5 - 15b^5d^2n^4x^5 + 5a\,b^4d^2n^4x^4 - 2b^5cd\,n^5x^3 - 85b^5d^2n^3x^5 + 50a\,b^4d^2n^3x^4 - 34b^5cd\,n^4x^3 - 225b^5d^2n^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^5*d^2*n^5*x^5-15*b^5*d^2*n^4*x^5+5*a*b^4*d^2*n^4*x^4-2*b
^5*c*d*n^5*x^3-85*b^5*d^2*n^3*x^5+50*a*b^4*d^2*n^3*x^4-34*b^5*c*d*n^4*x^3-2
25*b^5*d^2*n^2*x^5-20*a^2*b^3*d^2*n^3*x^3+6*a*b^4*c*d*n^4*x^2+175*a*b^4*d^2
*n^2*x^4-b^5*c^2*n^5*x-214*b^5*c*d*n^3*x^3-274*b^5*d^2*n*x^5-120*a^2*b^3*d^
2*n^2*x^3+84*a*b^4*c*d*n^3*x^2+250*a*b^4*d^2*n*x^4-19*b^5*c^2*n^4*x-614*b^5
*c*d*n^2*x^3-120*b^5*d^2*x^5+60*a^3*b^2*d^2*n^2*x^2-12*a^2*b^3*c*d*n^3*x-22
0*a^2*b^3*d^2*n*x^3+a*b^4*c^2*n^4+390*a*b^4*c*d*n^2*x^2+120*a*b^4*d^2*x^4-1
37*b^5*c^2*n^3*x-792*b^5*c*d*n*x^3+180*a^3*b^2*d^2*n*x^2-144*a^2*b^3*c*d*n^
2*x-120*a^2*b^3*d^2*x^3+18*a*b^4*c^2*n^3+672*a*b^4*c*d*n*x^2-461*b^5*c^2*n^
2*x-360*b^5*c*d*x^3-120*a^4*b*d^2*n*x+12*a^3*b^2*c*d*n^2+120*a^3*b^2*d^2*x^
2-492*a^2*b^3*c*d*n*x+119*a*b^4*c^2*n^2+360*a*b^4*c*d*x^2-702*b^5*c^2*n*x-1
20*a^4*b*d^2*x+132*a^3*b^2*c*d*n-360*a^2*b^3*c*d*x+342*a*b^4*c^2*n-360*b^5*
c^2*x+120*a^5*d^2+360*a^3*b^2*c*d+360*a*b^4*c^2)/b^6/(n^6+21*n^5+175*n^4+73
5*n^3+1624*n^2+1764*n+720)
```

maxima [A] time = 0.50, size = 335, normalized size = 1.81

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^nc^2}{\left(n^2+3\,n+2\right)b^2} + \frac{2\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4 + \left(n^3+3\,n^2+2\,n\right)ab^3x^3 - 3\left(n^2+n\right)a^2b^2x^2}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*
b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^
3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^
2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6 + 21*n^
5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

mupad [B] time = 3.05, size = 723, normalized size = 3.91

$$\frac{d^2\,x^6\,{\left(a+b\,x\right)}^n\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720} - \frac{a^2\,{\left(a+b\,x\right)}^n\,\left(120\,a^4\,d^2+12\,a^2\,b^2\,c\,d\,n^2+132\,a^2\,b^2}{b^6\,\left(n^6+21\,n^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (a^2*(a + b*x)^n*
(120*a^4*d^2 + 360*b^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n
^3 + b^4*c^2*n^4 + 360*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2
))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^
2*(n + 1)*(a + b*x)^n*(360*b^4*c^2 - 60*a^4*d^2*n + 342*b^4*c^2*n + 119*b^4
*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 - 180*a^2*b^2*c*d*n - 66*a^2*b^2*c*
d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6 + 720)) + (d*x^4*(a + b*x)^n*(60*b^2*c + 2*b^2*c*n^2 - 5*a^2*d*
n + 22*b^2*c*n)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3
 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*n*x*(a + b*x)^n*(120*a^4*d^2 + 360*b
^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 + 3
60*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(1764*n + 16
24*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d^2*n*x^5*(a + b*x)^
n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(10
*a^2*d + 30*b^2*c + b^2*c*n^2 + 11*b^2*c*n))/(b^3*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [A] time = 13.80, size = 8940, normalized size = 48.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), Eq(b, 0)), (60*a*
*5*d**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/(60
*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 30
0*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*log(a/b + x)/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d
**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d
**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*d*x/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*log(a/b + x)/(60*a**5*b*
*6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 60*a*b**4*c*
d*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4/(60*a**5*b**6
+ 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x
**4 + 60*b**11*x**5) - 15*b**5*c**2*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) -
60*b**5*c*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600
*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d**2*x**5*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d**2*lo
g(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x
**3 + 12*b**10*x**4) - 125*a**5*d**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
```

*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d**2*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*d**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*a**3*b**2*c*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*d**2*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 24*a**2*b**3*c*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d**2*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d**2*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a*b**4*c**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 36*a*b**4*c*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*b**5*c**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 24*b**5*c*d*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*d**2*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*d**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*d**2*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4*b*d**2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*a**3*b**2*c*d*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 22*a**3*b**2*c*d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a*2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a**2*b**3*c*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 54*a**2*b**3*c*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d**2*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - a*b**4*c**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a*b**4*c*d*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*d**2*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 3*b**5*c**2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*b**5*c*d*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*d**2*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5*d**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5*d**2/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d**2*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 36*a**3*b**2*c*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 54*a**3*b**2*c*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a**2*b**3*c*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a**2*b**3*c*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d**2*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 3*a*b**4*c**2/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4*d**2*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 6*b**5*c**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 12*b**5*c*d*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d**2*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d**2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5*d**2/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*d**2*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*

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c*d*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*c*d/(12*a*b**6 + 12
*b**7*x) - 30*a**3*b**2*d**2*x**2/(12*a*b**6 + 12*b**7*x) + 72*a**2*b**3*c*
d*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*d**2*x**3/(12*a*b**
6 + 12*b**7*x) + 12*a*b**4*c**2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 12*a
*b**4*c**2/(12*a*b**6 + 12*b**7*x) - 36*a*b**4*c*d*x**2/(12*a*b**6 + 12*b**
7*x) - 5*a*b**4*d**2*x**4/(12*a*b**6 + 12*b**7*x) + 12*b**5*c**2*x*log(a/b
+ x)/(12*a*b**6 + 12*b**7*x) + 12*b**5*c*d*x**3/(12*a*b**6 + 12*b**7*x) + 3
*b**5*d**2*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d**2*log(a/b +
x)/b**6 + a**4*d**2*x/b**5 - 2*a**3*c*d*log(a/b + x)/b**4 - a**3*d**2*x**2/
(2*b**4) + 2*a**2*c*d*x/b**3 + a**2*d**2*x**3/(3*b**3) - a*c**2*log(a/b + x
)/b**2 - a*c*d*x**2/b**2 - a*d**2*x**4/(4*b**2) + c**2*x/b + 2*c*d*x**3/(3*
b) + d**2*x**5/(5*b), Eq(n, -1)), (-120*a**6*d**2*(a + b*x)**n/(b**6*n**6 +
 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) + 120*a**5*b*d**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
+ 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
- 12*a**4*b**2*c*d*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 132*a**4*b
**2*c*d*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a**4*b**2*c*d*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d**2*n**2*x**2*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d**2*n*x**2*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 12*a**3*b**3*c*d*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 132*a**3*b**3*c*d*n**2*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) +
 360*a**3*b**3*c*d*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**
3*d**2*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*d**2
*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**
6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*d**2*n*x**
3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - a**2*b**4*c**2*n**4*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 18*a**2*b**4*c**2*n**3*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 119*a**2*b**4*c**2*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) - 342*a**2*b**4*c**2*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a
**2*b**4*c**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*
b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 6*a**2*b**4*c*d*n**4
*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**
3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 72*a**2*b**4*c*d*n**3*x**2*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 246*a**2*b**4*c*d*n**2*x**2*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) - 180*a**2*b**4*c*d*n*x**2*(a + b*x)**n/(b*
*6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) - 5*a**2*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 30*a**2*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) - 55*a**2*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) - 30*a**2*b**4*d**2*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
```

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*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b
**5*c**2*n**5*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*a*b**5*c**2*n**
4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 119*a*b**5*c**2*n**3*x*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 342*a*b**5*c**2*n**2*x*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 360*a*b**5*c**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 2*a*b**5*c*d*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 28
*a*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 130*a*b**5*c
*d*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 224*a*b**5*c*d*n**2*x
**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a*b**5*c*d*n*x**3*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + a*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 10*a*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 35*a*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
 + 50*a*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b
**5*d**2*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*c**2*n**5*x**
2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 19*b**6*c**2*n**4*x**2*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 137*b**6*c**2*n**3*x**2*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 461*b**6*c**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 702*b**6*c**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 3
60*b**6*c**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 2*b**6*c*d*n**5*
x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 34*b**6*c*d*n**4*x**4*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 214*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 614*b**6*c*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 792*b**6*c*d*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 36
0*b**6*c*d*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d**2*n**5*x**
6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d**2*n**4*x**6*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 225*b**6*d**2*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 274*b**6*d**2*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
```

```
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 12
0*b**6*d**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

3.357 $\int (a+bx)^n \left(c+dx^2\right)^2 dx$

Optimal. Leaf size=140

$$\frac{\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)}{b^5(n+5)}$$

```
[Out] (a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^5/(1+n)-4*a*d*(a^2*d+b^2*c)*(b*x+a)^(2+n)/b
^5/(2+n)+2*d*(3*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d^2*(b*x+a)^(4+n)/
b^5/(4+n)+d^2*(b*x+a)^(5+n)/b^5/(5+n)
```

Rubi [A] time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {697}

$$\frac{\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a + b
*x)^(5 + n))/(b^5*(5 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n\left(c+dx^2\right)^2 dx &= \int \left(\frac{\left(b^2c+a^2d\right)^2(a+bx)^n}{b^4}-\frac{4ad\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^4}+\frac{2d\left(b^2c+3a^2d\right)(a+bx)^2}{b^4}\right.\\ &= \frac{\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^5(1+n)}-\frac{4ad\left(b^2c+a^2d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{2d\left(b^2c+3a^2d\right)(a+bx)^{3+}}{b^5(3+n)}\end{aligned}$$

Mathematica [A] time = 0.18, size = 160, normalized size = 1.14

$$\frac{(a+bx)^{n+1}\left(\frac{4\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)}{b^4(n+1)(n+2)(n+3)}-\frac{4ad(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)}{b^4(n+2)(n+3)(n+4)}+\left(c+\right.\right.}{b(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^2 + (4*(b^2*c + a^2*d)*(2*a^2*d - 2*a*b*d*(
1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)))/(b^4*(1 + n)*(2 + n)*(
3 + n)) - (4*a*d*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4
 + n) + d*(2 + n)*x^2)))/(b^4*(2 + n)*(3 + n)*(4 + n))))/(b*(5 + n))
```

fricas [B] time = 0.89, size = 519, normalized size = 3.71

$$\big(ab^4c^2n^4 + 14\,ab^4c^2n^3 + 120\,ab^4c^2 + 80\,a^3b^2cd + 24\,a^5d^2 + \big(b^5d^2n^4 + 10\,b^5d^2n^3 + 35\,b^5d^2n^2 + 50\,b^5d^2n + 24$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^4*c^2*n^4 + 14*a*b^4*c^2*n^3 + 120*a*b^4*c^2 + 80*a^3*b^2*c*d + 24*a^5
*d^2 + (b^5*d^2*n^4 + 10*b^5*d^2*n^3 + 35*b^5*d^2*n^2 + 50*b^5*d^2*n + 24*b
^5*d^2)*x^5 + (a*b^4*d^2*n^4 + 6*a*b^4*d^2*n^3 + 11*a*b^4*d^2*n^2 + 6*a*b^4
*d^2*n)*x^4 + 2*(b^5*c*d*n^4 + 40*b^5*c*d + 2*(6*b^5*c*d - a^2*b^3*d^2)*n^3
 + (49*b^5*c*d - 6*a^2*b^3*d^2)*n^2 + 2*(39*b^5*c*d - 2*a^2*b^3*d^2)*n)*x^3
 + (71*a*b^4*c^2 + 4*a^3*b^2*c*d)*n^2 + 2*(a*b^4*c*d*n^4 + 10*a*b^4*c*d*n^3
 + (29*a*b^4*c*d + 6*a^3*b^2*d^2)*n^2 + 2*(10*a*b^4*c*d + 3*a^3*b^2*d^2)*n)
*x^2 + 2*(77*a*b^4*c^2 + 18*a^3*b^2*c*d)*n + (b^5*c^2*n^4 + 120*b^5*c^2 + 2
*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n^2 + 2*(7
7*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

giac [B] time = 0.20, size = 851, normalized size = 6.08

$$(bx+a)^n b^5d^2n^4x^5 + (bx+a)^n ab^4d^2n^4x^4 + 10\,(bx+a)^n b^5d^2n^3x^5 + 2\,(bx+a)^n b^5cdn^4x^3 + 6\,(bx+a)^n ab^4d^2n^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d^2*n^4*x^5 + (b*x + a)^n*a*b^4*d^2*n^4*x^4 + 10*(b*x + a)
^n*b^5*d^2*n^3*x^5 + 2*(b*x + a)^n*b^5*c*d*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d^
2*n^3*x^4 + 35*(b*x + a)^n*b^5*d^2*n^2*x^5 + 2*(b*x + a)^n*a*b^4*c*d*n^4*x^
2 + 24*(b*x + a)^n*b^5*c*d*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d^2*n^3*x^3 + 11
*(b*x + a)^n*a*b^4*d^2*n^2*x^4 + 50*(b*x + a)^n*b^5*d^2*n*x^5 + (b*x + a)^n
*b^5*c^2*n^4*x + 20*(b*x + a)^n*a*b^4*c*d*n^3*x^2 + 98*(b*x + a)^n*b^5*c*d*
n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d^2*n^2*x^3 + 6*(b*x + a)^n*a*b^4*d^2*n*x^
4 + 24*(b*x + a)^n*b^5*d^2*x^5 + (b*x + a)^n*a*b^4*c^2*n^4 + 14*(b*x + a)^n
*b^5*c^2*n^3*x - 4*(b*x + a)^n*a^2*b^3*c*d*n^3*x + 58*(b*x + a)^n*a*b^4*c*d
*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n^2*x^2 + 156*(b*x + a)^n*b^5*c*d*n*x
^3 - 8*(b*x + a)^n*a^2*b^3*d^2*n*x^3 + 14*(b*x + a)^n*a*b^4*c^2*n^3 + 71*(b
*x + a)^n*b^5*c^2*n^2*x - 36*(b*x + a)^n*a^2*b^3*c*d*n^2*x + 40*(b*x + a)^n
*a*b^4*c*d*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n*x^2 + 80*(b*x + a)^n*b^5*c*
d*x^3 + 71*(b*x + a)^n*a*b^4*c^2*n^2 + 4*(b*x + a)^n*a^3*b^2*c*d*n^2 + 154*
(b*x + a)^n*b^5*c^2*n*x - 80*(b*x + a)^n*a^2*b^3*c*d*n*x - 24*(b*x + a)^n*a
^4*b*d^2*n*x + 154*(b*x + a)^n*a*b^4*c^2*n + 36*(b*x + a)^n*a^3*b^2*c*d*n +
 120*(b*x + a)^n*b^5*c^2*x + 120*(b*x + a)^n*a*b^4*c^2 + 80*(b*x + a)^n*a^3
*b^2*c*d + 24*(b*x + a)^n*a^5*d^2)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

maple [B] time = 0.01, size = 420, normalized size = 3.00

$$\big(b^4d^2n^4x^4 + 10b^4d^2n^3x^4 - 4a\,b^3d^2n^3x^3 + 2b^4cd\,n^4x^2 + 35b^4d^2n^2x^4 - 24a\,b^3d^2n^2x^3 + 24b^4cd\,n^3x^2 + 50b^4d^2n\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*d^2*n^4*x^4+10*b^4*d^2*n^3*x^4-4*a*b^3*d^2*n^3*x^3+2*b^4
*c*d*n^4*x^2+35*b^4*d^2*n^2*x^4-24*a*b^3*d^2*n^2*x^3+24*b^4*c*d*n^3*x^2+50*
```

```
b^4*d^2*n*x^4+12*a^2*b^2*d^2*n^2*x^2-4*a*b^3*c*d*n^3*x-44*a*b^3*d^2*n*x^3+b
^4*c^2*n^4+98*b^4*c*d*n^2*x^2+24*b^4*d^2*x^4+36*a^2*b^2*d^2*n*x^2-40*a*b^3*
c*d*n^2*x-24*a*b^3*d^2*x^3+14*b^4*c^2*n^3+156*b^4*c*d*n*x^2-24*a^3*b*d^2*n*
x+4*a^2*b^2*c*d*n^2+24*a^2*b^2*d^2*x^2-116*a*b^3*c*d*n*x+71*b^4*c^2*n^2+80*
b^4*c*d*x^2-24*a^3*b*d^2*x+36*a^2*b^2*c*d*n-80*a*b^3*c*d*x+154*b^4*c^2*n+24
*a^4*d^2+80*a^2*b^2*c*d+120*b^4*c^2)/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+1
20)
```

maxima [A] time = 0.48, size = 235, normalized size = 1.68

$$\frac{(bx+a)^{n+1}c^2}{b(n+1)}+\frac{2\left(\left(n^2+3n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^ncd}{\left(n^3+6n^2+11n+6\right)b^3}+\frac{\left(\left(n^4+10n^3+35n^2+50n\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c*d/((n^3 + 6*n^2 + 11*n + 6)*
b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2
 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^
2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d^2/((n^5 + 15*n^4 + 85*n^3 + 22
5*n^2 + 274*n + 120)*b^5)
```

mupad [B] time = 2.84, size = 496, normalized size = 3.54

$$(a+b\,x)^n\left(\frac{a\,\left(24\,a^4\,d^2+4\,a^2\,b^2\,c\,d\,n^2+36\,a^2\,b^2\,c\,d\,n+80\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+14\,b^4\,c^2\,n^3+71\,b^4\,c^2\,n^2+154\,b^4\,c\right.}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((a*(24*a^4*d^2 + 120*b^4*c^2 + 154*b^4*c^2*n + 71*b^4*c^2*n^2
+ 14*b^4*c^2*n^3 + b^4*c^2*n^4 + 80*a^2*b^2*c*d + 36*a^2*b^2*c*d*n + 4*a^2*
b^2*c*d*n^2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d^2*
x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4
 + n^5 + 120) + (x*(120*b^5*c^2 + 154*b^5*c^2*n + 71*b^5*c^2*n^2 + 14*b^5*c
^2*n^3 + b^5*c^2*n^4 - 24*a^4*b*d^2*n - 80*a^2*b^3*c*d*n - 36*a^2*b^3*c*d*n
^2 - 4*a^2*b^3*c*d*n^3))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (2*d*x^3*(3*n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 2*a^2*d*n + 9*b^2*c*
n))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d^2*n*x^4*(1
1*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (2*a*d*n*x^2*(n + 1)*(6*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

sympy [A] time = 7.13, size = 5097, normalized size = 36.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Eq(b, 0)), (12*a**4*
d**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a
*b**8*x**3 + 12*b**9*x**4) + 25*a**4*d**2/(12*a**4*b**5 + 48*a**3*b**6*x +
72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d**2*x*log(a
/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 88*a**3*b*d**2*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 2*a**2*b**2*c*d/(12*a**4*b**
```

```
5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 7
2*a**2*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2
*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**2*d**2*x**2/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) - 8*a*b**3*c*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d**2*x**3*log(a/b + x)/(12*a**4*b
**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) +
 48*a*b**3*d**2*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) - 3*b**4*c**2/(12*a**4*b**5 + 48*a**3*b**6*x
+ 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 12*b**4*c*d*x**2/(12
*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*
x**4) + 12*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d**2
*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
 22*a**4*d**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
 36*a**3*b*d**2*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2
 + 3*b**8*x**3) - 54*a**3*b*d**2*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*
x**2 + 3*b**8*x**3) - 2*a**2*b**2*c*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b*
*7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**5 +
 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2/(3*a
**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 6*a*b**3*c*d*x/(3
*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d**2*
x**3*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) - b**4*c**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3)
- 6*b**4*c*d*x**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) + 3*b**4*d**2*x**4/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3), Eq(n, -4)), (12*a**4*d**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
2*b**7*x**2) + 18*a**4*d**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a
**3*b*d**2*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**
3*b*d**2*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a**2*b**2*c*d*log(a
/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 6*a**2*b**2*c*d/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d**2*x**2*log(a/b + x)/(2*a
**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x*log(a/b + x)/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x/(2*a**2*b**5 + 4*a*b**6*x
 + 2*b**7*x**2) - 4*a*b**3*d**2*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**
2) - b**4*c**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*b**4*c*d*x**2*l
og(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d**2*x**4/(2*a*
*2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3)), (-12*a**4*d**2*log(a/b + x
)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d**2/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*
*2*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d*log(a/b + x)/(3*
a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d
**2*x**2/(3*a*b**5 + 3*b**6*x) - 12*a*b**3*c*d*x*log(a/b + x)/(3*a*b**5 + 3
*b**6*x) - 2*a*b**3*d**2*x**3/(3*a*b**5 + 3*b**6*x) - 3*b**4*c**2/(3*a*b**5
 + 3*b**6*x) + 6*b**4*c*d*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d**2*x**4/(3*a*
b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d**2*log(a/b + x)/b**5 - a**3*d**2*x/b*
*4 + 2*a**2*c*d*log(a/b + x)/b**3 + a**2*d**2*x**2/(2*b**3) - 2*a*c*d*x/b**
2 - a*d**2*x**3/(3*b**2) + c**2*log(a/b + x)/b + c*d*x**2/b + d**2*x**4/(4*
b), Eq(n, -1)), (24*a**5*d**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d**2*n*x*(a +
 b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**
5*n + 120*b**5) + 4*a**3*b**2*c*d*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 36*a**3*b**2*c
*d*n*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*a**3*b**2*c*d*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b
**2*d**2*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d**2*n*x**2*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) - 4*a**2*b**3*c*d*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
```

```
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 36*a**2*b**3*c*d*
n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) - 80*a**2*b**3*c*d*n*x*(a + b*x)**n/(b**5*n**5 +
 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a
**2*b**3*d**2*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n*
*3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d**2*n**2*x**3*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) - 8*a**2*b**3*d**2*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c
**2*n**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n
**2 + 274*b**5*n + 120*b**5) + 14*a*b**4*c**2*n**3*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 71
*a*b**4*c**2*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 154*a*b**4*c**2*n*(a + b*x)**n/(b**
5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**
5) + 120*a*b**4*c**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 2*a*b**4*c*d*n**4*x**2*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 20*a*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 58*a*b**4*c*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 40*a*b**4*c*d*n*x**2*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*
b**4*d**2*n**4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n**3*x**4*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 11*a*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n
*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
 + 274*b**5*n + 120*b**5) + b**5*c**2*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 14*b**5*
c**2*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 71*b**5*c**2*n**2*x*(a + b*x)**n/(b**5*n*
*5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
 154*b**5*c**2*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 120*b**5*c**2*x*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 2*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n
**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*c*d*n**3*x**3*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 98*b**5*c*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 156*b**5*c*d*n*
x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*b**5*c*d*x**3*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d**2
*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 35*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*b**5*d**2*n*x**5*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 24*b**5*d**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 8
5*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), True))
```

3.358 $\int \frac{(a+bx)^n\left(c+dx^2\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$-\frac{ad\left(a^2d+2b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{d\left(3a^2d+2b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)}-\frac{c^2(a+bx)^{n+1}\,{}_2F}{}$$

```
[Out] -a*d*(a^2*d+2*b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+d*(3*a^2*d+2*b^2*c)*(b*x+a)^(2
+n)/b^4/(2+n)-3*a*d^2*(b*x+a)^(3+n)/b^4/(3+n)+d^2*(b*x+a)^(4+n)/b^4/(4+n)-c
^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {952, 1620, 65}

$$-\frac{ad\left(a^2d+2b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{d\left(3a^2d+2b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)}-\frac{c^2(a+bx)^{n+1}\,{}_2F}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

```
[Out] -((a*d*(2*b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (d*(2*b^2*c +
3*a^2*d)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^(3 + n))/(b^
4*(3 + n)) + (d^2*(a + b*x)^(4 + n))/(b^4*(4 + n)) - (c^2*(a + b*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n \left(c+dx^2\right)^2}{x} dx = \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \frac{(a+bx)^n\left(b^4c^2(4+n)-a^3bd^2(4+n)x+b^2d\left(2b^2c-3a^2d\right)(4+n)x^2-3ab^3d^2(4+n)x^3\right)}{x} dx}{b^4(4+n)}$$

$$= \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \left(-abd\left(2b^2c+a^2d\right)(4+n)(a+bx)^n + \frac{\left(4b^4c^2+b^4c^2n\right)(a+bx)^n}{x} + bd\left(2b\right.\right.}{b^4(4+}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2}{}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2}{}$$

Mathematica [A] time = 0.13, size = 132, normalized size = 0.89

$$(a+bx)^{n+1}\left(\frac{d(a+bx)\left(3a^2d+2b^2c\right)}{b^4(n+2)} - \frac{ad\left(a^2d+2b^2c\right)}{b^4(n+1)} + \frac{d^2(a+bx)^3}{b^4(n+4)} - \frac{3ad^2(a+bx)^2}{b^4(n+3)} - \frac{c^2\,{}_2F_1\left(1,n+1;n+2;\frac{a+}{}\right.}{an+a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*(-((a*d*(2*b^2*c + a^2*d))/(b^4*(1 + n))) + (d*(2*b^2*c +
 3*a^2*d)*(a + b*x))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^2)/(b^4*(3 + n)) +
(d^2*(a + b*x)^3)/(b^4*(4 + n)) - (c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*x)/a])/(a + a*n))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^4 + 2\,cdx^2 + c^2\right)(bx+a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x + a)^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^2+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^2 (bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^2\,{\left(a+b\,x\right)}^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^2*(a + b*x)^n)/x,x)
```

```
[Out] int(((c + d*x^2)^2*(a + b*x)^n)/x, x)
```

sympy [B] time = 11.19, size = 1678, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 2*c*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*
b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x),
Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n
+ 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2
*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + d**2*Piecewise
((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*
x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 18
*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b + x)
/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*
x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3
*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*
x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6
*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*log(
a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4
*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x
**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3/(2*a*
b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2
*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)),
(-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b), Eq(n
, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x)**n/(b
```

```
**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2
*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b**4*n**4
 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x**3*(a
+ b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
 b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*
b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x**4*(a
+ b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4),
True)) - b*b**n*c**2*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n
 + 1)/(a*gamma(n + 2)) - b*b**n*c**2*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1,
n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

3.359 $\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=343

$$-\frac{2ad^2\left(28a^2d+9b^2c\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{d^2\left(28a^2d+3b^2c\right)(a+bx)^{n+7}}{b^9(n+7)}+\frac{a^2\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(a^2d+b^2c\right)}{}$$

```
[Out] a^2*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(a^2*d+b^2*c)^2*(4*a^2*d+b^
2*c)*(b*x+a)^(2+n)/b^9/(2+n)+(a^2*d+b^2*c)*(28*a^4*d^2+17*a^2*b^2*c*d+b^4*c
^2)*(b*x+a)^(3+n)/b^9/(3+n)-4*a*d*(14*a^4*d^2+15*a^2*b^2*c*d+3*b^4*c^2)*(b*
x+a)^(4+n)/b^9/(4+n)+d*(70*a^4*d^2+45*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(5+n)/
b^9/(5+n)-2*a*d^2*(28*a^2*d+9*b^2*c)*(b*x+a)^(6+n)/b^9/(6+n)+d^2*(28*a^2*d+
3*b^2*c)*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a
)^(9+n)/b^9/(9+n)
```

Rubi [A] time = 0.21, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {948}

$$\frac{\left(a^2d+b^2c\right)\left(17a^2b^2cd+28a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad\left(15a^2b^2cd+14a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^9(n+4)}+\frac{d\left(45a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^2
*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*d)
*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) -
 (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b^9*(
4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 + n))/(
b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*(6 + n
)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*
(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx &= \int\left(\frac{a^2\left(b^2c+a^2d\right)^3(a+bx)^n}{b^8}-\frac{2a\left(b^2c+a^2d\right)^2\left(b^2c+4a^2d\right)(a+bx)^{1+n}}{b^8}+\frac{\left(b^2\right.}{}\right.\\ &= \frac{a^2\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^2c+a^2d\right)^2\left(b^2c+4a^2d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^2c\right.}{}\end{aligned}$$

Mathematica [A] time = 0.25, size = 302, normalized size = 0.88

$$\frac{(a+bx)^{n+1}\left(\frac{d^2(a+bx)^6\left(28a^2d+3b^2c\right)}{n+7}-\frac{2ad^2(a+bx)^5\left(28a^2d+9b^2c\right)}{n+6}-\frac{2a(a+bx)\left(a^2d+b^2c\right)^2\left(4a^2d+b^2c\right)}{n+2}+\frac{a^2\left(a^2d+b^2c\right)^3}{n+1}+\frac{d(a+bx)^4\left(70a^4d\right.}{}\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^2*c + a^2*d)^3)/(1 + n) - (2*a*(b^2*c + a^2*d)^
2*(b^2*c + 4*a^2*d)*(a + b*x))/(2 + n) + ((b^2*c + a^2*d)*(b^4*c^2 + 17*a^2
*b^2*c*d + 28*a^4*d^2)*(a + b*x)^2)/(3 + n) - (4*a*d*(3*b^4*c^2 + 15*a^2*b^
2*c*d + 14*a^4*d^2)*(a + b*x)^3)/(4 + n) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d +
 70*a^4*d^2)*(a + b*x)^4)/(5 + n) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)
^5)/(6 + n) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^6)/(7 + n) - (8*a*d^3*(a
+ b*x)^7)/(8 + n) + (d^3*(a + b*x)^8)/(9 + n)))/b^9
```

fricas [B] time = 1.03, size = 2165, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 217728*a^5*b
^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*d^3*n^8 + 36*b^9*d^3
*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 22449*b^9*d^3*n^4 + 67284*b^9*d
^3*n^3 + 118124*b^9*d^3*n^2 + 109584*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (a*b^
8*d^3*n^8 + 28*a*b^8*d^3*n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5 + 676
9*a*b^8*d^3*n^4 + 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*b^8*d^
3*n)*x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4*a^2*b^
7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1119*b^9*c*d^2 -
 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a^2*b^7*d^3)*n^4 + 28*(881
7*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*(12303*b^9*c*d^2 - 392*a^2*b^7*d^3)
*n^2 + 144*(2901*b^9*c*d^2 - 40*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c*d^2*n^8 +
96*a*b^8*c*d^2*n^7 + 4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 + 30*(275*a*b
^8*c*d^2 + 28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3*b^6*d^3)*n^4
 + 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b^8*c*d^2 + 191
8*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^3)*n)*x^6 + 3*(b^9*c
^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d - 3*a^2*b^7*c*d^2)*n^7 + 2*(33
5*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6 + 2*(3050*b^9*c^2*d - 831*a^2*b^7*c*d^2
 - 56*a^4*b^5*d^3)*n^5 + (32773*b^9*c^2*d - 8190*a^2*b^7*c*d^2 - 1120*a^4*b
^5*d^3)*n^4 + 4*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 - 980*a^4*b^5*d^3)*n^
3 + 4*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a^4*b^5*d^3)*n^2 + 48*(3
975*b^9*c^2*d - 216*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n)*x^5 + 2*(625*a^3*b^6
*c^3 + 36*a^5*b^4*c^2*d)*n^4 + 3*(a*b^8*c^2*d*n^8 + 36*a*b^8*c^2*d*n^7 + 2*
(263*a*b^8*c^2*d + 15*a^3*b^6*c*d^2)*n^6 + 6*(666*a*b^8*c^2*d + 115*a^3*b^6
*c*d^2)*n^5 + (16789*a*b^8*c^2*d + 5550*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^
4 + 6*(6384*a*b^8*c^2*d + 3125*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^3 + 4*(10
791*a*b^8*c^2*d + 6705*a^3*b^6*c*d^2 + 1540*a^5*b^4*d^3)*n^2 + 96*(189*a*b^
8*c^2*d + 135*a^3*b^6*c*d^2 + 35*a^5*b^4*d^3)*n)*x^4 + 270*(39*a^3*b^6*c^3
+ 8*a^5*b^4*c^2*d)*n^3 + (b^9*c^3*n^8 + 120960*b^9*c^3 + 6*(7*b^9*c^3 - 2*a
^2*b^7*c^2*d)*n^7 + 12*(62*b^9*c^3 - 33*a^2*b^7*c^2*d)*n^6 + 6*(1203*b^9*c^
3 - 854*a^2*b^7*c^2*d - 60*a^4*b^5*c*d^2)*n^5 + 3*(13873*b^9*c^3 - 10860*a^
2*b^7*c^2*d - 2400*a^4*b^5*c*d^2)*n^4 + 12*(12039*b^9*c^3 - 8644*a^2*b^7*c^
2*d - 3750*a^4*b^5*c*d^2 - 560*a^6*b^3*d^3)*n^3 + 4*(72569*b^9*c^3 - 37116*
a^2*b^7*c^2*d - 22500*a^4*b^5*c*d^2 - 5040*a^6*b^3*d^3)*n^2 + 48*(6289*b^9*
c^3 - 1512*a^2*b^7*c^2*d - 1080*a^4*b^5*c*d^2 - 280*a^6*b^3*d^3)*n)*x^3 + 4
*(12287*a^3*b^6*c^3 + 6030*a^5*b^4*c^2*d + 540*a^7*b^2*c*d^2)*n^2 + (a*b^8*
c^3*n^8 + 40*a*b^8*c^3*n^7 + 4*(166*a*b^8*c^3 + 9*a^3*b^6*c^2*d)*n^6 + 62*(
95*a*b^8*c^3 + 18*a^3*b^6*c^2*d)*n^5 + (29839*a*b^8*c^3 + 13140*a^3*b^6*c^2
*d + 1080*a^5*b^4*c*d^2)*n^4 + 10*(8479*a*b^8*c^3 + 7146*a^3*b^6*c^2*d + 19
44*a^5*b^4*c*d^2)*n^3 + 24*(5029*a*b^8*c^3 + 7011*a^3*b^6*c^2*d + 4005*a^5*
b^4*c*d^2 + 840*a^7*b^2*d^3)*n^2 + 576*(105*a*b^8*c^3 + 189*a^3*b^6*c^2*d +
 135*a^5*b^4*c*d^2 + 35*a^7*b^2*d^3)*n)*x^2 + 48*(2509*a^3*b^6*c^3 + 2475*a
```

```
^5*b^4*c^2*d + 765*a^7*b^2*c*d^2)*n - 2*(a^2*b^7*c^3*n^7 + 39*a^2*b^7*c^3*n
^6 + (625*a^2*b^7*c^3 + 36*a^4*b^5*c^2*d)*n^5 + 135*(39*a^2*b^7*c^3 + 8*a^4
*b^5*c^2*d)*n^4 + 2*(12287*a^2*b^7*c^3 + 6030*a^4*b^5*c^2*d + 540*a^6*b^3*c
*d^2)*n^3 + 24*(2509*a^2*b^7*c^3 + 2475*a^4*b^5*c^2*d + 765*a^6*b^3*c*d^2)*
n^2 + 576*(105*a^2*b^7*c^3 + 189*a^4*b^5*c^2*d + 135*a^6*b^3*c*d^2 + 35*a^8
*b*d^3)*n)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^
6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 102
6576*b^9*n + 362880*b^9)
```

giac [B] time = 0.30, size = 3713, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^3*n^8*x^9 + (b*x + a)^n*a*b^8*d^3*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^3*n^7*x^9 + 3*(b*x + a)^n*b^9*c*d^2*n^8*x^7 + 28*(b*x + a)^n*a*b^8
*d^3*n^7*x^8 + 546*(b*x + a)^n*b^9*d^3*n^6*x^9 + 3*(b*x + a)^n*a*b^8*c*d^2*
n^8*x^6 + 114*(b*x + a)^n*b^9*c*d^2*n^7*x^7 - 8*(b*x + a)^n*a^2*b^7*d^3*n^7
*x^7 + 322*(b*x + a)^n*a*b^8*d^3*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^3*n^5*x^9
 + 3*(b*x + a)^n*b^9*c^2*d*n^8*x^5 + 96*(b*x + a)^n*a*b^8*c*d^2*n^7*x^6 + 1
812*(b*x + a)^n*b^9*c*d^2*n^6*x^7 - 168*(b*x + a)^n*a^2*b^7*d^3*n^6*x^7 + 1
960*(b*x + a)^n*a*b^8*d^3*n^5*x^8 + 22449*(b*x + a)^n*b^9*d^3*n^4*x^9 + 3*(
b*x + a)^n*a*b^8*c^2*d*n^8*x^4 + 120*(b*x + a)^n*b^9*c^2*d*n^7*x^5 - 18*(b*
x + a)^n*a^2*b^7*c*d^2*n^7*x^5 + 1236*(b*x + a)^n*a*b^8*c*d^2*n^6*x^6 + 56*
(b*x + a)^n*a^3*b^6*d^3*n^6*x^6 + 15666*(b*x + a)^n*b^9*c*d^2*n^5*x^7 - 140
0*(b*x + a)^n*a^2*b^7*d^3*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^3*n^4*x^8 + 67
284*(b*x + a)^n*b^9*d^3*n^3*x^9 + (b*x + a)^n*b^9*c^3*n^8*x^3 + 108*(b*x +
a)^n*a*b^8*c^2*d*n^7*x^4 + 2010*(b*x + a)^n*b^9*c^2*d*n^6*x^5 - 486*(b*x +
a)^n*a^2*b^7*c*d^2*n^6*x^5 + 8250*(b*x + a)^n*a*b^8*c*d^2*n^5*x^6 + 840*(b*
x + a)^n*a^3*b^6*d^3*n^5*x^6 + 80157*(b*x + a)^n*b^9*c*d^2*n^4*x^7 - 5880*(
b*x + a)^n*a^2*b^7*d^3*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^3*n^3*x^8 + 1181
24*(b*x + a)^n*b^9*d^3*n^2*x^9 + (b*x + a)^n*a*b^8*c^3*n^8*x^2 + 42*(b*x +
a)^n*b^9*c^3*n^7*x^3 - 12*(b*x + a)^n*a^2*b^7*c^2*d*n^7*x^3 + 1578*(b*x + a
)^n*a*b^8*c^2*d*n^6*x^4 + 90*(b*x + a)^n*a^3*b^6*c*d^2*n^6*x^4 + 18300*(b*x
 + a)^n*b^9*c^2*d*n^5*x^5 - 4986*(b*x + a)^n*a^2*b^7*c*d^2*n^5*x^5 - 336*(b
*x + a)^n*a^4*b^5*d^3*n^5*x^5 + 30657*(b*x + a)^n*a*b^8*c*d^2*n^4*x^6 + 476
0*(b*x + a)^n*a^3*b^6*d^3*n^4*x^6 + 246876*(b*x + a)^n*b^9*c*d^2*n^3*x^7 -
12992*(b*x + a)^n*a^2*b^7*d^3*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^3*n^2*x^8
 + 109584*(b*x + a)^n*b^9*d^3*n*x^9 + 40*(b*x + a)^n*a*b^8*c^3*n^7*x^2 + 74
4*(b*x + a)^n*b^9*c^3*n^6*x^3 - 396*(b*x + a)^n*a^2*b^7*c^2*d*n^6*x^3 + 119
88*(b*x + a)^n*a*b^8*c^2*d*n^5*x^4 + 2070*(b*x + a)^n*a^3*b^6*c*d^2*n^5*x^4
 + 98319*(b*x + a)^n*b^9*c^2*d*n^4*x^5 - 24570*(b*x + a)^n*a^2*b^7*c*d^2*n^
4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^3*n^4*x^5 + 62934*(b*x + a)^n*a*b^8*c*d^
2*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^3*n^3*x^6 + 442908*(b*x + a)^n*b^9*
c*d^2*n^2*x^7 - 14112*(b*x + a)^n*a^2*b^7*d^3*n^2*x^7 + 5040*(b*x + a)^n*a*
b^8*d^3*n*x^8 + 40320*(b*x + a)^n*b^9*d^3*x^9 - 2*(b*x + a)^n*a^2*b^7*c^3*n
^7*x + 664*(b*x + a)^n*a*b^8*c^3*n^6*x^2 + 36*(b*x + a)^n*a^3*b^6*c^2*d*n^6
*x^2 + 7218*(b*x + a)^n*b^9*c^3*n^5*x^3 - 5124*(b*x + a)^n*a^2*b^7*c^2*d*n^
5*x^3 - 360*(b*x + a)^n*a^4*b^5*c*d^2*n^5*x^3 + 50367*(b*x + a)^n*a*b^8*c^2
*d*n^4*x^4 + 16650*(b*x + a)^n*a^3*b^6*c*d^2*n^4*x^4 + 1680*(b*x + a)^n*a^5
*b^4*d^3*n^4*x^4 + 316380*(b*x + a)^n*b^9*c^2*d*n^3*x^5 - 61092*(b*x + a)^n
*a^2*b^7*c*d^2*n^3*x^5 - 11760*(b*x + a)^n*a^4*b^5*d^3*n^3*x^5 + 65304*(b*x
 + a)^n*a*b^8*c*d^2*n^2*x^6 + 15344*(b*x + a)^n*a^3*b^6*d^3*n^2*x^6 + 41774
4*(b*x + a)^n*b^9*c*d^2*n*x^7 - 5760*(b*x + a)^n*a^2*b^7*d^3*n*x^7 - 78*(b*
x + a)^n*a^2*b^7*c^3*n^6*x + 5890*(b*x + a)^n*a*b^8*c^3*n^5*x^2 + 1116*(b*x
 + a)^n*a^3*b^6*c^2*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^3*n^4*x^3 - 32580*(
b*x + a)^n*a^2*b^7*c^2*d*n^4*x^3 - 7200*(b*x + a)^n*a^4*b^5*c*d^2*n^4*x^3 +
 114912*(b*x + a)^n*a*b^8*c^2*d*n^3*x^4 + 56250*(b*x + a)^n*a^3*b^6*c*d^2*n
```

```
^3*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n^3*x^4 + 589140*(b*x + a)^n*b^9*c^2
*d*n^2*x^5 - 72144*(b*x + a)^n*a^2*b^7*c*d^2*n^2*x^5 - 16800*(b*x + a)^n*a^
4*b^5*d^3*n^2*x^5 + 25920*(b*x + a)^n*a*b^8*c*d^2*n*x^6 + 6720*(b*x + a)^n*
a^3*b^6*d^3*n*x^6 + 155520*(b*x + a)^n*b^9*c*d^2*x^7 + 2*(b*x + a)^n*a^3*b^
6*c^3*n^6 - 1250*(b*x + a)^n*a^2*b^7*c^3*n^5*x - 72*(b*x + a)^n*a^4*b^5*c^2
*d*n^5*x + 29839*(b*x + a)^n*a*b^8*c^3*n^4*x^2 + 13140*(b*x + a)^n*a^3*b^6*
c^2*d*n^4*x^2 + 1080*(b*x + a)^n*a^5*b^4*c*d^2*n^4*x^2 + 144468*(b*x + a)^n
*b^9*c^3*n^3*x^3 - 103728*(b*x + a)^n*a^2*b^7*c^2*d*n^3*x^3 - 45000*(b*x +
a)^n*a^4*b^5*c*d^2*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^3*n^3*x^3 + 129492*
(b*x + a)^n*a*b^8*c^2*d*n^2*x^4 + 80460*(b*x + a)^n*a^3*b^6*c*d^2*n^2*x^4 +
 18480*(b*x + a)^n*a^5*b^4*d^3*n^2*x^4 + 572400*(b*x + a)^n*b^9*c^2*d*n*x^5
 - 31104*(b*x + a)^n*a^2*b^7*c*d^2*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^3*n*x
^5 + 78*(b*x + a)^n*a^3*b^6*c^3*n^5 - 10530*(b*x + a)^n*a^2*b^7*c^3*n^4*x -
 2160*(b*x + a)^n*a^4*b^5*c^2*d*n^4*x + 84790*(b*x + a)^n*a*b^8*c^3*n^3*x^2
 + 71460*(b*x + a)^n*a^3*b^6*c^2*d*n^3*x^2 + 19440*(b*x + a)^n*a^5*b^4*c*d^
2*n^3*x^2 + 290276*(b*x + a)^n*b^9*c^3*n^2*x^3 - 148464*(b*x + a)^n*a^2*b^7
*c^2*d*n^2*x^3 - 90000*(b*x + a)^n*a^4*b^5*c*d^2*n^2*x^3 - 20160*(b*x + a)^
n*a^6*b^3*d^3*n^2*x^3 + 54432*(b*x + a)^n*a*b^8*c^2*d*n*x^4 + 38880*(b*x +
a)^n*a^3*b^6*c*d^2*n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n*x^4 + 217728*(b*
x + a)^n*b^9*c^2*d*x^5 + 1250*(b*x + a)^n*a^3*b^6*c^3*n^4 + 72*(b*x + a)^n*
a^5*b^4*c^2*d*n^4 - 49148*(b*x + a)^n*a^2*b^7*c^3*n^3*x - 24120*(b*x + a)^n
*a^4*b^5*c^2*d*n^3*x - 2160*(b*x + a)^n*a^6*b^3*c*d^2*n^3*x + 120696*(b*x +
 a)^n*a*b^8*c^3*n^2*x^2 + 168264*(b*x + a)^n*a^3*b^6*c^2*d*n^2*x^2 + 96120*
(b*x + a)^n*a^5*b^4*c*d^2*n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n^2*x^2 +
 301872*(b*x + a)^n*b^9*c^3*n*x^3 - 72576*(b*x + a)^n*a^2*b^7*c^2*d*n*x^3 -
 51840*(b*x + a)^n*a^4*b^5*c*d^2*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d^3*n*x^
3 + 10530*(b*x + a)^n*a^3*b^6*c^3*n^3 + 2160*(b*x + a)^n*a^5*b^4*c^2*d*n^3
- 120432*(b*x + a)^n*a^2*b^7*c^3*n^2*x - 118800*(b*x + a)^n*a^4*b^5*c^2*d*n
^2*x - 36720*(b*x + a)^n*a^6*b^3*c*d^2*n^2*x + 60480*(b*x + a)^n*a*b^8*c^3*
n*x^2 + 108864*(b*x + a)^n*a^3*b^6*c^2*d*n*x^2 + 77760*(b*x + a)^n*a^5*b^4*
c*d^2*n*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n*x^2 + 120960*(b*x + a)^n*b^9*
c^3*x^3 + 49148*(b*x + a)^n*a^3*b^6*c^3*n^2 + 24120*(b*x + a)^n*a^5*b^4*c^2
*d*n^2 + 2160*(b*x + a)^n*a^7*b^2*c*d^2*n^2 - 120960*(b*x + a)^n*a^2*b^7*c^
3*n*x - 217728*(b*x + a)^n*a^4*b^5*c^2*d*n*x - 155520*(b*x + a)^n*a^6*b^3*c
*d^2*n*x - 40320*(b*x + a)^n*a^8*b*d^3*n*x + 120432*(b*x + a)^n*a^3*b^6*c^3
*n + 118800*(b*x + a)^n*a^5*b^4*c^2*d*n + 36720*(b*x + a)^n*a^7*b^2*c*d^2*n
 + 120960*(b*x + a)^n*a^3*b^6*c^3 + 217728*(b*x + a)^n*a^5*b^4*c^2*d + 1555
20*(b*x + a)^n*a^7*b^2*c*d^2 + 40320*(b*x + a)^n*a^9*d^3)/(b^9*n^9 + 45*b^9
*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680
*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

maple [B] time = 0.02, size = 2232, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^8*d^3*n^8*x^8+36*b^8*d^3*n^7*x^8-8*a*b^7*d^3*n^7*x^7+3*b^8
*c*d^2*n^8*x^6+546*b^8*d^3*n^6*x^8-224*a*b^7*d^3*n^6*x^7+114*b^8*c*d^2*n^7*
x^6+4536*b^8*d^3*n^5*x^8+56*a^2*b^6*d^3*n^6*x^6-18*a*b^7*c*d^2*n^7*x^5-2576
*a*b^7*d^3*n^5*x^7+3*b^8*c^2*d*n^8*x^4+1812*b^8*c*d^2*n^6*x^6+22449*b^8*d^3
*n^4*x^8+1176*a^2*b^6*d^3*n^5*x^6-576*a*b^7*c*d^2*n^6*x^5-15680*a*b^7*d^3*n
^4*x^7+120*b^8*c^2*d*n^7*x^4+15666*b^8*c*d^2*n^5*x^6+67284*b^8*d^3*n^3*x^8-
336*a^3*b^5*d^3*n^5*x^5+90*a^2*b^6*c*d^2*n^6*x^4+9800*a^2*b^6*d^3*n^4*x^6-1
2*a*b^7*c^2*d*n^7*x^3-7416*a*b^7*c*d^2*n^5*x^5-54152*a*b^7*d^3*n^3*x^7+b^8*
c^3*n^8*x^2+2010*b^8*c^2*d*n^6*x^4+80157*b^8*c*d^2*n^4*x^6+118124*b^8*d^3*n
^2*x^8-5040*a^3*b^5*d^3*n^4*x^5+2430*a^2*b^6*c*d^2*n^5*x^4+41160*a^2*b^6*d^
3*n^3*x^6-432*a*b^7*c^2*d*n^6*x^3-49500*a*b^7*c*d^2*n^4*x^5-105056*a*b^7*d^
3*n^2*x^7+42*b^8*c^3*n^7*x^2+18300*b^8*c^2*d*n^5*x^4+246876*b^8*c*d^2*n^3*x
```

```
^6+109584*b^8*d^3*n*x^8+1680*a^4*b^4*d^3*n^4*x^4-360*a^3*b^5*c*d^2*n^5*x^3-
28560*a^3*b^5*d^3*n^3*x^5+36*a^2*b^6*c^2*d*n^6*x^2+24930*a^2*b^6*c*d^2*n^4*
x^4+90944*a^2*b^6*d^3*n^2*x^6-2*a*b^7*c^3*n^7*x-6312*a*b^7*c^2*d*n^5*x^3-18
3942*a*b^7*c*d^2*n^3*x^5-104544*a*b^7*d^3*n*x^7+744*b^8*c^3*n^6*x^2+98319*b
^8*c^2*d*n^4*x^4+442908*b^8*c*d^2*n^2*x^6+40320*b^8*d^3*x^8+16800*a^4*b^4*d
^3*n^3*x^4-8280*a^3*b^5*c*d^2*n^4*x^3-75600*a^3*b^5*d^3*n^2*x^5+1188*a^2*b^
6*c^2*d*n^5*x^2+122850*a^2*b^6*c*d^2*n^3*x^4+98784*a^2*b^6*d^3*n*x^6-80*a*b
^7*c^3*n^6*x-47952*a*b^7*c^2*d*n^4*x^3-377604*a*b^7*c*d^2*n^2*x^5-40320*a*b
^7*d^3*x^7+7218*b^8*c^3*n^5*x^2+316380*b^8*c^2*d*n^3*x^4+417744*b^8*c*d^2*n
*x^6-6720*a^5*b^3*d^3*n^3*x^3+1080*a^4*b^4*c*d^2*n^4*x^2+58800*a^4*b^4*d^3*
n^2*x^4-72*a^3*b^5*c^2*d*n^5*x-66600*a^3*b^5*c*d^2*n^3*x^3-92064*a^3*b^5*d^
3*n*x^5+2*a^2*b^6*c^3*n^6+15372*a^2*b^6*c^2*d*n^4*x^2+305460*a^2*b^6*c*d^2*
n^2*x^4+40320*a^2*b^6*d^3*x^6-1328*a*b^7*c^3*n^5*x-201468*a*b^7*c^2*d*n^3*x
^3-391824*a*b^7*c*d^2*n*x^5+41619*b^8*c^3*n^4*x^2+589140*b^8*c^2*d*n^2*x^4+
155520*b^8*c*d^2*x^6-40320*a^5*b^3*d^3*n^2*x^3+21600*a^4*b^4*c*d^2*n^3*x^2+
84000*a^4*b^4*d^3*n*x^4-2232*a^3*b^5*c^2*d*n^4*x-225000*a^3*b^5*c*d^2*n^2*x
^3-40320*a^3*b^5*d^3*x^5+78*a^2*b^6*c^3*n^5+97740*a^2*b^6*c^2*d*n^3*x^2+360
720*a^2*b^6*c*d^2*n*x^4-11780*a*b^7*c^3*n^4*x-459648*a*b^7*c^2*d*n^2*x^3-15
5520*a*b^7*c*d^2*x^5+144468*b^8*c^3*n^3*x^2+572400*b^8*c^2*d*n*x^4+20160*a^
6*b^2*d^3*n^2*x^2-2160*a^5*b^3*c*d^2*n^3*x-73920*a^5*b^3*d^3*n*x^3+72*a^4*b
^4*c^2*d*n^4+135000*a^4*b^4*c*d^2*n^2*x^2+40320*a^4*b^4*d^3*x^4-26280*a^3*b
^5*c^2*d*n^3*x-321840*a^3*b^5*c*d^2*n*x^3+1250*a^2*b^6*c^3*n^4+311184*a^2*b
^6*c^2*d*n^2*x^2+155520*a^2*b^6*c*d^2*x^4-59678*a*b^7*c^3*n^3*x-517968*a*b^
7*c^2*d*n*x^3+290276*b^8*c^3*n^2*x^2+217728*b^8*c^2*d*x^4+60480*a^6*b^2*d^3
*n*x^2-38880*a^5*b^3*c*d^2*n^2*x-40320*a^5*b^3*d^3*x^3+2160*a^4*b^4*c^2*d*n
^3+270000*a^4*b^4*c*d^2*n*x^2-142920*a^3*b^5*c^2*d*n^2*x-155520*a^3*b^5*c*d
^2*x^3+10530*a^2*b^6*c^3*n^3+445392*a^2*b^6*c^2*d*n*x^2-169580*a*b^7*c^3*n^
2*x-217728*a*b^7*c^2*d*x^3+301872*b^8*c^3*n*x^2-40320*a^7*b*d^3*n*x+2160*a^
6*b^2*c*d^2*n^2+40320*a^6*b^2*d^3*x^2-192240*a^5*b^3*c*d^2*n*x+24120*a^4*b^
4*c^2*d*n^2+155520*a^4*b^4*c*d^2*x^2-336528*a^3*b^5*c^2*d*n*x+49148*a^2*b^6
*c^3*n^2+217728*a^2*b^6*c^2*d*x^2-241392*a*b^7*c^3*n*x+120960*b^8*c^3*x^2-4
0320*a^7*b*d^3*x+36720*a^6*b^2*c*d^2*n-155520*a^5*b^3*c*d^2*x+118800*a^4*b^
4*c^2*d*n-217728*a^3*b^5*c^2*d*x+120432*a^2*b^6*c^3*n-120960*a*b^7*c^3*x+40
320*a^8*d^3+155520*a^6*b^2*c*d^2+217728*a^4*b^4*c^2*d+120960*a^2*b^6*c^3)/b
^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+723680*n^3+1172700*n^2
+1026576*n+362880)
```

maxima [B] time = 0.55, size = 795, normalized size = 2.32

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc^3}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{3\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3*((n^
6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b
^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 -
720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*
n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^8 + 36*n^7 + 546*n^
6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x
^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764
```

```
*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x
^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*
n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)
*(b*x + a)^n*d^3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n
^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*b^9)
```

mupad [B] time = 3.81, size = 1796, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574
*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^
3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n
+ 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2
+ 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^9*(1026576*n + 1172700
*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
n^9 + 362880)) - (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(6720*a^6*d^3*n - 60480*b
^6*c^3 - 60216*b^6*c^3*n - 24574*b^6*c^3*n^2 - 5265*b^6*c^3*n^3 - 625*b^6*c
^3*n^4 - 39*b^6*c^3*n^5 - b^6*c^3*n^6 + 36288*a^2*b^4*c^2*d*n + 25920*a^4*b
^2*c*d^2*n + 19800*a^2*b^4*c^2*d*n^2 + 6120*a^4*b^2*c*d^2*n^2 + 4020*a^2*b^
4*c^2*d*n^3 + 360*a^4*b^2*c*d^2*n^3 + 360*a^2*b^4*c^2*d*n^4 + 12*a^2*b^4*c^
2*d*n^5))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n
^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (3*d*x^5*(a + b*x)^n*(5
0*n + 35*n^2 + 10*n^3 + n^4 + 24)*(3024*b^4*c^2 - 112*a^4*d^2*n + 1650*b^4*
c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 + b^4*c^2*n^4 - 432*a^2*b^2*c*d*n
- 102*a^2*b^2*c*d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^
6*c^3*n + 24574*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c
^3*n^5 + b^6*c^3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a
^2*b^4*c^2*d*n + 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4
*b^2*c*d^2*n^2 + 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^8*(1026
576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*
n^7 + 45*n^8 + n^9 + 362880)) + (d^2*x^7*(a + b*x)^n*(216*b^2*c + 3*b^2*c*n
^2 - 8*a^2*d*n + 51*b^2*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^
5 + n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 6
3273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (a*n*x^2*(n + 1)*
(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574*b^6*c^
3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^3*n^6 +
 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n + 18360
*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2 + 1080*
a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^7*(1026576*n + 1172700*n^2 +
723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 3
62880)) + (a*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n
^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
+ (a*d^2*n*x^6*(a + b*x)^n*(56*a^2*d + 216*b^2*c + 3*b^2*c*n^2 + 51*b^2*c*n
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(560*a^
4*d^2 + 3024*b^4*c^2 + 1650*b^4*c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 +
b^4*c^2*n^4 + 2160*a^2*b^2*c*d + 510*a^2*b^2*c*d*n + 30*a^2*b^2*c*d*n^2))/(
```

```
b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n
^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.360 $\int x(a+bx)^n \left(c+dx^2\right)^3 \,dx$

Optimal. Leaf size=282

$$-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(a^2d+b^2c\right)^2\left(7a^2d+\right.}{b^8(n+}$$

```
[Out] -a*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^8/(1+n)+(a^2*d+b^2*c)^2*(7*a^2*d+b^2*c)*
(b*x+a)^(2+n)/b^8/(2+n)-3*a*d*(a^2*d+b^2*c)*(7*a^2*d+3*b^2*c)*(b*x+a)^(3+n)
/b^8/(3+n)+d*(35*a^4*d^2+30*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(4+n)/b^8/(4+n)-
5*a*d^2*(7*a^2*d+3*b^2*c)*(b*x+a)^(5+n)/b^8/(5+n)+3*d^2*(7*a^2*d+b^2*c)*(b*
x+a)^(6+n)/b^8/(6+n)-7*a*d^3*(b*x+a)^(7+n)/b^8/(7+n)+d^3*(b*x+a)^(8+n)/b^8/
(8+n)
```

Rubi [A] time = 0.17, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {772}

$$\frac{d\left(30a^2b^2cd+35a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^8(n+4)}-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)
^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2
*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 +
30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3
*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*
d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 +
 n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^2\right)^3\,dx &= \int\left(-\frac{a\left(b^2c+a^2d\right)^3(a+bx)^n}{b^7}+\frac{\left(b^2c+a^2d\right)^2\left(b^2c+7a^2d\right)(a+bx)^{1+n}}{b^7}+\frac{3ad\left(-3b^2\right.}{}\right.\\ &= -\frac{a\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^2c+a^2d\right)^2\left(b^2c+7a^2d\right)(a+bx)^{2+n}}{b^8(2+n)}-\frac{3ad\left(b^2c+\right.}{}\end{aligned}$$

Mathematica [B] time = 1.47, size = 709, normalized size = 2.51

$$\frac{(a+bx)^{n+1}\left(6(n+1)(a+bx)\left((n+7)\left(a^2d+b^2c\right)\left(4(n+5)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7
+ n)*(a + b*x)*(c + d*x^2)^3 - a*(8 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*(5 + n)*(6 + n)*(c + d*x^2)^3 + 6*(b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n)*(
2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d -
 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 +
n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 +
 n)*x^2))) - 6*a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(
c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2
*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))) + 6*(1 + n)*
(a + b*x)*((b^2*c + a^2*d)*(7 + n)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(c
+ d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(
3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*
a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2))) - a*d*(2 + n)*(
a + b*x)*(b^4*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^
2*d)*(6 + n)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 +
 n)*x^2)) - 4*a*d*(3 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(4 + n)*x + b^2*(5 +
 n)*(c*(6 + n) + d*(4 + n)*x^2))))))/(b^8*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(
5 + n)*(6 + n)*(7 + n)*(8 + n))
```

fricas [B] time = 0.95, size = 1675, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^3*n^6 + 33*a^2*b^6*c^3*n^5 + 20160*a^2*b^6*c^3 + 30240*a^4*b^4*
c^2*d + 20160*a^6*b^2*c*d^2 + 5040*a^8*d^3 - (b^8*d^3*n^7 + 28*b^8*d^3*n^6
+ 322*b^8*d^3*n^5 + 1960*b^8*d^3*n^4 + 6769*b^8*d^3*n^3 + 13132*b^8*d^3*n^2
 + 13068*b^8*d^3*n + 5040*b^8*d^3)*x^8 - (a*b^7*d^3*n^7 + 21*a*b^7*d^3*n^6
+ 175*a*b^7*d^3*n^5 + 735*a*b^7*d^3*n^4 + 1624*a*b^7*d^3*n^3 + 1764*a*b^7*d
^3*n^2 + 720*a*b^7*d^3*n)*x^7 - (3*b^8*c*d^2*n^7 + 20160*b^8*c*d^2 + (90*b^
8*c*d^2 - 7*a^2*b^6*d^3)*n^6 + 3*(366*b^8*c*d^2 - 35*a^2*b^6*d^3)*n^5 + 5*(
1404*b^8*c*d^2 - 119*a^2*b^6*d^3)*n^4 + 9*(2803*b^8*c*d^2 - 175*a^2*b^6*d^3
)*n^3 + 2*(25245*b^8*c*d^2 - 959*a^2*b^6*d^3)*n^2 + 24*(2143*b^8*c*d^2 - 35
*a^2*b^6*d^3)*n)*x^6 - 3*(a*b^7*c*d^2*n^7 + 25*a*b^7*c*d^2*n^6 + (241*a*b^7
*c*d^2 + 14*a^3*b^5*d^3)*n^5 + 5*(227*a*b^7*c*d^2 + 28*a^3*b^5*d^3)*n^4 + 2
*(1367*a*b^7*c*d^2 + 245*a^3*b^5*d^3)*n^3 + 20*(158*a*b^7*c*d^2 + 35*a^3*b^
5*d^3)*n^2 + 336*(4*a*b^7*c*d^2 + a^3*b^5*d^3)*n)*x^5 + (445*a^2*b^6*c^3 +
18*a^4*b^4*c^2*d)*n^4 - 3*(b^8*c^2*d*n^7 + 10080*b^8*c^2*d + (32*b^8*c^2*d
- 5*a^2*b^6*c*d^2)*n^6 + (418*b^8*c^2*d - 105*a^2*b^6*c*d^2)*n^5 + (2864*b^
8*c^2*d - 785*a^2*b^6*c*d^2 - 70*a^4*b^4*d^3)*n^4 + (10993*b^8*c^2*d - 2535
*a^2*b^6*c*d^2 - 420*a^4*b^4*d^3)*n^3 + 2*(11656*b^8*c^2*d - 1765*a^2*b^6*c
*d^2 - 385*a^4*b^4*d^3)*n^2 + 12*(2073*b^8*c^2*d - 140*a^2*b^6*c*d^2 - 35*a
^4*b^4*d^3)*n)*x^4 + 3*(1045*a^2*b^6*c^3 + 156*a^4*b^4*c^2*d)*n^3 - 3*(a*b^
7*c^2*d*n^7 + 29*a*b^7*c^2*d*n^6 + (331*a*b^7*c^2*d + 20*a^3*b^5*c*d^2)*n^5
 + (1871*a*b^7*c^2*d + 360*a^3*b^5*c*d^2)*n^4 + 20*(269*a*b^7*c^2*d + 103*a
^3*b^5*c*d^2 + 14*a^5*b^3*d^3)*n^3 + 4*(1793*a*b^7*c^2*d + 990*a^3*b^5*c*d^
2 + 210*a^5*b^3*d^3)*n^2 + 560*(6*a*b^7*c^2*d + 4*a^3*b^5*c*d^2 + a^5*b^3*d
^3)*n)*x^3 + 2*(6077*a^2*b^6*c^3 + 2259*a^4*b^4*c^2*d + 180*a^6*b^2*c*d^2)*
n^2 - (b^8*c^3*n^7 + 20160*b^8*c^3 + (34*b^8*c^3 - 9*a^2*b^6*c^2*d)*n^6 + (
478*b^8*c^3 - 243*a^2*b^6*c^2*d)*n^5 + (3580*b^8*c^3 - 2493*a^2*b^6*c^2*d -
 180*a^4*b^4*c*d^2)*n^4 + (15289*b^8*c^3 - 11853*a^2*b^6*c^2*d - 2880*a^4*b
^4*c*d^2)*n^3 + 2*(18353*b^8*c^3 - 12357*a^2*b^6*c^2*d - 6390*a^4*b^4*c*d^2
 - 1260*a^6*b^2*d^3)*n^2 + 72*(621*b^8*c^3 - 210*a^2*b^6*c^2*d - 140*a^4*b^
4*c*d^2 - 35*a^6*b^2*d^3)*n)*x^2 + 36*(682*a^2*b^6*c^3 + 533*a^4*b^4*c^2*d
+ 150*a^6*b^2*c*d^2)*n - (a*b^7*c^3*n^7 + 33*a*b^7*c^3*n^6 + (445*a*b^7*c^3
 + 18*a^3*b^5*c^2*d)*n^5 + 3*(1045*a*b^7*c^3 + 156*a^3*b^5*c^2*d)*n^4 + 2*(
6077*a*b^7*c^3 + 2259*a^3*b^5*c^2*d + 180*a^5*b^3*c*d^2)*n^3 + 36*(682*a*b^
```

```
7*c^3 + 533*a^3*b^5*c^2*d + 150*a^5*b^3*c*d^2)*n^2 + 5040*(4*a*b^7*c^3 + 6*
a^3*b^5*c^2*d + 4*a^5*b^3*c*d^2 + a^7*b*d^3)*n)*x)*(b*x + a)^n/(b^8*n^8 + 3
6*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 11
8124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

giac [B] time = 0.25, size = 2851, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^3*n^7*x^8 + (b*x + a)^n*a*b^7*d^3*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^3*n^6*x^8 + 3*(b*x + a)^n*b^8*c*d^2*n^7*x^6 + 21*(b*x + a)^n*a*b^7
*d^3*n^6*x^7 + 322*(b*x + a)^n*b^8*d^3*n^5*x^8 + 3*(b*x + a)^n*a*b^7*c*d^2*
n^7*x^5 + 90*(b*x + a)^n*b^8*c*d^2*n^6*x^6 - 7*(b*x + a)^n*a^2*b^6*d^3*n^6*
x^6 + 175*(b*x + a)^n*a*b^7*d^3*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^3*n^4*x^8
+ 3*(b*x + a)^n*b^8*c^2*d*n^7*x^4 + 75*(b*x + a)^n*a*b^7*c*d^2*n^6*x^5 + 10
98*(b*x + a)^n*b^8*c*d^2*n^5*x^6 - 105*(b*x + a)^n*a^2*b^6*d^3*n^5*x^6 + 73
5*(b*x + a)^n*a*b^7*d^3*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^3*n^3*x^8 + 3*(b*x
 + a)^n*a*b^7*c^2*d*n^7*x^3 + 96*(b*x + a)^n*b^8*c^2*d*n^6*x^4 - 15*(b*x +
a)^n*a^2*b^6*c*d^2*n^6*x^4 + 723*(b*x + a)^n*a*b^7*c*d^2*n^5*x^5 + 42*(b*x
+ a)^n*a^3*b^5*d^3*n^5*x^5 + 7020*(b*x + a)^n*b^8*c*d^2*n^4*x^6 - 595*(b*x
+ a)^n*a^2*b^6*d^3*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^3*n^3*x^7 + 13132*(b*
x + a)^n*b^8*d^3*n^2*x^8 + (b*x + a)^n*b^8*c^3*n^7*x^2 + 87*(b*x + a)^n*a*b
^7*c^2*d*n^6*x^3 + 1254*(b*x + a)^n*b^8*c^2*d*n^5*x^4 - 315*(b*x + a)^n*a^2
*b^6*c*d^2*n^5*x^4 + 3405*(b*x + a)^n*a*b^7*c*d^2*n^4*x^5 + 420*(b*x + a)^n
*a^3*b^5*d^3*n^4*x^5 + 25227*(b*x + a)^n*b^8*c*d^2*n^3*x^6 - 1575*(b*x + a)
^n*a^2*b^6*d^3*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^3*n^2*x^7 + 13068*(b*x +
a)^n*b^8*d^3*n*x^8 + (b*x + a)^n*a*b^7*c^3*n^7*x + 34*(b*x + a)^n*b^8*c^3*n
^6*x^2 - 9*(b*x + a)^n*a^2*b^6*c^2*d*n^6*x^2 + 993*(b*x + a)^n*a*b^7*c^2*d*
n^5*x^3 + 60*(b*x + a)^n*a^3*b^5*c*d^2*n^5*x^3 + 8592*(b*x + a)^n*b^8*c^2*d
*n^4*x^4 - 2355*(b*x + a)^n*a^2*b^6*c*d^2*n^4*x^4 - 210*(b*x + a)^n*a^4*b^4
*d^3*n^4*x^4 + 8202*(b*x + a)^n*a*b^7*c*d^2*n^3*x^5 + 1470*(b*x + a)^n*a^3*
b^5*d^3*n^3*x^5 + 50490*(b*x + a)^n*b^8*c*d^2*n^2*x^6 - 1918*(b*x + a)^n*a^
2*b^6*d^3*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^3*n*x^7 + 5040*(b*x + a)^n*b^8*
d^3*x^8 + 33*(b*x + a)^n*a*b^7*c^3*n^6*x + 478*(b*x + a)^n*b^8*c^3*n^5*x^2
- 243*(b*x + a)^n*a^2*b^6*c^2*d*n^5*x^2 + 5613*(b*x + a)^n*a*b^7*c^2*d*n^4*
x^3 + 1080*(b*x + a)^n*a^3*b^5*c*d^2*n^4*x^3 + 32979*(b*x + a)^n*b^8*c^2*d*
n^3*x^4 - 7605*(b*x + a)^n*a^2*b^6*c*d^2*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4
*d^3*n^3*x^4 + 9480*(b*x + a)^n*a*b^7*c*d^2*n^2*x^5 + 2100*(b*x + a)^n*a^3*
b^5*d^3*n^2*x^5 + 51432*(b*x + a)^n*b^8*c*d^2*n*x^6 - 840*(b*x + a)^n*a^2*b
^6*d^3*n*x^6 - (b*x + a)^n*a^2*b^6*c^3*n^6 + 445*(b*x + a)^n*a*b^7*c^3*n^5*
x + 18*(b*x + a)^n*a^3*b^5*c^2*d*n^5*x + 3580*(b*x + a)^n*b^8*c^3*n^4*x^2 -
 2493*(b*x + a)^n*a^2*b^6*c^2*d*n^4*x^2 - 180*(b*x + a)^n*a^4*b^4*c*d^2*n^4
*x^2 + 16140*(b*x + a)^n*a*b^7*c^2*d*n^3*x^3 + 6180*(b*x + a)^n*a^3*b^5*c*d
^2*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^3*n^3*x^3 + 69936*(b*x + a)^n*b^8*c^
2*d*n^2*x^4 - 10590*(b*x + a)^n*a^2*b^6*c*d^2*n^2*x^4 - 2310*(b*x + a)^n*a^
4*b^4*d^3*n^2*x^4 + 4032*(b*x + a)^n*a*b^7*c*d^2*n*x^5 + 1008*(b*x + a)^n*a
^3*b^5*d^3*n*x^5 + 20160*(b*x + a)^n*b^8*c*d^2*x^6 - 33*(b*x + a)^n*a^2*b^6
*c^3*n^5 + 3135*(b*x + a)^n*a*b^7*c^3*n^4*x + 468*(b*x + a)^n*a^3*b^5*c^2*d
*n^4*x + 15289*(b*x + a)^n*b^8*c^3*n^3*x^2 - 11853*(b*x + a)^n*a^2*b^6*c^2*
d*n^3*x^2 - 2880*(b*x + a)^n*a^4*b^4*c*d^2*n^3*x^2 + 21516*(b*x + a)^n*a*b^
7*c^2*d*n^2*x^3 + 11880*(b*x + a)^n*a^3*b^5*c*d^2*n^2*x^3 + 2520*(b*x + a)^
n*a^5*b^3*d^3*n^2*x^3 + 74628*(b*x + a)^n*b^8*c^2*d*n*x^4 - 5040*(b*x + a)^
n*a^2*b^6*c*d^2*n*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n*x^4 - 445*(b*x + a)^
n*a^2*b^6*c^3*n^4 - 18*(b*x + a)^n*a^4*b^4*c^2*d*n^4 + 12154*(b*x + a)^n*a*
b^7*c^3*n^3*x + 4518*(b*x + a)^n*a^3*b^5*c^2*d*n^3*x + 360*(b*x + a)^n*a^5*
b^3*c*d^2*n^3*x + 36706*(b*x + a)^n*b^8*c^3*n^2*x^2 - 24714*(b*x + a)^n*a^2
*b^6*c^2*d*n^2*x^2 - 12780*(b*x + a)^n*a^4*b^4*c*d^2*n^2*x^2 - 2520*(b*x +
```

```
a)^n*a^6*b^2*d^3*n^2*x^2 + 10080*(b*x + a)^n*a*b^7*c^2*d*n*x^3 + 6720*(b*x
+ a)^n*a^3*b^5*c*d^2*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^3*n*x^3 + 30240*(b*
x + a)^n*b^8*c^2*d*x^4 - 3135*(b*x + a)^n*a^2*b^6*c^3*n^3 - 468*(b*x + a)^n
*a^4*b^4*c^2*d*n^3 + 24552*(b*x + a)^n*a*b^7*c^3*n^2*x + 19188*(b*x + a)^n*
a^3*b^5*c^2*d*n^2*x + 5400*(b*x + a)^n*a^5*b^3*c*d^2*n^2*x + 44712*(b*x + a
)^n*b^8*c^3*n*x^2 - 15120*(b*x + a)^n*a^2*b^6*c^2*d*n*x^2 - 10080*(b*x + a)
^n*a^4*b^4*c*d^2*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^3*n*x^2 - 12154*(b*x +
a)^n*a^2*b^6*c^3*n^2 - 4518*(b*x + a)^n*a^4*b^4*c^2*d*n^2 - 360*(b*x + a)^n
*a^6*b^2*c*d^2*n^2 + 20160*(b*x + a)^n*a*b^7*c^3*n*x + 30240*(b*x + a)^n*a^
3*b^5*c^2*d*n*x + 20160*(b*x + a)^n*a^5*b^3*c*d^2*n*x + 5040*(b*x + a)^n*a^
7*b*d^3*n*x + 20160*(b*x + a)^n*b^8*c^3*x^2 - 24552*(b*x + a)^n*a^2*b^6*c^3
*n - 19188*(b*x + a)^n*a^4*b^4*c^2*d*n - 5400*(b*x + a)^n*a^6*b^2*c*d^2*n -
 20160*(b*x + a)^n*a^2*b^6*c^3 - 30240*(b*x + a)^n*a^4*b^4*c^2*d - 20160*(b
*x + a)^n*a^6*b^2*c*d^2 - 5040*(b*x + a)^n*a^8*d^3)/(b^8*n^8 + 36*b^8*n^7 +
 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 118124*b^8*n^
2 + 109584*b^8*n + 40320*b^8)
```

maple [B] time = 0.02, size = 1639, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^7*d^3*n^7*x^7-28*b^7*d^3*n^6*x^7+7*a*b^6*d^3*n^6*x^6-3*b
^7*c*d^2*n^7*x^5-322*b^7*d^3*n^5*x^7+147*a*b^6*d^3*n^5*x^6-90*b^7*c*d^2*n^6
*x^5-1960*b^7*d^3*n^4*x^7-42*a^2*b^5*d^3*n^5*x^5+15*a*b^6*c*d^2*n^6*x^4+122
5*a*b^6*d^3*n^4*x^6-3*b^7*c^2*d*n^7*x^3-1098*b^7*c*d^2*n^5*x^5-6769*b^7*d^3
*n^3*x^7-630*a^2*b^5*d^3*n^4*x^5+375*a*b^6*c*d^2*n^5*x^4+5145*a*b^6*d^3*n^3
*x^6-96*b^7*c^2*d*n^6*x^3-7020*b^7*c*d^2*n^4*x^5-13132*b^7*d^3*n^2*x^7+210*
a^3*b^4*d^3*n^4*x^4-60*a^2*b^5*c*d^2*n^5*x^3-3570*a^2*b^5*d^3*n^3*x^5+9*a*b
^6*c^2*d*n^6*x^2+3615*a*b^6*c*d^2*n^4*x^4+11368*a*b^6*d^3*n^2*x^6-b^7*c^3*n
^7*x-1254*b^7*c^2*d*n^5*x^3-25227*b^7*c*d^2*n^3*x^5-13068*b^7*d^3*n*x^7+210
0*a^3*b^4*d^3*n^3*x^4-1260*a^2*b^5*c*d^2*n^4*x^3-9450*a^2*b^5*d^3*n^2*x^5+2
61*a*b^6*c^2*d*n^5*x^2+17025*a*b^6*c*d^2*n^3*x^4+12348*a*b^6*d^3*n*x^6-34*b
^7*c^3*n^6*x-8592*b^7*c^2*d*n^4*x^3-50490*b^7*c*d^2*n^2*x^5-5040*b^7*d^3*x^
7-840*a^4*b^3*d^3*n^3*x^3+180*a^3*b^4*c*d^2*n^4*x^2+7350*a^3*b^4*d^3*n^2*x^
4-18*a^2*b^5*c^2*d*n^5*x-9420*a^2*b^5*c*d^2*n^3*x^3-11508*a^2*b^5*d^3*n*x^5
+a*b^6*c^3*n^6+2979*a*b^6*c^2*d*n^4*x^2+41010*a*b^6*c*d^2*n^2*x^4+5040*a*b^
6*d^3*x^6-478*b^7*c^3*n^5*x-32979*b^7*c^2*d*n^3*x^3-51432*b^7*c*d^2*n*x^5-5
040*a^4*b^3*d^3*n^2*x^3+3240*a^3*b^4*c*d^2*n^3*x^2+10500*a^3*b^4*d^3*n*x^4-
486*a^2*b^5*c^2*d*n^4*x-30420*a^2*b^5*c*d^2*n^2*x^3-5040*a^2*b^5*d^3*x^5+33
*a*b^6*c^3*n^5+16839*a*b^6*c^2*d*n^3*x^2+47400*a*b^6*c*d^2*n*x^4-3580*b^7*c
^3*n^4*x-69936*b^7*c^2*d*n^2*x^3-20160*b^7*c*d^2*x^5+2520*a^5*b^2*d^3*n^2*x
^2-360*a^4*b^3*c*d^2*n^3*x-9240*a^4*b^3*d^3*n*x^3+18*a^3*b^4*c^2*d*n^4+1854
0*a^3*b^4*c*d^2*n^2*x^2+5040*a^3*b^4*d^3*x^4-4986*a^2*b^5*c^2*d*n^3*x-42360
*a^2*b^5*c*d^2*n*x^3+445*a*b^6*c^3*n^4+48420*a*b^6*c^2*d*n^2*x^2+20160*a*b^
6*c*d^2*x^4-15289*b^7*c^3*n^3*x-74628*b^7*c^2*d*n*x^3+7560*a^5*b^2*d^3*n*x^
2-5760*a^4*b^3*c*d^2*n^2*x-5040*a^4*b^3*d^3*x^3+468*a^3*b^4*c^2*d*n^3+35640
*a^3*b^4*c*d^2*n*x^2-23706*a^2*b^5*c^2*d*n^2*x-20160*a^2*b^5*c*d^2*x^3+3135
*a*b^6*c^3*n^3+64548*a*b^6*c^2*d*n*x^2-36706*b^7*c^3*n^2*x-30240*b^7*c^2*d*
x^3-5040*a^6*b*d^3*n*x+360*a^5*b^2*c*d^2*n^2+5040*a^5*b^2*d^3*x^2-25560*a^4
*b^3*c*d^2*n*x+4518*a^3*b^4*c^2*d*n^2+20160*a^3*b^4*c*d^2*x^2-49428*a^2*b^5
*c^2*d*n*x+12154*a*b^6*c^3*n^2+30240*a*b^6*c^2*d*x^2-44712*b^7*c^3*n*x-5040
*a^6*b*d^3*x+5400*a^5*b^2*c*d^2*n-20160*a^4*b^3*c*d^2*x+19188*a^3*b^4*c^2*d
*n-30240*a^2*b^5*c^2*d*x+24552*a*b^6*c^3*n-20160*b^7*c^3*x+5040*a^7*d^3+201
60*a^5*b^2*c*d^2+30240*a^3*b^4*c^2*d+20160*a*b^6*c^3)/b^8/(n^8+36*n^7+546*n
^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)
```

maxima [B] time = 0.53, size = 625, normalized size = 2.22

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n c^3}{\left(n^2+3\,n+2\right)b^2}+\frac{3\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2\right.}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 +
 35*n^2 + 50*n + 24)*b^4) + 3*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 1
20)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 +
6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60
*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d^2/((n^6 +
 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^7 + 28*n^
6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (
n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 -
 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^
5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^
2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)
*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^3/((n^8 + 36*n^7 +
546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)
*b^8)
```

mupad [B] time = 3.48, size = 1459, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^3 +
20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 + 44
5*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 20160*
a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b^4
*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c^2
*d*n^4))/(b^8*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 5
46*n^6 + 36*n^7 + n^8 + 40320)) - (x^2*(n + 1)*(a + b*x)^n*(2520*a^6*d^3*n
- 20160*b^6*c^3 - 24552*b^6*c^3*n - 12154*b^6*c^3*n^2 - 3135*b^6*c^3*n^3 -
445*b^6*c^3*n^4 - 33*b^6*c^3*n^5 - b^6*c^3*n^6 + 15120*a^2*b^4*c^2*d*n + 10
080*a^4*b^2*c*d^2*n + 9594*a^2*b^4*c^2*d*n^2 + 2700*a^4*b^2*c*d^2*n^2 + 225
9*a^2*b^4*c^2*d*n^3 + 180*a^4*b^2*c*d^2*n^3 + 234*a^2*b^4*c^2*d*n^4 + 9*a^2
*b^4*c^2*d*n^5))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (d^2*x^6*(a + b*x)^n*(168*b^2*c +
 3*b^2*c*n^2 - 7*a^2*d*n + 45*b^2*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 +
 n^5 + 120))/(b^2*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5
 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 +
n^3 + 6)*(1680*b^4*c^2 - 70*a^4*d^2*n + 1066*b^4*c^2*n + 251*b^4*c^2*n^2 +
26*b^4*c^2*n^3 + b^4*c^2*n^4 - 280*a^2*b^2*c*d*n - 75*a^2*b^2*c*d*n^2 - 5*a
^2*b^2*c*d*n^3))/(b^4*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)^n*(5040*a^6*d^3
+ 20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 +
445*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 2016
0*a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b
^4*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c
```

```
^2*d*n^4))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 +
 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^3*n*x^7*(a + b*x)^n*(1764*n + 1624
*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 +
 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a
*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(280*a^4*d^2 + 1680*b^4*c^2 + 1066*b^4
*c^2*n + 251*b^4*c^2*n^2 + 26*b^4*c^2*n^3 + b^4*c^2*n^4 + 1120*a^2*b^2*c*d
+ 300*a^2*b^2*c*d*n + 20*a^2*b^2*c*d*n^2))/(b^5*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a*d^
2*n*x^5*(a + b*x)^n*(14*a^2*d + 56*b^2*c + b^2*c*n^2 + 15*b^2*c*n)*(50*n +
35*n^2 + 10*n^3 + n^4 + 24))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 2244
9*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.361 $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=223

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+}{b^7(n+2)}$$

```
[Out] (a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^7/(1+n)-6*a*d*(a^2*d+b^2*c)^2*(b*x+a)^(2+n)
/b^7/(2+n)+3*d*(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^7/(3+n)-4*a*d^
2*(5*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+3*d^2*(5*a^2*d+b^2*c)*(b*x+a)^(
5+n)/b^7/(5+n)-6*a*d^3*(b*x+a)^(6+n)/b^7/(6+n)+d^3*(b*x+a)^(7+n)/b^7/(7+n)
```

Rubi [A] time = 0.13, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {697}

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+}{b^7(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*d
)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^2*
d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/(b^7
*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)^(7 +
 n))/(b^7*(7 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \int \left(\frac{\left(b^2c+a^2d\right)^3(a+bx)^n}{b^6}-\frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a\right.}{b^6}\right.$$

$$=\frac{\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^7(1+n)}-\frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{2+n}}{b^7(2+n)}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a^2\right.}{b^7(3+n)}$$

Mathematica [A] time = 0.50, size = 347, normalized size = 1.56

$$\frac{(a+bx)^{n+1}\left(\frac{6\left((n+6)\left(a^2d+b^2c\right)\left(4(n+4)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)-4ad(n+1)(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^3 + (6*((b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n
)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*
d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1
 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(
2 + n)*x^2))) - a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*
(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^
2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 +
n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))))/(b*(7 + n))
```

fricas [B] time = 0.83, size = 1244, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^3*n^6 + 27*a*b^6*c^3*n^5 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3
024*a^5*b^2*c*d^2 + 720*a^7*d^3 + (b^7*d^3*n^6 + 21*b^7*d^3*n^5 + 175*b^7*d
^3*n^4 + 735*b^7*d^3*n^3 + 1624*b^7*d^3*n^2 + 1764*b^7*d^3*n + 720*b^7*d^3)
*x^7 + (a*b^6*d^3*n^6 + 15*a*b^6*d^3*n^5 + 85*a*b^6*d^3*n^4 + 225*a*b^6*d^3
*n^3 + 274*a*b^6*d^3*n^2 + 120*a*b^6*d^3*n)*x^6 + 3*(b^7*c*d^2*n^6 + 1008*b
^7*c*d^2 + (23*b^7*c*d^2 - 2*a^2*b^5*d^3)*n^5 + (207*b^7*c*d^2 - 20*a^2*b^5
*d^3)*n^4 + 5*(185*b^7*c*d^2 - 14*a^2*b^5*d^3)*n^3 + 4*(536*b^7*c*d^2 - 25*
a^2*b^5*d^3)*n^2 + 12*(201*b^7*c*d^2 - 4*a^2*b^5*d^3)*n)*x^5 + (295*a*b^6*c
^3 + 6*a^3*b^4*c^2*d)*n^4 + 3*(a*b^6*c*d^2*n^6 + 19*a*b^6*c*d^2*n^5 + (131*
a*b^6*c*d^2 + 10*a^3*b^4*d^3)*n^4 + (401*a*b^6*c*d^2 + 60*a^3*b^4*d^3)*n^3
+ 10*(54*a*b^6*c*d^2 + 11*a^3*b^4*d^3)*n^2 + 12*(21*a*b^6*c*d^2 + 5*a^3*b^4
*d^3)*n)*x^4 + 3*(555*a*b^6*c^3 + 44*a^3*b^4*c^2*d)*n^3 + 3*(b^7*c^2*d*n^6
+ 1680*b^7*c^2*d + (25*b^7*c^2*d - 4*a^2*b^5*c*d^2)*n^5 + (247*b^7*c^2*d -
64*a^2*b^5*c*d^2)*n^4 + (1219*b^7*c^2*d - 332*a^2*b^5*c*d^2 - 40*a^4*b^3*d^
3)*n^3 + 8*(389*b^7*c^2*d - 76*a^2*b^5*c*d^2 - 15*a^4*b^3*d^3)*n^2 + 4*(949
*b^7*c^2*d - 84*a^2*b^5*c*d^2 - 20*a^4*b^3*d^3)*n)*x^3 + 2*(2552*a*b^6*c^3
+ 537*a^3*b^4*c^2*d + 36*a^5*b^2*c*d^2)*n^2 + 3*(a*b^6*c^2*d*n^6 + 23*a*b^6
*c^2*d*n^5 + 3*(67*a*b^6*c^2*d + 4*a^3*b^4*c*d^2)*n^4 + (817*a*b^6*c^2*d +
168*a^3*b^4*c*d^2)*n^3 + 2*(739*a*b^6*c^2*d + 330*a^3*b^4*c*d^2 + 60*a^5*b^
2*d^3)*n^2 + 24*(35*a*b^6*c^2*d + 21*a^3*b^4*c*d^2 + 5*a^5*b^2*d^3)*n)*x^2
+ 12*(669*a*b^6*c^3 + 319*a^3*b^4*c^2*d + 78*a^5*b^2*c*d^2)*n + (b^7*c^3*n^
6 + 5040*b^7*c^3 + 3*(9*b^7*c^3 - 2*a^2*b^5*c^2*d)*n^5 + (295*b^7*c^3 - 132
*a^2*b^5*c^2*d)*n^4 + 3*(555*b^7*c^3 - 358*a^2*b^5*c^2*d - 24*a^4*b^3*c*d^2
)*n^3 + 4*(1276*b^7*c^3 - 957*a^2*b^5*c^2*d - 234*a^4*b^3*c*d^2)*n^2 + 36*(
223*b^7*c^3 - 140*a^2*b^5*c^2*d - 84*a^4*b^3*c*d^2 - 20*a^6*b*d^3)*n)*x)*(b
*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3
 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

giac [B] time = 0.24, size = 2085, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^3*n^6*x^7 + (b*x + a)^n*a*b^6*d^3*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^3*n^5*x^7 + 3*(b*x + a)^n*b^7*c*d^2*n^6*x^5 + 15*(b*x + a)^n*a*b^6
*d^3*n^5*x^6 + 175*(b*x + a)^n*b^7*d^3*n^4*x^7 + 3*(b*x + a)^n*a*b^6*c*d^2*
n^6*x^4 + 69*(b*x + a)^n*b^7*c*d^2*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^3*n^5*
x^5 + 85*(b*x + a)^n*a*b^6*d^3*n^4*x^6 + 735*(b*x + a)^n*b^7*d^3*n^3*x^7 +
3*(b*x + a)^n*b^7*c^2*d*n^6*x^3 + 57*(b*x + a)^n*a*b^6*c*d^2*n^5*x^4 + 621*
(b*x + a)^n*b^7*c*d^2*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^3*n^4*x^5 + 225*(b
*x + a)^n*a*b^6*d^3*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^3*n^2*x^7 + 3*(b*x + a
```

```
)^n*a*b^6*c^2*d*n^6*x^2 + 75*(b*x + a)^n*b^7*c^2*d*n^5*x^3 - 12*(b*x + a)^n
*a^2*b^5*c*d^2*n^5*x^3 + 393*(b*x + a)^n*a*b^6*c*d^2*n^4*x^4 + 30*(b*x + a)
^n*a^3*b^4*d^3*n^4*x^4 + 2775*(b*x + a)^n*b^7*c*d^2*n^3*x^5 - 210*(b*x + a)
^n*a^2*b^5*d^3*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^3*n^2*x^6 + 1764*(b*x + a)
^n*b^7*d^3*n*x^7 + (b*x + a)^n*b^7*c^3*n^6*x + 69*(b*x + a)^n*a*b^6*c^2*d*n
^5*x^2 + 741*(b*x + a)^n*b^7*c^2*d*n^4*x^3 - 192*(b*x + a)^n*a^2*b^5*c*d^2*
n^4*x^3 + 1203*(b*x + a)^n*a*b^6*c*d^2*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^
3*n^3*x^4 + 6432*(b*x + a)^n*b^7*c*d^2*n^2*x^5 - 300*(b*x + a)^n*a^2*b^5*d^
3*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^3*n*x^6 + 720*(b*x + a)^n*b^7*d^3*x^7 +
 (b*x + a)^n*a*b^6*c^3*n^6 + 27*(b*x + a)^n*b^7*c^3*n^5*x - 6*(b*x + a)^n*a
^2*b^5*c^2*d*n^5*x + 603*(b*x + a)^n*a*b^6*c^2*d*n^4*x^2 + 36*(b*x + a)^n*a
^3*b^4*c*d^2*n^4*x^2 + 3657*(b*x + a)^n*b^7*c^2*d*n^3*x^3 - 996*(b*x + a)^n
*a^2*b^5*c*d^2*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^3*n^3*x^3 + 1620*(b*x +
a)^n*a*b^6*c*d^2*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^3*n^2*x^4 + 7236*(b*x
+ a)^n*b^7*c*d^2*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^3*n*x^5 + 27*(b*x + a)^n
*a*b^6*c^3*n^5 + 295*(b*x + a)^n*b^7*c^3*n^4*x - 132*(b*x + a)^n*a^2*b^5*c^
2*d*n^4*x + 2451*(b*x + a)^n*a*b^6*c^2*d*n^3*x^2 + 504*(b*x + a)^n*a^3*b^4*
c*d^2*n^3*x^2 + 9336*(b*x + a)^n*b^7*c^2*d*n^2*x^3 - 1824*(b*x + a)^n*a^2*b
^5*c*d^2*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^3*n^2*x^3 + 756*(b*x + a)^n*a*
b^6*c*d^2*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^3*n*x^4 + 3024*(b*x + a)^n*b^7*
c*d^2*x^5 + 295*(b*x + a)^n*a*b^6*c^3*n^4 + 6*(b*x + a)^n*a^3*b^4*c^2*d*n^4
 + 1665*(b*x + a)^n*b^7*c^3*n^3*x - 1074*(b*x + a)^n*a^2*b^5*c^2*d*n^3*x -
72*(b*x + a)^n*a^4*b^3*c*d^2*n^3*x + 4434*(b*x + a)^n*a*b^6*c^2*d*n^2*x^2 +
 1980*(b*x + a)^n*a^3*b^4*c*d^2*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^3*n^2*x
^2 + 11388*(b*x + a)^n*b^7*c^2*d*n*x^3 - 1008*(b*x + a)^n*a^2*b^5*c*d^2*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^3*n*x^3 + 1665*(b*x + a)^n*a*b^6*c^3*n^3 + 1
32*(b*x + a)^n*a^3*b^4*c^2*d*n^3 + 5104*(b*x + a)^n*b^7*c^3*n^2*x - 3828*(b
*x + a)^n*a^2*b^5*c^2*d*n^2*x - 936*(b*x + a)^n*a^4*b^3*c*d^2*n^2*x + 2520*
(b*x + a)^n*a*b^6*c^2*d*n*x^2 + 1512*(b*x + a)^n*a^3*b^4*c*d^2*n*x^2 + 360*
(b*x + a)^n*a^5*b^2*d^3*n*x^2 + 5040*(b*x + a)^n*b^7*c^2*d*x^3 + 5104*(b*x
+ a)^n*a*b^6*c^3*n^2 + 1074*(b*x + a)^n*a^3*b^4*c^2*d*n^2 + 72*(b*x + a)^n*
a^5*b^2*c*d^2*n^2 + 8028*(b*x + a)^n*b^7*c^3*n*x - 5040*(b*x + a)^n*a^2*b^5
*c^2*d*n*x - 3024*(b*x + a)^n*a^4*b^3*c*d^2*n*x - 720*(b*x + a)^n*a^6*b*d^3
*n*x + 8028*(b*x + a)^n*a*b^6*c^3*n + 3828*(b*x + a)^n*a^3*b^4*c^2*d*n + 93
6*(b*x + a)^n*a^5*b^2*c*d^2*n + 5040*(b*x + a)^n*b^7*c^3*x + 5040*(b*x + a)
^n*a*b^6*c^3 + 5040*(b*x + a)^n*a^3*b^4*c^2*d + 3024*(b*x + a)^n*a^5*b^2*c*
d^2 + 720*(b*x + a)^n*a^7*d^3)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b
^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

maple [B] time = 0.01, size = 1140, normalized size = 5.11

$$\frac{\left(b^6d^3n^6x^6+21b^6d^3n^5x^6-6a\,b^5d^3n^5x^5+3b^6c\,d^2n^6x^4+175b^6d^3n^4x^6-90a\,b^5d^3n^4x^5+69b^6c\,d^2n^5x^4+735b^6d^3n\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^6*d^3*n^6*x^6+21*b^6*d^3*n^5*x^6-6*a*b^5*d^3*n^5*x^5+3*b^6
*c*d^2*n^6*x^4+175*b^6*d^3*n^4*x^6-90*a*b^5*d^3*n^4*x^5+69*b^6*c*d^2*n^5*x^
4+735*b^6*d^3*n^3*x^6+30*a^2*b^4*d^3*n^4*x^4-12*a*b^5*c*d^2*n^5*x^3-510*a*b
^5*d^3*n^3*x^5+3*b^6*c^2*d*n^6*x^2+621*b^6*c*d^2*n^4*x^4+1624*b^6*d^3*n^2*x
^6+300*a^2*b^4*d^3*n^3*x^4-228*a*b^5*c*d^2*n^4*x^3-1350*a*b^5*d^3*n^2*x^5+7
5*b^6*c^2*d*n^5*x^2+2775*b^6*c*d^2*n^3*x^4+1764*b^6*d^3*n*x^6-120*a^3*b^3*d
^3*n^3*x^3+36*a^2*b^4*c*d^2*n^4*x^2+1050*a^2*b^4*d^3*n^2*x^4-6*a*b^5*c^2*d*
n^5*x-1572*a*b^5*c*d^2*n^3*x^3-1644*a*b^5*d^3*n*x^5+b^6*c^3*n^6+741*b^6*c^2
*d*n^4*x^2+6432*b^6*c*d^2*n^2*x^4+720*b^6*d^3*x^6-720*a^3*b^3*d^3*n^2*x^3+5
76*a^2*b^4*c*d^2*n^3*x^2+1500*a^2*b^4*d^3*n*x^4-138*a*b^5*c^2*d*n^4*x-4812*
a*b^5*c*d^2*n^2*x^3-720*a*b^5*d^3*x^5+27*b^6*c^3*n^5+3657*b^6*c^2*d*n^3*x^2
+7236*b^6*c*d^2*n*x^4+360*a^4*b^2*d^3*n^2*x^2-72*a^3*b^3*c*d^2*n^3*x-1320*a
```

```
^3*b^3*d^3*n*x^3+6*a^2*b^4*c^2*d*n^4+2988*a^2*b^4*c*d^2*n^2*x^2+720*a^2*b^4
*d^3*x^4-1206*a*b^5*c^2*d*n^3*x-6480*a*b^5*c*d^2*n*x^3+295*b^6*c^3*n^4+9336
*b^6*c^2*d*n^2*x^2+3024*b^6*c*d^2*x^4+1080*a^4*b^2*d^3*n*x^2-1008*a^3*b^3*c
*d^2*n^2*x-720*a^3*b^3*d^3*x^3+132*a^2*b^4*c^2*d*n^3+5472*a^2*b^4*c*d^2*n*x
^2-4902*a*b^5*c^2*d*n^2*x-3024*a*b^5*c*d^2*x^3+1665*b^6*c^3*n^3+11388*b^6*c
^2*d*n*x^2-720*a^5*b*d^3*n*x+72*a^4*b^2*c*d^2*n^2+720*a^4*b^2*d^3*x^2-3960*
a^3*b^3*c*d^2*n*x+1074*a^2*b^4*c^2*d*n^2+3024*a^2*b^4*c*d^2*x^2-8868*a*b^5*
c^2*d*n*x+5104*b^6*c^3*n^2+5040*b^6*c^2*d*x^2-720*a^5*b*d^3*x+936*a^4*b^2*c
*d^2*n-3024*a^3*b^3*c*d^2*x+3828*a^2*b^4*c^2*d*n-5040*a*b^5*c^2*d*x+8028*b^
6*c^3*n+720*a^6*d^3+3024*a^4*b^2*c*d^2+5040*a^2*b^4*c^2*d+5040*b^6*c^3)/b^7
/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)
```

maxima [B] time = 0.51, size = 472, normalized size = 2.12

$$\frac{(bx+a)^{n+1}c^3}{b(n+1)}+\frac{3\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc^2d}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{3\left(\left(n^4+10\,n^3+35\,n^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c^2*d/((n^3 + 6*n^2 + 11*n + 6
)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11
*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^
3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d^2/((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624
*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 +
 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*
b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*
d^3/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n +
5040)*b^7)
```

mupad [B] time = 3.16, size = 1144, normalized size = 5.13

$$\frac{(a+b\,x)^n\,\left(720\,a^7\,d^3+72\,a^5\,b^2\,c\,d^2\,n^2+936\,a^5\,b^2\,c\,d^2\,n+3024\,a^5\,b^2\,c\,d^2+6\,a^3\,b^4\,c^2\,d\,n^4+132\,a^3\,b^4\,c^2\,d\,n^3+\right.}{b^7\,\left(n^7+28\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(720*a^7*d^3 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3024*a^5*
b^2*c*d^2 + 5104*a*b^6*c^3*n^2 + 1665*a*b^6*c^3*n^3 + 295*a*b^6*c^3*n^4 + 2
7*a*b^6*c^3*n^5 + a*b^6*c^3*n^6 + 8028*a*b^6*c^3*n + 3828*a^3*b^4*c^2*d*n +
 936*a^5*b^2*c*d^2*n + 1074*a^3*b^4*c^2*d*n^2 + 72*a^5*b^2*c*d^2*n^2 + 132*
a^3*b^4*c^2*d*n^3 + 6*a^3*b^4*c^2*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (x*(a + b*x)^n*(720*a^6*
b*d^3*n - 8028*b^7*c^3*n - 5104*b^7*c^3*n^2 - 1665*b^7*c^3*n^3 - 295*b^7*c^
3*n^4 - 27*b^7*c^3*n^5 - b^7*c^3*n^6 - 5040*b^7*c^3 + 5040*a^2*b^5*c^2*d*n
+ 3024*a^4*b^3*c*d^2*n + 3828*a^2*b^5*c^2*d*n^2 + 936*a^4*b^3*c*d^2*n^2 + 1
074*a^2*b^5*c^2*d*n^3 + 72*a^4*b^3*c*d^2*n^3 + 132*a^2*b^5*c^2*d*n^4 + 6*a^
2*b^5*c^2*d*n^5))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
 + 28*n^6 + n^7 + 5040)) + (d^3*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^
3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (3*d^2*x^5*(a + b*x)^n*(42*b^2*c + b^
2*c*n^2 - 2*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2
*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040
)) + (3*d*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(840*b^4*c^2 - 40*a^4*d^2*n + 638
```

```
*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 - 168*a^2*b^2*c
*d*n - 52*a^2*b^2*c*d*n^2 - 4*a^2*b^2*c*d*n^3))/(b^4*(13068*n + 13132*n^2 +
 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^3*n*x^6*(a +
b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132
*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (3*a*d^2*n*x
^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(10*a^2*d + 42*b^2*c + b^2*c*n^2 +
13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28
*n^6 + n^7 + 5040)) + (3*a*d*n*x^2*(n + 1)*(a + b*x)^n*(120*a^4*d^2 + 840*b
^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 5
04*a^2*b^2*c*d + 156*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.362 $\int \frac{(a+bx)^n\left(c+dx^2\right)^3}{x}\,dx$

Optimal. Leaf size=246

$$-\frac{ad^2\left(10a^2d+9b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{d^2\left(10a^2d+3b^2c\right)(a+bx)^{n+4}}{b^6(n+4)}-\frac{ad\left(a^4d^2+3a^2b^2cd+3b^4c^2\right)(a+bx)^{n+1}}{b^6(n+1)}+\frac{d\left(\right.}{}$$

```
[Out] -a*d*(a^4*d^2+3*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(1+n)/b^6/(1+n)+d*(5*a^4*d^2
+9*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(2+n)/b^6/(2+n)-a*d^2*(10*a^2*d+9*b^2*c)*
(b*x+a)^(3+n)/b^6/(3+n)+d^2*(10*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*
d^3*(b*x+a)^(5+n)/b^6/(5+n)+d^3*(b*x+a)^(6+n)/b^6/(6+n)-c^3*(b*x+a)^(1+n)*h
ypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] time = 0.34, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {952, 1620, 65}

$$-\frac{ad\left(3a^2b^2cd+a^4d^2+3b^4c^2\right)(a+bx)^{n+1}}{b^6(n+1)}+\frac{d\left(9a^2b^2cd+5a^4d^2+3b^4c^2\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{ad^2\left(10a^2d+9b^2c\right)(a+}{b^6(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] -((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^(1 + n))/(b^6*(1 + n
))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^(2 + n))/(b^6*(2
 + n)) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (d^
2*(3*b^2*c + 10*a^2*d)*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^3*(a + b*x
)^(5 + n))/(b^6*(5 + n)) + (d^3*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^3*(a
+ b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n \left(c+dx^2\right)^3}{x} dx = \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \frac{(a+bx)^n\left(b^6c^3(6+n)-a^5bd^3(6+n)x+b^2d\left(3b^4c^2-5a^4d^2\right)(6+n)x^2-10a^3b^3d^3(6+n)x^3+b^4d^2}{x}}{b^6(6+n)}$$

$$= \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \left(-abd\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(6+n)(a+bx)^n + \frac{\left(6b^6c^3+b^6c^3n\right)(a}{x}\right.}{}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)^{2+}}{b^6(2+n)}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)^{2+}}{b^6(2+n)}$$

Mathematica [A] time = 0.20, size = 226, normalized size = 0.92

$$(a+bx)^{n+1}\left(\frac{d^2(a+bx)^3\left(10a^2d+3b^2c\right)}{b^6(n+4)} - \frac{ad^2(a+bx)^2\left(10a^2d+9b^2c\right)}{b^6(n+3)} + \frac{d(a+bx)\left(5a^4d^2+9a^2b^2cd+3b^4c^2\right)}{b^6(n+2)} - \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*(-((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2))/(b^6*(1 +
n))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x))/(b^6*(2 + n))
- (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^2)/(b^6*(3 + n)) + (d^2*(3*b^2*c +
10*a^2*d)*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^3*(a + b*x)^4)/(b^6*(5 + n))
+ (d^3*(a + b*x)^5)/(b^6*(6 + n)) - (c^3*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*x)/a])/(a + a*n))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3\right)(bx+a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x + a)^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^3(bx+a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^2+c\right)^3(bx+a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^2+c\right)^3\,\left(a+b\,x\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^3*(a + b*x)^n)/x,x)
```

```
[Out] int(((c + d*x^2)^3*(a + b*x)^n)/x, x)
```

sympy [B] time = 17.73, size = 5692, normalized size = 23.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3/x,x)
```

```
[Out] -b**n*c**3*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**3*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 3*c**2*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/
(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x
), Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n
/(b**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2
*n + 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b
**2*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + 3*c*d**2*Pi
ecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b
**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*
a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b
*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a
**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**
3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3
*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n,
 -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b
), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n*
*2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x
```

```
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a*
*2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b
**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
+ 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x
**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**
2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n
/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x
**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4), True)) + d**3*Piecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b
**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**
11*x**5) + 300*a**4*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
25*a**4*b*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2
*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b
 + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*
x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**
6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10
*x**4 + 60*b**11*x**5) + 600*a**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) + 900*a**2*b**3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) +
300*a*b**4*x**4*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**
8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**
4*x**4/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a
**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*
a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*
b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4)
 - 125*a**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*
x**3 + 12*b**10*x**4) - 240*a**4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b
**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*x/
(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b*
*10*x**4) - 360*a**3*b**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x
+ 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*x**2/
(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b*
*10*x**4) - 240*a**2*b**3*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x
+ 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3/
(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b*
*10*x**4) - 60*a*b**4*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72
*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*x**5/(12*a**4*b
**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4),
 Eq(n, -5)), (60*a**5*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**
8*x**2 + 6*b**9*x**3) + 110*a**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*
x**2 + 6*b**9*x**3) + 180*a**4*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7
*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4*b*x/(6*a**3*b**6 + 18*a**2*b*
*7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/(6*a
**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x
**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2
*b**3*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*
b**9*x**3) - 15*a*b**4*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2
+ 6*b**9*x**3) + 3*b**5*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2
 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**
7*x + 6*b**8*x**2) - 90*a**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 12
0*a**4*b*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**
```

```
4*b*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*x**2*log(a/b
 + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*x**3/(6*a**2
*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4*x**4/(6*a**2*b**6 + 12*a*b**7
*x + 6*b**8*x**2) + 2*b**5*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2),
Eq(n, -3)), (60*a**5*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5/(12*a*b
**6 + 12*b**7*x) + 60*a**4*b*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a*
*3*b**2*x**2/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*x**3/(12*a*b**6 + 12*b*
*7*x) - 5*a*b**4*x**4/(12*a*b**6 + 12*b**7*x) + 3*b**5*x**5/(12*a*b**6 + 12
*b**7*x), Eq(n, -2)), (-a**5*log(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2
*b**4) + a**2*x**3/(3*b**3) - a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1)), (-1
20*a**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*n*x*(a + b*x)**
n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**
2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n**2*x**2*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) - 60*a**4*b**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n*
*5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**
6) + 20*a**3*b**3*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**
3*b**3*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*n*x*
*3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 5*a**2*b**4*n**4*x**4*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n**3*x**4*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) - 55*a**2*b**4*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) - 30*a**2*b**4*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*
b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b*
*5*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*n**4*x**5*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*n**3*x**5*(a + b*x)**n/(b
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) + 50*a*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 24*a*b**5*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*
n**5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*n**4*x**6*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*n**3*x**6*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 225*b**6*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 274*b**6*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
+ 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*
(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 16
24*b**6*n**2 + 1764*b**6*n + 720*b**6), True)) - b*b**n*c**3*n*x*(a/b + x)*
*n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c**
3*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)
)
```

3.363 $\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=250

$$\frac{(cd^2-ae^2)(d+ex)^{n+1}}{c^2e^3(n+1)}+\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{2c^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] (-a*e^2+c*d^2)*(e*x+d)^(1+n)/c^2/e^3/(1+n)-2*d*(e*x+d)^(2+n)/c/e^3/(2+n)+(e
*x+d)^(3+n)/c/e^3/(3+n)+1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+
n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(-e*(-a)^(1/2)+d*c^
(1/2))-1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2
)/(e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.40, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{1629, 712, 68\}$

$$\frac{(cd^2-ae^2)(d+ex)^{n+1}}{c^2e^3(n+1)}+\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{2c^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((c*d^2 - a*e^2)*(d + e*x)^(1 + n))/(c^2*e^3*(1 + n)) - (2*d*(d + e*x)^(2 +
 n))/(c*e^3*(2 + n)) + (d + e*x)^(3 + n)/(c*e^3*(3 + n)) + ((-a)^(3/2)*(d +
 e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[
c]*d - Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((-a)^(3/2)
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{\left(cd^2-ae^2\right)(d+ex)^n}{c^2e^2} - \frac{2d(d+ex)^{1+n}}{ce^2} + \frac{(d+ex)^{2+n}}{ce^2} + \frac{a^2(d+ex)^n}{c^2\left(a+cx^2\right)}\right)dx$$

$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c^2}$$

$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right.}\right.}{c^2}$$

$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} - \frac{(-a)^{3/2}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2c^2} - \frac{(-a)^{3/2}}{}$$

$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{(-a)^{3/2}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;\right.}{2c^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1}$$

Mathematica [A] time = 0.50, size = 217, normalized size = 0.87

$$\frac{(d+ex)^{n+1}\left(\frac{2\left(cd^2-ae^2\right)}{e^3(n+1)} + \frac{(-a)^{3/2}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\sqrt{-a}\,a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{2c(d+ex)^2}{e^3(n+3)} - \frac{4cd(d+ex)}{e^3(n+2)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(c*d^2 - a*e^2))/(e^3*(1 + n)) - (4*c*d*(d + e*x))/(
e^3*(2 + n)) + (2*c*(d + e*x)^2)/(e^3*(3 + n)) + ((-a)^(3/2)*Hypergeometric
2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[
c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt[-a]*a*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*
(1 + n))))/(2*c^2)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x^4}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^4/(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int((x^4*(d + e*x)^n)/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x)**n/(a + c*x**2), x)
```

3.364 $\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=209

$$\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{d(d+ex)^{n+1}}{ce^2(n+1)}+\frac{(d+}{ce^2(}$$

```
[Out] -d*(e*x+d)^(1+n)/c/e^2/(1+n)+(e*x+d)^(2+n)/c/e^2/(2+n)+1/2*a*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)
/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*a*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+
n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)/(1+n)/(e*(-a)^(1/2)+d*
c^(1/2))
```

Rubi [A] time = 0.23, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1629, 831, 68}

$$\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{d(d+ex)^{n+1}}{ce^2(n+1)}+\frac{(d+}{ce^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d*(d + e*x)^(1 + n))/(c*e^2*(1 + n))) + (d + e*x)^(2 + n)/(c*e^2*(2 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)
)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \left(-\frac{d(d+ex)^n}{ce} + \frac{(d+ex)^{1+n}}{ce} - \frac{ax(d+ex)^n}{c\left(a+cx^2\right)}\right)dx$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} - \frac{a\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} - \frac{a\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2c^{3/2}} - \frac{a\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2c^{3/2}}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{a(d+ex)^{1+n}\,{}_2\cdots}{2c^{3/2}\left(\cdots\right.}$$

Mathematica [A] time = 0.21, size = 168, normalized size = 0.80

$$\frac{(d+ex)^{n+1}\left(\frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e} + \frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d} - \frac{2\sqrt{c}(d-e(n+1)x)}{e^2(n+2)}\right)}{2c^{3/2}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*Sqrt[c]*(d - e*(1 + n)*x))/(e^2*(2 + n)) + (a*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/(Sqrt[c]*d - Sqrt[-a]*e) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(2*c^(3/2
)*(1 + n))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x^3}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^3/(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a),x)
[Out] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^n)/(a + c*x^2),x)
[Out] int((x^3*(d + e*x)^n)/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a),x)
[Out] Integral(x**3*(d + e*x)**n/(a + c*x**2), x)
```

3.365 $\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{-a}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{\sqrt{-a}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}+\frac{(d+ex)^{n+1}}{ce(n+1)}$$

```
[Out] (e*x+d)^(1+n)/c/e/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*
c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(-e*(-a)^(1/2)+d*c^(1
/2))-1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/
2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.23, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1629, 712, 68}

$$\frac{\sqrt{-a}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{\sqrt{-a}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}+\frac{(d+ex)^{n+1}}{ce(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] (d + e*x)^(1 + n)/(c*e*(1 + n)) + (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c*(
Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c*
(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{(d+ex)^n}{c} - \frac{a(d+ex)^n}{c\left(a+cx^2\right)}\right)dx$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{a\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c}$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{a\int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx}{c}$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2c} - \frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2c}$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} + \frac{\sqrt{-a}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} - \frac{\sqrt{-a}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1\right.}{2c\left(\sqrt{c}\,d+\sqrt{-}\right.}$$

Mathematica [A] time = 0.13, size = 170, normalized size = 0.88

$$\frac{(d+ex)^{n+1}\left(2\left(ae^2+cd^2\right)+e\left(\sqrt{-a}\sqrt{c}\,d-ae\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)-e\left(\sqrt{-a}\sqrt{c}\,d+ae\right){}_2F_1\left(1,n+\right.\right.}{2ce(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(2*(c*d^2 + a*e^2) + e*(Sqrt[-a]*Sqrt[c]*d - a*e)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
- e*(Sqrt[-a]*Sqrt[c]*d + a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/(2*c*e*(c*d^2 + a*e^2)*(1 + n))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x^2}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^2/(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int((x^2*(d + e*x)^n)/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x)**n/(a + c*x**2), x)
```

3.366 $\int \frac{x(d+ex)^n}{a+cx^2} dx$

Optimal. Leaf size=163

$$-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] -1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+
d*c^(1/2)))/(1+n)/c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyper
geom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/c^(1/2)
/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {831, 68}

$$-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)^n}{a+cx^2}\,dx &= \int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx\\
&= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{c}}+\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{c}}\\
&= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 151, normalized size = 0.93

$$-\frac{(d+ex)^{n+1}\left(\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)+\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)\right)}{2\sqrt{c}\,(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -1/2*((d + e*x)^(1 + n)*((Sqrt[c]*d + Sqrt[-a]*e)*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (Sqrt[c]*d - Sqrt
[-a]*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
 Sqrt[-a]*e)]))/(Sqrt[c]*(c*d^2 + a*e^2)*(1 + n))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x}{cx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x/(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\,(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x*(e*x+d)^n/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int((x*(d + e*x)^n)/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x*(d + e*x)**n/(a + c*x**2), x)
```

3.367 $\int \frac{(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=167

$$\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{-a}\,(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{-a}\,(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}$$

```
[Out] 1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2)))/(1+n)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyp
ergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/(-a)^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {712, 68}

$$\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{-a}\,(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{-a}\,(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^n}{a+cx^2}\,dx &= \int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx\\
&= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2\sqrt{-a}}\\
&= \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 145, normalized size = 0.87

$$\frac{(d+ex)^{n+1}\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d}\right)}{2\sqrt{-a}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[c]*d - Sqrt[-a]*e) - Hypergeometric2F1[1, 1
 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[c]*d + Sqr
t[-a]*e)))/(2*Sqrt[-a]*(1 + n))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n}{cx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^n}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^n}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(a + c*x^2),x)
```

```
[Out] int((d + e*x)^n/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**n/(a + c*x**2), x)
```

3.368 $\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=207

$$\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2a(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1}{\cdots}$$

```
[Out] -(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],1+e*x/d)/a/d/(1+n)+1/2*(e*x+d)^(1+n
)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/
2)/a/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[
2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a/(1+n)/(e*(-a)^(1/2
)+d*c^(1/2))
```

Rubi [A] time = 0.18, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {961, 65, 831, 68}

$$\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2a(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] (Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) +
(Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) -
((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a*d*(1
 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx = \int \left(\frac{(d+ex)^n}{ax} - \frac{cx(d+ex)^n}{a\left(a+cx^2\right)}\right)dx$$
$$= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a}$$
$$= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} - \frac{c\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx}{a}$$
$$= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} + \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2a} - \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2a}$$
$$= \frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2a\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}$$

Mathematica [A] time = 0.12, size = 189, normalized size = 0.91

$$\frac{(d+ex)^{n+1}\left(-2\left(ae^2+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{ex}{d}+1\right)+\left(\sqrt{-a}\,\sqrt{c}\,de+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)+\left(cd\right.\right.}{2ad(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*((c*d^2 + Sqrt[-a]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (c*d^2 - Sqrt[-a
]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)] - 2*(c*d^2 + a*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
 1 + (e*x)/d]))/(2*a*d*(c*d^2 + a*e^2)*(1 + n))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n}{cx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c*x^3 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(c\,x^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/x/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x*(a + c*x^2)),x)
```

```
[Out] int((d + e*x)^n/(x*(a + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)), x)
```

3.369 $\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx$

Optimal. Leaf size=207

$$\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1(2,n-}{ad^2(n+}$$

```
[Out] e*(e*x+d)^(1+n)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a/d^2/(1+n)+1/2*c*(e*x+d)
^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/
(-a)^(3/2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*c*(e*x+d)^(1+n)*hypergeom([1
, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(3/2)/(1+n)/(e*
(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.22, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {961, 65, 712, 68}

$$\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1(2,n-}{ad^2(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 +
n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])
/(a*d^2*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)}\,dx &= \int \left(\frac{(d+ex)^n}{ax^2}-\frac{c(d+ex)^n}{a\left(a+cx^2\right)}\right)dx\\
&= \frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a}-\frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a}\\
&= \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}-\frac{c\int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx}{a}\\
&= \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}-\frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2(-a)^{3/2}}-\frac{c\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2(-a)^{3/2}}\\
&= \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2(-a)^{3/2}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 167, normalized size = 0.81

$$\frac{(d+ex)^{n+1}\left(-\frac{c\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{-a}\,\sqrt{c}\,d+ae}+\frac{c\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{-a}\,\sqrt{c}\,d-ae}+\frac{2e\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{d^2}\right)}{2a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*(-((c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[c]*d + a*e)) + (c*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sq
rt[-a]*Sqrt[c]*d - a*e) + (2*e*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)
/d])/d^2))/(2*a*(1 + n))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n}{cx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{n}}{\left(c\,x^{2}+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x^2/(c*x^2+a),x)
[Out] int((e*x+d)^n/x^2/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{n}}{\left(cx^{2}+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{n}}{x^{2}\,\left(c\,x^{2}+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x^2*(a + c*x^2)),x)
[Out] int((d + e*x)^n/(x^2*(a + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a),x)
[Out] Timed out
```

3.370 $\int \frac{x^4(d+ex)^n}{(a+cx^2)^2} dx$

Optimal. Leaf size=332

$$\frac{(d+ex)^{n+1}\left(3\sqrt{-a}\,cd^2+a\sqrt{c}\,den+\sqrt{-a}\,ae^2(n+3)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4c^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(3\sqrt{-a}\,cd^2-\right.}{4}$$

```
[Out] (e*x+d)^(1+n)/c^2/e/(1+n)+1/2*a*(c*d*x+a*e)*(e*x+d)^(1+n)/c^2/(a*e^2+c*d^2)
/(c*x^2+a)-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-
a)^(1/2)+d*c^(1/2)))*(3*c*d^2*(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)-a*d*e*n*c^(
1/2))/c^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/4*(e*x+d)^(1+n)*hy
pergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(3*c*d^2*
(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)+a*d*e*n*c^(1/2))/c^2/(a*e^2+c*d^2)/(1+n)/
(-e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.45, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1649, 1629, 68}

$$\frac{(d+ex)^{n+1}\left(3\sqrt{-a}\,cd^2+a\sqrt{c}\,den+\sqrt{-a}\,ae^2(n+3)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4c^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(3\sqrt{-a}\,cd^2-\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (d + e*x)^(1 + n)/(c^2*e*(1 + n)) + (a*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*
c^2*(c*d^2 + a*e^2)*(a + c*x^2)) + ((3*Sqrt[-a]*c*d^2 + a*Sqrt[c]*d*e*n + S
qrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d - Sqrt[-a
]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((3*Sqrt[-a]*c*d^2 - a*Sqrt[c]*d*e*n + Sqrt
[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d + Sqrt[-a]*e
)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
```

```
+ e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
&& RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^4(d+ex)^n}{\left(a+cx^2\right)^2}dx = \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2\left(cd^2+ae^2(1+n)\right)}{c^2}+\frac{a^2denx}{c}-2a\left(d^2+\frac{ae^2}{c}\right)x^2\right)}{a+cx^2}dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(-\frac{2a\left(cd^2+ae^2\right)(d+ex)^n}{c^2} + \frac{\left(-\frac{a^3den}{c^{3/2}}+\sqrt{-a}\left(\frac{3a^2d^2}{c}+\frac{3a^3e^2}{c^2}+\frac{a^3e^2n}{c^2}\right)\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(\frac{a^3c}{c^3}\right.}{}\right.}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(3\sqrt{-a}\,cd^2-a\sqrt{c}\,den+\sqrt{-a}\,ae^2(3+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}}}{4c^2\left(cd^2+ae^2\right)}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(3\sqrt{-a}\,cd^2+a\sqrt{c}\,den+\sqrt{-a}\,ae^2(3+n)\right)(d+ex)^{1}}{4c^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+a\right.}$$

Mathematica [A] time = 0.83, size = 413, normalized size = 1.24

$$\frac{(d+ex)^{n+1}\left(\frac{a\left(\frac{\left(\sqrt{-a}\,\sqrt{c}\,den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{c}\,d-\sqrt{-a}\,e}-\frac{\left(-\sqrt{-a}\,\sqrt{c}\,den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{-a}\,e+\sqrt{c}\,d}\right)}{\sqrt{-a}\,(n+1)\left(ae^2+cd^2\right)}+\frac{2a(ae+cdx)}{\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*(4/(e + e*n) + (2*a*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a +
 c*x^2)) + (4*Sqrt[-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (4*Sqrt[
-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e
^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(Sq
rt[-a]*(c*d^2 + a*e^2)*(1 + n))))/(4*c^2)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x^4}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^4/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^4}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^4 (ex+d)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
[Out] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^4}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (d+e\,x)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x^4*(d + e*x)^n)/(a + c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

3.371 $\int \frac{x^3(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=297

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(\sqrt{-a}\,den-\frac{ae^2(n+2)+}{\sqrt{c}}\right.}{4c(n+1)\left(\sqrt{c}d\right.}$$

```
[Out] 1/2*a*(-e*x+d)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*hy
pergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(d*e*n*(-
a)^(1/2)+(-2*c*d^2-a*e^2*(2+n))/c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/
2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e
*(-a)^(1/2)+d*c^(1/2)))*(2*c*d^2+a*e^2*(2+n)+d*e*n*(-a)^(1/2)*c^(1/2))/c^(3
/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.41, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1649, 831, 68}

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(\sqrt{-a}\,den-\frac{ae^2(n+2)+}{\sqrt{c}}\right.}{4c(n+1)\left(\sqrt{c}d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (a*(d - e*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[
-a]*d*e*n - (2*c*d^2 + a*e^2*(2 + n))/Sqrt[c])*(d + e*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*
c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((2*c*d^2 + Sqrt[-a]*
Sqrt[c]*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^(3/2)*(Sqrt[c]
*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
```

```
x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
&& RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^3(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2den}{c}-\frac{a\left(2cd^2+ae^2(2+n)\right)x}{c}\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\frac{\sqrt{-a}\,a^2den}{c}+\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(\frac{\sqrt{-a}\,a^2den}{c}-\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+e\right.}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(2cd^2+\sqrt{-a}\sqrt{c}\,den+ae^2(2+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{4c^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(\sqrt{-a}\,den\right.}{}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(\sqrt{-a}\,den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d}{\sqrt{c}\,d-}\right.}{4c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] time = 0.92, size = 247, normalized size = 0.83

$$-\frac{(d+ex)^{n+1}\left(\frac{\left(-\sqrt{-a}\sqrt{c}\,den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)} + \frac{\left(\sqrt{-a}\sqrt{c}\,den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)} + \right.}{4c^{3/2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/4*((d + e*x)^(1 + n)*((2*a*Sqrt[c]*(-d + e*x))/(a + c*x^2) + ((2*c*d^2 -
 Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(
1 + n)) + ((2*c*d^2 + Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt
[c]*d + Sqrt[-a]*e)*(1 + n))))/(c^(3/2)*(c*d^2 + a*e^2))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x^3}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^3/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^3}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^3 (ex+d)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^3}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(d+e\,x)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^n)/(a + c*x^2)^2,x)
```

```
[Out] int((x^3*(d + e*x)^n)/(a + c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.372 $\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=306

$$\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}\,den+ae^2(n+1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\,c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2\right.}{4\sqrt{-a}\,c(n+1}$$

```
[Out] -1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+
a*e^2*(1+n)-d*e*n*(-a)^(1/2)*c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(-e*
(-a)^(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1+n)+d*e*n*(-a)^(1/2)*c^(1/2)
)/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.53, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1649, 831, 68}

$$\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}\,den+ae^2(n+1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\,c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2\right.}{4\sqrt{-a}\,c(n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((c*
d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*S
qrt[-a]*c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((c*d^2 + Sqr
t[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*Sqrt[-a]*c
*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
```

```
x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
&& RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^2(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-\frac{a\left(cd^2+ae^2(1+n)\right)}{c}-adenx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}\,a\left(cd^2+ae^2(1+n)\right)}{c}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(-\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}\,a\left(cd^2+ae^2(1+n)\right)}{c}\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right.}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(cd^2-\sqrt{-a}\sqrt{c}\,den+ae^2(1+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{4\sqrt{-a}\,c\left(cd^2+ae^2\right)} - \frac{\left(cd^2+\sqrt{-a}\right.}{}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2-\sqrt{-a}\sqrt{c}\,den+ae^2(1+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n\right.}{4\sqrt{-a}\,c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] time = 0.67, size = 403, normalized size = 1.32

$$\frac{(d+ex)^{n+1}\left(\frac{a\left(\frac{\left(\sqrt{-a}\sqrt{c}\,den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{\left(-\sqrt{-a}\sqrt{c}\,den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d}\right)}{(-a)^{3/2}(n+1)\left(ae^2+cd^2\right)}-\frac{2(ae+cdx)}{\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a + c*x^2)) + (2*H
ypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]
*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (2*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a]*(Sq
rt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n) + Sqrt[-a]*Sq
rt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c
]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e^2*(-1 + n) - S
qrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/((-a)^(3/2)*(c*d^2
 + a*e^2)*(1 + n))))/(4*c)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x^2}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^2/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2 (ex+d)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^n)/(a + c*x^2)^2,x)
```

```
[Out] int((x^2*(d + e*x)^n)/(a + c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.373 $\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=279

$$\frac{en\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{c}\,(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}+\frac{en\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{4a\sqrt{c}\,(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right.}$$

```
[Out] -1/2*(-e*x+d)*(e*x+d)^(1+n)/(a*e^2+c*d^2)/(c*x^2+a)+1/4*e*n*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(e*(-a)^
(1/2)+d*c^(1/2))/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/c^(1/2)/(-e*(-a)^(1/2)+d*c^
(1/2))+1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-
a)^(1/2)+d*c^(1/2)))*(a*e+d*(-a)^(1/2)*c^(1/2))/a/(a*e^2+c*d^2)/(1+n)/c^(1/
2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.30, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {823, 831, 68}

$$\frac{en\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{c}\,(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}+\frac{en\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{4a\sqrt{c}\,(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -((d - e*x)*(d + e*x)^(1 + n))/(2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(Sqrt[c
]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*Sqrt[-a]*Sqrt[c]*(Sqrt[c]*d
 - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(Sqrt[-a]*Sqrt[c]*d + a*e)*n*(
d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)])/(4*a*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^
2)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-acden+ace^2nx\right)}{a+cx^2}\,dx}{2ac\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(-\sqrt{-a}\,acden-a^2\sqrt{c}\,e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(-\sqrt{-a}\,acden+a^2\sqrt{c}\,e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right) dx}{2ac\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(e\left(\sqrt{-a}\,\sqrt{c}\,d-ae\right)n\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{4a\sqrt{c}\left(cd^2+ae^2\right)} + \frac{\left(e\left(\sqrt{-a}\,d+\frac{ae}{\sqrt{c}}\right)n\right)\int -}{4a\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{c}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)} +$$

Mathematica [A] time = 0.34, size = 230, normalized size = 0.82

$$\frac{(d+ex)^{n+1}\left(-\frac{\left(\sqrt{-a}\,cden-a\sqrt{c}\,e^2n\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)} + \frac{\left(\sqrt{-a}\,cden+a\sqrt{c}\,e^2n\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)} - \frac{2ac(d-ex)}{a+cx^2}\right)}{4ac\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*a*c*(d - e*x))/(a + c*x^2) - ((Sqrt[-a]*c*d*e*n - a
*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqr
t[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + ((Sqrt[-a]*c*d*
e*n + a*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*c*(
c*d^2 + a*e^2))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n x}{c^2x^4+2\,acx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n x}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x\,(ex+d)^{n}}{\left(c\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
[Out] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{n}x}{\left(cx^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(d+e\,x)^{n}}{\left(c\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x*(d + e*x)^n)/(a + c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

3.374 $\int \frac{(d+ex)^n}{(a+cx^2)^2} dx$

Optimal. Leaf size=304

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(1-n)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}\,den+ae\right.}{4(-a)^{3/2}(n+}$$

```
[Out] 1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*
e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^
(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-
a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.42, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {741, 831, 68}

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(1-n)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}\,den+ae\right.}{4(-a)^{3/2}(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + ((c*d^2 + a*e^
2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\int \frac{(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-cd^2-ae^2(1-n)+cdenx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-n)\right)-a\sqrt{c}\,den\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-n)\right)+a\sqrt{c}\,den\right)(d+e\cdots}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right.}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}\,den\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{4(-a)^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(cd^2+ae^2(1-n\cdots\right.}{4(\cdots}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}\,den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\cdots\right.}{4(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] time = 0.37, size = 253, normalized size = 0.83

$$\frac{(d+ex)^{n+1}\left(\frac{\left(\sqrt{-a}\sqrt{c}\,den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\left(\sqrt{-a}\sqrt{c}\,den+ae^2(n-1)-cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{2(ae+cdx\cdots}{a+cx^2}\right.}{4a\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(a*e + c*d*x))/(a + c*x^2) + ((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 +
n)) + ((-(c*d^2) + a*e^2*(-1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2
F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a
]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*(c*d^2 + a*e^2))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(ex+d)^n}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^n/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(a + c*x^2)^2,x)
```

```
[Out] int((d + e*x)^n/(a + c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.375 $\int \frac{(d+ex)^n}{x(a+cx^2)^2}\,dx$

Optimal. Leaf size=489

$$-\frac{\sqrt{c}\,en\left(\sqrt{-a}\sqrt{c}\,d+ae\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4a^2(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right.}{2a^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}$$

```
[Out] 1/2*c*(-e*x+d)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)-(e*x+d)^(1+n)*hyperg
eom([1, 1+n],[2+n],1+e*x/d)/a^2/d/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n
],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(-e*(-
a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/
4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)
+d*c^(1/2)))*c^(1/2)*(e*(-a)^(1/2)+d*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n
)/(-e*(-a)^(1/2)+d*c^(1/2))-1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],
(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)*(a*e+d*(-a)^(1/2)*c^(1/2)
)/a^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A] time = 0.60, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {961, 65, 823, 831, 68}

$$-\frac{\sqrt{c}\,en\left(\sqrt{-a}\sqrt{c}\,d+ae\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4a^2(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right.}{2a^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

```
[Out] (c*(d - e*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c
]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt
[c]*e*(Sqrt[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(3/2)*(Sq
rt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (Sqrt[c]*(d + e*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)])/(2*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) - (Sqrt[c]*e*(Sqrt[-a]*Sqr
t[c]*d + a*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(c
*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, 1 + (e*x)/d])/(a^2*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx = \int \left(\frac{(d+ex)^n}{a^2x} - \frac{cx(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{cx(d+ex)^n}{a^2\left(a+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2}\,dx}{a}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} - \frac{c\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)}\right.}{a}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} + \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2a^2} -$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}(d+ex)^{1-}}{2a^2}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}e\left(\sqrt{-a}\sqrt{}\right.}{}$$

Mathematica [A] time = 0.89, size = 391, normalized size = 0.80

$$\frac{(d+ex)^{n+1}\left(\frac{\sqrt{c}en\left(\left(\sqrt{-a}cd^2-2a\sqrt{c}de+(-a)^{3/2}e^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(-\sqrt{-a}cd^2-2a\sqrt{c}de+\sqrt{-a}ae^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)\right)}{(n+1)\left(ae^2+cd^2\right)^2}\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*a*c*(d - e*x))/((c*d^2 + a*e^2)*(a + c*x^2)) - (4*Hy
pergeometric2F1[1, 1 + n, 2 + n, (d + e*x)/d])/(d + d*n) + (2*Sqrt[c]*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (2*Sqrt[c]*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt
[-a]*e)*(1 + n)) + (Sqrt[c]*e*n*((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*e + (-a)^(
3/2)*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d
 - Sqrt[-a]*e)] + (-(Sqrt[-a]*c*d^2) - 2*a*Sqrt[c]*d*e + Sqrt[-a]*a*e^2)*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*
e)]))/((c*d^2 + a*e^2)^2*(1 + n))))/(4*a^2)
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n}{c^2x^5+2\,acx^3+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c^2*x^5 + 2*a*c*x^3 + a^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(c\,x^2+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x\,\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x*(a + c*x^2)^2),x)
```

```
[Out] int((d + e*x)^n/(x*(a + c*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x/(c*x**2+a)**2,x)
```

```
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)**2), x)
```

3.376 $\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=513

$$-\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(1-n)+cd\right.}{4(-a)^{5/2}(n+1)\left(\sqrt{c}\,d-\sqrt{\right.}}$$

```
[Out] -1/2*c*(c*d*x+a*e)*(e*x+d)^(1+n)/a^2/(a*e^2+c*d^2)/(c*x^2+a)+e*(e*x+d)^(1+n
)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a^2/d^2/(1+n)-1/2*c*(e*x+d)^(1+n)*hyper
geom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(
1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n]
,(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(1+n)/(e*(-a)^(1/2)+d
*c^(1/2))+1/4*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(
-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/
2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))-1/4*c*(e*x+d)^(1+n)*hyperge
om([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(
1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2
)+d*c^(1/2))
```

Rubi [A] time = 0.70, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {961, 65, 741, 831, 68, 712}

$$-\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(1-n)+cd\right.}{4(-a)^{5/2}(n+1)\left(\sqrt{c}\,d-\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] -(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) -
(c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*
e)])/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c*(
d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (
c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])
/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d +
e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 +
 n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx = \int \left(\frac{(d+ex)^n}{a^2x^2} - \frac{c(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{c(d+ex)^n}{a^2\left(a+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{\left(a+cx^2\right)^2}\,dx}{a}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} - \frac{c\int\left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} \cdots\right.}{\cdots}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} + \frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx \cdots}{2(-a)^{5/2}}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{5/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{c(d+ex)^{1+n} \cdots}{2(-a) \cdots}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{5/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} - \frac{c\left(cd^2+ae^2 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.67, size = 437, normalized size = 0.85

$$\frac{1}{4}(d+ex)^{n+1}\left(-\frac{2c(ae+cdx)}{a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{4e\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}+\frac{ac\left(\frac{\left(\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\right.}{\sqrt{c}d-\sqrt{-a}e}\right.}{(-a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*c*(a*e + c*d*x))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
+ (2*c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d -
Sqrt[-a]*e)])/((-a)^(5/2)*(-(Sqrt[c]*d) + Sqrt[-a]*e)*(1 + n)) + (2*c*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]
)/((-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*c*(((c*d^2 - a*e^2*(-1
 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 -
 a*e^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))
/((-a)^(7/2)*(c*d^2 + a*e^2)*(1 + n)) + (4*e*Hypergeometric2F1[2, 1 + n, 2
+ n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))))/4
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n}{c^2x^6+2\,acx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c^2*x^6 + 2*a*c*x^4 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(c\,x^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x^2\,\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x^2*(a + c*x^2)^2),x)
[Out] int((d + e*x)^n/(x^2*(a + c*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)
[Out] Timed out
```

3.377 $\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=399

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(a^2e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)+cd^2(m+1)(m+2)\left(2ae^2\left(m^2\right.\right.\right.}{e^4g(m+1)(m+n+2)(m+n+3)(m+n+}$$

```
[Out] -c*d*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(1
+m)*(e*x+d)^(1+n)/e^4/g/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)+c*(c*d^2*(m^2+7*m+1
2)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(2+m)*(e*x+d)^(1+n)/e^3/g^2/(3
+m+n)/(4+m+n)/(5+m+n)-c^2*d*(4+m)*(g*x)^(3+m)*(e*x+d)^(1+n)/e^2/g^3/(4+m+n)
/(5+m+n)+c^2*(g*x)^(4+m)*(e*x+d)^(1+n)/e/g^4/(5+m+n)+(a^2*e^4*(2+m+n)*(3+m+
n)*(4+m+n)*(5+m+n)+c*d^2*(1+m)*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*
n+n^2+m*(9+2*n))))*(g*x)^(1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/
e^4/g/(1+m)/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)/((1+e*x/d)^n)
```

Rubi [A] time = 0.76, antiderivative size = 377, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {952, 1623, 80, 66, 64}

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(\frac{a^2}{m+1}+\frac{cd^2(m+2)\left(2ae^2\left(m^2+m(2n+9)+n^2+9n+20\right)+cd^2\left(m^2+7m+12\right)\right)}{e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)}\right) {}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] -((c*d*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*
(9 + 2*n)))*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^4*g*(2 + m + n)*(3 + m + n)
*(4 + m + n)*(5 + m + n))) + (c*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2
 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e^3*g^2*(3 +
 m + n)*(4 + m + n)*(5 + m + n)) - (c^2*d*(4 + m)*(g*x)^(3 + m)*(d + e*x)^(
1 + n))/(e^2*g^3*(4 + m + n)*(5 + m + n)) + (c^2*(g*x)^(4 + m)*(d + e*x)^(1
 + n))/(e*g^4*(5 + m + n)) + ((a^2/(1 + m) + (c*d^2*(2 + m)*(c*d^2*(12 + 7*
m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*(9 + 2*n))))/(e^4*(2 + m + n)*
(3 + m + n)*(4 + m + n)*(5 + m + n)))*(g*x)^(1 + m)*(d + e*x)^n*Hypergeomet
ric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k*(a + b*
x)^(m + q)*(c + d*x)^(n + 1))/(d*b^q*(m + n + q + 1)), x] + Dist[1/(d*b^q*(
m + n + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q +
 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)^
(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
x] && PolyQ[Px, x] && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx = \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(a^2eg^4(5+m+n)+2aceg^4(5+m\right.}{eg^4(5+m+n)}$$

$$= -\frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n}{}$$

$$= \frac{c\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)(gx)^{2+m}(d}{e^3g^2(3+m+n)(4+m+n)(5+m+n)}$$

$$= -\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$

$$= -\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$

$$= -\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$

Mathematica [A] time = 0.14, size = 275, normalized size = 0.69

$$\frac{x(gx)^m(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(a^2e^4\,{}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)+2acd^2e^2\,{}_2F_1\left(m+1,-n-2;m+2;-\frac{ex}{d}\right)-4acd^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c^2*d^4*Hypergeometric2F1[1 + m, -4 - n, 2 + m, -((
e*x)/d)] - 4*c^2*d^4*Hypergeometric2F1[1 + m, -3 - n, 2 + m, -((e*x)/d)] +
6*c^2*d^4*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] + 2*a*c*d^2*e
^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] - 4*c^2*d^4*Hypergeo
```

```
metric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] - 4*a*c*d^2*e^2*Hypergeometric2
F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + c^2*d^4*Hypergeometric2F1[1 + m, -n,
 2 + m, -((e*x)/d)] + 2*a*c*d^2*e^2*Hypergeometric2F1[1 + m, -n, 2 + m, -((
e*x)/d)] + a^2*e^4*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^4*(
1 + m)*(1 + (e*x)/d)^n)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^2x^4 + 2\,acx^2 + a^2\right)(ex+d)^n\left(gx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^n*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2 + a\right)^2 (ex+d)^n \left(gx\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(c\,x^2 + a\right)^2 \left(gx\right)^m (ex+d)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2 + a\right)^2 (ex+d)^n \left(gx\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,x\right)}^m\,{\left(c\,x^2+a\right)}^2\,{\left(d+e\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n,x)
[Out] int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n, x)
```

sympy [C] time = 38.73, size = 131, normalized size = 0.33

$$\frac{a^2d^ng^mxx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-n, m+1\\ m+2\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)} + \frac{2acd^ng^mx^3x^m\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}-n, m+3\\ m+4\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)} + \frac{c^2d^ng^mx^5x^m\Gamma(m+5)}{\Gamma(m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**n*g**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_pol
ar(I*pi)/d)/gamma(m + 2) + 2*a*c*d**n*g**m*x**3*x**m*gamma(m + 3)*hyper((-n
, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4) + c**2*d**n*g**m*x*
*5*x**m*gamma(m + 5)*hyper((-n, m + 5), (m + 6,), e*x*exp_polar(I*pi)/d)/ga
mma(m + 6)
```

3.378 $\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx$

Optimal. Leaf size=164

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(ae^2(m+n+2)(m+n+3)+cd^2(m+1)(m+2)\right){}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{e^2g(m+1)(m+n+2)(m+n+3)}-\frac{cd(m}{e^2g(}$$

```
[Out] -c*d*(2+m)*(g*x)^(1+m)*(e*x+d)^(1+n)/e^2/g/(2+m+n)/(3+m+n)+c*(g*x)^(2+m)*(e
*x+d)^(1+n)/e/g^2/(3+m+n)+(c*d^2*(1+m)*(2+m)+a*e^2*(2+m+n)*(3+m+n))*(g*x)^(
1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/e^2/g/(1+m)/(2+m+n)/(3+m+n
)/((1+e*x/d)^n)
```

Rubi [A] time = 0.13, antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {952, 80, 66, 64}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\frac{a}{m+1}+\frac{cd^2(m+2)}{e^2(m+n+2)(m+n+3)}\right){}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{g}-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{e^2g(m+n+2)(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] -((c*d*(2 + m)*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^2*g*(2 + m + n)*(3 + m +
 n))) + (c*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + ((a/(1 +
m) + (c*d^2*(2 + m))/(e^2*(2 + m + n)*(3 + m + n)))*(g*x)^(1 + m)*(d + e*x)
^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
```

```
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\int (gx)^m (d+ex)^n \left(a+cx^2\right) dx = \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\int (gx)^m (d+ex)^n \left(aeg^2(3+m+n) - cdg^2(2+m)x\right) d}{eg^2(3+m+n)}$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(a + \frac{cd^2(1+m)}{e^2(2+m+n)(}\right.$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(\left(a + \frac{cd^2(1+m}{e^2(2+m+n}\right.\right.$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\left(a + \frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+}\right.}{}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.69

$$\frac{x(gx)^m(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\left(ae^2+cd^2\right) {}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)+cd^2\,{}_2F_1\left(m+1,-n-2;m+2;-\frac{ex}{d}\right)-2c\right.}{e^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c*d^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*
x)/d)] - 2*c*d^2*Hypergeometric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + (c*d
^2 + a*e^2)*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^2*(1 + m)*
(1 + (e*x)/d)^n)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(cx^2+a\right)(ex+d)^n\left(gx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+a\right)(ex+d)^n\left(gx\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+a\right)\left(gx\right)^m(ex+d)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2 + a\right)(ex + d)^n \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\,x\right)^m \,\left(c\,x^2 + a\right)\,{\left(d + e\,x\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)*(d + e*x)^n,x)
```

```
[Out] int((g*x)^m*(a + c*x^2)*(d + e*x)^n, x)
```

sympy [C] time = 15.75, size = 82, normalized size = 0.50

$$\frac{ad^n g^m x x^m \Gamma\left(m+1\right) {}_2F_1\left(\left.\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\right| \frac{exe^{i\pi}}{d}\right)}{\Gamma\left(m+2\right)} + \frac{cd^n g^m x^3 x^m \Gamma\left(m+3\right) {}_2F_1\left(\left.\begin{matrix} -n, m+3 \\ m+4 \end{matrix}\right| \frac{exe^{i\pi}}{d}\right)}{\Gamma\left(m+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a),x)
```

```
[Out] a*d**n*g**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_polar(
I*pi)/d)/gamma(m + 2) + c*d**n*g**m*x**3*x**m*gamma(m + 3)*hyper((-n, m + 3
), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4)
```

3.379 $\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=148

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;\right.}{2ag(m+1)}$$

```
[Out] 1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2
))/a/g/(1+m)/((1+e*x/d)^n)+1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+m,
x*c^(1/2)/(-a)^(1/2),-e*x/d)/a/g/(1+m)/((1+e*x/d)^n)
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {912, 135, 133}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;\right.}{2ag(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(2*a*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e
*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(2*a
*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx$$
$$= -\frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{-a}} - \frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2\sqrt{-a}}$$
$$= -\frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{-a}} - \frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2\sqrt{-a}}$$
$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\right.}{}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2), x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^n\,(gx)^m}{cx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
[Out] integral((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n\,(gx)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\,(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n (gx)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2),x)
[Out] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.380 $\int \frac{(gx)^m(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=295

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m\right.}{4a^2g(m+1)}$$

```
[Out] 1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2
))/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+
m,x*c^(1/2)/(-a)^(1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e
*x+d)^n*AppellF1(1+m,-n,2,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2))/a^2/g/(1+m)/((1
+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,2,-n,2+m,x*c^(1/2)/(-a)^(
1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)
```

Rubi [A] time = 0.42, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {961, 135, 133, 912}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m\right.}{4a^2g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d +
 e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4
*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m
, -n, 2, 2 + m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 +
 (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((
e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \int \left(-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\,\sqrt{c}-cx\right)^2}-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\,\sqrt{c}+cx\right)^2}-\frac{c(gx)^m(d+ex)^n}{2a\left(-ac-c^2x^2\right)}\right)dx$$

$$=-\frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\,\sqrt{c}-cx\right)^2}\,dx}{4a}-\frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\,\sqrt{c}+cx\right)^2}\,dx}{4a}-\frac{c\int \frac{(gx)^m(d+ex)^n}{-ac-c^2x^2}\,dx}{2a}$$

$$=-\frac{c\int \left(-\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}-\sqrt{c}\,x\right)}-\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx}{2a}-\frac{\left(c(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\left(\sqrt{-a}\,\sqrt{c}-cx\right)^2}\,d}{4a}$$

$$=\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)}+\frac{(gx)^{1+m}(d+ex)^n\left(1\right.}{}$$

$$=\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)}+\frac{(gx)^{1+m}(d+ex)^n\left(1\right.}{}$$

$$=\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)}+\frac{(gx)^{1+m}(d+ex)^n\left(1\right.}{}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex+d)^n\,(gx)^m}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((e*x + d)^n*(g*x)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n\,(gx)^m}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^{m} \left(ex+d\right)^{n}}{\left(c\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{n} \left(gx\right)^{m}}{\left(cx^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,x\right)^{m}\,\left(d+e\,x\right)^{n}}{\left(c\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

3.381 $\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{a^2d\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ad\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{d\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*d*(b*x^2+a)^(1+p)/b^3/(1+p)-a*d*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*d*(b*
x^2+a)^(3+p)/b^3/(3+p)+1/7*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-b*x
^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 43, 365, 364}

$$\frac{a^2d\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ad\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{d\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*d*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (d*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*x^7*(a + b*x^2)^p
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx = d\int x^5\left(a+bx^2\right)^p\,dx + e\int x^6\left(a+bx^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\mathrm{Subst}\left(\int x^2(a+bx)^p\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^2}{a}\right)^p\,d$$
$$= \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}d\,\mathrm{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2} - \right.\right.$$
$$= \frac{a^2d\left(a+bx^2\right)^{1+p}}{2b^3(1+p)} - \frac{ad\left(a+bx^2\right)^{2+p}}{b^3(2+p)} + \frac{d\left(a+bx^2\right)^{3+p}}{2b^3(3+p)} + \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.90

$$\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)} + 2ex^7\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*e*x^7*Hypergeometric2F1[7/2
, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex^6+dx^5\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^6 + d*x^5)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)\left(bx^2+a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^5, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int(ex+d)\,x^5\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int\left(bx^2+a\right)^p x^6\,dx + \frac{\left(\left(p^2+3\,p+2\right)b^3x^6+\left(p^2+p\right)ab^2x^4-2\,a^2bpx^2+2\,a^3\right)\left(bx^2+a\right)^p d}{2\left(p^3+6\,p^2+11\,p+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^6, x) + 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p
)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*d/((p^3 + 6*p^2 + 11*p +
 6)*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(b\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^5*(a + b*x^2)^p*(d + e*x), x)
```

sympy [C] time = 29.68, size = 1012, normalized size = 8.10

$$\frac{a^p e x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -p \\ \frac{9}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right)}{7} + d \begin{cases} \frac{a^p x^6}{6} & \\ \frac{2a^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \cdots & \\ -\frac{2a^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} - \frac{2a^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} - \cdots & \\ \frac{a^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3} + \frac{a^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} & \\ \frac{2a^3\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \cdots & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*
b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4
*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/
(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b
) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*lo
g(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)
*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2)
- 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b
*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*
a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b
**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/
(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22
*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3
*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**
3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*
b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**
2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a
```

```
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x
**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), Tru
e))
```

3.382 $\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{a^2e\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ae\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{e\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*e*(b*x^2+a)^(1+p)/b^3/(1+p)-a*e*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e*(b*
x^2+a)^(3+p)/b^3/(3+p)+1/5*d*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x
^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 365, 364, 266, 43}

$$\frac{a^2e\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ae\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{e\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)\left(a+bx^2\right)^p dx = d\int x^4\left(a+bx^2\right)^p dx + e\int x^5\left(a+bx^2\right)^p dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int x^2(a+bx)^p\,dx, x, x^2\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p a$$
$$= \frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2} - \right.\right.$$
$$= \frac{a^2e\left(a+bx^2\right)^{1+p}}{2b^3(1+p)} - \frac{ae\left(a+bx^2\right)^{2+p}}{b^3(2+p)} + \frac{e\left(a+bx^2\right)^{3+p}}{2b^3(3+p)} + \frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.90

$$\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5e\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)} + 2dx^5\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeometric2F1[5/2
, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^5+dx^4\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)\,x^4\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(b\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^4*(a + b*x^2)^p*(d + e*x), x)
```

sympy [C] time = 21.96, size = 1012, normalized size = 8.10

$$\frac{a^p d x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+e\begin{cases}\frac{a^px^6}{6}\\ \frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\\ -\frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\text{lo}}{2a}\\ \frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3}+\frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}\\ \frac{2a^3\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}-\frac{2a^2bpx^2\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2p^2x^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{ab^2p}{2b^3p^3+12b}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + e*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*
b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4
*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/
(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b
) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*lo
g(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)
*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2)
- 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b
*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*
a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b
**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/
(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22
*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3
*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**
3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*
b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**
2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x
**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), Tru
e))
```

3.383 $\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{ad\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{d\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*d*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*d*(b*x^2+a)^(2+p)/b^2/(2+p)+1/5*e*x^
5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 43, 365, 364}

$$-\frac{ad\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{d\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -(a*d*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b^2
*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/
a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx = d\int x^3\left(a+bx^2\right)^p\,dx + e\int x^4\left(a+bx^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int x(a+bx)^p\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p$$
$$= \frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}d\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}\right.\right.$$
$$= -\frac{ad\left(a+bx^2\right)^{1+p}}{2b^2(1+p)} + \frac{d\left(a+bx^2\right)^{2+p}}{2b^2(2+p)} + \frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2}\right.$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.87

$$\frac{1}{10}\left(a+bx^2\right)^p\left(2ex^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right) - \frac{5d\left(a+bx^2\right)\left(a-b(p+1)x^2\right)}{b^2(p+1)(p+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*e*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/10
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^4+dx^3\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)\,x^3\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int\left(bx^2+a\right)^p x^4\,dx + \frac{\left(b^2(p+1)x^4+abpx^2-a^2\right)\left(bx^2+a\right)^p d}{2\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^4, x) + 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)
*(b*x^2 + a)^p*d/((p^2 + 3*p + 2)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(b\,x^2+a\right)^p\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^p*(d + e*x), x)
```

sympy [C] time = 16.78, size = 394, normalized size = 3.94

$$\frac{a^p e x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+d\left(\begin{cases}\frac{a^px^4}{4}\\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}\\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}+\frac{x^2}{2b}\\ -\frac{a^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2
*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(
2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 +
2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
, Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*
sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2
*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p +
 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), Tru
e))
```

3.384 $\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{ae\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*e*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*e*(b*x^2+a)^(2+p)/b^2/(2+p)+1/3*d*x^
3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 365, 364, 266, 43}

$$-\frac{ae\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -(a*e*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e*(a + b*x^2)^(2 + p))/(2*b^2
*(2 + p)) + (d*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/
a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)\left(a+bx^2\right)^p dx = d\int x^2\left(a+bx^2\right)^p dx + e\int x^3\left(a+bx^2\right)^p dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int x(a+bx)^p dx, x, x^2\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p dx$$
$$= \frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}e\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\right.\right.$$
$$= -\frac{ae\left(a+bx^2\right)^{1+p}}{2b^2(1+p)} + \frac{e\left(a+bx^2\right)^{2+p}}{2b^2(2+p)} + \frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\right.$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.87

$$\frac{1}{6}\left(a+bx^2\right)^p\left(2dx^3\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right) - \frac{3e\left(a+bx^2\right)\left(a-b(p+1)x^2\right)}{b^2(p+1)(p+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((-3*e*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/6
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^3+dx^2\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^3 + d*x^2)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex+d)\,x^2\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(b x^2 + a\right)^p (d + e x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^p*(d + e*x), x)
```

sympy [C] time = 11.88, size = 394, normalized size = 3.94

$$\frac{a^p d x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right)}{3} + e \left(\begin{cases} \frac{a^p x^4}{4} & \\ \frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2ab^2 + 2b^3x^2} & \\ -\frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2b^2} - \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x\right)}{2b^2} + \frac{x^2}{2b} & \\ -\frac{a^2\left(a + bx^2\right)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{abpx^2\left(a + bx^2\right)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2px^4\left(a + bx^2\right)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2x^4\left(a + bx^2\right)^p}{2b^2p^2 + 6b^2p + 4b^2} & \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + e*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2
*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(
2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 +
2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
, Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*
sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2
*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p +
 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), Tru
e))
```

3.385 $\int x(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*d*(b*x^2+a)^(1+p)/b/(1+p)+1/3*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/
2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {764, 261, 365, 364}

$$\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e*x^3*(a + b*x^2)^p*Hypergeometric
2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx = d\int x\left(a+bx^2\right)^p\,dx + e\int x^2\left(a+bx^2\right)^p\,dx$$
$$= \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)} + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$= \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)} + \frac{1}{3}ex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.95

$$\frac{1}{6}\left(a+bx^2\right)^p\left(\frac{3d\left(a+bx^2\right)}{b(p+1)}+2ex^3\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((3*d*(a + b*x^2))/(b*(1 + p)) + (2*e*x^3*Hypergeometric2F1[
3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/6
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex^2+dx\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^2 + d*x)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (ex+d)\,x\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(b*x^2+a)^p,x)
[Out] int(x*(e*x+d)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int\left(bx^2+a\right)^p x^2\,dx + \frac{\left(bx^2+a\right)^{p+1}d}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^2, x) + 1/2*(b*x^2 + a)^(p + 1)*d/(b*(p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(d + e*x),x)
```

```
[Out] int(x*(a + b*x^2)^p*(d + e*x), x)
```

sympy [A] time = 9.01, size = 65, normalized size = 0.87

$$\frac{a^{p}ex^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}+d\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\: b=0\\\frac{\begin{cases}\frac{\left(a+bx^{2}\right)^{p+1}}{p+1} & \text{for}\: p\neq -1\\\log\left(a+bx^{2}\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

3.386 $\int (d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=70

$$dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] 1/2*e*(b*x^2+a)^(1+p)/b/(1+p)+d*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*
x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {641, 246, 245}

$$dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (e*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (d*x*(a + b*x^2)^p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(a+bx^2\right)^p\,dx &= \frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+d\int \left(a+bx^2\right)^p\,dx\\
&= \frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx\\
&= \frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 1.40

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2bd(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+bex^2\left(\frac{bx^2}{a}+1\right)^p+ae\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(b*e*x^2*(1 + (b*x^2)/a)^p + a*e*(-1 + (1 + (b*x^2)/a)^p) +
2*b*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left((ex+d)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p, x)
```

mupad [B] time = 3.36, size = 65, normalized size = 0.93

$$\frac{e\left(b\,x^2+a\right)^{p+1}}{2\,b\,\left(p+1\right)}+\frac{d\,x\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\;\frac{3}{2};\;-\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(d + e*x),x)
```

```
[Out] (e*(a + b*x^2)^(p + 1))/(2*b*(p + 1)) + (d*x*(a + b*x^2)^p*hypergeom([1/2,
-p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

sympy [A] time = 6.41, size = 61, normalized size = 0.87

$$a^p d x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + e\left(\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(a + bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + e*Piecewise((
a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(a + b*x**2), True))/(2*b), True))
```

3.387 $\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx$

Optimal. Leaf size=88

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[Out] e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d*(
b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 65, 246, 245}

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x}\,dx + e\int \left(a+bx^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$= ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) - \frac{d\left(a+bx^2\right)^{1+p} {}_2F_1\left(1,1+p;2+\right.}{2a(1+p)}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.00

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) - \frac{d\left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)\left(bx^2+a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(b\,x^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x))/x,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x))/x, x)
```

sympy [C] time = 9.67, size = 65, normalized size = 0.74

$$a^p ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^p dx^{2p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x,x)
```

```
[Out] a**p*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*x**(2*
p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma
(1 - p))
```

3.388 $\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=91

$$-\frac{d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[Out] -d*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*e
*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {764, 365, 364, 266, 65}

$$-\frac{d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^2}\,dx &= d\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx+e\int \frac{\left(a+bx^2\right)^p}{x}\,dx\\
&= \frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx\\
&= -\frac{d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\right.}{2a(1+p)}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 91, normalized size = 1.00

$$
-\frac{d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{(ex+d)\left(bx^2+a\right)^p}{x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{(ex+d)\left(b\,x^2+a\right)^p}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x))/x^2,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x))/x^2, x)
```

sympy [C] time = 10.87, size = 68, normalized size = 0.75

$$-\frac{a^p d\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{x}-\frac{b^p e x^{2p}\Gamma\left(-p\right){}_2F_1\left(\left.\begin{matrix}-p,-p\\ 1-p\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*e*x**(
2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gam
ma(1 - p))
```

3.389 $\int \frac{(d+ex)(a+bx^2)^p}{x^3} dx$

Optimal. Leaf size=92

$$\frac{bd\left(a+bx^2\right)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

```
[Out] -e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)+1/2*b
*d*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],1+b*x^2/a)/a^2/(1+p)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 65, 365, 364}

$$\frac{bd\left(a+bx^2\right)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -((e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) + (b*d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^3}\,dx + e\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx$$
$$= -\frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} + \frac{bd\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(2,1+p;2\right.}{2a^2(1+p)}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.97

$$\frac{1}{2}\left(a+bx^2\right)^p\left(\frac{bd\left(a+bx^2\right)\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{a^2(p+1)} - \frac{2e\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^3,x]
[Out] ((a + b*x^2)^p*((-2*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1
 + (b*x^2)/a)^p) + (b*d*(a + b*x^2)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(a^2*(1 + p))))/2
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)\left(bx^2+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="fricas")
[Out] integral((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(b\,x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x^3,x)
[Out] int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x))/x^3,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x))/x^3, x)
```

sympy [C] time = 13.92, size = 71, normalized size = 0.77

$$-\frac{a^p e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}-\frac{b^p d x^{2p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\ 2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma\left(2-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**3,x)
```

```
[Out] -a**p*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*x**(
2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*x**2*gamma(2 - p))
```

3.390 $\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=188

$$\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{2}{7}dex^7$$

```
[Out] 1/2*a^2*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^4/(1+p)-1/2*a*(-3*a*e^2+2*b*d^2)*(
b*x^2+a)^(2+p)/b^4/(2+p)+1/2*(-3*a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2
*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)+2/7*d*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],
[9/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1652, 446, 77, 12, 365, 364}

$$\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{2}{7}dex^7$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) - (a*(2*b*d^2 - 3
*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) + ((b*d^2 - 3*a*e^2)*(a + b*x^
2)^(3 + p))/(2*b^4*(3 + p)) + (e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (
2*d*e*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(
1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^6\left(a+bx^2\right)^p\,dx+\int x^5\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)dx\\
&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^6\left(a+bx^2\right)^p\,dx\\
&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2\left(-bd^2+ae^2\right)(a+bx)^p}{b^3}+\frac{a\left(-2bd^2+3ae^2\right)(a+bx)^{1+p}}{b^3}+\frac{\left(bd^2-\right.}{}\right.\right.\\
&=\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx\right.}{2b^4(3+p)}\end{aligned}$$

Mathematica [A] time = 0.20, size = 205, normalized size = 1.09

$$\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d^2\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{7e^2\left(a+bx^2\right)\left(-6a^3+6a^2b(p+1)x^2-\right.}{b^4(p+1\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*
p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*e^2*(a + b*x^2)*(-6*a^3 +
 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6*p^2
+ p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (4*d*e*x^7*Hypergeomet
ric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^7+2\,dex^6+d^2x^5\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^2\left(bx^2+a\right)^px^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^5, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^5 \left(b x^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
[Out] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(p^2+3p+2\right)b^3x^6+\left(p^2+p\right)ab^2x^4-2a^2bpx^2+2a^3\right)\left(bx^2+a\right)^p d^2}{2\left(p^3+6p^2+11p+6\right)b^3}+\int \left(e^2x^7+2dex^6\right)\left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^2*x^7 + 2*
d*e*x^6)*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(b x^2+a\right)^p (d+e x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^5*(a + b*x^2)^p*(d + e*x)^2, x)
```

sympy [C] time = 46.59, size = 3046, normalized size = 16.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2*(b*x**2+a)**p,x)
[Out] 2*a**p*d*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**2
*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b)
 + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a
**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b
) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x
)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*s
qrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*
a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*
sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4
*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2)
- 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x
**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) +
x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2)
```

```
+ x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 +
12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b
**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a
+ b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*
x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2
*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**
3), True)) + e**2*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)
*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**
7*x**6) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*
x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqr
t(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x*
*6) + 18*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 +
36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*
sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 +
 12*b**7*x**6) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4
+ 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a
**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6
*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**
6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*
b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*
a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x*
*4) - 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*
b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b
*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x
**4) - 12*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*
x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*
x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5
*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b
**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x*
*2 + 4*b**6*x**4), Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b
**4 + 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5
*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqr
t(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b
) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) +
b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqrt(
1/b) + x)/(2*b**4) - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2
/(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)
**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*
a**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 +
100*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4
+ 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*
(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 4
8*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70
*b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*
b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*
p*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**
4*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3
 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/
(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**
4*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b
**4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3
+ 70*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

3.391 $\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=177

$$\frac{a^2de\left(a+bx^2\right)^{p+1}}{b^3(p+1)}-\frac{2ade\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{de\left(a+bx^2\right)^{p+3}}{b^3(p+3)}-\frac{x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(5ae^2-bd^2(2p+7)\right){}_2F_1\left(\frac{5}{2},\right.}{5b(2p+7)}$$

```
[Out] a^2*d*e*(b*x^2+a)^(1+p)/b^3/(1+p)+e^2*x^5*(b*x^2+a)^(1+p)/b/(7+2*p)-2*a*d*e
*(b*x^2+a)^(2+p)/b^3/(2+p)+d*e*(b*x^2+a)^(3+p)/b^3/(3+p)-1/5*(5*a*e^2-b*d^2
*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1
+b*x^2/a)^p)
```

Rubi [A] time = 0.16, antiderivative size = 169, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$\frac{a^2de\left(a+bx^2\right)^{p+1}}{b^3(p+1)}-\frac{2ade\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{de\left(a+bx^2\right)^{p+3}}{b^3(p+3)}+\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{5ae^2}{2bp+7b}\right){}_2F_1\left(\frac{5}{2},\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*e*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^2*x^5*(a + b*x^2)^(1 + p))
/(b*(7 + 2*p)) - (2*a*d*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (d*e*(a + b*
x^2)^(3 + p))/(b^3*(3 + p)) + ((d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x
^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^5\left(a+bx^2\right)^p\,dx+\int x^4\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+(2de)\int x^5\left(a+bx^2\right)^p\,dx-\left(-d^2+\frac{5ae^2}{7b+2bp}\right)\int x^4\left(a+bx^2\right)^p\\
&= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+(de)\operatorname{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)-\left(\left(-d^2+\frac{5ae^2}{7b+2bp}\right)\left(a\right.\right.\\
&= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{5}\left(d^2-\frac{5ae^2}{7b+2bp}\right)x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\right.\\
&= \frac{a^2de\left(a+bx^2\right)^{1+p}}{b^3(1+p)}+\frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}-\frac{2ade\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{de\left(a+bx^2\right)^{3+p}}{b^3(3+p)}+\frac{1}{5}\end{aligned}$$

Mathematica [A] time = 0.16, size = 156, normalized size = 0.88

$$\frac{1}{35}\left(a+bx^2\right)^p\left(\frac{35de\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+7d^2x^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx}{a}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*d^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (5*e^2*x^7*Hypergeometri
c2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/35
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(e^2x^6+2\,dex^5+d^2x^4\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^4 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
[Out] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(b\,x^2+a\right)^p(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x)^2, x)
```

sympy [C] time = 38.89, size = 1047, normalized size = 5.92

$$\frac{a^p d^2 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{a^p e^2 x^7 {}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}+2de\begin{cases}\frac{a^px^6}{6} \\ \frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+\cdots} \\ -\frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\cdots \\ \frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3}+\frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b} \\ \frac{2a^3\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}-\frac{2a^2bpx^2\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3}+\frac{\cdots}{2b^3\cdots}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*
e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 2*d*e*Piec
ewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b*
*4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x
)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*
a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1
/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*
log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(
a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2
) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a
*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(
2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**
4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b*
*3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p
**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(
2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x
**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*
(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3
*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), T
rue))
```

3.392 $\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=149

$$-\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\ {}_2F_1\Big($$

```
[Out] -1/2*a*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^3/(1+p)+1/2*(-2*a*e^2+b*d^2)*(b*x^2
+a)^(2+p)/b^3/(2+p)+1/2*e^2*(b*x^2+a)^(3+p)/b^3/(3+p)+2/5*d*e*x^5*(b*x^2+a)
^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1652, 446, 77, 12, 365, 364}

$$-\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\ {}_2F_1\Big($$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + ((b*d^2 - 2*a*e^
2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^3*
(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^4\left(a+bx^2\right)^p\,dx+\int x^3\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^4\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a\left(-bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^2-2ae^2\right)(a+bx)^{1+p}}{b^2}+\frac{e^2(a+bx)^{2+p}}{b^2}\right.\right.\\ &= -\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{2}{5}de\end{aligned}$$

Mathematica [A] time = 0.14, size = 152, normalized size = 1.02

$$\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5e^2\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{5d^2\left(a+bx^2\right)\left(b(p+1)x^2-a\right)}{b^2(p+1)(p+2)}+4dex^5\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
p)) + (5*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x
^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (4*d*e*x^5*Hypergeometric2F1[5/2, -p,
7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^5+2\,dex^4+d^2x^3\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^2\left(bx^2+a\right)^px^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^3 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2(p+1)x^4+abpx^2-a^2\right)\left(bx^2+a\right)^p d^2}{2\left(p^2+3\,p+2\right)b^2}+\int \left(e^2x^5+2\,dex^4\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^2/((p^2 + 3*p + 2)*
b^2) + integrate((e^2*x^5 + 2*d*e*x^4)*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^3*(a + b*x^2)^p*(d + e*x)^2, x)
```

sympy [C] time = 25.95, size = 1386, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**2
*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2
) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*
b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**
3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2
)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2
*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b
**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**
2), True)) + e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)
*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*
sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I
*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b
*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt
(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**
4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4),
 Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2
```

```
) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(
2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**
3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b*
*4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqr
t(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) -
 a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*
p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a
+ b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*
x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b
**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*
b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3
*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 2
2*b**3*p + 12*b**3), True))
```

3.393 $\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=152

$$-\frac{ade\left(a+bx^2\right)^{p+1}}{b^2(p+1)}+\frac{de\left(a+bx^2\right)^{p+2}}{b^2(p+2)}-\frac{x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2-bd^2(2p+5)\right)\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)}{3b(2p+5)}+\frac{e^2x^3}{b}$$

```
[Out] -a*d*e*(b*x^2+a)^(1+p)/b^2/(1+p)+e^2*x^3*(b*x^2+a)^(1+p)/b/(5+2*p)+d*e*(b*x
^2+a)^(2+p)/b^2/(2+p)-1/3*(3*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hypergeom
([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.14, antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$-\frac{ade\left(a+bx^2\right)^{p+1}}{b^2(p+1)}+\frac{de\left(a+bx^2\right)^{p+2}}{b^2(p+2)}+\frac{1}{3}x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+5b}\right)\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{e^2x^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -((a*d*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (e^2*x^3*(a + b*x^2)^(1 + p)
)/(b*(5 + 2*p)) + (d*e*(a + b*x^2)^(2 + p))/(b^2*(2 + p)) + ((d^2 - (3*a*e^
2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^
2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^3\left(a+bx^2\right)^p\,dx+\int x^2\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+(2de)\int x^3\left(a+bx^2\right)^p\,dx-\left(-d^2+\frac{3ae^2}{5b+2bp}\right)\int x^2\left(a+bx^2\right)^{\dots}\\ &= \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+(de)\,\mathrm{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)-\left(\left(-d^2+\frac{3ae^2}{5b+2bp}\right)\left(a+\dots\right.\right.\\ &= \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\dots\right.\\ &= -\frac{ade\left(a+bx^2\right)^{1+p}}{b^2(1+p)}+\frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{de\left(a+bx^2\right)^{2+p}}{b^2(2+p)}+\frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3\dots\end{aligned}$$

Mathematica [A] time = 0.15, size = 139, normalized size = 0.91

$$\frac{1}{15}\left(a+bx^2\right)^p\left(\frac{3e\left(e\left(p^2+3p+2\right)x^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)-\frac{5d\left(a+bx^2\right)\left(a-b(p+1)x^2\right)}{b^2}\right)}{(p+1)(p+2)}+5d^2x^3\left(\frac{bx^2}{a}+1\right)^{-p}\dots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p + (3*e*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/b^2 + (e*(2
 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/((1 + p)*(2 + p))))/15
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^2x^4+2\,dex^3+d^2x^2\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^2 x^2 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
[Out] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^p (d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^2*(a + b*x^2)^p*(d + e*x)^2, x)
```

sympy [C] time = 21.18, size = 430, normalized size = 2.83

$$\frac{a^p d^2 x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+\frac{a^p e^2 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+2de\left(\begin{cases}\frac{a^p x^4}{4} \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2 l}{} \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}+\frac{x^2}{2b} \\ -\frac{a^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b}{2b} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(b*x**2+a)**p,x)
[Out] a**p*d**2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*
e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 2*d*e*Piec
ewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 +
```

```
 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a
/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2
+ 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**
2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a
)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2
 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p
 + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), T
rue))
```

3.394 $\int x(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=113

$$\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*e^2*(b*x^2+a)^(2+p)/b^2/(2
+p)+2/3*d*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a
)^p)
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1652, 444, 43, 12, 365, 364}

$$\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2
 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p
, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int x(d+ex)^2\left(a+bx^2\right)^p\,dx=\int 2dex^2\left(a+bx^2\right)^p\,dx+\int x\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)dx$$
$$=\frac{1}{2}\text{Subst}\left(\int(a+bx)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^2\left(a+bx^2\right)^p\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(bd^2-ae^2\right)(a+bx)^p}{b}+\frac{e^2(a+bx)^{1+p}}{b}\right)dx,x,x^2\right)+\left(2de\left(a+bx^2\right)^p\left(1\right.\right.$$
$$=\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2}\right.$$

Mathematica [A] time = 0.18, size = 184, normalized size = 1.63

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(-3a^2e^2\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)+3b^2x^2\left(\frac{bx^2}{a}+1\right)^p\left(d^2(p+2)+e^2(p+1)x^2\right)+4b^2de\left(p^2+3p+2\right.\right.}{6b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + e^2*(1 + p)*x^2)
 - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + 3*a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 4*b^2*d*e*(2 + 3*p + p^2)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(6*b^2*(1 + p)*(2 + p)*(1 + (b
*x^2)/a)^p)
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^3+2\,dex^2+d^2x\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^2\left(bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int(ex+d)^2\,x\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bx^2+a\right)^{p+1}d^2}{2\,b\left(p+1\right)}+\int\left(e^2x^3+2\,dex^2\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)*d^2/(b*(p + 1)) + integrate((e^2*x^3 + 2*d*e*x^2)*(
b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x*(a + b*x^2)^p*(d + e*x)^2, x)
```

sympy [A] time = 12.91, size = 439, normalized size = 3.88

$$\frac{2a^pdex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+d^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for } p\neq-1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}\frac{a^px^4}{4}\\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(i\sqrt{a}\sqrt{\ldots\right.}}{2ab^2+2b\ldots}\\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}-\frac{a\log\left(i\sqrt{a}\ldots\right.}{2b\ldots}\\ -\frac{a^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2\left(a+bx\ldots\right.}{2b^2p^2+6b^2p+\ldots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d**2
*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p +
1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**2*Piecewise((a*
*p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x
**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**
2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*
x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p,
 -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/
b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p
**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2
*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2
) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

3.395 $\int (d+ex)^2 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=133

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-bd^2(2p+3)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}+\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}+\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{b(p+1)(2p+3)}$$

```
[Out] d*e*(2+p)*(b*x^2+a)^(1+p)/b/(2*p^2+5*p+3)+e*(e*x+d)*(b*x^2+a)^(1+p)/b/(3+2*
p)-(a*e^2-b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/
b/(3+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {743, 641, 246, 245}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}+\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{b(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (d*e*(2 + p)*(a + b*x^2)^(1 + p))/(b*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\int (d+ex)^2 \left(a+bx^2\right)^p dx = \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \frac{\int \left(-ae^2+bd^2(3+2p)+2bde(2+p)x\right)\left(a+bx^2\right)^p dx}{b(3+2p)}$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2 - \frac{ae^2}{3b+2bp}\right)\int \left(a+bx^2\right)^p$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(\left(d^2 - \frac{ae^2}{3b+2bp}\right)\left(a+bx^2\right)^p \left(1\right.\right.$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2 - \frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p \left(\right.$$

Mathematica [A] time = 0.12, size = 133, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3bd^2(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(3d\left(bx^2\left(\frac{bx^2}{a}+1\right)^p+a\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)\right)+be(p+\right.\right.}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + e*(3*d*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*e*
(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(3*b*(1 + p)*(
1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(b\,x^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int((a + b*x^2)^p*(d + e*x)^2, x)
```

sympy [A] time = 11.61, size = 97, normalized size = 0.73

$$a^p d^2 x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{a^p e^2 x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+2de\left(\begin{cases}\frac{a^p x^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e**2*
x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 2*d*e*Piecewise
((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

3.396 $\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x}\,dx$

Optimal. Leaf size=118

$$-\frac{d^2\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+2dex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e^2\left(a+bx^2\right.}{2b(p+1}$$

```
[Out] 1/2*e^2*(b*x^2+a)^(1+p)/b/(1+p)+2*d*e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/
2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+
p],1+b*x^2/a)/a/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1652, 446, 80, 65, 12, 246, 245}

$$-\frac{d^2\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+2dex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e^2\left(a+bx^2\right.}{2b(p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] (e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (2*d*e*x*(a + b*x^2)^p*Hypergeome
tric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x}\,dx &= \int 2de\left(a+bx^2\right)^p\,dx+\int \frac{\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^p\left(d^2+e^2x\right)}{x}\,dx,x,x^2\right)+(2de)\int \left(a+bx^2\right)^p\,dx\\ &= \frac{e^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{1}{2}d^2\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\\ &= \frac{e^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+2dex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d^2\left(a+bx^2\right)^1}{}\end{aligned}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.86

$$\frac{1}{2}\left(a+bx^2\right)^p\left(\frac{\left(a+bx^2\right)\left(ae^2-bd^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)\right)}{ab(p+1)}+4dex\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] ((a + b*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p + ((a + b*x^2)*(a*e^2 - b*d^2*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 + (b*x^2)/a]))/(a*b*(1 + p))))/2
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(bx^2+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2(bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(b\,x^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x,x)
[Out] int((e*x+d)^2*(b*x^2+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2(bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="maxima")
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,(d+e\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x, x)
```

sympy [A] time = 12.06, size = 109, normalized size = 0.92

$$2a^p dex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^p d^2x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}+e^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq-1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x,x)
[Out] 2*a**p*d*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**2
*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(
2*gamma(1 - p)) + e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a +
b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)
)
```

3.397 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx$

Optimal. Leaf size=127

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{d^2\left(a+bx^2\right)^{p+1}}{ax}-\frac{de\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p\right.}{a(p+1)}$$

[Out] -d^2*(b*x^2+a)^(1+p)/a/x+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-d*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 764, 266, 65, 246, 245}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{d^2\left(a+bx^2\right)^{p+1}}{ax}-\frac{de\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p\right.}{a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]

[Out] -((d^2*(a + b*x^2)^(1 + p))/(a*x)) + ((a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(a*(1 + p))

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x^2}\,dx &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}-\frac{\int \frac{\left(-2ade-\left(ae^2+bd^2(1+2p)\right)x\right)\left(a+bx^2\right)^p}{x}\,dx}{a}\\ &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}+(2de)\int \frac{\left(a+bx^2\right)^p}{x}\,dx+\frac{\left(ae^2+bd^2(1+2p)\right)\int \left(a+bx^2\right)^p\,dx}{a}\\ &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}+(de)\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\frac{\left(\left(ae^2+bd^2(1+2p)\right)\left(a\right.\right.}{}\\ &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}+\frac{\left(ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b}{}\right.}{a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 1.06

$$-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ad^2(p+1)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)+ex\left(d\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+\right.\right.\right.}{a(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(a*d^2*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/
a)] + e*x*(-(a*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]) +
 d*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^2)/a])))/(a*(1 + p)*x*(1 + (b*x^2)/a)^p))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(bx^2+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(b\,x^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,(d+e\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x^2,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x^2, x)
```

sympy [C] time = 13.57, size = 95, normalized size = 0.75

$$-\frac{a^p d^2 {}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, -p\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2 e^{i\pi}}{a}\right)}{x}+a^p e^2 x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2 e^{i\pi}}{a}\right)-\frac{b^p d e x^{2p}\Gamma\left(-p\right){}_2F_1\left(\left.\begin{matrix}-p, -p\\ 1-p\end{matrix}\right|\frac{a e^{i\pi}}{bx^2}\right)}{\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**2*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p*e**
2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*e*x**(2*p)*
gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/gamma(1 - p
)
```

3.398 $\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(a+bx^2\right)^{p+1}\left(ae^2+bd^2p\right){}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2ax^2}-\frac{2de\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},\right.}{x}$$

```
[Out] -1/2*d^2*(b*x^2+a)^(1+p)/a/x^2-2*d*e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2]
,-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*(b*d^2*p+a*e^2)*(b*x^2+a)^(1+p)*hypergeom(
[1, 1+p],[2+p],1+b*x^2/a)/a^2/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 764, 365, 364, 266, 65}

$$-\frac{\left(a+bx^2\right)^{p+1}\left(ae^2+bd^2p\right){}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^2\left(a+bx^2\right)^{p+1}}{2ax^2}-\frac{2de\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -(d^2*(a + b*x^2)^(1 + p))/(2*a*x^2) - (2*d*e*(a + b*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - ((a*e^2 + b*d^2*p)
*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*
a^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x^3}\,dx = -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{\int \frac{\left(-4ade-2\left(ae^2+bd^2p\right)x\right)\left(a+bx^2\right)^p}{x^2}\,dx}{2a}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2} + (2de)\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx + \frac{\left(ae^2+bd^2p\right)\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{a}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2} + \frac{\left(ae^2+bd^2p\right)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2a} + \left(2de\left(a+bx^2\right)^p\left(1+\frac{b}{\cdots}\right.\right.$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{\left(ae^2+bd^2p\right)\cdots}{\cdots}$$

Mathematica [A] time = 0.09, size = 119, normalized size = 0.94

$$\frac{1}{2}\left(a+bx^2\right)^p\left(-\frac{\left(a+bx^2\right)\left(ae^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)-bd^2\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)\right)}{a^2(p+1)} - \frac{4de\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^p) - ((a + b*x^2)*(a*e^2*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a]))
/(a^2*(1 + p))))/2
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(bx^2+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)
```

maple **[F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2 \left(b\,x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2 \left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,(d+e\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x^3,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x^3, x)
```

sympy **[C]** time = 18.62, size = 119, normalized size = 0.94

$$-\frac{2a^p d e\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d^2 x^{2p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma\left(2-p\right)} - \frac{b^p e^2 x^{2p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**3,x)
```

```
[Out] -2*a**p*d*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*
*2*x**(2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*
x**2))/(2*x**2*gamma(2 - p)) - b**p*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p),
 (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

3.399 $\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=247

$$\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{d\left(bd^2-9ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{3de^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}-\frac{e}{}$$

```
[Out] 1/2*a^2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^4/(1+p)+e^3*x^7*(b*x^2+a)^(1+p
)/b/(9+2*p)-1/2*a*d*(-9*a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+1/2*d*(-9*
a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+3/2*d*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)-
1/7*e*(7*a*e^2-3*b*d^2*(9+2*p))*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-
b*x^2/a)/b/(9+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.25, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1652, 446, 77, 459, 365, 364}

$$\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{d\left(bd^2-9ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{3de^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{1}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (e^3*x^7*(a
 + b*x^2)^(1 + p))/(b*(9 + 2*p)) - (a*d*(2*b*d^2 - 9*a*e^2)*(a + b*x^2)^(2
+ p))/(2*b^4*(2 + p)) + (d*(b*d^2 - 9*a*e^2)*(a + b*x^2)^(3 + p))/(2*b^4*(3
 + p)) + (3*d*e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (e*(3*d^2 - (7*a*e
^2)/(9*b + 2*b*p))*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x
^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^5\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^6\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\mathrm{Subst}\left(\int x^2(a+bx)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\left(e\left(3d^2\right.\right.$$

$$=\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\mathrm{Subst}\left(\int\left(-\frac{a^2d\left(-bd^2+3ae^2\right)(a+bx)^p}{b^3}+\frac{ad\left(-2bd^2+\right.}{}\right.\right.$$

$$=\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right.}{2b^4(2+p)}$$

Mathematica [A] time = 0.22, size = 249, normalized size = 1.01

$$\frac{1}{126}\left(a+bx^2\right)^p\left(\frac{63d^3\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{189de^2\left(a+bx^2\right)\left(-6a^3+6a^2b(p\right.}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((63*d^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (189*d*e^2*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (54*d^2*e*x^7*Hyp
ergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (14*e^3*x^9
*Hypergeometric2F1[9/2, -p, 11/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/126
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^3x^8+3\,de^2x^7+3\,d^2ex^6+d^3x^5\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^5, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^5 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(p^2+3\,p+2\right)b^3x^6+\left(p^2+p\right)ab^2x^4-2\,a^2bpx^2+2\,a^3\right)\left(bx^2+a\right)^p d^3}{2\left(p^3+6\,p^2+11\,p+6\right)b^3}+\int \left(e^3x^8+3\,de^2x^7+3\,d^2ex^6\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^3*x^8 + 3*
d*e^2*x^7 + 3*d^2*e*x^6)*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^5*(a + b*x^2)^p*(d + e*x)^3, x)
```

sympy [C] time = 74.61, size = 3082, normalized size = 12.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] 3*a**p*d**2*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + a
**p*e**3*x**9*hyper((9/2, -p), (11/2,), b*x**2*exp_polar(I*pi)/a)/9 + d**3*
Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*
a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b)
 + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sq
```

```
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a
**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*
x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x*
*4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x
)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) +
 x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b*
*3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)*
*p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
 b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x
**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*
b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3
), True)) + 3*d*e**2*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt
(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*
b**7*x**6) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b*
*5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*
b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*
sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7
*x**6) + 18*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**
2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4
 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(
-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**
4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b*
*4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(1
2*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x
**6*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*
b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a*
*3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (
-6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6
*x**4) - 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 +
 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**
2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 12*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b*
*5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b*
*6*x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b
**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**
2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5
*x**2 + 4*b**6*x**4), Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*
a*b**4 + 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b
**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*
sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(
1/b) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2)
 + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sq
rt(1/b) + x)/(2*b**4) - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x
**2/(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x*
*2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) +
 6*a**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2
 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p*
*4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x*
*4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p
+ 48*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 +
 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/
(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b*
*3*p*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*
b**4*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p
**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)*
```

```
*p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*
b**4*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 10
0*b**4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p*
*3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

3.400 $\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=249

$$\frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{3ae\left(2bd^2-ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{3e\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^3\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}-\frac{d\dots}{}$$

```
[Out] 1/2*a^2*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(1+p)/b^4/(1+p)+3*d*e^2*x^5*(b*x^2+a)^
(1+p)/b/(7+2*p)-3/2*a*e*(-a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+3/2*e*(-
a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*e^3*(b*x^2+a)^(4+p)/b^4/(4+p)-1/
5*d*(15*a*e^2-b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x
^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.24, antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1652, 459, 365, 364, 446, 77}

$$\frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{3ae\left(2bd^2-ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{3e\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^3\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{1}{5}\dots$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*d*e^2*x^
5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2)*(a + b*x^2)
^(2 + p))/(2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2)*(a + b*x^2)^(3 + p))/(2*b^
4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (d*(d^2 - (15*a*e^
2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^4\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^5\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=\frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\left(d\left(d^2-\right.\right.$$

$$=\frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^2e\left(-3bd^2+ae^2\right)(a+bx)^p}{b^3}+\frac{3ae\left(-2bd^2+\right.}{}\right.\right.$$

$$=\frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}-\frac{3ae\left(2bd^2-ae^2\right)\left(a+bx^2\right)}{2b^4(2+p)}$$

Mathematica [A] time = 0.20, size = 249, normalized size = 1.00

$$\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{105d^2e\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{35e^3\left(a+bx^2\right)\left(-6a^3+6a^2b(p+1)\right.}{b^4(p}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((105*d^2*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2
+ 3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (35*e^3*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyper
geometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (30*d*e^2*x^7
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(e^3x^7+3\,de^2x^6+3\,d^2ex^5+d^3x^4\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^4 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
[Out] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(b\,x^2+a\right)^p (d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x)^3, x)
```

sympy [C] time = 57.34, size = 3082, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)
[Out] a**p*d**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*a**
p*d*e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 3*d**2
*e*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/
b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1
/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) +
 x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)
*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-
2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(
I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b*
```

```
*4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2
) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2
*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2
) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3
+ 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2
*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**
2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(
a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*
p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b
**3), True)) + e**3*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(
a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b
**7*x**6) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*s
qrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*
x**6) + 18*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4
+ 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-
I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4
 + 12*b**7*x**6) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**
4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12
*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x*
*6*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b
**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**
3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-
6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*
x**4) - 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 +
4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2
*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6
*x**4) - 12*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**
5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b*
*5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2
*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*
x**2 + 4*b**6*x**4), Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a
*b**4 + 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b*
*5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*s
qrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1
/b) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2)
+ b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqr
t(1/b) + x)/(2*b**4) - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x*
*2/(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**
2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) +
6*a**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2
+ 100*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**
4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**
4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p +
 48*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 +
70*b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(
2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**
3*p*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b
**4*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p*
*3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**
p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b
**4*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100
*b**4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**
```

```
3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

3.401 $\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=207

$$-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{3de^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}-\frac{ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(5ae^2\right.}{5b(}$$

```
[Out] -1/2*a*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^3/(1+p)+e^3*x^5*(b*x^2+a)^(1+p)
/b/(7+2*p)+1/2*d*(-6*a*e^2+b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+3/2*d*e^2*(b*x^
2+a)^(3+p)/b^3/(3+p)-1/5*e*(5*a*e^2-3*b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hyperg
eom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.20, antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1652, 446, 77, 459, 365, 364}

$$-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{3de^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (e^3*x^5*(a
+ b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p))
/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*(3*d^
2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7
/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^3\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^4\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$
$$=\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\left(e\left(3d^2-\right.\right.$$
$$=\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{ad\left(-bd^2+3ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^3-6ade^2\right)(}{b^2}\right.\right.$$
$$=-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)^2}{2b^3(2+p)}$$

Mathematica [A] time = 0.15, size = 196, normalized size = 0.95

$$\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{105de^2\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{35d^3\left(a+bx^2\right)\left(b(p+1)x^2-a\right)}{b^2(p+1)(p+2)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d^3*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
 p)) + (105*d*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p
^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (42*d^2*e*x^5*Hypergeometric2F1[5
/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (10*e^3*x^7*Hypergeometric2
F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^6+3\,de^2x^5+3\,d^2ex^4+d^3x^3\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^3\left(bx^2+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^3 \left(b x^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2\left(p+1\right)x^4+abpx^2-a^2\right)\left(bx^2+a\right)^p d^3}{2\left(p^2+3\,p+2\right)b^2}+\int \left(e^3x^6+3\,de^2x^5+3\,d^2ex^4\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^3/((p^2 + 3*p + 2)*
b^2) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(b\,x^2+a\right)^p \left(d+e\,x\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^3*(a + b*x^2)^p*(d + e*x)^3, x)
```

sympy [C] time = 42.20, size = 1421, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] 3*a**p*d**2*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a
**p*e**3*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**3*P
iecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**
2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
+ a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b*
*2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*
x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqr
t(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)*
*p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p
**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**
2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2)
, True)) + 3*d*e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(
I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a*
*2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*
```

```
sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log
(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a
*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sq
rt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x
**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x*
*2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2
/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*
b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*s
qrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3)
 - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**
3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(
a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*
p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
 b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 1
2*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b*
*3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
 22*b**3*p + 12*b**3), True))
```

3.402 $\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=210

$$-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{e\left(3bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^3\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}-\frac{dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(9ae^2-\right.}{3b(2p}$$

```
[Out] -1/2*a*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(1+p)/b^3/(1+p)+3*d*e^2*x^3*(b*x^2+a)^(
1+p)/b/(5+2*p)+1/2*e*(-2*a*e^2+3*b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e^3*(
b*x^2+a)^(3+p)/b^3/(3+p)-1/3*d*(9*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hype
rgeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.20, antiderivative size = 202, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1652, 459, 365, 364, 446, 77}

$$-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{e\left(3bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^3\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (3*d*e^2*x^3
*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^(2
 + p))/(2*b^3*(2 + p)) + (e^3*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*(d^
2 - (9*a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5
/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^2\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^3\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$
$$=\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\left(d\left(d^2\right.\right.$$
$$=\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{ae\left(-3bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{\left(3bd^2e-2ae\right.}{}\right.\right.$$
$$=-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{e\left(3bd^2-2ae^2\right)\left(a+bx\right.}{2b^3(2+p)}$$

Mathematica [A] time = 0.14, size = 196, normalized size = 0.93

$$\frac{1}{30}\left(a+bx^2\right)^p\left(\frac{15e^3\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{45d^2e\left(a+bx^2\right)\left(b(p+1)x^2-a\right)}{b^2(p+1)(p+2)}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((45*d^2*e*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2
 + p)) + (15*e^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^
2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (10*d^3*x^3*Hypergeometric2F1[3/2,
 -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (18*d*e^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/30
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^3\left(bx^2+a\right)^px^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x^2 \left(b\,x^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(b\,x^2+a\right)^p (d+e\,x)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^2*(a + b*x^2)^p*(d + e*x)^3, x)
```

sympy [C] time = 29.71, size = 1421, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 3*a**
p*d*e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*d**2
*e*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a
*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x*
*2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*
a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b
**3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I
*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x*
*2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b
**2), True)) + e**3*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(
I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a*
*2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*
sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log
(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a
*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sq
```

```
rt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x
**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x*
*2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2
/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*
b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*s
qrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3)
 - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**
3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(
a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*
p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
 b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 1
2*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b*
*3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
 22*b**3*p + 12*b**3), True))
```

3.403 $\int x(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=167

$$\frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{3de^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-bd^2(2p+5)\right)\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\cdots\right.}{b(2p+5)}$$

```
[Out] 1/2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^2/(1+p)+e^3*x^3*(b*x^2+a)^(1+p)/b/
(5+2*p)+3/2*d*e^2*(b*x^2+a)^(2+p)/b^2/(2+p)-e*(a*e^2-b*d^2*(5+2*p))*x^3*(b*
x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.15, antiderivative size = 159, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1652, 444, 43, 459, 365, 364}

$$\frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{3de^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+5b}\right)\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b
*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)
) + (e*(d^2 - (a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^2\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int(a+bx)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\left(3e\left(d^2-\frac{}{5b}\right.\right.$$

$$=\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(bd^3-3ade^2\right)(a+bx)^p}{b}+\frac{3de^2(a+bx)^{1+p}}{b}\right)d\right.$$

$$=\frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{3de^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+e\left(d^2-\frac{}{5}\right.$$

Mathematica [A] time = 0.23, size = 228, normalized size = 1.37

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(5d\left(-3a^2e^2\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)+b^2x^2\left(\frac{bx^2}{a}+1\right)^p\left(d^2(p+2)+3e^2(p+1)x^2\right)+ab\left(d^2(p+2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(5*d*(b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + 3*e^2*(1 + p)
*x^2) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(3*e^2*p*x^2*(1 + (b*x^2)/
a)^p + d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p))) + 10*b^2*d^2*e*(2 + 3*p + p^2
)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 2*b^2*e^3*(2 + 3*p +
p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)]))/(10*b^2*(1 + p)*(2
 + p)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(ex+d)^3\left(bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^3 x \left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
[Out] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bx^2+a\right)^{p+1} d^3}{2\,b\left(p+1\right)} + \int \left(e^3x^4+3\,de^2x^3+3\,d^2ex^2\right)\left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + a)^(p + 1)*d^3/(b*(p + 1)) + integrate((e^3*x^4 + 3*d*e^2*x^3
+ 3*d^2*e*x^2)*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(bx^2+a\right)^p (d+ex)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x*(a + b*x^2)^p*(d + e*x)^3, x)
```

sympy [A] time = 22.80, size = 471, normalized size = 2.82

$$a^p d^2 e x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + d^3 \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\begin{cases} \frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + 3de^2 \left(\begin{cases} \frac{a^p}{4} \ldots \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(b*x**2+a)**p,x)
[Out] a**p*d**2*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + a**p*
e**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**3*Piece
wise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne
(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + 3*d*e**2*Piecewise((a**p
*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**
2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2
+ 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x*
```

```
*2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -
2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b)
 + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**
2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p
 + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2)
+ b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

3.404 $\int (d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=176

$$-\frac{e\left(a+bx^2\right)^{p+1}\left(ae^2-3bd^2(p+2)\right)}{2b^2(p+1)(p+2)}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2-bd^2(2p+3)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}+\frac{3de^2x\left(\right.}{b(2}$$

[Out] -1/2*e*(a*e^2-3*b*d^2*(2+p))*(b*x^2+a)^(1+p)/b^2/(1+p)/(2+p)+3*d*e^2*x*(b*x^2+a)^(1+p)/b/(3+2*p)+1/2*e^3*x^2*(b*x^2+a)^(1+p)/b/(2+p)-d*(3*a*e^2-b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1+b*x^2/a)^p)

Rubi [A] time = 0.15, antiderivative size = 169, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {743, 780, 246, 245}

$$-\frac{e\left(a+bx^2\right)^{p+1}\left((2p+3)\left(ae^2-bd^2(2p+5)\right)-2bde(p+1)(p+3)x\right)}{2b^2(p+2)\left(2p^2+5p+3\right)}+dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+3b}\right)$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3*(a + b*x^2)^p,x]

[Out] (e*(d + e*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - (e*((3 + 2*p)*(a*e^2 - b*d^2*(5 + 2*p)) - 2*b*d*e*(1 + p)*(3 + p)*x)*(a + b*x^2)^(1 + p))/(2*b^2*(2 + p)*(3 + 5*p + 2*p^2)) + (d*(d^2 - (3*a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m - 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} + \frac{\int (d+ex)\left(-2\left(ae^2-bd^2(2+p)\right)+2bde(3+p)x\right)\left(a+b\right.}{2b(2+p)}$$

$$= \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e\left((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x\right)\left(\right.}{2b^2(2+p)\left(3+5p+2p^2\right)}$$

$$= \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e\left((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x\right)\left(\right.}{2b^2(2+p)\left(3+5p+2p^2\right)}$$

$$= \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e\left((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x\right)\left(\right.}{2b^2(2+p)\left(3+5p+2p^2\right)}$$

Mathematica [A] time = 0.21, size = 223, normalized size = 1.27

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(e\left(-a^2e^2\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)+b^2x^2\left(\frac{bx^2}{a}+1\right)^p\left(3d^2(p+2)+e^2(p+1)x^2\right)+2b^2de\left(p^2+3p\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(2*b^2*d^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*x^2)/a)] + e*(b^2*x^2*(1 + (b*x^2)/a)^p*(3*d^2*(2 + p) + e^2*(1 + p)*
x^2) - a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ 3*d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*d*e*(2 + 3*p + p^2)*x^3*H
ypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(2*b^2*(1 + p)*(2 + p)*(1 +
 (b*x^2)/a)^p)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(b\,x^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p,x)
[Out] int((e*x+d)^3*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^p\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(d + e*x)^3,x)
[Out] int((a + b*x^2)^p*(d + e*x)^3, x)
```

sympy [C] time = 16.26, size = 468, normalized size = 2.66

$$a^p d^3 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + a^p d e^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + 3d^2 e \left(\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + e^3 \left(\begin{cases} \frac{a^p}{4} \ldots \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p,x)
[Out] a**p*d**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*e**
2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*d**2*e*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**3*Piecewise((a**p*x**4
/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) +
a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b
**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) +
 b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)),
(-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)
/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*
b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**
2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

3.405 $\int \frac{(d+ex)^3(a+bx^2)^p}{x} \, dx$

Optimal. Leaf size=171

$$-\frac{d^3\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-3bd^2(2p+3)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b}{}\right.}{b(2p+3)}$$

```
[Out] 3/2*d*e^2*(b*x^2+a)^(1+p)/b/(1+p)+e^3*x*(b*x^2+a)^(1+p)/b/(3+2*p)-e*(a*e^2-
3*b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p
)/((1+b*x^2/a)^p)-1/2*d^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/
a)/a/(1+p)
```

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1652, 446, 80, 65, 388, 246, 245}

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{ae^2}{2bp+3b}\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d^3\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right.}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] (3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(
b*(3 + 2*p)) + (e*(3*d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^3*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x}\,dx &= \int \frac{\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)}{x}\,dx+\int \left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\ &= \frac{e^3x\left(a+bx^2\right)^{1+p}}{b(3+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^p\left(d^3+3de^2x\right)}{x}\,dx,x,x^2\right)+\left(e\left(3d^2-\frac{ae^2}{3b+2\right.\right.\\ &= \frac{3de^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{e^3x\left(a+bx^2\right)^{1+p}}{b(3+2p)}+\frac{1}{2}d^3\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\left(e\left(3d^2\right.\right.\\ &= \frac{3de^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{e^3x\left(a+bx^2\right)^{1+p}}{b(3+2p)}+e\left(3d^2-\frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\end{aligned}$$

Mathematica [A] time = 0.13, size = 170, normalized size = 0.99

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(-3bd^3\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)+18abd^2e(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\right.\right.}{6ab(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] ((a + b*x^2)^p*(18*a*b*d^2*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)] - 3*b*d^3*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*x^2)/a] + a*e^2*(9*d*(a + b*x^2)*(1 + (b*x^2)/a)^p + 2*b*e
*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(6*a*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="fricas")
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3(bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(b\,x^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x,x)
[Out] int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3(bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,(d+e\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x, x)
```

sympy [A] time = 17.62, size = 144, normalized size = 0.84

$$3a^pd^2ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{a^pe^3x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}-\frac{b^pd^3x^{2p}\Gamma\left(-p\right)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}+3de^2\left(\left\{\begin{matrix}\frac{a^px^2}{2}\\ \frac{\begin{cases}\frac{\left(a+bx^2\right)^{p+1}}{p+1}\\ \log\left(a+bx^2\right)\end{cases}}{2b}\end{matrix}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x,x)
```

```
[Out] 3*a**p*d**2*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e
**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - b**p*d**3*x
**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*
gamma(1 - p)) + 3*d*e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a
+ b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), Tru
e))
```

3.406 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx$

Optimal. Leaf size=159

$$-\frac{d^3\left(a+bx^2\right)^{p+1}}{ax}+\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(\right.}{2a(p}$$

```
[Out] 1/2*e^3*(b*x^2+a)^(1+p)/b/(1+p)-d^3*(b*x^2+a)^(1+p)/a/x+d*(3*a*e^2+b*d^2*(1
+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-
3/2*d^2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A] time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1807, 1652, 446, 80, 65, 12, 246, 245}

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}\right.}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] (e^3*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (d^3*(a + b*x^2)^(1 + p))/(a*x) +
 (d*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (3*d^2*e*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^2}\,dx &= -\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}-\frac{\int \frac{\left(a+bx^2\right)^p\left(-3ad^2e-d\left(3ae^2+bd^2(1+2p)\right)x-ae^3x^2\right)}{x}\,dx}{a}\\
&= -\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}+\frac{\int d\left(3ae^2+bd^2(1+2p)\right)\left(a+bx^2\right)^p\,dx}{a}-\frac{\int \frac{\left(a+bx^2\right)^p\left(-3ad^2e-ae^3x^2\right)}{x}}{a}\\
&= -\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}-\frac{\text{Subst}\left(\int \frac{(a+bx)^p\left(-3ad^2e-ae^3x\right)}{x}\,dx,x,x^2\right)}{2a}+\frac{\left(d\left(3ae^2+bd^2(1+2p)\right)\right.}{a}\\
&= \frac{e^3\left(a+bx^2\right)^{1+p}}{2b(1+p)}-\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}+\frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\frac{\left(d\left(3a\right.\right.}{}\\
&= \frac{e^3\left(a+bx^2\right)^{1+p}}{2b(1+p)}-\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}+\frac{d\left(3ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}}{a}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 154, normalized size = 0.97

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ex\left(\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\left(ae^2-3bd^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)\right)+6abde(p+1)x\,{}_2F_1\left(\frac{1}{2},\right.\right.\right.}{2ab(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*a*b*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^
2)/a)] + e*x*(6*a*b*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
```

```
/a)] + (a + b*x^2)*(1 + (b*x^2)/a)^p*(a*e^2 - 3*b*d^2*Hypergeometric2F1[1,
1 + p, 2 + p, 1 + (b*x^2)/a]))))/(2*a*b*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}\right)\left(bx^{2}+a\right)^{p}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^{3}\left(bx^{2}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^{3}\left(b\,x^{2}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^{3}\left(bx^{2}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^{2}+a\right)^{p}\,(d+e\,x)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x^2,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x^2, x)
```

sympy [A] time = 14.91, size = 143, normalized size = 0.90

$$-\frac{a^{p}d^{3}{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{x}+3a^{p}de^{2}x{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)-\frac{3b^{p}d^{2}ex^{2p}\Gamma\left(-p\right){}_{2}F_{1}\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma\left(1-p\right)}+e^{3}\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{f}\\ \frac{\begin{cases}\frac{\left(a+bx^{2}\right)^{p+1}}{p+1} & \text{f}\\ \log\left(a+bx^{2}\right) & \text{o}\end{cases}}{2b} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**3*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + 3*a**p*d
*e**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - 3*b**p*d**2*e*
x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2
*gamma(1 - p)) + e**3*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b
*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

3.407 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=168

$$-\frac{d\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2p\right)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^3\left(a+bx^2\right)^{p+1}}{2ax^2}+\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+\right.}{}$$

```
[Out] -1/2*d^3*(b*x^2+a)^(1+p)/a/x^2-3*d^2*e*(b*x^2+a)^(1+p)/a/x+e*(a*e^2+3*b*d^2
*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^
p)-1/2*d*(b*d^2*p+3*a*e^2)*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2
/a)/a^2/(1+p)
```

Rubi [A] time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 764, 266, 65, 246, 245}

$$-\frac{d\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2p\right)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}+\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+3bd^2(2p+1)\right)\,{}_2F}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -(d^3*(a + b*x^2)^(1 + p))/(2*a*x^2) - (3*d^2*e*(a + b*x^2)^(1 + p))/(a*x)
+ (e*(a*e^2 + 3*b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (d*(3*a*e^2 + b*d^2*p)*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*(1 +
 p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^3}\,dx = -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{\int \frac{\left(a+bx^2\right)^p\left(-6ad^2e-2d\left(3ae^2+bd^2p\right)x-2ae^3x^2\right)}{x^2}\,dx}{2a}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{\int \frac{\left(2ad\left(3ae^2+bd^2p\right)+2ae\left(ae^2+3bd^2(1+2p)\right)x\right)\left(a+bx^2\right)^p}{x}\,d}{2a^2}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{\left(d\left(3ae^2+bd^2p\right)\right)\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{a} + \frac{\left(e\left(ae^2+\right.\right.}{}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{\left(d\left(3ae^2+bd^2p\right)\right)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{e\left(ae^2+3bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right.}{a}$$

Mathematica [A] time = 0.12, size = 174, normalized size = 1.04

$$-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(x\left(d\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\left(3ae^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)-bd^2\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}\right.\right.\right.\right.}{2a^2(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -1/2*((a + b*x^2)^p*(6*a^2*d^2*e*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -
((b*x^2)/a)] + x*(-2*a^2*e^3*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)] + d*(a + b*x^2)*(1 + (b*x^2)/a)^p*(3*a*e^2*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x^2)/a]))))/(a^2*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (bx^2+a)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (b\,x^2+a)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (bx^2+a)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x^3,x)
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x^3, x)
```

sympy [C] time = 21.01, size = 150, normalized size = 0.89

$$-\frac{3a^p d^2 e\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + a^p e^3 x\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d^3 x^{2p}\Gamma(1-p)\, {}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma(2-p)} - \frac{3b^p de^2 x^{2p}\Gamma(-p)\, {}_2 \cdots}{2\Gamma(1 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**3,x)
[Out] -3*a**p*d**2*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p
*e**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**3*x**(
2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*x**2*gamma(2 - p)) - 3*b**p*d*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1
 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

3.408 $\int \frac{x^4(a+bx^2)^p}{d+ex} dx$

Optimal. Leaf size=199

$$\frac{x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}+\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2e^3(p+1)}+\frac{\left(a+bx^2\right)^{p+2}}{2b^2e(p+2)}-\frac{d^4\left(a+bx^2\right)^{p+1} {}_2F_1}{2e^3(p+}$$

```
[Out] 1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^2/e^3/(1+p)+1/2*(b*x^2+a)^(2+p)/b^2/e/
(2+p)+1/5*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,7/2,e^2*x^2/d^2,-b*x^2/a)/d/((1
+b*x^2/a)^p)-1/2*d^4*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)
/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)/(1+p)
```

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = $\{959, 511, 510, 446, 88, 68\}$

$$\frac{x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}+\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2e^3(p+1)}+\frac{\left(a+bx^2\right)^{p+2}}{2b^2e(p+2)}-\frac{d^4\left(a+bx^2\right)^{p+1} {}_2F_1}{2e^3(p+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*e^3*(1 + p)) + (a + b*x^2)^(2
+ p)/(2*b^2*e*(2 + p)) + (x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*
x^2)/a), (e^2*x^2)/d^2])/(5*d*(1 + (b*x^2)/a)^p) - (d^4*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
e^3*(b*d^2 + a*e^2)*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^2\right)^p}{d+ex}\,dx = d\int \frac{x^4 \left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^5 \left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{x^2(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)\right) + \left(d\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4 \left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{x^5 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\,\text{Subst}\left(\int \left(\frac{\left(-bd^2+ae^2\right)(a+b}{be^4}\right.\right.$$

$$= \frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2e^3(1+p)} + \frac{\left(a+bx^2\right)^{2+p}}{2b^2e(2+p)} + \frac{x^5 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a}\right.}{5d}$$

$$= \frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2e^3(1+p)} + \frac{\left(a+bx^2\right)^{2+p}}{2b^2e(2+p)} + \frac{x^5 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a}\right.}{5d}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(a+bx^2\right)^p}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x),x]
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x), x]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^4}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

[Out] integral((b*x^2 + a)^p*x^4/(e*x + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^4 (b\,x^2+a)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^2+a)^p/(e*x+d),x)

[Out] int(x^4*(b*x^2+a)^p/(e*x+d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^p x^4}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 (b\,x^2+a)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(a + b*x^2)^p)/(d + e*x),x)

[Out] int((x^4*(a + b*x^2)^p)/(d + e*x), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b*x**2+a)**p/(e*x+d),x)

[Out] Timed out

3.409 $\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=163

$$-\frac{ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d^3\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2e^2(p+1)\left(ae^2+bd^2\right)}-\frac{d\left(a+b\right.}{2be^2(p}$$

```
[Out] -1/2*d*(b*x^2+a)^(1+p)/b/e^2/(1+p)-1/5*e*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,
7/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/a)^p)+1/2*d^3*(b*x^2+a)^(1+p)*hyper
geom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)/(1+p)
```

Rubi [A] time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {959, 446, 80, 68, 511, 510}

$$-\frac{ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d^3\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2e^2(p+1)\left(ae^2+bd^2\right)}-\frac{d\left(a+b\right.}{2be^2(p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -(d*(a + b*x^2)^(1 + p))/(2*b*e^2*(1 + p)) - (e*x^5*(a + b*x^2)^p*AppellF1[
5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1 + (b*x^2)/a)^p) +
(d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2
))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^p}{d+ex}\,dx = d\int \frac{x^3\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^4\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= -\frac{d\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} - \frac{ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3\,\text{Subst}\left(\int \frac{(a+bx}{d^2-e^2}\cdots\right.}{2e^2}$$

$$= -\frac{d\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} - \frac{ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3\left(a+bx^2\right)^{1+p}\cdots}{2e^2\left(b\cdots\right.}$$

Mathematica [A] time = 0.33, size = 260, normalized size = 1.60

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(\frac{bx^2}{a}+1\right)^{-p}\left(6bd^2(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(2be(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)-3d\left(bx^2\left(\frac{bx^2}{a}+1\right)^p+a\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)\right)\right)\right)}{b(p+1)} - \frac{3d^3\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\cdots\right.}{\cdots}\right)}{6e^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*((-3*d^3*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(
d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + (e*(6*b*d^2*(1 + p)*x*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + e*(-3*d*(b*x^2*(1 + (b*x^2)/
a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + 2*b*e*(1 + p)*x^3*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])))/(b*(1 + p)*(1 + (b*x^2)/a)^p)))/(6*e^4)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^3}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^3/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(b\,x^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d),x)
[Out] int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(b\,x^2+a\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

3.410 $\int \frac{x^2(a+bx^2)^p}{d+ex} dx$

Optimal. Leaf size=161

$$\frac{x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)\left(ae^2+bd^2\right)}+\frac{\left(a+bx^2\right)^{p+1}}{2be(p+1)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/e/(1+p)+1/3*x^3*(b*x^2+a)^p*AppellF1(3/2,1,-p,5/2,e^2
*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(1+p)*hypergeom([1,
1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)/(1+p)
```

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {959, 511, 510, 446, 80, 68}

$$\frac{x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)\left(ae^2+bd^2\right)}+\frac{\left(a+bx^2\right)^{p+1}}{2be(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e*(1 + p)) + (x^3*(a + b*x^2)^p*AppellF1[3/2, -p,
1, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d*(1 + (b*x^2)/a)^p) - (d^2*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*e*(b*d^2 + a*e^2)*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^p}{d+ex}\,dx = d\int \frac{x^2\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^3\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx$$
$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx,x,x^2\right)\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$
$$= \frac{\left(a+bx^2\right)^{1+p}}{2be(1+p)}+\frac{x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2\,\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\right.}{2e}$$
$$= \frac{\left(a+bx^2\right)^{1+p}}{2be(1+p)}+\frac{x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2\left(a+bx^2\right)^{1+p}\,{}_2F}{2e\left(bd\right.}$$

Mathematica [A] time = 0.24, size = 227, normalized size = 1.41

$$\frac{\left(a+bx^2\right)^p\left(bd^2(p+1)\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)-2bdep(p+1)x\left(\right.\right.}{2be^3p(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(a*e^2*p + b*e^2*p*x^2 - (a*e^2*p)/(1 + (b*x^2)/a)^p + (b*d^
2*(1 + p)*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (
d + Sqrt[-(a/b)]*e)/(d + e*x)])/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*
(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*b*d*e*p*(1 + p)*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*b*e^3*p*(1 + p))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^2}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b\,x^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d),x)
[Out] int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(b\,x^2+a\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^2\right)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d),x)
[Out] Integral(x**2*(a + b*x**2)**p/(d + e*x), x)
```

3.411 $\int \frac{x\left(a+bx^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=173

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e}+\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)}{}$$

```
[Out] -x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/((1+b*x^2/a)^p
)+x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/((1+b*x^2/a)^p)+1/2*d
*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e
^2+b*d^2)/(1+p)
```

Rubi [A] time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e}+\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(e*(1 + (b*x^2)/a)^p)) + (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -
((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p) + (d*(a + b*x^2)^(1 + p)*Hypergeometric
2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)
*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx^2\right)^p}{d+ex}\,dx &= \frac{\int \left(a+bx^2\right)^p\,dx}{e} - \frac{d\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e}\\
&= -\frac{d\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e} + \frac{\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx}{e}\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - d\int \frac{x\left(a+bx^2\right)^p}{-d^2+e^2x^2}\,dx - \frac{d^2\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx}{e}\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right) - \frac{\left(d^2\left(a+\right.\right.}{}\\
&= -\frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} + \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{e}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 172, normalized size = 0.99

$$
\frac{\left(a+bx^2\right)^p\left(2ex\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}\right)}{2e^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(-((d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(
d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d +
 e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*e*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^2)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\left(b\,x^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(b\,x^2+a\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^p)/(d + e*x),x)
```

```
[Out] int((x*(a + b*x^2)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^2\right)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d),x)
[Out] Integral(x*(a + b*x**2)**p/(d + e*x), x)
```

3.412 $\int \frac{(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=125

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)
-1/2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2)
)/(a*e^2+b*d^2)/(1+p)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {757, 430, 429, 444, 68}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx &= \int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx\\ &= d\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx+e\int \frac{x\left(a+bx^2\right)^p}{-d^2+e^2x^2}\,dx\\ &= \frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\ &= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx}{bd^2+ae}\right.}{2\left(bd^2+ae^2\right)(1+p)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 131, normalized size = 1.05

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{2ep}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*
x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{p}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(d + e*x),x)
```

```
[Out] int((a + b*x^2)^p/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{p}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Integral((a + b*x**2)**p/(d + e*x), x)
```

3.413 $\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$

Optimal. Leaf size=176

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2d(p+1)\left(ae^2+bd^2\right)}-\frac{\left(a+bx^2\right)^{p+1}}{}$$

```
[Out] -e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/
a)^p)+1/2*e^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2
+b*d^2))/d/(a*e^2+b*d^2)/(1+p)-1/2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p]
,1+b*x^2/a)/a/d/(1+p)
```

Rubi [A] time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {959, 446, 86, 65, 68, 430, 429}

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2d(p+1)\left(ae^2+bd^2\right)}-\frac{\left(a+bx^2\right)^{p+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^2*(1 + (b*x^2)/a)^p)) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1,
1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d*(b*d^2 + a*e^2)*(1 +
 p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/
a])/(2*a*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^p}{x(d+ex)}\,dx &= d\int \frac{\left(a+bx^2\right)^p}{x\left(d^2-e^2x^2\right)}\,dx - e\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx\\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x\left(d^2-e^2x\right)}\,dx,x,x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&= -\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2d} + \frac{e^2\,\text{Sub}}{}\\
&= -\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\right.}{2d\left(bd^2+ae^2\right)(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 170, normalized size = 0.97

$$\frac{\left(a+bx^2\right)^p\left(\left(\frac{a}{bx^2}+1\right)^{-p}\,{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)-\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\right.\right.}{2dp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p
, -(a/(b*x^2))]/(1 + a/(b*x^2))^p))/(2*d*p)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^p}{(ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2 + a\right)^p}{(ex + d)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^p}{(ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^p}{x\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x*(d + e*x)),x)
```

```
[Out] int((a + b*x^2)^p/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^p}{x\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d),x)
```

```
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)), x)
```

3.414 $\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$

Optimal. Leaf size=178

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{e^3(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)}+\frac{e(a+bx^2)}{}$$

```
[Out] -(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d^2,-b*x^2/a)/d/x/((1+b*x^2/a)^
p)-1/2*e^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*
d^2))/d^2/(a*e^2+b*d^2)/(1+p)+1/2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p
],1+b*x^2/a)/a/d^2/(1+p)
```

Rubi [A] time = 0.17, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {959, 511, 510, 446, 86, 65, 68}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{e^3(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)}+\frac{e(a+bx^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -(((a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d*x*(1 + (b*x^2)/a)^p)) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 +
p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)
/a])/(2*a*d^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{x^2(d+ex)}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^2\left(d^2-e^2x^2\right)}\,dx - e\int \frac{\left(a+bx^2\right)^p}{x\left(d^2-e^2x^2\right)}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x\left(d^2-e^2x\right)}\,dx, x, x^2\right)\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2\left(d^2-e^2x^2\right)}\,dx$$

$$= -\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e^3\,\text{Subst}}{\cdots}$$

$$= -\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e^3\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a\cdots}{bd^2\cdots}\right)}{2d^2\left(bd^2+ae^2\right)(1+p)}$$

Mathematica [A] time = 0.35, size = 214, normalized size = 1.20

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} - \frac{2d\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(\frac{a}{bx^2}+1\right)^{-p}{}_2F_1\left(-p,\cdots\right)}{p}\cdots\right)}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*((e*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*d*Hypergeometric2F1[-1/2,
 -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -
p, 1 - p, -(a/(b*x^2))])/(p*(1 + a/(b*x^2))^p)))/(2*d^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{(ex+d)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d),x)
[Out] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^p}{x^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d),x)
[Out] Integral((a + b*x**2)**p/(x**2*(d + e*x)), x)
```

3.415 $\int \frac{(a+bx^2)^p}{x^3(d+ex)} dx$

Optimal. Leaf size=213

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)}+\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^4}{}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)/a/d/x^2+e*(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d
^2,-b*x^2/a)/d^2/x/((1+b*x^2/a)^p)+1/2*e^4*(b*x^2+a)^(1+p)*hypergeom([1, 1+
p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(1+p)-1/2*(b*d^2*p+
a*e^2)*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a^2/d^3/(1+p)
```

Rubi [A] time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {959, 446, 103, 156, 65, 68, 511, 510}

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)}+\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*d*x^2) + (e*(a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1
/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^2*x*(1 + (b*x^2)/a)^p) + (e^4*(a + b*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a
*e^2)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^(1
 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*d^3*(1 + p)
)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{x^3(d+ex)}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^3\left(d^2-e^2x^2\right)}\,dx - e\int \frac{\left(a+bx^2\right)^p}{x^2\left(d^2-e^2x^2\right)}\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^2\left(d^2-e^2x\right)}\,dx, x, x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2\left(d^2-e^2x^2\right)}\,dx$$

$$= -\frac{\left(a+bx^2\right)^{1+p}}{2adx^2} + \frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(-a\cdots}{x(\cdots}\right.}{2\cdots}$$

$$= -\frac{\left(a+bx^2\right)^{1+p}}{2adx^2} + \frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4\,\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\cdots\right.}{2d^3}$$

$$= -\frac{\left(a+bx^2\right)^{1+p}}{2adx^2} + \frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4\left(a+bx^2\right)^{1+p}\,{}_2F_1\cdots}{2d^3\left(bd^2\cdots\right.}$$

Mathematica [A] time = 0.28, size = 256, normalized size = 1.20

$$\frac{\left(a+bx^2\right)^p\left(-\frac{e^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}+\left(\frac{a}{bx^2}+1\right)^{-p}\left(\frac{d^2\,{}_2F_1\left(1-p,-p;2-p;-\frac{a}{bx^2}\right)}{(p-1)x^2}+\frac{e^2\,{}_2F_1(-p\cdots}{\cdots}\right.\right.}{2d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-((e^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)
/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d
 + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*d*e*Hypergeometric2F
1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) + ((d^2*Hypergeometri
c2F1[1 - p, -p, 2 - p, -(a/(b*x^2))])/((-1 + p)*x^2) + (e^2*Hypergeometric2
F1[-p, -p, 1 - p, -(a/(b*x^2))])/p)/(1 + a/(b*x^2))^p))/(2*d^3)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^p}{(ex+d)\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^3/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x^3/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^3*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**3/(e*x+d),x)
[Out] Timed out
```

3.416 $\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=392

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4(2p+3)\left(ae^2+bd^2\right)}-\frac{2d^2x\left(a+bx^2\right)^p\left(\right.}{}$$

```
[Out] -d*(4+3*p)*(b*x^2+a)^(1+p)/b/e^3/(1+p)/(3+2*p)-d^4*(b*x^2+a)^(1+p)/e^3/(a*e
^2+b*d^2)/(e*x+d)+(e*x+d)*(b*x^2+a)^(1+p)/b/e^3/(3+2*p)-2*d^2*(2*a*e^2+b*d^
2*(2+p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e
^2+b*d^2)/((1+b*x^2/a)^p)-(a^2*e^4-2*a*b*d^2*e^2*(4+3*p)-2*b^2*d^4*(2*p^2+7
*p+6))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/e^4/(a*e^2+b*d^2
)/(3+2*p)/((1+b*x^2/a)^p)+d^3*(2*a*e^2+b*d^2*(2+p))*(b*x^2+a)^(1+p)*hyperge
om([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A] time = 0.89, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1651, 1654, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4(2p+3)\left(ae^2+bd^2\right)}-\frac{2d^2x\left(a+bx^2\right)^p\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d*(4 + 3*p)*(a + b*x^2)^(1 + p))/(b*e^3*(1 + p)*(3 + 2*p))) - (d^4*(a +
b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*(a + b*x^2)^(1
 + p))/(b*e^3*(3 + 2*p)) - (2*d^2*(2*a*e^2 + b*d^2*(2 + p))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^
2)*(1 + (b*x^2)/a)^p) - ((a^2*e^4 - 2*a*b*d^2*e^2*(4 + 3*p) - 2*b^2*d^4*(6
+ 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b*e^4*(b*d^2 + a*e^2)*(3 + 2*p)*(1 + (b*x^2)/a)^p) + (d^3*(2*a*e^2 + b
*d^2*(2 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(e^3*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^2\right)^p}{(d+ex)^2} \, dx = -\frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(a+bx^2\right)^p \left(\frac{ad^3}{e^2}-\frac{d^2\left(ae^2+2bd^2(1+p)\right)x}{e^3}+d\left(a+\frac{bd^2}{e^2}\right)x^2-\frac{\left(bd^2+ae^2\right)x^3}{e}\right)}{d+ex} \, dx}{bd^2+ae^2}$$

$$= -\frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\int \frac{\left(a+bx^2\right)^p \left(ade\left(ae^2+2bd^2(2+p)\right)+\left(a^2e^4-4b^2d^4(1+p\right.\right.}{d+ex}}{be^3\left(bd^2+ae^2\right)(3}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\int \left(2abde^3(1+p)(a\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(2d^3\left(2ae^2+b\right.\right.}{e^4}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(2d^3\left(2ae^2+b\right.\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(a^2e^4-2abd^2\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(a^2e^4-2abd^2\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4 \left(a+bx^2\right)^{1+p}}{e^3 \left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{2d^2\left(2ae^2+bd\right.}{}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(a+bx^2\right)^p}{(d+ex)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2, x]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^4}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

maple **[F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(b\,x^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\left(b\,x^2+a\right)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x)^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

3.417 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=321

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+3)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)}-\frac{d^2\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2(2p+3)\right)\,{}_2F_1}{2e^2(p+1)\left(ae^2+\right.}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/e^2/(1+p)+d^3*(b*x^2+a)^(1+p)/e^2/(a*e^2+b*d^2)/(e*x+
d)+d*(3*a*e^2+b*d^2*(3+2*p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^
2,-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/a)^p)-d*(2*a*e^2+b*d^2*(3+2*p))*x*(
b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/
a)^p)-1/2*d^2*(3*a*e^2+b*d^2*(3+2*p))*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2
+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A] time = 0.55, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1651, 1654, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+3)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2\right.}{e^3\left(ae^2+bd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^2*(1 + p)) + (d^3*(a + b*x^2)^(1 + p))/(e^2*(b*d
^2 + a*e^2)*(d + e*x)) + (d*(3*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*App
ellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2 + a*e^2)*(
1 + (b*x^2)/a)^p) - (d*(2*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)
^p) - (d^2*(3*a*e^2 + b*d^2*(3 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^
2)^2*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3 \left(a+bx^2\right)^p}{(d+ex)^2}\,dx = \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(a+bx^2\right)^p \left(-\frac{ad^2}{e}+d\left(a+\frac{2bd^2(1+p)}{e^2}\right)x-\frac{\left(bd^2+ae^2\right)x^2}{e}\right)}{d+ex}\,dx}{bd^2+ae^2}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(-2abd^2e(1+p)+2bd(1+p)\left(2ae^2+bd^2(3+2p)\right)x\right)\left(a+bx^2\right)^p}{d+ex}\,dx}{2be^2\left(bd^2+ae^2\right)(1+p)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} - \frac{\left(d\left(2ae^2+bd^2(3+2p)\right)\right)\int \left(a+bx^2\right)^p\,dx}{e^3\left(bd^2+ae^2\right)} + \frac{\left(d^2 \left(\right.\right.}{}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} + \frac{\left(d^2\left(3ae^2+bd^2(3+2p)\right)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right.}{e^3\left(bd^2+ae^2\right)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} - \frac{d\left(2ae^2+bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F}{e^3\left(bd^2+ae^2\right)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} - \frac{d\left(2ae^2+bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F}{e^3\left(bd^2+ae^2\right)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3 \left(a+bx^2\right)^{1+p}}{e^2 \left(bd^2+ae^2\right)(d+ex)} + \frac{d\left(3ae^2+bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1}{e^3\left(bd^2+ae^2\right)}$$

Mathematica [A] time = 0.69, size = 343, normalized size = 1.07

$$\frac{\left(a+bx^2\right)^p\left(-\frac{2d^3\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)}+\frac{3d^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p,\right.}{p}\right.}{2e^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((-2*d^3*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a
/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (
3*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(S
qrt[-(a/b)] + x))/(d + e*x))^p) + e*((e*(a + b*x^2 - a/(1 + (b*x^2)/a)^p))/
(b + b*p) - (4*d*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p)))/(2*e^4)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^p x^3}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(b\,x^2+a\right)^p}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(b\,x^2+a\right)^p}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

3.418 $\int \frac{x^2\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=281

$$-\frac{2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+2bd^2(p+\right.}{e^2\left(ae^2+bd^2\right)}$$

```
[Out] -d^2*(b*x^2+a)^(1+p)/e/(a*e^2+b*d^2)/(e*x+d)-2*(a*e^2+b*d^2*(1+p))*x*(b*x^2
+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x
^2/a)^p)+(a*e^2+2*b*d^2*(1+p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x
^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+d*(a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+
p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)^2/
(1+p)
```

Rubi [A] time = 0.33, antiderivative size = 277, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1651, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a+\frac{2bd^2(p+1)}{e^2}\right)\,2}{ae^2+bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x))) - (2*(a*e^2 + b*
d^2*(1 + p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e^2*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) + ((a + (2*b*d^2*(1 + p))
/e^2)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/((b*d^
2 + a*e^2)*(1 + (b*x^2)/a)^p) + (d*(a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 +
 p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/
(e*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^p}{(d+ex)^2}\,dx = -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(ad-\frac{(ae^2+2bd^2(1+p))x}{e}\right)\left(a+bx^2\right)^p}{d+ex}\,dx}{bd^2+ae^2}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e^2\left(bd^2+ae^2\right)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)\int \left(a+bx^2\right.}{bd^2+ae^2}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2} + \frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e^2\left(bd^2+ae^2\right)} + \frac{\left(\left(a+\frac{2bd^2}{e}\right.\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2} - \frac{(2a}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2} - \frac{(d}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{2\left(ae^2+bd^2(1+p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\right.}{e^2\left(bd^2+ae^2\right)}$$

Mathematica [A] time = 0.33, size = 300, normalized size = 1.07

$$\frac{\left(a+bx^2\right)^p\left(\frac{d^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)} - \frac{d\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-}}{d+e}\right.}{p}\right.}{e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) + (e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(1 + (b*x^2)/a)^p))/e^3
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^2}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b\,x^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(b\,x^2+a\right)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

3.419 $\int \frac{x(a+bx^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=273

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\left(ae^2+bd^2\right)}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},\right.}{e\left(ae^2+bd^2\right)}$$

```
[Out] d*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(e*x+d)+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/e/(a*e^2+b*d^2)/((1+b*x^2/a)
^p)-b*d*(1+2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/(a*e^2+
b*d^2)/((1+b*x^2/a)^p)-1/2*(a*e^2+b*d^2*(1+2*p))*(b*x^2+a)^(1+p)*hypergeom(
[1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A] time = 0.23, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {835, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\left(ae^2+bd^2\right)}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},\right.}{e\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x)) + ((a*e^2 + b*d^2*(1 +
2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2
])/(d*e*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*x*(a + b*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)*(1 + (b*
x^2)/a)^p) - ((a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^
2*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^p}{(d+ex)^2}\,dx = \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{(-ae+bd(1+2p)x)\left(a+bx^2\right)^p}{d+ex}\,dx}{bd^2+ae^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{(bd(1+2p))\int \left(a+bx^2\right)^p\,dx}{e\left(bd^2+ae^2\right)} + \frac{\left(ae^2+bd^2(1+2p)\right)\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e\left(bd^2+ae^2\right)}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(ae^2+bd^2(1+2p)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2} + \frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e\left(bd^2+ae^2\right)} - \frac{\left(bd(1+2p)\left(a+b\right.\right.}{}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{bd(1+2p)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e\left(bd^2+ae^2\right)} + \frac{\left(ae^2+bd^2\right.}{}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{bd(1+2p)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e\left(bd^2+ae^2\right)} + \frac{\left(ae^2+bd^2\right.}{}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right.}{de\left(bd^2+ae^2\right)}$$

Mathematica [A] time = 0.18, size = 223, normalized size = 0.82

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left((2p-1)(d+ex)F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)-2dpF_1\left(1-2p\right.\right.}{2e^2p(2p-1)(d+ex)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*d*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + (-1 + 2*p)*(d + e*x)*Appel
lF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)
]*e)/(d + e*x)]))/(2*e^2*p*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\left(b\,x^{2}+a\right)^{p}}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{p}x}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(b\,x^{2}+a\right)^{p}}{\left(d+e\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x*(a + b*x^2)^p)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^{2}\right)^{p}}{\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x*(a + b*x**2)**p/(d + e*x)**2, x)
```

3.420 $\int \frac{(a+bx^2)^p}{(d+ex)^2} \, dx$

Optimal. Leaf size=244

$$-\frac{2bpx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{ae^2+bd^2}+\frac{b(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{ae^2+bd^2}-$$

```
[Out] e^2*x*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)-2*b*p*x*(b*x^2+a)^p*Appe
llF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+b*(1+
2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/(a*e^2+b*d^2)/((1+b*
x^2/a)^p)-b*d*e*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e
^2+b*d^2))/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A] time = 0.18, antiderivative size = 191, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {757, 430, 429, 444, 68, 511, 510}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4}-\frac{bde\left(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
^2*(1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -(
(b*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (b*x^2)/a)^p) - (b*d*e*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^p}{(d+ex)^2}\,dx &= \int \left(\frac{d^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}-\frac{2dex\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}+\frac{e^2x^2\left(a+bx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\right)dx\\
&= d^2\int \frac{\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx-(2de)\int \frac{x\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx+e^2\int \frac{x^2\left(a+bx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\,dx\\
&= -\left((de)\,\text{Subst}\left(\int \frac{(a+bx)^p}{\left(d^2-e^2x\right)^2}\,dx,x,x^2\right)\right)+\left(d^2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,\right.}{3d^4}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 141, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{e(2p-1)(d+ex)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(e*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d)^2,x)
[Out] int((b*x^2+a)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(d + e*x)^2,x)
[Out] int((a + b*x^2)^p/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d)**2,x)
[Out] Integral((a + b*x**2)**p/(d + e*x)**2, x)
```

3.421 $\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{e^3x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^5}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-$$

```
[Out] -e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/
a)^p)-e*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b
*x^2/a)^p)-1/3*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2
/a)/d^5/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)/(1+p)-1/2*(b*x^2+a)^(1+p)*hype
rgeom([1, 1+p],[2+p],1+b*x^2/a)/a/d^2/(1+p)+b*e^2*(b*x^2+a)^(1+p)*hypergeom
([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A] time = 0.43, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 266, 65, 757, 430, 429, 444, 68, 511, 510}

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^3*(1 + (b*x^2)/a)^p)) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -
((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)
^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^5*(1 + (b*x
^2)/a)^p) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^2*(1
 + p)) + (b*e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{x(d+ex)^2}\,dx = \int \left(\frac{\left(a+bx^2\right)^p}{d^2x} - \frac{e\left(a+bx^2\right)^p}{d(d+ex)^2} - \frac{e\left(a+bx^2\right)^p}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{d^2} - \frac{e\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{d^2} - \frac{e\int \frac{\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2} + \frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{d^2} - \frac{e\int \left(\frac{d^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2} - \frac{2dex\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2} +\right.}{d}$$

$$= -\frac{\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} - \frac{e\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx}{d} - (de)\int \frac{\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx + \left(2\right.$$

$$= -\frac{\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} + e^2\,\text{Subst}\left(\int \frac{(a+bx)^p}{\left(d^2-e^2x\right)^2}\,dx, x, x^2\right) - \frac{e^2\,\text{Subs}}{}$$

$$= -\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,\right.}{d^3}$$

Mathematica [A] time = 0.35, size = 303, normalized size = 0.82

$$\frac{\left(a+bx^2\right)^p\left(\frac{\left(\frac{a}{bx^2}+1\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)-\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} - \frac{2d\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e(\sqrt{}}{}\right.}{}\right.}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (-(
AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-
(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b
)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1
 + a/(b*x^2))^p)/p))/(2*d^2)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^2x^3+2\,dex^2+d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{\left(ex+d\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d)^2,x)
[Out] int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{p}}{\left(ex+d\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{x\,\left(d+e\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x*(d + e*x)^2),x)
[Out] int((a + b*x^2)^p/(x*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{p}}{x\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**2,x)
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)**2), x)
```

3.422 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=421

$$\frac{e^4x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}+\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\cdots$$

```
[Out] 2*e^2*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x
^2/a)^p)+e^2*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/
((1+b*x^2/a)^p)+1/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-
b*x^2/a)/d^6/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/
a)/d^2/x/((1+b*x^2/a)^p)-e^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(
b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(1+p)+e*(b*x^2+a)^(1+p)*hypergeom
([1, 1+p],[2+p],1+b*x^2/a)/a/d^3/(1+p)-b*e^3*(b*x^2+a)^(1+p)*hypergeom([2,
1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A] time = 0.45, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {961, 365, 364, 266, 65, 757, 430, 429, 444, 68, 511, 510}

$$\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^
2])/(d^4*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x
^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1 + (
b*x^2)/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(d^2*x*(1 + (b*x^2)/a)^p) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1
, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^3*(b*d^2 + a*e^2)*(1
 + p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x
^2)/a])/(a*d^3*(1 + p)) - (b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d*(b*d^2 + a*e^2)^2*(1 +
p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
```

gerQ[p] && ILtQ[m, 0]

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}dx = \int \left(\frac{(a+bx^2)^p}{d^2x^2} - \frac{2e(a+bx^2)^p}{d^3x} + \frac{e^2(a+bx^2)^p}{d^2(d+ex)^2} + \frac{2e^2(a+bx^2)^p}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x^2}dx}{d^2} - \frac{(2e)\int \frac{(a+bx^2)^p}{x}dx}{d^3} + \frac{(2e^2)\int \frac{(a+bx^2)^p}{d+ex}dx}{d^3} + \frac{e^2\int \frac{(a+bx^2)^p}{(d+ex)^2}dx}{d^2}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}dx, x, x^2\right)}{d^3} + \frac{(2e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^3} + \frac{e^2\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2de}{(d}\right.}{}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} + \frac{e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} + \frac{e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)}$$

$$= \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-\right.}{d^4}$$

Mathematica [A] time = 0.46, size = 342, normalized size = 0.81

$$\frac{(a+bx^2)^p\left(\frac{de\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)} + \frac{e\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\dots\right)}{p}\right.}{d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (e*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) - (d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))
])/(p*(1 + a/(b*x^2))^p)))/d^3
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^2x^4+2\,dex^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^p}{(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^p}{(ex+d)^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^p}{(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(b\,x^2+a\right)^p}{x^2\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)^2),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**2,x)
[Out] Timed out
```

3.423 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=449

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(ae^2+bd^2\right)^2}-\frac{dx\left(a+\right.}{}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/e^3/(1+p)-1/2*d^4*(b*x^2+a)^(1+p)/e^3/(a*e^2+b*d^2)/(
e*x+d)^2+d^3*(4*a*e^2+b*d^2*(3+p))*(b*x^2+a)^(1+p)/e^3/(a*e^2+b*d^2)^2/(e*x
+d)+d*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/
a)^p)-d*(3*a^2*e^4+2*a*b*d^2*e^2*(5+4*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a)
^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-
1/2*d^2*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*(b*x^2+a)^(
1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2
)^3/(1+p)
```

Rubi [A] time = 0.94, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1651, 1654, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(ae^2+bd^2\right)^2}-\frac{dx\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^3*(1 + p)) - (d^4*(a + b*x^2)^(1 + p))/(2*e^3*(b
*d^2 + a*e^2)*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^(1
+ p))/(e^3*(b*d^2 + a*e^2)^2*(d + e*x)) + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4
+ 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/
2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p)
- (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^4*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2
*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^3*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = -\frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{\left(a+bx^2\right)^p\left(\frac{2ad^3}{e^2}-\frac{2d^2\left(ae^2+bd^2(1+p)\right)x}{e^3}+2d\left(a+\frac{bd^2}{e^2}\right)x^2-2\left(\frac{bd^2}{e}+ae\right)x^3\right)}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= -\frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \int \frac{\left(a+bx^2\right)^p\left(2ad^2\left(3a+\frac{bd^2}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \int \frac{\left(4ab\cdots\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(d^2\left(6\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(d^2\left(6\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} - \frac{d\left(3a^2\cdots\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} - \frac{d\left(3a^2\cdots\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{d\left(6a^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.93, size = 462, normalized size = 1.03

$$\frac{\left(a+bx^2\right)^p\left(\frac{d^4\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(p-1)(d+ex)^2} - \frac{8d^3\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2\cdots\right.}{(2p-1)(d+ex)}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((a*e^2)/(b + b*p) + (e^2*x^2)/(1 + p) - (8*d^3*AppellF1[1 -
 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)
/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(
a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d^4*AppellF1[2 - 2*p, -p, -p, 3 - 2*
p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 +
p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))
```

```
^p*(d + e*x)^2) + (6*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/
(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (6*d*e*x*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^5)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^4}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^4}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(b\,x^2+a\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^4}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\left(b\,x^2+a\right)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.424 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=416

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)^2}+\frac{d\left(a+bx^2\right)^p}{}$$

```
[Out] 1/2*d^3*(b*x^2+a)^(1+p)/e^2/(a*e^2+b*d^2)/(e*x+d)^2-d^2*(3*a*e^2+b*d^2*(2+p
))*(b*x^2+a)^(1+p)/e^2/(a*e^2+b*d^2)^2/(e*x+d)-(3*a^2*e^4+a*b*d^2*e^2*(6+7*
p)+b^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-
b*x^2/a)/e^3/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+(a^2*e^4+a*b*d^2*e^2*(5+6*p)+b
^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3
/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*d*(3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d
^4*(2*p^2+5*p+3))*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a
*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^3/(1+p)
```

Rubi [A] time = 0.59, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1651, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)^2}+\frac{x\left(a+bx^2\right)^p}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^3*(a + b*x^2)^(1 + p))/(2*e^2*(b*d^2 + a*e^2)*(d + e*x)^2) - (d^2*(3*a*e
^2 + b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)^2*(d + e*x))
- ((3*a^2*e^4 + a*b*d^2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*x*(a + b
*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2
 + a*e^2)^2*(1 + (b*x^2)/a)^p) + ((a^2*e^4 + a*b*d^2*e^2*(5 + 6*p) + b^2*d^
4*(3 + 5*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x
^2)/a)])/(e^3*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (d*(3*a^2*e^4 + a*b*d^
2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeome
tric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2
+ a*e^2)^3*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{\left(a+bx^2\right)^p\left(-\frac{2ad^2}{e}+2d\left(a+\frac{bd^2(1+p)}{e^2}\right)x-2\left(\frac{bd^2}{e}+ae\right)x^2\right)}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(-\frac{2ad\left(2ae^2+bd^2(1+p)\right)}{e}+\frac{2\left(a^2e^4+\right.}{}\right.}{}}{2\left(\right.}$$

$$= \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(5+6p)+\right.}{e}$$

$$= \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(d\left(3a^2e^4+abd^2e^2(6+7\right.\right.}{}$$

$$= \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(5+6p)+\right.}{}$$

$$= \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(5+6p)+\right.}{}$$

$$= \frac{d^3\left(a+bx^2\right)^{1+p}}{2e^2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(3a^2e^4+abd^2e^2(6+7p)\right.}{}$$

Mathematica [A] time = 0.74, size = 436, normalized size = 1.05

$$\frac{\left(a+bx^2\right)^p\left(-\frac{d^3\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(p-1)(d+ex)^2}+\frac{6d^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p,\right.}{(2p-1)(d+ex)}\right.}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((6*d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d
^3*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) - (3*d*AppellF1[-2*p, -p
, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*
x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
e*x))^p) + (2*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/(2*e^4)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^p x^3}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^3}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(b\,x^2+a\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p x^3}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(b\,x^2+a\right)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.425 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=396

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2\left(ae^2+bd^2\right)^2}-\frac{\left(a+bx^2\right)^{p+1}(a}{}$$

```
[Out] -1/2*d^2*(b*x^2+a)^(1+p)/e/(a*e^2+b*d^2)/(e*x+d)^2+d*(2*a*e^2+b*d^2*(1+p))*
(b*x^2+a)^(1+p)/e/(a*e^2+b*d^2)^2/(e*x+d)+(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*
d^4*(2*p^2+3*p+1))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a
)/d/e^2/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-b*d*(1+2*p)*(2*a*e^2+b*d^2*(1+p))*x
*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^2/(a*e^2+b*d^2)^2/((1+b*
x^2/a)^p)-1/2*(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*d^4*(2*p^2+3*p+1))*(b*x^2+a)
^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2
)^3/(1+p)
```

Rubi [A] time = 0.57, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1651, 835, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2\left(ae^2+bd^2\right)^2}-\frac{\left(a+bx^2\right)^{p+1}(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -(d^2*(a + b*x^2)^(1 + p))/(2*e*(b*d^2 + a*e^2)*(d + e*x)^2) + (d*(2*a*e^2
+ b*d^2*(1 + p))*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)^2*(d + e*x)) + ((a
^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 + 3*p + 2*p^2))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*e^2*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*(2*a*e^2 + b*d^2*(1 + p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^2*(b*d^2 + a*e^
2)^2*(1 + (b*x^2)/a)^p) - ((a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 +
3*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(2*e*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{\left(2ad-\frac{2\left(ae^2+bd^2(1+p)\right)x}{e}\right)\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(2a\left(ae^2+bd^2p\right)-\frac{2bd(1+2p)(2ae^2}{e}\right.}{d+ex}}{2\left(bd^2+a\right.}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(bd(1+2p)\left(2ae^2+bd^2(1\right.\right.}{e^2\left(bd^2\right.}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(2+5p)+\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)} - \frac{bd(1+2p)\left(2ae^2+bd^2(1\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)} - \frac{bd(1+2p)\left(2ae^2+bd^2(1\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(2+5p)+\right.}{}$$

Mathematica [A] time = 0.34, size = 290, normalized size = 0.73

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{d^2F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(p-1)(d+ex)^2} - \frac{4dF_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)} + F_1\right.}{2e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*(d + e*x)) + (d^
2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + S
qrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*(d + e*x)^2) + AppellF1[-2*p, -p, -p,
1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/p)
)/(2*e^3*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
 e*x))^p)
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p x^2}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^p x^2}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(b\,x^2 + a\right)^p}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^p x^2}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(b\,x^2 + a\right)^p}{(d + e\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.426 $\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=336

$$-\frac{bpx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e\left(ae^2+bd^2\right)^2}+\frac{b(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae\right.}{e\left(ae^2+bd\right.}$$

```
[Out] 1/2*d*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(e*x+d)^2-(b*d^2*p+a*e^2)*(b*x^2+a)^(1+
p)/(a*e^2+b*d^2)^2/(e*x+d)-b*p*(3*a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*Appell
F1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+b*(
1+2*p)*(b*d^2*p+a*e^2)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/
(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*b*d*p*(3*a*e^2+b*d^2*(1+2*p))*(b*x^2+a)
^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^
3/(1+p)
```

Rubi [A] time = 0.40, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {835, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{bpx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e\left(ae^2+bd^2\right)^2}+\frac{b(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae\right.}{e\left(ae^2+bd\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p
)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^
2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b
*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(
b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d
^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{(-2ae+2bdpx)\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(2abde(1-p)+2b(1+2p)\left(ae^2+bd^2p\right)x\right)\left(a+bx^2\right)^p}{d+ex}}{2\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(b(1+2p)\left(ae^2+bd^2p\right)\right)\int\left(a+bx^2\right)^p}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(bdp\left(3ae^2+bd^2(1+2p)\right)\right)\int\left(\frac{d\left(a+bx^2\right.}{d^2-e^2x^2}\right.}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{b(1+2p)\left(ae^2+bd^2p\right)x\left(a+bx^2\right)^p(1}{e\left(bd^2+ae\right.}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{b(1+2p)\left(ae^2+bd^2p\right)x\left(a+bx^2\right)^p(1}{e\left(bd^2+ae\right.}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} - \frac{bp\left(3ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p(}{e\left(bd^2+\right.}$$

Mathematica [A] time = 0.21, size = 229, normalized size = 0.68

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(2(p-1)(d+ex)F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)+d(1-2p)F_1\right.}{2e^2(p-1)(2p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*(2*(-1 + p)*(d + e*x)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
- Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + d*(1 - 2*p)*
AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqr
t[-(a/b)]*e)/(d + e*x)]))/(2*e^2*(-1 + p)*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x
))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

maple **[F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x\,(b\,x^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^3,x)
[Out] int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x*(a + b*x^2)^p)/(d + e*x)^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

3.427 $\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=322

$$\frac{e^2x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{3b^2d^2e}{}$$

```
[Out] -1/4*d^2*e*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+x*(b*x^2+a)^p*App
ellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/a)^p)+e^2*x^3*(b*x^2
+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+1/4*b
*e*(2*a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^
2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)-3/2*b^2*d^2*e*(b*x^2+a)^(1+p)*hyp
ergeom([3, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)
```

Rubi [A] time = 0.33, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {757, 430, 429, 444, 68, 511, 510, 446, 78}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{3b^2d^2e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -(d^2*e*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(a
+ b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1
+ (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x^2
)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (b*e*(2*a*e^2 + b*d^2*(1 +
p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2)
)/(b*d^2 + a*e^2)])/(4*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*d^2*e*(a + b*x^2
)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e
^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = \int \left(\frac{d^3\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^3} - \frac{3d^2ex\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^3} + \frac{3de^2x^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^3} + \frac{e^3x^3\left(a+bx^2\right)^p}{\left(-d^2+e^2x^2\right)^3}\right) dx$$

$$= d^3\int \frac{\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx - \left(3d^2e\right)\int \frac{x\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx + \left(3de^2\right)\int \frac{x^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx + e^3\int \frac{x^3\left(a+\right.}{\left(-d^2+\right.}$$

$$= -\left(\frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{(a+bx)^p}{\left(d^2-e^2x\right)^3}\,dx, x, x^2\right)\right) + \frac{1}{2}e^3\,\text{Subst}\left(\int \frac{x(a+bx)^p}{\left(-d^2+e^2x\right)^3}\,dx, x, x^2\right) + \left(d^3\left(a\right.\right.$$

$$= -\frac{d^2e\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2} + \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3\left(a+\right.}{}$$

$$= -\frac{d^2e\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2} + \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3\left(a+\right.}{}$$

Mathematica [A] time = 0.11, size = 142, normalized size = 0.44

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2e(p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*(-1 + p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{\left(ex+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d)^3,x)
[Out] int((b*x^2+a)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{p}}{\left(ex+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{\left(d+e\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(d + e*x)^3,x)
[Out] int((a + b*x^2)^p/(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

3.428 $\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$

Optimal. Leaf size=700

$$-\frac{e^3x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}-\frac{e^3x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}-\frac{ex}{}$$

```
[Out] 1/4*d*e^2*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2-e*x*(b*x^2+a)^p*Ap
pellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x^2+a)
^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x
^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-1/3
*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^6/((1+b*
x^2/a)^p)-e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d
^6/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*
x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(1+p)-1/2*(b*x^2+a)^(1+p)*hypergeom
([1, 1+p],[2+p],1+b*x^2/a)/a/d^3/(1+p)+b*e^2*(b*x^2+a)^(1+p)*hypergeom([2,
1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(1+p)-1/4*b*e^2*(
2*a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^2+a)
/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^3/(1+p)+3/2*b^2*d*e^2*(b*x^2+a)^(1+p)*hyper
geom([3, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)
```

Rubi [A] time = 0.82, antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {961, 266, 65, 757, 430, 429, 444, 68, 511, 510, 446, 78}

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (d*e^2*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) - (e*x*(a
 + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1
 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a
), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/
2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e^3
*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(3*d^6*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^6*(1 + (b*x^2)/a)^p) + (e^2*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^3*(1 + p)) + (b*e^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)
])/(d*(b*d^2 + a*e^2)^2*(1 + p)) - (b*e^2*(2*a*e^2 + b*d^2*(1 + p))*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(4*d*(b*d^2 + a*e^2)^3*(1 + p)) + (3*b^2*d*e^2*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
```

```
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{x(d+ex)^3}\,dx=\int\left(\frac{\left(a+bx^2\right)^p}{d^3x}-\frac{e\left(a+bx^2\right)^p}{d(d+ex)^3}-\frac{e\left(a+bx^2\right)^p}{d^2(d+ex)^2}-\frac{e\left(a+bx^2\right)^p}{d^3(d+ex)}\right)dx$$

$$=\frac{\int\frac{\left(a+bx^2\right)^p}{x}\,dx}{d^3}-\frac{e\int\frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{d^3}-\frac{e\int\frac{\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{d^2}-\frac{e\int\frac{\left(a+bx^2\right)^p}{(d+ex)^3}\,dx}{d}$$

$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2d^3}-\frac{e\int\left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{d^3}-\frac{e\int\left(\frac{d^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}-\frac{2dex\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}+\frac{e^2}{(}\cdots\right.}{d^2}$$

$$=-\frac{\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}-e\int\frac{\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx-\frac{e\int\frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx}{d^2}-\left(d^2e\right)\int\cdots$$

$$=-\frac{\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}-\frac{e^2\,\text{Subst}\left(\int\frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2d^3}+\frac{e^2\,\text{Subst}\left(\int\frac{(a+\cdots}{(d^2-\cdots}\right.}{d\cdots}$$

$$=\frac{de^2\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}-\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\cdots\right.}{\cdots}$$

$$=\frac{de^2\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}-\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.72, size = 434, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^p\left(-\frac{d^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(p-1)(d+ex)^2}+\frac{\left(\frac{a}{bx^2}+1\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)-\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\right.}{}\right.}{2d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d^
2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + S
qrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*
((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (-(AppellF1[-2*p, -p, -
p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]
/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^
p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1 + a/(b*x^2))^p)/p))
/(2*d^3)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^p}{e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{(ex+d)^3\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x*(d + e*x)^3),x)
```

```
[Out] int((a + b*x^2)^p/(x*(d + e*x)^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.429 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$

Optimal. Leaf size=754

$$\frac{2e^4x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7}+\frac{e^4x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7}+$$

```
[Out] -1/4*e^3*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+3*e^2*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+2*e^2*x*(b
*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+e
^2*x*(b*x^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/
a)^p)+2/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d
^7/((1+b*x^2/a)^p)+e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b
*x^2/a)/d^7/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a
)/d^3/x/((1+b*x^2/a)^p)-3/2*e^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^4/(a*e^2+b*d^2)/(1+p)+3/2*e*(b*x^2+a)^(1+p)*hy
pergeom([1, 1+p],[2+p],1+b*x^2/a)/a/d^4/(1+p)-2*b*e^3*(b*x^2+a)^(1+p)*hyper
geom([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^2/(1+p)+
1/4*b*e^3*(2*a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^3/(1+p)-3/2*b^2*e^3*(b*x^2+a)^
(1+p)*hypergeom([3, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3
/(1+p)
```

Rubi [A] time = 0.86, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {961, 365, 364, 266, 65, 757, 430, 429, 444, 68, 511, 510, 446, 78}

$$\frac{3e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] -(e^3*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (3*e^2*x
*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5
*(1 + (b*x^2)/a)^p) + (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b
*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*Ap
pellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^
p) + (2*e^4*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*
x^2)/d^2])/(3*d^7*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x^2)^p*AppellF1[3/2,
 -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^7*(1 + (b*x^2)/a)^p) - ((a +
b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(d^3*x*(1 + (b*x^2
)/a)^p) - (3*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^4*(b*d^2 + a*e^2)*(1 + p)) + (3*e*(a
+ b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^
4*(1 + p)) - (2*b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)^2*(1 + p)) + (b*
e^3*(2*a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 +
p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(4*d^2*(b*d^2 + a*e^2)^3*(1 +
 p)) - (3*b^2*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e
^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^p}{x^2(d+ex)^3}\,dx = \int \left(\frac{\left(a+bx^2\right)^p}{d^3x^2} - \frac{3e\left(a+bx^2\right)^p}{d^4x} + \frac{e^2\left(a+bx^2\right)^p}{d^2(d+ex)^3} + \frac{2e^2\left(a+bx^2\right)^p}{d^3(d+ex)^2} + \frac{3e^2\left(a+bx^2\right)^p}{d^4(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx^2\right)^p}{x^2}dx}{d^3} - \frac{(3e)\int \frac{\left(a+bx^2\right)^p}{x}dx}{d^4} + \frac{\left(3e^2\right)\int \frac{\left(a+bx^2\right)^p}{d+ex}dx}{d^4} + \frac{\left(2e^2\right)\int \frac{\left(a+bx^2\right)^p}{(d+ex)^2}dx}{d^3} + \frac{e^2\int \frac{\left(a+bx^2\right)^p}{(d+ex)^3} d}{d^2}$$

$$= -\frac{(3e)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}dx, x, x^2\right)}{2d^4} + \frac{\left(3e^2\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2} + \frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{d^4} + \frac{\left(2e^2\right)\int \left(\frac{d^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2} - \right.}{}$$

$$= -\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} + \frac{3e\left(a+bx^2\right)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)}$$

$$= -\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} + \frac{3e\left(a+bx^2\right)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)}$$

$$= -\frac{e^3\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2} + \frac{3e^2x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{2e^2x\,(}{}$$

$$= -\frac{e^3\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2} + \frac{3e^2x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{2e^2x\,(}{}$$

Mathematica [A] time = 0.77, size = 478, normalized size = 0.63

$$\left(a+bx^2\right)^p\left(\frac{d^2e\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(p-1)(d+ex)^2} + \frac{4de\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\right.}{(2p-1)(d+ex)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((a + b*x^2)^p*((4*d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d
^2*e*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d
+ Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (3*e*AppellF1[-2*p,
-p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d +
e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d
+ e*x))^p) - (2*d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (
b*x^2)/a)^p) - (3*e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))])/(p*(1 +
 a/(b*x^2))^p)))/(2*d^4)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2 + a)^p}{(ex + d)^3\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2 + a)^p}{x^2\,(d + e\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)^3),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)
[Out] Timed out
```

3.430 $\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=276

$$-\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+2)-3cd^2(m+2p+4)\right)\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{cg^2(m+2)(m+2p+4)}-\frac{d(gx)^{m+1}\left(a+cx^2\right)^p}{}$$

[Out] 3*d*e^2*(g*x)^(1+m)*(c*x^2+a)^(1+p)/c/g/(3+m+2*p)+e^3*(g*x)^(2+m)*(c*x^2+a)^(1+p)/c/g^2/(4+m+2*p)-d*(3*a*e^2*(1+m)-c*d^2*(3+m+2*p))*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/g/(1+m)/(3+m+2*p)/((c*x^2/a+1)^p)-e*(a*e^2*(2+m)-3*c*d^2*(4+m+2*p))*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/c/g^2/(2+m)/(4+m+2*p)/((c*x^2/a+1)^p)

Rubi [A] time = 0.47, antiderivative size = 254, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1809, 808, 365, 364}

$$\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{3d^2}{m+2}-\frac{ae^2}{c(m+2p+4)}\right)\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2}+\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]

[Out] (3*d*e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + (e^3*(g*x)^(2 + m)*(a + c*x^2)^(1 + p))/(c*g^2*(4 + m + 2*p)) + (d*(d^2/(1 + m) - (3*a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (e*((3*d^2)/(2 + m) - (a*e^2)/(c*(4 + m + 2*p)))*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(1 + (c*x^2)/a)^p)

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q - 1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m + q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
```

```
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx = \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(a+cx^2\right)^p\left(cd^3(4+m+2p)-e\left(ae^2(2+\right.\right.}{c(4+}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(-cd(4+m+\right.}{}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2 - \frac{3ae^2(1+m)}{c(3+m+2p}\right.\right.$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2 - \frac{3ae^2(1+m)}{c(3+m+2p}\right.\right.$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)(gx}{}$$

Mathematica [A] time = 0.21, size = 182, normalized size = 0.66

$$x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{m+1} + ex\left(\frac{3d^2\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{m+2} + ex\left(\frac{3d\,{}_2F_1\left(\right.}{}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p, (4 +
m)/2, -((c*x^2)/a)])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2, -p, (
5 + m)/2, -((c*x^2)/a)])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2, -p, (6
 + m)/2, -((c*x^2)/a)])/(4 + m)))))/(1 + (c*x^2)/a)^p
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(cx^2+a\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + a)^p*(g*x)^m, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(cx^2+a\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(gx\right)^m \left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
[Out] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(cx^2+a\right)^p \left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g\,x\right)^m \left(c\,x^2+a\right)^p (d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3,x)
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(c*x**2+a)**p,x)
[Out] Timed out
```

3.431 $\int (gx)^m (d+ex)^2 \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=205

$$-\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+1)-cd^2(m+2p+3)\right)\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{cg(m+1)(m+2p+3)}+\frac{2de(gx)^{m+2}\left(a+cx\right.}{}$$

```
[Out] e^2*(g*x)^(1+m)*(c*x^2+a)^(1+p)/c/g/(3+m+2*p)-(a*e^2*(1+m)-c*d^2*(3+m+2*p))
*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/
g/(1+m)/(3+m+2*p)/((c*x^2/a+1)^p)+2*d*e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([
-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/g^2/(2+m)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.20, antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1809, 808, 365, 364}

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}-\frac{ae^2}{c(m+2p+3)}\right)\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g}+\frac{2de(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + ((d^2/(1 + m)
 - (a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1
[(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (2*d*e*(g
*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*
x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx=\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\int (gx)^m\left(-ae^2(1+m)+cd^2(3+m+2p)+2cde(3+\right.}{c(3+m+2p)}$$

$$=\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{(2de)\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g}+\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right.$$

$$=\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(2de\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^{1+m}\left(1+\frac{cx^2}{a}\right)^p\,dx}{g}$$

$$=\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\right.}{g(1+m)}$$

Mathematica [A] time = 0.10, size = 158, normalized size = 0.77

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{m+2}{2},-p;\right.\right.\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)/2,
-p, (3 + m)/2, -((c*x^2)/a)] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric2F1[(
2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)] + e*(2 + m)*x*Hypergeometric2F1[(3 +
 m)/2, -p, (5 + m)/2, -((c*x^2)/a)])))/((1 + m)*(2 + m)*(3 + m)*(1 + (c*x^2
)/a)^p)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(cx^2+a\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + a)^p*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(cx^2+a\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(gx\right)^m\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \left(cx^2+a\right)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,x)^m \left(c\,x^2+a\right)^p (d+e\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2, x)
```

sympy [C] time = 170.87, size = 172, normalized size = 0.84

$$\frac{a^p d^2 g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{a^p d e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{a^p e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\right.}{2\mathrm{I}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**2*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
 c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*d*e*g**m*x**2*x**m*g
amma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/a)/ga
mma(m/2 + 2) + a**p*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3
/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2))
```

3.432 $\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=135

$$\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}+\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx}{a}\right.}{g^2(m+2)}$$

```
[Out] d*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g
/(1+m)/((c*x^2/a+1)^p)+e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([-p, 1+1/2*m],[2
+1/2*m],-c*x^2/a)/g^2/(2+m)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {808, 365, 364}

$$\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}+\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx}{a}\right.}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) + (e*(g*x)^(2 + m)*(a + c*x^2)
^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*
(1 + (c*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = d\int (gx)^m\left(a+cx^2\right)^p\,dx + \frac{e\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g}$$

$$= \left(d\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx + \frac{\left(e\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right.}{}$$

$$= \frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(a+cx^2\right.}{}$$

Mathematica [A] time = 0.03, size = 106, normalized size = 0.79

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d(m+2)\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)+e(m+1)x\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3 + m
)/2, -((c*x^2)/a)] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2
, -((c*x^2)/a)]))/((1 + m)*(2 + m)*(1 + (c*x^2)/a)^p)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((ex+d)\left(cx^2+a\right)^p(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(cx^2+a\right)^p(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)\,(gx)^m\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)\left(cx^2+a\right)^p(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x),x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x), x)
```

sympy [C] time = 88.94, size = 109, normalized size = 0.81

$$\frac{a^p d g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{a^p e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(c*x**2+a)**p,x)
```

```
[Out] a**p*d*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), c*
x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*e*g**m*x**2*x**m*gamma(
m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/a)/(2*gamm
a(m/2 + 2))
```

3.433 $\int (gx)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=66

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}$$

```
[Out] (g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g/(
1+m)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(a + c*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m\left(a+cx^2\right)^p\,dx &= \left(\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx\\ &= \frac{(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.97

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;-\frac{cx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(c*x^2)/a)])/((1 + m)*(1 + (c*x^2)/a)^p)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2+a\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2+a\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\left(gx\right)^m\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p,x)
[Out] int((g*x)^m*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2+a\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(g\,x\right)^m\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p,x)
[Out] int((g*x)^m*(a + c*x^2)^p, x)
```

sympy [C] time = 22.60, size = 54, normalized size = 0.82

$$\frac{a^p g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p,x)
```

```
[Out] a**p*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), c*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

3.434 $\int \frac{(gx)^m(a+cx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=157

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)}-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,1;\frac{m+4}{2}\right.}{d^2(m+2)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d/(1+m)/((c*x^2/a+1)^p)-e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,1,-p,
2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^2/(2+m)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 511, 510}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)}-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,1;\frac{m+4}{2}\right.}{d^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d*(1 + m)*(1 + (c*x^2)/a)^p) - (e*x^2*(g*x)^m*(a + c*x^2
)^p*AppellF1[(2 + m)/2, -p, 1, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^
2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{(gx)^m \left(a+cx^2\right)^p}{d+ex} dx = \left(dx^{-m}(gx)^m\right) \int \frac{x^m \left(a+cx^2\right)^p}{d^2-e^2x^2} dx - \left(ex^{-m}(gx)^m\right) \int \frac{x^{1+m} \left(a+cx^2\right)^p}{d^2-e^2x^2} dx$$

$$= \left(dx^{-m}(gx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right) \int \frac{x^m \left(1+\frac{cx^2}{a}\right)^p}{d^2-e^2x^2} dx - \left(ex^{-m}(gx)^m \left(a+cx^2\right)^p \left(1\right.\right.$$

$$= \frac{x(gx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1+m}{2}; -p, 1; \frac{3+m}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d(1+m)} - \frac{ex^2(gx)^m \left(a+cx^2\right)^p \left(\right.}{}$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m \left(a+cx^2\right)^p}{d+ex} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x), x]
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p (gx)^m}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^p (gx)^m}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m \left(c\,x^2+a\right)^p}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^p (gx)^m}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(gx)^m (cx^2+a)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x),x)
```

```
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

3.435 $\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=238

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,2;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)}+\frac{e^2x^3(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+3}{2};-p,2;\right.}{d^4(m+3)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,2,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d^2/(1+m)/((c*x^2/a+1)^p)-2*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,2
,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^3/(2+m)/((c*x^2/a+1)^p)+e^2*x^3*(g*x)^m
*(c*x^2+a)^p*AppellF1(3/2+1/2*m,2,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(3
+m)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.28, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {962, 511, 510}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,2;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)}-\frac{2ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,2;\right.}{d^3(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^2*(1 + m)*(1 + (c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 2, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^3*(2 + m)*(1 + (c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF1[
(3 + m)/2, -p, 2, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^4*(3 + m)*(1
+ (c*x^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 962

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx = \left(x^{-m}(gx)^m\right)\int\left(\frac{d^2x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2} - \frac{2dex^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2} + \frac{e^2x^{2+m}\left(a+cx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\right)dx$$

$$= \left(d^2x^{-m}(gx)^m\right)\int \frac{x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx - \left(2dex^{-m}(gx)^m\right)\int \frac{x^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx + \left(e^2x^{-m}(gx\right.$$

$$= \left(d^2x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx - \left(2dex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1\right.\right.$$

$$= \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,2;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)} - \frac{2ex^2(gx)^m\left(a+cx^2\right)^p\left(1\right.}{}$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x]
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p\left(gx\right)^m}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^p\left(gx\right)^m}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(gx\right)^m\left(c\,x^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + a\right)^p \left(gx\right)^m}{\left(ex + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,x\right)^m\,\left(c\,x^2 + a\right)^p}{\left(d + e\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.436 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=321

$$-\frac{e^3x^4(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+4}{2};-p,3;\frac{m+6}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6(m+4)}+\frac{3e^2x^3(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+3}{2};-p,\right.}{d^5(m+3)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,3,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d^3/(1+m)/((c*x^2/a+1)^p)-3*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,3
,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(2+m)/((c*x^2/a+1)^p)+3*e^2*x^3*(g*x)
^m*(c*x^2+a)^p*AppellF1(3/2+1/2*m,3,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^5/
(3+m)/((c*x^2/a+1)^p)-e^3*x^4*(g*x)^m*(c*x^2+a)^p*AppellF1(2+1/2*m,3,-p,3+1
/2*m,e^2*x^2/d^2,-c*x^2/a)/d^6/(4+m)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.37, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {962, 511, 510}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,3;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+1)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,3;\frac{m+}{2}\right.}{d^4(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 3, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^3*(1 + m)*(1 + (c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 3, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^4*(2 + m)*(1 + (c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF
1[(3 + m)/2, -p, 3, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^5*(3 + m)*(
1 + (c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a + c*x^2)^p*AppellF1[(4 + m)/2, -p,
3, (6 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^6*(4 + m)*(1 + (c*x^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 962

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^3}\,dx=\left(x^{-m}(gx)^m\right)\int\left(\frac{d^3x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}-\frac{3d^2ex^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}+\frac{3de^2x^{2+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}+\frac{e^3}{}\right.$$

$$=\left(d^3x^{-m}(gx)^m\right)\int\frac{x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx-\left(3d^2ex^{-m}(gx)^m\right)\int\frac{x^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx+\left(3de^2x\right.$$

$$=\left(d^3x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx-\left(3d^2ex^{-m}(gx)^m\left(a+cx^2\right)\right.$$

$$=\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,3;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(1+m)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p}{}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int\frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x]
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(cx^2+a\right)^p(gx)^m}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2+a\right)^p(gx)^m}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{(gx)^m\left(c\,x^2+a\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^p \left(gx\right)^m}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,x\right)^m\,\left(c\,x^2+a\right)^p}{\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.437 $\int \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(-15a^3e^6-2cdex\left(-5a^2e^4-6acd^2e^2+35c^2d^4\right)-17a^2cd^2e^4-25ac^2d^4e^2+105c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{192c^3d^3e^4}$$

```
[Out] 1/128*(-a*e^2+c*d^2)*(5*a^3*e^6+9*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+35*c^3*d^6
)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)+1/24*(a/c/d-7*d/e^2)*x^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/e-1/192*(105*c^3*d^6-25*a*c^2*d^4*e^2-17*a^2*c*d^2*e^4-15*a^
3*e^6-2*c*d*e*(-5*a^2*e^4-6*a*c*d^2*e^2+35*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A] time = 0.51, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 832, 779, 621, 206}

$$-\frac{\left(-2cdex\left(-5a^2e^4-6acd^2e^2+35c^2d^4\right)-17a^2cd^2e^4-15a^3e^6-25ac^2d^4e^2+105c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{192c^3d^3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] ((a/(c*d) - (7*d)/e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/24
+ (x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((105*c^3*d^6 -
 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*c^2*d^4 - 6
*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
192*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*
c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(7/2)*d^(7
/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \int \frac{x^3(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e} + \frac{\int \frac{x^2\left(-3acd^2e-\frac{1}{2}cd\left(7cd^2-ae^2\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4cde}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

Mathematica [A] time = 1.59, size = 304, normalized size = 0.88

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\sqrt{c}\sqrt{d}\sqrt{e}\left(-15a^3e^6+a^2cd\right.\right.}{192c^{7/2}d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^3*e^6 + a^
2*c*d*e^4*(-17*d + 10*e*x) + a*c^2*d^2*e^2*(-25*d^2 + 12*d*e*x - 8*e^2*x^2)
```

```
 + c^3*d^3*(105*d^3 - 70*d^2*e*x + 56*d*e^2*x^2 - 48*e^3*x^3))) + (3*Sqrt[c
*d]*Sqrt[c*d^2 - a*e^2]*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 +
5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*S
qrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])))/(192*c^(7/2)*d^(7/2)*e^(9/2))
```

fricas [A] time = 1.24, size = 678, normalized size = 1.97

$$\left[-\frac{3\left(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^
4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*
e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d
^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^4*d^4*e^5), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^
4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2
*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^4*e^4*x^
3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7
- 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d^4*e
^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
4*d^4*e^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Evaluation time: 1.91Error: Bad Argument Type
```

maple [B] time = 0.03, size = 946, normalized size = 2.74

$$-\frac{5a^4e^4\ln\left(\frac{cdex+\frac{1}{2}a e^2+\frac{1}{2}c d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}\right)}{128\sqrt{cde}\;c^3d^3}-\frac{a^3e^2\ln\left(\frac{cdex+\frac{1}{2}a e^2+\frac{1}{2}c d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a e^2+c\right.}\right.}{32\sqrt{cde}\;c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 19/64/c^2/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+29/32/e^3*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/32*e^2/c^2*a^3/d*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e
)^(1/2)+5/32*e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-5/128*
e^4/c^3/d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^4+1/4/e^2*x*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/c/d-5/24/e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)*a+7/16/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+5/64*e^2/c^3/d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+43/64/e^2/c*d*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)*a-3/64/c*d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^2-29/128/e^4*
c*d^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+11/32/e^2*a*d^3*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)
-d^3/e^4*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-13/24/e^3/c*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+29/64/e^4*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-1/2*d^3/e^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)
+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2*d^5/e^4
*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
[Out] Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.438 $\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=251

$$-\frac{\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cd\right.}{}$$

```
[Out] -1/16*(-a*e^2+c*d^2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e
*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/e+1/24*((-3*a*e^2+5*c*d^2)*(a*e^2+3*c*d^2)-2*c*d*e*(-a*e^2+5*c*d^2)*x)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A] time = 0.26, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 832, 779, 621, 206}

$$-\frac{\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) + (((5*c*d^2 - 3*a*
e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(24*c^2*d^2*e^3) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/2)*d^(5/
2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx=\int \frac{x^2(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\int \frac{x\left(-2acd^2e-\frac{1}{2}cd\left(5cd^2-ae^2\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3cde}$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2c\ldots}{\ldots}$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2c\ldots}{\ldots}$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2c\ldots}{\ldots}$$

Mathematica [A] time = 0.82, size = 245, normalized size = 0.98

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-3a^2e^4+2acde^2(ex-2d)+c^2d^2\left(15d^2-10dex+8e^2x^2\right)\right)-\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(a^2e^4\ldots\right.}{\ldots}\right.}{24c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*c
*d*e^2*(-2*d + e*x) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (3*Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(24*c^(5/2)*d^(5/2
)*e^(7/2))
```

fricas [A] time = 1.09, size = 536, normalized size = 2.14

$$\left[-\frac{3\left(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade+\left(cd^2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4*a*c
^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*(5*c^3*d^6 - 3*a
*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*
e^3*x^2 + 15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2
 - a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*
e^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Evaluation time: 1.92Error: Bad Argument Type
```

maple [B] time = 0.01, size = 713, normalized size = 2.84

$$\frac{a^3e^3\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}\right)}{16\sqrt{cde}\;c^2d^2}+\frac{a^2e\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)}\right.}{16\sqrt{cde}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/c/d-1/4/c/d*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x*a-3/4/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x-1/8*e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-1/2/e/c*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-3/8/e^3*d^2*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)+1/16*e^3/c^2/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+1/16*e/c*ln
((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2))/(c*d*e)^(1/2)*a^2-5/16/e*d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+3/16/e
^3*c*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+d^2/e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)+1/2*d^2/e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a-1/2*d^4/e
^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{(d+ex)(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.439 $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=207

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{\cdots}$$

```
[Out] 1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e/(e*x+d)+1/8*(-a*e^2+c*d^2
)*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/4*(a/
c/d+3*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {794, 664, 621, 206}

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] -((a/(c*d) + (3*d)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4 + (a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*c*d*e*(d + e*x)) + ((c*d^2 -
 a*e^2)*(3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(
3/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)}+\frac{1}{4}\left(-\frac{3d}{e}-\frac{ae}{cd}\right)\int \frac{\sqrt{ade+\left(cd^2+a\right.}}{d+e}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+\right.}{2cde(d+ex}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+\right.}{2cde(d+ex}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+\right.}{2cde(d+ex}$$

Mathematica [A] time = 0.66, size = 197, normalized size = 0.95

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{cd}\sqrt{cd^2-ae^2}\left(ae^2+3cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\sqrt{c}\sqrt{d}\sqrt{e}\left(ae^2+cd(2ex-3d)\right)\right)}{4c^{3/2}d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(-3*d
+ 2*e*x)) + (Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(3*c*d^2 + a*e^2)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(4*c^(3/2)*d^(3/2)
*e^(5/2))
```

fricas [A] time = 1.01, size = 418, normalized size = 2.02

$$\left[-\frac{\left(3\,c^2d^4-2\,acd^2e^2-a^2e^4\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\left(2\right.\right.}{16\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2
*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*
d*e^3)*x) - 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/8*((3*c^2*d^4 - 2*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^
```

```
2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*
e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Evaluati
on time: 1.9Error: Bad Argument Type
```

maple [B] time = 0.01, size = 516, normalized size = 2.49

$$-\frac{a^{2}e^{2}\ln\left(\frac{cdex+\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}}{\sqrt{cde}}+\sqrt{cde\,x^{2}+ade+\left(a\,e^{2}+c\,d^{2}\right)x}\right)}{8\sqrt{cde}\;cd}-\frac{ad\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2}cde+\left(a\,e^{2}-c\right.}\right.}{2\sqrt{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/4/c/d*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)*a+1/4/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/
8*e^2/c/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+1/4*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a
-1/8/e^2*c*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-d/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d
^2)*(x+d/e))^(1/2)-1/2*d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2*d^3/e
^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
[Out] Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.440 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

```
[Out] -1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {664, 621, 206}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right.}}}{2e^2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cde-}\right.}{e^2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{cd^2+ae^2+2c}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2\right.}}\right.}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Mathematica [A] time = 0.75, size = 155, normalized size = 1.18

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{e}-\frac{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] - (c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e
^2]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d
^2 - a*e^2])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 -
 a*e^2)])))/e^(3/2)
```

fricas [A] time = 0.98, size = 337, normalized size = 2.57

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,cde-\left(cd^2-ae^2\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\sqrt{cdex^2+}\right.}{4\,cde^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sq
rt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/(c
*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 205, normalized size = 1.56

$$\frac{ae\ln\left(\frac{\frac{ae^2}{2}-\frac{cd^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}}-\frac{cd^2\ln\left(\frac{\frac{ae^2}{2}-\frac{cd^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a \cdots\right.}\right.}{2\sqrt{cde}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*e*ln((1/2*a*e^2-1/2*c
*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)*a-1/2/e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.441 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

```
[Out] arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-arctanh(1/2*(2*a*d*e+(a*e
^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))*a^(1/2)*e^(1/2)/d^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 843, 621, 206, 724}

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x(d+ex)}\,dx = \int \frac{ae+cdx}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= (cd)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx + (ae)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x}}$$

$$= (2cd)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right) - (2ae)\,\text{Su}$$

$$= \frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{d}\sqrt{e}}\right.}{\sqrt{d}}$$

Mathematica [A] time = 0.18, size = 210, normalized size = 1.25

$$-\frac{2\sqrt{ae+cdx}\left(\sqrt{a}\sqrt{c}\,e\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)-\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[a*e + c*d*x]*(-(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt
[c*d]*Sqrt[c*d^2 - a*e^2])]) + Sqrt[a]*Sqrt[c]*e*Sqrt[d + e*x]*ArcTanh[(Sqr
t[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.31, size = 947, normalized size = 5.64

$$\left[\frac{1}{2}\sqrt{\frac{cd}{e}}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\left(2cde^2x+cd^2e+ae^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{\frac{cd}{e}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/2*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
+ 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
```

```
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^
2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 1/2*sqrt(c*d/e)*log(8*c^
2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*
e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 +
 a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d
)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 439, normalized size = 2.61

$$-\frac{a e^2 \ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}\, d}+\frac{a e^2 \ln\left(\frac{cdex+\frac{1}{2}a e^2+\frac{1}{2}c d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+}\right)}{2\sqrt{cde}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x/(e*x+d),x)
```

```
[Out] 1/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/2/d*ln((c*d*e*x+1/2*a*e^2+1/2
*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2
)*a*e^2+1/2*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-1
/d*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/2/d*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(c*d*e)^(1/2)*a*e^2+1/2*d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

3.442 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

```
[Out] -1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {849, 806, 724, 206}

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
```

```
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2(d+ex)}\,dx=\int \frac{ae+cdx}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$
$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{}{x\sqrt{ade+(cd^2}}}{2ade}$$
$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}+\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\mathrm{Subst}\left(\int \frac{}{4a}\right.}{ade}$$
$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{2ade+\left(cd^2+ae^2\right.}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+\right.}}\right.}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.85

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\left(ae^2-cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[d]/x) + ((-(c*d^2) + a*e^2)*ArcTanh[
(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt
[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/d^(3/2)
```

fricas [A] time = 1.08, size = 355, normalized size = 2.59

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,ade+\left(cd^2-ae^2\right)\sqrt{ade}\,x\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(2\right.}{x^2}\right.}{4ad^2ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^2
)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a*d^2*e*x), -1/2*(2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e - (c*d^2 - a*e^2)*sqrt(-a
*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e
+ a^2*d*e^3)*x)))/(a*d^2*e*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)*a*exp(2)+2*exp(1)^3*a)/d/2/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*a
tan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(-a*ex
p(2)+2*exp(1)^2*a-c*d^2)/d/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-((sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a*exp(2)+
c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)-2*d*exp(1)*sqrt(c*d*exp(1))*a)/2/d/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a))
```

maple [B] time = 0.02, size = 594, normalized size = 4.34

$$\frac{a\,e^{2}\ln\left(\frac{2ade+\left(a\,e^{2}+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{cde\,x^{2}+ade+\left(a\,e^{2}+c\,d^{2}\right)x}}{x}\right)}{2\sqrt{ade}\,d}+\frac{a\,e^{3}\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2}cde+\left(a\,e^{2}-c\,d^{2}\right)\left(\right.}\right.}{2\sqrt{cde}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d^2/a/e/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/a/e*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*c+1/2*e*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1
/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+1/2/d*a*e^2/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/x)-1/2*d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+1/d*c/a*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/2*e^3/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a
+e/d^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*e^3/d^2*ln((1/2*a*
e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)*a-1/2*e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)
/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{(ex+d)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}}{x^{2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x^2(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)), x)
```

3.443 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=202

$$\frac{\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4x}-\sqrt{x}$$

```
[Out] 1/8*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^
(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5
/2)/e^(3/2)-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2-1/4*(c/a/e-3*
e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A] time = 0.28, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$\frac{\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4x}-\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*d*x^2) - ((c/(a*e) - (3*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*x) + ((c*d^2 - a*e^2)*
(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(5/2)*e
^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3(d+ex)}\,dx = \int \frac{ae+cdx}{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2} - \frac{\int \frac{-\frac{1}{2}ae\left(cd^2-3ae^2\right)+acde^2x}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2ade}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

Mathematica [A] time = 0.16, size = 162, normalized size = 0.80

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}+\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(ae(3ex-2d)-cd^2x\right)}{x^2}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x) + a*e*
(-2*d + 3*e*x)))/x^2 + ((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(Sqrt
[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]
*Sqrt[d + e*x])))/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

fricas [A] time = 1.94, size = 442, normalized size = 2.19

$$\left[-\frac{\left(c^2d^4+2\,acd^2e^2-3\,a^2e^4\right)\sqrt{ade}\,x^2\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\,(a\right.}{x^2}\right.}{16\,a^2d^3e^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^
2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3
*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^2), -1/8*((c^2*d
^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^
2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(a^2*d^3*e^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^2*a*exp(2)-2*exp(1)^4*a)/2/d^2/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)
)*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(-a
^2*exp(2)^2-4*exp(1)^2*a^2*exp(2)+8*exp(1)^4*a^2-2*c*d^2*a*exp(2)-c^2*d^4)/
4/d^2/exp(1)/a/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-((sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)^2-4*exp(
1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^3*a^2*exp(2)+2*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^3*a*exp(2)+c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3-8*d*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^2*exp(2)
+8*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^2*a^2-8*c*d^3*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a+d*exp(1)
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^
3*exp(2)^2+4*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)*a^3*exp(2)+2*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^2*exp(2)+8*c*d^3*exp(1)^3*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^2+c^
2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)*a-8*d^2*exp(1)^4*sqrt(c*d*exp(1))*a^3)/8/d^2/exp(1)/a/((sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)
^2)
```

maple [B] time = 0.02, size = 882, normalized size = 4.37

$$-\frac{3 a e^{3} \ln\left(\frac{2ade+\left(a e^{2}+c d^{2}\right)x+2\sqrt{ade}\,\sqrt{cde\,x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}}{x}\right)}{8\sqrt{ade}\, d^{2}}-\frac{a e^{4} \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} cde+\left(a e^{2}-c d^{2}\right.}\right.}{2\sqrt{cde}\, d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] 5/4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/4*e^2/d^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/d/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
c-1/2*e^2/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-3/8*e^3/d^2*a/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/x)+1/4*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-5/4*e/d^2*c/a*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)*x-1/2/d^2/a/e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3
/2)+1/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-1/4*d/a^2/e^2
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+1/8*d^2/a/e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2))/x)*c^2-1/4/a^2/e*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/2/d^
3*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a-1/d^3*e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)-1/2/d^3*e^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2/d*
e^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^3\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^3\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**3*(d + e*x)), x)
```

3.444 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=286

$$\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24a^2d^3e^2x}-\frac{\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{\ }{2\sqrt{a}\sqrt{}}\right.}{16a^{5/2}d^{7/2}e^{5/2}}$$

```
[Out] -1/16*(-a*e^2+c*d^2)*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e
+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/d/x^3-1/12*(c/a/e-5*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2+1/2
4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^2/d^3/e^2/x
```

Rubi [A] time = 0.40, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$-\frac{\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x}}{24a^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*d*x^3) - ((c/(a*e) - (5*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*x^2) + ((3*c*d^2 - 5*
a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*a
^2*d^3*e^2*x) - ((c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^4(d + ex)}\,dx = \int \frac{ae + cdx}{x^4\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{3dx^3} - \frac{\int \frac{-\frac{1}{2}ae(cd^2-5ae^2)+2acde^2x}{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx}{3ade}$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{12x^2}$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{12x^2}$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{12x^2}$$

$$= -\frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{12x^2}$$

Mathematica [A] time = 0.24, size = 210, normalized size = 0.73

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^2(-8d^2+10dex-15e^2x^2)-2acd^2ex(d-2ex)+3c^2d^4x^2\right)}{x^3} - \frac{3(-5a^3e^6+3a^2cd^2e^4+ac^2d^4e^2+c^3d^6)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a}}{\sqrt{a}\sqrt{e}}\right.}{\sqrt{d+ex}\sqrt{ae+cdx}}\right.}{24a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 - 2
*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3 -
 (3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

fricas [A] time = 4.39, size = 558, normalized size = 1.95

$$\left[-\frac{3\left(c^3d^6+ac^2d^4e^2+3\,a^2cd^2e^4-5\,a^3e^6\right)\sqrt{ade}\,x^3\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2+4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(c\ldots\right.\right.}{x^2}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c^2*d
^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e
^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3), 1/48
*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d
*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^3*a*exp(2)+2*exp(1)^5*a)/2/d^3/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2
))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(-
a^3*exp(2)^3-2*exp(1)^2*a^3*exp(2)^2-8*exp(1)^4*a^3*exp(2)+16*exp(1)^6*a^3-
3*c*d^2*a^2*exp(2)^2-3*c^2*d^4*a*exp(2)+2*c^2*d^4*exp(1)^2*a-c^3*d^6)/8/d^3
/exp(1)^2/a^2/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))+(3*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^3+6*exp
(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^5*a^3*exp(2)^2-24*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)+9*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2*exp(2)^2+9*c^2*d^4*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a*exp(
2)-6*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^5*a+3*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^5-48*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3*exp(2)+48
*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^4*a^3-8*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)^3+48*d*exp(1)^5*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp
(2)-24*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^3*a^3*exp(2)^2-48*c*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*exp(2)+48*c*d^3*exp(1)
^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
```

```
3*a^3-24*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)-48*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2-8*c^3*d^7*exp(1)*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a+
48*d^2*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4*exp(2)^2+48*d^2*exp(1)^4*sqrt(c*d*exp(1
))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
2*a^4*exp(2)-96*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4+96*c*d^4*exp(1)^2*sqrt(c*d
*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^2*a^3*exp(2)+48*c*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^3+48*c^2*d^6*exp(1)^
2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt
(c*d*exp(1))*x)^2*a^2-3*d^2*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)^3-6*d^2*exp(1)^4*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)^2-24*
d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)*a^5*exp(2)-9*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4*exp(2)^2-48*c*d^4*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4*exp(2)-4
8*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^4-9*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3*exp(2)-42*c^2*d^6*exp(1)^4*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3-3*c^3*d^8
*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)*a^2+48*d^3*exp(1)^7*sqrt(c*d*exp(1))*a^5+16*c*d^5*exp(1)^5*sqrt(c*d*
exp(1))*a^4)/48/d^3/exp(1)^2/a^2/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^3)
```

maple [B] time = 0.02, size = 1165, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] 3/4/d^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/8/e/a^2*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+1/4/d/a^2/e^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(3/2)*c-1/16*d^3/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+1/8*d/a^3
/e^2*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/2/e/d^2/a^2/x*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+11/8/d^3*e^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*x+1/d^4*e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3
/8/d^4*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/16*d/a/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2))/x)*c^2+1/2/d/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/3
/d^2/a/e/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/8/a^3/e^3/x*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^2+1/8*d^2/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*c^3-1/2/d^4*e^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+1/2/d^4*e^5*l
n((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a-1/2/d^2*e^3*ln((1/2*a*e^2-1/2*c*d^2
+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2)*c-11/8/d^4*e/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+9/
8/d^2*e/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+1/2/d^2*e^3*ln((c*d*e*x
+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
)/(c*d*e)^(1/2)*c+5/16/d^3*e^4*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+
2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-3/16/d*e^2/(a*d
*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/x)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^4\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^4\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**4*(d + e*x)), x)
```

3.445 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=389

$$\frac{\left(-35a^2e^4+6acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96a^2d^3e^2x^2}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(\right.}}{192a^3d^4e^3x}$$

```
[Out] 1/128*(-a*e^2+c*d^2)*(35*a^3*e^6+15*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+5*c^3*d^6
)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/4*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/d/x^4-1/24*(c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/x^3+1/96*(-35*a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/192*(-105*a^3*e^6+25*a^2*c*d^2
*e^4+17*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a
^3/d^4/e^3/x
```

Rubi [A] time = 0.59, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$-\frac{\left(25a^2cd^2e^4-105a^3e^6+17ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192a^3d^4e^3x}+\frac{\left(-35a^2e^4+6acd^2e^2+5c^2d^4\right)\sqrt{x\left(\right.}}{96a^2d^3e^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*d*x^4) - ((c/(a*e) - (7*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*x^3) + ((5*c^2*d^4 +
6*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9
6*a^2*d^3*e^2*x^2) - ((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 - 1
05*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192*a^3*d^4*e^3*x
) + ((c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35*a
^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
```

```
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5(d+ex)}\,dx = \int \frac{ae+cdx}{x^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\int \frac{-\frac{1}{2}ae\left(cd^2-7ae^2\right)+3acde^2x}{x^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4ade}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{24x^3}$$

Mathematica [A] time = 0.36, size = 273, normalized size = 0.70

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{a}\,\sqrt{d}\,\sqrt{e}\left(a^3e^3\left(-48d^3+56d^2ex-70de^2x^2+105e^3x^3\right)+a^2cd^2e^2x\left(-8d^2+12dex-25e^2x^2\right)+ac^2d^4ex^2(10d-17ex)-15c^3d^6x^3\right.}{x^4}\right.}{192a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3 +
 a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 25*
e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/x^
4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(7/2))
```

fricas [A] time = 21.12, size = 702, normalized size = 1.80

$$\left[-\frac{3\left(5\,c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+20\,a^3cd^2e^6-35\,a^4e^8\right)\sqrt{ade}\,x^4\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{cdex^2+ade+(}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*
e^6 - 35*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*
(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d^3*e^5 -
 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a^4*d^2*e
^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4), -1/384*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2
+ 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*sqrt(-a*d*e)*x^4*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*
x)) + 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d
^3*e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a
^4*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^4*a*exp(2)-2*exp(1)^6*a)/2/d^4/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)
)*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(-5
*a^4*exp(2)^4-8*exp(1)^2*a^4*exp(2)^3-16*exp(1)^4*a^4*exp(2)^2-64*exp(1)^6*
a^4*exp(2)+128*exp(1)^8*a^4-20*c*d^2*a^3*exp(2)^3-30*c^2*d^4*a^2*exp(2)^2+2
4*c^2*d^4*exp(1)^2*a^2*exp(2)-20*c^3*d^6*a*exp(2)+16*c^3*d^6*exp(1)^2*a-5*c
^4*d^8)/64/d^4/exp(1)^3/a^3/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))+(-15*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^
4*exp(2)^4-24*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3-48*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2+192*exp(1)^6*(sq
```

```
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*
exp(2)-60*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^7*a^3*exp(2)^3-90*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)^2+72*c^2*d^4*exp(1)^2*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*e
xp(2)-60*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^7*a*exp(2)+48*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a-15*c^4*d^8*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7+384*d*exp(1)^5*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^6*a^4*exp(2)-384*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4+55*d*exp(1)*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*ex
p(2)^4+88*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^5*a^5*exp(2)^3+48*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^2-576*d*exp(1)^7*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^
5*exp(2)+220*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^3+384*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)-384*c*d^
3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^5*a^4+330*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^2-264*c^2*d^5*exp(1)^3*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)
+220*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a^2*exp(2)-176*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2+55*c^4*d^9*exp(1)*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a-38
4*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^2-768*d^2*exp(1)^6*sqrt(c*d*exp(1
))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
4*a^5*exp(2)+1152*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5+384*c*d^4*exp(1)^4*sqrt(
c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^4*a^4*exp(2)-384*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4+768*c^2*d^6*ex
p(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^4*a^3-73*d^2*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^4-40*d^2*exp(1)^4*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*e
xp(2)^3+48*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+576*d^2*exp(1)^8*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)-292*c*d^4*ex
p(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^3*a^5*exp(2)^3-768*c*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^2+768*c*d^4*exp(1)^8*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5-438
*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^3*a^4*exp(2)^2-1416*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)-384*c^2*d^6*e
xp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^3*a^4-292*c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*exp(2)-688*c^3*d^8*exp(1)^4*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3-73*c^4*d^
10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^3*a^2+384*d^3*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)^3+384*d^3*exp(1)
^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^2*a^6*exp(2)^2+384*d^3*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d
```

*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)
-1152*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d
*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6+1152*c*d^5*exp(1)^3*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*
a^5*exp(2)^2+1024*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^5*exp(2)+256*c*d^5*exp(1
)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^2*a^5+1152*c^2*d^7*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4*exp(2)+64
0*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d
*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4+384*c^3*d^9*exp(1)^3*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2
*a^3-15*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)*a^7*exp(2)^4-24*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3-48*d^3*exp(1)^7*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*e
xp(2)^2-192*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)*a^7*exp(2)-60*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^3-384*c*d^5*exp(
1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)*a^6*exp(2)^2-384*c*d^5*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)-384*c*d^5*exp(1)^9*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6-90*c^2*d^7*ex
p(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)*a^5*exp(2)^2-696*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)-384*c^2*d^7*exp(1)^7*(sqrt(a*
d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5-60*c^3*
d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)*a^4*exp(2)-336*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4-15*c^4*d^11*exp(1)^3*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3+384*d^4*ex
p(1)^10*sqrt(c*d*exp(1))*a^7+128*c*d^6*exp(1)^6*sqrt(c*d*exp(1))*a^6*exp(2)
+128*c*d^6*exp(1)^8*sqrt(c*d*exp(1))*a^6+128*c^2*d^8*exp(1)^6*sqrt(c*d*exp(
1))*a^5)/384/d^4/exp(1)^3/a^3/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^4)

maple [B] time = 0.02, size = 1494, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^5/(e*x+d),x)

[Out] 19/64/e^2/d/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^2+1/32/e*d^2/a^
2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-43/64/d^2*e/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2)*x-93/64/d^4*e^3*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x+13/24/d^3/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-17/32/d/a^2*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+5/128*d^4/a^3/e^3/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2))/x)*c^4-5/64*d^2/a^4/e^3*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*x+5/64*d/a^4/e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^3+5/24/d/a^2
/e^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-7/16/e/d^2/a^2/x^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-1/d^5*e^4*((x+d/e)^2*c*d*e+(a*e^2-c*d^
2)*(x+d/e))^(1/2)-29/64/d^5*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/4
/d^2/a/e/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/2/d^5*e^6*ln((c*d*e*
x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
))/(c*d*e)^(1/2)*a-1/2/d^5*e^6*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2
/d^3*e^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*

```
d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c+93/64/d^5*e^2/a/x*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-39/32/d^3*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)*c-1/2/d^3*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-35/128/d^4*e^5*a/(
a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2))/x)+5/32/d^2*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d
^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-29/32/d
^4*e/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+43/64/d^3/a^2/x*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+3/64*e/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-7
/32/e^2*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-19/64/e/a^3*c^3*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-5/32/a^3/e^3/x^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(3/2)*c^2-5/64*d^3/a^4/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^5\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^5\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**5*(d + e*x)), x)
```

3.446 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=449

$$-\frac{\left(-35a^3e^6-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{960c^3d^3e^4}$$

```
[Out] 1/20*(a/c/d-3*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/6*x^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e-1/960*(105*c^3*d^6-21*a*c^2*d^4*e^2
-33*a^2*c*d^2*e^4-35*a^3*e^6-6*c*d*e*(-7*a^2*e^4-6*a*c*d^2*e^2+21*c^2*d^4)*
x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4-1/1024*(-a*e^2+c*d^2
)^3*(7*a^3*e^6+15*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+21*c^3*d^6)*arctanh(1/2*(2
*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/512*(-7*a^4*e^8-8*a^3*c*d^2*e^6-6*a^
2*c^2*d^4*e^4+21*c^4*d^8)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^4/d^4/e^5
```

Rubi [A] time = 0.57, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 832, 779, 612, 621, 206}

$$-\frac{\left(-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-35a^3e^6-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{960c^3d^3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^4*d^4*
e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((10
5*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*
c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d
^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
1024*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int x^3(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$=\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}+\frac{\int x^2\left(-3acd^2e-\frac{3}{2}cd\left(3cd^2-\right.\right.}{}$$

$$=\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2\right.\right.}{}$$

$$=\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2\right.\right.}{}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ad}}{512c^4d^4e^5}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ad}}{512c^4d^4e^5}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ad}}{512c^4d^4e^5}$$

Mathematica [A] time = 2.29, size = 425, normalized size = 0.95

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-105a^5e^{10}+5a^4cde^8(11d+14ex)+2a^3c^2d^2e^6\left(27d^2-16dex-28e^2x^2\right)+6a^2c^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5*
a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e^2
*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3) +
a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3 + 1
664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*d^2*e
^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*Sqrt[c*d]*(c*d^2 - a*e^2)^(5/
2)*(21*c^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcSinh[(
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]
)/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(7680*c^(9/2)
*d^(9/2)*e^(11/2))
```

fricas [A] time = 1.16, size = 1044, normalized size = 2.33

$$\left[-\frac{15\left(21\,c^6d^{12}-42\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4+4\,a^3c^3d^6e^6+3\,a^4c^2d^4e^8+6\,a^5cd^2e^{10}-7\,a^6e^{12}\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^3
*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e -
525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^
3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9
*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^
3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*
c^6*d^10*e^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8
- 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5
*d^5*e^6), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^
4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e - 525*a*c^5*
d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^3*e^9 - 10
5*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9*c^6*d^8*e
^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^3 - 33*a*c
^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*c^6*d^10*e
^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8 - 35*a^4*c
^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Error: Bad Argument Type
```

maple [B] time = 0.03, size = 1883, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/8*d^6/e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-43/512/e^5*c*d^
6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+43/96/e^3*d^2*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)*x+21/512/e^3*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*a-7/256*e/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+29/192/c^2/d*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2-75/512/e^3*c*d^6*ln((c*d*e*x+1/2
*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c
*d*e)^(1/2)*a+1/16*d^2*e*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*
d^6/e^3*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2
*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/32*e^2/c^2*a^3/d*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+7/96*e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(3/2)*x*a^2-7/256*e^4/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*x*a^4+7/1024*e^7/c^4/d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^6-3/512*e^5/c^3
/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^5-17/256*e/c*d^2*ln((c*d*e*x+1/2*a*e^2+1/
2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/
2)*a^3-3/16*d^4/e*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*d^5/e^4*c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/16*d^8/e^5*c^2*ln((1/2*a*
e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)-3/128/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x*a^2+1/4/e^2*a*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+11/48/e/c
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a-3/1024*e^3/c^2*ln((c*d*e*x+1/2
*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c
```

```
*d*e)^(1/2)*a^4+177/1024/e*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+65/192/e^2/c*
d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a+7/192*e^2/c^3/d^3*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3-43/256/e^4*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x-7/512*e^5/c^4/d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^
5-15/512*e^3/c^3/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4+29/256/e/c
*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+43/1024/e^5*c^2*d^8*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)-7/60/e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
*a+1/6/e^2*x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/c/d-1/3*d^3/e^4*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-19/60/e^3/c*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(5/2)+43/192/e^4*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/4
*d^3/e^2*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8*d^2/e*a^2/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(\left(d + e x\right) \left(a e + c d x\right)\right)^{\frac{3}{2}}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.447 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=352

$$\frac{\left(-15a^2e^4-6cdex\left(7cd^2-3ae^2\right)-12acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240c^2d^2e^3}+\frac{\left(3a^2e^4+6acd^2e^2+7c\right.}{}$$

```
[Out] 1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/240*(35*c^2*d^4-12*a*c*
d^2*e^2-15*a^2*e^4-6*c*d*e*(-3*a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/c^2/d^2/e^3+1/256*(-a*e^2+c*d^2)^3*(3*a^2*e^4+6*a*c*d^2*e^2+7
*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-1/128*(-a*e^2+c
*d^2)*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A] time = 0.33, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 832, 779, 612, 621, 206}

$$-\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^4}+\frac{\left(-15a^2e^4-6cdex\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*e^2 +
2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^3*d^3*e^4) +
 (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) + ((35*c^2*d^4 -
 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(240*c^2*d^2*e^3) + ((c*d^2 - a*e^2)^3*(7
*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(
2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
56*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int x^2(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\int x\left(-2acd^2e-\frac{1}{2}cd\left(7cd^2-3ae\right.\right.}{}$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\left(35c^2d^4-12acd^2e^2-15a^2e^4-\right.}{}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(\right.}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(\right.}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(\right.}}{128c^3d^3e^4}$$

Mathematica [A] time = 2.77, size = 497, normalized size = 1.41

$$\sqrt{(d+ex)(ae+cdx)}\left(\frac{\frac{5\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(8c^3d^3e^3\sqrt{cd}\sqrt{cd^2-ae^2}\,(ae+cdx)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-cd\left(cd^2-ae^2\right)\left(-3c^{5/2}d^{5/2}\sqrt{e}\left(cd^2-ae^2\right)^2\sqrt{ae+cdx}\,\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}}{\sqrt{cd}\sqrt{cd}}\right.\right.\right.}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}}{384c^5d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(x*(a*e + c*d*x)^2*(d + e*x) + (-48*c^4*d^4*
e^3*(7*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^3*(d + e*x) + (5*(7*c^2*d^4 + 6*a*c*d
^2*e^2 + 3*a^2*e^4)*(8*c^3*d^3*Sqrt[c*d]*e^3*Sqrt[c*d^2 - a*e^2]*(a*e + c*d
*x)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - c*d*(c*d^2 - a*e^2)*(3*(c*d)^
(5/2)*e*(c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a
*e^2)] - 2*(c*d)^(5/2)*e^2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^2*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)] - 3*c^(5/2)*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*Sqr
t[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2])])))/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e
*x))/(c*d^2 - a*e^2)]))/(384*c^5*d^5*e^4*(a*e + c*d*x))))/(5*c*d*e)
```

fricas [A] time = 0.85, size = 846, normalized size = 2.40

$$\left[-\frac{15\left(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8-3\,a^5e^{10}\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2
*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*
c^3*d^5*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*
c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)
*x^2 + 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^
2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -
1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d
^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(384*c^5*d^5
*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5*e^5 - 30*a^3*
c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*c^4*d^4*e^6)*x^3 - 8*
(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^2 + 2*(35*c^5*d^8*
e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^2*e^8)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
```

```
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Evaluation time: 0.42Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1560, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] -21/128*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/32/c*d*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*a^2+9/128/e^4*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)-1/4/e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a-3/64/e^2*d^3*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+1/5/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
5/2)/c/d-3/8/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8*d*a^2/c*((
x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16*d^3*a^2*ln((1/2*a*e^2-1/2*
c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2))/(c*d*e)^(1/2)-1/8*d^5/e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-1/16*d*e^2*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)
+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/16*d^5/e^2*
a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*e^3/c^2/d^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3-3/256*e^6/c^3/d^3*ln((c*d*e*x+1/2*a*e^2+1/2*
c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)
*a^5+3/256*e^4/c^2/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+9/128*e^2/c*d*ln((c*d*e
*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/(c*d*e)^(1/2)*a^3+33/256/e^2*c*d^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*
d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+1/16*d^
7/e^4*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16*e/c^2/d^2*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2+9/64/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x+3/128*e^4/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^
4-1/8/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a+3/64*e/c*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+1/3*d^2/e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(3/2)-3/16/e^3*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/4*d
^2/e*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/4*d^4/e^3*c*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/64*e^2/c^2/d*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)*a^3-15/64/e*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x*a-9/256/e^4*c^2*d^7*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(\left(d + e x\right) \left(a e + c d x\right)\right)^{\frac{3}{2}}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.448 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=295

$$-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right.}{64c^2d^2e^3}$$

```
[Out] -1/24*(3*a/c/d+5*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/4*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e/(e*x+d)-1/128*(-a*e^2+c*d^2)^3*(3*a*
e^2+5*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/64*(-a*e^2
+c*d^2)*(3*a*e^2+5*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A] time = 0.28, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {794, 664, 612, 621, 206}

$$\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^3}-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\right.}{128c^{5/2}d^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^3) - (((3*a)/(c*d) + (5*d)
/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/24 + (a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2)/(4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*
d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(128*c^(5/2)*d^(5/2)*e^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4cde(d+ex)}+\frac{1}{8}\left(-\frac{5d}{e}-\frac{3ae}{cd}\right)\int \frac{\left(ade+\left(cd^2+\right.\right.}{}$$

$$=-\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{\left(ade+\left(cd^2+ae\right.\right.}{4cde(d}$$

$$=\frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{64c^2d^2e^3}$$

$$=\frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{64c^2d^2e^3}$$

$$=\frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{64c^2d^2e^3}$$

Mathematica [A] time = 1.24, size = 276, normalized size = 0.94

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-9a^3e^6+3a^2cde^4(3d+2ex)+ac^2d^2e^2\left(-31d^2+20dex+72e^2x^2\right)+c^3d^3\left(15d\right.\right.\right.}{192c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^2
*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2) +
c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*Sqrt[c*d]*(c
*d^2 - a*e^2)^(5/2)*(5*c*d^2 + 3*a*e^2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(192*c^(5/2)*d^(5/2)*e^(7/2))
```

fricas [A] time = 1.16, size = 676, normalized size = 2.29

$$\left[-\frac{3\left(5c^4d^8-12ac^3d^6e^2+6a^2c^2d^4e^4+4a^3cd^2e^6-3a^4e^8\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*
e^6 - 3*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*
d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^4
*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7
+ 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e
^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
3*d^3*e^4), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 + 15
*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7 + 8*(c^4*
d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 - 3*a^
2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1279, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/4/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8/c/d*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)*a+1/8/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/
32*e^2/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+3/16*d*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a-3/32/e^2*c*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x-3/64*e^3/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3
```

```
+3/64*e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+3/64/e*d^2*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-3/64/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)+3/128*e^5/c^2/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2
)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4-3/32*e^3/c*ln(
(c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2))/(c*d*e)^(1/2)*a^3+9/64*e*d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*
d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/32/
e*c*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+3/128/e^3*c^2*d^6*ln((c*d*e*x+1/2*a*e^
2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)
^(1/2)-1/3*d/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/4*d*a*((x+
d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8*e*a^2/c*((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*e^3*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c
*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^
(1/2)-3/16*d^2*e*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*d^4/e*a*c*
ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*d^3/e^2*c*((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2)*x+1/8*d^4/e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(1/2)-1/16*d^6/e^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(x*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.449 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+\right.$$

```
[Out] 1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/16*(-a*e^2+c*d^2)^3*arctanh
(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+1/8*(a/c/d-d/e^2)*(2*c*d*e*x+a*e
^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
```

```
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \sqrt{ade+}}{2e^2}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{(ac}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{(ac}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{(ac}{}$$

Mathematica [A] time = 0.67, size = 264, normalized size = 1.31

$$\frac{\sqrt{c}\sqrt{d}\left(3\left(cd^2-ae^2\right)^{7/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}\,(d+ex)\left(-3a^3e^5-a^2cd\right.\right.}{24e^{5/2}(cd)^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-3*a^3*e^5
 - a^2*c*d*e^3*(8*d + 17*e*x) + a*c^2*d^2*e*(3*d^2 - 10*d*e*x - 22*e^2*x^2)
 + c^3*d^3*x*(3*d^2 - 2*d*e*x - 8*e^2*x^2))) + 3*(c*d^2 - a*e^2)^(7/2)*Sqrt
[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(24*(c*d)^(
5/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.26, size = 532, normalized size = 2.65

$$\left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\sqrt{cdex^2+ade+(c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 - 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8*c^3*d^3*
```

```
e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 +
7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e
^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Error: Bad Argument Type
```

maple [B] time = 0.01, size = 566, normalized size = 2.82

$$-\frac{a^{3}e^{4}\ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2}cde+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{16\sqrt{cde}\,cd}+\frac{3a^{2}d\,e^{2}\ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2}cd\right.}{16\sqrt{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/4*e*a*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8*e^2*a^2/c/d*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)-1/16*e^4*a^3/c/d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)
/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
+3/16*e^2*a^2*d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/16*a*c*d^3*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/4/e*c*d^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)*x-1/8/e^2*c*d^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2
)+1/16/e^2*c^2*d^5*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [B] time = 0.50, size = 462, normalized size = 2.30

$$\frac{3\,a^{2}cd^{2}\log\left(2\,cdx+\frac{cd^{2}}{e}+ae+2\,\sqrt{cdex^{2}+cd^{2}x+ae^{2}x+ade}\,\sqrt{\frac{cd}{e}}\right)}{16\,\left(\frac{cd}{e}\right)^{\frac{3}{2}}}+\frac{c^{3}d^{6}\log\left(2\,cdx+\frac{cd^{2}}{e}+ae+2\,\sqrt{cdex^{2}+cd^{2}\right.}{16\,\left(\frac{cd}{e}\right)^{\frac{3}{2}}e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 3/16*a^2*c*d^2*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e)*sqrt(c*d/e))/(c*d/e)^(3/2) + 1/16*c^3*d^6*log(2*c*d*x + c*d
^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/((c
*d/e)^(3/2)*e^4) - 3/16*a*c^2*d^4*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/((c*d/e)^(3/2)*e^2) - 1/16*
a^3*e^2*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e)*sqrt(c*d/e))/(c*d/e)^(3/2) - 1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e)*c*d^2*x/e + 1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a*e*
x - 1/8*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d^3/e^2 + 1/8*sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a^2*e^2/(c*d) + 1/3*(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e)^(3/2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.450 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=251

$$-a^{3/2}\sqrt{d}\,e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+\cdots}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\cdots}}\right)}{8\sqrt{c}\sqrt{d}\,e^{3/2}}$$

```
[Out] -1/8*(-3*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)
/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c
^(1/2)/d^(1/2)-a^(3/2)*e^(3/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2
)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*d^(1/2)+1/4*(2*c
*d*e*x+5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A] time = 0.28, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {849, 814, 843, 621, 206, 724}

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}\,e^{3/2}}-a^{3/2}\sqrt{d}\,e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)\cdots}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd\cdots\right.}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]])])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) - a^(3/2)*Sqrt[d]*e^(3/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}\,dx$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\int \frac{-4a^2cd^2e^3+\frac{1}{2}cd\ldots}{x\sqrt{ade+(\ldots}}}{\ldots}$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}+\left(a^2de^2\right)\int \frac{\ldots}{x\sqrt{\ldots}}$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\left(2a^2de^2\right)\text{Subs}\ldots$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\left(c^2d^4-6acd^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.85, size = 275, normalized size = 1.10

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{8a^{3/2}\sqrt{d}e^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}-\frac{\sqrt{c}\sqrt{d}\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\sqrt{e}\sqrt{ae+c\ldots}\right.}{4e^{3/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(5*a*e^2 + c*d*(d
 + 2*e*x)) - (Sqrt[c]*Sqrt[d]*(c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcSinh
[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]
)])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) -
 (8*a^(3/2)*Sqrt[d]*e^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(4*e^(3/2)*Sqrt[a*e + c*d*x])
```

fricas [A] time = 11.02, size = 1327, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/16*(8*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (c^2*
d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4
*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c*d*e^2), 1/8*(4*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)
) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c*d*e^2), 1/16*(16*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - (
c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(c*d*e^2), 1/8*(8*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + (c^2
*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^
2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x
+ c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*
d*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 1130, normalized size = 4.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x/(e*x+d),x)
```

```
[Out] 1/3/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/4/d*a*e^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x+1/8/d^2*a^2*e^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)+5/4*a*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/16/d^2*a^3*e^5/c*
ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2))/(c*d*e)^(1/2)+9/16*a^2*e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+9/16*d^2
*a*e*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/4*d*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*x+1/8*d^2*c/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/16*d^4*c^2/e
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)-d*a^2*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-1/3/d*((x+
d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/4/d*a*e^2*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^2*a^2*e^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)+1/16/d^2*a^3*e^5/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(
c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3
/16*a^2*e^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2
*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*d^2*a*e*c*ln((1/2*a
*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*d*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)*x+1/8*d^2*c/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/16*d
^4*c^2/e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)), x)
```

3.451 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(ae-cdx)}{x}+\frac{\sqrt{c}\,\sqrt{d}\,\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,(\ldots}{}$$

```
[Out] 1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-1/2*(
a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)*e^(1/2)/d^(1/2)-(-c*d*x+
a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A] time = 0.27, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {849, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(ae-cdx)}{x}+\frac{\sqrt{c}\,\sqrt{d}\,\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (Sqrt[c]
*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*S
qrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e]) -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(
2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[d])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
```

```
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2}\,dx$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}-\frac{1}{2}\int \frac{-ae\left(3cd^2+ae^2\right)-}{x\sqrt{ade+\left(cd^2+\right.}}$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}+\frac{1}{2}\left(ae\left(3cd^2+ae^2\right)\right)\int -$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}-\left(ae\left(3cd^2+ae^2\right)\right)\text{Subst}$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}+\frac{\sqrt{c}\sqrt{d}\left(cd^2+3ae^2\right)\tan}{}$$

Mathematica [A] time = 1.20, size = 263, normalized size = 1.10

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{c}\,d\sqrt{cd}\left(3ae^2+cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\frac{\sqrt{d}\sqrt{ae+cdx}\,(cdx-a}{x}\right.}{\sqrt{d}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[d]*(-(a*e) + c*d*x)*Sqrt[a*e + c*d*x]
)/x + (Sqrt[c]*d*Sqrt[c*d]*(c*d^2 + 3*a*e^2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
```

```
e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*Sqrt[c*d^2
 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (Sqrt[a]*Sqrt[e]*(3*c*d^
2 + a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*
x])])/Sqrt[d + e*x]))/(Sqrt[d]*Sqrt[a*e + c*d*x])
```

fricas [A] time = 4.37, size = 1221, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/4*((c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c
*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e +
(c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/4*(2*(c*d^2
+ 3*a*e^2)*sqrt(-c*d/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e +
 (c^2*d^3 + a*c*d*e^2)*x)) - (3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2
*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(c*d*x - a*e))/x, 1/4*(2*(3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + (c*d^2 + 3*a
*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/2*((c*d^2 + 3*a*e^2)*sqrt(-c*d
/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^
2)*x)) - (3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2
*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1310, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^2/(e*x+d),x)
```

```
[Out] -1/4*e*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/16*d^3*c^2*ln((1
/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d
```

```
^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/d*a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)+1/a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+5/4*e*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-1/2*a^2*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+7/16
*d^3*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-1/16*e^6/d^3*a^3/c*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(c*d*e)^(1/2)-3/16*e^2*d*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c
*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/
16*e^6/d^3*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+27/16*d*a*e^2*ln((c*d*e*x+1/2*a
*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d
*e)^(1/2)*c-3/2*d^2*a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+1/3*e/d^2*((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/8*d*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(1/2)+2/3/d^2*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+17/8*d*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-1/4*e^3/d^2*a*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*x-3/16/d*a^2*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/d*c/a*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/d^2/a/e/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(5/2)-1/8*e^4/d^3*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/4*e
^3/d^2*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8*e^4/d^3*a^2/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16*e^4/d*a^2*ln((1/2*a*e^2-
1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{3}{2}}}{x^{2} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**2*(d + e*x)), x)
```

3.452 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=256

$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{a}\,d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd\right.}}\right.$$

```
[Out] -1/8*(-a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)
/a^(1/2)/e^(1/2)+c^(3/2)*d^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2
)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/4*(2*a
*d*e+(a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2
```

Rubi [A] time = 0.28, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 810, 843, 621, 206, 724}

$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{a}\,d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd\right.}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*d*x^2) + c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])] -
 ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*
x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(8*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
```

```
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}\,dx$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}-\frac{\int \frac{-\frac{1}{2}ae(3c^2\cdots}{x\sqrt{\cdots}}}{\cdots}$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+\left(c^2d^2e\right)\int\cdots$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+\left(2c^2d^2e\right)\mathrm{S}\cdots$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+c^{3/2}d^{3/2}\sqrt{e}\cdots$$

Mathematica [A] time = 2.45, size = 285, normalized size = 1.11

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d+ex}\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}+\frac{8e(cd)^{5/2}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{c^{3/2}}-\frac{\sqrt{d}\sqrt{e}(\cdots}{\cdots}\right.}{4d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(5*c*d^2
*x + a*e*(2*d + e*x)))/x^2) + (8*(c*d)^(5/2)*e*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*
d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c
*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]])/c^(3/2) - ((3*c^2*d^4 + 6*a*c*d^2*
e^2 - a^2*e^4)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*S
qrt[e]*Sqrt[d + e*x])]])/Sqrt[a]))/(4*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

fricas [A] time = 5.63, size = 1375, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(8*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (3*c^2*d^4
+ 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*d^3*e*x^
2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*
e^3)*x)) + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2
*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), 1/8*(4*sqrt(c
*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c^2*d^4 + 6*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*
d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a
^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1
/8*(8*sqrt(-c*d*e)*a*c*d^3*e*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*
e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2
 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.42Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 1604, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^3/(e*x+d),x)
```

```
[Out] 1/16/d^4*e^7*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16/d^4*e^7*a^3/
c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2))/(c*d*e)^(1/2)-1/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(5/2)*c-3/4*e/d^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-3/4*e^2
*d*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-3/8*d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2+3/
4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4*e^3/d^2*a*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/4/a^2/e*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(3/2)*x+3/4*d^2/a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-1/3/d^3*e
^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/8*e*c*((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2)+7/8*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c
-5/12*e^2/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/4*d/a^2/e^2*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^2+3/4*d/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*x*c^2-1/2/d^2/a/e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4
/d^3*e^4*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^4*e^5*a^2/
c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/16/d^2*e^5*a^2*ln((1/2*a*
e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*e^3*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c
*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^
(1/2)+1/4/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/16*d^2*
e*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4/d^3*e^4*a*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*x+1/8/d^4*e^5*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)+3/16*e^5/d^2*a^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+17/16*e*d^2*c^2*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)-3/16*e^3*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1
/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-1/2*e^2/d*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c+1/8*e^4/d*a^2/(a*d*e)^(1/2)*ln((2*a
*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{x^{3}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**3*(d + e*x)), x)
```

3.453 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=211

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{8x^2}$$

```
[Out] -1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^3+1/16*(-a*e^2+c*d^2)^3*ar
ctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-1/8*(c/a/e-e/d^2)*(2*a*d*
e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 806, 720, 724, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] -((c/(a*e) - e/d^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(8*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
/(3*d*x^3) + ((c*d^2 - a*e^2)^3*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^
(3/2)*d^(5/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{ade+}}{}}{2ade}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{\left(ade\right.}{}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{\left(ade\right.}{}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{\left(ade\right.}{}$$

Mathematica [A] time = 0.28, size = 188, normalized size = 0.89

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^2\left(8d^2+2dex-3e^2x^2\right)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)}{x^3}\right)}{24a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 +
 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x - 3*e^2*x^2)))/x^3)
 + (3*(c*d^2 - a*e^2)^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*a^(3/2)*d^(5/2)*
e^(3/2))
```

fricas [A] time = 3.68, size = 558, normalized size = 2.64

$$\left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{ade}\,x^3\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+a\right.\right.}{x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (3*a*c^2*d
^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^
4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3), -1/48
*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (3*a*c^2*d^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*
e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^2*a^2*exp(2)^2-4*exp(1)^4*a^2*exp(2)+2*exp(1)^6*a^2)/2/d^2/sqrt(-a*
d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(
1)^3+a*d*exp(1)*exp(2)))-(a^3*exp(2)^3+6*exp(1)^2*a^3*exp(2)^2-24*exp(1)^4*
a^3*exp(2)+16*exp(1)^6*a^3+3*c*d^2*a^2*exp(2)^2+3*c^2*d^4*a*exp(2)-6*c^2*d^
4*exp(1)^2*a+c^3*d^6)/8/d^2/exp(1)/a/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)
))+(-3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^5*a^3*exp(2)^3+30*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^2-24*exp(1)^4*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)-9*c*d^2*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2
*exp(2)^2-9*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a*exp(2)-30*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a-3*c^3*d^6*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5+48*d*exp(1)*sqrt(
c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^4*a^3*exp(2)^2-96*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3*exp(2)+48*d*exp(1
)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^4*a^3+96*c*d^3*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^2*exp(2)+48*c^2*d
^5*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^4*a-8*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)^3-48*d*exp(1)^3*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)^
2+48*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^3*a^4*exp(2)-24*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*exp(2)^2-96*c*d^3*exp(1)^3*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*
exp(2)+48*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^3*a^3-24*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)-48*c^2*d^5*exp(1)^3
```

```
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*
a^2-8*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^3*a+144*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4*exp(2)-96*d^2*ex
p(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^2*a^4+48*c*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^3+3*d^2*exp(
1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)*a^5*exp(2)^3+18*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)^2-24*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)+9*c*d^4*ex
p(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)*a^4*exp(2)^2-48*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4+9*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3*exp(2)-18*c^2*d^6*e
xp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)*a^3+3*c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)*a^2-48*d^3*exp(1)^5*sqrt(c*d*exp(1))*a^5*exp(2)+48
*d^3*exp(1)^7*sqrt(c*d*exp(1))*a^5+16*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*a^4)/
48/d^2/exp(1)/a/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^2-d*exp(1)*a)^3)
```

maple [B] time = 0.02, size = 1945, normalized size = 9.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^4/(e*x+d),x)
```

```
[Out] -3/16/d*e^4*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/24*d/a^3/e^2*c^3*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/12/d/a^2/e^2/x^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)*c+1/16*d^4/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-1/8*
d^2/a^2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-1/3/e/d^2/a^2/x*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+17/24/d^3*e^2*c/a*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(3/2)*x+3/16/d*e^4*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*
e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+1/16/d^5*
e^8*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-1/16/d^5*e^8*a^3/c*ln((1/2*a*e^2-1/2*c
*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)-1/8/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2)+7/12/d^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/8*d/a*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+5/24/e/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(3/2)*c^2+1/8/d^3*e^4*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/3/d^
4*e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/8/d^4*e^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/24/a^3/e^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(5/2)*c^2-1/8*d^3/a^2/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-1/
24*d^2/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^3-1/3/d^2/a/e/x^3*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4/d^4*e^5*a*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*x-1/8/d^5*e^6*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)+3/16*e^3*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-17/24/d^4*e/a/x*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)+11/24/d^2*e/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3
/2)*c+3/8/d^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-1/16/d^2*e^5*
a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-3/16*e*d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2+1/
3/d/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/4/d^4*e^5*a*((x+d/e
)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8/d^5*e^6*a^2/c*((x+d/e)^2*c*d*e
```

```
+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16/d^3*e^6*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d
/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*
d*e)^(1/2)-1/4/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/
16*d*e^2*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^
2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/16/d^3*e^6*a^2*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)-1/16*d*e^2*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e
)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^4\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}}{x^4\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**4*(d + e*x)), x)
```

3.454 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=295

$$-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2a\right.}{64a^2d^3e^2x}$$

```
[Out] -1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^4-1/24*(3*c/a/e-5*e/d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3-1/128*(-a*e^2+c*d^2)^3*(5*a*e^2+
3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+1/64*(-a*e^2+c
*d^2)*(5*a*e^2+3*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2
```

Rubi [A] time = 0.39, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64a^2d^3e^2x^2}-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh}{128a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(3*c*d^2 + 5*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*a^2*d^3*e^2*x^2) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*d*x^4) - (((3*c)/(a*e) - (5*e)/d^2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*x^3) - ((c*d^2 - a*e^2)^3*(3*
c*d^2 + 5*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(5/2)*d^(7/2)*
e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(3cd^2-5ae^2\right)+acde^2x\right)\sqrt{ade+\left(cd^2\right.}}{x^4}}{4ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4}-\frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{24x^3}$$

$$=\frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)}}{64a^2d^3e^2x^2}$$

$$=\frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)}}{64a^2d^3e^2x^2}$$

$$=\frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)}}{64a^2d^3e^2x^2}$$

Mathematica [A] time = 0.30, size = 253, normalized size = 0.86

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{x\left(5ae^2+3cd^2\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2\left(8d^2+2dex-3e^2x^2\right)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)-3x^3\left(cd^2-ae^2\right)^3\tanh\right.}{a^{3/2}d^{5/2}e^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}}\right.}{192adex^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*(a*e + c*d*x)^2*(d + e*x) + ((3*c*d^2 +
 5*a*e^2)*x*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(3*c^2
*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x - 3*e^2*x
^2)) - 3*(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]
*Sqrt[e]*Sqrt[d + e*x])]))/(a^(3/2)*d^(5/2)*e^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[
d + e*x])))/(192*a*d*e*x^4)
```

fricas [A] time = 17.38, size = 704, normalized size = 2.39

$$\left[-\frac{3\left(3c^4d^8 - 4ac^3d^6e^2 - 6a^2c^2d^4e^4 + 12a^3cd^2e^6 - 5a^4e^8\right)\sqrt{ade}\,x^4\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2+4\sqrt{cdex^2+ade+(cd}}{x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*
e^6 - 5*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(
48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5 - 15
*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2*e^6)*
x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*d^4*e^3*x^4), 1/384*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a
^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) -
2*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5 -
 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2*e^
6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^3*a^2*exp(2)^2+4*exp(1)^5*a^2*exp(2)-2*exp(1)^7*a^2)/2/d^3/sqrt(-a
*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp
(1)^3+a*d*exp(1)*exp(2)))+(3*a^4*exp(2)^4+8*exp(1)^2*a^4*exp(2)^3+48*exp(1)
^4*a^4*exp(2)^2-192*exp(1)^6*a^4*exp(2)+128*exp(1)^8*a^4+12*c*d^2*a^3*exp(2
)^3+18*c^2*d^4*a^2*exp(2)^2-24*c^2*d^4*exp(1)^2*a^2*exp(2)+12*c^3*d^6*a*exp
(2)-16*c^3*d^6*exp(1)^2*a+3*c^4*d^8)/64/d^3/exp(1)^2/a^2/2/sqrt(-a*d*exp(1)
)*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)/sqrt(-a*d*exp(1)))+(9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^4+24*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3-240*exp(1)^4*
(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a
```

^4*exp(2)^2+192*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)+36*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)^3+54*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)^2-72*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)+36*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a*exp(2)-48*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a+9*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7-384*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)^2+768*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)-384*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4+384*c^2*d^5*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^2-33*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^4+40*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^3+624*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^2-576*d*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)-132*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^3-384*c*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^2+768*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)-384*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4-198*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^2-888*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)-132*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2*exp(2)-464*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2-33*c^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a+384*d^2*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^3+384*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^2-1920*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)+1152*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5+1152*c*d^4*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4*exp(2)^2+768*c*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4*exp(2)-384*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4+1152*c^2*d^6*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3*exp(2)+384*c^2*d^6*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3+384*c^3*d^8*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^2-33*d^2*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^4-88*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^3-528*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+576*d^2*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)-132*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^3-768*c*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^2-768*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+768*c*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*

```
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5-198*c^2*d^6*exp(1)
^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
3*a^4*exp(2)^2-1272*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)-384*c^2*d^6*exp(1)^6*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4-132*
c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^3*a^3*exp(2)-592*c^3*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3-33*c^4*d^10*exp(1)^2*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2+
1536*d^3*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)-1152*d^3*exp(1)^9*sqrt(c*d*exp
(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^2*a^6+768*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^5*exp(2)+256*c*d^5*exp(1)^7*sq
rt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^2*a^5+768*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4+9*d^3*exp(1)^3*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*
exp(2)^4+24*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3+144*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^2-192*d^3*exp(1
)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
*a^7*exp(2)+36*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^3-384*c*d^5*exp(1)^9*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6+54*c^2*d^7*exp(
1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)*a^5*exp(2)^2-72*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)-384*c^2*d^7*exp(1)^7*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5+36*c^3*d^9
*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)*a^4*exp(2)-48*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4+9*c^4*d^11*exp(1)^3*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3-384*d^4*exp(1)^
8*sqrt(c*d*exp(1))*a^7*exp(2)+384*d^4*exp(1)^10*sqrt(c*d*exp(1))*a^7+128*c*
d^6*exp(1)^8*sqrt(c*d*exp(1))*a^6)/384/d^3/exp(1)^2/a^2/((sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^4)
```

maple [B] time = 0.03, size = 2427, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^5/(e*x+d),x)
```

```
[Out] -3/64/d*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-3/32/d*e^4*a/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/x)*c-133/192/d^4*e^3*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(3/2)*x-3/16/d^2*e^5*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)
+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+3/64*d^3/a^3/e^2*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4+1/8/d/a^2/e^2/x^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+1/64*d^2/a^4/e^3*c^4*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(3/2)*x-1/64*d/a^4/e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
*c^3-3/128*d^5/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4+1/16/d^6*e^9*a^3/c*ln
((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16/d^2*e^5*a*c*ln((1/2*a*e^2-1/2*c*d
^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2))/(c*d*e)^(1/2)-1/16/d^6*e^9*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*
e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-13/48/e/d^2
/a^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+19/192/e^2/d/a^3/x*(c*d*
```

```
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2-91/192/d^2*e/a^2*c^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-1/16*e^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+11/24/d^3/a
/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/32*e/a*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*c^2-29/96/d/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*
c^2+1/16*e^3*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-5/64/d^4*e^5*a*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)-3/32/d^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*c-1/3/d^5*e^4*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-23/64/d^5*e^4
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/16/d^4*e^7*a^2*ln((c*d*e*x+1/2*a
*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d
*e)^(1/2)+133/192/d^5*e^2/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+3/64*
d*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/32/a^3/e^3/x^2*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(5/2)*c^2+3/64*d^4/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)*c^4+1/64*d^3/a^4/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4-1/4/d
^2/a/e/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-53/96/d^3*e^2/a*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-21/64/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x*c+5/128/d^3*e^6*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-59/96/d^4*e/a
/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+91/192/d^3/a^2/x*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)*c+1/4/d^5*e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*x+1/8/d^6*e^7*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/4/d^5
*e^6*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^6*e^7*a^2/c*((
x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/16/d^4*e^7*a^2*ln((1/2*a*e^2-
1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2))/(c*d*e)^(1/2)+1/4/d^3*e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2)*x+1/32/e*d^2/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-5/96
/e^2*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^3-19/192/e/a^3*c^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/32*d^3/a/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x
)*c^3+5/64*d/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^5\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{3}{2}}}{x^{5} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**5*(d + e*x)), x)
```

3.455 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=395

$$\frac{\left(-35a^2e^4+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240a^2d^3e^2x^3}+\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh}{256a^{7/2}d^{9/2}e^{7}}$$

```
[Out] -1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^5-1/40*(3*c/a/e-7*e/d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4+1/240*(-35*a^2*e^4+12*a*c*d^2*e^
2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^3+1/256
*(-a*e^2+c*d^2)^3*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+
(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/128*(-a*e^2+c*d^2)*(7*a^2*e^4+6*a*c*d^2*e^
2+3*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^3/d^4/e^3/x^2
```

Rubi [A] time = 0.51, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^3d^4e^3x^2}+\frac{\left(-35a^2e^4+12a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] -((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*a^3*d^4*e^3*
x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(5*d*x^5) - (((3*c)/(a
*e) - (7*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*x^4) +
((15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(240*a^2*d^3*e^2*x^3) + ((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a
*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(7/2)*d
^(9/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^6}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(3cd^2-7ae^2\right)+2acde^2x\right)\sqrt{ade+\left(cd^2+a\right.}}{x^5}}{5ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{40x^4}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{40x^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(\right.}}{128a^3d^4e^3x^2}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(\right.}}{128a^3d^4e^3x^2}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(\right.}}{128a^3d^4e^3x^2}$$

Mathematica [A] time = 0.49, size = 310, normalized size = 0.78

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{5x^2(7a^2e^4+6acd^2e^2+3c^2d^4)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2(-8d^2-2dex+3e^2x^2)-2acd^2ex(7d+4ex)-3c^2d^4x^2\right)+3x^3(\ldots}{a^{5/2}d^{7/2}e^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}}\right.}{1920adex^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-384*(a*e + c*d*x)^2*(d + e*x) + (48*(5*c*d
^2 + 7*a*e^2)*x*(a*e + c*d*x)^2*(d + e*x))/(a*d*e) + (5*(3*c^2*d^4 + 6*a*c*
d^2*e^2 + 7*a^2*e^4)*x^2*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
+ e*x]*(-3*c^2*d^4*x^2 - 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(-8*d^2 - 2*
d*e*x + 3*e^2*x^2)) + 3*(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c
*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(a^(5/2)*d^(7/2)*e^(5/2)*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x])))/(1920*a*d*e*x^5)
```

fricas [A] time = 40.54, size = 872, normalized size = 2.21

$$\left[-\frac{15\left(3c^5d^{10}-3ac^4d^8e^2-2a^2c^3d^6e^4-6a^3c^2d^4e^6+15a^4cd^2e^8-7a^5e^{10}\right)\sqrt{ade}\,x^5\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e\ldots}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*
d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2
 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e +
a^2*d*e^3)*x)/x^2) + 4*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3*d^7*
e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2*(15*a
^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e^8)*x^3
 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 48*(11*a^
4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(a^4*d^5*e^4*x^5), -1/3840*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^
6*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(-a*d*e)*x^5
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) + 2*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3*d^7*e^3 - 36*
a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2*(15*a^2*c^3*d^
8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e^8)*x^3 + 8*(3*a
^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 48*(11*a^4*c*d^6*e
^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*
e^4*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^4*a^2*exp(2)^2-4*exp(1)^6*a^2*exp(2)+2*exp(1)^8*a^2)/2/d^4/sqrt(-a*
d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(
1)^3+a*d*exp(1)*exp(2)))-(3*a^5*exp(2)^5+6*exp(1)^2*a^5*exp(2)^4+16*exp(1)^
4*a^5*exp(2)^3+96*exp(1)^6*a^5*exp(2)^2-384*exp(1)^8*a^5*exp(2)+256*exp(1)^
10*a^5+15*c*d^2*a^4*exp(2)^4+30*c^2*d^4*a^3*exp(2)^3-36*c^2*d^4*exp(1)^2*a^
3*exp(2)^2+30*c^3*d^6*a^2*exp(2)^2-48*c^3*d^6*exp(1)^2*a^2*exp(2)+16*c^3*d^
6*exp(1)^4*a^2+15*c^4*d^8*a*exp(2)-18*c^4*d^8*exp(1)^2*a+3*c^5*d^10)/128/d^
4/exp(1)^3/a^3/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-(45*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^5+90*
exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^9*a^5*exp(2)^4+240*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^3-2400*exp(1)^6*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^2+192
0*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^9*a^5*exp(2)+225*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^4+450*c^2*d^4*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^3-540*c^2*
d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^9*a^3*exp(2)^2+450*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(2)^2-720*c^3*d^6*exp(1)^2*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*ex
p(2)+240*c^3*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^9*a^2+225*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a*exp(2)-270*c^4*d^8*exp(1)^2*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a+45*c
^5*d^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^9-3840*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5*exp(2)^2+7680*d*exp(1)^7*sqrt(c*
d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^8*a^5*exp(2)-3840*d*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5-210*d*exp(1)*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp
(2)^5-420*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^7*a^6*exp(2)^4+160*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^3+8640*d*exp(1)^7
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*
a^6*exp(2)^2-7680*d*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)-1050*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^4-3840*c
*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^7*a^5*exp(2)^2+7680*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)-3840*c*d^3*exp(1)^9
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*
a^5-2100*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3+2520*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2-2100*
c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^7*a^3*exp(2)^2+3360*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)+4000*c^3*d^7*exp
(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^7*a^3-1050*c^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)+1260*c^4*d^9*exp(1)^3*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2-210*c^5*d^1
1*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^7*a+3840*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^3+7680*d^2*exp(1)^6*
```

```
sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^6*a^6*exp(2)^2-26880*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)+
15360*d^2*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6-3840*c*d^4*exp(1)^4*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6
*a^5*exp(2)^2+7680*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)-3840*c*d^4*exp
(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^6*a^5-19200*c^2*d^6*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)
-11520*c^3*d^8*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^3+384*d^2*exp(1)^2*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^5-12
80*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^5*a^7*exp(2)^3-11520*d^2*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^2+11520*d^2*exp(1)
^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
^5*a^7*exp(2)+1920*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^4+7680*c*d^4*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)
^3-3840*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^2-15360*c*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)+11520*c*d
^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^5*a^6+3840*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^3+23040*c^2*d^6*exp(1)^4*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5
*exp(2)^2+7680*c^2*d^6*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)-3840*c^2*d^6*exp(1)^8*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5+3840*c^3
*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^5*a^4*exp(2)^2+23040*c^3*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)+10240*c^3*d^8*ex
p(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^5*a^4+1920*c^4*d^10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)+7680*c^4*d^10*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3+384*c^
5*d^12*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^5*a^2-3840*d^3*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^4-3840*d^3*
exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^3-3840*d^3*exp(1)^7*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7
*exp(2)^2+34560*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)-23040*d^3*exp(1)^11
*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^4*a^7-15360*c*d^5*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)^3-1920
0*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)^2+6400*c*d^5*exp(1)^9*sqrt(c*d*
exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^4*a^6-23040*c^2*d^7*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^2-26880*c^2*d^
7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)-3840*c^2*d^7*exp(1)^7*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4
*a^5-15360*c^3*d^9*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4*exp(2)-11520*c^3*d^9*exp(1)
```

^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4-3840*c^4*d^11*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3+210*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^5+420*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^4+1120*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^3+6720*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^2-7680*d^3*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)+1050*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^4+7680*c*d^5*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^3+7680*c*d^5*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^2+7680*c*d^5*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)-11520*c*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7+2100*c^2*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^3+20520*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+19200*c^2*d^7*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)+2100*c^3*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^2+19680*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+12640*c^3*d^9*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5+1050*c^4*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)+6420*c^4*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4+210*c^5*d^13*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3-19200*d^4*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)+15360*d^4*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8-7680*c*d^6*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7*exp(2)^2-7680*c*d^6*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7*exp(2)-1280*c*d^6*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7-15360*c^2*d^8*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)-7680*c^2*d^8*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6-7680*c^3*d^10*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^5-45*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^5-90*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^4-240*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^3-1440*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^2+1920*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)-225*c*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^4+3840*c*d^6*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8-450*c^2*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3+540*c^2*d^8*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^2+3840*c^2*d^8*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)+3840*c^2*d^8*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7-450*c^3*d^10*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^2+720*c^3*d^10*e

```
xp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)*a^6*exp(2)+3600*c^3*d^10*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6-225*c^4*d^12*exp(1)^4*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)+270*c
^4*d^12*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^5-45*c^5*d^14*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4+3840*d^5*exp(1)^11*sqrt(c*d*exp(1))
*a^9*exp(2)-3840*d^5*exp(1)^13*sqrt(c*d*exp(1))*a^9-1280*c*d^7*exp(1)^11*sq
rt(c*d*exp(1))*a^8-768*c^2*d^9*exp(1)^9*sqrt(c*d*exp(1))*a^7)/3840/d^4/exp(
1)^3/a^3/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^2-d*exp(1)*a)^5)
```

maple [B] time = 0.03, size = 2888, normalized size = 7.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^6/(e*x+d),x)
```

```
[Out] 7/128/d^5*e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/32*d/a^2*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-3/128*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*
e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^
2-1/8/d^3*e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/8/d^3/a/x^4
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+7/48/e/a^3*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(3/2)*c^3+15/128/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
*c+1/3/d^6*e^5*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+45/128/d^6*e^5
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+121/192/d^5*e^2/a/x^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(5/2)+19/48/d^2*e/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(3/2)*c^2+1/32/d*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+1/4/d
^6*e^7*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8/d^7*e^8*a^2/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16/d^5*e^8*a^2*ln((1/2*a*e^
2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2))/(c*d*e)^(1/2)-1/4/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)*x+1/16/d*e^4*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)
^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16/a^
3/e^3/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2-3/64/a^4/e^3/x*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3-1/5/d^2/a/e/x^5*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(5/2)-3/128*d^3/a^3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*c^4+3/128*d^2/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4-1/128*d
^4/a^5/e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^5-3/128*d^5/a^4/e^4*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5-3/16/d^5*e^8*a^2*ln((c*d*e*x+1/2*
a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*
d*e)^(1/2)-1/16/d*e^4*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-263/384/d^6*e^3/a/x*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+227/384/d^4*e^3/a*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)*c+39/128/d^4*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x*c-7/256/d^4*e^7*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-1/32*e/a^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3+23/96/d/a^3*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)*x+73/192/d^3/a^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
*c-25/48/d^4*e/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4/d^6*e^7*a*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/8/d^7*e^8*a^2/c*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)+1/8/d/a^2/e^2/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(5/2)*c-1/4/d^2/a^2/e/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-1/128
*d^2*e/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-23/96/d^2/e/a^3/x*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(5/2)*c^2+1/16/d^7*e^10*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1
03/192/d^3*e^2/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-103/192/d^
4*e/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+3/64/d^2*e^3/a*(c*d*e*x
```

```
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2+263/384/d^5*e^4*c/a*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(3/2)*x+3/16/d^3*e^6*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+15/256
/d^2*e^5*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-3/64/e*d^2/a^3*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*x*c^4-1/16/d^7*e^10*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/
e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d
*e)^(1/2)-3/16/d^3*e^6*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(
1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/256*d^4
/a^2/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4+3/256*d^6/a^3/e^3/(a*d*e)^(1/2)*ln((2
*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2))/x)*c^5-3/128*d^4/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^5
+3/64*d/a^4/e^2*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/128*d^3/a^5
/e^4*c^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+9/64/d/a^3/e^2/x^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+1/64*d/a^4/e^4/x^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(5/2)*c^3+1/128*d^2/a^5/e^5/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(5/2)*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"maxima")
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^6\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
[Out] Timed out
```

3.456 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=498

$$\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4}+\frac{\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(x\left(\right.\right.}{512a}$$

```
[Out] -1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^6-1/20*(c/a/e-3*e/d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5+1/160*(-21*a^2*e^4+6*a*c*d^2*e^2+7
*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/960*(-1
05*a^3*e^6+21*a^2*c*d^2*e^4+33*a*c^2*d^4*e^2+35*c^3*d^6)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^3-1/1024*(-a*e^2+c*d^2)^3*(21*a^3*e^6+2
1*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d
^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(
9/2)/d^(11/2)/e^(9/2)+1/512*(-21*a^4*e^8+6*a^2*c^2*d^4*e^4+8*a*c^3*d^6*e^2+
7*c^4*d^8)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^4/d^5/e^4/x^2
```

Rubi [A] time = 0.72, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(21a^2cd^2e^4-105a^3e^6+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}+\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)}{160a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^4*d^
5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(6*d*x^6) - ((c/
(a*e) - (3*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*x^5)
+ ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 2
1*a^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
))/(960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2
 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*
Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(102
4*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^7}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\int \frac{\left(-\frac{3}{2}ae\left(cd^2-3ae^2\right)+3acde^2x\right)\sqrt{ade+\left(cd^2\right.}}{x^6}}{6ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{20x^5}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{20x^5}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{20x^5}$$

$$=\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ad}}{512a^4d^5e^4x^2}$$

$$=\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ad}}{512a^4d^5e^4x^2}$$

$$=\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ad}}{512a^4d^5e^4x^2}$$

Mathematica [A] time = 0.77, size = 380, normalized size = 0.76

$$-\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{16x^2(d+ex)\left(63a^2e^4+54acd^2e^2+35c^2d^4\right)(ae+cdx)^2}{a^2d^2e^2}+\frac{5x^3\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] -1/7680*(Sqrt[(a*e + c*d*x)*(d + e*x)]*(1280*(a*e + c*d*x)^2*(d + e*x) - (1
28*(7*c*d^2 + 9*a*e^2)*x*(a*e + c*d*x)^2*(d + e*x))/(a*d*e) + (16*(35*c^2*d
^4 + 54*a*c*d^2*e^2 + 63*a^2*e^4)*x^2*(a*e + c*d*x)^2*(d + e*x))/(a^2*d^2*e
^2) + (5*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*x^3
*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c^2*d^4*x^2 -
 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(-8*d^2 - 2*d*e*x + 3*e^2*x^2)) + 3*
(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*
Sqrt[d + e*x])]))/(a^(7/2)*d^(9/2)*e^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])
))/(a*d*e*x^6)
```

fricas [A] time = 112.22, size = 1072, normalized size = 2.15

$$\left[-\frac{15\left(7c^6d^{12}-6ac^5d^{10}e^2-3a^2c^4d^8e^4-4a^3c^3d^6e^6-15a^4c^2d^4e^8+42a^5cd^2e^{10}-21a^6e^{12}\right)\sqrt{ade}\,x^6\log\left(\frac{8a^2d^2e}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^
3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(a*d*
e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 - (105*a*
c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5*e^7 +
 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 - 16*a^3*
c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*e^10)*x^
4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 - 21*a^6*d^
3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4*e^8)*x^2
+ 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^5*d^6*e^5*x^6), 1/15360*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 -
 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*
e^10 - 21*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^
2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(1280*a^6*d^6*e^6 - (105*
a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5*e^7
 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 - 16*a^
3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*e^10)*
x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 - 21*a^6*
d^3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4*e^8)*x^
2 + 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a^5*d^6*e^5*x^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^5*a^2*exp(2)^2+4*exp(1)^7*a^2*exp(2)-2*exp(1)^9*a^2)/2/d^5/sqrt(-a
*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp
(1)^3+a*d*exp(1)*exp(2)))+(7*a^6*exp(2)^6+12*exp(1)^2*a^6*exp(2)^5+24*exp(1
)^4*a^6*exp(2)^4+64*exp(1)^6*a^6*exp(2)^3+384*exp(1)^8*a^6*exp(2)^2-1536*ex
p(1)^10*a^6*exp(2)+1024*exp(1)^12*a^6+42*c*d^2*a^5*exp(2)^5+105*c^2*d^4*a^4
*exp(2)^4-120*c^2*d^4*exp(1)^2*a^4*exp(2)^3+140*c^3*d^6*a^3*exp(2)^3-240*c^
3*d^6*exp(1)^2*a^3*exp(2)^2+96*c^3*d^6*exp(1)^4*a^3*exp(2)+105*c^4*d^8*a^2*
exp(2)^2-180*c^4*d^8*exp(1)^2*a^2*exp(2)+72*c^4*d^8*exp(1)^4*a^2+42*c^5*d^1
0*a*exp(2)-48*c^5*d^10*exp(1)^2*a+7*c^6*d^12)/512/d^5/exp(1)^4/a^4/2/sqrt(-
a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)/sqrt(-a*d*exp(1)))-(-105*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^6-180*exp(1)^2*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)
^5-360*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^11*a^6*exp(2)^4-960*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^3+9600*exp(1)^8*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*ex
p(2)^2-7680*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^11*a^6*exp(2)-630*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^5*exp(2)^5-1575*c^2*d^4*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^4*ex
p(2)^4+1800*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^11*a^4*exp(2)^3-2100*c^3*d^6*(sqrt(a*d*exp(1)+a*x
```

*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^3*exp(2)^3+3600*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^3*exp(2)^2-1440*c^3*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^3*exp(2)-1575*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^2*exp(2)^2+2700*c^4*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^2*exp(2)-1080*c^4*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^2-630*c^5*d^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a*exp(2)+720*c^5*d^10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a-105*c^6*d^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11+15360*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^10*a^6*exp(2)^2-30720*d*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^10*a^6*exp(2)+15360*d*exp(1)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^10*a^6+595*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^6+1020*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^5+2040*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^4+320*d*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^3-44160*d*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^2+38400*d*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)+3570*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6*exp(2)^5+15360*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6*exp(2)^2-30720*c*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6*exp(2)+15360*c*d^3*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6+8925*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^4-10200*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^3+11900*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^3-20400*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^2+8160*c^3*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)+8925*c^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^2-15300*c^4*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)+6120*c^4*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3+3570*c^5*d^11*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(2)-4080*c^5*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2+595*c^6*d^13*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a-15360*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^7*exp(2)^3-46080*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^7*exp(2)^2+138240*d^2*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^7*exp(2)-76800*d^2*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^7+15360*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^6*exp(2)^2-30720*c*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^6*exp(2)+15360*c*d^4*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^6-30720*c^3*d^8*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^4-1386*d^2*exp(1)^2*(sqrt(a*d*

exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp(2)^
6-2376*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^7*a^8*exp(2)^5-1680*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp(2)^4+5760*d^2*exp(
1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^7*a^8*exp(2)^3+80640*d^2*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp(2)^2-76800*d^2*exp(1)^12*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp
(2)-8316*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^7*a^7*exp(2)^5-30720*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^7*exp(2)^3+92160*
c*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^7*a^7*exp(2)-61440*c*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^7-20790*c^2*d^6*exp(1)^2*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^
6*exp(2)^4+23760*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^3+15360*c^2*d^6*exp(1)^6*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp
(2)^2-30720*c^2*d^6*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)+15360*c^2*d^6*exp(1)^10*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6-27720*c^3
*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^7*a^5*exp(2)^3+47520*c^3*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^2+116160*c^3*d^8
exp(1)^6(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^7*a^5*exp(2)-20790*c^4*d^10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2+35640*c^4*d^10*exp(
1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^7*a^4*exp(2)+71760*c^4*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4-8316*c^5*d^12*exp(1)^2*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)+
9504*c^5*d^12*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^7*a^3-1386*c^6*d^14*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2+15360*d^3*exp(1)^5*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^6*a^8*exp(2)^4+30720*d^3*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^8*exp(2)^3+46
080*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^8*exp(2)^2-245760*d^3*exp(1)^11*sqrt(c*d
exp(1))(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^6*a^8*exp(2)+153600*d^3*exp(1)^13*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^8-51200*c*d^5*exp
(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^6*a^7*exp(2)^3+61440*c*d^5*exp(1)^7*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^7
*exp(2)^2+30720*c*d^5*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^7*exp(2)-40960*c*d^5*exp(1
)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^6*a^7-245760*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)
^2-30720*c^2*d^7*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)+15360*c^2*d^7*exp(1)^9
*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^6*a^6-276480*c^3*d^9*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)-614
40*c^3*d^9*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5-97280*c^4*d^11*exp(1)^5*sqrt(c*d*ex
p(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^6*a^4+1686*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(

1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^6+696*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^5-1680*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^4-9600*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^3-72960*d^3*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^2+76800*d^3*exp(1)^13*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)+10116*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8*exp(2)^5+46080*c*d^5*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8*exp(2)^4-46080*c*d^5*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8*exp(2)^2-92160*c*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8*exp(2)+92160*c*d^5*exp(1)^13*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8+25290*c^2*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^4+177360*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^3+138240*c^2*d^7*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^2-46080*c^2*d^7*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)-15360*c^2*d^7*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7+33720*c^3*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^3+262560*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^2+269760*c^3*d^9*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)+15360*c^3*d^9*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6+25290*c^4*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^2+173880*c^4*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)+133200*c^4*d^11*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5+10116*c^5*d^13*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)+43296*c^5*d^13*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4+1686*c^6*d^15*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3-15360*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^5-15360*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^4-15360*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^3-15360*d^4*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^2+215040*d^4*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)-153600*d^4*exp(1)^14*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9-76800*c*d^6*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)^4-122880*c*d^6*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)^3-46080*c*d^6*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)^2+30720*c*d^6*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)+30720*c*d^6*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^8-153600*c^2*d^8*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^3-276480*c^2*d^8*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^2-138240*c^2*d^8*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4* a^7*exp(2)+15360*c^2*d^8*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7-153600*c^3*d^10*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)^2-245760*c^3*d^10*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)-107520*c^3*d^10*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6-76800*c^4*d^12*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)-76800*c^4*d^12*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5-15360*c^5*d^14*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4+595*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^6+1020*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^5+2040*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^4+5440*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^3+32640*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^2-38400*d^4*exp(1)^14*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)+3570*c*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^5+30720*c*d^6*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^4+30720*c*d^6*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^3+30720*c*d^6*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^2+30720*c*d^6*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)-61440*c*d^6*exp(1)^14*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9+8925*c^2*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^4+112680*c^2*d^8*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^3+138240*c^2*d^8*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^2+61440*c^2*d^8*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)-15360*c^2*d^8*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8+11900*c^3*d^10*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^3+163920*c^3*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^2+192480*c^3*d^10*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)+51200*c^3*d^10*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7+8925*c^4*d^12*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+107580*c^4*d^12*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)+82920*c^4*d^12*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6+3570*c^5*d^14*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+26640*c^5*d^14*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5+595*c^6*d^16*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4-92160*d^5*exp(1)^13*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^10*exp(2)+76800*d^5*exp(1)^15*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^10-30720*c*d^7*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^9*exp(2)^3-30720*c*d^7*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^9*exp(2)^2-30720*c*d^7*exp(1)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^9*exp(2)-92160*c^2*d^9*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1

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)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)^2-737
28*c^2*d^9*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)-27648*c^2*d^9*exp(1)^11*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^2*a^8-92160*c^3*d^11*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7*exp(2)-43008*c^
3*d^11*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7-30720*c^4*d^13*exp(1)^7*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2
*a^6-105*d^5*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)*a^11*exp(2)^6-180*d^5*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^11*exp(2)^5-360*d^5*exp(1)
^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^11*exp(2)^4-960*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)*a^11*exp(2)^3-5760*d^5*exp(1)^13*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^11*exp(2)^2+76
80*d^5*exp(1)^15*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^11*exp(2)-630*c*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^10*exp(2)^5+15360*c*d^7*exp(1)^
15*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^10-1575*c^2*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^4+1800*c^2*d^9*exp(1)^7*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^3+15360*
c^2*d^9*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^9*exp(2)^2+15360*c^2*d^9*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)+15360*c^2*d^9*ex
p(1)^13*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)*a^9-2100*c^3*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^3+3600*c^3*d^11*exp(1)^7*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^2+
29280*c^3*d^11*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)*a^8*exp(2)+15360*c^3*d^11*exp(1)^11*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8-1575*c^4*d^13*exp
(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)*a^7*exp(2)^2+2700*c^4*d^13*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)+14280*c^4*d^13*exp(1)^9*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7-630
*c^5*d^15*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt
(c*d*exp(1))*x)*a^6*exp(2)+720*c^5*d^15*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6-105*c^6*d^17*exp(1)^5*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5+15
360*d^6*exp(1)^14*sqrt(c*d*exp(1))*a^11*exp(2)-15360*d^6*exp(1)^16*sqrt(c*d
*exp(1))*a^11-5120*c*d^8*exp(1)^14*sqrt(c*d*exp(1))*a^10-3072*c^2*d^10*exp(
1)^10*sqrt(c*d*exp(1))*a^9*exp(2)-3072*c^2*d^10*exp(1)^12*sqrt(c*d*exp(1))*
a^9-3072*c^3*d^12*exp(1)^10*sqrt(c*d*exp(1))*a^8)/15360/d^5/exp(1)^4/a^4/((
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*
exp(1)*a)^6)
```

maple [B] time = 0.04, size = 3387, normalized size = 6.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^7/(e*x+d),x)
```

```
[Out] 1/8/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+19/60/d^3/a/x^5
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/64*e/a^2*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*c^3-23/96/d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^
3-21/512/d^6*e^7*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/8/d^4*e^5*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-15/512/d^2*e^3/a*(c*d*e*x^2+a*d*e+(a*
```

e^2+c*d^2)*x)^(1/2)*c^2-703/1536/d^3*e^2/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(3/2)*c^2-491/768/d^6*e^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
+257/768/d^3/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+107/192/d^5*
e^2/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/3/d^7*e^6*((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-533/1536/d^7*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)+1/4/d^7*e^8*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/8/
d^8*e^9*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/16/d^6*e^9*a^2*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4/d^5*e^6*c*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)*x-1/16/d^2*e^5*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*
e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/
2)+7/1536*d^5/a^6/e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^6+7/512*d^6
/a^5/e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^6+7/256*d/a^3*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4-1/6/d^2/a/e/x^6*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(5/2)-7/96/a^3/e^3/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c
^2-1/4/d^7*e^8*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^8*e^
9*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/12/e^3/a^4/x^2*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3-109/768/e/a^4*c^4*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(3/2)*x+3/1024*d^3/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4+65/1
92/d^3/a^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-5/384/e^4*d^3/a^5*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^5-43/96*e/d^4/a/x^4*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)+13/512/e*d^2/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)*c^4+1/64/e^3*d^4/a^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5-1
31/1536/e^2*d/a^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4+3/16/d^6*e^9*
a^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/16/d^2*e^5*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c
*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+
1045/1536/d^7*e^4/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-235/384/d^5*e
^4/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-149/512/d^5*e^6*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-91/384/e/d^2/a^3/x^2*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(5/2)*c^2+29/192/e^2/d/a^3/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(5/2)*c^2+109/768/e^2/d/a^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3
+41/1536/e^4*d/a^5/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^4+3/512/e^2*
d^5/a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^5-41/1536/e^3*d^2/a^5*c^5*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+15/512/e^2*d^3/a^4*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)*x*c^5-11/48/e/d^2/a^2/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
5/2)*c-1/16/d^8*e^11*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-43/96/d^4*e/a^2/x^2*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+1/256*d*e^2/a/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/x)*c^3+3/256/d*e^2/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-25
7/768/d^2*e/a^3*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-877/1536/d^4*
e^3/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+877/1536/d^5*e^2/a^2/
x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-21/512/d^3*e^4/a*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-1045/1536/d^6*e^5*c/a*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(3/2)*x-21/512/d^3*e^6*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-3/16/d^4
*e^7*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+7/192*d/a^4/e^4/x^3*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(5/2)*c^3-7/1536*d^3/a^6/e^6/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(5/2)*c^5-7/768*d^2/a^5/e^5/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*c^4-7/1024*d^7/a^4/e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^6+7/1536*d^4/a^6/e^5*
c^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+7/60/d/a^2/e^2/x^5*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+7/512*d^5/a^5/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*c^6+1/16/d^8*e^11*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*
d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(

```
1/2)+3/16/d^4*e^7*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+21/1024/d^5*e^
8*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+15/1024/d*e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c
^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^7\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

3.457 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=574

$$-\frac{\left(-105a^3e^6-10cdex\left(-15a^2e^4-10acd^2e^2+33c^2d^4\right)-95a^2cd^2e^4-15ac^2d^4e^2+231c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+\right.}{4480c^3d^3e^4}$$

```
[Out] 1/2048*(-a*e^2+c*d^2)*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*
d^6)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^
4/e^5+1/112*(5*a/c/d-11*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+
1/8*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-1/4480*(231*c^3*d^6-15*a*
c^2*d^4*e^2-95*a^2*c*d^2*e^4-105*a^3*e^6-10*c*d*e*(-15*a^2*e^4-10*a*c*d^2*e
^2+33*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e^4+3/327
68*(-a*e^2+c*d^2)^5*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^
6)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(11/2)/d^(11/2)/e^(13/2)-3/16384*(-a*e^2+c*d
^2)^3*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^6)*(2*c*d*e*x+
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^6
```

Rubi [A] time = 0.69, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {849, 832, 779, 612, 621, 206}

$$-\frac{3\left(35a^2cd^2e^4+15a^3e^6+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^6}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 1
5*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)*(33*c^3*d^6 + 45*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (
11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/112 + (x^3*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((231*c^3*d^6 - 15*a*c
^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c
*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35
*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(1
1/2)*d^(11/2)*e^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \int x^3(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}dx$$

$$= \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e} + \frac{\int x^2\left(-3acd^2e-\frac{1}{2}cd\left(11cd^2-5\right.\right.}{}$$

$$= \frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{x^3\left(ade+\left(cd^2\right.\right.}{}$$

$$= \frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{x^3\left(ade+\left(cd^2\right.\right.}{}$$

$$= \frac{\left(cd^2-ae^2\right)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+\right.}{2048c^4d^4e^5}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae\right.}{16384c^5d^5e^6}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae\right.}{16384c^5d^5e^6}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae\right.}{16384c^5d^5e^6}$$

Mathematica [A] time = 3.61, size = 681, normalized size = 1.19

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{105\sqrt{c}\sqrt{d}\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\frac{\sqrt{e}\left(1575a^8e^{15}-525a^7cde^{1}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*(1575*a^8*e^15 - 525*a^7*c*d*e^13*
(7*d - e*x) + 35*a^6*c^2*d^2*e^11*(29*d^2 - 37*d*e*x - 6*e^2*x^2) + 5*a^5*c
^3*d^3*e^9*(185*d^3 + 93*d^2*e*x + 100*d*e^2*x^2 + 24*e^3*x^3) + 5*a^4*c^4*
d^4*e^7*(265*d^4 + 65*d^3*e*x - 30*d^2*e^2*x^2 - 56*d*e^3*x^3 - 16*e^4*x^4)
 + a^3*c^5*d^5*e^5*(-11193*d^5 + 8359*d^4*e*x - 6088*d^3*e^2*x^2 + 5040*d^2
*e^3*x^3 + 139200*d*e^4*x^4 + 104320*e^5*x^5) + a^2*c^6*d^6*e^3*(11445*d^6
- 18669*d^5*e*x + 12962*d^4*e^2*x^2 - 10544*d^3*e^3*x^3 + 9120*d^2*e^4*x^4
+ 350080*d*e^5*x^5 + 272640*e^6*x^6) + c^8*d^8*x*(-3465*d^7 + 2310*d^6*e*x
- 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e^4*x^4 + 1280*d^2*e^5*x^5
 + 87040*d*e^6*x^6 + 71680*e^7*x^7) + a*c^7*d^7*e*(-3465*d^7 + 13755*d^6*e*
x - 9324*d^5*e^2*x^2 + 7512*d^4*e^3*x^3 - 6464*d^3*e^4*x^4 + 5760*d^2*e^5*x
^5 + 299520*d*e^6*x^6 + 240640*e^7*x^7)))/(a*e + c*d*x) + (105*Sqrt[c]*Sqrt
[d]*(c*d^2 - a*e^2)^(9/2)*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4
 + 15*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)])))/(573440*c^5*d^5*e^(13/2))
```

fricas [A] time = 1.23, size = 1524, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 -
40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*
d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x
^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*
e^3)*x) + 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3
 - 11193*a^2*c^6*d^11*e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 10
15*a^5*c^3*d^5*e^11 - 3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*
c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^
8 + 81*a^2*c^6*d^6*e^10)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 10
75*a^2*c^6*d^7*e^9 - 5*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*
c^7*d^10*e^6 + 290*a^2*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*
e^12)*x^3 - 8*(231*c^8*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7
- 50*a^3*c^5*d^7*e^9 + 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2
*(1155*c^8*d^14*e^2 - 3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3
*c^5*d^8*e^8 - 275*a^4*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d
^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7), -1/
1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 - 40*
a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*d^4
*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/
(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(71680*c^8
*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3 - 11193*a^2*c^6*d^11*
e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 1015*a^5*c^3*d^5*e^11 -
3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*c^8*d^9*e^7 + 33*a*c^7
*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^8 + 81*a^2*c^6*d^6*e^1
0)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 1075*a^2*c^6*d^7*e^9 - 5
*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*c^7*d^10*e^6 + 290*a^2
*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*e^12)*x^3 - 8*(231*c^8
*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7 - 50*a^3*c^5*d^7*e^9 +
 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 -
3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3*c^5*d^8*e^8 - 275*a^4
*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
```

```
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Warning, replacing 0 by ` u`, a substitutio
n variable should perhaps be purged.Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Warning, replacing 0 by ` u`, a sub
stitution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
 substitution variable should perhaps be purged.Warning, replacing 0 by ` u
`, a substitution variable should perhaps be purged.Warning, replacing 0 by
 ` u`, a substitution variable should perhaps be purged.Evaluation time: 0.
5Error: Bad Argument Type
```

maple [B] time = 0.03, size = 3178, normalized size = 5.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] -975/16384*e^2/c*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+195/4096/e^4*c^2*d^9*ln
((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2))/(c*d*e)^(1/2)*a+3/64*e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(5/2)*x*a^2+9/64*d^6/e^3*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
*x-3/256*d*e^4*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)
+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128*d^7/e^
2*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^9/e^4*a*c^2*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*d^2*e*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)*x+15/256*d^3*e^2*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*
c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)
^(1/2)-5/256*e^2/c^2*a^3/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/102
4*e^4/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^4+45/8192*e^7/c^4
/d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^6-855/8192/e^2*c*d^7*ln((c
*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2))/(c*d*e)^(1/2)*a^2-495/4096/e^3*c*d^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*x*a-15/4096*e^5/c^3/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x
*a^5-105/2048*e/c*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3-45/3276
8*e^10/c^5/d^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^8+15/4096*e^8/c^4/d^3*ln((c*d*e
*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/(c*d*e)^(1/2)*a^7-15/8192*e^6/c^3/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^6+45/4
096*e^4/c^2*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^5-3/64*d^3*a^3/c*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*d^6/e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(3/2)-3/128*d^9/e^6*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-15/128*d^5*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-95/2048/e^5*c*d
^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+285/16384/e^6*c^2*d^9*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+45/2048/e^3*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(3/2)*a+165/16384/e^2*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+
19/64/e^3*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-15/1024*e/c^2*(c*d*
```

```
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3+465/4096*d^5*ln((c*d*e*x+1/2*a*e^2+1
/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1
/2)*a^3+735/16384/c*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+13/128/
c^2/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*a^2-1/5*d^3/e^4*((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+19/128/e^4*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(5/2)-25/112/e^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+65/1024/e
/c*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2+285/8192/e^5*c^2*d^8*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/8*d^3/e^2*a*((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(3/2)*x+1/8*d^5/e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(3/2)*x-1/16*d^2/e*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)
-9/64*d^4/e*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/128*d*e^2
*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/64*d^7/e^4*a*c*((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64*d^8/e^5*c^2*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d^11/e^6*c^3*ln((1/2*a*e^2-1/2*c*d^2
+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2)+45/16384*e^8/c^5/d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*a^7+15/16384*e^6/c^4/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^6-75/1
6384*e^4/c^3/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^5-465/16384*e^2/c^
2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-285/32768/e^6*c^3*d^11*ln((
c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2))/(c*d*e)^(1/2)+3/128*e^2/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(5/2)*a^3+29/128/e^2/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*a-5/512/c*
d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^2+35/256/e^2*a*d^3*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+5/32/e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(5/2)*x*a-45/8192*e^3/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^4+115
5/8192/e*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+1/8/e^2*x*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)/c/d-705/16384/e^4*c*d^7*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*a-9/112/e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(7/2)*a-95/1024/e^4*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/2048
*e^5/c^4/d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^5-35/2048*e^3/c^3/d^
2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

3.458 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=452

$$\frac{\left(-35a^2e^4-10cdex\left(9cd^2-5ae^2\right)-20acd^2e^2+63c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{840c^2d^2e^3}-\frac{\left(5a^2e^4+10acd^2e^2+\right.}{}$$

```
[Out] -1/384*(-a*e^2+c*d^2)*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4+1/7*x^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e+1/840*(63*c^2*d^4-20*a*c*d^2*e^2-35*a^2*
e^4-10*c*d*e*(-5*a*e^2+9*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
c^2/d^2/e^3-1/2048*(-a*e^2+c*d^2)^5*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*ar
ctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/1024*(-a*e^2+c*d^2)^3*(5
*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5
```

Rubi [A] time = 0.41, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 832, 779, 612, 621, 206}

$$\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^4d^4e^5}-\frac{\left(5a^2e^4+10acd^2e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^5
) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*
e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) + ((63*c^2
*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*c^2*d^2*e^3) - ((c*d^2 - a*e^
2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(2048*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \int x^2(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$= \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e} + \frac{\int x\left(-2acd^2e-\frac{1}{2}cd\left(9cd^2-5ae\right.\right.}{}$$

$$= \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e} + \frac{\left(63c^2d^4-20acd^2e^2-35a^2e^4-\right.}{}$$

$$= -\frac{\left(cd^2-ae^2\right)\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(\right.\right.}{384c^3d^3e^4}$$

$$= \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{1024c^4d^4e^5}$$

$$= \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{1024c^4d^4e^5}$$

$$= \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{1024c^4d^4e^5}$$

Mathematica [A] time = 5.71, size = 562, normalized size = 1.24

$$((d+ex)(ae+cdx))^{3/2}\left(\frac{7\sqrt{cd}\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(15\sqrt{e}\sqrt{cd}\left(cd^2-ae^2\right)^{11/2}(ae+cdx)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-15\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^6\sqrt{ae+cdx}\sinh^{-1}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-1/12*((9*c*d^2 + 7*a*e^2)*(a*e + c*d*x)^
2*(d + e*x))/(c*d*e) + x*(a*e + c*d*x)^2*(d + e*x) + (7*Sqrt[c*d]*(9*c^2*d^
4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(15*Sqrt[c*d]*Sqrt[e]*(c*d^2 - a*e^2)^(11/2
)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - 10*Sqrt[c*d]*e^(3/2
)*(c*d^2 - a*e^2)^(9/2)*(a*e + c*d*x)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2
)] + 8*Sqrt[c*d]*e^(5/2)*(c*d^2 - a*e^2)^(7/2)*(a*e + c*d*x)^3*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)] + 16*Sqrt[c*d]*e^(7/2)*(c*d^2 - a*e^2)^(3/2)*(a*e
 + c*d*x)^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*(-3*a*e^2 + c*d*(11*d + 8
*e*x)) - 15*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^6*Sqrt[a*e + c*d*x]*ArcSinh[(Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))
/(15360*c^3*d^3*e^(9/2)*(c*d^2 - a*e^2)^(5/2)*(a*e + c*d*x)^2*((c*d*(d + e*
x))/(c*d^2 - a*e^2))^(3/2))))/(7*c*d*e)
```

fricas [A] time = 0.84, size = 1272, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*
a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^1
2 - 5*a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d
^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15360*c^7*d^7*e
^7*x^6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*
a^3*c^4*d^7*e^7 - 525*a^4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d
*e^13 + 1280*(15*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 +
 380*a*c^6*d^7*e^7 + 185*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*
c^6*d^8*e^6 - 2095*a^2*c^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d
^11*e^3 - 218*a*c^6*d^9*e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35
*a^4*c^3*d^3*e^11)*x^2 - 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a
^2*c^5*d^8*e^6 - 150*a^3*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d
^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6), 1/2
15040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*a^3*c
^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^12 - 5
*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15360*c^7*d^7*e^7*x^6 + 945*c^7*d^13*e
 - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*a^3*c^4*d^7*e^7 - 525*a^
4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d*e^13 + 1280*(15*c^7*d^8
*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 + 380*a*c^6*d^7*e^7 + 185
*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*c^6*d^8*e^6 - 2095*a^2*c
^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d^11*e^3 - 218*a*c^6*d^9*
e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35*a^4*c^3*d^3*e^11)*x^2 -
 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a^2*c^5*d^8*e^6 - 150*a^3
*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Warning, replacing 0 by ` u`, a substitutio
n variable should perhaps be purged.Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Warning, replacing 0 by ` u`, a sub
stitution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
 substitution variable should perhaps be purged.Evaluation time: 0.52Error:
 Bad Argument Type
```

maple [B] time = 0.02, size = 2731, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] 195/2048/e*c*d^6*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-5/512*e^6/c^3/d^3*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^5-3/64*d*e^2*a^3/c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)*x-9/64*d^5/e^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)*x-15/128*d^6/e*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/
(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+
15/256*d^8/e^3*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^2*e^3*
a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+5/512*e^4/c^2/d*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^4+15/256*e^2/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)*x*a^3-85/2048/e^3*c^2*d^8*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d
*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+5/192*e^
3/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^3+5/2048*e^9/c^4/d^4*
ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2))/(c*d*e)^(1/2)*a^7-15/2048*e^7/c^3/d^2*ln((c*d*e*x+1/2*a*e^2+1/
```

```
2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/
2)*a^6+55/512/e^2*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a+125/204
8*e^3/c*d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4-1/16*d^5/e^4*c*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128*d^8/e^5*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^
2)*(x+d/e))^(1/2)+9/64*d^3*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2
)*x-3/128*e^3*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*d*
a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/6/e/c*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)*a-15/1024/e*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*a^2-5/192/e^2*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a-15/1024/e
^5*c^2*d^8*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+5/128/e^4*c*d^5*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/7/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
7/2)/c/d+35/1024*e^3/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-5/96/c
*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2-35/256*d^3*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x*a^2-1/4/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5
/2)*x+1/5*d^2/e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)-1/8/e^3*d^2
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/12/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(5/2)*x*a-25/192/e*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a
+15/2048/e^5*c^3*d^10*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+5/384*e^4/c^3/d^3*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^4+3/256*e^5*a^5/c^2*ln((1/2*a*e^2-1/2*c*d
^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2))/(c*d*e)^(1/2)+1/8*d^2/e*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)
*x+3/64*d^2*e*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*d^4/e
^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-3/64*d^6/e^3*a*c*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/128*d^4*e*a^3*ln((1/2*a*e^2-1/2
*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))
^(1/2))/(c*d*e)^(1/2)+3/64*d^7/e^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2)*x-3/256*d^10/e^5*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/51
2/e^4*c^2*d^7*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-5/1024*e^7/c^4/d^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^6+5/128/e^3*c*d^6*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*a-5/128*e/c*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*a^3+5/64/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/24*e/c
^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*a^2+5/192*e^2/c^2/d*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3-15/2048*e^5/c^2*ln((c*d*e*x+1/2*a*e^2+1
/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1
/2)*a^5-225/2048*e*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+5/192*e/c*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
[Out] Timed out
```

3.459 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=381

$$\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cd\right.}{512c^3d^3e^4}$$

```
[Out] 1/192*(-a*e^2+c*d^2)*(5*a*e^2+7*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^3-1/60*(5*a/c/d+7*d/e^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
/e/(e*x+d)+1/1024*(-a*e^2+c*d^2)^5*(5*a*e^2+7*c*d^2)*arctanh(1/2*(2*c*d*e*x
+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/c^(7/2)/d^(7/2)/e^(9/2)-1/512*(-a*e^2+c*d^2)^3*(5*a*e^2+7*c*d^2)*(2*c*d
*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A] time = 0.39, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {794, 664, 612, 621, 206}

$$-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^4}+\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)\left(ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*
(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^3) - (((5*a)/(c*d) + (7*d)/e^2)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/60 + (a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2)/(6*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^2)
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)}+\frac{1}{12}\left(-\frac{7d}{e}-\frac{5ae}{cd}\right)\int \frac{\left(ade+\left(cd^2+a\right.\right.}{d}$$

$$= -\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{\left(ade+\left(cd^2+ae^2\right)\right.}{6cde(d+}$$

$$= \frac{\left(cd^2-ae^2\right)\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+c\right.}{192c^2d^2e^3}$$

$$= -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512c^3d^3e^4}$$

$$= -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512c^3d^3e^4}$$

$$= -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512c^3d^3e^4}$$

Mathematica [A] time = 2.74, size = 506, normalized size = 1.33

$$(ae+cdx)((d+ex)(ae+cdx))^{5/2}\left(7-\frac{7\sqrt{cd}\sqrt{cd^2-ae^2}\left(5ae^2+7cd^2\right)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/2}\left(15\sqrt{e}\sqrt{cd}\left(cd^2-ae^2\right)^{11/2}(ae+cdx)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-15\sqrt{c}\sqrt{d}\left(c\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(7 - (7*Sqrt[c*d]*Sqrt[c*d^2
 - a*e^2]*(7*c*d^2 + 5*a*e^2)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/2)*(15*S
qrt[c*d]*Sqrt[e]*(c*d^2 - a*e^2)^(11/2)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/
(c*d^2 - a*e^2)] - 10*Sqrt[c*d]*e^(3/2)*(c*d^2 - a*e^2)^(9/2)*(a*e + c*d*x)
^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 8*Sqrt[c*d]*e^(5/2)*(c*d^2 - a*e
^2)^(7/2)*(a*e + c*d*x)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 16*Sqrt[c
```

```
*d]*e^(7/2)*(c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)^4*Sqrt[(c*d*(d + e*x))/(c*d
^2 - a*e^2)]*(-3*a*e^2 + c*d*(11*d + 8*e*x)) - 15*Sqrt[c]*Sqrt[d]*(c*d^2 -
a*e^2)^6*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(1280*c^5*d^5*e^(7/2)*(a*e + c*d*x)^
4*(d + e*x)^4)))/(42*c*d*e)
```

fricas [A] time = 1.03, size = 1046, normalized size = 2.75

$$\left[-\frac{15\left(7c^6d^{12}-30ac^5d^{10}e^2+45a^2c^4d^8e^4-20a^3c^3d^6e^6-15a^4c^2d^4e^8+18a^5cd^2e^{10}-5a^6e^{12}\right)\sqrt{cde}\log\left(8c^2d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3
*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
 + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e
+ 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4*c
^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^4
+ 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7*c
^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*x^
2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3*c
^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^4*d^4*e^5), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2
*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
- 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*
e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4
*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^
4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7
*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*
x^2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3
*c^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c^4*d^4*e^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
```

```
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Warning, replacing 0 by ` u`, a substitutio
n variable should perhaps be purged.Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Evaluation time: 0.45Error: Bad Arg
ument Type
```

maple [B] time = 0.01, size = 2411, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] 9/64*d^4/e*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d*e^6*
a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^7/e^2*a*c^2*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/256*d*e^4*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+
(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))
/(c*d*e)^(1/2)-75/1024*e^4/c*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4-5/1024*e^8/c^
3/d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^6+5/256*e^5/c^2/d^2*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*x*a^4-5/64/e*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x*a-5/96*e^2/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^2+15/512/e
^2*c^2*d^7*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+15/512*e^6/c^2/d*ln((c*d*e*x+1/2*a*
e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*
e)^(1/2)*a^5+5/512/e^4*c^2*d^7*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+5/19
2*e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2+5/192/e*d^2*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(3/2)*a+1/12/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/
2)*a+5/48*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a-1/16*e*a^2/c*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-5/192/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)+1/16*d^4/e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))
^(3/2)-3/128*d^7/e^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*
d*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-5/192*e^3/c^2/d^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3-75/1024*c*d^5*ln((c*d*e*x+1/2*a*e^2+
1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(
1/2)*a^2+1/12/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/6/e*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+5/256*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*a^2-1/5*d/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)-15/512/
e^2*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-5/96/e^2*c*d^3*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+5/256*e^2/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*a^3-5/1024/e^4*c^3*d^9*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e
)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-5/64*e^3/c*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3+15/128*e*d^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x*a^2+25/256*e^2*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2
)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+
5/512*e^6/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^5-15/512*e^4/c^
```

```
2/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4+5/256/e^3*c^2*d^6*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/64*e^3*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d
^2)*(x+d/e))^(1/2)*x+15/128*d^5*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e
)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2
)-3/64*d*e^2*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/8*d^3/e^
2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-9/64*d^2*e*a^2*((x+d/e)
^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/128/d*e^4*a^4/c^2*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/64*d^5/e^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d
^2)*(x+d/e))^(1/2)-15/128*d^3*e^2*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e
)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2
)-3/64*d^6/e^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d^
9/e^4*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Integral(x*((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x), x)
```

3.460 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=274

$$-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade\ }}{128c^2d^2e^3}$$

```
[Out] 1/16*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-3/256*(-a*e^2+c*d^2)^
5*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+3/128*(-a*e^2+c*d^2)^3*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A] time = 0.19, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\left(ade+\left(\right.\right.}{2e^2}$$

$$= \frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} +$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{1}$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{1}$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{1}$$

Mathematica [A] time = 1.22, size = 384, normalized size = 1.40

$$\sqrt{cd}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(-15a^5e^9+5a^4cde^7(14d-ex)+2a^3c^2d^2e^5\left(64d^2+268dex+129e^2x^2\right)+2a^2c^3d^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c*d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-15*a^5*e^9 + 5*a^
4*c*d*e^7*(14*d - e*x) + 2*a^3*c^2*d^2*e^5*(64*d^2 + 268*d*e*x + 129*e^2*x^
2) + 2*a^2*c^3*d^3*e^3*(-35*d^3 + 87*d^2*e*x + 489*d*e^2*x^2 + 292*e^3*x^3)
 + c^5*d^5*x*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*e^4
*x^4) + a*c^4*d^4*e*(15*d^4 - 80*d^3*e*x + 54*d^2*e^2*x^2 + 688*d*e^3*x^3 +
 464*e^4*x^4)) - 15*(c*d^2 - a*e^2)^(11/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])
/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(640*c^(7/2)*d^(7/2)*e^(7/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)])
```

fricas [A] time = 1.00, size = 844, normalized size = 3.08

$$\left[\frac{15\left(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
```

```
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d
^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^
4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^
2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d
^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/12
80*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*c^5*d^5*e^5*x^
4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 + 70*a^3*c^2*d^3*
e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*
d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 2
3*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Evaluation time: 0.45Error: Bad Argument Ty
pe
```

maple [B] time = 0.01, size = 1123, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+15/256/e*a*c^2*d^6*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64*e^4*a^3/c/d*((x+d/e)^2*c*d*e+(a*e^2
```

```
-c*d^2)*(x+d/e))^(1/2)*x-9/64*a*c*d^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)*x+3/256*e^7*a^5/c^2/d^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*
d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/
128*e*a^2*c*d^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8*e*a*((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/64*e^3*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2)-1/8/e*c*d^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
3/2)*x-1/16/e^2*c*d^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128/e
^3*c^2*d^6*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/256/e^3*c^3*d^8*
ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64/e*a*c*d^4*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)-15/256*e^5*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c
*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^
(1/2)+15/128*e^3*a^3*d^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64/e^2*c^
2*d^5*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+9/64*e^2*a^2*d*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/128*e^5*a^4/c^2/d^2*((x+d/e)^2*
c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*e^2*a^2/c/d*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(3/2)
```

maxima [B] time = 0.57, size = 915, normalized size = 3.34

$$-\frac{3\,c^4 d^9 \log\left(2\,cdx + \frac{cd^2}{e} + ae + 2\,\sqrt{cdex^2 + cd^2x + ae^2x + ade}\,\sqrt{\frac{cd}{e}}\right)}{256\,\left(\frac{cd}{e}\right)^{\frac{3}{2}} e^5} + \frac{15\,ac^3d^7 \log\left(2\,cdx + \frac{cd^2}{e} + ae + 2\,\sqrt{cdex\right.}}{256\,\left(\frac{cd}{e}\right)^{\frac{3}{2}} e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] -3/256*c^4*d^9*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e)*sqrt(c*d/e))/((c*d/e)^(3/2)*e^5) + 15/256*a*c^3*d^7*log(2*c
*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c
*d/e))/((c*d/e)^(3/2)*e^3) - 15/128*a^2*c^2*d^5*log(2*c*d*x + c*d^2/e + a*e
 + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/((c*d/e)^(3/2
)*e) + 15/128*a^3*c*d^3*e*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/(c*d/e)^(3/2) - 15/256*a^4*d*e^3*lo
g(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*s
qrt(c*d/e))/(c*d/e)^(3/2) + 3/256*a^5*e^5*log(2*c*d*x + c*d^2/e + a*e + 2*s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/(c*d*(c*d/e)^(3/2))
 - 9/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a*c*d^3*x + 3/64*sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c^2*d^5*x/e^2 + 9/64*sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e)*a^2*d*e^2*x - 3/64*sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e)*a^3*e^4*x/(c*d) + 3/128*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e)*c^2*d^6/e^3 - 3/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a*c
*d^4/e + 3/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a^3*e^3/c - 3/128
*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a^4*e^5/(c^2*d^2) - 1/8*(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e)^(3/2)*c*d^2*x/e + 1/8*(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e)^(3/2)*a*e*x - 1/16*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e)^(3/2)*c*d^3/e^2 + 1/16*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)^(3/2)*a
^2*e^2/(c*d) + 1/5*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)^(5/2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x), x)
```

3.461 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=394

$$-a^{5/2}d^{3/2}e^{5/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex\left(c\right.\right.}{}$$

```
[Out] 1/24*(6*c*d*e*x+11*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e
-a^(5/2)*d^(3/2)*e^(5/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1
/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/128*(-5*a^4*e^8+60*a
^3*c*d^2*e^6+90*a^2*c^2*d^4*e^4-20*a*c^3*d^6*e^2+3*c^4*d^8)*arctanh(1/2*(2*
c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/64*(3*c^3*d^6-11*a*c^2*d^4*e^2-83*a^2*
c*d^2*e^4-5*a^3*e^6+2*c*d*e*(-5*a*e^2+c*d^2)*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2
```

Rubi [A] time = 0.45, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 814, 843, 621, 206, 724}

$$-\frac{\left(-83a^2cd^2e^4-5a^3e^6-11ac^2d^4e^2+2cdex\left(cd^2-5ae^2\right)\left(ae^2+3cd^2\right)+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] -((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 - 5*a^3*e^6 + 2*c*d*e*(c
*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(64*c*d*e^2) + ((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(24*e) + ((3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^
2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(128*c^(3/2)*d^(3/2)*e^(5/2)) - a^(5/2)*d^(3/2)*e^(5/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x}\,dx$$

$$=\frac{\left(3cd^2+11ae^2+6cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e}-\int\frac{\left(-8a^2cd^2e\right.}{}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+\right.\right.}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+\right.\right.}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+\right.\right.}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+\right.\right.}{64cde^2}$$

Mathematica [A] time = 2.01, size = 390, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{384a^{5/2}cd^{5/2}e^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\sqrt{e}\sqrt{ae+cdx}\left(15a^3e^6+a^2cde^4(337d+118ex)+ac^2d^2e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(15*a^3*e^6 + a^2
*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^2*(57*d^2 + 244*d*e*x + 136*e^2*x^
2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^2*x^2 + 48*d^3*e^3*x^3)) + (3*Sqrt[
c]*Sqrt[d]*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^
2*e^6 - 5*a^4*e^8)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqr
t[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]) - (384*a^(5/2)*c*d^(5/2)*e^5*ArcTanh[(Sqrt[d]*Sqrt
[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(192*c*d*e
^(5/2)*Sqrt[a*e + c*d*x])
```

fricas [A] time = 93.32, size = 1873, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/768*(384*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
```

```
) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6
 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^
3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 +
 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^
4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3), 1/384*(192*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 6
0*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 -
9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(
9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^
4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
2*d^2*e^3), 1/768*(768*sqrt(-a*d*e)*a^2*c^2*d^3*e^5*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/
(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 3*(3*c^4*d^8
 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sq
rt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c
*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^3 - 9*c^4*d^7*e
+ 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^
3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^4 + 59*a^2*c
^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3),
1/384*(384*sqrt(-a*d*e)*a^2*c^2*d^3*e^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2
*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 3*(3*c^4*d^8 - 20*a*c^3*
d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*a
rctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3
)*x)) + 2*(48*c^4*d^4*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^
2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(
3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 2180, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x/(e*x+d),x)
```

```
[Out] 3/64/d^2*a^3*e^5/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+9/64*d^2
*a*e*x*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d^3*a^5*e^8/c^
2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128*d^3*a^2*e^2*c*ln((1/2*a*e^2
-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
```

```
/e))^(1/2))/(c*d*e)^(1/2)+15/256/d*a^4*e^6/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e
)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*
e)^(1/2)+75/128*d^3*a^2*e^2*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2
)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-25/256/d*a^4*e^6/c
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)+19/64*d^2*a*e*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)*x+3/256/d^3*a^5*e^8/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-3/64/d^2*a^3*e^
5/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/8*d*c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(3/2)*x+1/16*d^2*c/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(3/2)-3/128*d^5*c^2/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-
9/64*a^2*e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/64*d^3*a*c*(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/8*d^3*a*c*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)-3/128*d^5*c^2/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)+83/64*d*a^2*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/8*d*c*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/16*d^2*c/e*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(3/2)+19/64*a^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/5/d
*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/5/d*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(5/2)-15/128*d*a^3*e^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(
c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3
/64*d^4*c^2/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d^7*c^3
/e^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/16/d^2*a^2*e^3/c*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/128/d^3*a^4*e^6/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)+75/128*d*a^3*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1
/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-3/64*d^4*c^2/e*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/256*d^7*c^3/e^2*ln((c*d*e*x+1/2*
a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*
d*e)^(1/2)-1/8/d*a*e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-3/64
/d*a^3*e^4/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-15/256*d^5*a*c^2
*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8/d*a^3*e^4/c*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)+1/8/d*a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-
d^2*a^3*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-25/256*d^5*a*c^2*ln((c*d*e*x+1/2*a*e
^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e
)^(1/2)+11/24*a*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/16/d^2*a^2*e^3/
c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128/d^3*a^4*e^6/c^2*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)), x)
```

3.462 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=352

$$-\frac{1}{2}a^{3/2}\sqrt{d}\,e^{3/2}\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)+\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right.\right.}{}$$

```
[Out] -1/3*(-c*d*x+3*a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x-1/16*(-5*a^3*
e^6-45*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+c^3*d^6)*arctanh(1/2*(2*c*d*e*x+a*e^2
+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^
(3/2)/c^(1/2)/d^(1/2)-1/2*a^(3/2)*e^(3/2)*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2*
a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))*d^(1/2)+1/8*(c^2*d^4+28*a*c*d^2*e^2+19*a^2*e^4+2*c*d*e*(7*a*e^
2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A] time = 0.43, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {849, 812, 814, 843, 621, 206, 724}

$$\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e}-\frac{\left(-45a^2cd^2e^4-5a^3e^6-15ac^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] ((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e) - ((3*a*e - c*d*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*x) - ((c^3*d^6 - 15*a*c^2*d^4*e^2
- 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(16*Sqrt
[c]*Sqrt[d]*e^(3/2)) - (a^(3/2)*Sqrt[d]*e^(3/2)*(5*c*d^2 + 3*a*e^2)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2}\,dx$$

$$=-\frac{(3ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3x}-\frac{1}{2}\int \frac{\left(-ae\left(5cd^2+3a\right.\right.}{}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{8e}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{8e}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{8e}$$

$$=\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{8e}$$

Mathematica [A] time = 2.07, size = 350, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{24a^{3/2}\sqrt{d}\,e^3\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\frac{\sqrt{e}\sqrt{ae+cdx}\left(3a^2e^3(11ex-8d)+2acde^2x(34d+13ex)+c^2d^2x\left(3d^2+\right.\right.}{x}\right.}{24e^{3/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*Sqrt[a*e + c*d*x]*(3*a^2*e^3*(-8*d
 + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x) + c^2*d^2*x*(3*d^2 + 14*d*e*x +
8*e^2*x^2)))/x - (3*Sqrt[c]*Sqrt[d]*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*
d^2*e^4 - 5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(S
qrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)]) - (24*a^(3/2)*Sqrt[d]*e^3*(5*c*d^2 + 3*a*e^2)*Ar
cTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d
+ e*x]))/(24*e^(3/2)*Sqrt[a*e + c*d*x])
```

fricas [A] time = 30.73, size = 1717, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(
c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*
sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2
```

```
- 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*c^3*d^3*e^3*x^3 -
 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e
 + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c*d*e^2*x), 1/48*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*
e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 12*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^
5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(8*c^3*d^3*e^3*x^
3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^
5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(c*d*e^2*x), 1/96*(48*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(
-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*
e + a^2*d*e^3)*x)) - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a
^3*e^6)*sqrt(c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^3*x^3 - 2
4*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e +
 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c*d*e^2*x), 1/48*(24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(-a*d*
e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a
^2*d*e^3)*x)) + 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^
6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^
2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*e^3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c
^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a
^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.48Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 2364, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^2/(e*x+d),x)
```

```
[Out] -3/64*e^6/d^3*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-9/64*e^
2*d*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*e^9/d^4*a^5/c
^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/128*e^3*d^2*a^2*c*ln((1/2*a*e^
2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2))/(c*d*e)^(1/2)+15/256*e*d^4*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/
e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d
*e)^(1/2)-15/256*e^7/d^2*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
```

```
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*
e^6/d^3*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-3/256*e^9/d^4*a^5/c
^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2))/(c*d*e)^(1/2)+121/64*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x*a*c*e^2+15/256/d^2/c*e^7*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+375/128*d^2*c
*e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+225/256*d^4*e*ln((c*d*e*x+1/2*a*e^2+1/2
*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2
)*a*c^2-5/2*d^3*a^2*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+15/128*e^5*a^3*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*d^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)*x-1/8*e*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3
/128/e*d^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+13/64*d^3*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2+13/128*d^4/e*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^2+1/d*a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+9
/8*e*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+25/128*e^5*ln((c*d*e*x+1/2
*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c
*d*e)^(1/2)*a^3+1/a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+4/5/d^2*e*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/5*e/d^2*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(5/2)-1/16*d*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+
67/48*d*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+19/8*e^3*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*a^2-13/256*d^6*c^3/e*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^
2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/8
*e^3/d^2*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/64*e^5/d^2*a^3
/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*e^4/d^3*a^2/c*((x+d/e
)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+9/64*e^4/d*a^2*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)*x-3/128*e^7/d^4*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)-3/64*e*d^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-3/256/e*d^6*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/d^2/a/e/x*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)-3/2*d*a^3*e^4/(a*d*e)^(1/2)*ln((2*a*d
*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))
/x)+1/d*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-9/64/d*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2*e^4-1/8*e^3/d^2*a*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)*x-1/16*e^4/d^3*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)
+3/128*e^7/d^4*a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/64/d^2/c*e
^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+227/64*d^2*c*e*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^2\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{x^2(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x**2*(d + e*x)), x)
```

3.463 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=339

$$\frac{3\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\sqrt{e}\left(a^2e^4+10acd^2e^2+5c^2d^4\right)}{}$$

```
[Out] -1/2*(-c*d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2+3/8*(5*a^2*e^
4+10*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/
2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)
-3/8*(a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)
*e^(1/2)/d^(1/2)-3/4*(a*e*(a*e^2+3*c*d^2)-c*d*(3*a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A] time = 0.39, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 812, 843, 621, 206, 724}

$$\frac{3\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\sqrt{e}\left(a^2e^4+10acd^2e^2+5c^2d^4\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(2*x^2) + (3*Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^2 +
 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[e]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*Sqrt[d])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3}\,dx$$

$$=-\frac{(ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{\left(-2ae\left(3cd^2+ae^2\right)\right.}{}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}$$

Mathematica [A] time = 2.25, size = 334, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{d}\sqrt{ae+cdx}\left(-a^2e^2(2d+5ex)-9acdex(d-ex)+c^2d^2x^2(5d+2ex)\right)}{x^2}+\frac{3\sqrt{c}\,d\sqrt{cd}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{}}{\sqrt{cd}\sqrt{cd^2-}}\right.}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right.}{4\sqrt{d}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[d]*Sqrt[a*e + c*d*x]*(-9*a*c*d*e*x*(d
 - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d + 5*e*x)))/x^2 + (3*Sqrt
[c]*d*Sqrt[c*d]*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcSinh[(Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*
Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d
*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(4*Sqrt[d]*Sqrt[a*e
+ c*d*x])
```

fricas [A] time = 14.80, size = 1569, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e +
 a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) + 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*e/d)*
x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqr
t(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*c^2*d^2*e*x^3 - 2*a^2*d
*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/16*(6*(c^2*d^4 + 10*a*c*d^2*e
^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*
d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4
)*sqrt(a*e/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a
*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*c^2*d^2*e
*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, 1/16*(6*(5*c^2*d^4
 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 3*(c^2*d^4 + 10*a*c*d^2*
e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*
d^2*e*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*
a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/8*(3*(c^2*
d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^
2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c
*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2
+ a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*(2*c^2*d^2*e*x^3 - 2*a^2*d*e^2
+ (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.53Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 2688, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^3/(e*x+d),x)
```

```
[Out] -3/256/d^5*e^10*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2
)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128*d*e^4
*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/256/d^3*e^8*a^4/c*ln((1/2*
a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^3*e^2*a*c^2*ln((1/2*a*e^2-1/2*c*d^2
+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2)+3/64/d^4*e^7*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)*x-3/64/d^4*e^7*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/256/
d^5*e^10*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-3/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(7/2)*c-15/8*d^4*a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/4*
e/d^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+975/256*e^2*d^3*ln((c*d
*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2))/(c*d*e)^(1/2)*a*c^2-15/256*e^8/d^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)
/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+2
25/128*e^4*d*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-15/4*e^3*d^2*a^2/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2))/x)*c+3/256*d^5*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*e
^3/d^2*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/d/a*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(5/2)*c-3/8*e^5*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+93/256*d^5*c^3
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)+3/4*e^4/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*a^2+1/4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+39/64*e^3*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a*c-1/16/d^4*e^5*a^2/c*((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(3/2)-9/64/d^2*e^5*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^
2)*(x+d/e))^(1/2)*x+3/128/d^5*e^8*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(1/2)+3/64*d*e^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1
5/128/d*e^6*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64*d^2*e*c^2*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/4/a^2/e*c^2*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(5/2)*x+5/4*d/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c
^2-1/2/d^2/a/e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+3/4*d/a^2/e^2*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+5/4*d^2/a/e*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(3/2)*c^2+1/8/d^3*e^4*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2
)*x+1/16/d^4*e^5*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/128/d^5*e^
8*a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/8*e^2/d*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x*c+15/128*e^6/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/
(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+3/
64*e^6/d^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+333/64*e^2*d*c*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+9/64*e^3*a*c*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^3*e^4*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
```

```
e))^(3/2)*x-3/64/d^3*e^6*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2
)+1/8/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-1/5/d^3*e^2*(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/16*e*c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(3/2)-3/128*d^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2)+31/16*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+387/128*d^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-1/20*e^2/d^3*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(5/2)+9/64*e^5/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a
^2+147/64*e*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^3\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

3.464 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=371

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4\right.}{}$$

```
[Out] -1/12*(4*a*d*e+3*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
/d/x^3-1/16*(-a^3*e^6+15*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+5*c^3*d^6)*arctanh(
1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)+1/2*c^(3/2)*d^(3/2)*(5*a*e^2+3*
c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/8*(5*c^2*d^4+12*a*c*d^2*e^2-a^2*
e^4-2*c*d*e*(a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

Rubi [A] time = 0.47, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {849, 810, 812, 843, 621, 206, 724}

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(15a^2cd^2e^4-a^3e^6+45ac^2d^4e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] -((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*e*(7*c*d^2 + a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*d*x) - ((4*a*d*e + 3*(3*c*d^2 +
 a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*d*x^3) + (c^(
3/2)*d^(3/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])
/2 - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(
2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4}\,dx$$

$$=-\frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12dx^3}-\int \frac{\left(-\frac{1}{2}ae(5c\right.}{}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

Mathematica [A] time = 3.04, size = 357, normalized size = 0.96

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(a^2e^2\left(8d^2+14dex+3e^2x^2\right)+2acd^2ex(13d+34ex)+3c^2d^3x^2(11d-8ex)\right)}{x^3}-\frac{3\sqrt{d+ex}\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e\right.}{\sqrt{a}}\right.}{24d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(3*c^2*d
^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d + 34*e*x) + a^2*e^2*(8*d^2 + 14
*d*e*x + 3*e^2*x^2)))/x^3) + (24*(c*d)^(5/2)*e*Sqrt[c*d^2 - a*e^2]*(3*c*d^2
 + 5*a*e^2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/c^(3/2) - (3*(
5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[d + e*x]*Ar
cTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[a]
))/(24*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 18.60, size = 1741, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/96*(24*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e
^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) - 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)
```

```
*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(24*a*c^2*d^4*e^2
*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^
2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a*d^2*e*x^3), -1/96*(48*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sq
rt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d
^3*e + a*c*d*e^3)*x)) + 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4
- a^3*e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(24*a*c
^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3
*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e^
2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 12*(3*a*c^2*d^5
*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 2*(24*a*
c^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^
3*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e
^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a
*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 24*(3*a*c^2*d^
5*e + 5*a^2*c*d^3*e^3)*sqrt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(24*a*c^2*d^4*e^2*x^3 -
8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2 - 2*(
13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(a*d^2*e*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.11Error: Bad Argument Typ
e
```

maple [B] time = 0.03, size = 3144, normalized size = 8.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^4/(e*x+d),x)
```

```
[Out] -1/16/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128*d^2*e*c^2
*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64*e^3*a*c*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-5/16*d^5/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+35/2
4*d/a*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+25/24/e/a^2*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+5/12/d^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
```

```
)*x)^(7/2)+493/128*d^2*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-1/8/d^
2*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+37/48/d*e^2*c*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/24/d^3*e^4*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(3/2)+107/64*e^3*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+1/5/d^4*e^3
*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+7/40/d^4*e^3*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)-1/8/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(3/2)*x+1/16/d^5*e^6*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2
)+9/64/d^3*e^6*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/128/d^
6*e^9*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/128/d^2*e^7*
a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*d*e^2*c^2*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256*d^4*e*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d
/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*
d*e)^(1/2)-15/128*e^5*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+5/8*d^4/
a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-1/3/d^2/a/e/x^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(7/2)+5/24*d^3/a^2/e^2*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)+1/8*d^2/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3+
5/8*d^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-1/8/a^3/e^3/x*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-9/64/d^3*e^6*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*a^2+93/64*d*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
x*c^2+15/128*e^5*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/8/d^4*e/a/x*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(7/2)+1/16/d*e^6*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-3/64
/d^4*e^7/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+1/12/d^2*e^3*c*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/128/d^2*e^7*ln((c*d*e*x+1/2*a*e^2+1
/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1
/2)*a^3+387/256*d^4*e*c^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+19/24/d^2*e/a*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+5/6*e/a*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(3/2)*x+5/6/d/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-1/8/d^
4*e^5*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/16/d^5*e^6*a^2/c*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/128/d^6*e^9*a^4/c^2*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)+3/8/d^3*e^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*x-15/16*d*e^4*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/
2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+1/64/d*e^4*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x*a*c+15/256/d^4*e^9/c*ln((c*d*e*x+1/2*a*e^2+1/2*c
*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*
a^4+625/256*d^2*e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2+5/24*d^2/a^2/e*c^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8*d/a^3/e^2*c^3*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(5/2)*x-1/12/d/a^2/e^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c+3/64/d^5*e^8*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-3/25
6/d^6*e^11*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-15/256/d^4*e^9*a^4/c*ln((1/2*
a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64/d^5*e^8*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c
*d^2)*(x+d/e))^(1/2)*x-9/64/d*e^4*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)*x+3/256/d^6*e^11*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c
*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15
/256*d^2*e^3*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8/d^4*e^5*a*((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/64/d^4*e^7*a^3/c*((x+d/e)^2*
c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-5/6/e/d^2/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(7/2)*c-45/16*e^2*d^3*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^4\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

3.465 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=404

$$-\frac{\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}+$$

```
[Out] -1/24*(6*a*d*e+(3*a*e^2+11*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/d/x^4+1/128*(-3*a^4*e^8+20*a^3*c*d^2*e^6-90*a^2*c^2*d^4*e^4-60*a*c^3*d^6*
e^2+5*c^4*d^8)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+c^(5/2)*
d^(5/2)*e^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-1/64*(2*a*d*e*(-a*e^2+5*c*d^2)*(3
*a*e^2+c*d^2)+(-3*a^3*e^6+11*a^2*c*d^2*e^4+83*a*c^2*d^4*e^2+5*c^3*d^6)*x)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/x^2
```

Rubi [A] time = 0.46, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 810, 843, 621, 206, 724}

$$-\frac{\left(x\left(11a^2cd^2e^4-3a^3e^6+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}+-$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] -((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2*d^4*
e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(64*a*d^2*e*x^2) - ((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*d*x^4) + c^(5/2)*d^(5/2)*e^(3/2)*Ar
cTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])] + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c
^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^
2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(128*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5}\,dx$$

$$=-\frac{\left(6ade+\left(11cd^2+3ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24dx^4}-\int\frac{\left(-\frac{1}{2}ae(5\right.}{}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a\right.\right.}{64ad^2ex^2}$$

Mathematica [A] time = 3.48, size = 404, normalized size = 1.00

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(3a^3e^3\left(16d^3+24d^2ex+2de^2x^2-3e^3x^3\right)+a^2cd^2e^2x\left(136d^2+244dex+57e^2x^2\right)+ac^2d^4ex^2(118d+337ex)+15c^3d^6x^3\right)}{ax^4}\right.}{192d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(15*c^3*
d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^2*c*d^2*e^2*x*(136*d^2 + 24
4*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*d^2*e*x + 2*d*e^2*x^2 - 3*e^
3*x^3)))/(a*x^4)) + 384*c^(3/2)*d^4*Sqrt[c*d]*e^3*Sqrt[c*d^2 - a*e^2]*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e
+ c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])] + (3*(5*c^4*d^8 - 60*a*c^3*d^6*e
^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*Sqrt[d + e*x]*ArcTa
nh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/a^(3/2)))/
(192*d^(5/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 48.64, size = 1917, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/768*(384*sqrt(c*d*e)*a^2*c^2*d^5*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^4
 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 3*
```

```
(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a
^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(48*a^4*d^
4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 - 9*a^4*d*
e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 +
 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/768*(768*sqrt(-c*d*e)*a^2*c^2*d^5*e^3*x^4*
arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^
3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^
2*e^6 - 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4
*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5
 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2
*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), 1/384*(192*sqrt(c*d*e)*a^2*c^2*d^5
*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*
a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) -
 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e
^5 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d
^2*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/384*(384*sqrt(-c*d*e)*a^2*c^2
*d^5*e^3*x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4
+ 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(48*a^4*d^4*e
^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 - 9*a^4*d*e^7
)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 + 8*
(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(a^2*d^3*e^2*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.26Error: Bad Argument Typ
e
```

maple [B] time = 0.03, size = 3646, normalized size = 9.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^5/(e*x+d),x)
```

```
[Out] 1/16/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/128*d*e^2*c^
2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64*e^3*c^2*((x+d/e)^2*c*d
```

*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+25/32*d^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2)*c^3+3/8/d^3/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+1/96/d
/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+35/96*e/a*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(3/2)*c^2-1/8*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
*x*c^2+127/128*d*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+3/64/d^3*e
^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+1/64/d^4*e^5*a*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(3/2)-19/96/d^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(3/2)*c-1/5/d^5*e^4*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)-61/320/d^
5*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+15/256/d^5*e^10*a^4/c*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64/d^6*e^9*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2)*x+9/64/d^2*e^5*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)*x-3/256/d^7*e^12*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)
/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
+15/128/d*e^6*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((
x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d*e^4*a*c
^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-45/64*d^2*e^3*a/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/x)*c^2-31/64/d^2*e/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x
-35/192/d*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^2-3/32/d^2*e^5*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a*c-15/256/d^5*e^10/c*ln((c*d*e*x
+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
)/(c*d*e)^(1/2)*a^4-15/128/d*e^6*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)
^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+15/256*d*
e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2-25/64/d^4*e^3*c/a*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(5/2)*x-5/64*d^4/a^2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*x*c^4+5/128*d^6/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4-5/192*d^3/a^3/e^2*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^4-1/64*d^2/a^4/e^3*c^4*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+1/64*d/a^4/e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(7/2)*c^3+1/24/d/a^2/e^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*
c-3/64/d^6*e^9*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/256/d^7*e^
12*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+45/64*e*d^2/a*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*c^3-13/48/e/d^2/a^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c-43/192/e^2/d/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-5/
192*d^4/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4-1/64*d^3/a^4/e^
4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^4-5/64*d^5/a^2/e^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4+1/96/a^3/e^3/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(7/2)*c^2+5/32*e^5*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+9/64/d^4*e^7*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+25/64/d^5*e^2/a/x*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(7/2)-7/64/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
3/2)*x*c+15/128/d^3*e^8*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3-3/128/d^2*e^7*a^3/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/x)-15/32/d^3*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(5/2)*c-17/64/d*e^4*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+253/256*d^3
*e^2*c^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+3/64/d^5*e^8/c*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*a^3+1/8/d^5*e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x
+1/16/d^6*e^7*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/128/d^7*e^10*
a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/8/d^3*e^4*c*((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-1/16/d^6*e^7*a^2/c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(3/2)-9/64/d^4*e^7*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(1/2)*x+3/128/d^7*e^10*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)+3/64/d*e^4*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-15/12

```
8/d^3*e^8*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)
^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/256*d^3*e^2*c^3*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+31/64/d^3/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(7/2)*c-13/32/d^4*e/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)
-15/32*e*d^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+35/96/e*d^2/a^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*c^3+19/96/e^2*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(5/2)*c^3+85/192*d/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^3+43
/192/e/a^3*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-1/4/d^2/a/e/x^4*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)-1/8/d^5*e^6*a*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(3/2)*x-3/64/d^5*e^8*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^5\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

3.466 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=289

$$-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}+\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ac}}{128a^2d^3e^2x^2}$$

```
[Out] -1/16*(c/a/e-e/d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/x^4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^5-3/256*(-a*
e^2+c*d^2)^5*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+3/128*(-a*
e^2+c*d^2)^3*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^2/d^3/e^2/x^2
```

Rubi [A] time = 0.33, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 806, 720, 724, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^2d^3e^2x^2}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+a}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(128*a^2*d^3*e^2*x^2) - ((c/(a*e) - e/d^2)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*x^4)
- (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*d*x^5) - (3*(c*d^2 - a*e
^2)^5*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5dx^5}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{(ad}{}}{2ade}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{16x^4}-$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-$$

Mathematica [A] time = 0.94, size = 295, normalized size = 1.02

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5ex)-3cd^2x\right)\right)}{a^{5/2}\sqrt{d}e^{5/2}}-8(d\cdots\right)}{d}\right)}{64dx^4(d+ex)^{3/2}(ae+cdx)^{3/2}}\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((-2*(a*e + c*d*x)*(d + e*x))/x^5 + (5*(c*
d^2 - a*e^2)*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a*e^2)*x*
(-8*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt[a]*Sqrt
[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)
) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sq
rt[e]*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d))/(64*d*x^4*(a*e + c*
d*x)^(3/2)*(d + e*x)^(3/2))))/(10*d)
```

fricas [A] time = 39.55, size = 872, normalized size = 3.02

$$\left[\frac{15\left(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10}\right)\sqrt{ade}\,x^5\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\ldots}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2 + (
c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*
d*e^3)*x)/x^2) - 4*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^3
- 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*c^3
*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3 + 8*
(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4*c*d^
6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^
3*d^4*e^3*x^5), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4
 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-a*d*e)*x^5*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) - 2*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^3 - 128*a^3*c^
2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*c^3*d^8*e^2 + 2
33*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3 + 8*(31*a^3*c^2*
d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4*c*d^6*e^4 + 11*a
^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^
5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^3*a^3*exp(2)^3+6*exp(1)^5*a^3*exp(2)^2-6*exp(1)^7*a^3*exp(2)+2*exp
(1)^9*a^3)/2/d^3/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*ex
p(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(-3*a^5*exp(2)^5-10*exp(1
)^2*a^5*exp(2)^4-80*exp(1)^4*a^5*exp(2)^3+480*exp(1)^6*a^5*exp(2)^2-640*exp
(1)^8*a^5*exp(2)+256*exp(1)^10*a^5-15*c*d^2*a^4*exp(2)^4-30*c^2*d^4*a^3*exp
(2)^3+60*c^2*d^4*exp(1)^2*a^3*exp(2)^2-30*c^3*d^6*a^2*exp(2)^2+80*c^3*d^6*e
xp(1)^2*a^2*exp(2)-80*c^3*d^6*exp(1)^4*a^2-15*c^4*d^8*a*exp(2)+30*c^4*d^8*e
xp(1)^2*a-3*c^5*d^10)/128/d^3/exp(1)^2/a^2/2/sqrt(-a*d*exp(1))*atan((sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*
```

exp(1)))-(-45*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^9*a^5*exp(2)^5-150*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^4+2640*exp(1)^4*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^
3-4320*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^9*a^5*exp(2)^2+1920*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)-225*c*d^2*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^4-4
50*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^9*a^3*exp(2)^3+900*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^2-450*c^3*d^6*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(
2)^2+1200*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(2)+2640*c^3*d^6*exp(1)^4*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2-225*c^4*d^8*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a*
exp(2)+450*c^4*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)^9*a-45*c^5*d^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9+3840*d*exp(1)^3*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5
*exp(2)^3-11520*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5*exp(2)^2+11520*d*exp(1)^7*sq
rt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^8*a^5*exp(2)-3840*d*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5-11520*c^2*d^5*
exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^8*a^3*exp(2)-7680*c^3*d^7*exp(1)^3*sqrt(c*d*exp(1))*
(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a
^2+210*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^7*a^6*exp(2)^5-580*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^4-8480*d*exp(1)^5*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*e
xp(2)^3+16320*d*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^2-7680*d*exp(1)^9*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)+1050*c*d^3*e
xp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^7*a^5*exp(2)^4+3840*c*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^3-11520*c*d^3*exp(1)^5*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*e
xp(2)^2+11520*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)-3840*c*d^3*exp(1)^9*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5+2100*c^2*d^5*
exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^7*a^4*exp(2)^3+15000*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2+2100*c^3*d^7*exp(1)*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^
3*exp(2)^2+16160*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)+3040*c^3*d^7*exp(1)^5*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3+1050*c
^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^7*a^2*exp(2)+5580*c^4*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2+210*c^5*d^11*exp(1)*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a-3840*
d^2*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^4-3840*d^2*exp(1)^4*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6
*a^6*exp(2)^3+34560*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^2-42240*d^2*exp
(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-

sqrt(c*d*exp(1))*x)^6*a^6*exp(2)+15360*d^2*exp(1)^10*sqrt(c*d*exp(1))*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6-15
360*c*d^4*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d
*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)^3-11520*c*d^4*exp(1)^4*sqrt(c
*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^6*a^5*exp(2)^2+11520*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)-3840
*c*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5-23040*c^2*d^6*exp(1)^2*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*
a^4*exp(2)^2-11520*c^2*d^6*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)-15360*c^3*d^
8*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)^6*a^3*exp(2)-3840*c^3*d^8*exp(1)^4*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6
*a^3-3840*c^4*d^10*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^2+384*d^2*exp(1)^2*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^
5+1280*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a^7*exp(2)^4+10240*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^3-23040*d^2*ex
p(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^5*a^7*exp(2)^2+11520*d^2*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)+1920*c*d^4*exp(1)^2*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp
(2)^4+11520*c*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^3+11520*c*d^4*exp(1)^6*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^2-268
80*c*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^5*a^6*exp(2)+11520*c*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6+3840*c^2*d^6*exp(1)^2*
(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a
^5*exp(2)^3+26880*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^2+11520*c^2*d^6*exp(1)^6*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*ex
p(2)-3840*c^2*d^6*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^5*a^5+3840*c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^2+24320*c^3*d
^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^5*a^4*exp(2)+10240*c^3*d^8*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4+1920*c^4*d^10*exp(1)^2*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*e
xp(2)+7680*c^4*d^10*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^5*a^3+384*c^5*d^12*exp(1)^2*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2-38400*d^3*exp(1)^7
*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^4*a^7*exp(2)^2+57600*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)
-23040*d^3*exp(1)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7-19200*c*d^5*exp(1)^5*sqrt(c*d*exp(
1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
^4*a^6*exp(2)^2-6400*c*d^5*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)+6400*c*d^5*e
xp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^4*a^6-38400*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(
2)-3840*c^2*d^7*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5-19200*c^3*d^9*exp(1)^5*sqrt(c*
d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(

1))*x)^4*a^4-210*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^5-700*d^3*exp(1)^5*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^4-5600
*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^3*a^8*exp(2)^3+14400*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^2-7680*d^3*exp(1)^11
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*
a^8*exp(2)-1050*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^4+19200*c*d^5*exp(1)^9*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)-1
1520*c*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^3*a^7-2100*c^2*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^3+4200*c^2*d^7*exp(
1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^3*a^6*exp(2)^2+19200*c^2*d^7*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)-2100*c^3*d^9*exp(1)^3*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*
exp(2)^2+5600*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+13600*c^3*d^9*exp(1)^7*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5-1050*c^4
*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^3*a^4*exp(2)+2100*c^4*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4-210*c^5*d^13*exp(1)^3*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3
+19200*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)^2-34560*d^4*exp(1)^10*sqrt(c
*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^2*a^8*exp(2)+15360*d^4*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8-6400*c*d^6*exp
(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^2*a^7*exp(2)-1280*c*d^6*exp(1)^10*sqrt(c*d*exp(1))*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7-7
680*c^2*d^8*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6+45*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^5+150*d^4*
exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)*a^9*exp(2)^4+1200*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^3-3360*d^4*exp(1)^10*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^2
+1920*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)*a^9*exp(2)+225*c*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^4-3840*c*d^6*exp(1)^
10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^8*exp(2)+3840*c*d^6*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)*a^8+450*c^2*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3-900*c^2*d^8
exp(1)^6(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)*a^7*exp(2)^2+3840*c^2*d^8*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7+450*c^3*d^10*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^2
-1200*c^3*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)*a^6*exp(2)+1200*c^3*d^10*exp(1)^8*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6+225*c^4*d^12*exp(1)
^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^5*exp(2)-450*c^4*d^12*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)*a^5+45*c^5*d^14*exp(1)^4*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4-3840*d^5*exp(1)^9*s
qrt(c*d*exp(1))*a^9*exp(2)^2+7680*d^5*exp(1)^11*sqrt(c*d*exp(1))*a^9*exp(2)
-3840*d^5*exp(1)^13*sqrt(c*d*exp(1))*a^9+1280*c*d^7*exp(1)^9*sqrt(c*d*exp(1

```
))*a^8*exp(2)-1280*c*d^7*exp(1)^11*sqrt(c*d*exp(1))*a^8-768*c^2*d^9*exp(1)^
9*sqrt(c*d*exp(1))*a^7)/3840/d^3/exp(1)^2/a^2/((sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^5)
```

maple [B] time = 0.04, size = 3991, normalized size = 13.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^6/(e*x+d),x)
```

```
[Out] -1/16/d^3*e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+13/40/d^3/a/x
^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+17/160/e/a^3*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(5/2)*c^3-3/128/d^4*e^7*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*a^2+15/128/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-1/128/d^5*
e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/5/d^6*e^5*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(5/2)+3/128*e^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2)-15/128*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+25/128
/d^6*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-31/80/d^4*e/a/x^3*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+3/32*d^2*e/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*c^3-15/128*d^3*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+1/5/d/a^3*c^3*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+109/320/d^3/a^2/x^2*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(7/2)*c+1/8/d^6*e^7*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(3/2)*x+3/64/d^6*e^9*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2)-1/8/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+1/16/d^7
*e^8*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+9/64/d^5*e^8*a^2*(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/128/d^8*e^11*a^4/c^2*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64/d^2*e^5*a*c*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/128/d^4*e^9*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+
d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c
*d*e)^(1/2)+3/64/d*e^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-
3/256*d^2*e^3*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+
d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/256*e^5*a*c^2*l
n((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+11/320/a^4/e^3/x*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(7/2)*c^3-1/80/a^3/e^3/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(7/2)*c^2-9/128*d^4/a^2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4+3/128
*d^6/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5+1/128*d^5/a^4/e^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^5+3/640*d^4/a^5/e^5*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)*c^5-7/128*d^3/a^3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(3/2)*c^4-17/640*d^2/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*c^4+15/256*d^5/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4-3/64*d^3/a^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4-1/5/d^2/a/e/x^5*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(7/2)-1/8/d^6*e^7*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/2
56*e^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2-9/64/d^5*e^8*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2)*x*a^2-253/640/d^6*e^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(7/2)+15/128/d^4*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c-15/128/
d^4*e^9*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3-3/64/d^6*e^9/c*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*a^3+15/128/d^2*e^5*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)*a+3/256*d^2*e^3*c^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+3/256/d^3*e^8*a^3/(a*d*
e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/x)+273/640/d^4*e^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
5/2)*c+129/320/d^5*e^2/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+47/160
/d^2*e/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+139/320/d^3*e^2/a^2*
c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+15/128*d*e^4*a/(a*d*e)^(1/2)*
```

```
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2))/x)*c^2-139/320/d^4*e/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2
)*c-15/256/d*e^6*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+15/128/d^2*e^7*c*ln((c*d*
e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2))/(c*d*e)^(1/2)*a^2+15/128/d^3*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x*a*c+15/256/d^6*e^11/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+253/640/d^5*e^4*c
/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-5/64*d^2/a^3/e*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x*c^4-1/5/d^2/a^2/e/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(7/2)*c-11/320*d/a^4/e^2*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*x+3/640*d^3/a^5/e^4*c^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-3/640*d
^2/a^5/e^5/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^4+19/320/d/a^3/e^2/x
^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-1/320*d/a^4/e^4/x^2*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3+1/128*d^4/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(3/2)*x*c^5+3/128*d^5/a^3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*x*c^5-3/256*d^7/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^5+3/40/d/a^2/e
^2/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c-3/64/d^7*e^10*a^3/c*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-9/64/d^3*e^6*a*c*((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256/d^8*e^13*a^5/c^2*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(c*d*e)^(1/2)-15/128/d^2*e^7*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d
*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1
/2)-15/256/d^6*e^11*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1
/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/256/d^8*
e^13*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+3/64/d^7*e^10*a^3/c*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x-1/5/d^2/e/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c^2+5/64/d^2*e^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^2-1/
16/d^7*e^8*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/128/d^8*e^11*a^4
/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
[Out] Timed out
```

3.467 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=386

$$\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+\right.}{512a^3d^4e}$$

```
[Out] 1/192*(-a*e^2+c*d^2)*(7*a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(5/2)/d/x^6-1/60*(5*c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(5/2)/x^5+1/1024*(-a*e^2+c*d^2)^5*(7*a*e^2+5*c*d^2)*arctanh(1/2*(2*a*d*e
+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/512*(-a*e^2+c*d^2)^3*(7*a*e^2+5*c*d^2)*(2
*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3
/x^2
```

Rubi [A] time = 0.49, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^3d^4e^3x^2}+\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^3*d^4*e^3*x^2) + ((c*d^2 - a
*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(192*a^2*d^3*e^2*x^4) - (a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2)/(6*d*x^6) - (((5*c)/(a*e) - (7*e)/d^2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*x^5) + ((c*d^2 - a*e^2)^5*(5*c*d^2 +
7*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(1024*a^(7/2)*d^(9/2)*e^(7/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6dx^6}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-7ae^2\right)+acde^2x\right)\left(ade+\left(cd^2+ae^2\right)\right.}{x^6}}{6ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6dx^6}-\frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{60x^5}$$

$$=\frac{\left(cd^2-ae^2\right)\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{192a^2d^3e^2x^4}$$

$$=-\frac{\left(cd^2-ae^2\right)^3\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512a^3d^4e^3x^2}$$

$$=-\frac{\left(cd^2-ae^2\right)^3\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512a^3d^4e^3x^2}$$

$$=-\frac{\left(cd^2-ae^2\right)^3\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512a^3d^4e^3x^2}$$

Mathematica [A] time = 0.99, size = 344, normalized size = 0.89

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(-\frac{(7ae^2+5cd^2)\left(5x(cd^2-ae^2)\left(\frac{x(cd^2-ae^2)\left(\frac{x(ae^2-cd^2)\left(3x^2(cd^2-ae^2)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d\cdots\right.\right.}{a^{5/2}\sqrt{d}e^{5/2}}}{d}\cdots}{1280d^2x^5(d+ex)^{3/2}(ae+cdx)^3\cdots}\right.}{6ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^6) - ((5*
c*d^2 + 7*a*e^2)*(-128*d*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) + 5*(c*d^2 - a
*e^2)*x*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a*e^2)*x*(-8*S
qrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)) + 3
*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]
*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d)))/(1280*d^2*x^5*(a*e + c*
d*x)^(3/2)*(d + e*x)^(3/2))))/(6*a*d*e)
```

fricas [A] time = 114.75, size = 1072, normalized size = 2.78

$$\left[-\frac{15\left(5c^6d^{12}-18ac^5d^{10}e^2+15a^2c^4d^8e^4+20a^3c^3d^6e^6-45a^4c^2d^4e^8+30a^5cd^2e^{10}-7a^6e^{12}\right)\sqrt{ade}\,x^6\log\left(\frac{8a^2\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3
*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(a*
d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 + (75*a
*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546*a^4*c^2*d^5*e
^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*d^10*e^2 - 80*
a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 35*a^6*d^2*e^10
)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c*d^5*e^7 - 7*a
^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6 + 3*a^6*d^4*e
^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6), -1/15360*(15*(5*c^6*d^12 - 18*a*c
^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8
+ 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(
a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(1280*a^6*d
^6*e^6 + (75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546
*a^4*c^2*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*
d^10*e^2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 3
```

```
5*a^6*d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c
*d^5*e^7 - 7*a^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6
 + 3*a^6*d^4*e^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 4735, normalized size = 12.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^7/(e*x+d),x)
```

```
[Out] 1/16/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/128/d*e^4*c^
2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+7/1536/d^6*a*e^7*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+7/512/d^5*a^2*e^8*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)-35/384/d^4*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+17/60
/d^3/a/x^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+1/512*d^3/a^2*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4-59/320/d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(5/2)*c^3+25/512/d*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+9/
64/d^6*e^9*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+15/128/d^5*e^10*ln
((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2))/(c*d*e)^(1/2)*a^3-101/512/d^7*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(5/2)-1/5/d^7*e^6*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+3/64/d^
7*e^10/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3-3/256*d*e^4*c^3*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)+1017/2560/d^7/a*e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)-1/512*d^5/a^6/e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^6-5/512
*d^7/a^4/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^6-397/960/d^5/a*e^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-35/1536/d^2/a*e^3*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*c^2+49/1536*d^2/a^3/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(3/2)*c^4-5/64/d^3*a*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-26
81/7680/d^3/a^2*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+1/64*d^5/a^
3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5-221/7680*d/a^4/e^2*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^4-57/160/d^4/a*e/x^4*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(7/2)+7/384*d^4/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
3/2)*c^5-1/64*d/a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-5/256/a*e
^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-11/480/a^4/e^3/x^2*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3-45/1024*a*e^5/(a*d*e)^(1/2)*ln((2*a*d*
e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/
x)*c^2-1/32/a^3/e^3/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-81/1280
/a^4/e*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-1/6/d^2/a/e/x^6*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+35/768*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(3/2)*x*c^4+5/256*d^2*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-185/1536/d^5
*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c+381/1280/d^3/a^3/x*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2+15/512/d^2*e^5*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*c^2+89/320/d^3/a^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(7/2)*c-7/1024/d^4*a^3*e^9/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+1/120*d^3/a^5/e^4*(c
```

*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^5-1543/3840/d^6/a*e^3/x^2*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)-5/1536*d^6/a^5/e^5*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)*c^6+377/960/d^5/a*e^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(7/2)+1/8/d^5*e^6*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-1/8/d^7
*e^8*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-3/64/d^7*e^10*a^3/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/16/d^8*e^9*a^2/c*((x+d/e)^2
*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-9/64/d^6*e^9*a^2*((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2)*x+3/128/d^9*e^12*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)+3/64/d^3*e^6*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e)
)^(1/2)-15/128/d^5*e^10*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64/d^2
*e^5*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d*e^4*c^3*ln
((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8/d^7*e^8*a*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(3/2)*x+1/16/d^8*e^9*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2
)-3/128/d^9*e^12*a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+3/64/d^8*e
^11*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+9/64/d^4*e^7*a*c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d^9*e^14*a^5/c^2*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128/d^3*e^8*a^2*c*ln((1/2*a*e^2-1/2*c
*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)-15/256/d*e^6*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*
e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/
2)+15/256/d^7*e^12*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/128/d^3*
e^8*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/64/d^8*e^11*a^3/c*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)*x+3/256/d^9*e^14*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1
5/256/d*e^6*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2-15/256/d^7*e^12/c*ln((c*d*e*x+
1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))
/(c*d*e)^(1/2)*a^4+15/512/d^2*a^2*e^7/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+3/512*d^
4/a^3/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^5-5/512*d^6/a^4/e^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^6-25/768/d/a^2*e^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x*c^3+89/7680*d^2/a^5/e^3*c^5*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(5/2)*x-1/512*d^4/a^6/e^5*c^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(5/2)*x+1/768*d^2/a^5/e^5/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^
4+43/1536*d^3/a^4/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^5-5/1536*
d^5/a^5/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^6+29/320/d/a^3/e^2/
x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2+1/192*d/a^4/e^4/x^3*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3-9/512*d^6/a^2/e/(a*d*e)^(1/2)*ln((2*a*
d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
)/x)*c^5+5/1024*d^8/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^6+81/1280/d/a^4/e^
2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3-89/7680*d/a^5/e^4/x*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^4+1/512*d^3/a^6/e^6/x*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(7/2)*c^5-11/30/d^4/a^2*e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(7/2)*c+3211/7680/d^5/a^2*e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2
)*c-1017/2560/d^6/a*e^5*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-3211/76
80/d^4/a^2*e^3*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-381/1280/d^2/a
^3*e*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-65/512/d^4*a*e^7*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c+7/256*d^2/a^2*e*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2)*x*c^4+1/12/d/a^2/e^2/x^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c+15/1024*d^4/a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4-65/1536/d^3/a*e^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^2-43/240/d^2/a^2/e/x^4*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c-113/640/d^2/a^3/e/x^2*(c*d*e*x^2+a*d*e+(a

```
*e^2+c*d^2)*x)^(7/2)*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^7\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

3.468 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=500

$$\frac{\left(-63a^2e^4+20acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}-\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^5\tan}{2048a^{9/2}d^{11/2}}$$

```
[Out] -1/384*(-a*e^2+c*d^2)*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2+
c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^4-1/7*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^7-1/84*(5*c/a/e-9*e/d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6+1/840*(-63*a^2*e^4+20*a*c*d^2*e^2+35*c^2*d
^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/e^2/x^5-1/2048*(-a*e^2+
c*d^2)^5*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c
*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a
^(9/2)/d^(11/2)/e^(9/2)+1/1024*(-a*e^2+c*d^2)^3*(9*a^2*e^4+10*a*c*d^2*e^2+5
*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/a^4/d^5/e^4/x^2
```

Rubi [A] time = 0.64, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {849, 834, 806, 720, 724, 206}

$$\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024a^4d^5e^4x^2}-\left(9a^2e^4+10acd\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e
^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*
a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(7*d*x^7)
- (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(5/2))/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c
^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
2048*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^8}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-9ae^2\right)+2acde^2x\right)\left(ade+\left(cd^2\right.\right.}{x^7}}{7ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{84x^6}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{84x^6}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade\right.}{384a^3d^4e^3x^4}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}}{1024a^4d^5e^4x^2}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}}{1024a^4d^5e^4x^2}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}}{1024a^4d^5e^4x^2}$$

Mathematica [A] time = 0.75, size = 408, normalized size = 0.82

$$((d+ex)(ae+cdx))^{3/2}\left(\frac{7\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(5x\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{\ }\right.}{a^{5/2}\sqrt{d}e^{5/2}}}{d}\right.\right.}{15360ad^3ex^5(d+ex)^{3/2}(ae}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^7) + ((7*
c*d^2 + 9*a*e^2)*(a*e + c*d*x)^2*(d + e*x))/(12*a*d*e*x^6) + (7*(5*c^2*d^4
+ 10*a*c*d^2*e^2 + 9*a^2*e^4)*(-128*d*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) +
 5*(c*d^2 - a*e^2)*x*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a
*e^2)*x*(-8*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt
[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d
+ 5*e*x)) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sq
rt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d)))/(15360*a*d
^3*e*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7*a*d*e)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.09, size = 5353, normalized size = 10.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^8\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] Timed out
```

3.469 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=628

$$\frac{\left(-33a^2e^4+10acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6}-\frac{\left(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3\right.}{4480a^3d^4e^3}$$

```
[Out] 1/2048*(-a*e^2+c*d^2)*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*
d^6)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^4/
d^5/e^4/x^4-1/8*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^8-1/112*(5*c/a/
e-11*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7+1/448*(-33*a^2*e^4+
10*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/
e^2/x^6-1/4480*(-231*a^3*e^6+15*a^2*c*d^2*e^4+95*a*c^2*d^4*e^2+105*c^3*d^6)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^3/d^4/e^3/x^5+3/32768*(-a*e^2+c*
d^2)^5*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*arctanh(1/
2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/a^(11/2)/d^(13/2)/e^(11/2)-3/16384*(-a*e^2+c*d^2)^3*(33*a
^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*(2*a*d*e+(a*e^2+c*d^2)
*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^5/d^6/e^5/x^2
```

Rubi [A] time = 0.89, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{3\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{16384a^5d^6e^5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 3
3*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^
2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)
/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d
^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*
a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45
*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(32768*a^
(11/2)*d^(13/2)*e^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^9}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-11ae^2\right)+3acde^2x\right)\left(ade+\left(cd\right.\right.}{x^8}}{8ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{112x^7}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{112x^7}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{112x^7}$$

$$=\frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2\right.\right.}{2048a^4d^5e^4x^4}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\right.}{16384a^5d^6e^5x^2}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\right.}{16384a^5d^6e^5x^2}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\right.}{16384a^5d^6e^5x^2}$$

Mathematica [A] time = 1.46, size = 512, normalized size = 0.82

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(-\frac{(d+ex)\left(33a^2e^4+34acd^2e^2+21c^2d^4\right)(ae+cdx)^2}{56a^2d^2e^2x^6}+\frac{\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(128a^{5/2}d^{5/2}e^{5/2}(d+ex)^5\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^8) + ((9*
c*d^2 + 11*a*e^2)*(a*e + c*d*x)^2*(d + e*x))/(14*a*d*e*x^7) - ((21*c^2*d^4
+ 34*a*c*d^2*e^2 + 33*a^2*e^4)*(a*e + c*d*x)^2*(d + e*x))/(56*a^2*d^2*e^2*x
^6) + ((15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(128
*a^(5/2)*d^(5/2)*e^(5/2)*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) + 5*(c*d^2 - a
*e^2)*x*(16*a^(5/2)*d^(3/2)*e^(5/2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + (
c*d^2 - a*e^2)*x*(8*a^(5/2)*Sqrt[d]*e^(5/2)*Sqrt[a*e + c*d*x]*(d + e*x)^(5/
2) + (c*d^2 - a*e^2)*x*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sq
rt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])))))/(10240*a^(9/
2)*d^(11/2)*e^(9/2)*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(8*a*d*e)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.13, size = 6030, normalized size = 9.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^9/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^9\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] Timed out
```

3.470 $\int \frac{x^3}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=271

$$\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{3\left(ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4c^2d^2e^3}$$

```
[Out] 3/8*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c
^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(
5/2)/e^(7/2)-3/4*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^
2/d^2/e^3-2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)/
(e*x+d)+1/2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3
```

Rubi [A] time = 0.34, antiderivative size = 298, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 779, 621, 206}

$$\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2\right.\right.}{4c^2d^2e^3\left(cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((5*c*d^2 - 3*a*e^2)*(3*
c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(4*c^2*d^2*e^3*(c*d^2 - a*e^2)) + (3*(5*c^2*d^4 + 2*a*c*d^2
*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(7/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\int \frac{x\left(2acd^2e\left(cd^2-ae^2\right)\right.}{\sqrt{ade+}}}{cde}$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(3\right.\right.}{}$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(3\right.\right.}{}$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(3\right.\right.}{}$$

Mathematica [A] time = 0.51, size = 331, normalized size = 1.22

$$\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(-a^3e^6-a^2cd^2e^4-3ac^2d^4e^2+5c^3d^6\right)\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)+c^{3/2}d^{3/2}\sqrt{}}{4c^{7/2}d^{7/2}e^{7/2}\left(cd^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (c^(3/2)*d^(3/2)*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e*
x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(15*
```

```
d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]
*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*
Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(4*c^(7/2)*d^(7/2)*e^(7/2)
*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 2.29, size = 758, normalized size = 2.80

$$\left[\frac{3\left(5\,c^3d^7 - 3\,ac^2d^5e^2 - a^2cd^3e^4 - a^3de^6 + \left(5\,c^3d^6e - 3\,ac^2d^4e^3 - a^2cd^2e^5 - a^3e^7\right)x\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2 + c^2d^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*
d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)*log(8*c^2
*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*
e + a*c*d*e^3)*x) - 4*(15*c^3*d^6*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2
*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a
^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6*e^4 -
a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)*x), -1/8*(3*(5*c^3*d^7 - 3*a*
c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*d^6*e - 3*a*c^2*d^4*e^3 -
a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*
x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*c^3*d^6*e - 4*a*c^2
*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d
^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c^4*d^6*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)
*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Evaluation time:
 0.41Error: Bad Argument Type
```

maple [A] time = 0.02, size = 391, normalized size = 1.44

$$\frac{3a^2e\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}\right)}{8\sqrt{cde}\;c^2d^2}+\frac{3a\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right.}\right.}{4\sqrt{cde}\;ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/2/e^2*x/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/4/e/c^2/d^2*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-7/4/e^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)+3/8*e/c^2/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+3/4/e/c*ln((c*d*e*x+
1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))
/(c*d*e)^(1/2)*a+15/8*d^2/e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2
)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2*d^3/e^4/(a*e^2-c
*d^2)/(x+d/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.471 $\int \frac{x^2}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+a}}{cde^2}$$

```
[Out] -1/2*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^
(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1638, 792, 621, 206}

$$-\frac{\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+a}}{cde^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d*e^2) + (2*d^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(d + e*x)) - ((3*c*d^
2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{\int \frac{-\frac{1}{2}de\left(cd^2+ae^2\right)-\frac{1}{2}e^2\left(3cd^2+ae^2\right)x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cde^3}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}$$

Mathematica [A] time = 0.36, size = 255, normalized size = 1.31

$$\frac{c^{3/2}d^{3/2}\sqrt{e}\left(-a^2e^3(d+ex)+acde\left(3d^2-e^2x^2\right)+c^2d^3x(3d+ex)\right)-\sqrt{cd}\sqrt{cd^2-ae^2}\left(-a^2e^4-2acd^2e^2+3c^2d^4\right)\sqrt{a}}{c^{5/2}d^{5/2}e^{5/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (c^(3/2)*d^(3/2)*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a*
c*d*e*(3*d^2 - e^2*x^2)) - Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(3*c^2*d^4 - 2*a*c
*d^2*e^2 - a^2*e^4)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]
*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2
- a*e^2])])/(c^(5/2)*d^(5/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

fricas [A] time = 1.11, size = 586, normalized size = 3.01

$$\left[\frac{\left(3\,c^2d^5-2\,acd^3e^2-a^2de^4+\left(3\,c^2d^4e-2\,acd^2e^3-a^2e^5\right)x\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cd}\right.}{4\left(c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/4*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^2*d^4*e - a*
c*d^2*e^3 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x),
 1/2*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2
*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3 + (c^2*d^
3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^5
*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Evaluation time: 0.41Err
or: Bad Argument Type
```

maple [A] time = 0.01, size = 241, normalized size = 1.24

$$-\frac{a\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}\right)}{2\sqrt{cde}\,cd}-\frac{3d\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right.}\right.}{2\sqrt{cde}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/c/d/e^2-1/2/c/d*ln((c*d*e*x+1/2*a*e
^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e
)^(1/2)*a-3/2/e^2*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-2*d^2/e^3/(a*e^2-c*d^2)/(x+d
/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.472 $\int \frac{x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

```
[Out] arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)-2*d*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {792, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)}+\frac{\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{e}$$

$$= -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)}+\frac{2\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{\ldots}{\sqrt{ade\ldots}}\right)}{e}$$

$$= -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)}+\frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cde\ldots}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+a\ldots\right.}}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Mathematica [A] time = 0.42, size = 189, normalized size = 1.36

$$\frac{2\sqrt{cd}\left(cd^2-ae^2\right)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-2c^{3/2}d^{5/2}\sqrt{e}(ae+cdx)}{c^{3/2}d^{3/2}e^{3/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*c^(3/2)*d^(5/2)*Sqrt[e]*(a*e + c*d*x) + 2*Sqrt[c*d]*(c*d^2 - a*e^2)^(3/
2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(c^(3
/2)*d^(3/2)*e^(3/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.38, size = 443, normalized size = 3.19

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,cd^2e-\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2\ldots\right.}{2\left(c^2d^4e^2-acd^2e^4+\left(c^2d^3e^3-ac\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^
4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x), -(2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*c*d^2*e + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)))/(c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)
*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 131, normalized size = 0.94

$$\frac{\ln\left(\frac{cdex+\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}}{\sqrt{cde}}+\sqrt{cde\,x^{2}+ade+\left(a\,e^{2}+c\,d^{2}\right)x}\right)}{\sqrt{cde}\;e}+\frac{2\sqrt{\left(x+\frac{d}{e}\right)^{2}cde+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}\;d}{\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/e*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(x+d/e)*((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.473 $\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\left(cd^2-ae^2\right)(d+ex)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{2(ae+cdx)}{\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.24, size = 59, normalized size = 1.13

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{cd^3-ade^2+\left(cd^2e-ae^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e -
 a*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2/sqr
t(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(c*d*exp(
1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d
*exp(1)^3+a*d*exp(1)*exp(2)))
```

maple [A] time = 0.01, size = 51, normalized size = 0.98

$$-\frac{2\left(cdx+ae\right)}{\left(a\,e^{2}-c\,d^{2}\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2*(c*d*x+a*e)/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 2.64, size = 50, normalized size = 0.96

$$-\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(a\,e^2-c\,d^2\right)\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] -(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e*
x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.474 $\int \frac{1}{x(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}$$

```
[Out] -arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-2*e*(c*d*x+a*e)/d/(-a*
e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 12, 724, 206}

$$-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\int -\frac{ae(}{2x\sqrt{ade+(}}}{ade(c}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\int \frac{1}{x\sqrt{ade+(cd^2+}}}{d}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{}{4a}\right.}{}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{}$$

Mathematica [A] time = 0.13, size = 131, normalized size = 0.92

$$\frac{2\left(-\frac{\sqrt{d}e^{3/2}(ae+cdx)}{cd^2-ae^2}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 2.02, size = 454, normalized size = 3.17

$$\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,ade^2-\left(cd^3-ade^2+(cd^2e-ae^3)x\right)\sqrt{ade}\log\left(\frac{8a^2d^2e^2+(c^2d^4+6acd^2e^2+a^2e^4)x^2-4\sqrt{}}{}\right.}{2\left(acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 - a^2*d^2*e^4)*x), -(2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d*e^2 + (c*d^2*e -
a*e^3)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^
2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 -
a^2*d^2*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 136, normalized size = 0.95

$$-\frac{\ln\left(\frac{2ade+\left(a\,e^{2}+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{cde\,x^{2}+ade+\left(a\,e^{2}+c\,d^{2}\right)x}}{x}\right)}{\sqrt{ade}\,d}+\frac{2\sqrt{\left(x+\frac{d}{e}\right)^{2}cde+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+2/d/(a*e^2-c*d^2)/(x+d/e)*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.475 $\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=229

$$\frac{\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ad^2ex\left(cd^2-ae^2\right)}-\frac{}{dx\left(cd^2-ae^2\right)}$$

```
[Out] 1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-2*e
*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(-3
*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/(-a*e^2+c*d^2
)/x
```

Rubi [A] time = 0.29, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{851, 822, 806, 724, 206\}$

$$\frac{\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ad^2ex\left(cd^2-ae^2\right)}-\frac{}{dx\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(2*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(cd^2\right.}{x}}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right.}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right.}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right.}{}$$

Mathematica [A] time = 0.12, size = 201, normalized size = 0.88

$$\frac{x\sqrt{d+ex}\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^3(d+3ex)-acde\left(d^2-\right.\right.}{a^{3/2}d^{5/2}e^{3/2}x\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a*
c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

fricas [A] time = 3.30, size = 610, normalized size = 2.66

$$\left[\frac{\sqrt{ade}\left(\left(c^2d^4e+2\,acd^2e^3-3\,a^2e^5\right)x^2+\left(c^2d^5+2\,acd^3e^2-3\,a^2de^4\right)x\right)\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2+4\,\sqrt{cdex^2+ade+}}{}\right.}{4\left(\left(a^2cd^5e^3-a^3d^3e^5\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(sqrt(a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (c^2*d^5 +
 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(
a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*
e^2 - a^3*d^4*e^4)*x), -1/2*(sqrt(-a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a
^2*e^5)*x^2 + (c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*
d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(a*c*
d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*e^2
- a^3*d^4*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(2*
exp(1)^2/2/d^2/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(
1))+(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)
*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(a*exp(2)+2*exp(1)^2*a+c*d^
2)/d^2/exp(1)/a/2/sqrt(-a*d*exp(1))*atan((sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c
*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-((sqrt(c*d*exp(1)*
x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a*exp(2)+c*d^2*(sqrt
(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)-2*d*exp(
1)*sqrt(c*d*exp(1))*a)/2/d^2/exp(1)/a/((sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d
^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a))
```

maple [A] time = 0.02, size = 253, normalized size = 1.10

$$\frac{c\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{2\sqrt{ade}\,ae}+\frac{3e\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{2\sqrt{ade}\,d^2}-\frac{2\sqrt{\left(x+\frac{d}{e}\right)^2}}{\left(a\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/d^2/a/e/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+3/2*e/d^2/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2))/x)+1/2/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
```

```
(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-2/d^2*e/(a*e^2-c*d^2)/(
x+d/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.476 $\int \frac{1}{x^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=329

$$\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4a^2d^3e^2x\left(cd^2-ae^2\right)}-\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2\cdots}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\cdots\right.}}\right.}{8a^{5/2}d^{7/2}e^{5/2}}\cdots$$

```
[Out] -3/8*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x
)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/
d^(7/2)/e^(5/2)-2*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-1/2*(-5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a/d^2/e/(-a*e^2+c*d^2)/x^2+1/4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)/x
```

Rubi [A] time = 0.51, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+\cdots}}{4a^2d^3e^2x\left(cd^2-ae^2\right)}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e
^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae(cd^2-5\ldots}{x^3\ldots}}{\ldots}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\ldots}{2\ldots}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\ldots}{2\ldots}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\ldots}{2\ldots}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\ldots}{2\ldots}$$

Mathematica [A] time = 0.19, size = 283, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^3e^4\left(2d^2-5dex-15e^2x^2\right)-a^2cde^2\left(2d^3-4d^2ex+de^2x^2+15e^3x^3\right)+ac^2d^3ex\left(d^2+5dex+4e^2x^2\right)+\ldots\right.}{4a^{5/2}d^{7/2}e^{5/2}x^2\left(cd^2-ae^2\right)\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e*
x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2*(
2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2)*d^(
7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 7.60, size = 792, normalized size = 2.41

$$\left[\frac{3\left(\left(c^3d^6e+ac^2d^4e^3+3\,a^2cd^2e^5-5\,a^3e^7\right)x^3+\left(c^3d^7+ac^2d^5e^2+3\,a^2cd^3e^4-5\,a^3de^6\right)x^2\right)\sqrt{ade}\,\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/16*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 + (
c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(a*d*e)*l
```

```
og((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e)
+ 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 -
(3*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2
*a^2*c*d^4*e^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/((a^3*c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2
), 1/8*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 +
(c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(-a*d*e)
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) - 2*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 - (3*a*c^2*d^5*e^2 + 4*a^2*c
*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2*a^2*c*d^4*e^3 - 5*a^3*d^2
*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c*d^6*e^4 - a^4
*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(-2
*exp(1)^3/2/d^3/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp
(1))+(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x
)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(3*a^2*exp(2)^2+4*exp(1)^2
*a^2*exp(2)+8*exp(1)^4*a^2+6*c*d^2*a*exp(2)+3*c^2*d^4)/4/d^3/exp(1)^2/a^2/2
/sqrt(-a*d*exp(1))*atan((sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)
-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-(-3*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)
+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)^2-4*exp(1)^2*(sqrt(c*
d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^3*a^2*exp(2
)-6*c*d^2*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(
1))*x)^3*a*exp(2)-3*c^2*d^4*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2)
)*x)-sqrt(c*d*exp(1))*x)^3+8*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(c*d*exp(1)*x
^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^2*a^2+5*d*exp(1)*(sqr
t(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a^3*exp
(2)^2+4*d*exp(1)^3*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt
(c*d*exp(1))*x)*a^3*exp(2)+10*c*d^3*exp(1)*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+
(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a^2*exp(2)+8*c*d^3*exp(1)^3*(sqrt(c
*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a^2+5*c^2*
d^5*exp(1)*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp
(1))*x)*a-8*d^2*exp(1)^2*sqrt(c*d*exp(1))*a^3*exp(2)-8*d^2*exp(1)^4*sqrt(c*
d*exp(1))*a^3-8*c*d^4*exp(1)^2*sqrt(c*d*exp(1))*a^2)/8/d^3/exp(1)^2/a^2/((s
qrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^2-d*e
xp(1)*a)^2)
```

maple [A] time = 0.02, size = 414, normalized size = 1.26

$$-\frac{3c\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{4\sqrt{ade}\,ad}-\frac{3c^2d\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{8\sqrt{ade}\,a^2e^2}-\frac{15e^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 7/4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-15/8/d^3*e^2/(a*d*e)^(1
/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d
```

```
^2)*x)^(1/2))/x)-3/4/d/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-1/2/d^2/a/e/x^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+3/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*c-3/8*d/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2+2/d^3*e^2/(a
*e^2-c*d^2)/(x+d/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{(d + ex)\,(ae + cdx)}\,\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.477 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$\frac{5\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}-\frac{2x^2\left(ade\left(cd^2-ae^2\right)\left(-3a^2e^4-12acd^2e^2+\right.\right.}{3cde^2\left(cd^2-\right.}$$

```
[Out] -2/3*d*x^4*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/8*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4
)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-2/3*x^2*(a*d*e*(-a*e^2+
c*d^2)*(-3*a^2*e^4-12*a*c*d^2*e^2+7*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2
*c*d^2*e^4-11*a*c^2*d^4*e^2+7*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/12*(105*c^4*d^8-190*a*c^3*d^6*e^2+36*a^2*
c^2*d^4*e^4+30*a^3*c*d^2*e^6-45*a^4*e^8-2*c*d*e*(-15*a^3*e^6+9*a^2*c*d^2*e^
4-61*a*c^2*d^4*e^2+35*c^3*d^6)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^3/d^3/e^4/(-a*e^2+c*d^2)^3
```

Rubi [A] time = 0.62, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 779, 621, 206}

$$-\frac{2x^2\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-11ac^2d^4e^2+7c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x^2*(a*d*e*(c*d^2
- a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*
d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 -
 a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 19
0*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*
d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3)
+ (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(8*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x^3(4a}{}}{}$$

$$= -\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x^2\left(ade\right.}{}$$

$$= -\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x^2\left(ade\right.}{}$$

$$= -\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x^2\left(ade\right.}{}$$

$$= -\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x^2\left(ade\right.}{}$$

Mathematica [A] time = 5.63, size = 296, normalized size = 0.57

$$\frac{\frac{2(d+ex)^2(ae+cdx)^2\left(\frac{24a^5e^9}{c^3\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{3\left(7ae^2+11cd^2\right)}{c^3}+\frac{8d^8}{(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{40\left(3ad^7e^2-2cd^9\right)}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{6dex}{c^2}\right)}{3d^3e^4}+\frac{5(d+ex)^{3/2}\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)(ae+cdx)^{3/}}{c^7}}{8((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((2*(a*e + c*d*x)^2*(d + e*x)^2*((-3*(11*c*d^2 + 7*a*e^2))/c^3 + (6*d*e*x)/
c^2 + (24*a^5*e^9)/(c^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)) + (8*d^8)/((c*d^2
- a*e^2)^2*(d + e*x)^2) + (40*(-2*c*d^9 + 3*a*d^7*e^2))/((c*d^2 - a*e^2)^3*
(d + e*x))))/(3*d^3*e^4) + (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(a*e
+ c*d*x)^(3/2)*(d + e*x)^(3/2)*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a
*e + c*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)])/(c^(7/2)*d^(7/2)*e^(9/2)))/(8
*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 10.29, size = 2120, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/48*(15*(7*a*c^5*d^12*e - 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4
*c^2*d^6*e^7 + 3*a^5*c*d^4*e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^
5*d^9*e^4 + 6*a^2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*
```

```
a^5*c*d*e^12)*x^3 + (14*c^6*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5
+ 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x
^2 + (7*c^6*d^13 - a*c^5*d^11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6
 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(c*d*e)*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x) - 4*(105*a*c^5*d^11*e^2 - 190*a^2*c^4*d^9*e^4 + 36*a
^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c*d^3*e^10 - 6*(c^6*d^9*e^4 -
3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*d^3*e^10)*x^4 + 3*(7*c^6*d^10
*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 + 8*a^3*c^3*d^4*e^9 - 5*a^4*c^2
*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5*d^9*e^4 + 12*a^2*c^4*d^7*e^6
 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 + (105*c^6*d^12*e - 50*a*c^5*d
^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6*e^7 + 45*a^4*c^2*d^4*e^9 - 9
0*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^7*d^
12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^10 - a^4*c^4*d^6*e^12 + (c^8*
d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^11 - a^3*c^5*d^5*e^13)*x^3 + (
2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*d^8*e^10 + a^3*c^5*d^6*e^12 -
 a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7*d^11*e^7 - 3*a^2*c^6*d^9*e^9
 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x), -1/24*(15*(7*a*c^5*d^12*e -
 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4*c^2*d^6*e^7 + 3*a^5*c*d^4*
e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^5*d^9*e^4 + 6*a^2*c^4*d^7*e
^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*a^5*c*d*e^12)*x^3 + (14*c^6
*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^
4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x^2 + (7*c^6*d^13 - a*c^5*d^
11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^
5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(105*a*c^5*d^11*e^2
 - 190*a^2*c^4*d^9*e^4 + 36*a^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c
*d^3*e^10 - 6*(c^6*d^9*e^4 - 3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*
d^3*e^10)*x^4 + 3*(7*c^6*d^10*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 +
8*a^3*c^3*d^4*e^9 - 5*a^4*c^2*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5
*d^9*e^4 + 12*a^2*c^4*d^7*e^6 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 +
 (105*c^6*d^12*e - 50*a*c^5*d^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6
*e^7 + 45*a^4*c^2*d^4*e^9 - 90*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^7*d^12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^
10 - a^4*c^4*d^6*e^12 + (c^8*d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^1
1 - a^3*c^5*d^5*e^13)*x^3 + (2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*
d^8*e^10 + a^3*c^5*d^6*e^12 - a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7
*d^11*e^7 - 3*a^2*c^6*d^9*e^9 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.03, size = 1680, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 15/8*e^4/c^3/d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*a^4+5/2*e^2/c^2/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3-7/16/e^3/c^2/(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*a+51/16/e^5/c*d^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-
9/4/e^3/c*x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+51/8/e^4*c*d^5/(-a^2*
e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+15/16*
e^5/c^4/d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*a^5+35/16*e^3/c^3/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-15/4/e^2/c^2/d*x/(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*a+15/4/e^2/c^2/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a-8/3*d^6/e^3*c
/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a-16/3*d^7/e
^4*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/4/
c*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*x*a^2-5/4/e/c^2/d^2*x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+21/8/e/
c*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*a^2+11/2/e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*x*a+9/8*e/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+95/16/e^3*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^
2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+2*d^4/e^5*(2*c*d*e*x+a*e^2
+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)+15/8/c^3/d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-15/8/c^3/d^3*x/(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-35/8/e^4/c*d*x/(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)+15/16*e/c^4/d^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+51
/16/e^5*c*d^6/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)+35/8/e^4/c*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2/3*d^5/e^6/(a*e^2-c*d^
2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-8/3*d^8/e^5*c^2/(a
*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+5/16/e/c^3/d^2/
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+1/2/e^2*x^3/c/d/(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.478 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=438

$$-\frac{2x\left(ade\left(cd^2-ae^2\right)\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)+x\left(cd^2-ae^2\right)\left(-3a^3e^6-a^2cd^2e^4-9ac^2d^4e^2+5c^3d^6\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(-}{}$$

```
[Out] -2/3*d*x^3*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2*c
*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)-2/3*x*(a*d*e*(-a*e^2+c*d^2)*(-3*a^2*e^4-1
0*a*c*d^2*e^2+5*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2*c*d^2*e^4-9*a*c^2*d
^4*e^2+5*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/3*(-9*a^3*e^6+9*a^2*c*d^2*e^4-31*a*c^2*d^4*e^2+15*c^3*d^6)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3/(-a*e^2+c*d^2)^3
```

Rubi [A] time = 0.54, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {849, 818, 640, 621, 206}

$$-\frac{2x\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-9ac^2d^4e^2+5c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(9}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x*(a*d*e*(c*d^2 -
a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^
6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*
e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c
^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]])])/(2*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x^2\left(3acd^2e \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\left(cd^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\left(cd^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\left(cd^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\left(cd^2 \cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 1.34, size = 387, normalized size = 0.88

$$\frac{(ae+cdx)\left(-\frac{ae(3ae^2-cd^2)\left(a^2e^2(8d^2+12dex+3e^2x^2)+2acd^2ex(2d+3ex)-c^2d^4x^2\right)}{cd(cd^2-ae^2)^3}-\frac{(3ae^2+5cd^2)\sqrt{ae+cdx}\left(c^{3/2}d^{7/2}\sqrt{e}(cd^2-ae^2)\sqrt{ae+cdx}-(d+\cdots\right.}{\cdots}\right.}{3cde((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((a*e + c*d*x)*(3*x^3 - (a*e*(-(c*d^2) + 3*a*e^2)*(-(c^2*d^4*x^2) + 2*a*c*d
^2*e*x*(2*d + 3*e*x) + a^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2)))/(c*d*(c*d^2
 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*Sqrt[a*e + c*d*x]*(c^(3/2)*d^(7/2)*Sqrt
[e]*(c*d^2 - a*e^2)*Sqrt[a*e + c*d*x] - (d + e*x)*(2*c^(3/2)*d^(5/2)*Sqrt[e
]*(2*c*d^2 - 3*a*e^2)*Sqrt[a*e + c*d*x] - 3*Sqrt[c*d]*(c*d^2 - a*e^2)^(5/2)
*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])))/(c^(5/2)*d^(5/2)*e^(5/2
)*(c*d^2 - a*e^2)^2)))/(3*c*d*e*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 4.63, size = 1782, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c
*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^
5*e^6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4
*d^8*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*
d^11 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*
d^3*e^8 - 6*a^5*d*e^10)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(15*a
*c^4*d^9*e^2 - 31*a^2*c^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3
*(c^5*d^8*e^3 - 3*a*c^4*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3
+ (20*c^5*d^9*e^2 - 39*a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^
8 - 9*a^4*c*d*e^10)*x^2 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^
6*e^5 + 15*a^3*c^2*d^4*e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9
 - a^4*c^3*d^5*e^11 + (c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10
- a^3*c^4*d^4*e^12)*x^3 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7
*e^9 + a^3*c^4*d^5*e^11 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^1
0*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x), 1/
6*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c*d^4
*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^5*e^
6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4*d^8
*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*d^11
 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*d^3*
e^8 - 6*a^5*d*e^10)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
 a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*a*c^4*d^9*e^2 - 31*a^2*c
^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3*(c^5*d^8*e^3 - 3*a*c^4
*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3 + (20*c^5*d^9*e^2 - 39*
a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^8 - 9*a^4*c*d*e^10)*x^2
 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^6*e^5 + 15*a^3*c^2*d^4*
e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*
c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9 - a^4*c^3*d^5*e^11 + (
```

```
c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10 - a^3*c^4*d^4*e^12)*x^3
 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7*e^9 + a^3*c^4*d^5*e^11
 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^10*e^6 - 3*a^2*c^5*d^8*e
^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 1266, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -3/2*e^3/c^2/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*a^3+5/2/e^3/c*x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/
4/c^3/d^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-9/4/e^4/c*d/(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-5/2/e^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(
c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+16/3*d^
6/e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+8
/3*d^5/e^2*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*
a-9/2/e*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*x*a-9/2/e^3*c*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x-3/4*e^4/c^3/d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4
)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-3/2*e^2/c^2/d/(-a^2*e^4+2*a*c
*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3-3/2/e/c^2/d^2
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)*a+3/2/e/c^2/d^2*x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2)*a-3/2*e/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*x*a^2-2/3*d^4/e^5/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d^7/e^4*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/e^2*x^2/c
/d/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/c*d/(-a^2*e^4+2*a*c*d^2*e^2-c^
2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-9/2/e^2*d^3/(-a^2*e^4+2*
a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-9/4/e^4*c*d^
5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.479 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\left(x\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+c}}\right.}{c^{3/2}d^{3/2}e^{5/2}}$$

```
[Out] -2/3*d*x^2*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^
(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3
/2)/e^(5/2)-2/3*(a*d*e*(-3*a*e^2+c*d^2)*(a*e^2+3*c*d^2)+(-3*a^3*e^6-a^2*c*d
^2*e^4-7*a*c^2*d^4*e^2+3*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 777, 621, 206}

$$-\frac{2\left(x\left(-a^2cd^2e^4-3a^3e^6-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+c}}\right.}{c^{3/2}d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(a*d*e*(c*d^2 - 3*
a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3
*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x(2ac\ldots}{(\ldots}}{\ldots}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(c\ldots\right.\right.}{\ldots}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(c\ldots\right.\right.}{\ldots}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(c\ldots\right.\right.}{\ldots}$$

Mathematica [C] time = 4.64, size = 1443, normalized size = 4.86

$$a^3e^3(ae+cdx)^2\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{5/2}\left(\frac{56\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)(ae+cdx)^5}{a^3e\left(cd^2-ae^2\right)^2} - \frac{280\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{e(ae+cdx)}{ae^2-cd^2}\right)(ae+cdx)^4}{a^2\left(cd^2-ae^2\right)^2} + \frac{96\,{}_4F_3\left(\frac{1}{2},2,2,\frac{7}{2};1,1,\frac{9}{2};\frac{e(ae+cd\ldots}{ae^2-cd\ldots}\right)}{a^3e^2\left(ae^2-cd^2\right)\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (a^3*e^3*(a*e + c*d*x)^2*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(5/2)*((2520*c*d
*(d + e*x))/(a*e^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (1330*c*d*(d +
e*x))/(e*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (1050*c*d*(
a*e + c*d*x)*(d + e*x))/(a^2*e^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) + (
196*c*d*(a*e + c*d*x)^2*(d + e*x))/(a^3*e^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a
*e^2)]) + 1568*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + (1575*(c*d^2 - a*e^2
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a^2*e^4) + (1995*(c*d^2 - a*e^2
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e^2*(a*e + c*d*x)^2) - (3780*(c
*d^2 - a*e^2)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a*e^3*(a*e + c*d*x)
) - (294*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])/(a^3*e^5) - 504*(1 + (c*d*x)/(a*e))*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)] + 336*(1 + (c*d*x)/(a*e))^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - 56*
(1 + (c*d*x)/(a*e))^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - (1995*ArcSin[
Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/((e*(a*e + c*d*x))/(-(c*d^2) +
 a*e^2))^(5/2) + (3780*(a*e + c*d*x)*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2
) + a*e^2)]])/(a*e*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) - (1575*(a
*e + c*d*x)^2*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a^2*e^2*
((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) + (294*(a*e + c*d*x)^3*ArcSin
[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a^3*e^3*((e*(a*e + c*d*x))/(
-(c*d^2) + a*e^2))^(5/2)) - (168*e^2*(a*e + c*d*x)^2*Hypergeometric2F1[3/2,
 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d^2 - a*e^2)^2 + (392
*e*(a*e + c*d*x)^3*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c
*d^2) + a*e^2)])/(a*(c*d^2 - a*e^2)^2) - (280*(a*e + c*d*x)^4*Hypergeometri
c2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^2*(c*d^2 - a
*e^2)^2) + (56*(a*e + c*d*x)^5*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e +
c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e*(c*d^2 - a*e^2)^2) - (96*e*(a*e + c*d*x
)*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d
^2) + a*e^2)])/(-(c*d^2) + a*e^2) + (288*(a*e + c*d*x)^2*HypergeometricPFQ[
{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a*(-
(c*d^2) + a*e^2)) - (288*(a*e + c*d*x)^3*HypergeometricPFQ[{1/2, 2, 2, 7/2}
, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^2*(-(c*d^2*e) + a*
e^3)) + (96*(a*e + c*d*x)^4*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}
, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e^2*(-(c*d^2) + a*e^2))))/(25
2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2))
```

fricas [B] time = 5.33, size = 1466, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^7*e^2 - 8
*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c^3*d^5*e^4 - 3*a
^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^2*d^4*e^5 - 6*a^
3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^5*d^10*e^
4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^10 + (c^6*d^9*e^5
 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*x^3 + (2*c^6*d^1
0*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e^10 - a^4*c^2*d^
2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7*e^7 + 5*a^3*c^3
*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x), -1/3*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3
 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c
^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*
```

```
d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c
^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*a
*c^3*d^7*e^2 - 8*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c
^3*d^5*e^4 - 3*a^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^
2*d^4*e^5 - 6*a^3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(a*c^5*d^10*e^4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^1
0 + (c^6*d^9*e^5 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*
x^3 + (2*c^6*d^10*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e
^10 - a^4*c^2*d^2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7
*e^7 + 5*a^3*c^3*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [**B**] time = 0.01, size = 977, normalized size = 3.29

$$-\frac{16c^2d^5x}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}\,e^2}-\frac{8ac\,d^4}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/e^2*x/c/d/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/2/e/c^2/d^2/(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+3/2/e^3/c/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)+e^2/c/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*a^2+4*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x*a+3/e^2*c*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/2*e^3/c^2/d^2/(-a^2*e^4+2*a*c*d^2*
e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+5/2*e/c/(-a^2*e^4+
2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+7/2/e*d^
2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
a+3/2/e^3*c*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)+1/e^2/c/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2*d^2/e^3*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)+2/3*d^3/e^4/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-16/3*d^5/e^2*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)*x-8/3*d^4/e*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2)*a-8/3*d^6/e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.480 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] 2/3*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
a*e*(2*a*d*e+(a*e^2+c*d^2)*x)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {854, 12, 636}

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\int -\frac{\cdots}{(ade\cdots}}{3a\cdots}$$

$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(4ae)\int -\cdots}{\cdots}$$

$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\cdots}{3\left(cd^2-\cdots\right.}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.79

$$\frac{-2a^2e^2\left(8d^2+12dex+3e^2x^2\right)-4acd^2ex(2d+3ex)+2c^2d^4x^2}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*c^2*d^4*x^2 - 4*a*c*d^2*e*x*(2*d + 3*e*x) - 2*a^2*e^2*(8*d^2 + 12*d*e*x
+ 3*e^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 4.27, size = 308, normalized size = 2.44

$$-\frac{2\left(8\,a^2d^2e^2-\left(c^2d^4-6\,acd^2e^2-3\,a^2e^4\right)x^2+4\left(acd^3e+3\,a\cdots\right.\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2/3*(8*a^2*d^2*e^2 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*x^2 + 4*(a*c*d^
3*e + 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*
a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c
^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 -
a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 145, normalized size = 1.15

$$\frac{2\,(cdx+ae)\left(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*a^2*e^4*x^2+6*a*c*d^2*e^2*x^2-c^2*d^4*x^2+12*a^2*d*e^3*x
+4*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*
d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 3.60, size = 1071, normalized size = 8.50

$$\frac{4\,c\,d^3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^5\,e^3+3\,x\,a\,c^2\,d^4\,e^4-c^3\,d^7\,e-x\,c^3\,d^6\,e^2\right)}-\frac{\cdots}{3\,a^2\,d^2\,e^5+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*c*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^3
*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*d^3*e^5 - c^3*d^6*e^2*x + 3*
a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) - (2*d^2*(a*d*e + a*e^2*x + c*d^2*x +
 c*d*e*x^2)^(1/2))/(3*c^2*d^6*e + 3*a^2*d^2*e^5 + 3*a^2*e^7*x^2 + 6*c^2*d^5
*e^2*x + 3*c^2*d^4*e^3*x^2 - 6*a*c*d^4*e^3 + 6*a^2*d*e^6*x - 12*a*c*d^3*e^4
*x - 6*a*c*d^2*e^5*x^2) - (4*a*d*e^2*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2
)^(1/2))/(3*(a^3*d*e^7 - c^3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*
d^3*e^5 - c^3*d^6*e^2*x + 3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) + (2*c^4*
d^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*
d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) + (22*a^3*c
*d^2*e^5)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a
*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) - (28*
a^2*c^2*d^4*e^3)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (2*a*c^3*d^6*e)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (10*a^2*c^2*d^3*e^4*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*
(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*
c*d*e^9)) + (2*a^3*c*d*e^6*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2
)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^
4*c*d*e^9)) - (22*a*c^3*d^5*e^2*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^
2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e
^7 + a^4*c*d*e^9))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.481 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] -2/3*d/e/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2/3
*(3*a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {792, 613}

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a
*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd\right.}{}$$

$$= -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{}{3e\left(\right.}$$

Mathematica [A] time = 0.04, size = 100, normalized size = 0.72

$$\frac{2\left(a^2e^3(2d+3ex)+2acde\left(3d^2+5dex+3e^2x^2\right)+c^2d^3x(3d+2ex)\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5*
d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d +
 e*x)])
```

fricas [B] time = 4.13, size = 314, normalized size = 2.28

$$\frac{2\left(6\,acd^3e+2\,a^2de^3+2\left(c^2d^3e+3\,acde^3\right)x^2+\left(3\,c^2d^4+10\,acd^2e^2\right.\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*d
^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^
4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4
*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2
 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d
*e^8)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 149, normalized size = 1.08

$$-\frac{2\,(cdx+ae)\left(6acd\,e^3x^2+2c^2d^3e\,x^2+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2a^2d\,e^3+6ac\,d^3e\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e^
2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d
^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 3.32, size = 499, normalized size = 3.62

$$\frac{4\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,c^2\,d^4\,x\,\sqrt{c\,d\,e\,x^2+\left(\right.}}{-3\,a^4\,d^2\,e^7-6\,a^4\,d\,e^8\,x-3\,a^4\,e^9\,x^2+9\,a^3\,c\,d^4\,e^5+15\,a^3\,c\,d^3\,e^6\,x+3\,a^3\,c\,d^2\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2) + 4*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*d*e +
 c*d*e*x^2)^(1/2) + 12*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 12*a
*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*c^4*d^9*x -
3*a^4*d^2*e^7 - 3*a^4*e^9*x^2 + 9*a^3*c*d^4*e^5 + 6*c^4*d^8*e*x^2 - 9*a^2*c
^2*d^6*e^3 + 3*c^4*d^7*e^2*x^3 + 3*a*c^3*d^8*e - 6*a^4*d*e^8*x + 9*a^2*c^2*
d^4*e^5*x^2 + 9*a^2*c^2*d^3*e^6*x^3 - 3*a*c^3*d^7*e^2*x + 15*a^3*c*d^3*e^6*
x - 3*a^3*c*d*e^8*x^3 - 9*a^2*c^2*d^5*e^4*x - 15*a*c^3*d^6*e^3*x^2 + 3*a^3*
c*d^2*e^7*x^2 - 9*a*c^3*d^5*e^4*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.482 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] 2/3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*c*d*
(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(4cd)\int}{}$$

$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{}{3\left(cd^2-\right.}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.79

$$\frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2\left(3d^2+12dex+8e^2x^2\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e^
2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 3.73, size = 306, normalized size = 2.53

$$-\frac{2\left(8\,c^2d^2e^2x^2+3\,c^2d^4+6\,acd^2e^2-a^2e^4+4\left(3\,c^2d^3e+\right.\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d^
3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^8*
e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*
c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3
*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*
c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 138, normalized size = 1.14

$$-\frac{2\,(cdx+ae)\left(-8c^2d^2e^2x^2-4acd\,e^3x-12c^2d^3ex+a^2e^4-6ac\,d^2e^2-3c^2d^4\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-6
*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 2.88, size = 120, normalized size = 0.99

$$\frac{2\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+4\,a\,c\,d\,e^3\,x+3\,c^2\,d^4+12\,c^2\,d^3\,e\,x+8\,c^2\,d^2\,e^2\,x^2\right)}{3\,\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\,d^2\right)^3\,\left(d+e\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*c
^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e +
 c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.483 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}+\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3\right.}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
-arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+2/3*(3*c^3*d^6+a*c^2*d
^4*e^2+7*a^2*c*d^2*e^4-3*a^3*e^6+c*d*e*(-a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*x)/
a/d^2/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 12, 724, 206}

$$\frac{2\left(7a^2cd^2e^4-3a^3e^6+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}\right.}{a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6
 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)
^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])]/(a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
```

```
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{3}{2}ae\left(cd^2\right.}{x\left(ade\right.}}{3a}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+\right.}{}$$

Mathematica [A] time = 0.41, size = 262, normalized size = 0.97

$$\frac{2\left(-\frac{(d+ex)(ae+cdx)^{3/2}\left(\sqrt{a}\sqrt{d}\sqrt{e}\left(3a^2e^5-8acd^2e^3-3c^2d^4e\right)\sqrt{ae+cdx}+3\sqrt{d+ex}\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)}{3\sqrt{a}\,d^{5/2}\sqrt{e}\left(cd^2-ae^2\right)^2}+\frac{\left(ae^3+3cd^2e\right)(ae+cdx)^2}{3cd^3-3ade^2}+cd(\right.}{ae\left(cd^2-ae^2\right)((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c*d*(a*e + c*d*x) + ((3*c*d^2*e + a*e^3)*(a*e + c*d*x)^2)/(3*c*d^3 - 3*
a*d*e^2) - ((a*e + c*d*x)^(3/2)*(d + e*x)*(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-3*c^2*
```

```
d^4*e - 8*a*c*d^2*e^3 + 3*a^2*e^5)*Sqrt[a*e + c*d*x] + 3*(c*d^2 - a*e^2)^3*
Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])]))/(3*Sqrt[a]*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)^2)))/(a*e*(c*d^2 - a*e
^2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 10.62, size = 1476, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*a*c^3*d
^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3 + 8*a^2*c^2*d^4*e
^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d^5*e^4 + 4*a^3*c*
d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3
*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*d^5*e^9 + (a^2*c^
4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c*d^4*e^10)*x^3 +
(2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^7 + a^5*c*d^5*e^9
 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^4 - 3*a^4*c^2*d^8
*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x), 1/3*(3*(a*c^3*d^8*e - 3*a^2*c^
2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4
+ 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3
*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2
- 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-a*d*e)*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) + 2*(3*a*c^3*d^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3
+ 8*a^2*c^2*d^4*e^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d
^5*e^4 + 4*a^3*c*d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*
d^5*e^9 + (a^2*c^4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c
*d^4*e^10)*x^3 + (2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^
7 + a^5*c*d^5*e^9 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^
4 - 3*a^4*c^2*d^8*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.02, size = 682, normalized size = 2.52

$$-\frac{16c^2 d\,e^2 x}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{8ac\,e^3}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 1/d^2/a/e/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-2/d*e^2/(-a^2*e^4+2*a*c*d
^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-2*d/a/(-a^2*e^4
+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-1/d^2
*a*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)-2*e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*c-d^2/a/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*c^2-1/d^2/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+2/3/d/(a*e^2-c*d^2)
/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-16/3*d*c^2*e^2/(a*e^
2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-8/3*c*e^3/(a*e^2
-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a-8/3*d^2*c^2*e/(a*
e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.484 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=394

$$\frac{\left(5ae^2+3cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}+\frac{2\left(-5a^3e^6+cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2 \cdots\right.}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade \cdots}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)+1/2*(5*a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1
/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2
)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+9*a^2*c*d^2*e^4-5*a^3*e^6+c*d*e*(-5*a^2*e^4+
10*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/x/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-1/3*(-15*a^3*e^6+31*a^2*c*d^2*e^4-9*a*c^2*d^4*e^2+9*c^
3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3
/x
```

Rubi [A] time = 0.59, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 806, 724, 206}

$$-\frac{\left(31a^2cd^2e^4-15a^3e^6-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x\left(cd^2-ae^2\right)^3}+\frac{2\left(cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4 \cdots\right.\right.}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e
^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 -
 a*e^2)^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((9*c^3*d^6 - 9*
a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x) + ((3*c*d^2 + 5*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
```

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ae(}{}}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6\right.}{}$$

Mathematica [A] time = 0.57, size = 370, normalized size = 0.94

$$(ae+cdx)\left(3a^{3/2}d^{5/2}e^{3/2}\left(ae^2-cd^2\right)^3+\sqrt{a}\,d^{3/2}\sqrt{e}\,x\left(ae^2-cd^2\right)\left(5a^2e^5-6acd^2e^3+9c^2d^4e\right)(ae+cdx)+x(d+e$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(3*a^(3/2)*d^(5/2)*e^(3/2)*(-(c*d^2) + a*e^2)^3 + 3*Sqrt[a]*
c*d^(7/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-3*c*d^2 + a*e^2)*x + Sqrt[a]*d^(3/2)*
Sqrt[e]*(-(c*d^2) + a*e^2)*(9*c^2*d^4*e - 6*a*c*d^2*e^3 + 5*a^2*e^5)*x*(a*e
 + c*d*x) + x*Sqrt[a*e + c*d*x]*(d + e*x)*(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*
d^6*e + 9*a*c^2*d^4*e^3 - 31*a^2*c*d^2*e^5 + 15*a^3*e^7)*Sqrt[a*e + c*d*x]
+ 3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sq
rt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])))/(3*a^(5/2)*d^(7/2)*e^(
5/2)*(c*d^2 - a*e^2)^3*x*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 30.79, size = 1812, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/12*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^2
*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 16*a^2*c
^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*a^5*e^11)*x^3 + (3*c^
5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4*c*d^3*e^8 - 10*a^5*d
*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 + 12*a
^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4
+ 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x
)/x^2) - 4*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5
*d^3*e^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a
^4*c*d^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5
*e^6 + 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*
d^8*e^3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^
7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c
^3*d^10*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3
*c^4*d^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*
a^7*d^5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^
8 - a^7*d^6*e^10)*x), -1/6*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3
*d^5*e^6 + 12*a^3*c^2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c
^4*d^8*e^3 - 16*a^2*c^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*
a^5*e^11)*x^3 + (3*c^5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4
*c*d^3*e^8 - 10*a^5*d*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a
^3*c^2*d^6*e^5 + 12*a^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(-a*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5*d^3*e
^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a^4*c*d
^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5*e^6 +
 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*d^8*e^
3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^7 + 3*
```

```
a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c^3*d^1
0*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3*c^4*d
^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*a^7*d^
5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^8 - a^
7*d^6*e^10)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.41Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.02, size = 912, normalized size = 2.31

$$\frac{16c^2e^3x}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8ac\,e^4}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}\,d}+- \quad 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/d^2/a/e/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-5/2/d^3/a/(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/2/d/a^2/e^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*c+5/d^2*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*c+3*d^2/a^2/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3+5/2/d^3*a*e^4/(-a^2*e^4+2*a*c*d^2*e^2-
c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+5/2/d*e^2/(-a^2*e^4+2*a*c*
d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+3/2*d/a/(-a^2*e^
4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+3/2*d^
3/a^2/e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*c^3+5/2/d^3/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+3/2/d/a^2/e^2/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2))/x)*c-2/3*e/d^2/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)+16/3*e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)*x+8/3*e^4/d*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c
*d^2)*(x+d/e))^(1/2)*a+8/3*e^2*d*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{3}{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.485 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=522

$$-\frac{5\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}+\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2\right.\right.}{3ad^2ex^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2\right.}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)-5/8*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*
d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^
(7/2)/d^(9/2)/e^(7/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+11*a^2*c*d^2*e^4-7*a^3*e
^6+c*d*e*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/
x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/6*(-35*a^3*e^6+61*a^2*c*d^2*e
^4-9*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/
d^3/e^2/(-a*e^2+c*d^2)^3/x^2+1/12*(-105*a^4*e^8+190*a^3*c*d^2*e^6-36*a^2*c^
2*d^4*e^4-30*a*c^3*d^6*e^2+45*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x
```

Rubi [A] time = 0.80, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{\left(61a^2cd^2e^4-35a^3e^6-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6a^2d^3e^2x^2\left(cd^2-ae^2\right)^3}+\frac{\left(-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^
6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8
 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^
2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NegQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}a}{}}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\right.}{}$$

Mathematica [A] time = 0.93, size = 467, normalized size = 0.89

$$\frac{(ae+cdx)\left(6a^{5/2}d^{7/2}e^{5/2}\left(ae^2-cd^2\right)^3+x\left(3a^{3/2}d^{5/2}e^{3/2}\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3-3\sqrt{a}\,d^{5/2}\sqrt{e}\,x\left(7a^2cde^4-15c^3d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(6*a^(5/2)*d^(7/2)*e^(5/2)*(-(c*d^2) + a*e^2)^3 + x*(3*a^(3/
2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2) - 3*Sqrt[a]*d^(5/2
)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-15*c^3*d^5 + 7*a^2*c*d*e^4)*x - Sqrt[a]*d^(3/
2)*Sqrt[e]*(-(c*d^2) + a*e^2)*(45*c^3*d^6*e - 15*a*c^2*d^4*e^3 - 33*a^2*c*d
^2*e^5 + 35*a^3*e^7)*x*(a*e + c*d*x) - x*Sqrt[a*e + c*d*x]*(d + e*x)*(Sqrt[
a]*Sqrt[d]*Sqrt[e]*(-45*c^4*d^8*e + 30*a*c^3*d^6*e^3 + 36*a^2*c^2*d^4*e^5 -
 190*a^3*c*d^2*e^7 + 105*a^4*e^9)*Sqrt[a*e + c*d*x] + 15*(c*d^2 - a*e^2)^3*
(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt
[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))))/(12*a^(7/2)*d^(9/2)*e^(
7/2)*(c*d^2 - a*e^2)^3*x^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 64.97, size = 2162, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/48*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c^
3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3*a
*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e^9
 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 - 8*a^
2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^10
- 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c^3*d^8
*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*sqrt(a*d
*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*
d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(6*a^3*c^3*d^10*e^3 - 18*a^4*c
^2*d^8*e^5 + 18*a^5*c*d^6*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e^3 - 30*a^2
*c^4*d^8*e^5 - 36*a^3*c^3*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a^5*c*d^2*e^1
1)*x^4 - (90*a*c^5*d^11*e^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*d^7*e^6 + 222
*a^4*c^2*d^5*e^8 + 50*a^5*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 - (45*a*c^5*d^12
*e - 66*a^3*c^3*d^8*e^5 - 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4*e^9 - 140*a^6*
d^2*e^11)*x^2 - 3*(5*a^2*c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 - 6*a^4*c^2*d^7*e
^6 + 16*a^5*c*d^5*e^8 - 7*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/((a^4*c^4*d^12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*a^6*c^2*d^8*e^10 -
 a^7*c*d^6*e^12)*x^5 + (2*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d^11*e^7 + 3*a^6*c^2
*d^9*e^9 + a^7*c*d^7*e^11 - a^8*d^5*e^13)*x^4 + (a^4*c^4*d^14*e^4 - a^5*c^3
*d^12*e^6 - 3*a^6*c^2*d^10*e^8 + 5*a^7*c*d^8*e^10 - 2*a^8*d^6*e^12)*x^3 + (
a^5*c^3*d^13*e^5 - 3*a^6*c^2*d^11*e^7 + 3*a^7*c*d^9*e^9 - a^8*d^7*e^11)*x^2
), 1/24*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*
c^3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3
*a*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e
^9 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 - 8*
a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^1
0 - 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c^3*d
^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*sqrt(-
a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e +
 a^2*d*e^3)*x)) - 2*(6*a^3*c^3*d^10*e^3 - 18*a^4*c^2*d^8*e^5 + 18*a^5*c*d^6
*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e^3 - 30*a^2*c^4*d^8*e^5 - 36*a^3*c^3
*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a^5*c*d^2*e^11)*x^4 - (90*a*c^5*d^11*e
^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*d^7*e^6 + 222*a^4*c^2*d^5*e^8 + 50*a^5
*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 - (45*a*c^5*d^12*e - 66*a^3*c^3*d^8*e^5 -
 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4*e^9 - 140*a^6*d^2*e^11)*x^2 - 3*(5*a^2*
c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 - 6*a^4*c^2*d^7*e^6 + 16*a^5*c*d^5*e^8 - 7
*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^4*c^4*d^
12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*a^6*c^2*d^8*e^10 - a^7*c*d^6*e^12)*x^5 + (2
*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d^11*e^7 + 3*a^6*c^2*d^9*e^9 + a^7*c*d^7*e^11
 - a^8*d^5*e^13)*x^4 + (a^4*c^4*d^14*e^4 - a^5*c^3*d^12*e^6 - 3*a^6*c^2*d^1
0*e^8 + 5*a^7*c*d^8*e^10 - 2*a^8*d^6*e^12)*x^3 + (a^5*c^3*d^13*e^5 - 3*a^6*
c^2*d^11*e^7 + 3*a^7*c*d^9*e^9 - a^8*d^7*e^11)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.02, size = 1319, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -15/4*d^3/a^3/e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*x*c^4+7/4*e^2/d/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-8/3*e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^
2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/8/a^3/e^3/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*c^2+9/4/d^3/a/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+35/8
*e/d^4/a/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-16/3/d*e^4*c^2/(a*e^2-c*d^
2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-8/3/d^2*e^5*c/(a*e^2-c
*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a-15/8*d^4/a^3/e^3/(-
a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4+
5/4/d/a^2/e^2/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-35/4*e^4/d^3/(-a^
2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-5/
4*d/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*x*c^3-5/2/e*d^2/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*c^3-15/4/e/d^2/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-15/
8/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/2/d^2/a/e/x^2/(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)+2/3/d^3*e^2/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2)+15/4/e/d^2/a^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*c-35/8*e^5/d^4*a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)-7/2*e^3/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+1/4*e/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-35/8*e/d^4/a/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{3}{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.486 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=664

$$\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{3ad^2ex^3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2\right.}{3a^2d^3e^2}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+5/16*(21*a^3*e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(
1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+1
3*a^2*c*d^2*e^4-9*a^3*e^6+c*d*e*(-9*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*x)/a/
d^2/e/(-a*e^2+c*d^2)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/3*(-21
*a^3*e^6+33*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+7*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3/x^3+1/12*(-105*a^4*e^8+168*a
^3*c*d^2*e^6-18*a^2*c^2*d^4*e^4-16*a*c^3*d^6*e^2+35*c^4*d^8)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x^2-1/24*(-315*a^5*e
^10+525*a^4*c*d^2*e^8-78*a^3*c^2*d^4*e^6-54*a^2*c^3*d^6*e^4-55*a*c^4*d^8*e^
2+105*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^4/(-a*e^2
+c*d^2)^3/x
```

Rubi [A] time = 1.17, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{\left(33a^2cd^2e^4-21a^3e^6-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x^3\left(cd^2-ae^2\right)^3}-\frac{\left(-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6
 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8
- 16*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2
)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*
c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(24*a^4*d^5*e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15
*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 +
 a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}dx = \int \frac{ae+cdx}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{3}{2}a\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.39, size = 593, normalized size = 0.89

$$\frac{(ae+cdx)\left(24a^{7/2}d^{9/2}e^{7/2}\left(ae^2-cd^2\right)^3+x\left(6a^{5/2}d^{7/2}e^{5/2}\left(cd^2-ae^2\right)^3\left(9ae^2+7cd^2\right)+3a^{3/2}d^{5/2}e^{3/2}x\left(ae^2-cd^2\right)^3\left(6\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(24*a^(7/2)*d^(9/2)*e^(7/2)*(-(c*d^2) + a*e^2)^3 + x*(6*a^(5
/2)*d^(7/2)*e^(5/2)*(c*d^2 - a*e^2)^3*(7*c*d^2 + 9*a*e^2) + 3*a^(3/2)*d^(5/
2)*e^(3/2)*(-(c*d^2) + a*e^2)^3*(35*c^2*d^4 + 54*a*c*d^2*e^2 + 63*a^2*e^4)*
x + x^2*(9*Sqrt[a]*c*d^(7/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-35*c^3*d^6 - 5*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 21*a^3*e^6) + 3*Sqrt[a]*d^(3/2)*Sqrt[e]*(-(c
*d^2) + a*e^2)*(105*c^4*d^8*e - 20*a*c^3*d^6*e^3 - 42*a^2*c^2*d^4*e^5 - 84*
a^3*c*d^2*e^7 + 105*a^4*e^9)*(a*e + c*d*x) + Sqrt[a*e + c*d*x]*(d + e*x)*(3
*Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*e + 55*a*c^4*d^8*e^3 + 54*a^2*c^3*d
^6*e^5 + 78*a^3*c^2*d^4*e^7 - 525*a^4*c*d^2*e^9 + 315*a^5*e^11)*Sqrt[a*e +
c*d*x] + 45*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*
```

```
e^4 + 21*a^3*e^6)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a
]*Sqrt[e]*Sqrt[d + e*x])])))))/(72*a^(9/2)*d^(11/2)*e^(9/2)*(c*d^2 - a*e^2)
^3*x^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.42Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.02, size = 1705, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 35/8*d^4/a^4/e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*c^5+25/12/e*d^2/a^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4-41/12/d^2*e^3/a/(-a^2*e^4+2*a*c*d^
2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2+75/16/e^2/d/a^
3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/6*e/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)
/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-43/24/d*e^2/a/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+105/8/d^4*e^
5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
x*c+35/16*d^5/a^4/e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^5+35/16*d/a^4/e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+16/3
/d^2*e^5*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*
x+8/3/d^3*e^6*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2)*a-17/6/e/d^2/a^2/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+155/48/e^2*
d^3/a^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*c^4+7/12/d/a^2/e^2/x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+13/1
2/d^3/a/x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-105/16/d^5*e^2/a/(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-105/16/d^3/a^2/(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)*c-89/24/d^4*e/a/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+105
/16/d^5*e^6*a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)+233/48/d^3*e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*c+23/24*d/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3+105/16/d^5*e^2/a/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/x)+105/16/d^3/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
```

```
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-75/16/e^2/d/a^3/(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-35/24/a^3/e^3/x/(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^2-1/3/d^2/a/e/x^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)-35/16*d/a^4/e^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-2/3/d^4*e^
3/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3/d
*e^4*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.487 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=259

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(-2a^3e^6+a^2cd^2e^4+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right.\right.}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] 2/5*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/15
*(a*d*e*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)+(-2*a^3*e^6+a^2*c*d^2*e^4+c^3*d^6)*x
)/e/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8/15*(5*a^2*e^
4+10*a*c*d^2*e^2+c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {854, 777, 613}

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(a^2cd^2e^4-2a^3e^6+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right.}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d
^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x(-}{}}{}$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8\left(ade\right.}{1}$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8\left(ade\right.}{1}$$

Mathematica [A] time = 0.12, size = 235, normalized size = 0.91

$$\frac{2\left(a^4e^6\left(8d^2+20dex+15e^2x^2\right)+4a^3cde^4\left(20d^3+53d^2ex+45de^2x^2+15e^3x^3\right)+2a^2c^2d^2e^2\left(20d^4+110d^3ex+18\right.\right.}{15(d+ex)\left(cd^2-ae^2\right)^{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e^3
*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3) +
 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*x^3
+ 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(
3/2))
```

fricas [B] time = 88.31, size = 820, normalized size = 3.17

$$\frac{}{15\left(a^2c^5d^{13}e^2-5\,a^3c^4d^{11}e^4+10\,a^4c^3d^9e^6-10\,a^5c^2d^7e^8+5\,a^6cd^5e^{10}-a^7d^3e^{12}+\left(c^7d^{12}e^3-5\,ac^6d^{10}e^5+10\,a^2c^5d^8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")
```

```
[Out] 2/15*(40*a^2*c^2*d^6*e^2 + 80*a^3*c*d^4*e^4 + 8*a^4*d^2*e^6 + 8*(c^4*d^6*e^
2 + 10*a*c^3*d^4*e^4 + 5*a^2*c^2*d^2*e^6)*x^4 + 4*(5*c^4*d^7*e + 53*a*c^3*d
^5*e^3 + 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7)*x^3 + 3*(5*c^4*d^8 + 60*a*c^3
*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 + 5*a^4*e^8)*x^2 + 4*(15*
a*c^3*d^7*e + 55*a^2*c^2*d^5*e^3 + 53*a^3*c*d^3*e^5 + 5*a^4*d*e^7)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e
^4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e
^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^
6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a
*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^1
0 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12
*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*
c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2
- 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d
^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^
5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 +
 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.5Unable to transpose Erro
r: Bad Argument Value
```

maple [A] time = 0.01, size = 366, normalized size = 1.41

$$-\frac{2\left(cdx+ae\right)\left(40a^{2}c^{2}d^{2}e^{6}x^{4}+80a\,c^{3}d^{4}e^{4}x^{4}+8c^{4}d^{6}e^{2}x^{4}+60a^{3}cd\,e^{7}x^{3}+220a^{2}c^{2}d^{3}e^{5}x^{3}+212a\,c^{3}d^{5}e^{3}x^{3}+20\cdots\right.}{15\left(a^{5}e^{10}-5a^{4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(40*a^2*c^2*d^2*e^6*x^4+80*a*c^3*d^4*e^4*x^4+8*c^4*d^6*e^
2*x^4+60*a^3*c*d*e^7*x^3+220*a^2*c^2*d^3*e^5*x^3+212*a*c^3*d^5*e^3*x^3+20*c
^4*d^7*e*x^3+15*a^4*e^8*x^2+180*a^3*c*d^2*e^6*x^2+378*a^2*c^2*d^4*e^4*x^2+1
80*a*c^3*d^6*e^2*x^2+15*c^4*d^8*x^2+20*a^4*d*e^7*x+212*a^3*c*d^3*e^5*x+220*
a^2*c^2*d^5*e^3*x+60*a*c^3*d^7*e*x+8*a^4*d^2*e^6+80*a^3*c*d^4*e^4+40*a^2*c^
2*d^6*e^2)/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+
5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 4.33, size = 3099, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

```
[Out] (((6*a*e^2 - 10*c*d^2)/(15*(a*e^2 - c*d^2)^4) - (4*c*d^2)/(5*(a*e^2 - c*d^2
)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((e*(
2*a*e^3 - 2*c*d^2*e))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (4*c*d^
2*e^2)/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(2*c*d^3 + 2*a*
d*e^2))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*
x^2)^(1/2)*(x*((((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(a*e^2 + c*d^2))/(c*d*e)
```

```
 - (6*c^2*d^2*e*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) + (8*a*c^3*d^4*e^3)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^2*d^2*e*(46*a^2*e^4 + 4*c^2*d^4 +
 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5))) + (a*((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - (c*d*(a*e^2 + c*d^2
)*(46*a^2*e^4 + 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 + c*d^2)*((4*c^4*d^4
*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^
3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d
^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/
(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4
*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*
e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^4*d^4*e^
3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2
*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^3*d
^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3*(c
^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d^2)
*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^
2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36*a^2*c*d^2*e^3))/(15
*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a
*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 -
c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*a^3*c^
2*d^3*e^6)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)
) - (c*d*e*(12*a^3*e^5 - 36*a^2*c*d^2*e^3)*(a*e^2 + c*d^2))/(15*(a*e^2 - c*
d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) + (a*((a*((4*c^4*d^4*e
^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^
2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c
*d^2)*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e
^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c
^2*d^2*e^2*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d
^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*
(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3
*e^2*(a*e^2 + c*d^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36
*a^2*c*d^2*e^3))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c
*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d
*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))
/c + (4*a^3*c*d^2*e^5*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e*x)^2) - (2*d^2*e*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*x)^3*(5*a^3*e^7 - 5*c^
3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c*d^2*e^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.488 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8(\cdots}{35\cdots}$$

```
[Out] 2/7*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-8/35
*(2*a*d*e*(2*a*e^2+c*d^2)+(3*a^2*e^4+a*c*d^2*e^2+2*c^2*d^4)*x)/e/(-a*e^2+c*
d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+16/105*(7*a^2*e^4+14*a*c*d^2
*e^2+3*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-128/105*c*d*(7*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*(2
*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)
```

Rubi [A] time = 0.29, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{854, 777, 614, 613\}$

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8(\cdots}{35\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (2*x^2)/(7*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)) - (8*(2*a*d*e*(c*d^2 + 2*a*e^2) + (2*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4)*x))/(35*e*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)) + (16*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(105*e*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) - (128*c*d*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*
c*d*e*x))/(105*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && LtQ[p, -1]
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} + \frac{2\int}{}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8(2}{35}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8(2}{35}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8(2}{35}$$

Mathematica [A] time = 0.19, size = 433, normalized size = 1.27

$$-\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-a^6e^{10}\left(8d^2+28dex+35e^2x^2\right)+2a^5cde^8\left(112d^3+382d^2ex+455de^2x^2+140e^3x^3\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (-2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a^6*e^10*(8*d^2 + 28*d*e*x + 35*e^2*x^
2)) + 2*a^5*c*d*e^8*(112*d^3 + 382*d^2*e*x + 455*d*e^2*x^2 + 140*e^3*x^3) +
 3*c^6*d^8*x^2*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2 + 448*d*e^3*x^3 + 12
8*e^4*x^4) + 5*a^4*c^2*d^2*e^6*(336*d^4 + 1288*d^3*e*x + 1859*d^2*e^2*x^2 +
 1288*d*e^3*x^3 + 336*e^4*x^4) + 20*a^3*c^3*d^3*e^4*(56*d^5 + 406*d^4*e*x +
 1001*d^3*e^2*x^2 + 1084*d^2*e^3*x^3 + 560*d*e^4*x^4 + 112*e^5*x^5) + 2*a*c
^5*d^6*e*x*(70*d^5 + 1295*d^4*e*x + 4060*d^3*e^2*x^2 + 5600*d^2*e^3*x^3 + 3
616*d*e^4*x^4 + 896*e^5*x^5) + a^2*c^4*d^4*e^2*(56*d^6 + 2996*d^5*e*x + 131
95*d^4*e^2*x^2 + 24080*d^3*e^3*x^3 + 20320*d^2*e^4*x^4 + 7616*d*e^5*x^5 + 8
96*e^6*x^6)))/(105*(c*d^2 - a*e^2)^7*(a*e + c*d*x)^3*(d + e*x)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.6Unable to transpose Erro
r: Bad Argument Value
```

maple [B] time = 0.02, size = 663, normalized size = 1.94

$$-\frac{2\,(cdx+ae)\left(-896a^2c^4d^4e^8x^6-1792a\,c^5d^6e^6x^6-384c^6d^8e^4x^6-2240a^3c^3d^3e^9x^5-7616a^2c^4d^5e^7x^5-7232a\,c^5d^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(-896*a^2*c^4*d^4*e^8*x^6-1792*a*c^5*d^6*e^6*x^6-384*c^6
*d^8*e^4*x^6-2240*a^3*c^3*d^3*e^9*x^5-7616*a^2*c^4*d^5*e^7*x^5-7232*a*c^5*d
^7*e^5*x^5-1344*c^6*d^9*e^3*x^5-1680*a^4*c^2*d^2*e^10*x^4-11200*a^3*c^3*d^4
*e^8*x^4-20320*a^2*c^4*d^6*e^6*x^4-11200*a*c^5*d^8*e^4*x^4-1680*c^6*d^10*e^
2*x^4-280*a^5*c*d*e^11*x^3-6440*a^4*c^2*d^3*e^9*x^3-21680*a^3*c^3*d^5*e^7*x
^3-24080*a^2*c^4*d^7*e^5*x^3-8120*a*c^5*d^9*e^3*x^3-840*c^6*d^11*e*x^3+35*a
^6*e^12*x^2-910*a^5*c*d^2*e^10*x^2-9295*a^4*c^2*d^4*e^8*x^2-20020*a^3*c^3*d
^6*e^6*x^2-13195*a^2*c^4*d^8*e^4*x^2-2590*a*c^5*d^10*e^2*x^2-105*c^6*d^12*x
^2+28*a^6*d*e^11*x-764*a^5*c*d^3*e^9*x-6440*a^4*c^2*d^5*e^7*x-8120*a^3*c^3*
d^7*e^5*x-2996*a^2*c^4*d^9*e^3*x-140*a*c^5*d^11*e*x+8*a^6*d^2*e^10-224*a^5*
c*d^4*e^8-1680*a^4*c^2*d^6*e^6-1120*a^3*c^3*d^8*e^4-56*a^2*c^4*d^10*e^2)/(a
^7*e^14-7*a^6*c*d^2*e^12+21*a^5*c^2*d^4*e^10-35*a^4*c^3*d^6*e^8+35*a^3*c^4*
d^8*e^6-21*a^2*c^5*d^10*e^4+7*a*c^6*d^12*e^2-c^7*d^14)/(c*d*e*x^2+a*e^2*x+c
*d^2*x+a*d*e)^(7/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 7.72, size = 11469, normalized size = 33.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)),x)
```

```
[Out] ((6*c^3*d^5 + 36*a*c^2*d^3*e^2 - 10*a^2*c*d*e^4)/(105*(a*e^2 - c*d^2)^6) -
x*((16*c^2*d^2*e)/(105*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))/(
105*(a*e^2 - c*d^2)^6)) + (8*a*c^2*d^3*e^2)/(105*(a*e^2 - c*d^2)^6))/(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + (x*((a*((64*c^5*d^5*e^4*(a*e^2 +
c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))
 - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*(
(64*c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^
2*d^3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c
^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a
*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*
e^3*(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(60*c^4*d^7 -
204*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^3*d^3*e^2*(a*e^
2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((64*
c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^
3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 -
c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^4*d
^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*e^3*
(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(60*c^4*d^7 - 20
4*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))/(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2) + (x*((a*((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2
 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6)))/
c + (36*c^4*d^7*e - 76*a*c^3*d^5*e^3 - 36*a^2*c^2*d^3*e^5 + 12*a^3*c*d*e^7)
/(105*e*(a*e^2 - c*d^2)^6) + ((a*e^2 + c*d^2)*((8*a*c^3*d^4*e^3)/(105*(a*e^
2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6) -
 (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6))*(a*e^2 + c*d^2)
)/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^4 + 14*a*c*d^2*e^2))/(105*(
a*e^2 - c*d^2)^6)))/(c*d*e)) + (30*a^4*e^8 - 22*c^4*d^8 + 20*a*c^3*d^6*e^2
- 132*a^3*c*d^2*e^6 + 72*a^2*c^2*d^4*e^4)/(105*e*(a*e^2 - c*d^2)^6) + (a*((
8*a*c^3*d^4*e^3)/(105*(a*e^2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2)
)/(105*(a*e^2 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 -
 c*d^2)^6))*(a*e^2 + c*d^2))/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^
4 + 14*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6)))/c)/(x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(3/2) - (((d*((e*(2*a*e^4 - 2*c*d^2*e^2))/(7*(a*e^2 - c*d^2)
^4*(5*a*e^3 - 5*c*d^2*e)) - (4*c*d^2*e^3)/(7*(a*e^2 - c*d^2)^4*(5*a*e^3 - 5
*c*d^2*e))))/e + (e*(2*a*d*e^3 + 2*c*d^3*e))/(7*(a*e^2 - c*d^2)^4*(5*a*e^3
- 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 +
 (((e*(10*a*e^3 - 14*c*d^2*e))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e))
 - (4*c*d^2*e^2)/(7*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*((a*((a*((4*c^5*d^5*e^4
*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d
^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4
*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 -
c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e
```

^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^
5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c
^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)*(
(4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*
d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*
e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2
)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 +
c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^
3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^5*d^5*e^4*(a*e^2 + c*d^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*
c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^5*d^5
e^4(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2
*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*
a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*
d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*
c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*
e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(16*a^4*e^7 - 64*a^3*c*d^2*e^5))/(
35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*
(a*e^2 + c*d^2)*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^
3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^
5))))/(c*d*e) + (32*a^4*c^2*d^3*e^8)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(16*a^4*e^7 - 64*a^3*c*d^2*e^5)*(a*e^
2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^
5))) - (a*((a*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(
7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*
a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d
*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d
^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*(
(a*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2
- c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d
^2)*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^
4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 -
c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))
)/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*
e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*
e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (
2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3
*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4
*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2
- c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a
*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^
4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(
16*a^4*e^7 - 64*a^3*c*d^2*e^5))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*

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d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(14*c^4*d^7 - 56*a*c^3*d^5
*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (16*a^4*c*d^2*e^7*(a*e^2 + c*d^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(x*(
a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e + c*d*x)^3*(d + e*x)^3) -
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*((16*c^5*d^5*e^4*(a*e
^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e
^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2
)*((16*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e
^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*
c^4*d^4*e^3*(c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^5*d^6*e^5)/(35*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e^3*(
a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(484*c^4*d^7 + 1228*
a*c^3*d^5*e^2 - 1092*a^2*c^2*d^3*e^4 - 812*a^3*c*d*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*c^3*d^3*e^2*(a*e^2 +
 c*d^2)*(c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((16*c^5*d^
5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*c^4*d^4*e^3*(
c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e^3*(a*e^2 + c*d^2
)*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(484*c^4*d^7 + 1228*a*c^3*d^5*e
^2 - 1092*a^2*c^2*d^3*e^4 - 812*a^3*c*d*e^6))/(105*(a*e^2 - c*d^2)^6*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a
*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*((a*((16*c^6*d^6*e^5*(a*e^2
 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^
5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*
((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^
5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^4*
(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e^6 + 5*c^3*
d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 + c*d^2)*(2*c
^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)*
((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^
5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^4*
(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((16*c^6*d^6*e^5*(a*
e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d
*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^
2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
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a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*
e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32
*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^
6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a
*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e
^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e^6 + 5*c
^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 + c*d^2)*(
2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^3*d^3*e^2*(7*a^4*e^8 - 3
*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2
)*(25*a^3*e^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e
^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c
^2*d^2*e*(88*c^5*d^10 - 152*a^5*e^10 + 80*a*c^4*d^8*e^2 + 272*a^4*c*d^2*e^8
 - 520*a^2*c^3*d^6*e^4 + 296*a^3*c^2*d^4*e^6))/(105*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^2*d^2*e*(a*e^2 + c*d^2)*(7*
a^4*e^8 - 3*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 -
 c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*((a*(((a*e^2 +
 c*d^2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(10
5*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e)
- (32*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(1
05*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*
d^5*e^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((16*c^6*d^6
*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*
(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^
2 + c*d^2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/
(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*
e) - (32*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 -
 c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)
/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c
^5*d^5*e^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e
^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)
^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 +
c*d^2)*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3
*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^3*d^3*e^2*(7*a^4
*e^8 - 3*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2
 + c*d^2)*(25*a^3*e^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(
105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c
*d*(a*e^2 + c*d^2)*(88*c^5*d^10 - 152*a^5*e^10 + 80*a*c^4*d^8*e^2 + 272*a^4
*c*d^2*e^8 - 520*a^2*c^3*d^6*e^4 + 296*a^3*c^2*d^4*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e
*x)^2) - (2*d^2*e^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*
x)^4*(7*a^4*e^9 + 7*c^4*d^8*e - 28*a*c^3*d^6*e^3 - 28*a^3*c*d^2*e^7 + 42*a^
2*c^2*d^4*e^5)) + (8*c*d*e*(5*a*e^2 + c*d^2)*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(105*(a*e^2 - c*d^2)^6*(d + e*x))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
[Out] Timed out
```

3.489 $\int x^3\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=170

$$\frac{6}{55}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x+\frac{2}{11}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x}{x}\right.\right.}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}$$

```
[Out] 6/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)-4/5
5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*
(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)
/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 279, 321, 218}

$$\frac{2}{11}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4+\frac{6}{55}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x}{x}\right.\right.}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (6*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/11 - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 +
x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{11\sqrt{1+x^3}}$$

$$= \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\left(6\sqrt{1+x}\sqrt{1-x+x^2}\right)\int}{55\sqrt{1+x^3}}$$

$$= \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}}{}$$

Mathematica [C] time = 0.92, size = 221, normalized size = 1.30

$$\frac{2\left(x\sqrt{x+1}\left(5x^5-5x^4+5x^3+3x^2-3x+3\right)+\sqrt{-\frac{6i}{\sqrt{3}+3i}}\left(\sqrt{3}+3i\right)(x+1)\sqrt{\frac{\left(\sqrt{3}-3i\right)x+\sqrt{3}+3i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{\frac{\left(\sqrt{3}+3i\right)x+\sqrt{3}-3}{\left(\sqrt{3}+3i\right)(x+1)}}\right.}{55\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(3 - 3*x + 3*x^2 + 5*x^3 - 5*x^4 + 5*x^5) + Sqrt[(-6*I)/(
3*I + Sqrt[3])]*(3*I + Sqrt[3])*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I + Sqrt[
3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt[3])*x
)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3]
)]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(55*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2-x+1}\,\sqrt{x+1}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

maple [A] time = 0.14, size = 257, normalized size = 1.51

$$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(5x^7+8x^4+3x+3i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\right.\right.}{55\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x+1)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] 2/55*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(5*x^7+3*I*3^(1/2)*(-2*(x+1)/(-3+I*3^(1/2)
))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3
^(1/2)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(
I*3^(1/2)+3))^(1/2))-9*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(
I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*
(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+8*x^4+3*
x)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2-x+1}\,\sqrt{x+1}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

```
[Out] int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.490 $\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

```
[Out] 2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

fricas [A] time = 1.02, size = 22, normalized size = 0.96

$$\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

giac [B] time = 0.24, size = 67, normalized size = 2.91

$$\frac{2}{315}\left((5(7x-23)(x+1)+258)(x+1)-213\right)\sqrt{(x+1)^2-3x}\,\sqrt{x+1}+\frac{2}{105}\left(3(5x-12)(x+1)+71\right)\sqrt{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sq
rt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x
+ 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{2(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x+1)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] 2/9*(x+1)^(3/2)*(x^2-x+1)^(3/2)
```

maxima [A] time = 0.95, size = 22, normalized size = 0.96

$$\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

mupad [B] time = 2.62, size = 22, normalized size = 0.96

$$\frac{2\left(x^3+1\right)\sqrt{x+1}\,\sqrt{x^2-x+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

```
[Out] (2*(x^3 + 1)*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.491 $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=294

$$\frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}\,x^2+\frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right.}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

```
[Out] 2/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+6/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^
(1/2))+2/7*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-3/7*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2
))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 279, 303, 218, 1877}

$$\frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}\,x^2+\frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right.}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (6*Sqrt[1 + x]*Sqrt[1 - x + x^2])
/(7*(1 + Sqrt[3] + x)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1
- x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*(1 + x^3)) + (2*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1
 + x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
```

```
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}}$$

$$= \frac{2}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}} + \frac{\left(3\sqrt{2\left(2-\sqrt{3}\right)}\cdots\right.}{\cdots}$$

$$= \frac{2}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{6\sqrt{1+x}\,\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x\cdots}}{7\sqrt{\cdots}}$$

Mathematica [C] time = 0.51, size = 347, normalized size = 1.18

$$\frac{\sqrt{x+1}\left(4\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\left(x^2-x+1\right)x^2+3\sqrt{2}\left(\sqrt{3}-i\right)\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\cdots}{14\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*(1 - x + x^2) - 3*
Sqrt[2]*(-3*I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt
[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])] + 3*Sq
rt[2]*(-I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I
 + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((
```

```
-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(14*Sqrt
[((-I)*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2-x+1}\,\sqrt{x+1}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

maple [A] time = 0.03, size = 361, normalized size = 1.23

$$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(2x^5+2x^2-18\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\text{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+1)^(1/2)*(x^2-x+1)^(1/2),x)
[Out] 1/7*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2
)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)
))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1
/2)+3))^(1/2))+2*x^5+9*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/
(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2
*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-18*(-2*
(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+
I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2
),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+2*x^2)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2-x+1}\,\sqrt{x+1}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

```
[Out] int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.492 $\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=144

$$\frac{2}{5}x\sqrt{x^2-x+1}\,\sqrt{x+1}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}$$

```
[Out] 2/5*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^
(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {713, 195, 218}

$$\frac{2}{5}x\sqrt{x^2-x+1}\,\sqrt{x+1}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x
)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$
$$= \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x^3}}$$
$$= \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\right.}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Mathematica [C] time = 0.48, size = 169, normalized size = 1.17

$$\frac{2x\sqrt{x+1}\left(x^2-x+1\right)+\frac{i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{5\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3]
)*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]
)/Sqrt[(-I)/(3*I + Sqrt[3])])/(5*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^2-x+1}\sqrt{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

maple [B] time = 0.03, size = 252, normalized size = 1.75

$$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(-2x^4-2x+3i\sqrt{3}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{}}{i\sqrt{3}+}}\right.\right.}{5\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] -1/5*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(x+1)/(-3+I*3^(1/2)))^(1/
2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2
)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(
1/2)+3))^(1/2))-9*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^
(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1
)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^4-2*x)/(
x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2-x+1}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

```
[Out] int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.493 $\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 50, 63, 207}

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x^3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.41, size = 197, normalized size = 2.98

$$\frac{\sqrt{x+1}\left(2\left(x^2-x+1\right)+\frac{3i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\Pi\left(\frac{3}{2}-\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{3\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (Sqrt[1 + x]*(2*(1 - x + x^2) + ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (2*I)*x)
/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticP
i[3/2 - (I/2)*Sqrt[3], I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
))/(3*Sqrt[1 - x + x^2])
```

fricas [A] time = 1.08, size = 60, normalized size = 0.91

$$\frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) +
 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

maple [A] time = 0.03, size = 43, normalized size = 0.65

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(\operatorname{arctanh}\left(\sqrt{x^3+1}\right)-\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x,x)
```

```
[Out] -2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(x
^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

3.494 $\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^2}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}+\frac{3\sqrt{x^2-x+1}\,\sqrt{x+1}}{x+\sqrt{3}+1}+\frac{\sqrt{2}\,3^{3/4}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

```
[Out] -(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+3*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+
3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^
(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(
1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 277, 303, 218, 1877}

$$-\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}+\frac{3\sqrt{x^2-x+1}\,\sqrt{x+1}}{x+\sqrt{3}+1}+\frac{\sqrt{2}\,3^{3/4}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + x]*Sqrt[1 - x + x^2])/(1
 + Sqrt[3] + x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*(1 + x^3)) + (Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
```

```
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^2}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}} + \frac{\left(3\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{1\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+x}\sqrt{1-x+x^2}}{1+\sqrt{3}+x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x\ldots}}{2\sqrt{\frac{\ldots}{(1\ldots}}}$$

Mathematica [C] time = 0.40, size = 349, normalized size = 1.22

$$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}+\frac{3\sqrt{1+\frac{2i(x+1)}{\sqrt{3}-3i}}\sqrt{1-\frac{2i(x+1)}{\sqrt{3}+3i}}\left(\frac{\left(\sqrt{3}-i\right)\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x+1}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}-\frac{\left(\sqrt{3}-3i\right)\sqrt{-\ldots}}{\ldots}\right.}{2\sqrt{2}\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{(x+1)^2-3(x+1)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))/(-3*I +
Sqrt[3])]*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-(((-3*I + Sqrt[3])*Sq
rt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)
*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(
```

```
1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt
[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]],
(3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(2
*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

maple [A] time = 0.03, size = 363, normalized size = 1.26

$$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-2x^3-18\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x^2,x)
```

```
[Out] 1/2*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(3*I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x
+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)
*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^
(1/2))*3^(1/2)*x+9*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3
^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+
1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-18*(-2*(x
+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*
3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),
(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

3.495 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx$

Optimal. Leaf size=146

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{2x^2}$$

```
[Out] -1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2+1/2*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {915, 277, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] -(Sqrt[1 + x]*Sqrt[1 - x + x^2])/(2*x^2) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 +
x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^3}\,dx}{\sqrt{1+x^3}}$$
$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x^3}}$$
$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1}{1}\right.\right.}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Mathematica [C] time = 0.41, size = 185, normalized size = 1.27

$$\frac{\sqrt{x+1}\left(-\frac{2\left(x^2-x+1\right)}{x^2}-\frac{3i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{4\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((-2*(1 - x + x^2))/x^2 - ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (
2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqr
t[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(4*Sqrt[1 -
x + x^2])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

maple [B] time = 0.03, size = 259, normalized size = 1.77

$$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(2x^3+3i\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\sqrt{3}\,x^2\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-\right.}{4\left(x^3+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x^3,x)
```

```
[Out] -1/4*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(3*I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*
x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2
)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))
^(1/2))*3^(1/2)*x^2-9*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(
I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*
(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+2*x^
3+2)/(x^3+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)
```

3.496 $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$\frac{54}{935}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x+\frac{18}{187}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-}\right.}{935\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3+1\right)}$$

```
[Out] 54/935*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+18/187*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)
+2/17*x^4*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-36/935*3^(3/4)*(1+x)^(3/2)*El
lipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(
1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 279, 321, 218}

$$\frac{2}{17}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)x^4+\frac{18}{187}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4+\frac{54}{935}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (54*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/935 + (18*x^4*Sqrt[1 + x]*Sqrt[1 - x +
 x^2])/187 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/17 - (36*3^(3/
4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(935*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x^3\left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right) + \frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,d}{17\sqrt{1+x^3}}$$

$$= \frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right) + \frac{\left(27\sqrt{1+\right.}{}$$

$$= \frac{54}{935}x\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-}$$

$$= \frac{54}{935}x\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-}$$

Mathematica [C] time = 0.88, size = 235, normalized size = 1.17

$$\frac{2\left(x\sqrt{x+1}\,\left(55x^8-55x^7+55x^6+100x^5-100x^4+100x^3+27x^2-27x+27\right)-\frac{9i\sqrt{6}(x+1)\sqrt{\frac{\left(\sqrt{3}-3i\right)x+\sqrt{3}+3i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{\frac{\left(\sqrt{3}\right.}{\left(\sqrt{\right.}}}}{\sqrt{}}\right.}{935\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(27 - 27*x + 27*x^2 + 100*x^3 - 100*x^4 + 100*x^5 + 55*x^
6 - 55*x^7 + 55*x^8) - ((9*I)*Sqrt[6]*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I +
 Sqrt[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt
[3])*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + S
qrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + S
qrt[3])]))/(935*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(x^6+x^3\right)\sqrt{x^2-x+1}\,\sqrt{x+1}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] integral((x^6 + x^3)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 - x + 1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

maple [A] time = 0.03, size = 262, normalized size = 1.30

$$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(55x^{10}+155x^7+127x^4+27x+27i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{935\left(x^3+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x+1)^(3/2)*(x^2-x+1)^(3/2),x)
[Out] 2/935*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(55*x^10+155*x^7+27*I*(-2*(x+1)/(-3+I*3^(
1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-
3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/
2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)-81*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x
+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)
*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^
(1/2))+127*x^4+27*x)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 - x + 1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(x+1\right)}^{3/2}\,{\left(x^2-x+1\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
[Out] int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(x + 1\right)^{\frac{3}{2}} \left(x^{2} - x + 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
[Out] Integral(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.497 $\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

```
[Out] 2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

fricas [A] time = 1.09, size = 27, normalized size = 1.17

$$\frac{2}{15}\left(x^6+2x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

giac [B] time = 0.61, size = 173, normalized size = 7.52

$$\frac{2}{45045}(((7(3(11(13x-80)(x+1)+3165)(x+1)-16442)(x+1)+121227)(x+1)-80187)(x+1)+34077)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(((7*(3*(11*(13*x - 80)*(x + 1) + 3165)*(x + 1) - 16442)*(x + 1) +
121227)*(x + 1) - 80187)*(x + 1) + 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
 + 2/45045*((5*(7*(9*(11*x - 57)*(x + 1) + 1601)*(x + 1) - 15837)*(x + 1) +
 65172)*(x + 1) - 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/315*((5*(7*x
 - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/
105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{2\,(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x+1)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] 2/15*(x+1)^(5/2)*(x^2-x+1)^(5/2)
```

maxima [A] time = 0.97, size = 27, normalized size = 1.17

$$\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

mupad [B] time = 0.12, size = 25, normalized size = 1.09

$$\frac{2\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}\,\left(x^2+2\,x+1\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

```
[Out] (2*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2)*(2*x + x^2 + 1))/15
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\,(x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.498 $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=325

$$\frac{18}{91}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2+\frac{54\sqrt{x+1}\,\sqrt{x^2-x+1}}{91\left(x+\sqrt{3}+1\right)}+\frac{2}{13}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)x^2+\frac{18\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2}}{}$$

```
[Out] 18/91*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/13*x^2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1
)^(1/2)+54/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+18/91*3^(3/4)*(1+x)
^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+
1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^
(1/2)-27/91*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 279, 303, 218, 1877}

$$\frac{2}{13}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)x^2+\frac{18}{91}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2+\frac{54\sqrt{x+1}\,\sqrt{x^2-x+1}}{91\left(x+\sqrt{3}+1\right)}+\frac{18\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/91 + (54*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/(91*(1 + Sqrt[3] + x)) + (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3)
)/13 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + S
qrt[3] + x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x
^3)) + (18*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}$$

$$=\frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{13\sqrt{1+x^3}}$$

$$=\frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(27\sqrt{1+x}\,\sqrt{1\right.}{9}$$

$$=\frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(27\sqrt{1+x}\,\sqrt{1\right.}{9}$$

$$=\frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\,\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}$$

Mathematica [C] time = 0.49, size = 244, normalized size = 0.75

$$\frac{\sqrt{x+1}\left(4x^2\left(x^2-x+1\right)\left(7x^3+16\right)-\frac{27\sqrt{2}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left(\left(\sqrt{3}-3i\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-\left(\sqrt{3}-i\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right.\right.\right.}{\sqrt{-\frac{i(x+1)}{-2ix+\sqrt{3}+i}}}\right.}{182\sqrt{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*(1 - x + x^2)*(16 + 7*x^3) - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(182*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(x^4+x\right)\sqrt{x^2-x+1}\,\sqrt{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + x)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

maple [A] time = 0.03, size = 366, normalized size = 1.13

$$\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(14x^8+46x^5+32x^2-162\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+1)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] 1/91*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(14*x^8+27*I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/
2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2
)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(
1/2)+3))^(1/2))*3^(1/2)+46*x^5+81*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*
3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*El
lipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/
2))-162*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^
(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^
(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+32*x^2)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

```
[Out] int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\,(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.499 $\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx$

Optimal. Leaf size=173

$$\frac{18}{55}x\sqrt{x^2-x+1}\,\sqrt{x+1}+\frac{2}{11}x\sqrt{x^2-x+1}\,\left(x^3+1\right)\sqrt{x+1}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3}}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3\right.}$$

[Out] 18/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)+18/55*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {713, 195, 218}

$$\frac{2}{11}x\sqrt{x^2-x+1}\,\left(x^3+1\right)\sqrt{x+1}+\frac{18}{55}x\sqrt{x^2-x+1}\,\sqrt{x+1}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3}}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(x^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2
]*(1 + x^3))/11 + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x +
x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*(1 + x^3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] && !IntegerQ[p]
```

Rubi steps

$$\int (1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}$$
$$=\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{11\sqrt{1+x^3}}$$
$$=\frac{18}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(27\sqrt{1+x}\,\sqrt{1-x\right.}{55\sqrt{1}}$$
$$=\frac{18}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,(}{}$$

Mathematica [C] time = 0.64, size = 176, normalized size = 1.02

$$\frac{2x\sqrt{x+1}\,\left(x^2-x+1\right)\left(5x^3+14\right)+\frac{9i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\,\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{55\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2)*(14 + 5*x^3) + ((9*I)*(1 + x)*Sqrt[1 + (6*I)
/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(55*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

maple [A] time = 0.02, size = 257, normalized size = 1.49

$$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-10x^7-38x^4-28x+27i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\right.\right.}{55\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] -1/55*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-10*x^7+27*I*(-2*(x+1)/(-3+I*3^(1/2)))^(
1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1
/2)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3
^(1/2)+3))^(1/2))*3^(1/2)-81*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/
2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*Ellipti
cF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-3
8*x^4-28*x)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 - x + 1\right)^{\frac{3}{2}} (x + 1)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x + 1)^{3/2} \left(x^2 - x + 1\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

```
[Out] int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x + 1)^{\frac{3}{2}} \left(x^2 - x + 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.500 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x} dx$

Optimal. Leaf size=94

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}+\frac{2}{9}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/9*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3
*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 50, 63, 207}

$$\frac{2}{9}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)+\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 + (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 +
 x^3))/9 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt
[1 + x^3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
```

```
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}$$

$$= \frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,\right.}{3\sqrt{1+x^3}}$$

$$= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x\right.}{}$$

$$= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)+\frac{\left(2\sqrt{1+x}\,\sqrt{1-x+\right.}{}$$

$$= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)-\frac{2\sqrt{1+x}\,\sqrt{1-x+x}{3\sqrt{1}}$$

Mathematica [C] time = 0.30, size = 201, normalized size = 2.14

$$\frac{\sqrt{x+1}\left(\frac{2}{9}\left(x^2-x+1\right)\left(x^3+4\right)+\frac{i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\,\Pi\left(\frac{3}{2}-\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(4 + x^3))/9 + (I*Sqrt[2]*Sqrt[(I + Sqrt[3]
- (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]
*EllipticPi[3/2 - (I/2)*Sqrt[3], I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I +
 Sqrt[3])]))/Sqrt[1 - x + x^2]
```

fricas [A] time = 0.69, size = 65, normalized size = 0.69

$$\frac{2}{9}\left(x^3+4\right)\sqrt{x^2-x+1}\,\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

maple [A] time = 0.01, size = 57, normalized size = 0.61

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-\sqrt{x^3+1}\,x^3+3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)-4\sqrt{x^3+1}\right)}{9\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x,x)
```

```
[Out] -2/9*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/2
))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x,x)
```

```
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

3.501 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=323

$$\frac{9}{7}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2+\frac{27\sqrt{x+1}\,\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)}{x}+\frac{9\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}}{7\sqrt{}}$$

```
[Out] 9/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)-(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+2
7/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+9/7*3^(3/4)*(1+x)^(3/2)*Ellip
ticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-27/14*
3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x
^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(
x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {915, 277, 279, 303, 218, 1877}

$$\frac{9}{7}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2+\frac{27\sqrt{x+1}\,\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)}{x}+\frac{9\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}}{7\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (9*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (27*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)/(7*(1 + Sqrt[3] + x)) - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/x - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(14*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3)) + (9*
Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^2}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}{x} + \frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{2\sqrt{1+x^3}}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}{x} + \frac{\left(27\sqrt{1+x}\,\sqrt{1-x+x\right.}{14\sqrt{1+}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}{x} + \frac{\left(27\sqrt{1+x}\,\sqrt{1-x+x\right.}{14\sqrt{1+}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{27\sqrt{1+x}\,\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right.}{x}$$

Mathematica [C] time = 0.49, size = 244, normalized size = 0.76

$$\frac{\sqrt{x+1}\left(\frac{4(x^2-x+1)(2x^3-7)}{x}-\frac{27\sqrt{2}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left((\sqrt{3}-3i)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-(\sqrt{3}-i)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)}{\sqrt{-\frac{i(x+1)}{-2ix+\sqrt{3}+i}}}\right.}{28\sqrt{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[1 + x]*((4*(1 - x + x^2)*(-7 + 2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(28*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

maple [A] time = 0.02, size = 368, normalized size = 1.14

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(4x^6-10x^3-162\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x\,\text{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x^2,x)
```

```
[Out] 1/14*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(27*I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2
*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/
2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3)
)^(1/2))*3^(1/2)*x+4*x^6+81*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2
)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*Elliptic
F((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-
162*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2
)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2
)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-10*x^3-14)/x/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2,x)
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)
```

3.502 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3} dx$

Optimal. Leaf size=175

$$\frac{9}{10}x\sqrt{x^2-x+1}\sqrt{x+1}-\frac{\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+1}}{2x^2}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}F\left(\mathrm{s}\right.}{10\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

```
[Out] 9/10*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)-1/2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/
x^2+9/10*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2
)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {915, 277, 195, 218}

$$-\frac{\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+1}}{2x^2}+\frac{9}{10}x\sqrt{x^2-x+1}\sqrt{x+1}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}F}{10\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (9*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/10 - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
+ x^3))/(2*x^2) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(10*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*(1 + x^3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^3}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2} + \frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{4\sqrt{1+x^3}}$$

$$= \frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2} + \frac{\left(27\sqrt{1+x}\sqrt{1-x+\right.}{20\sqrt{1+}$$

$$= \frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+}{}$$

Mathematica [C] time = 0.35, size = 192, normalized size = 1.10

$$\frac{\sqrt{x+1}\left(\frac{2\left(x^2-x+1\right)\left(4x^3-5\right)}{x^2}-\frac{27i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{20\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(-5 + 4*x^3))/x^2 - ((27*I)*Sqrt[2]*Sqrt[(I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))
/(20*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

maple [A] time = 0.02, size = 264, normalized size = 1.51

$$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-8x^6+2x^3+27i\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\sqrt{3}\,x^2\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{\right.\right.}{20\left(x^3+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x^3,x)
```

```
[Out] -1/20*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(27*I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-
2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1
/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3
))^(1/2))*3^(1/2)*x^2-81*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1
)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((
-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-8
*x^6+2*x^3+10)/(x^3+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3,x)
```

```
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)
```

3.503 $\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=142

$$\frac{2x\left(x^3+1\right)}{5\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[Out] 2/5*x*(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-4/15*EllipticF((1+x-3^(1/2))/(1+x
+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {915, 321, 218}

$$\frac{2x\left(x^3+1\right)}{5\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*x*(1 + x^3))/(5*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (4*Sqrt[2 + Sqrt[3]]*Sq
rt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 +
 Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x\left(1+x^3\right)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x\left(1+x^3\right)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.61, size = 169, normalized size = 1.19

$$\frac{6x\sqrt{x+1}\left(x^2-x+1\right) - \frac{2i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{15\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (6*x*Sqrt[1 + x]*(1 - x + x^2) - ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqr
t[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcS
inh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(15*Sqrt[1 - x + x^2])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}\,x^3}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

maple [B] time = 0.04, size = 248, normalized size = 1.75

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\left(x^4+x+i\sqrt{3}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right.}{5\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 2/5*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I
*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*E
llipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1
/2))*3^(1/2)-3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/
2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/(
-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+x^4+x)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2-x+1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.504 $\int \frac{x^2}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

fricas [A] time = 0.97, size = 17, normalized size = 0.74

$$\frac{2}{3}\,\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

giac [A] time = 0.18, size = 18, normalized size = 0.78

$$\frac{2}{3}\,\sqrt{(x+1)^2-3\,x}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)
```

maxima [A] time = 0.97, size = 22, normalized size = 0.96

$$\frac{2\left(x^3+1\right)}{3\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 0.15, size = 9, normalized size = 0.39

$$\frac{2\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] (2*(x^3 + 1)^(1/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.505 $\int \frac{x}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=253

$$\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right.\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[Out] 2*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/3*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4)*
EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/
2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {809, 303, 218, 1877}

$$\frac{2\left(x^3+1\right)}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*(1 + x^3))/(Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (2*Sqrt[2]*Sqrt[1 + x]*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= \frac{2\left(1+x^3\right)}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\cdots}{1+\cdots}\right.\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

Mathematica [C] time = 1.00, size = 375, normalized size = 1.48

$$\frac{(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2} + \frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+\cdots}{\sqrt{3}+3i}}\cdots}{\cdots}\right.}{6\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (
3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[
3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*Ar
cSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqr
t[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I
)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + S
qrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*
I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(6*Sqrt[(-I)/(3*I + Sqrt[3])]*
Sqrt[1 - x + x^2])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 - x + 1}\,\sqrt{x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

maple [A] time = 0.04, size = 275, normalized size = 1.09

$$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-3+i\sqrt{3}\right)\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\left(i\sqrt{3}\ \mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{}}{i\sqrt{3}+}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 1/2*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-3+I*3^(1/2))*(-2*(x+1)/(-3+I*3^(1/2)))^(1
/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/
2)))^(1/2)*(I*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I
*3^(1/2)+3))^(1/2))*3^(1/2)-I*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(
-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)+3*EllipticE((-2*(x+1)/(-3+I*3^(
1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-EllipticF((-2*(x+1)/(-3
+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)))/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 - x + 1}\,\sqrt{x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.506 $\int \frac{1}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=110

$$\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

```
[Out] 2/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {713, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.14, size = 148, normalized size = 1.35

$$\frac{i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{\frac{2}{3}-\frac{4i}{\left(\sqrt{3}+3i\right)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[2/3 - (4*I)/((3*
I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqr
t[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/(Sqrt[(-I)/(3*I + Sqrt[3])]*Sq
rt[1 - x + x^2])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

maple [A] time = 0.02, size = 137, normalized size = 1.25

$$\frac{\left(3-i\sqrt{3}\right)\sqrt{x+1}\sqrt{x^2-x+1}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)}{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] (3-I*3^(1/2))*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*(
(-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^
(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)
+3))^(1/2))/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 - x + 1}\,\sqrt{x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x + 1}\,\sqrt{x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + 1}\,\sqrt{x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.507 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 266, 63, 207}

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\sqrt{1+x^3}\ \mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\left(2\sqrt{1+x^3}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 12.80, size = 2463, normalized size = 58.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] 2*(((-I)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]]
, (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sq
rt[3 - 3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sq
rt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x
])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1
/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(S
qrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sq
rt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqr
t[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6
/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3
])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*
(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqr
t[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sq
rt[3]))/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/S
qrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-S
qrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[
3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 +
 x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])]
 - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3
 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1
/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt
[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) - (Sqrt[
3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt
[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*
(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] +
 Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*
Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x
]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (
I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sq
rt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] +
Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1 + Sqr
t[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[
```

```
3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqr
t[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]],
(Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3
 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt[1/2 -
 (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/(
(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I
/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(S
qrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I
*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3
]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2])
)/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/
2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3
 - 3*(1 + x) + (1 + x)^2]))
```

fricas [A] time = 0.70, size = 43, normalized size = 1.02

$$-\frac{1}{3}\,\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,+1\right)+\frac{1}{3}\,\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

maple [A] time = 0.03, size = 33, normalized size = 0.79

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(x+1)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.508 $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right.}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[Out] (-x^3-1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x
^2-x+1)^(1/2)+1/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1
/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/(
(1+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/
2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 325, 303, 218, 1877}

$$-\frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{x^3+1}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-\right.}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((1 + x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]*(1
+ Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 - x + x^2]) + (Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+}}}{\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+}}{}\end{aligned}$$

Mathematica [C] time = 0.83, size = 400, normalized size = 1.42

$$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}+\frac{(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2}+\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}}\right)}{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I +
 Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I
+ Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1
```

```
 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/S
qrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*
I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3
*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
1 + x]))/(12*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^5+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^5 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

maple [A] time = 0.02, size = 363, normalized size = 1.29

$$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-2x^3-6\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x\,\text{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+i\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
[Out] 1/2*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I
*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*E
llipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1
/2))*3^(1/2)*x+3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(
1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)
/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-6*(-2*(x+1)
/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(
1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(
-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.509 $\int \frac{1}{x^3 \sqrt{1+x} \sqrt{1-x+x^2}} \, dx$

Optimal. Leaf size=146

$$\frac{-x^3-1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[Out] 1/2*(-x^3-1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-1/6*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {915, 325, 218}

$$-\frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -(1 + x^3)/(2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (Sqrt[2 + Sqrt[3]]*Sqrt[
1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.65, size = 171, normalized size = 1.17

$$\frac{-\frac{6\sqrt{x+1}\left(x^2-x+1\right)}{x^2}-\frac{i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{12\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((-6*Sqrt[1 + x]*(1 - x + x^2))/x^2 - (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sq
rt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*Arc
Sinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt
[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(12*Sqrt[1 - x + x^2])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^6+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

maple [B] time = 0.02, size = 259, normalized size = 1.77

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(-2x^3+i\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\sqrt{3}\,x^2\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-\right.}{4\left(x^3+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 1/4*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I
*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*E
llipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1
/2))*3^(1/2)*x^2-3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3
^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+
1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-2*x^3-2
)/(x^3+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.510 $\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}-\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

```
[Out] -2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+4/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)
),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {915, 288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}-\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (-2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.57, size = 161, normalized size = 1.18

$$\frac{-\frac{6x}{\sqrt{x+1}} + \frac{2i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{9\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*x)/Sqrt[1 + x] + ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 +
x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
(-I)/(3*I + Sqrt[3])])/(9*Sqrt[1 - x + x^2])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^3}{x^6+2\,x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3/(x^6 + 2*x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

maple [B] time = 0.07, size = 245, normalized size = 1.79

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\left(x+i\sqrt{3}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right) - \right.}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+
I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*
EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(
1/2))*3^(1/2)-3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1
/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/
(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+x)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.511 $\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out] -2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

fricas [A] time = 0.95, size = 24, normalized size = 1.04

$$-\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
[Out] -2/3/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

maxima [A] time = 0.98, size = 17, normalized size = 0.74

$$-\frac{2}{3\,\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] -2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 2.69, size = 17, normalized size = 0.74

$$-\frac{2}{3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] -2/(3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.512 $\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right.}}}{3^{3/4}\sqrt{\frac{x+}{\left(x+\sqrt{3}\right.}}}$$

```
[Out] 2/3*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(
x^2-x+1)^(1/2)-2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(
1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)+1/3*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1
/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3
^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 290, 303, 218, 1877}

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\left(x^3+1\right)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}-\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-\right.}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*(1 + x^3))/(3*Sqrt[1 + x]*(1
 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) + (Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1
 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sq
rt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 - x + x^2]) - (2*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3
]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
```

```
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+}}}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2\left(1+x^3\right)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} + \frac{\sqrt{2-\sqrt{3}}\sqrt{1+}}{}$$

Mathematica [C] time = 0.71, size = 402, normalized size = 1.43

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2} + \frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}}\right.}{18\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - ((1 + x)^(3/2)*((12*Sqrt[(-I)/(
3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[
```

```
(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*
I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3
])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2
]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqr
t[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sq
rt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/
Sqrt[1 + x]))/(18*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x}{x^6+2\,x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x/(x^6 + 2*x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

maple [A] time = 0.05, size = 356, normalized size = 1.26

$$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(-2x^2-6\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+i\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -1/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+
I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*
EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(
1/2))*3^(1/2)+3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1
/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/
(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-6*(-2*(x+1)/(-
3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2
)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+
I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^2)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.513 $\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

```
[Out] 2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2))
,I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {713, 199, 218}

$$\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 - x + x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7}{3\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.36, size = 216, normalized size = 1.58

$$\sqrt{(x+1)^2-3(x+1)+3}\left(\frac{2(x+1)^{3/2}}{9\left((x+1)^2-3(x+1)+3\right)}-\frac{2}{9\sqrt{x+1}}\right)+\frac{i\sqrt{\frac{2}{3}}\,(x+1)\sqrt{1-\frac{6}{\left(3-i\sqrt{3}\right)(x+1)}}\,\sqrt{1-\frac{6}{\left(3+i\sqrt{3}\right)(x+}}}{3\sqrt{-\frac{1}{3-i\sqrt{3}}}\,\sqrt{(x+1)^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[3 - 3*(1 + x) + (1 + x)^2]*(-2/(9*Sqrt[1 + x]) + (2*(1 + x)^(3/2))/(9*
(3 - 3*(1 + x) + (1 + x)^2))) + ((I/3)*Sqrt[2/3]*(1 + x)*Sqrt[1 - 6/((3 - I
*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSi
nh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])
/(Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^6+2\,x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

maple [B] time = 0.04, size = 247, normalized size = 1.80

$$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-2x+i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -1/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+
I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*
EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(
1/2))*3^(1/2)-3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1
/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/
(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x)/(x^3+1)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.514 $\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] 2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1
+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 51, 63, 207}

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3
]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\sqrt{1+x^3}\,\mathrm{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(2\sqrt{1+x^3}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 6.08, size = 2511, normalized size = 38.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2/(9*(1 + x)) - (2*(-2 + x))/(9*(1 - x + x^2
))) + 2*(((-I)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3
 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1
 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-
1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]
] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] +
1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-
Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))
/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]
] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])
)]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3]
)] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(
3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*Ellipti
cF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(S
qrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 +
 I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3
- I*Sqrt[3]))/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (
I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]
])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[(
(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sq
rt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqr
t[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(
Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 -
Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2
)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) -
```

```
(Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*
(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]
))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt
[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 +
x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1
/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3
+ I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[
3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1
 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] -
 Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 -
 I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]
))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] -
Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt
[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3
]]))/((1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/
2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3
]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt
[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*
Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]
])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2
 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*
Sqrt[3 - 3*(1 + x) + (1 + x)^2]))
```

fricas [A] time = 1.32, size = 78, normalized size = 1.18

$$-\frac{\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}\,+1\right)-\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}\,-1\right)-2\,\sqrt{x^{2}-x+1}\,\sqrt{x+1}}{3\left(x^{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqrt
(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^{2}-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

maple [A] time = 0.04, size = 43, normalized size = 0.65

$$-\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\left(\sqrt{x^{3}+1}\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)-1\right)}{3\left(x^{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/(
x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x+1)^{\frac{3}{2}}\,\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.515 $\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{2}{3x\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{5\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}-\frac{5\sqrt{2-\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}{2\,3^{3/4}\,\sqrt{\frac{x+}{\left(x+\sqrt{3}\right.}}}$$

```
[Out] 2/3/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-5/3*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)
+5/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+5/9*EllipticF((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-5/6*3
^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/
2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {915, 290, 325, 303, 218, 1877}

$$-\frac{5\left(x^3+1\right)}{3x\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{5\left(x^3+1\right)}{3\sqrt{x+1}\,\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{2}{3x\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{5\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}{3\sqrt[4]{3}\,\sqrt{\frac{x}{\left(x+\sqrt{\right.}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (5*(1 + x^3))/(3*x*Sqrt[1 + x]*Sqrt
[1 - x + x^2]) + (5*(1 + x^3))/(3*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x
+ x^2]) - (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (5*S
qrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{5\left(1+x^3\right)}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{5\left(1+x^3\right)}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{5\left(1+x^3\right)}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{5\left(1+x^3\right)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)}$$

Mathematica [C] time = 0.77, size = 409, normalized size = 1.29

$$-\frac{5x^3+3}{3x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{5(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2}+\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)|\frac{3i+}{3i-}\right.}{\sqrt{x+1}}\right.}{36\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -1/3*(3 + 5*x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (5*(1 + x)^(3/2)*((12*
Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqr
t[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sq
rt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3
*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]
+ (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sq
rt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[1 + x]))/(36*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^8+2x^5+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^8 + 2*x^5 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

maple [A] time = 0.04, size = 363, normalized size = 1.15

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(-10x^3-30\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x\operatorname{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)+5i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] 1/6*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(5*I*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1
/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x*(-2*(x+1)/(-3+I*3^(1/2
)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I
*3^(1/2)))^(1/2)+15*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*
```

```
3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x
+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-30*(-2*(
x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I
*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2)
,(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-10*x^3-6)/(x^3+1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,(x+1)^{\frac{3}{2}}\,\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(1/(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.516 $\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{7\left(x^3+1\right)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] 2/3/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-7/6*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(
1/2)-7/18*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {915, 290, 325, 218}

$$-\frac{7\left(x^3+1\right)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (7*(1 + x^3))/(6*x^2*Sqrt[1 + x]*
Sqrt[1 - x + x^2]) - (7*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(6*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\left(1+x^3\right)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}}{12\sqrt{1+x}\sqrt{1-x+x}}$$
$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\left(1+x^3\right)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{}}{6\sqrt[4]{3}}$$

Mathematica [C] time = 0.39, size = 170, normalized size = 1.00

$$\frac{-\frac{6\left(7x^3+3\right)}{x^2\sqrt{x+1}} - \frac{7i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{36\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*(3 + 7*x^3))/(x^2*Sqrt[1 + x]) - ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I
+ Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(36*Sqrt[1 - x + x^2])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^9+2x^6+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^9 + 2*x^6 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

maple [A] time = 0.04, size = 259, normalized size = 1.52

$$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-14x^3+7i\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\sqrt{3}\,x^2\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right.}{12\left(x^3+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] 1/12*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(7*I*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(
1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x^2*(-2*(x+1)/(-3+I*3^(
1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-
3+I*3^(1/2)))^(1/2)-21*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/
(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2
*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-14*
x^3-6)/(x^3+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(x+1)^{\frac{3}{2}}\,\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.517 $\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$\frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] 4/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2
)+4/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^
(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 288, 199, 218}

$$\frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}+\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (4*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
+ x^2]*(1 + x^3)) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{4x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}}{27\sqrt{1+x}\,\sqrt{1-x+x}}$$

$$= \frac{4x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} + \frac{4\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{}}{27\sqrt[4]{3}}$$

Mathematica [C] time = 0.50, size = 178, normalized size = 1.06

$$\frac{\frac{6x\left(2x^3-1\right)}{(x+1)^{3/2}\left(x^2-x+1\right)}+\frac{2i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\,\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{81\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(-1 + 2*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((2*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^3}{x^9+3x^6+3x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3/(x^9 + 3*x^6 + 3*x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

maple [B] time = 0.07, size = 467, normalized size = 2.78

$$-\frac{2\left(-2x^4+i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x^3\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -2/27*(I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))
/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)
+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)-3*Ellipti
cF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x
^3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)
*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)+I*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)
*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2))
)^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/
2)+3))^(1/2))*3^(1/2)-3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)
/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-
2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^4+
x)/(x^2-x+1)^(3/2)/(x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x**3/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.518 $\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

```
[Out] -2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

fricas [A] time = 1.01, size = 29, normalized size = 1.26

$$-\frac{2\sqrt{x^2-x+1}\,\sqrt{x+1}}{9\left(x^6+2\,x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] -2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2 - x + 1\right)^{\frac{5}{2}} (x + 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2}{9\, (x + 1)^{\frac{3}{2}} \left(x^2 - x + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
[Out] -2/9/(x+1)^(3/2)/(x^2-x+1)^(3/2)
```

maxima [A] time = 0.97, size = 24, normalized size = 1.04

$$-\frac{2}{9\left(x^3 + 1\right)\sqrt{x^2 - x + 1}\,\sqrt{x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] -2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 2.88, size = 82, normalized size = 3.57

$$\frac{18\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}-18\,x\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}}{(x+1)\,\left(81\,x\,\left(x^2-x+1\right)^4-162\,\left(x^2-x+1\right)^4+81\,\left(x^2-x+1\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] (18*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2) - 18*x*(x + 1)^(1/2)*(x^2 - x + 1)^(5
/2))/((x + 1)*(81*x*(x^2 - x + 1)^4 - 162*(x^2 - x + 1)^4 + 81*(x^2 - x + 1
)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x + 1)^{\frac{5}{2}} \left(x^2 - x + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.519 $\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=318

$$\frac{10x^2}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{10\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{5\sqrt{2-\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right.}}}{9\,3^{3/4}\,\sqrt{\frac{x\cdots}{\left(x+\sqrt{\cdots}\right.}}}$$

```
[Out] 10/27*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)-10/27*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)-10/81*Ellipt
icF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)+5/27*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x
)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 290, 303, 218, 1877}

$$\frac{10x^2}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2x^2}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)}-\frac{10\left(x^3+1\right)}{27\sqrt{x+1}\,\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}-\frac{10\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{\cdots}{(x\cdots}}}{27\sqrt[4]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (10*x^2)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x^2)/(9*Sqrt[1 + x]*Sqrt[1
 - x + x^2]*(1 + x^3)) - (10*(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*S
qrt[1 - x + x^2]) + (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(9*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^
2]) - (10*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+ \cdots}}}{27\sqrt{1+x}\sqrt{1-x \cdots}}$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1-\cdots}{\sqrt{1\cdots}}}{27\sqrt{1+x}\sqrt{1-x \cdots}}$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{10\left(1 \cdots\right.}{27\sqrt{1+x}\left(1+\sqrt{3}\cdots\right.}$$

Mathematica [C] time = 0.76, size = 409, normalized size = 1.29

$$\frac{2x^2\left(5x^3+8\right)}{27(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}} - \frac{5(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2} + \frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{\cdots}}}}{\sqrt{x+1}}\cdots\right.\right.}{\sqrt{x+1}}\cdots\right.}{162\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (2*x^2*(8 + 5*x^3))/(27*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) - (5*(1 + x)^(3/
2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1
 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-
3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(
-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt
[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(
3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(162*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x
 + x^2])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\,\sqrt{x + 1}\,x}{x^9 + 3\,x^6 + 3\,x^3 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x/(x^9 + 3*x^6 + 3*x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2 - x + 1\right)^{\frac{5}{2}} (x + 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

maple [B] time = 0.06, size = 688, normalized size = 2.16

$$-\frac{-10x^5 - 30\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x^3\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right) + 5i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -1/27*(5*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2
))/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/
2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)+15*Elli
pticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
)*x^3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1
/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)-30*EllipticE((-2*(x+1)/(-3+I*3
^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(x+1)/(-3+I*3
^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/
(-3+I*3^(1/2)))^(1/2)+5*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2)
,(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-
2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1
/2)-10*x^5+15*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2
)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/(-
```

```
3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-30*(-2*(x+1)/(-3
+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)
-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I
*3^(1/2))/(I*3^(1/2)+3))^(1/2))-16*x^2)/(x^2-x+1)^(3/2)/(x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.520 $\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$\frac{14x}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{14\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{2x}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,(}$$

```
[Out] 14/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)+14/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1
)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {713, 199, 218}

$$\frac{14x}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2x}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)}+\frac{14\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7\right.}{27\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (14*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) + (14*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x +
 x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+}}}{27\sqrt{1+x}\sqrt{1-x}}$$

$$= \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{1+}}{2}$$

Mathematica [C] time = 0.47, size = 178, normalized size = 1.06

$$\frac{\frac{6x\left(7x^3+10\right)}{(x+1)^{3/2}\left(x^2-x+1\right)} + \frac{7i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{81\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(10 + 7*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((7*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^9+3\,x^6+3\,x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^9 + 3*x^6 + 3*x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

maple [B] time = 0.05, size = 469, normalized size = 2.79

$$-\frac{-14x^4+7i\sqrt{3}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x^3\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-21\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -1/27*(7*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2
))/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/
2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)-21*Elli
pticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
)*x^3*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1
/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)+7*I*3^(1/2)*EllipticF((-2*(x+1
)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*(-2*(x+1)/(-
3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2
)-1)/(-3+I*3^(1/2)))^(1/2)-21*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1
/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*Ellipt
icF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-
14*x^4-20*x)/(x^2-x+1)^(3/2)/(x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.521 $\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] 2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3
*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 51, 63, 207}

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
 + x^3)) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 -
 x + x^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
```

```
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{5/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} + \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} + \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{\ldots}}\right.}{3\sqrt{1+x}\,\sqrt{1-\ldots}}$$

$$= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int\ldots\right.}{3\sqrt{1+x}\ldots}$$

$$= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)} - \frac{2\sqrt{1+x^3}\,\tanh^{-1}\left(\sqrt{\ldots}\right.}{3\sqrt{1+x}\,\sqrt{1-x+\ldots}}$$

Mathematica [C] time = 6.09, size = 2539, normalized size = 26.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2/(81*(1 + x)^2) + 22/(81*(1 + x)) - (2*(-1
+ x))/(27*(1 - x + x^2)^2) - (2*(-21 + 11*x))/(81*(1 - x + x^2))) + 2*(((-I
)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])
*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 -
I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 -
3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2
+ (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sq
rt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]
))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I
/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2
 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2
+ (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqr
t[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1
 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3]
)] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqr
t[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6
/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^
2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))
/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]]
 + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2
```

```
 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*S
qrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/(
(Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sq
rt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sq
rt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I
/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] -
Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) - (Sqrt[3/2]*(S
qrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 -
(I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[
1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1
/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(S
qrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((S
qrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sq
rt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])
*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3
+ I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1 + Sqrt[1/2 -
 (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*S
qrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] +
 Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3
 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sq
rt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt[1/2 - (I/2)/
Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((1 - Sq
rt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqr
t[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(
3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3
]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sq
rt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt
[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt
[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1
 + x) + (1 + x)^2]))
```

fricas [A] time = 1.03, size = 101, normalized size = 1.05

$$\frac{2\left(3x^{3}+4\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}\,-3\left(x^{6}+2x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}\,+1\right)+3\left(x^{6}+2x^{3}+1\right)\log\left(\sqrt{x^{2}\cdots}\right.}{9\left(x^{6}+2x^{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log(
sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x +
 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

maple [A] time = 0.06, size = 69, normalized size = 0.72

$$-\frac{2\left(3\sqrt{x^{3}+1}\,x^{3}\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)-3x^{3}+3\sqrt{x^{3}+1}\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)-4\right)}{9\left(x^{3}+1\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -2/9*(3*(x^3+1)^(1/2)*arctanh((x^3+1)^(1/2))*x^3-3*x^3+3*arctanh((x^3+1)^(1
/2))*(x^3+1)^(1/2)-4)/(x^3+1)/(x^2-x+1)^(1/2)/(x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(x+1)^{5/2}\,\left(x^2-x+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x+1)^{\frac{5}{2}}\,\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.522 $\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{22}{27x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{55\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{55\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\cdots}}{18\ 3^{3/4}\cdots}$$

```
[Out] 22/27/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)-55/27*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/27*(x^3+1)/(1+x+3^(1/2))/
(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3
^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(
x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-55/54*3^(1/4)*EllipticE((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {915, 290, 325, 303, 218, 1877}

$$-\frac{55\left(x^3+1\right)}{27x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{55\left(x^3+1\right)}{27\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{22}{27x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9x\sqrt{x+1}\sqrt{x^2-x\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 22/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x*Sqrt[1 + x]*Sqrt[1 - x + x
^2]*(1 + x^3)) - (55*(1 + x^3))/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (55*
(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (55*Sqrt[
2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (55*Sqrt[2]*Sqrt[1 +
x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2}{9x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(11\sqrt{1+x^3}\right)\int \frac{1}{x^2\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(55\sqrt{1+x^3}\right)}{27\sqrt{1+x}}$$

$$= \frac{22}{27x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{55\,(1+}{27x\sqrt{1+x}\,\sqrt{}}$$

$$= \frac{22}{27x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{55\,(1+}{27x\sqrt{1+x}\,\sqrt{}}$$

$$= \frac{22}{27x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{55\,(1+}{27x\sqrt{1+x}\,\sqrt{}}$$

Mathematica [C] time = 0.86, size = 414, normalized size = 1.19

$$-\frac{55x^6+88x^3+27}{27x(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}} + \frac{55(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2} + \frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\,F\left(i\sinh^{-1}\left(\right.\right.}{\sqrt{x+1}}\right.}{324\sqrt{-\frac{i}{\sqrt{3}+3i}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -1/27*(27 + 88*x^3 + 55*x^6)/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) + (55*(1
 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*S
qrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])
]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSi
nh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3
])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(
1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt
[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(324*Sqrt[(-I)/(3*I + Sqrt[3])]*S
qrt[1 - x + x^2])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^{11}+3\,x^8+3\,x^5+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^11 + 3*x^8 + 3*x^5 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 - x + 1\right)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

maple [B] time = 0.06, size = 695, normalized size = 1.99

$$\frac{-110x^6 - 330\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\, x^4 \,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right) + 55i\sqrt{3}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\ \sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] 1/54*(55*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2
))/(I*3^(1/2)+3))^(1/2))*x^4*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/
2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)+165*Ell
ipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2
))*x^4*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(
1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)-330*EllipticE((-2*(x+1)/(-3+I
*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^4*(-2*(x+1)/(-3+I
*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1
)/(-3+I*3^(1/2)))^(1/2)+55*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1
/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2
)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)
))^(1/2)-110*x^6+165*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I
*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(
x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-330*(-2
*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x
+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/
2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-176*x^3-54)/x/(x^2-x+1)^(3/2)/(
x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 - x + 1\right)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{(x+1)}^{5/2}\,{\left(x^2-x+1\right)}^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (x+1)^{\frac{5}{2}} (x^2 - x + 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.523 $\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{26}{27x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{91\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}-\frac{91\left(x^3+1\right)}{54x^2\sqrt{x+1}\,\sqrt{x^2-x}}$$

```
[Out] 26/27/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)-91/54*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-91/162*EllipticF((1+x-
3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))
*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {915, 290, 325, 218}

$$-\frac{91\left(x^3+1\right)}{54x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{26}{27x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2}{9x^2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)}-\frac{91\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\cdots}}{54\sqrt[4]{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 26/(27*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x^2*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) - (91*(1 + x^3))/(54*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])
- (91*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]
*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5
4*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(13\sqrt{1+x^3}\right)\int \frac{1}{x^3\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(91\sqrt{1+x^3 \cdots}\right.}{27\sqrt{1+x \cdots}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{91\,(1 \cdots}{54x^2\sqrt{1+x \cdots}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{91\,(1 \cdots}{54x^2\sqrt{1+x \cdots}}$$

Mathematica [C] time = 0.65, size = 183, normalized size = 0.90

$$\frac{-\frac{6\left(91x^6+130x^3+27\right)}{x^2(x+1)^{3/2}} - \frac{91i(x+1)\left(x^2-x+1\right)\sqrt{6+\frac{36i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{1-\frac{6i}{\left(\sqrt{3}+3i\right)(x+1)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{324\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((-6*(27 + 130*x^3 + 91*x^6))/(x^2*(1 + x)^(3/2)) - ((91*I)*(1 + x)*(1 - x
+ x^2)*Sqrt[6 + (36*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[1 - (6*I)/((3*I + S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 +
 x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(324*(1
 - x + x^2)^(3/2))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x^{12}+3\,x^9+3\,x^6+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^12 + 3*x^9 + 3*x^6 + x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")

[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)

maple [B] time = 0.06, size = 481, normalized size = 2.37

$$\frac{-182x^6+91i\sqrt{3}\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\sqrt{\frac{-2x+i\sqrt{3}+1}{i\sqrt{3}+3}}\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x^5\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)-273\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\ \sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)

[Out] 1/108*(91*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^5*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)-273*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^5*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)+91*I*3^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)-273*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(I*3^(1/2)+3))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-182*x^6-260*x^3-54)/x^2/(x^2-x+1)^(3/2)/(x+1)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{(x+1)}^{5/2}\,{\left(x^2-x+1\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)

[Out] int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (x+1)^{\frac{5}{2}} \left(x^2-x+1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.524 $\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx$

Optimal. Leaf size=97

$$\frac{44x+39}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{11\log\left(4x^2+5x+3\right)}{4608}-\frac{97}{4416(1-x)}-\frac{21}{736(1-x)^2}+\frac{11\log(1-x)}{2304}+\frac{6023\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

```
[Out] -21/736/(1-x)^2-97/4416/(1-x)+1/276*(39+44*x)/(1-x)^2/(4*x^2+5*x+3)+11/2304
*ln(1-x)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(1/2))
*23^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {822, 800, 634, 618, 204, 628}

$$\frac{44x+39}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{11\log\left(4x^2+5x+3\right)}{4608}-\frac{97}{4416(1-x)}-\frac{21}{736(1-x)^2}+\frac{11\log(1-x)}{2304}+\frac{6023\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -21/(736*(1 - x)^2) - 97/(4416*(1 - x)) + (39 + 44*x)/(276*(1 - x)^2*(3 + 5
*x + 4*x^2)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23]) + (11*Log
[1 - x])/2304 - (11*Log[3 + 5*x + 4*x^2])/4608
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}dx = \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{1}{276}\int \frac{57+132x}{(-1+x)^3\left(3+5x+4x^2\right)}dx$$

$$= \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{1}{276}\int \left(\frac{63}{4(-1+x)^3}-\frac{97}{16(-1+x)^2}+\frac{253}{192(-1$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{6023\tan^{-1}\left(\frac{5}{\sqrt{}}\right.}{52992\sqrt{23}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.80

$$\frac{\frac{184(2204x+975)}{4x^2+5x+3}-17457\log\left(4x^2+5x+3\right)+\frac{59248}{x-1}-\frac{25392}{(x-1)^2}+34914\log(1-x)+36138\sqrt{23}\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{7312896}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^2
) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*Lo
g[3 + 5*x + 4*x^2])/7312896
```

fricas [A] time = 1.01, size = 134, normalized size = 1.38

$$\frac{214176\,x^3+12046\,\sqrt{23}\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(8\,x+5)\right)-224664\,x^2-5819\left(4\,x^4-3\,x^3\right.}{2437632\left(4\,x^4-3\,x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arct
an(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
+ 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x - 1
) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

giac [A] time = 0.18, size = 71, normalized size = 0.73

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)+\frac{388x^3-407x^2-120x-45}{4416\left(4x^2+5x+3\right)(x-1)^2}-\frac{11}{4608}\log\left(4x^2+5x+3\right)+\frac{11}{2304}\log(|x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 + 5*
x + 3) + 11/2304*log(abs(x - 1))
```

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$\frac{6023\sqrt{23}\arctan\left(\frac{(8x+5)\sqrt{23}}{23}\right)}{1218816}+\frac{11\ln(x-1)}{2304}-\frac{11\ln\left(4x^2+5x+3\right)}{4608}-\frac{-\frac{2204x}{23}-\frac{975}{23}}{6912\left(x^2+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{1}{288(x-1)^2}+\frac{7}{864(x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)^3/(4*x^2+5*x+3)^2,x)
```

```
[Out] -1/6912*(-2204/23*x-975/23)/(x^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/12
18816*arctan(1/23*(5+8*x)*23^(1/2))*23^(1/2)-1/288/(-1+x)^2+7/864/(-1+x)+11
/2304*ln(-1+x)
```

maxima [A] time = 0.96, size = 75, normalized size = 0.77

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)+\frac{388x^3-407x^2-120x-45}{4416\left(4x^4-3x^3-3x^2-x+3\right)}-\frac{11}{4608}\log\left(4x^2+5x+3\right)+\frac{11}{2304}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(x - 1)
```

mupad [B] time = 0.13, size = 84, normalized size = 0.87

$$\frac{11\ln(x-1)}{2304}+\frac{-\frac{97x^3}{4416}+\frac{407x^2}{17664}+\frac{5x}{736}+\frac{15}{5888}}{-x^4+\frac{3x^3}{4}+\frac{3x^2}{4}+\frac{x}{4}-\frac{3}{4}}-\ln\left(x+\frac{5}{8}-\frac{\sqrt{23}\,1i}{8}\right)\left(\frac{11}{4608}+\frac{\sqrt{23}\,6023i}{2437632}\right)+\ln\left(x+\frac{5}{8}+\frac{\sqrt{23}\,1i}{8}\right)\left(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

```
[Out] (11*log(x - 1))/2304 + ((5*x)/736 + (407*x^2)/17664 - (97*x^3)/4416 + 15/58
88)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)/8 + 5
/8)*((23^(1/2)*6023i)/2437632 + 11/4608) + log(x + (23^(1/2)*1i)/8 + 5/8)*(
(23^(1/2)*6023i)/2437632 - 11/4608)
```

sympy [A] time = 0.24, size = 88, normalized size = 0.91

$$\frac{388x^3 - 407x^2 - 120x - 45}{17664x^4 - 13248x^3 - 13248x^2 - 4416x + 13248} + \frac{11\log(x-1)}{2304} - \frac{11\log\left(x^2 + \frac{5x}{4} + \frac{3}{4}\right)}{4608} + \frac{6023\sqrt{23}\operatorname{atan}\left(\frac{8\sqrt{23}\,x}{23}\right.}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] (388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 + 60
23*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

3.525 $\int \frac{x^4 \sqrt{d+ex}}{a+bx+cx^2} \, dx$

Optimal. Leaf size=490

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right.}}\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] 2/3*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*(e*x+d)^(3/2)/c^3/e^3-2/5*(b*e+2*c*d)*
(e*x+d)^(5/2)/c^2/e^3+2/7*(e*x+d)^(7/2)/c/e^3-2*b*(-2*a*c+b^2)*(e*x+d)^(1/2
)/c^4+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*(b^3*c*d-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(5*a^2*b*
c^2*e-2*a^2*c^3*d-5*a*b^3*c*e+4*a*b^2*c^2*d+b^5*e-b^4*c*d)/(-4*a*c+b^2)^(1/
2))/c^(9/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*c*d-2*a*
b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a*b^3*c*e
-4*a*b^2*c^2*d-b^5*e+b^4*c*d)/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 14.85, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d-4ab^2c^2d+5ab^3ce+b^4cd+b^5(-e)}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right.}}\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*(b^2 - 2*a*c)*Sqrt[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d -
 a*e))*(d + e*x)^(3/2))/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^(5/2))/(5*
c^2*e^3) + (2*(d + e*x)^(7/2))/(7*c*e^3) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
- b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d -
 b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d -
 b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{\left(b^3-2abc\right)e}{c^4}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)x^2}{c^3e^2}-\frac{(2cd+be)x^4}{c^2e^2}+\frac{x^6}{ce^2}+\frac{b\left(b^2-2ac\right)\left(cd^2-bde+ae^2\right)-\left(b^3 \cdots\right.}{c^4e\left(\frac{cd^2-bde+ae^2}{e^2} \cdots\right.}\right.\right.}{e}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex \cdots}{5c^2e^3}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex \cdots}{5c^2e^3}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex \cdots}{5c^2e^3}$$

Mathematica [A] time = 0.75, size = 568, normalized size = 1.16

$$-\frac{\sqrt{2}\left(a^2c^2\left(e\sqrt{b^2-4ac}+2cd\right)+abc^2\left(2d\sqrt{b^2-4ac}-5ae\right)-ab^2c\left(3e\sqrt{b^2-4ac}+4cd\right)+b^4\left(e\sqrt{b^2-4ac}+ \cdots\right.\right.}{c^{9/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd \cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e*
x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a*e
 + b*(d + e*x))))/(105*c^4*e^3) - (Sqrt[2]*(-(b^5*e) + a*b*c^2*(2*Sqrt[b^2
- 4*a*c]*d - 5*a*e) + b^3*c*(-(Sqrt[b^2 - 4*a*c]*d) + 5*a*e) + b^4*(c*d + S
qrt[b^2 - 4*a*c]*e) + a^2*c^2*(2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*c*(4*c*
d + 3*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^5*e - b^3*c*(Sqrt[b^2 - 4*a*c]*d
 + 5*a*e) + a*b*c^2*(2*Sqrt[b^2 - 4*a*c]*d + 5*a*e) + a*b^2*c*(4*c*d - 3*Sq
rt[b^2 - 4*a*c]*e) + a^2*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b^4*(-(c*d) +
 Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

fricas [B] time = 2.52, size = 5507, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16
*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^
3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c
^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 +
 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^
3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^
8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4
*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^
2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^12*c - 12*
a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^
2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4
*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (b^6*c^9 - 8*a*b^4*c^10 + 18*a
^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c
^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4
*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)
*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c
^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c
^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*
c^3 + a^8*c^4)*e)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b
^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c
+ 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*s
qrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^
4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^
2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 +
 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
- 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 -
20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)
)*log(-sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4
+ 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^
```

2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (
b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12
*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*
b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4
*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
+ 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^
8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4
*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12
*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*
b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4
*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
+ 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^
8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c -
 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c +
 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) + 105*sqrt(2)
*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a
^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^
4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c
^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d
^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^
4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 1
4*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5
*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a
*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^
2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 -
 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*
b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^
3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5
 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*
a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*
b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*
b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b
^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*
c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9
*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*
a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5
 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*
a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*
b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*
b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b
^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 -
4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*
d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*
sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^
4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 1
2*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5
*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^
3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 -
4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a
^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b
^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5
- 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^
3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*
c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a

```
^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b
^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4
 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e +
(b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c
^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*
c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9
*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a
^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b
^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4
 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e +
(b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c
^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 1
0*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 -
10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) + 4*(15*c^3*e^3*x^3 + 8*c^3*d^3
 + 14*b*c^2*d^2*e + 35*(b^2*c - a*c^2)*d*e^2 - 105*(b^3 - 2*a*b*c)*e^3 + 3*
(c^3*d*e^2 - 7*b*c^2*e^3)*x^2 - (4*c^3*d^2*e + 7*b*c^2*d*e^2 - 35*(b^2*c -
a*c^2)*e^3)*x)*sqrt(e*x + d))/(c^4*e^3)
```

giac [B] time = 0.83, size = 1171, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*
c^2 - 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c
^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d +
 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^
2 - (3*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e + (b^6*c^2 - 5*a*b^4*c^3
+ 5*a^2*b^2*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^24
 - b*c^7*e^25 + sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^26)*c^8*e^24
 + (2*c^8*d*e^24 - b*c^7*e^25)^2))*e^(-24)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^
2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 1/4*(
sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2 +
2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c^3)*sq
rt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt
(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*
c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2 - (3
*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^
2*b^2*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^24 - b*c
^7*e^25 - sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^26)*c^8*e^24 + (2*
c^8*d*e^24 - b*c^7*e^25)^2))*e^(-24)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sq
rt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 2/105*(15*(
x*e + d)^(7/2)*c^6*e^18 - 42*(x*e + d)^(5/2)*c^6*d*e^18 + 35*(x*e + d)^(3/2
)*c^6*d^2*e^18 - 21*(x*e + d)^(5/2)*b*c^5*e^19 + 35*(x*e + d)^(3/2)*b*c^5*d
*e^19 + 35*(x*e + d)^(3/2)*b^2*c^4*e^20 - 35*(x*e + d)^(3/2)*a*c^5*e^20 - 1
05*sqrt(x*e + d)*b^3*c^3*e^21 + 210*sqrt(x*e + d)*a*b*c^4*e^21)*e^(-21)/c^7
```

maple [B] time = 0.11, size = 2218, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/7*(e*x+d)^(7/2)/c/e^3-2/5/e^2/c^2*(e*x+d)^(5/2)*b-2/3/e/c^2*(e*x+d)^(3/2)
*a+2/3/e/c^3*(e*x+d)^(3/2)*b^2-4/5/e^3/c*(e*x+d)^(5/2)*d+2/3/e^3/c*(e*x+d)^
(3/2)*d^2+4/c^3*a*b*(e*x+d)^(1/2)+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d+4*e/c^2/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*a*b^2*d+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d-e/c^3/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*b^4*d-2/c^4*b^3*(e*x+d)^(1/2)+2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2))*a*b*d-2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*a*b*d+e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5+3*e/c^3*2^(1/2)/((-b*e
+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2+e^2/c^4/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c
*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5-
3*e/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*
x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2-5*
e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*a*b^3+e/c^4*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*b^4+1/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*b^3*d-1/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b^3*d-e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*a^2-e/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b^4+e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a
^2+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*a^2*b-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e
-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d-5*e^2/c^3/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c
*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^
3-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*b^4*d+2/3/e^2/c^2*(e*x+d)^(3/2)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\, x^4}{cx^2+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^4/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.86, size = 13879, normalized size = 28.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (d + e*x)^(3/2)*((4*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c^2
*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d
*e^3))/(3*c*e^3)) - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d
*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^
7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^
(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d -
 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e
- 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2
) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d
*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2
*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*
d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^1
0*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7
*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^
3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^
3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 -
8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a
^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^
2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e
^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a
*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^
11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6
*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b
^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^
9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2)
- 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c -
 b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1
/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*
c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2
*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e +
 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d
 + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3
*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e
+ 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^
3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(
4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b
^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b
^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^
6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*
```

b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b ^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^ (1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^ 3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e ^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c ^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6* d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b ^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^ 4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8* c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^ 4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((((8*(a* b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6 *d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^ 8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4 *d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3* c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(- (4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2 *b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^ 2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6 *d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3* b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a ^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8 *c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(- (4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c ^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^ 2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e* x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^ 4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e ^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a *b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3* d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a* c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4 *b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4 *c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b* c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^ 2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e* (-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(1 6*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (16*(a^5*b^4*e^5 + a^7*c^2*e ^5 - 3*a^6*b^2*c*e^5 - a^4*b^5*d*e^4 + a^6*c^3*d^2*e^3 - a^4*b^3*c^2*d^3*e^ 2 - 5*a^5*b^2*c^2*d^2*e^3 + 2*a^5*b^3*c*d*e^4 + a^6*b*c^2*d*e^4 + 2*a^4*b^4 *c*d^2*e^3 + 2*a^5*b*c^3*d^3*e^2))/c^7 + (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7* e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e ^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/ 2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63

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*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c
 - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*
c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^
4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*
e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)
^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5
*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(
-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e -
b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6
*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1
/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5
*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2
*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^
2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*
c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e
^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c
*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^
2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^
(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*
a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c
^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) -
 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a
*b^2*c^10)))^(1/2)))*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/
2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63
*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c
 - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*
c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^
4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*
e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e
^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7
 - (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*
e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*
a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c
 - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*
(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c
^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10
*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4
*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e
^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d -
 b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d
 - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(
4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^
7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a
*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c
^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a
^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*
```

b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6

```
*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*
a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^
(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a
*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*
(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4
*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*
b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2
 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b
^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d +
66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e
 + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*
a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2
)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*
c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*
c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))
/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (16*(a^5*b^4*e^5 + a^7
*c^2*e^5 - 3*a^6*b^2*c*e^5 - a^4*b^5*d*e^4 + a^6*c^3*d^2*e^3 - a^4*b^3*c^2*
d^3*e^2 - 5*a^5*b^2*c^2*d^2*e^3 + 2*a^5*b^3*c*d*e^4 + a^6*b*c^2*d*e^4 + 2*a
^4*b^4*c*d^2*e^3 + 2*a^5*b*c^3*d^3*e^2))/c^7 + (((8*(a*b^5*c^5*e^4 + 8*a^3*
b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c
^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3)
)/c^7 + (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d -
 b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d
 - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(
4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^
7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a
*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*
c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c
^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2
*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4
*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*
e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3
)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^
2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2
*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4
 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 2
0*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^
4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4
*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^1
0*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7
*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^
3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^
3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 -
8*a*b^2*c^10)))^(1/2)))*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^
11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d -
63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a
*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c
*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^
5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) -
 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^
```

```
2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 +
b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - ((8*d)/(5*c*e^3) + (2*(b*e^4 - 2*c*d*e
^3))/(5*c^2*e^6))*(d + e*x)^(5/2) - (d + e*x)^(1/2)*((8*d^3)/(c*e^3) - (((8
*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e
^4))/(c*e^3) + ((b*e^4 - 2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e
^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e
^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3)) + (2*(d + e*x)^(7/2))/(7*c*e^3
)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.526 $\int \frac{x^3 \sqrt{d+ex}}{a+bx+cx^2}\, dx$

Optimal. Leaf size=326

$$\frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}+\frac{\left(-\sqrt{b^2-4ac}\left(b^2-ac\right)-3abc+b^3\right)\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{b^2-4ac}}$$

```
[Out] -2/3*(b*e+c*d)*(e*x+d)^(3/2)/c^2/e^2+2/5*(e*x+d)^(5/2)/c/e^2+2*(-a*c+b^2)*(
e*x+d)^(1/2)/c^3+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*
a*c+b^2)^(1/2)))^(1/2))*(b^3-3*a*b*c-(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*arct
anh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(
b^3-3*a*b*c+(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 7.47, antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {897, 1287, 1166, 208}

$$-\frac{\sqrt{2}\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\frac{-2a^2c^2e+\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2 - a*c)*Sqrt[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^(3/2))/(3*c^2*
e^2) + (2*(d + e*x)^(5/2))/(5*c*e^2) - (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e
+ 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d
- b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d-\right.}{e}$$

$$= \frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}-\frac{2\,\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+a}{\frac{cd^2-bde+}{e^2}}\right.}{}$$

$$= \frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}+\frac{\left(b^2cd-ac^2d-b^3e+2abce\right.}{}$$

$$= \frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}-\frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2a\right.}{}$$

Mathematica [A] time = 0.53, size = 466, normalized size = 1.43

$$\frac{2\sqrt{d+ex}\left(-5ce(3ae+b(d+ex))+15b^2e^2+c^2\left(-2d^2+dex+3e^2x^2\right)\right)}{15c^3e^2}+\frac{\sqrt{2}\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^2c\left(4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) - 5*c*e*(3*
a*e + b*(d + e*x))))/(15*c^3*e^2) + (Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b^2 -
4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d + Sqr
t[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*
```

```
Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^4
*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*
c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e])
```

fricas [B] time = 1.88, size = 4245, normalized size = 13.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3
*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4
*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9
*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c
^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*
c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 +
 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e - (b^5*c^7 -
 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b
^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*
b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e
 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*
a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3
 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*
a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9
*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
- 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^
7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (
b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^
5*c^5)*e - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8
*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9
*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6
)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c -
 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 +
 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9
*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e
+ (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))
 + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c
+ 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e^2*sqrt(((b^
6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^
2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*
c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*
```

```
a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)
*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*
c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 +
 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 2
9*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)
*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b
^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^
15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10
*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 -
 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5
 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/
(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^
3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(
2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt((
(b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*
d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^
3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^1
5)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c
^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8
+ 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4
*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 -
 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 +
 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5
*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^
2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 +
 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt
(e*x + d)) + 4*(3*c^2*e^2*x^2 - 2*c^2*d^2 - 5*b*c*d*e + 15*(b^2 - a*c)*e^2
+ (c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e^2)
```

giac [B] time = 0.56, size = 1045, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2
 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2 - 2*((b^2*c^3
- a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e
+ (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*(2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d
*e + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*
e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e
^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/
((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*
c)*a*c^5*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*
```

```
((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^
2)*c^2 + 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*s
qrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c -
 sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b
^2*c^4 + 4*a^2*c^5)*d*e + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2))*arcta
n(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6
*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^1
3)^2))*e^(-12)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*
d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2) + 2/15*(3*(x*e + d)^(5/2)*c^4*e^8 -
 5*(x*e + d)^(3/2)*c^4*d*e^8 - 5*(x*e + d)^(3/2)*b*c^3*e^9 + 15*sqrt(x*e +
d)*b^2*c^2*e^10 - 15*sqrt(x*e + d)*a*c^3*e^10)*e^(-10)/c^5
```

maple [B] time = 0.06, size = 1764, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/5*(e*x+d)^(5/2)/c/e^2-2/3/e/c^2*(e*x+d)^(3/2)*b-2/3/e^2/c*(e*x+d)^(3/2)*d
-2/c^2*a*(e*x+d)^(1/2)+2/c^3*b^2*(e*x+d)^(1/2)-2*e^2/c/(-(4*a*c-b^2)*e^2)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d
)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2+4*e^
2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*c)*a*b^2-3*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*
c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d-e^2/c^3/(-(4*a*c-b^2)*e^2)^
(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+
d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4+e/c
^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*c)*b^3*d-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*a*b+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(
1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*c)*a*d+e/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(
1/2)*c)*b^3-1/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*b^2*d-2*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2+4*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2-3*e/c/(-(4*a*c-b^2
)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan(
(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b
*d-e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*c)*b^4+e/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d+2*e/c^2*2^(1/2)/((b*e-2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*d-e/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*b^3+1/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/
```

```
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^3/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.37, size = 11143, normalized size = 34.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e
^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*
d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5
*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b
^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*
c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^
(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)
*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2
*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c
^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^
4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*
c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 +
20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e
^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d
*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^
5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a
^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3
*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*
c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5
*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*
b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x
)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d -
 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e
+ a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a
^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9
 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^
8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)
^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*
e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e +
```

10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5 + (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1

/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i - ((2*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(3*c^2*e^4))*(d + e*x)^(3/2) + atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c

- b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5 + (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*

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d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*
c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*
b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^
7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*((8*a^4*c^
5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d -
 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*2i + (d + e*x)^(1/2)*((6*d^2)/(c*e^2) - (2*(a*e^4 + c*d^2*e^
2 - b*d*e^3))/(c^2*e^4) + (((6*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(c^2*e^
4))*(b*e^3 - 2*c*d*e^2))/(c*e^2)) + (2*(d + e*x)^(5/2))/(5*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.527 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=316

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+\right.}{c^{5/2}\sqrt{2cd-}}$$

```
[Out] 2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(
-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^
2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2)
```

Rubi [A] time = 3.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+\right.}{c^{5/2}\sqrt{2cd-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c*e) + (Sqrt[2]*(b*c*d -
b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b\left(cd^2-bde+ae^2\right)-\left(bcd-b^2e+ace\right)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx,x,\sqrt{d+ex}\right)}{e}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{2\,\text{Subst}\left(\int \frac{b\left(cd^2-bde+ae^2\right)+\left(-bcd+b^2e-ace\right)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\cdots}\right.}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{2cd-(\cdots}}\right.}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.44, size = 375, normalized size = 1.19

$$\frac{\sqrt{2}\left(-b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(d\sqrt{b^2-4ac}-3ae\right)+ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2c\cdots}}\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/(3*c^2*e) + (Sqrt[2]*(b^3*e + b*c*
(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*
d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d + 3
*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]
*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4
```

```
*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])
```

fricas [B] time = 1.02, size = 2966, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^6*c - 6*a*b^4*c
^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e
- (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^
2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a
^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*e*sqrt(((
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b
^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^
7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*
c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 - 6*a*b^2*c^6 +
8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5
*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^
5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*e)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^
6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)
*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c
^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 -
4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c -
 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*s
qrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*
a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*
```

```
c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*
c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^
2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*
a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) + 4*(c*e*x + c*d - 3*b*e)*sqrt(e
*x + d))/(c^2*e)
```

giac [B] time = 0.45, size = 868, normalized size = 2.75

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^3c-4\,abc^2\right)de-\left(b^4-5\,ab^2c+4\,a^2c^2\right)e^2\right)c^2-2\left(\sqrt{b^2-4\,ac}\,bc^3d^2-\sqrt{b\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2)
*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^3*
d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c
^4)*d*e + (b^4*c^2 - 3*a*b^2*c^3)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sq
rt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e
^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((sqrt(b^2 - 4*a*c
)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2)
 + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^
2)*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt
(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*
c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b
*c^4)*d*e + (b^4*c^2 - 3*a*b^2*c^3)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/
sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3
*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((sqrt(b^2 - 4*a
*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^
2) + 2/3*((x*e + d)^(3/2)*c^2*e^2 - 3*sqrt(x*e + d)*b*c*e^3)*e^(-3)/c^3
```

maple [B] time = 0.04, size = 1329, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2-3*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b+2*e/(-(
4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^
(1/2)*c)*a*d+e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2
```

```
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d+e/c*2^(1/2)/((-b
*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a-e/c^2*2^(1/2)/((-b*e+2*
c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e
+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2+1/c*2^(1/2)/((-b*e+2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d-3*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b+2*e/(-(4*a
*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)*a*d+e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^3-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c
*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d-e/c*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a+e/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b^2-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^2/(c*x^2 + b*x + a), x)
```

mupad [B] time = 3.91, size = 8171, normalized size = 25.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c*e) - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 -
 b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/
c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^
(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-
(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^
7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c
^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
+ 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4
- 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 -
 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 -
4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3
)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
```

```
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*1i - (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*
e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b
^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*
c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
 + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*
b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2
*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^
2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(
4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^
3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c
^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^
3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-
(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^
2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*
a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b
^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*
d^2*e^3))/c^3 + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3
*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(
1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18
*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/
2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2
*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d +
b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*
a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b
^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2)
)/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d -
18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*
e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2
 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^
4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*
e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a
^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a
*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) -
 b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c
- b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a
*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d
 + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*
e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3
- 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d +
b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20
```

*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*

```
a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(
d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2
*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3
 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*
d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*
(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^
6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e
^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*
c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a
^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c
^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7
 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3
*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*
e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3
*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^
6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*((b^4*e*
(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d
 - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*
a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2
*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - ((4*d)/(c*e) + (2*(b*e^2 - 2*
c*d*e))/(c^2*e^2))*(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.528 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=287

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{}}$$

```
[Out] 2*(e*x+d)^(1/2)/c+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(
1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arc
tanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 3.20, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {824, 826, 1166, 208}

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d
- b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{d+ex}}{c} + \frac{\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c}$$
$$= \frac{2\sqrt{d+ex}}{c} + \frac{2\,\mathrm{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$
$$= \frac{2\sqrt{d+ex}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{c\sqrt{b^2-4ac}}$$
$$= \frac{2\sqrt{d+ex}}{c} + \frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} -$$

Mathematica [A] time = 0.41, size = 301, normalized size = 1.05

$$\frac{\frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd-\ldots}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x] + (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e
+ 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d +
b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

fricas [B] time = 1.28, size = 1721, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
```

```
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c
- 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sq
rt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2
)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^
2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*
d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d
 - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a
*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d
*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c -
2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) +
 sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c - 4*
a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((
b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^
3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c
 + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d -
(a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (
b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c
^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*
c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3
*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) + 4*s
qrt(e*x + d))/c
```

giac [B] time = 0.41, size = 753, normalized size = 2.62

$$\frac{2\sqrt{xe+d}}{c} + \frac{\left(\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2c - 4\,ac^2\right)de - \left(b^3 - 4\,abc\right)e^2\right)c^2 - 2\left(\sqrt{b^2-4\,ac}\,c^3d^2 - \sqrt{b^2-\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - 2*(sqrt(b^2 - 4*a*c)*c^3
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^4*d^2 - (3*b^2*c^3
- 4*a*c^4)*d*e + (b^3*c^2 - 2*a*b*c^3)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e
+ sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/(
(sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c
)*a*c^3*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 + 2*(sqrt(b^2 - 4*a*c)*c^3
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^4*d^2 - (3*b^2*c^3
- 4*a*c^4)*d*e + (b^3*c^2 - 2*a*b*c^3)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b
```

```
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e
- sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/(
(sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c
)*a*c^3*e^2)*c^2)
```

maple [B] time = 0.04, size = 926, normalized size = 3.23

$$\frac{2\sqrt{2}\,a\,e^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}+\frac{2\sqrt{2}\,a\,e^{2}\,\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*(e*x+d)^(1/2)/c+2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^2-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^2+1/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arcta
nh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
*b*d*e+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*
e-2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+2/(-(4*a*
c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a*e^2-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*b^2*e^2+1/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d*e-1/c*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*e+2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*c)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x}{cx^{2}+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a), x)
```

mupad [B] time = 3.82, size = 5664, normalized size = 19.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4
*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d +
```

```
e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d
 + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b
^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a
^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b
^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b
^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*
a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a
^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 -
2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e
- b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c -
 b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*
c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)
*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e
^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(
4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1
/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^
2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d
*e^3))/c - (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)
^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*
e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(
1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 -
2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c
^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*
c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^
4 - 2*a*b*c*d^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2
*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)
^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7
*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c
)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^
3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6
*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c
^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c
^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c
*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a
^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
- atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e
^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d + e*x)^(1/2)*(-(b^5*e -
```

8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.529 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/
2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2))*2^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(
1/2)
```

Rubi [A] time = 0.27, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {699, 1130, 208}

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= -\left(\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)\right)+\Big(e$$

$$= -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right.}}{\sqrt{c}\sqrt{}}$$

Mathematica [A] time = 0.40, size = 175, normalized size = 0.88

$$\frac{\sqrt{2}\left(\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right.\right.}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*(-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 1.18, size = 715, normalized size = 3.61

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^
3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*
e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log
(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e
 + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*
sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e
^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*s
qrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2
/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) - 1/2*sqrt(2
)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2
*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))
*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*
c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

giac [A] time = 0.27, size = 223, normalized size = 1.13

$$-\frac{\sqrt{-4\,c^{2}d+2\left(bc-\sqrt{b^{2}-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,cd-be+\sqrt{(2\,cd-be)^{2}-4\left(cd^{2}-bde+ae^{2}\right)c}}{c}}}\right)}{\sqrt{b^{2}-4\,ac}\,|c|}+\frac{\sqrt{-4\,c^{2}d+2\left(bc+\sqrt{b^{2}-4\,ac}\,c\right)\cdots}}{\sqrt{b\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x
*e + d)/sqrt(-(2*c*d - b*e + sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^
2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)) + sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c*d - b*e - sqrt((2
*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)
)
```

maple [B] time = 0.03, size = 545, normalized size = 2.75

$$\frac{\sqrt{2}\,b\,e^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}+\frac{\sqrt{2}\,b\,e^{2}\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] e^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*b-2*c*e/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-e*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)+e^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b-2*c*e/(-(4*a*c-b^2)*e^2)^(1/2)*2^
(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2
^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+e*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ex+d}}{cx^{2}+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

mupad [B] time = 2.99, size = 709, normalized size = 3.58

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)+\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,c\right.}{2\left(16\,a^2\,c^3\right.}\right.}{16\,c^2\,d^2\,e^3-16\,b\,c\,d\,e^4+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(a + b*x + c*x^2),x)
```

```
[Out] - 2*atanh((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2
+ 16*b*c^2*d*e^3) + ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32
*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^
2*d - 2*b^2*c*d - 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^
3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16
*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d
- 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d
 + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2 + 16*b*c^2*d*e^3
) - ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 6
4*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d
+ 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3
)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3
 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16*b*c*d*e^4))*((e*
(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [A] time = 50.10, size = 155, normalized size = 0.78

$$2e\,\mathrm{RootSum}\left(t^4\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^3+32ac^2de^2+4b^3e^3-8b^2cde^2\right)+ae^2-bde+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] 2*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e*
*4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e
**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16
*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))
```

3.530 $\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=275

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2cd}}\right.}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+\right.}}$$

```
[Out] -2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e+d
*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1
/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.13, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {897, 1287, 206, 1166, 208}

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2cd}}\right.}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a + (Sqrt[2]*Sqrt[c]*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{de}{a\left(d-x^2\right)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{a\sqrt{b^2-4ac}}$$

$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.94, size = 267, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b
^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*A
```

```
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a
```

fricas [B] time = 2.15, size = 2446, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^
2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log
(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) + 2*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, 1/2*(sqrt(2)
*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*a
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + 4*sqrt(-d
)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/a]
```

giac [B] time = 0.39, size = 712, normalized size = 2.59

$$\frac{2\,d\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(b^2-4\,ac\right)a^2de-2\left(\sqrt{b^2-4\,ac}\,acd^2-\sqrt{b^2-4\,ac}\,abde+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d*arctan(sqrt(x*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2
*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2
 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d
^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*
a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*a
bs(a)*abs(c)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2
- 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e
 + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d
)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a
*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

maple [B] time = 0.06, size = 581, normalized size = 2.11

$$\frac{\sqrt{2}\,bcde\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,a}+\frac{\sqrt{2}\,bcde\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x)
```

```
[Out] -2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a-2*e^2*c/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)+e/a*c/(-(4*
a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1
/2)*c)*b*d+1/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*ar
ctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*c)*d-2*e^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)+e/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d-1/a*c*2^(1/2)/((b*e-2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x), x)
```

mupad [B] time = 7.41, size = 10894, normalized size = 39.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b
*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a
^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*
a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*
c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e
^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^
8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 3
84*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b
^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)
^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576
*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d
*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e
^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*
c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*
a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^
3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10
) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 +
 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
 + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i - (((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*
(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*
((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*
c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d
^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9
) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*
a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d +
 e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11
+ 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*
c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*
d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-
(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2
*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^
10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b
^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^
(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^
2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
```

)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i)/((((b^4*d + 8*a^2*c^2
*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2
)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e +
a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
+ 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4
*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 +
64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64
*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*
c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c
^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 +
192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^
3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^
8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10)
)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4
*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^
4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11
+ 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*
c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*
b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/
2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*
e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4
*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e
^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*
d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c
^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8
+ 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 -
64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^
3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b
*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11
+ 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*
c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*
e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^1
0))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^
4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d
*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5
*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((
b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c
- b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)))^(1/2) - 64*c^4*d^3*e^10 + 64*b*c^3*d^2*e^11 - 64*a*c^3*d*e^12)
)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4
*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))^(1/2)*2i - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-
(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2
*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c
^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1
/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^
3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^

2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^
8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 +
480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*
b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b
^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 -
 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^
2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2
*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2
*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2
) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^
2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^1
1 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4
*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*
d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*
b^2*c)))^(1/2)*1i - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^
3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*
a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)
^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c -
b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(
2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b
^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d
^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e
^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96
*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*
a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e
^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b
^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^
3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^
2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(
1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))
^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 6
4*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*
c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c
^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*
d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)
*1i)/((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d
*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4
*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*
c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d
 + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^1
0 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
- 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384
*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3
*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(
1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a
^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e
^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9
 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c
- b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^
2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*

e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - 64*c^4*d^3*e^10 + 64*b*c^3*d^2*e^11 - 64*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - (2*d^(1/2)*atanh((640*c^4*d^(5/2)*e^10*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) + (576*c^5*d^(9/2)*e^8*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) + (64*b^3*c^2*d^(3/2)*e^11*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) + (192*b^2*c^3*d^(5/2)*e^10*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) - (64*b^4*c^2*d^(5/2)*e^10*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) + (192*b^3*c^3*d^(7/2)*e^9*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) - (128*b^2*c^4*d^(9/2)*e^8*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) + (64*a*c^3*d^(1/2)*e^12*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^

```
3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*
c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) - (38
4*b*c^3*d^(3/2)*e^11*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^1
1 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*
b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2
 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) - (896*b*c^4*d^(7/2)*
e^9*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e
^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128
*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 3
84*a*b*c^3*d^2*e^11)))/a
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.531 $\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)-\right.}{a^2\sqrt{b^2-}}$$

```
[Out] e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2*(-a*e+b*d)*arctanh((e*x+d)^(1/
2)/d^(1/2))/a^2/d^(1/2)-(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d
-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d-a*b*e-(-a*e+b*d)
*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

Rubi [A] time = 3.66, antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-\sqrt{b^2-4ac}\,(bd-ae)-a\right.}{a^2\sqrt{b^2-4ac}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/(a*x)) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a*Sqrt[d]) + (
2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt[2]*Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c
]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{de^2}{a\left(d-x^2\right)^2}-\frac{e(-bd+ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{a^2}+\frac{(2de)\,\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx,x,\right.}{a}$$

$$= -\frac{\sqrt{d+ex}}{ax}+\frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}+\frac{e\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx,x,\sqrt{d+ex}\right)}{a}-\frac{\left(c\left(b^2\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex}}{ax}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}+\frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd\right.}{a^2}$$

Mathematica [A] time = 1.59, size = 364, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*Sqrt[d + e*x])/x) + (a*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (2
*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (Sqrt[2]*Sqrt[c]*(-(
b^2*d) + 2*a*c*d - b*Sqrt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]
*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[
2]*Sqrt[c]*(-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2
 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]))/a^2
```

fricas [B] time = 25.23, size = 4860, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^
2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2
- 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*
a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b
^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 -
 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b
^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2
 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x
*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 -
 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*
c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8
*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e +
(a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5
- 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(
a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a
*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^
2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d
*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2
- 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a
^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6
- 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*
e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4
```

```
 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*s
qrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3
*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e
)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d -
(a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt
(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c
^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2
*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*
c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))
 + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - (
2*b*d - a*e)*sqrt(d)*x*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*sqr
t(e*x + d)*a*d)/(a^2*d*x), 1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b
^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^
2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*
c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c
 + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^
4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b
^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^
6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*
d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2
 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e
*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 -
 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^
6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e -
(a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d
- (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^
2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a
^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b
^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a
^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4
*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*
a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*
b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2
*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*
c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b
^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)
*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a
*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e
^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d
 - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4
*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4
*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5
 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
```

```
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt
(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^
5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c +
 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9
*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*
c^3)*e)*sqrt(e*x + d)) - 2*(2*b*d - a*e)*sqrt(-d)*x*arctan(sqrt(e*x + d)*sq
rt(-d)/d) - 2*sqrt(e*x + d)*a*d)/(a^2*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 1.17Done
```

maple [B] time = 0.05, size = 999, normalized size = 2.71

$$\frac{\sqrt{2}\, b c\, e^{2} \operatorname{arctanh}\left(\frac{\sqrt{e x+d}\, \sqrt{2}\, c}{\sqrt{\left(-b e+2 c d+\sqrt{-\left(4 a c-b^{2}\right) e^{2}}\right) c}}\right)}{\sqrt{-\left(4 a c-b^{2}\right) e^{2}}\, \sqrt{\left(-b e+2 c d+\sqrt{-\left(4 a c-b^{2}\right) e^{2}}\right) c}\, a}+\frac{\sqrt{2}\, b c\, e^{2} \arctan\left(\frac{\sqrt{e x+d}\, \sqrt{2}\, c}{\sqrt{\left(b e-2 c d+\sqrt{-\left(4 a c-b^{2}\right) e^{2}}\right) c}}\right)}{\sqrt{-\left(4 a c-b^{2}\right) e^{2}}\, \sqrt{\left(b e-2 c d+\sqrt{-\left(4 a c-b^{2}\right) e^{2}}\right) c}\, a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x)
```

```
[Out] -(e*x+d)^(1/2)/a/x-e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2/a^2*d^(1/2)
*arctanh((e*x+d)^(1/2)/d^(1/2))*b+e^2/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b+2*e/a*c^2/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)*d-e/a^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*b^2*d+e/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)-1/a^2*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b*d+e^2/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b+2*e/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-e/a^2*c/(-(4
*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*b^2*d-e/a*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
+1/a^2*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x
+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e x+d}}{\left(c x^{2}+b x+a\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x)
```

mupad [B] time = 6.81, size = 19887, normalized size = 54.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (atan((((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e
 - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8
*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^
4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b
^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d
^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^
11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*(
(8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e
^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*
a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108
*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*
a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 -
(((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c
^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4
*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^
8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*
c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 +
8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^
6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a
^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)) + ((a*
e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*
c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d
*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*(
(8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^
3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e
^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*
e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3
*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4
*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x
)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^
5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2
*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^
5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3
*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^
8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9
+ 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 -
64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*
b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 +
4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c
^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d
*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))
))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)))/((16*(a^3*c^5*e^
13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4
*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4
 - ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 +
```

6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 - (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)) + ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))*(a*e - 2*b*d)*1i)/(a^2*d^(1/2)) - atan((((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e

^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)

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^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/
2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b
^2*c)))^(1/2)*1i)/((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^
2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 -
 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9
+ 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a
*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^
4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(
32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2
*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e
^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24
*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3
*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*
b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 +
 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 -
32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b
^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3
*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b
^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c
*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c
^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8
+ 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 -
 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 +
 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^
4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a
^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c
*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3
*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*
d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3
*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)
*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e
^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 -
12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^
3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5
*b^2*c)))^(1/2) + (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2
*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 -
20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 +
 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*
b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4
*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(3
2*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*
e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^
9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*
a^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*
d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-
(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 3
2*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^
2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*
d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^
```

5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*
d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
 + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^
2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 +
 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 -
108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 +
87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4
)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^
2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*
e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c
- b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*
c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d
 + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*
b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(
1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*
(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^
8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 1
2*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3
)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1
/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2) + (16*(a^3*c^5*e^13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*
a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e
^12 + 2*a*b^2*c^5*d^2*e^11))/a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c
- b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2)*2i - (d + e*x)^(1/2)/(a*x) - atan(((((8*(16*a^5*b*c^
4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4
*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5
*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*
c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*
e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10
 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11
 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5
*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5
*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d +
e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*
e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*
(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(
64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2
*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9
 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d
 + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*
d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2
*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4
*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4
*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a
^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4
*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^
3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^
5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-
(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a
*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a
^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c -

b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8
 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^
3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3
*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*
d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3
*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((8*(16*a^5*b*
c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a
^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b
^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^
5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*
d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^
10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^
11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a
^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b
^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d
+ e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^
5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*
d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d
^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e
^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6
*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*
c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a
^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c
^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40
*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b
^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*
b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*
c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*
(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2
e(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4
*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16
*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c -
b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)
^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a
^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^
2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a
^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3
)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((8*(16*a^5*
b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20
*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2
*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*
b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^
2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*
e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*
e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6
*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5
*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(
d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*
b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -

```
7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*
c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/
2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5
*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d
*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b
^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^
4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e +
 a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*
(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b
*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 +
40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3
*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^
4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*
b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*
d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-
(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c
- b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4
*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 1
8*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8
*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*
c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12
*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)
^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((8*(16*a^5*b
*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*
a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*
b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b
^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2
*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e
^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e
^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*
a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*
b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d
 + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b
^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c
*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*
d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*
e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^
6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4
*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e +
a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*
c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 4
0*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*
b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4
*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b
*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d
*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(
4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 1
6*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c -
 b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2
```

```
)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 -
 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*
e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18
*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*
a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c
^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*
a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^
3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(a^3*c^5*e^
13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4
*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4
))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a
^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c
*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.532 $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=531

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2\right.}}$$

```
[Out] -3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(3/2)-e*(-a*e+b*d)*arctanh((e*x
+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-2*(-a*b*e-a*c*d+b^2*d)*arctanh((e*x+d)^(1/2)
/d^(1/2))/a^3/d^(1/2)-1/2*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*x+d)^(1/2)/a/d/x+(-a
*e+b*d)*(e*x+d)^(1/2)/a^2/d/x+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d-a*c*(-2*a*e+d*(-4*a
*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d+e*(-4*a*c+b^2)^(1
/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh
(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1
/2)*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))+a*c*(2*a*e+d*(-4*a*c+b^2)
^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 3.57, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -Sqrt[d + e*x]/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d - a*e)*Sqr
t[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*d^(3/2)
) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*d^(3/2)) - (2*(b^2*
d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[2]
*Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c
]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d - b^2
*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c
*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{de^3}{a\left(d-x^2\right)^3}+\frac{e^2(-bd+ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d+acd+abe\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right.\right.}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right.\right.}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{\left(2de^2\right)\text{Subst}\left(\int \frac{1}{(d-x}\right.}{a}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{2\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{\left(3e^2\right)\text{Subs}}{}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}-\frac{2\left(b^2d\right.}{}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{e(bd-ae)\tan}{a^2d}$$

Mathematica [A] time = 2.34, size = 516, normalized size = 0.97

$$-\frac{\frac{3a^2e\left(ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)-\sqrt{d}\sqrt{d+ex}\right)}{d^{3/2}x}+\frac{2a^2\sqrt{d+ex}}{x^2}+\frac{8\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{\sqrt{d}}+\frac{4\sqrt{2}\sqrt{c}\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right.\right.}{\sqrt{b^2-4ac}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/4*((2*a^2*Sqrt[d + e*x])/x^2 - (4*a*(b*d - a*e)*Sqrt[d + e*x])/(d*x) - (
4*a*e*(-(b*d) + a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/2) + (8*(b^2*d -
a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (3*a^2*e*(-(Sqrt[d
]*Sqrt[d + e*x]) + e*x*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]))/(d^(3/2)*x) + (4*Sq
rt[2]*Sqrt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b
^2 - 4*a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b
^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)
+ a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]))/a^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.59, size = 1041, normalized size = 1.96

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^4-5\,ab^2c+4\,a^2c^2\right)de-\left(ab^3-4\,a^2bc\right)e^2\right)a^2-2\left(\left(ab^2c-a^2c^2\right)\sqrt{b^2-4\,ac}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a) - sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c -
 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^
4*b*c)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e +
sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))
/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sq
rt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*
c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b
*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3
*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*arctan(2*sqrt(1/2)*sq
rt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4
*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*ab
s(c)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*arctan(sqrt(x*e +
 d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/4*(4*(x*e + d)^(3/2)*b*d*e - 4*sqrt(x*e
+ d)*b*d^2*e - (x*e + d)^(3/2)*a*e^2 - sqrt(x*e + d)*a*d*e^2)*e^(-2)/(a^2*d
*x^2)
```

maple [B] time = 0.05, size = 1486, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x)
```

```
[Out] -1/4/a/x^2/d*(e*x+d)^(3/2)+1/e/a^2/x^2*(e*x+d)^(3/2)*b-1/e/a^2/x^2*(e*x+d)^
(1/2)*b*d-1/4*(e*x+d)^(1/2)/a/x^2+1/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/
d^(3/2)+e/a^2/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b+2/a^2*d^(1/2)*arctan
h((e*x+d)^(1/2)/d^(1/2))*c-2/a^3*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b^2
+2*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)-e^2/a^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b
*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2-3*e/a^2*c^2/(-(4*a*c-b^2
)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*
```

```
b*d+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^3*d-e/a^2*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*b-1/a^2*c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*d+1/a^3*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*b^2*d+2*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b
*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-e^2/a^2*c/(-(4*a*c-b^2)*e^2
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2-3*e/
a^2*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*b*d+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d+e/a^2*c*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b+1/a^2*c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-1/a^3*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3), x)
```

mupad [B] time = 8.09, size = 33838, normalized size = 63.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)),x)
```

```
[Out] atan((((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^
10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*
e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*
d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*
b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^1
1*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*
d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b
^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*
a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*
a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 3
8*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
```

0*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c
^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^
2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*
c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 5
12*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1
152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^
10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3
*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b
^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 451
2*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 +
 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*
c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*
b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(
4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)
+ (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*
e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 12
8*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 +
 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^
10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d
^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*
c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a
^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12
+ 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d
^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e
^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e
^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-
(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25
*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(
-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(
1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^
6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 70
4*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10
- 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*
e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6
*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^
6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^
5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*
d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2
*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a
^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6
*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3
*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*
c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^1
0*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8
*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2
*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^
3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e
 + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b
^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c

- b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^1
2*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^1
0*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 6
4*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e
^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) -
3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(
8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e
^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8
- 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^
4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*
c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^
7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 +
568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2
*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^1
2 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d
+ 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*
d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^
4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c -
b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*
b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*
e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^
9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^
5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c
^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*
b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 140
8*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^
11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^
5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*
b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3
*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((
b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4
*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*
a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 +
16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2
*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*
e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e
^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*
d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*
c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*
b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*
c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^
5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1
/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^
3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3
)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((128*
a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^
4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*
b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a
^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^1

1 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^1
1 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*
c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^
3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*
b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
 + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*
a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8
+ 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^
2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5
d(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d
 - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a
*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*
c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a
^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c
)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*
b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*
e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4
*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b
^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*
a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9
+ 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d
^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*
d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4
*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/
2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2
)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^
14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*
c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d
^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^
5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9
*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a
^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 +
1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*
e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*
c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a
^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*
b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b
^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3
)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e
+ a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^
2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^
3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*
x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*
c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d
^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c
^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b
^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*
a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*
a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5
*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
 - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2
*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c
*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*

a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
 - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7
*b^2*c)))^(1/2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^1
1*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*
b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*
a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10
 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^
12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d +
e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e
 + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*
b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/
(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 10
24*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9
- 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2
*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*
((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b
^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e
+ 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(
4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4
 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12
- 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7
*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2
*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^
8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728
*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8
+ 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^
2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*
c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5
*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b
^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3
*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d +
 a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^
3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3
*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2
*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*
a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d
^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^
2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^
8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a
^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 +
 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*
e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c
^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a
^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5
*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7
*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*
d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^
2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*
c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4
*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2
) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^
7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 19
2*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*
d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c
^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*
b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128

*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2)))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (((a*e^2 + 4*b*d*e)*(d + e*x)^(1/2))/(4*a^2) + ((a*e^2 - 4*b*d*e)*(d + e*x)^(3/2))/(4*a^2*d))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4 *d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)

^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c -
b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5
*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a
^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^
4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6
*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*
b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 268
8*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12
 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d
^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b
^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 42
9*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a
^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^
7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2
)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2
*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a
c - b^2)^3)^(1/2))/(2(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d +
 e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a
^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^
7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^
2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^
3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 -
15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 3
84*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*
a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^
4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*
b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^
6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e -
 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*
a^7*b^2*c)))^(1/2)*1i)/((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 8
96*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 3
2*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9
+ 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^
3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^
2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) -
((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a
*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2)
 + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(
1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^
8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^
3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c
^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*
d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33
*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^
5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2
d(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a
^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d
*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a
^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b
^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*
a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9
- 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^

5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4 *a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2)))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + (atan((((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b

^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*
a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 +
224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*
e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^
6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5
*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 +
559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13
+ a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 14
08*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12
*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^
4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*
c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b
^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 -
256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 -
64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2
+ 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 +
128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4
*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c
^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2
- 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/
2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c
^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5
*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^
3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*
b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 409
6*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^1
0 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4
*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d
*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^
2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 -
8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)
*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*
e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 +
960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e
^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d
^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*
c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^
6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d
^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*
d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2)) - (((((4*a^9*c^5*e^14 - a^6*b^6*c^
2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 1
92*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2
*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512
*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 -
2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*
e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^
7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5
*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 +
3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*
e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 -
23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/
(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^
5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*
c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*
b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 1
92*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 +
384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)
^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8
+ 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3

*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2)))/((7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2) + (((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12)

```
)/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^
(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2))
- ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8
 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2
*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512
*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10
- 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*
e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e
^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9
 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 -
8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + (((((4*a^9*c^5*e^
14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*
c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d
^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^
5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9
*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a
^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 +
1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*
e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*
c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a
^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*
b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b
^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^
8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^
9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4
*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c
^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b
^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^
2) + ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a
^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a
^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 +
 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3
*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*
d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b
*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 +
512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 -
1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e
^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^
3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*
b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 45
12*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12
+ 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4
*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*
e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^
14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b
^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^
2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 7
52*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^1
0 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2
*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9
 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/
(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1
/2))))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(4*a^3*(d^3)^(1/2)
)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.533 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=650

$$-\frac{2\sqrt{d+ex}\left(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)}{c^5}+\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)-b^4c\left(cd^2-6ae^2\right)-10ab^3c^2de+ab^2c^2(\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{\cdots}$$

```
[Out] -2/3*b*(-2*a*c+b^2)*(e*x+d)^(3/2)/c^4+2/5*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*
(e*x+d)^(5/2)/c^3/e^3-2/7*(b*e+2*c*d)*(e*x+d)^(7/2)/c^2/e^3+2/9*(e*x+d)^(9/
2)/c/e^3-2*(-a^2*c^2*e+3*a*b^2*c*e-2*a*b*c^2*d-b^4*e+b^3*c*d)*(e*x+d)^(1/2)
/c^5+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2))*2^(1/2)*((a*c*e-b^2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(2
*b^5*c*d*e-10*a*b^3*c^2*d*e+10*a^2*b*c^3*d*e-b^6*e^2+a*b^2*c^2*(-9*a*e^2+4*
c*d^2)-b^4*c*(-6*a*e^2+c*d^2)-2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))
/c^(11/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e
*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((a*c*e-b^2*e+b
*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(-2*b^5*c*d*e+10*a*b^3*c^2*d*e-10
*a^2*b*c^3*d*e+b^6*e^2-a*b^2*c^2*(-9*a*e^2+4*c*d^2)+b^4*c*(-6*a*e^2+c*d^2)+
2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(11/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 2.68, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)+ab^2c^2\left(4cd^2-9ae^2\right)-10ab^3c^2de-b^4c\left(cd^2-6ae^2\right)+2b^5cde+b^6\left(-e^2\right)}{\sqrt{b^2-4ac}}+\left(ace+b^2(-e)+bcd\right)\left(3abce-2a\cdots\right.\right.}{c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d + e*x]
)/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^(3/2))/(3*c^4) + (2*(c^2*d^2 + b^2*e^2
 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*
x)^(7/2))/(7*c^2*e^3) + (2*(d + e*x)^(9/2))/(9*c*e^3) + (Sqrt[2]*((b*c*d -
b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 1
0*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2
) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((b*c
*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*
e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*
a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{e\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)}{c^5}-\frac{b\left(b^2-2ac\right)ex^2}{c^4}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)x^4}{c^3e^2}-\frac{(2cd+be)x^6}{c^2e^2}+\right.\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2+\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2+\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2+\right.}{}$$

Mathematica [A] time = 1.12, size = 808, normalized size = 1.24

$$\frac{2\sqrt{d+ex}\,\left((d+ex)^2\left(8d^2-20exd+35e^2x^2\right)c^4-9e(d+ex)^2(-2bd+7ae+5bex)c^3+21e^2\left(15a^2e^2+10ab(4d+e\right.\right.}{315c^5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*c
^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20*d
*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b*e*
(4*d + e*x))))/(315*c^5*e^3) + (Sqrt[2]*(-(b^6*e^2) + b^5*e*(2*c*d + Sqrt[b
^2 - 4*a*c]*e) + a*b^2*c^2*(4*c*d^2 + 6*Sqrt[b^2 - 4*a*c]*d*e - 9*a*e^2) +
b^3*c*(-4*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 10*a*e)) + a
*b*c^2*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - 2*c*d*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) -
b^4*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + 2*a^2*c^3*(-(c*d^2) + e
*(-(Sqrt[b^2 - 4*a*c]*d) + a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(11/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^6*e^2 + b^5*e*(-2*c*d + S
qrt[b^2 - 4*a*c]*e) + a*b^2*c^2*(-4*c*d^2 + 6*Sqrt[b^2 - 4*a*c]*d*e + 9*a*e
^2) - 2*a^2*c^3*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^4*c*(c*d^2 -
 2*e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + a*b*c^2*(3*a*Sqrt[b^2 - 4*a*c]*e^2 -
2*c*d*(Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + b^3*c*(-4*a*Sqrt[b^2 - 4*a*c]*e^2 +
c*d*(Sqrt[b^2 - 4*a*c]*d + 10*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(11/2)*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

fricas [B] time = 33.46, size = 14340, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/630*(315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 -
 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^
4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2
*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*
a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*
e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c
^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12
)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 2
39*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e +
 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 183
5*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*
a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 29
70*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 -
700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 +
 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^
7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^
2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1
068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 +
95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c
^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^
8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^
6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(sqrt(2)*((b^12*c^4 -
12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5
*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7
*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*
c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6
- 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*
b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^
5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 1
04*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 34
```

2*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) - 315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6

+ 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^1
0 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c
^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^
6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19
*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^
5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9
- 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^
14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^
7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 -
4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(-sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5
+ 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4
- (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4
*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^1
2*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*
c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360
*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 56
1*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^
2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6
- 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*
c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*
b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*
c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c
^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 -
166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 -
70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1
570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2
- 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 +
5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10
+ 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4
- 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*
c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c
- 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5
- 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 -
5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14
*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*
b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*
a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 +
2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5
+ 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*
b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b
^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 -
4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c
^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^
5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 1
66*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 7
0*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 15
70*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2
- 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 +
5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 +
49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4
- 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c
^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c
- 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 -
1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5
*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*
c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b
^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a
*c^23)))/(b^2*c^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*
b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c

^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3
+ 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c
- 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^
4*b^11 - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14
*a^9*b*c^5)*d*e^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c
^3 + 15*a^9*b^2*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) + 315*sqrt(2)*c^5*e^3*s
qrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*
d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*
c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 2
5*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77
*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*s
qrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^
4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^
13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*
c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^
5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*
c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^1
7*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^
9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c
^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*
b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a
^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17
*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b
^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10
)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*
a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 29
5*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b
^2*c^11 - 4*a*c^12))*log(sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^
6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 -
52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*
a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2
*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^
2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 -
1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 -
68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^1
0*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^
7 + 4*a^8*c^8)*e^4 + ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^
15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt
(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b
^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*
c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^1
0 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 +
395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9
+ 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c
^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c
^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11
)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^1
2*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*
b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^
2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*
c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d
*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4
*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a
^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(
((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3
- 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)
*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^
4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3
*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(

((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^
6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c
^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10
 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 +
395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9
+ 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^
3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^
7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)
*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12
*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b
^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2
 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c
^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*
e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*
b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^
8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c
^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7
*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2
*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^
4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^
2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*
b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e
^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2
*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) - 315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 -
 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^
2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*
(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 -
 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 +
55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6
- 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80
*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2
*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b
^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^1
2*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b
^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b
^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^
5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 +
3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855
*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 12
5*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*
b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a
^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20
 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 -
2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 -
 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c
^12))*log(-sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^
6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4
 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 +
40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 42
0*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*
c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*
c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)
*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^
4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)
*e^4 + ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c
^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 -
12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a
^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b
^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3

```
*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*
c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4
*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^1
5*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*
b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*
(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a
^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*
a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^
15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6
*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 -
 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 22
32*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 3
0*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*
a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 -
 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^
10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*
a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*
a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 1
2*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^
5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^
11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*
c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c
^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*
c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15
*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b
^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(
5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^
4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a
^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^1
5*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*
b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 -
18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 223
2*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30
*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12
)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 -
(4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c
^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3
*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6
*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*
b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10
 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^
5)*e^5)*sqrt(e*x + d)) - 4*(35*c^4*e^4*x^4 + 8*c^4*d^4 + 18*b*c^3*d^3*e + 6
3*(b^2*c^2 - a*c^3)*d^2*e^2 - 420*(b^3*c - 2*a*b*c^2)*d*e^3 + 315*(b^4 - 3*
a*b^2*c + a^2*c^2)*e^4 + 5*(10*c^4*d*e^3 - 9*b*c^3*e^4)*x^3 + 3*(c^4*d^2*e^
2 - 24*b*c^3*d*e^3 + 21*(b^2*c^2 - a*c^3)*e^4)*x^2 - (4*c^4*d^3*e + 9*b*c^3
*d^2*e^2 - 126*(b^2*c^2 - a*c^3)*d*e^3 + 105*(b^3*c - 2*a*b*c^2)*e^4)*x)*sq
rt(e*x + d))/(c^5*e^3)
```

giac [B] time = 0.69, size = 1577, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2*e - 2*(b^6*c - 7*a*b^4*c^2
 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^2 + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 -
12*a^3*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 - 2
*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^4 +
a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)*sqrt
```

```
(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqrt(b^2 - 4*a*
c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(b^4*c
^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3 - (5*b^5*c^4 - 24*a*b^3*c^5 + 22*a^2*b*c^
6)*d^2*e + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3*c^6)*d*e^2
- (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^3)*sqrt(-4*c^2*d
+ 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-
(2*c^10*d*e^30 - b*c^9*e^31 + sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*e^31 + a*c^9
*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))*e^(-30)/c^10))/((sqrt(b
^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 - 4*a*c)*a*c^7
*e^2)*c^2) + 1/4*(((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2*e - 2*(b^6*c -
 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^2 + (b^7 - 8*a*b^5*c + 19*a^
2*b^3*c^2 - 12*a^3*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c
)*e)*c^2 + 2*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*
a*b^2*c^4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2
*b*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqr
t(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)
 + (2*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3 - (5*b^5*c^4 - 24*a*b^3*c^5 +
 22*a^2*b*c^6)*d^2*e + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3
*c^6)*d*e^2 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^3)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e
 + d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 - sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*
e^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))*e^(-30)/c^1
0))/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 -
4*a*c)*a*c^7*e^2)*c^2) + 2/315*(35*(x*e + d)^(9/2)*c^8*e^24 - 90*(x*e + d)^
(7/2)*c^8*d*e^24 + 63*(x*e + d)^(5/2)*c^8*d^2*e^24 - 45*(x*e + d)^(7/2)*b*c
^7*e^25 + 63*(x*e + d)^(5/2)*b*c^7*d*e^25 + 63*(x*e + d)^(5/2)*b^2*c^6*e^26
 - 63*(x*e + d)^(5/2)*a*c^7*e^26 - 105*(x*e + d)^(3/2)*b^3*c^5*e^27 + 210*(
x*e + d)^(3/2)*a*b*c^6*e^27 - 315*sqrt(x*e + d)*b^3*c^5*d*e^27 + 630*sqrt(x
*e + d)*a*b*c^6*d*e^27 + 315*sqrt(x*e + d)*b^4*c^4*e^28 - 945*sqrt(x*e + d)
*a*b^2*c^5*e^28 + 315*sqrt(x*e + d)*a^2*c^6*e^28)*e^(-27)/c^9
```

maple [B] time = 0.08, size = 3685, normalized size = 5.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/9*(e*x+d)^(9/2)/c/e^3+2*e/c^3*a^2*(e*x+d)^(1/2)+2*e/c^5*b^4*(e*x+d)^(1/2)
-4/7/e^3/c*(e*x+d)^(7/2)*d+2/5/e^3/c*(e*x+d)^(5/2)*d^2+4/3/c^3*(e*x+d)^(3/2
)*a*b-2/c^4*b^3*d*(e*x+d)^(1/2)-2/7/e^2/c^2*(e*x+d)^(7/2)*b-2/5/e/c^2*(e*x+
d)^(5/2)*a+2/5/e/c^3*(e*x+d)^(5/2)*b^2-10*e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*
2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/
2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^3*d+4*e/c
^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*c)*a*b^2*d^2+10*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b*d-10*e^2/c^3/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*
b^3*d+4*e/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a*b^2*d^2+10*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b*d+4/c^3*a*b*d
*(e*x+d)^(1/2)+2*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^3-e^3/c^5/(-(4*a*c-b^2)*e^2)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
```

)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^6+2*e^3/c^2/
(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^
(1/2)*c)*a^3-e^3/c^5/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^6-3*e^2/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b+2*e/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d+4*e^2/c^4*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*a*b^3+2*e/c^4*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^4*d+3*e^2/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d-4*e^2/c^4*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^3-2*e/c^4*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d+2/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2-2/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2+2/5/e^2/c^2*(e*x+d)^(5/2)*b*d-6*e/c^4*
a*b^2*(e*x+d)^(1/2)+1/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*c)*b^3*d^2-1/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*c)*b^3*d^2+e^2/c^5*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2)
)*c)^(1/2)*c)*b^5-e^2/c^5*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^
(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*c)*b^5-2/3/c^4*(e*x+d)^(3/2)*b^3+6*e/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2*d-6*e/c^3*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2*d-2*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d^2-9*e^3/
c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*c)*a^2*b^2-2*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d^2+6*e^3/c^4/(-(4*a*c-b^2)*e^2)^(1
/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(
1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^4+2*e^2/
c^4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*c)*b^5*d-e/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d^2+6*e^3/c^4/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^4+2*e^2
/c^4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*b^5*d-e/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c

```
*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d^2-9*e^3/c^3/(-(4*a*c-b^2)*e^
2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e
*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*
b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x^4}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)
```

mupad [B] time = 7.97, size = 31485, normalized size = 48.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (d + e*x)^(1/2)*((2*d^4)/(c*e^3) - ((a*e^5 + c*d^2*e^3 - b*d*e^4)*((12*d^2)
/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) +
(2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3) +
 ((b*e^4 - 2*c*d*e^3)*((8*d^3)/(c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*
d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c*e^3) + ((b*e^4 - 2*c*d
*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((
8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e
^3)))/(c*e^3)))/(c*e^3)) - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*
c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b
^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d
*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a
^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*
e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12
*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 5
2*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^
2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a
^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*
b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e
^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5
*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e +
 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 30
6*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2
*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*
a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3
*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*
e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4
*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 +
363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^
```

(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d
*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^
4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b
^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 22
5*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a
^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^
6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^
4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*
d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6
 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c
^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b
^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2
 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e
^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^
2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*
a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*
b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13
*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e -
52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c
^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 +
a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a
*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*
e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e
+ 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 3
06*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^
2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i - (((8*(4*a^4*c^9*e^5 - a*b^6*c^6
*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*
d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*
a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^
2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/
2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^
2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88
*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*
c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4
*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b
^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b

^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*
c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5
*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) -
 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d
e^2(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2
*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12
*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3
 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44
*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 +
 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7
*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 -
 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*
d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4
*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*
e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2
 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a
^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a
*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*
b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^
6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^
3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*
c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^
2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4
*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^
4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5
 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c
^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c
^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 8
8*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3
*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^
4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*
b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5
*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^
5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i)/((16*(a^6*b^5*e^8 - 4
*a^7*b^3*c*e^8 + 3*a^8*b*c^2*e^8 - 2*a^5*b^6*d*e^7 - 2*a^8*c^3*d*e^7 + a^4*

b^7*d^2*e^6 - 2*a^6*c^5*d^5*e^3 - 4*a^7*c^4*d^3*e^5 + a^4*b^3*c^4*d^6*e^2 - 4*a^4*b^4*c^3*d^5*e^3 + 6*a^4*b^5*c^2*d^4*e^4 + 10*a^5*b^2*c^4*d^5*e^3 - 16*a^5*b^3*c^3*d^4*e^4 + 8*a^5*b^4*c^2*d^3*e^5 + 8*a^6*b^2*c^3*d^3*e^5 - 16*a^6*b^3*c^2*d^2*e^6 + 6*a^6*b^4*c*d*e^7 - 4*a^4*b^6*c*d^3*e^5 - 2*a^5*b*c^5*d^6*e^2 + 2*a^5*b^5*c*d^2*e^6 + 3*a^6*b*c^4*d^4*e^4 + 8*a^7*b*c^3*d^2*e^6))/c^9 + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^

10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d

^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)))*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*2i - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c

^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e

- 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i - (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9

*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^
3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 -
36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8
*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3
+ 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d
^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3
*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4
*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e
^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*
b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*
c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7
*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^
4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d
*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 -
 8*a*b^2*c^12)))^(1/2)*1i)/((16*(a^6*b^5*e^8 - 4*a^7*b^3*c*e^8 + 3*a^8*b*c^
2*e^8 - 2*a^5*b^6*d*e^7 - 2*a^8*c^3*d*e^7 + a^4*b^7*d^2*e^6 - 2*a^6*c^5*d^5
*e^3 - 4*a^7*c^4*d^3*e^5 + a^4*b^3*c^4*d^6*e^2 - 4*a^4*b^4*c^3*d^5*e^3 + 6*
a^4*b^5*c^2*d^4*e^4 + 10*a^5*b^2*c^4*d^5*e^3 - 16*a^5*b^3*c^3*d^4*e^4 + 8*a
^5*b^4*c^2*d^3*e^5 + 8*a^6*b^2*c^3*d^3*e^5 - 16*a^6*b^3*c^2*d^2*e^6 + 6*a^6
*b^4*c*d*e^7 - 4*a^4*b^6*c*d^3*e^5 - 2*a^5*b*c^5*d^6*e^2 + 2*a^5*b^5*c*d^2*
e^6 + 3*a^6*b*c^4*d^4*e^4 + 8*a^7*b*c^3*d^2*e^6))/c^9 + (((8*(4*a^4*c^9*e^5
 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 +
 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*
d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*
a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(
d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2
 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*
c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 -
231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^
2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 -
414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675
*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15
*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2
*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^
3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3
- 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8
*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*
c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b
^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 -
 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*
e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*

b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*

d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2
*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^
3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30
*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c
^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13
*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e -
52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c
^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 -
a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*
e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^
5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e
+ 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 3
06*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^
2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a
^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6
 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^
4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c
*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4
*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c
^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*
e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 1
76*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a
^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*
c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66
*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c
^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2
*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b
^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b
*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3
*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^
2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c
^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
0*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*
c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2))*(-(
b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*

```
e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b
^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^
3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^
2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*
a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^
2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2
 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6
3*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a
^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*2i - ((8*d)/(7*c*e^3) + (2*(b*e^
4 - 2*c*d*e^3))/(7*c^2*e^6))*(d + e*x)^(7/2) + (d + e*x)^(5/2)*((12*d^2)/(5
*c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(5*c^2*e^6) + (((8*d)/(c*e^3) +
 (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(5*c*e^3)) - (d +
e*x)^(3/2)*((8*d^3)/(3*c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(
c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c*e^3) + ((b*e^4 - 2*c*d*e^3)*(
(12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c
*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(
3*c*e^3)) + (2*(d + e*x)^(9/2))/(9*c*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.534 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=581

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-b^3c\left(cd^2-5ae^2\right)-8ab^2c^2de+abc^2\left(3cd^2-5ae^2\right)+b^5\left(-e^2\right)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+b\cdots\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] 2/3*(-a*c+b^2)*(e*x+d)^(3/2)/c^3-2/5*(b*e+c*d)*(e*x+d)^(5/2)/c^2/e^2+2/7*(e
*x+d)^(7/2)/c/e^2+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(e*x+d)^(1/2)/c^4+arc
tanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a
*e^2+c*d^2)+(-2*b^4*c*d*e+8*a*b^2*c^2*d*e-4*a^2*c^3*d*e+b^5*e^2+b^3*c*(-5*a
*e^2+c*d^2)-a*b*c^2*(-5*a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e
^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a*e^2+c*d^2)+(2*b^4*c*d*e-8*a*b^2*c^2*d*e
+4*a^2*c^3*d*e-b^5*e^2-b^3*c*(-5*a*e^2+c*d^2)+a*b*c^2*(-5*a*e^2+3*c*d^2))/(
-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 15.25, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-8ab^2c^2de-b^3c\left(cd^2-5ae^2\right)+abc^2\left(3cd^2-5ae^2\right)+2b^4cde+b^5\left(-e^2\right)}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+2\cdots\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + (2*(b^2 - a
*c)*(d + e*x)^(3/2))/(3*c^3) - (2*(c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e^2)
+ (2*(d + e*x)^(7/2))/(7*c*e^2) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b
^4*e^2 - b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8
*a*b^2*c^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^
2*(3*c*d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
 b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{e\left(b^2cd-ac^2d-b^3e+2abce\right)}{c^4}+\frac{\left(b^2-ac\right)ex^2}{c^3}-\frac{(cd+be)x^4}{c^2e}+\frac{x^6}{ce}-\frac{\left(b^2cd-ac^2d-b^3e+2abce\right)\left(cd^2-bde+ae^2\right)}{c^4e\left(\frac{cd^2}{\ldots}\right.}\right.\right.}{e}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\ldots$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\ldots$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\ldots$$

Mathematica [A] time = 0.90, size = 680, normalized size = 1.17

$$-\frac{2\sqrt{d+ex}\left(7c^2e\left(5ae(4d+ex)+3b(d+ex)^2\right)-35bce^2(6ae+4bd+bex)+105b^3e^3+3c^3(2d-5ex)(d+ex)^2\right)}{105c^4e^2}-\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*e
^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)))
)/(105*c^4*e^2) - (Sqrt[2]*(-(b^5*e^2) + b^4*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e
) + b^2*c*(-3*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e))
- b^3*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 5*a*e)) + a*b*c^2*(3*c*d^2 + e*
(4*Sqrt[b^2 - 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(-(
Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqr
t[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^5*e^2 + b^4*e*(-2*c*d + Sqrt
[b^2 - 4*a*c]*e) + a*c^2*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*
d + 4*a*e)) + b^3*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + a*b*c^2*(
-3*c*d^2 + e*(4*Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + b^2*c*(-3*a*Sqrt[b^2 - 4*a*
c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

fricas [B] time = 12.12, size = 11459, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/210*(105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 -
2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d
^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5
)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)
*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8
- 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2
*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^1
2*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8
 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 +
 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39
*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 64
0*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*
c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^
4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5
 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c
^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^
2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^9*c^4 - 9*a*b^7*
c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 4
0*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*
c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^
5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 2
16*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^
6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181
*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^10 - 7*a*b^3*c^1
1 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^
12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*
a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5
*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5
 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3
*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 91
0*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 +
 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a
^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^1
5*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 -
 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*
```

c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))* sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)* sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^

16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 3
14*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18
 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^
3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8
*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^
9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c
^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*
c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40
*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*
b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2
*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*
c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d
^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5
 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4
- 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b
^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14
*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*
b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*
c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c
^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4
e + 2(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*
e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^
7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 -
11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*
c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) + 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6
*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14
*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3
 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 -
8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^
11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3
*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*
a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(1
0*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b
^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^1
2*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*
c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*
a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6
*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230
*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^
7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log
(sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4
*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^
6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88
*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 -
(4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4
*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^
9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e
^4 + ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10
 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*
b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6
+ 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e +
3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4
*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^
11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*
c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*
c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8
- 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^
3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^

8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 + ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c

```
^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b
^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 +
9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4
- 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2
*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^
3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 +
 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*
b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*
d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 +
 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 -
110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*
a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a
^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*
b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
- 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*
a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a
^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^
4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21
*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^
5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5
*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^
5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c +
 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 4*(15*c^3
*e^3*x^3 - 6*c^3*d^3 - 21*b*c^2*d^2*e + 140*(b^2*c - a*c^2)*d*e^2 - 105*(b^
3 - 2*a*b*c)*e^3 + 3*(8*c^3*d*e^2 - 7*b*c^2*e^3)*x^2 + (3*c^3*d^2*e - 42*b*
c^2*d*e^2 + 35*(b^2*c - a*c^2)*e^3)*x)*sqrt(e*x + d))/(c^4*e^2)
```

giac [B] time = 0.58, size = 1362, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e - 2*(b^5*c - 6*a*b^3*c^2 +
8*a^2*b*c^3)*d*e^2 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 - 2*((b^2*c^4 - a*c^5)*s
qrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b
^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*
b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(c) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3 - (5*b^4*c^4 - 19*a*b^2*c^5 + 8*a
^2*c^6)*d^2*e + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^2 - (b^6*c^2
- 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^16 - b*c^7*e^1
7 + sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16 + (2*c^8*d*
e^16 - b*c^7*e^17)^2))*e^(-16)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2
 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) - 1/4*(((b^4*c^2 -
5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^
2 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*((b^2*c^4 - a*c^5)*sqrt(b^2 - 4*a*c)*
d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b^4*c^2 - a*b^2*c^
3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 -
 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(
b^3*c^5 - 3*a*b*c^6)*d^3 - (5*b^4*c^4 - 19*a*b^2*c^5 + 8*a^2*c^6)*d^2*e + 2
*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^2 - (b^6*c^2 - 5*a*b^4*c^3 + 5
*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan
(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^16 - b*c^7*e^17 - sqrt(-4*(c^8*
d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16 + (2*c^8*d*e^16 - b*c^7*e^17
```

```
)^2))*e^(-16)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d
*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 2/105*(15*(x*e + d)^(7/2)*c^6*e^12
 - 21*(x*e + d)^(5/2)*c^6*d*e^12 - 21*(x*e + d)^(5/2)*b*c^5*e^13 + 35*(x*e
+ d)^(3/2)*b^2*c^4*e^14 - 35*(x*e + d)^(3/2)*a*c^5*e^14 + 105*sqrt(x*e + d)
*b^2*c^4*d*e^14 - 105*sqrt(x*e + d)*a*c^5*d*e^14 - 105*sqrt(x*e + d)*b^3*c^
3*e^15 + 210*sqrt(x*e + d)*a*b*c^4*e^15)*e^(-14)/c^7
```

maple [B] time = 0.06, size = 2988, normalized size = 5.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/7*(e*x+d)^(7/2)/c/e^2-2/5/e^2/c*(e*x+d)^(5/2)*d-2/c^2*a*d*(e*x+d)^(1/2)+2
/c^3*b^2*d*(e*x+d)^(1/2)-2/5/e/c^2*(e*x+d)^(5/2)*b-2*e/c^4*b^3*(e*x+d)^(1/2
)+8*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*a*b^2*d-3*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2
*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2+8*e^2/c^2/(-(4*a*c-b^2)
*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh
((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a
*b^2*d-3*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2+e^3/c^4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^5+3*e^2/c^3*2^(1/2)/((-
b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2+2*e/c^3*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d-3*e^2/c^3*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2-2*e/c^3*2^(1/2)/(
(b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d+e^3/c^4/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arcta
nh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
*b^5+4*e/c^3*a*b*(e*x+d)^(1/2)+5*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b-4*e^2/c/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^
2*d-5*e^3/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a*b^3-2*e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b
*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d+e/c^2/(-(4*a*c-b^2)*e
^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*
x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^
2+5*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*a^2*b-4*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*a*b*d+4*e/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a*b*d-4*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d-5*e^3/c^3/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
```

```
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a*b^3-2*e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d+e/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^2-2/3/c^2*(e*x+
d)^(3/2)*a+2/3/c^3*(e*x+d)^(3/2)*b^2+1/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*a*d^2-1/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2-e^2/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2-e^2/c^4*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b^4+e^2/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a^2+e^2/c^4*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^4-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*c)*a*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

mupad [B] time = 7.14, size = 25497, normalized size = 43.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e
^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^
2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12
*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11
*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*
a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*
a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3
*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^
4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*
b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c
^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*
b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^
```

2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2
))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c
^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*
d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 3
6*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 +
 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^
3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 8
4*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126
*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^
3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^1
1 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*
c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*
a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3
+ 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^
2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e
^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b
^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^
6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*
c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^
5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^
2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3
+ a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^
6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c
^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*
d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^
9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*
b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c
^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 +
11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (
8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^
2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*
c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c
- b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3
*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^
3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
3*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^
2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*
c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5
*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*
c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^

(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((16*(a^5*b^4*e^8 + a^7*c^2*e^8 - 3*a^6*b^2*c*e^8 - 2*a^4*b^5*d*e^7 + a^3*b^6*d^2*e^6 - a^4*c^5*d^6*e^2 - a^5*c^4*d^4*e^4 + a^6*c^3*d^2*e^6 + a^3*b^2*c^4*d^6*e^2 - 4*a^3*b^3*c^3*d^5*e^3 + 6*a^3*b^4*c^2*d^4*e^4 - 10*a^4*b^2*c^3*d^4*e^4 + 4*a^4*b^3*c^2*d^3*e^5 - 12*a^5*b^2*c^2*d^2*e^6 + 4*a^5*b^3*c*d*e^7 + 2*a^6*b*c^2*d*e^7 - 4*a^3*b^5*c*d^3*e^5 + 6*a^4*b*c^4*d^5*e^3 + 3*a^4*b^4*c*d^2*e^6 + 8*a^5*b*c^3*d^3*e^5))/c^7 + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4* a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e

- 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*
e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 -
198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*
a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a
*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c
^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2
+ 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c
^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c -
b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*
b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3
e^3(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33
*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2
*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c
^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*
d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2
) + (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*
a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e
^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e
^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a
^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b
*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*
e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5)
)/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c
^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 1
38*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(
1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e
^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3
*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 -
189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*
a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*
a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^1
1*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10
*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33
*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^
3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b
^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a
*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b
^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*
c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e

*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a
*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - ((6*d)/(5*c*e^2)
 + (2*(b*e^3 - 2*c*d*e^2))/(5*c^2*e^4))*(d + e*x)^(5/2) + atan(((((8*(4*a^3
*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^
6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c
^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^
3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*
d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 3
6*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 +
 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^
3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 8
4*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126
*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^
3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^
3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^1
1 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b
*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3
 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 +
 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d
^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13
*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e
 + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*
e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 -
 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*
a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a
*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*
b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 +
 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e
^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^
5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2
*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 -
48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a
^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^
6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^
2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(
4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e
^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*
b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^

2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b
^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*
a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^
3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)
))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6
*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^
5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*
e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)
*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2
*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3
*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 1
5*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e
 + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a
^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^
5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^
2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 -
 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*
a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4
*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c
^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 12
9*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7
*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d
*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^
3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e
+ 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16
*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^6
- 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*
e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*
d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^
6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c
^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2
+ 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*
a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^
3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*
a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 3
8*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*
c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*
a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a

^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*
b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*
b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((16*(a^5*b^4*e^8 + a^7*c^2*e^8 - 3*a
^6*b^2*c*e^8 - 2*a^4*b^5*d*e^7 + a^3*b^6*d^2*e^6 - a^4*c^5*d^6*e^2 - a^5*c^
4*d^4*e^4 + a^6*c^3*d^2*e^6 + a^3*b^2*c^4*d^6*e^2 - 4*a^3*b^3*c^3*d^5*e^3 +
 6*a^3*b^4*c^2*d^4*e^4 - 10*a^4*b^2*c^3*d^4*e^4 + 4*a^4*b^3*c^2*d^3*e^5 - 1
2*a^5*b^2*c^2*d^2*e^6 + 4*a^5*b^3*c*d*e^7 + 2*a^6*b*c^2*d*e^7 - 4*a^3*b^5*c
*d^3*e^5 + 6*a^4*b*c^4*d^5*e^3 + 3*a^4*b^4*c*d^2*e^6 + 8*a^5*b*c^3*d^3*e^5)
)/c^7 + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d
*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*
d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 -
12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^
11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
0*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 3
3*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c
^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*
b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*
a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3
*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*
a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2
*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^
7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 -
 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3
 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*
b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 +
 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 1
26*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*
a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*
a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^
5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 -
2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^
3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*
e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2
*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a
*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*
c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^
8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*
c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*
b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^

3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*
c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5
*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^
4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*
c^9 - 8*a*b^2*c^10)))^(1/2) + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b
^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^
7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 1
1*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8
*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2
 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c
^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*
b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3
e^3(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33
*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2
*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c
^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*
d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2
)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*
(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2
) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*
e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*
b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15
*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e
+ 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^
3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2
*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^
3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^
(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 +
 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*
e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^
5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^
4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*
b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5
*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^
3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2
*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^

```
3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8
*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*
d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2))*(-(b^11*e^3 - 8*a^4*c^7
*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 -
36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3
+ 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b
^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 +
84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 12
6*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a
^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a
^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i + (d + e*x)^(3/2)*((2*d^2)/(c*e^2)
- (2*(a*e^4 + c*d^2*e^2 - b*d*e^3))/(3*c^2*e^4) + (((6*d)/(c*e^2) + (2*(b*e
^3 - 2*c*d*e^2))/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(3*c*e^2)) - (d + e*x)^(1/
2)*((2*d^3)/(c*e^2) - (((6*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(c^2*e^4))*
(a*e^4 + c*d^2*e^2 - b*d*e^3))/(c*e^2) + ((b*e^3 - 2*c*d*e^2)*((6*d^2)/(c*e
^2) - (2*(a*e^4 + c*d^2*e^2 - b*d*e^3))/(c^2*e^4) + (((6*d)/(c*e^2) + (2*(b
*e^3 - 2*c*d*e^2))/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(c*e^2)))/(c*e^2)) + (2*
(d + e*x)^(7/2))/(7*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.535 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=441

$$-\frac{2\sqrt{d+ex}\left(ace+b^2(-e)+bcd\right)}{c^3}+\frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)+\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+b^4\left(-e^2\right.}{\sqrt{b^2-4ac}}\right.}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -2/3*b*(e*x+d)^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2*(a*c*e-b^2*e+b*c*d)*(e*x+d
)^(1/2)/c^3+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2))*2^(1/2)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b^3*c*d*e-6*a*b
*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2
)^(1/2))/c^(7/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(
1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((-b*e+c
*d)*(2*a*c*e-b^2*e+b*c*d)+(-2*b^3*c*d*e+6*a*b*c^2*d*e+b^4*e^2+b^2*c*(-4*a*e
^2+c*d^2)-2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 2.15, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)}{\sqrt{b^2-4ac}}+(cd-be)\left(2ace+b^2(-e)+bcd\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c
^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e +
2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2)
 + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c
*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{e\left(bcd-b^2e+ace\right)}{c^3}-\frac{bex^2}{c^2}+\frac{x^4}{c}+\frac{\left(bcd-b^2e+ace\right)\left(cd^2-bde+ae^2\right)-(cd-be)\left(bcd-b^2e+2ace\right)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right) dx, x, \ldots\right.}{e}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{2\,\text{Subst}\left(\int \frac{\left(bcd-b^2e+ace\right)\left(cd^2-\ldots\right.}{\frac{cd^2-bde\ldots}{e^2}}\right.}{\ldots}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}-\frac{\left((cd-be)\left(bcd-b^2e+2ace\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+\ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 0.67, size = 538, normalized size = 1.22

$$\frac{2\sqrt{d+ex}\left(-5ce(3ae+4bd+bex)+15b^2e^2+3c^2(d+ex)^2\right)}{15c^3e}+\frac{\sqrt{2}\left(2ac^2\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)-b^2c\left(2e\left(d\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d + 3*a*e + b
*e*x)))/(15*c^3*e) + (Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c
```

```
]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6*a*e))
 - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^2 + e*(
Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sqrt[b^
2 - 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^2*c*
(c*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e
^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

fricas [B] time = 5.09, size = 8530, normalized size = 19.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(
b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2
*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)
*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5
*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*
e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29
*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*
a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2
 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 +
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*l
og(sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a
*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c
^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a
*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5
)*e^4 - ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 +
12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7
*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30
*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b
^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d
^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7
*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^
2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a
*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7
+ 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c
^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 +
 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*
a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^
2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11
*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5
*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c
^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a
^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b
*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d
^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3
```

*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) - 15*sqrt
(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*
b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2
*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*
c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*
a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^
9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^
3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*
c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53
*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b
^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 10
7*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 - ((b^4
*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)
*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 +
55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a
*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5
*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c
^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3
*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 -
3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*
a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*
b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c
^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3
*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d
^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 +
29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 1
20*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*
c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5
+ (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)
) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 +
3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2
- 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2
*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*
b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e*sqr
t(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2
- (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^
8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5
*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 5
5*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*
b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*
b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^
6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*
b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6
)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^6*c^4 - 6*a*
b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 -
20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^
3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3

- 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 + ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 + ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5

```
+ 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70
*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*
(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2
*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37
*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*
e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3
*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2
*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b
^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*
b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e^5)*sqrt(e*x + d)) - 4*(3*c^2*e^2*x^2 + 3*c^2*d^2 - 20*b*c*d*e + 15*
(b^2 - a*c)*e^2 + (6*c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e)
```

giac [B] time = 0.53, size = 1160, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^3*c^2 - 4*a*b*c^3)*d^2*e - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*
e^2 + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^
2 - 4*a*c)*c)*e)*c^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b
^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a
^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c
)*e)*abs(c) + (2*(b^2*c^5 - 2*a*c^6)*d^3 - (5*b^3*c^4 - 14*a*b*c^5)*d^2*e +
 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^2 - (b^5*c^2 - 4*a*b^3*c^3 + 2
*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2
*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7 + sqrt(-4*(c^6*d^2*
e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6 - b*c^5*e^7)^2))*e^(-
6)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b
^2 - 4*a*c)*a*c^5*e^2)*c^2) + 1/4*(((b^3*c^2 - 4*a*b*c^3)*d^2*e - 2*(b^4*c
- 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^2 + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^3)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*(sqrt(b^2 - 4*a*c)*b*
c^4*d^3 + sqrt(b^2 - 4*a*c)*b^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 -
4*a*c)*d^2*e - (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(b^2*c^5 - 2*a*c^6)*d^3 - (5*
b^3*c^4 - 14*a*b*c^5)*d^2*e + 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^2
 - (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^6 -
b*c^5*e^7 - sqrt(-4*(c^6*d^2*e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^
6*d*e^6 - b*c^5*e^7)^2))*e^(-6)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^
2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2) + 2/15*(3*(x*e + d
)^(5/2)*c^4*e^4 - 5*(x*e + d)^(3/2)*b*c^3*e^5 - 15*sqrt(x*e + d)*b*c^3*d*e^
5 + 15*sqrt(x*e + d)*b^2*c^2*e^6 - 15*sqrt(x*e + d)*a*c^3*e^6)*e^(-5)/c^5
```

maple [B] time = 0.06, size = 2358, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] -e^2/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x
+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3-1/c
*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2+1/c*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^
```

```
(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2+e^2/c^3*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3-2/3*b*(e*x+d)
^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2*e/c^2*a*(e*x+d)^(1/2)+2*e/c^3*b^2*(e*x+d
)^(1/2)-2/c^2*b*d*(e*x+d)^(1/2)-6*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d-6*e^2/c/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*
a*b*d-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^2*d^2+4*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2+2*e^2/c^2/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*b^3*d-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2+4*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2-2*e^3/c/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a^2-e^3/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b^4-2*e^2/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*a*b+2*e/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*a*d-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*b^2*d-2*e^3/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*
e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2-e^3/c^3/(-(4*a*c-b^2)*e^2
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4+2*e^
2/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b-2*e/c*
2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)
*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*d+2*e/c^2*2^(1
/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(
1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d+2*e/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a*d^2+2*e/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*a*d^2+2*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*
e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

mupad [B] time = 5.72, size = 19465, normalized size = 44.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e
^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^
3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*
(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e
- 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^
3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^
7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2
 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3
*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^
3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*
b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 -
 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b
^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^
3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
2*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x
)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6
+ 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*
e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^
2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*
a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^
3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2
 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c
^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^
5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 9
9*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*
a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5
*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3
+ 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d
 + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*
b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^
3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(
```

1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2
*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2
*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/
c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d
^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^
5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*
d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) -
3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)
+ (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^
3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 -
4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*
a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5
*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e
^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 -
b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24
*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3
- 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b
^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2
- 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3
*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*
b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5
*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^
4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3
))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*
e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*
c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*
a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60
*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*
d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c
^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7
- 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*
a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2
+ 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c
^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b

^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^

3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^8 - 2*a^3*b^4*d*e^7 + 2*a^5*c^2*d*e^7 + a^2*b^5*d^2*e^6 + 2*a^3*c^4*d^5*e^3 + 4*a^4*c^3*d^3*e^5 - 2*a^5*b*c*e^8 - 4*a^2*b^2*c^3*d^5*e^3 + 6*a^2*b^3*c^2*d^4*e^4 + 2*a^4*b^2*c*d*e^7 + a^2*b*c^4*d^6*e^2 - 4*a^2*b^4*c*d^3*e^5 - 4*a^3*b*c^3*d^4*e^4 + 4*a^3*b^3*c*d^2*e^6 - 7*a^4*b*c^2*d^2*e^6))/c^5))*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + atan((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c

^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d
e^5))/c^5)(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^
7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3
 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e
 - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b
^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c
^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))
)^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^
2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b
*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)
^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2
*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3
*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*
b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^
3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*
e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/
2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-
(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e -
18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*
(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*
c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*
c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 +
 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*
a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(
d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c
^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c
^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2
*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^
2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56
*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^
5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^
3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a
^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^
2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 -
 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*

e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d
^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5
- (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e
^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3
*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*
c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e -
 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*
b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d
e^2))/c^5)(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^
7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3
- a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e
 - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b
^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c
^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6
- 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4
*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^
5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*
b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^
5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^
3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e
^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c
^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2
*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^
2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*
a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^
4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 -
 b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 +
2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2
e^3))/c^5 + (8(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 +
b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*
a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 -
 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^
4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2
- 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*
c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b
^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4

```
*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3
+ 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d
*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b
^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*
b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e
 - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b
^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 +
b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4
*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d
*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 -
20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d
^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*
a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 4
2*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2
+ 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e +
30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2
 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^8 - 2*a^3*b
^4*d*e^7 + 2*a^5*c^2*d*e^7 + a^2*b^5*d^2*e^6 + 2*a^3*c^4*d^5*e^3 + 4*a^4*c^
3*d^3*e^5 - 2*a^5*b*c*e^8 - 4*a^2*b^2*c^3*d^5*e^3 + 6*a^2*b^3*c^2*d^4*e^4 +
 2*a^4*b^2*c*d*e^7 + a^2*b*c^4*d^6*e^2 - 4*a^2*b^4*c*d^3*e^5 - 4*a^3*b*c^3*
d^4*e^4 + 4*a^3*b^3*c*d^2*e^6 - 7*a^4*b*c^2*d^2*e^6))/c^5))*(-(b^9*e^3 + 8*
a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*
d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^
5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8
*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3
*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*
c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^
3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)
^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + (d + e*x)^(1/2
)*((2*d^2)/(c*e) - (2*(a*e^3 - b*d*e^2 + c*d^2*e))/(c^2*e^2) + (((4*d)/(c*e
) + (2*(b*e^2 - 2*c*d*e))/(c^2*e^2))*(b*e^2 - 2*c*d*e))/(c*e)) - ((4*d)/(3*
c*e) + (2*(b*e^2 - 2*c*d*e))/(3*c^2*e^2))*(d + e*x)^(3/2) + (2*(d + e*x)^(5
/2))/(5*c*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.536 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=453

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] 2/3*(e*x+d)^(3/2)/c+2*(-b*e+c*d)*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^2-b^2*
e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-
4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/2)/(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^
2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e
+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/
2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 4.53, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {824, 826, 1166, 208}

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)\right.}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c) + (Sqrt[2]*(b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - (Sqrt[2]*(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NegQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}\,(-ae+(cd-be)x)}{a+bx+cx^2}\,dx}{c}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{-ae(2cd-be)+\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{2\,\mathrm{Subst}\left(\int \frac{-ae^2(2cd-be)-d\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)+\left(c^2d^2+b^2e^2-ce\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\right.}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2-c\right.\right.}{}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2\right.\right.}{c^5}$$

Mathematica [A] time = 1.49, size = 779, normalized size = 1.72

$$2\left(\frac{3\sqrt{c}\,d\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{3\sqrt{c}\,d\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2\right.}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(-3*c*d*e*Sqrt[d + e*x] - 3*e*(-2*c*d + b*e)*Sqrt[d + e*x] + c*e*(d + e*
x)^(3/2) + (3*Sqrt[c]*d*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*
(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (3*(2*c^3*d^3 + b^2*(-b + Sqrt[b^2 -
4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b
```

```
^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 - (3*Sqrt[c]*d*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + S
qrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]) + (3*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^
3 - 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt
[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/(3*c^2*e)
```

fricas [B] time = 2.31, size = 5572, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)
*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^
6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3
 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*
a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^3
*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(
2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2
+ 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4
*a^3*b*c^3)*e^4 - ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2
*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 1
0*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*
c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^
2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*
a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3
*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^
3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^
4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c
^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^
6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2
*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^
4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2
*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*sqrt(((b^
2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*
a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*
a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^
5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*
e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b
^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 1
2*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4
)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 - ((b^3*c^6
- 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 -
 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4
```

```
*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2
 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2
*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^
2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sq
rt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5
 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^
3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(
b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(
b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*
(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*e^5)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^
3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5
 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6
*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e
^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 -
20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2
 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
*log(sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^
2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6
*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c +
13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6
*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e
+ 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b
^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c
^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e
^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*
a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3
*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5
- a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10
*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c
^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*
c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d
^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b
*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 3*sqr
t(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*
(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e
^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3
*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4
 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^
4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a
*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2
 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b
^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3
)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*s
qrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^
5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e
^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*
```

```
sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c -
4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2
*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*
c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19
*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*
c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a
^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^
2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a
^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 4*(c*e*x + 4*c*d - 3*b*e)*sqrt(e*
x + d))/c^2
```

giac [B] time = 0.46, size = 978, normalized size = 2.16

$$\left(\left(\left(b^2c^2-4\,ac^3\right)d^2e-2\left(b^3c-4\,abc^2\right)de^2+\left(b^4-5\,ab^2c+4\,a^2c^2\right)e^3\right)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,c^2-2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^
2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b
^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^5*d^3 - (5*b^2*c
^4 - 8*a*c^5)*d^2*e + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 - (b^4*c^2 - 3*a*b^2*
c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1
/2)*sqrt(x*e + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2 - 4*a*c)*c^5*d^2
 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2) - 1/4*((
(b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c
+ 4*a^2*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*
(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4
*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^5*d^3 - (5*b^2*c^4 - 8
*a*c^5)*d^2*e + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 - (b^4*c^2 - 3*a*b^2*c^3)*e
^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sq
rt(x*e + d)/sqrt(-(2*c^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3
*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqr
t(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2) + 2/3*((x*e +
d)^(3/2)*c^2 + 3*sqrt(x*e + d)*c^2*d - 3*sqrt(x*e + d)*b*c*e)/c^3
```

maple [B] time = 0.05, size = 1714, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c-2/c^2*b*e*(e*x+d)^(1/2)+2/c*d*(e*x+d)^(1/2)-3/c/(-(4*a*
c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*a*b*e^3+4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
```

```
2)*e^2)^(1/2))*c)^(1/2)*c)*a*d*e^2+1/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*e^3-2/c/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*b^2*d*e^2+1/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2*e+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*a*e^2-1/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^2*e^2+2/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*b*d*e-2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*d^2-3/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*e^3+4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*d*e^2+1/c^2/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*
b^3*e^3-2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*b^2*d*e^2+1/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c
*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*
c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2*e-1/c*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^2+1/c^2*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^2-2/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d*e+2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*c)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.72, size = 13841, normalized size = 30.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c) - ((2*d)/c + (2*(b*e - 2*c*d))/c^2)*(d + e*x)^(1/
2) - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*
c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5
*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*
c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3
- 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2
```

+ 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54
*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d
e^2(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(
-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4
*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 -
 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c
^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b
^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b
*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e
^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 3
6*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5
 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*
b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x
)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3
)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2
e^3(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*
a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 +
b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e
^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3
*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b
^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*
e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*
e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e
^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3
*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d
^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3
- b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e +
 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3
- 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2
*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2
+ 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2

)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3 + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 +

12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b

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^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 -
8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c
^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 -
 a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a
*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e
^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^
3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*
c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5
*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 2
4*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^
3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*
d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*
b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
)/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^8 - a^3
*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^
4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d
^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3 + (((8*(a*b^3*c^3*
e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 -
 b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^
4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*
b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*
c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*
a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^
6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 -
 b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*
b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c
^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*
b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b
^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*
c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4
*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*
(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*
e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*
d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2
*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^
2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^
3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^
2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) -
54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^
```

```
(1/2) + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5
*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^
2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5
*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 2
0*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3
*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^
2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b
^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6
*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24
*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^
2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*
d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*
c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^
4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4)
)/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a
^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 -
b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a
^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b
^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e +
24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b
^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.537 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\right.\right.}{}$$

```
[Out] 2*e*(e*x+d)^(1/2)/c-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*
e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))
-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {703, 826, 1166, 208}

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\,\text{Subst}\left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{}{-\frac{1}{2}}\right.}{c\sqrt{b^2-4ac}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.70, size = 317, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}\right.}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x] + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*
c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

fricas [B] time = 1.11, size = 2770, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2
 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3
*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (
b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*
a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3
)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3
+ 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt(2)*c*sqrt((2*
c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 +
 (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2
- 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*(3*(b^2*c^2
- 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4
*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c -
a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqr
t((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*
e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6
*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b
^2 - a^2*c)*e^5)*sqrt(e*x + d)) + sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e
 + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sq
rt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*
(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(
b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4
- 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d
^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4
)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^
2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*
x + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3
 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c
^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 -
 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d
*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 -
(b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - 4*sqrt(e*x +
d)*e)/c
```

giac [B] time = 0.42, size = 783, normalized size = 2.43

$$\frac{2\sqrt{xe+d}\,e}{c}+\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(2\left(b^2c-4\,ac^2\right)de^2-\left(b^3-4\,abc\right)e^3\right)c^2-2\left(\sqrt{b^2-4\,ac}\,c^3d^2e-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*e/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)
*(2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^2 - 2*(sqrt(b^2 - 4*a*
c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)
*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) - (4*c^5*d^3 - 6*b
*c^4*d^2*e + 4*(b^2*c^3 - a*c^4)*d*e^2 - (b^3*c^2 - 2*a*b*c^3)*e^3)*sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)
/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^
2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^
3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^
2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b
^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs
(c) - (4*c^5*d^3 - 6*b*c^4*d^2*e + 4*(b^2*c^3 - a*c^4)*d*e^2 - (b^3*c^2 - 2
*a*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*s
qrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e
+ a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 -
sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2)
```

maple [B] time = 0.04, size = 1138, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c+2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^3-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^3+2/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)*b*d*e^2-2*e*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b*e^2-2*e*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*d+2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^3-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^3+2/(-(4*a*c-b^2
)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan(
(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d
*e^2-2*e*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
```

```
^(1/2))*c)^(1/2)*c)*d^2-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*c)*b*e^2+2*e*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/
2)*c)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.44, size = 8334, normalized size = 25.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*e*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c
^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d
+ e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*
b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*
c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^
2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*1i - (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*
e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^
2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 -
 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a
^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*
d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a
^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2
*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
```

b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d^2*e^6))/c))*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3

- 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a
*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(
2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^
6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*
b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*
b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4
*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^
2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 -
2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a
^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*
d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*
a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 +
b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3
*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3
- 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c
^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b
^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 -
2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a
^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*
d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^5 - a*b^2*c^
2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4
))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d
e^2))/c)(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
- (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d
^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^
5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 +
b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*
(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*
e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a
*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*

```
e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 1
2*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4
*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +
 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*
(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(2*
c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*b*c^2*d^4*e^4 +
 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d^2*e^6))/c))*(-
(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b
^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a
*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.538 $\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$

Optimal. Leaf size=340

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4\right.\right.}{}$$

```
[Out] -2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a-arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(-b*(a*e^2+c*d^2)+a*
e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*(a*e^2+c*d
^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-
4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.58, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {897, 1287, 206, 1166, 208}

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2
]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 +
 a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{d^2e}{a\left(d-x^2\right)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d \cdots}\right.}{a}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\text{S}\cdots}{a\sqrt{b^2-4a\cdots}}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\cdots\right.\right.}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-\cdots}\right.}}$$

Mathematica [A] time = 1.16, size = 331, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}+4ae\right)-ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\cdots}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\cdots}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*(-(a*Sqrt[b^2 - 4*a*c
]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sq
```

```
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]
*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) - b*(c*d^2
 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]))/a
```

fricas [B] time = 15.30, size = 5167, normalized size = 15.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c
- 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*
e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4
*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*
b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((
a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e
^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*
c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c
*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) +
4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 -
(b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e -
6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)
*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^
3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a
^3*c^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^
3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2
*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2
+ 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))
)*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3
+ (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d
^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^
5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*
c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d))
 + sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e +
9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2
*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^
2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*
b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6
*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 +
a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 +
 a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*
d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2
*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b
*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2
*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^
2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqr
t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 -
6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c
^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e
+ (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c -
 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*
a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sq
rt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a
```

^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e
^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^
3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^
2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - 2
*d^(3/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, -1/2*(sqrt(2)*a*s
qrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*
e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c
^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*
b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*
b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e
 + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*
b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (
b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^
2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6
)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a
*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*
d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^
2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt
(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4
*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)
*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^
3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d
^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a
^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c
*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*
c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*
d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) + sqrt(2)*a*sqrt(
-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*
b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2
 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))
)/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*
c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^
2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9
*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*
c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*
c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^
5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a
^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c
*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*
e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*
e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6
*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 +
a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt(2)*
((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3
*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqr
t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 -
6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e
 - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c
^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3
*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c -
4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^
4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - 4*sqrt(-d)*d*arctan
(sqrt(e*x + d)*sqrt(-d)/d))/a]

giac [B] time = 0.41, size = 822, normalized size = 2.42

$$\frac{2\,d^2\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}-\frac{\left(\left(\left(b^2c-4\,ac^2\right)d^2e-\left(ab^2-4\,a^2c\right)e^3\right)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,a^2-2\left(\sqrt{b^2-4\,ac}\,ac\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(sqrt(x*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(((b^2*c - 4*a*c^2)
*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
+ sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 + 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2
*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2 -
a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*
a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*
abs(a)*abs(c)) + 1/4*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqr
t(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*a*
c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 +
6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt
(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d -
 a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2
*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))
```

maple [B] time = 0.04, size = 944, normalized size = 2.78

$$\frac{\sqrt{2}\,bc\,d^2e\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\;a}+\frac{\sqrt{2}\,bc\,d^2e\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\;a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x)
```

```
[Out] -2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a+e^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b-4*e^2*c/(-(4*
a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1
/2)*c)*d+e/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2-e^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)+1/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*d^2+e^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b-4*e^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+e/a*c/(-(4*a*c-b^2)*
```

```
e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e
*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2
+e^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^
(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-1/a*c*2^(1/
2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1
/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x), x)
```

mupad [B] time = 8.16, size = 20897, normalized size = 61.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c
*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
 + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(
512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5
*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4
*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^
8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9
 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11
 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3
*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8
 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2
*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*
b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*
b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d
^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b
^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*
a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5
*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14
 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^1
1 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a
^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^
4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^
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6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 +
 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*
b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e
^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*
c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d
*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e -
 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*
e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*(
(b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^
2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^
10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 38
4*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2
*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*
a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*
x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576
*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d
^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c
*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3
*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*
e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4
*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9
- 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*
a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a
^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d +
e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4
*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14
+ 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*
b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*
b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^
2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^
3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e
^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3
*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^
2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*
b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^
3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
+ 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((
d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^
```

2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^4*

d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e^1
2 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 - 16
0*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 + 12
8*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 2
56*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4
*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11
- 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*
a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8
*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*
c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^
2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2
*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 192*c^4*d^8
*e^10 + 448*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 512*b*c^3*d^7*e^11 - 128*b
^3*c*d^5*e^13 + 320*a^2*c^2*d^4*e^14 + 448*b^2*c^2*d^6*e^12 - 768*a*b*c^2*d
^5*e^13 + 320*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15))*((b^4*c*d^3 - a^2*b
^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3
 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e +
 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i
+ atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c
- 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b
*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^
4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a
^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2
*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)
*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c
^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c
^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*
e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e
^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^
11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a
^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e
^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a
^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*
a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^
2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c
*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)
))^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 73
6*a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d
^5*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^
14 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e
^11 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32
*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*
b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*
d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13
 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14
))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)

```
+ 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^
3*b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c
*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^
2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2
*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e
 + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*
d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*
b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*
e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e
^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) +
384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a
^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 12
8*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d +
e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 5
76*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2
*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4
*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d
^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*
d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c
- 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c
^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^
9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 3
2*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280
*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d
+ e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c
^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^1
4 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*
a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^
2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 -
 b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 +
a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*
c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d
*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e +
 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*
e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^
2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*
e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*
((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^
2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 -
256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a
^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b
^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4
```

*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d

^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^1
1 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 38
4*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 +
 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^
3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*
d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b
^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 192*c^4*d
^8*e^10 + 448*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 512*b*c^3*d^7*e^11 - 128
*b^3*c*d^5*e^13 + 320*a^2*c^2*d^4*e^14 + 448*b^2*c^2*d^6*e^12 - 768*a*b*c^2
*d^5*e^13 + 320*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15))*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
 + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2
i - (2*atanh((64*a^3*c*e^16*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10
 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*
c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*
e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d
^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 102
4*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*
c^4*d^9*e^9)/a) + (576*c^5*d^8*e^8*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^
10*e^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 192
0*a^2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*
c^2*d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^
4*c^2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*
b*c*d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b
^2*c*d^4*e^14) + (2304*c^4*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*
d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 -
192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*
c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b
^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a
^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 -
(1536*b*c^4*d^9*e^9)/a) - (128*b^2*c^4*d^8*e^8*(d^3)^(1/2)*(d + e*x)^(1/2))
/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1920*a^3
*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^3*c^3*d
^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*c^4*d^9
*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2*d^7*e^
11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c^2*d^5*
e^13 + 384*a^3*b^2*c*d^4*e^14) + (320*b^3*c^3*d^7*e^9*(d^3)^(1/2)*(d + e*x)
^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1
920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^
3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*
c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2
*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c
^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) - (192*b^4*c^2*d^6*e^10*(d^3)^(1/2)*(
d + e*x)^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*
e^10 + 1920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8
+ 320*b^3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1
536*a*b*c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a
*b^3*c^2*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024
*a^3*b*c^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) + (1920*a*c^3*d^4*e^12*(d^3)^
(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*
d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14
+ (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a +
(384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^
9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d
^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (3328*b*c^3*d^5*
e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12
+ 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^

2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8
*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3
*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 38
4*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (192*
b^3*c*d^3*e^13*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3
*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 +
256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b
^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2
+ (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5
*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/
a) + (640*b^2*c^3*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^10*e^8 +
2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^2*c^
3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*d^7*
e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^2*d^
8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*d^3*
e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c*d^4
*e^14) + (384*b^3*c^2*d^5*e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^10*e
^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^
2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*
d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^
2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*
d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c
*d^4*e^14) + (256*a^2*c^2*d^2*e^14*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d
^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 1
92*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c
^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^
2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^
2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (
1536*b*c^4*d^9*e^9)/a) + (640*b^2*c^2*d^4*e^12*(d^3)^(1/2)*(d + e*x)^(1/2))
/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*
d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a
+ 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)
/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*
d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*
d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (1536*b*c^4*d^7*e^9*(d^3)^(1/2)*(d + e
*x)^(1/2))/(576*c^5*d^10*e^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 15
36*b*c^4*d^9*e^9 + 1920*a^2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c
^3*d^8*e^10 + 384*b^3*c^2*d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^
3*d^9*e^9)/a - (192*b^4*c^2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3
*c*d^5*e^13 - 256*a^3*b*c*d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^
2*d^5*e^13 + 384*a^2*b^2*c*d^4*e^14) - (256*a^2*b*c*d*e^15*(d^3)^(1/2)*(d +
e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 -
3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5
*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c
^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (
192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 -
256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (1024*a*b*c^2*d^3*e^13*(d^
3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3
*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^
14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a
+ (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9
*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*
c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) + (384*a*b^2*c*
d^2*e^14*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e
^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^
2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3
*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320
*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13
+ 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a))*(d

```
^3)^(1/2))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.539 $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=403

$$-\frac{\sqrt{2}\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}}{}$$

```
[Out] e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*(-2*a*e+b*d)*arctanh((e*x+d)^(
1/2)/d^(1/2))*d^(1/2)/a^2-d*(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+
d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d^2+b
*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b^2)^(1/2))))
/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1
/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c
^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2-e*(a*e+d*(-4*a*
c+b^2)^(1/2))))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A] time = 3.07, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a +
 (2*Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2 - (Sqrt[2]*Sq
rt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(Sqr
t[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(Sqrt[
b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{d^2e^2}{a\left(d-x^2\right)^2}-\frac{de(-bd+2ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right.}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{\left(2d^2e\right)\mathrm{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\right.}{a}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\,\mathrm{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{2}\sqrt{c}\,\left(b^2\right.}{}$$

Mathematica [A] time = 1.54, size = 393, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2ac \cdots\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2 \cdots}\right.}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x])/x) + a*Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*
Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*Sqrt[c]*(-(
b^2*d^2) + b*d*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2
- 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*
e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 -
 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(-(b^2*d^2) + 2*a*c*d^2 - 2*a*e*(Sqrt[b^2 -
 4*a*c]*d + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a^2
```

fricas [B] time = 73.17, size = 8653, normalized size = 21.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 +
 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a
^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^
6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2
*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^
2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2
)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3
*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c +
8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b
^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)
*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 -
 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)
))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*
b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5
*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^
4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e
^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c
*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e +
2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt
(e*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2
 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c
^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a
^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*
a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(
-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c +
 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*
(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b
^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e
```

```
^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3
*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^
3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4
*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 -
 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(
-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^
5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*
c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c
)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4
- a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d
^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^
3)*sqrt(e*x + d)) + sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (
a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^
2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4
 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3
- 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c
))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b
^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^
2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6
*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 -
 a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c +
 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*
(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b
^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a
*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)
*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6
*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c +
3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2
*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*
d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*
c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*
d^2*e^3)*sqrt(e*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e
^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c
+ 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a
^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^
3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4
*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 2
1*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)
*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*
b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*
d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*
b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e
^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)
/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3
 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4
*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*
d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b
^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*
b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a
^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 -
 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^
3*c^2)*d^2*e^3)*sqrt(e*x + d)) + (2*b*d - 3*a*e)*sqrt(d)*x*log((e*x - 2*sqr
t(e*x + d)*sqrt(d) + 2*d)/x) + 2*sqrt(e*x + d)*a*d)/(a^2*x), -1/2*(sqrt(2)*
a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a
^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*
a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 -
3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)
```

*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d)) + sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e

```
*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d
^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 -
 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2
)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3
*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^
4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-s
qrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 2
0*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a
^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2
*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6
 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b
*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*
b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a
^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3
*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(
6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5
- 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^
2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*
d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 -
a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4
*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)
*sqrt(e*x + d)) + 2*(2*b*d - 3*a*e)*sqrt(-d)*x*arctan(sqrt(e*x + d)*sqrt(-d
)/d) + 2*sqrt(e*x + d)*a*d)/(a^2*x)]
```

giac [A] time = 0.55, size = 425, normalized size = 1.05

$$-\frac{\sqrt{xe+d}\,d}{ax}-\frac{\left(2\,bd^2-3\,ade\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a^2\sqrt{-d}}-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac+\sqrt{b^2-4\,ac}\,b\right)d-\cdots\right.}{2\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(x*e + d)*d/(a*x) - (2*b*d^2 - 3*a*d*e)*arctan(sqrt(x*e + d)/sqrt(-d))
/(a^2*sqrt(-d)) - 1/2*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^
2 - 2*a*c + sqrt(b^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(
2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 -
 a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2
 - 4*a*c)*a^2*abs(c)) + 1/2*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e
)*((b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)*d - (a*b - sqrt(b^2 - 4*a*c)*a)*e)*a
rctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c
*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sq
rt(b^2 - 4*a*c)*a^2*abs(c))
```

maple [B] time = 0.05, size = 1215, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x)
```

```
[Out] -d*(e*x+d)^(1/2)/a/x-3*e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*d^(3/2)
/a^2*arctanh((e*x+d)^(1/2)/d^(1/2))*b-2*e^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)+2*e^2/a*c/(-(4*a*
c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
```

```
)*c)*b*d+2*e/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2-e/a^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2+2*e/a*c*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-1/a^2*c*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2-2*e^3*c/(-(4*a
*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)+2*e^2/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*c)*b*d+2*e/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2-e/a^2*c/(-(4*a*c-b^2)*e^2)
^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2-2
*e/a*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+1/a^2*c
*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x)
```

mupad [B] time = 7.36, size = 29890, normalized size = 74.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8
*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b
^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2
*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*
a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3
*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*
d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 1
00*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11
*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 +
 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e
^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*
d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9
+ 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252
*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(
1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c
^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d
^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d
^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^
3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b
```

^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b *c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8* (80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4 *c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3* c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4* d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e* x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a ^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b *c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3 *a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^ 2) + (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a ^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8 *e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d ^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^ 3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e ^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13)) /a^4 + (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4 *d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c ^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b ^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161* a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7* c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4 *d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a ^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 5 6*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2 *e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e ^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^ 4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e ^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^ 10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2 *e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b *d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^2))/((16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c ^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^ 6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6* e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 1 32*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2* c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^ 2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5 *b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^ 7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 - (d^(1/2)*((8 *(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2* b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a ^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 2 2*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^1 2 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^1 4 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a ^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b ^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88* a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*

e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e
^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5
*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d
^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c
^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^
3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c
^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*
d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d
^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e
^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4
*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^
3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*
e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*
e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2) + (d^(1/2)*((8*(d + e*x)^(1/2
)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^
4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10
- 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^1
2 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15
- 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a
^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 + (d^(1/2)*((8*(56*a^4*c^6*
d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e
^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^
6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2
*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^
4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c
^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*
e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14
))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^1
2 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*
b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b
^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4
*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234
*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 +
16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^
4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 +
8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*
a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^
6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (
4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*
e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 5
6*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*
b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(
2*a^2))*(3*a*e - 2*b*d))/(2*a^2)))*(3*a*e - 2*b*d)*1i)/a^2 - atan((((((8*(
80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*
c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c
^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d
^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*
b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^
3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2
*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*
c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*
e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^
4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2
+ 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*

b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4* b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^

2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3
*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6
*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^
7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^
10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8
 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e
^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2
*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5
*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*
d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4
*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d
^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 3
6*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*
e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 -
 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^
9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3
*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^
2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d
^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8
+ 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a
^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1
/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2
 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a
*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^
2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3
*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a
^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3
*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b
^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c
^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5
*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^
3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*
a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2
e(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
) + (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8
- 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 +
36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112
*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^
6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^
2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*
b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e
- 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4
+ 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 -
 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*
b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^
3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^

4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4))*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - atan((((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b

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*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3
- a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*
c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a
^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a
^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^
8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^
4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^
9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*
d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e
+ 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3
*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))
/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a
^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8
 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 5
6*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 +
108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10
 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2
*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e
^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3
+ a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a
^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^
4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2
1*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
+ (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*
a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 1
08*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11
 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3
*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d
*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 -
32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a
^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*
(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e
^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3
- 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*
c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d +
e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a
^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*
d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^
4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c
^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^
9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b
^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3
 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*
c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2)*1i - (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9
+ 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 +
4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*
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a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^

10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*

e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*

```
d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*
e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11
 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19
*a^4*b*c^3*d^2*e^16))/a^4))*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e
^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*
d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e
^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b
^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a
*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d
*(d + e*x)^(1/2))/(a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.540 $\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=607

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4\right.\right.}{a^3\sqrt{}}$$

```
[Out] -3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)-e*(-2*a*e+b*d)*arctanh((e
*x+d)^(1/2)/d^(1/2))/a^2/d^(1/2)-2*(b^2*d^2-2*a*b*d*e-a*(-a*e^2+c*d^2))*arc
tanh((e*x+d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/2*d*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*
x+d)^(1/2)/a/x+(-2*a*e+b*d)*(e*x+d)^(1/2)/a^2/x+arctanh(2^(1/2)*c^(1/2)*(e*
x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d^2
+b^2*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+a*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*
e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2+e*(-a*e+2*d*(-4*a*c+b^2)^(1/2))))/a^3
/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*
c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/
2)*(b^3*d^2-b^2*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*(a*e^2*(-4*a*c+b^2)^(1/2)-
c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2-e*(a*e+2*d*(-4*a*c+b^2)^(1/2
))))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 3.93, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4\right.\right.}{a^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -(d*Sqrt[d + e*x])/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*e)
*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqrt[
d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2*(
b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a
^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*
e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*b*
(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*
d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d +
 a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int\left(-\frac{d^2e^3}{a(d-x^2)^3}+\frac{de^2(-bd+2ae)}{a^2(d-x^2)^2}+\frac{e\left(-b^2d^2+2abde+a\left(cd^2-ae^2\right)\right)}{a^3(d-x^2)}+\frac{e\left(\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)\right.}{a^3\left(cd^2-bde+ae^2\right.}\right.}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{\left(2d^2e^2\right)\mathrm{Su}}{}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{2\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}} -$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{2\left(b^2c\right.}{}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{e(bd-2ae)\,\mathrm{ta}}{a^2}$$

Mathematica [A] time = 2.86, size = 587, normalized size = 0.97

$$-\frac{2a^2d\sqrt{d+ex}}{x^2}+3a^2e\left(\frac{\sqrt{d+ex}}{x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}\right)-\frac{8\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde+a\left(ae^2-cd^2\right)+b^2d^2\right)}{\sqrt{d}}+\frac{4\sqrt{2}\sqrt{c}\left(ab\left(e\left(ae-2d\sqrt{b^2-4ac}\right)-3cd^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*a^2*d*Sqrt[d + e*x])/x^2 + (4*a*(b*d - 2*a*e)*Sqrt[d + e*x])/x + (4*a*
e*(-(b*d) + 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] - (8*(b^2*d^2 -
2*a*b*d*e + a*(-(c*d^2) + a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] +
 3*a^2*e*(Sqrt[d + e*x]/x - (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d]) + (
4*Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*b*(-3*
c*d^2 + e*(-2*Sqrt[b^2 - 4*a*c]*d + a*e)) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*
d*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*d*(Sqrt[b
^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d + a*e)) +
 a*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(4*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.61, size = 1121, normalized size = 1.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e - 2*(a*b^3 - 4*a^2*b*c)*d*e^2 +
(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*a
^2 + 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3 - (a*b^
2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c
)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) + (a^4*b^2
*e^3 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3 + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*
d^2*e - 2*(a^3*b^3 - a^4*b*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*
a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e + s
qrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/
(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqr
t(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/4*(((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*d^2*e - 2*(a*b^3 - 4*a^2*b*c)*d*e^2 + (a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2
*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^
3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*abs(a) + (a^4*b^2*e^3 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*
d^3 + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e - 2*(a^3*b^3 - a^4*b*c)*d*e^2
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt
(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4
*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c
*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(
c)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e + 3*a^2*e^2)*arctan(sqrt(x*e
+ d)/sqrt(-d))/(a^3*sqrt(-d)) + 1/4*(4*(x*e + d)^(3/2)*b*d*e - 4*sqrt(x*e +
 d)*b*d^2*e - 5*(x*e + d)^(3/2)*a*e^2 + 3*sqrt(x*e + d)*a*d*e^2)*e^(-2)/(a^
2*x^2)
```

maple [B] time = 0.06, size = 1880, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x)
```

```
[Out] -1/a^2*c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^
2+1/a^2*c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((
e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2+
e^2/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*
x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-e^2/a
*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1
/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-1/e/a^2/x^2*(
e*x+d)^(1/2)*b*d^2+1/e/a^2/x^2*(e*x+d)^(3/2)*b*d-2/a^3*d^(3/2)*arctanh((e*x
+d)^(1/2)/d^(1/2))*b^2+2/a^2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-3*e/a
^2*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*b*d^2+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d
```

```
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^2-2*e^2/a^2*c/(-(4*a*c-b^2)*e^
2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e
*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*
d-3*e/a^2*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^2-2*e^2/a^2*c/(-(
4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/
2)*c)*b^2*d-3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+3/4*d*(e*x+d)^
(1/2)/a/x^2-5/4/a/x^2*(e*x+d)^(3/2)+3*e/a^2*d^(1/2)*arctanh((e*x+d)^(1/2)/d
^(1/2))*b+2*e/a^2*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*c)*b*d+e^3/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*b+4*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-2*e/a^2*c*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d+e^3/a*c/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b
+4*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*c)*d+1/a^3*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^2*d^2-1/a^3*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^2*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x)
```

mupad [B] time = 8.19, size = 44649, normalized size = 73.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x)
```

```
[Out] (((3*a*d*e^2 - 4*b*d^2*e)*(d + e*x)^(1/2))/(4*a^2) - ((5*a*e^2 - 4*b*d*e)*(
d + e*x)^(3/2))/(4*a^2))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((1
92*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*
c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*
c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b
^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 14
08*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576
*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 +
8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5
*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*
c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c
```

- b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2* b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 3

28*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280 *a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b

^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3
497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*
e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b
^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 +
3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 +
867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14
)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^
3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a
*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*
d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a
^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8
- 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*
b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344
*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9
- 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*
d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5
*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 -
436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896
*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*
a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e
+ 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*
a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^
2*c)))^(1/2) + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12
*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*
d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^
5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*
b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^1
0*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8
*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a
^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 -
a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a
*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2
)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536
*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1
792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2
a^8))((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^
2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7
*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c -

b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3* d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + atan((((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2* d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d

^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e *(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^ 4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2* (-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3* a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^ 3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c ^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512* a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896* a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960 *a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^ 5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c *e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2 *b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3 *a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d *e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^ 2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^ 2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4 *a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^ 4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^ 13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 11 52*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128* a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e ^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^ 6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7 *b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3 *c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6* d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^ 2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c *d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27 *a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^ 3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75* a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^ 3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (2 16*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3* c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^ 12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5 *e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3 *d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4* b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 2 80*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e ^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6* c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912 *a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e ^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c ^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4 *c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^ 3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^ (1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^ 2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(

1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1

/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(87
6*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^
3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3
*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7
*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^
6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 +
4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*
e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^
3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8
*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^
8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e -
3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^
3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*
c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15
- 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932
*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a
^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 +
840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*
e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c
^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^
5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11
+ 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d
^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^
6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13
+ 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^1
1 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e
^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38
*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 -
3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c
^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*
e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 +
64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1
344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e
^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c
^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*
a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^1
4 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 +
896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/
(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*
b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b
^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^
2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c -

b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5 *d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (atan((((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) + (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*

d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b
^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164
*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e
^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^
6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*
e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 160
0*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867
*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14
)/2)/a^8 + ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 +
 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a
^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*
a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 -
3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e
^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^
5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^
8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 -
4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11
- 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) - (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4
*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^1
0 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^
10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^
2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2
*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 - ((d + e*x
)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^1
0 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 +
128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9
 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*
a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2
+ 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*
d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*
a*c*d^2 - 12*a*b*d*e)*1i)/(8*a^3*d^(1/2)) + ((((d + e*x)^(1/2)*(82*a^8*c^5*
e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 +
 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*
a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 -
 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^
5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b
^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1
110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 4
48*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 78
0*a^6*b*c^6*d^3*e^13))/(2*a^8) - (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14
 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672
*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*
b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a
^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1
088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e
^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^
7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5
*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^
13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*
b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^
12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6
*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^
8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 - ((((d + e*x)^(1/2)*(8
76*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b
^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^
3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^
7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a
^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 +
 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3

```
*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b
^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^
8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a
^8) + (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 +
384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*
a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 1
60*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^
10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9
 - 288*a^10*b^3*c^3*d*e^11)/a^8 + ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 -
 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a
^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a
^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*
a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2
- 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b
*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(
8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(8*a^3
*d^(1/2)))/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12
+ 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^
5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*
b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 20
88*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e
^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d
*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e
^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^
11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 - ((((d + e*x)^(1
/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^
6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7
*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b
^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 224
8*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e
^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*
c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6
*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c
^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) + (((108*a^9*b*c^4*e^15 + 30
2*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c
^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d
^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^
5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*
c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a
^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12
- 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5
*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^
5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^
11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^
7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*
e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e
^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 + ((((d
 + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*
e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10
- 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*
e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5
*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^
7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824
*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^
10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d
^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c
^4*d*e^12))/(2*a^8) - (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*
a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c
^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2
```

```
*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^1
0*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^
10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 - ((d + e*x)^(1/2)*(3*a^2*e
^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*
c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3
*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*
c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*
d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*
a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2
 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b
*d*e))/(8*a^3*d^(1/2)) + ((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*
d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^1
4 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9
 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d
^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b
^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3
748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2
*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^
9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^1
3))/(2*a^8) - (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*
e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 -
 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 1
6*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9
+ 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7
*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*
c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^
5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 +
 (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^
2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2
 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b
^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a
^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 -
(1971*a^8*b^2*c^4*d*e^14)/2)/a^8 - ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13
+ 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*
a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4
*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448
*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9
+ 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d
^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^
5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 3
28*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 -
4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) + (((96*a^11*b^
2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^
8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9
 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*
e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4
*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3
*d*e^11)/a^8 + ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b
*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 +
1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8
 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9)
)/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a
^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/
2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*
a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2))))*(3*a^2*e^2
 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(4*a^3*d^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{x^m(e+fx)^n}{a+bx+cx^2} dx$

Optimal. Leaf size=201

$$\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n\right.}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-\right.}}$$

```
[Out] 2*c*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-
f*x/e)/(1+m)/((1+f*x/e)^n)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^
(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b+(-4*a*c+b^2)^(1/2)),-f*x/e)
/(1+m)/((1+f*x/e)^n)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.36, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {911, 135, 133}

$$\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n\right.}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c*
x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1
+ m)*(1 + (f*x)/e)^n) - (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
 + m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b
+ Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx$$
$$= \frac{(2c)\int \frac{x^m(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^m(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b+\sqrt{b^2-4ac}+2cx}}{\sqrt{b^2-4ac}}$$
$$= \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}(e+fx)^r}{}$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2), x]
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^n x^m}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^m}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(fx+e\right)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
[Out] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x^{m}}{cx^{2}+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{m}\left(e+f\,x\right)^{n}}{c\,x^{2}+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

```
[Out] int((x^m*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.542 $\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=290

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,\right.}{c(n+1)\left(2ce-f\left(\sqrt{b^2}\right.\right.}$$

```
[Out] -(b*f+c*e)*(f*x+e)^(1+n)/c^2/f^2/(1+n)+(f*x+e)^(2+n)/c/f^2/(2+n)+(f*x+e)^(1
+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*
(a-1/c*b^2+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+n)/(2*c*e-f*(b-(-4*a*c
+b^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(
b+(-4*a*c+b^2)^(1/2))))*(a-1/c*b^2-b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(
1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.77, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1628, 68}

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,\right.}{c(n+1)\left(2ce-f\left(\sqrt{b^2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e + b*f)*(e + f*x)^(1 + n))/(c^2*f^2*(1 + n))) + (e + f*x)^(2 + n)/(c
*f^2*(2 + n)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (
b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))
 + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2
- 4*a*c])*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{(-ce-bf)(e+fx)^n}{c^2 f} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx} \right.$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,d$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n}$$

Mathematica [A] time = 0.82, size = 261, normalized size = 0.90

$$\frac{(e+fx)^{n+1}\left(\frac{c\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{(n+1)\left(f\left(\sqrt{b^2-4ac}-b\right)+2ce\right)} + \frac{c\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)} - \frac{bf+ce}{f^2(n+1)} + \right.}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((c*e + b*f)/(f^2*(1 + n))) + (c*(e + f*x))/(f^2*(2 +
n)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)
])/((2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(a - b^2/c - (b*(b^2
 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(
e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4
*a*c])*f)*(1 + n))))/c^2
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(fx+e)^n x^3}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^3(fx+e)^n}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

```
[Out] int((x^3*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.543 $\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=237

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\ldots}{2ce\ldots}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] (f*x+e)^(1+n)/c/f/(1+n)+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/
(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(1+n
)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2
*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))/c/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.38, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1628, 68}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\ldots}{2ce\ldots}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (e + f*x)^(1 + n)/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e -
 (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n
)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])
/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{(e+fx)^n}{c} + \frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx} \right) dx$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{c}$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{\left(b+\frac{b^2}{\sqrt{b^2}}\right.}{}$$

Mathematica [A] time = 0.37, size = 202, normalized size = 0.85

$$\frac{(e+fx)^{n+1}\left(\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)} + \frac{1}{f}\right)}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(f^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hyperg
eometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*
c])*f)])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f) + ((b + (b^2 - 2*a*c)/Sqrt[b^
2 - 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b
+ Sqrt[b^2 - 4*a*c])*f)])/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(c*(1 + n))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^2}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(fx+e\right)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
[Out] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x^2*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (e+fx)^n}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Integral(x**2*(e + f*x)**n/(a + b*x + c*x**2), x)
```

3.544 $\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1(1,n+1;n+2}{(n+1)\left(2ce-f(\sqrt{b^2-4ac}+\right.}$$

```
[Out] -(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2
)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-
(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)
^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {830, 68}

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1(1,n+1;n+2}{(n+1)\left(2ce-f(\sqrt{b^2-4ac}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b -
 Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{a+bx+cx^2} dx = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx} \right) dx$$
$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx} dx$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+}}{\left(2ce-\left(b\right.\right.}$$

Mathematica [A] time = 0.29, size = 183, normalized size = 0.92

$$\frac{(e+fx)^{n+1}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(((1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f)) - ((1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1
, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*
e - (b + Sqrt[b^2 - 4*a*c])*f)))/(1 + n)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x\left(fx+e\right)^n}{c\,x^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n x}{cx^2+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(e+f\,x)^n}{c\,x^2+b\,x+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(e + f*x)^n)/(a + b*x + c*x^2),x)
```

```
[Out] int((x*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Integral(x*(e + f*x)**n/(a + b*x + c*x**2), x)
```

3.545 $\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=191

$$\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] -2*c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c
+b^2)^(1/2))))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+2*
c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^
2)^(1/2))))/(1+n)/(-4*a*c+b^2)^(1/2)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.26, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {711, 68}

$$\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (-2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/
(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2 - 4*a*c]*(2*c*e - (b - Sqrt
[b^2 - 4*a*c])*f)*(1 + n)) + (2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2
 - 4*a*c]*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx$$

$$= \frac{(2c)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n\right.}{\sqrt{b^2-4ac}\left(2ce-\left(b+\sqrt{b^2-\right.\right.}$$

Mathematica [A] time = 0.25, size = 163, normalized size = 0.85

$$\frac{2c(e+fx)^{n+1}\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce}\right)}{(n+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x)
)/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)]/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f
)) + Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)]/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(Sqrt[b^2 - 4*a*c]*
(1 + n))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(a + b*x + c*x^2),x)
```

```
[Out] int((e + f*x)^n/(a + b*x + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Integral((e + f*x)**n/(a + b*x + c*x**2), x)
```

3.546 $\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx$

Optimal. Leaf size=242

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+\right.}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}\right.\right.}$$

```
[Out] -(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a/e/(1+n)+c*(f*x+e)^(1+n)*
hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(1+b
/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+c*(f*x+e)^(1+
n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(
1-b/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] time = 0.39, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+\right.}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b
 - Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^
(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sq
rt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 +
n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(e+fx)^n}{x\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{(e+fx)^n}{ax} + \frac{(-b-cx)(e+fx)^n}{a\left(a+bx+cx^2\right)}\right) dx$$

$$= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a} + \frac{\int \frac{(-b-cx)(e+fx)^n}{a+bx+cx^2}\,dx}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} + \frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right) \ldots}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a} - \frac{\left(c\left(1 \ldots\right.\right.}{\ldots}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+ \ldots}{a\left(2ce \ldots\right.}$$

Mathematica [A] time = 0.41, size = 207, normalized size = 0.86

$$\frac{(e+fx)^{n+1}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{e}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[
1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c
*e - (b + Sqrt[b^2 - 4*a*c])*f) - Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f
*x)/e]/e))/(a*(1 + n))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n}{cx^3+bx^2+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^3 + b*x^2 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(c\,x^2+bx+a)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
[Out] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x\,(c\,x^2+b\,x+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x*(a + b*x + c*x^2)),x)
[Out] int((e + f*x)^n/(x*(a + b*x + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(c*x**2+b*x+a),x)
[Out] Integral((e + f*x)**n/(x*(a + b*x + c*x**2)), x)
```

3.547 $\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=296

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+\right.\right.}$$

```
[Out] b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a^2/e/(1+n)+f*(f*x+e)^(1+
n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1+n)-c*(f*x+e)^(1+n)*hypergeom(
[1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2
)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-c*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/
2)))
```

Rubi [A] time = 0.48, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {960, 65, 830, 68}

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2\right.}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/
(a^2*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - (c*(b - (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*
c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(a^2*(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)*(1 + n)) + (b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + (f*x)/e])/(a^2*e*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}dx=\int\left(\frac{(e+fx)^n}{ax^2}-\frac{b(e+fx)^n}{a^2x}+\frac{\left(b^2-ac+bcx\right)(e+fx)^n}{a^2\left(a+bx+cx^2\right)}\right)dx$$

$$=\frac{\int\frac{\left(b^2-ac+bcx\right)(e+fx)^n}{a+bx+cx^2}dx}{a^2}+\frac{\int\frac{(e+fx)^n}{x^2}dx}{a}-\frac{b\int\frac{(e+fx)^n}{x}dx}{a^2}$$

$$=\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)}+\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}$$

$$=\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)}+\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}$$

$$=-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{a}$$

Mathematica [A] time = 0.43, size = 246, normalized size = 0.83

$$\frac{(e+fx)^{n+1}\left(-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce}-\frac{c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)}+\frac{af\,{}_2F_1\left(2,n+1;n+2;\right.}{e^2}\right.}{a^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)
])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[b^2
- 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b +
Sqrt[b^2 - 4*a*c])*f)])/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f) + (b*Hypergeome
tric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/e + (a*f*Hypergeometric2F1[2, 1 + n,
 2 + n, 1 + (f*x)/e])/e^2))/(a^2*(1 + n))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^n}{cx^4+bx^3+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(c*x^4 + b*x^3 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(c\,x^2+bx+a)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
[Out] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(cx^2+bx+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x^2\,(c\,x^2+b\,x+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x^2*(a + b*x + c*x^2)),x)
[Out] int((e + f*x)^n/(x^2*(a + b*x + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.548 $\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=141

$$-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{1}{2}egx^4(2dg+ef)-$$

```
[Out] -d^2*(8*d^2*g^2+16*d*e*f*g+7*e^2*f^2)*x/e^2-d*(4*d^2*g^2+7*d*e*f*g+2*e^2*f^
2)*x^2/e-1/3*(d*g+e*f)*(7*d*g+e*f)*x^3-1/2*e*g*(2*d*g+e*f)*x^4-1/5*e^2*g^2*
x^5-8*d^3*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{1}{2}egx^4(2dg+ef)-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*e
*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f +
2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)^3(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{d^2\left(7e^2f^2+16defg+8d^2g^2\right)}{e^2}-\frac{2d\left(2e^2f^2+7defg+4d^2g^2\right)x}{e}+(-ef-7dg)\right.\\ &= -\frac{d^2\left(7e^2f^2+16defg+8d^2g^2\right)x}{e^2}-\frac{d\left(2e^2f^2+7defg+4d^2g^2\right)x^2}{e}-\frac{1}{3}(ef+dg)(ef\end{aligned}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 0.95

$$-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{x\left(240d^4g^2+120d^3eg(4f+gx)+70d^2e^2\left(3f^2+3fgx+g^2x^2\right)+10de^3x\left(6f^2+8\right.\right.}{30e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/30*(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*g
*x + g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^2
+ 15*f*g*x + 6*g^2*x^2)))/e^2 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

fricas [A] time = 0.82, size = 176, normalized size = 1.25

$$-\frac{6\,e^{5}g^{2}x^{5}+15\,\left(e^{5}fg+2\,de^{4}g^{2}\right)x^{4}+10\,\left(e^{5}f^{2}+8\,de^{4}fg+7\,d^{2}e^{3}g^{2}\right)x^{3}+30\,\left(2\,de^{4}f^{2}+7\,d^{2}e^{3}fg+4\,d^{3}e^{2}g^{2}\right)x^{2}+3}{30\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*e^5*g^2*x^5 + 15*(e^5*f*g + 2*d*e^4*g^2)*x^4 + 10*(e^5*f^2 + 8*d*e
^4*f*g + 7*d^2*e^3*g^2)*x^3 + 30*(2*d*e^4*f^2 + 7*d^2*e^3*f*g + 4*d^3*e^2*g
^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*e*g^2)*x + 240*(d^3*e^
2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d))/e^3
```

giac [A] time = 0.16, size = 249, normalized size = 1.77

$$-4\left(d^{5}g^{2}e^{3}+2\,d^{4}fge^{4}+d^{3}f^{2}e^{5}\right)e^{(-6)}\log\left(\left|x^{2}e^{2}-d^{2}\right|\right)-\frac{1}{30}\left(6\,g^{2}x^{5}e^{12}+30\,dg^{2}x^{4}e^{11}+70\,d^{2}g^{2}x^{3}e^{10}+120\,d^{3}g^{2}x^{2}e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2
)) - 1/30*(6*g^2*x^5*e^12 + 30*d*g^2*x^4*e^11 + 70*d^2*g^2*x^3*e^10 + 120*d
^3*g^2*x^2*e^9 + 240*d^4*g^2*x*e^8 + 15*f*g*x^4*e^12 + 80*d*f*g*x^3*e^11 +
210*d^2*f*g*x^2*e^10 + 480*d^3*f*g*x*e^9 + 10*f^2*x^3*e^12 + 60*d*f^2*x^2*e
^11 + 210*d^2*f^2*x*e^10)*e^(-10) - 4*(d^6*g^2*e^4 + 2*d^5*f*g*e^5 + d^4*f^
2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(
d)
```

maple [A] time = 0.01, size = 186, normalized size = 1.32

$$-\frac{e^{2}g^{2}x^{5}}{5}-de\,g^{2}x^{4}-\frac{e^{2}fg\,x^{4}}{2}-\frac{7d^{2}g^{2}x^{3}}{3}-\frac{8defg\,x^{3}}{3}-\frac{e^{2}f^{2}x^{3}}{3}-\frac{4d^{3}g^{2}x^{2}}{e}-7d^{2}fg\,x^{2}-2de\,f^{2}x^{2}-\frac{8d^{5}g^{2}\ln\left(ex-d\right)}{e^{3}}-\frac{16d^{4}f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/5*e^2*g^2*x^5-e*x^4*d*g^2-1/2*e^2*x^4*f*g-7/3*x^3*d^2*g^2-8/3*e*x^3*d*f*
g-1/3*e^2*x^3*f^2-4/e*x^2*d^3*g^2-7*x^2*d^2*f*g-2*e*x^2*d*f^2-8/e^2*x*d^4*g
^2-16/e*x*d^3*f*g-7*x*d^2*f^2-8*d^5/e^3*ln(e*x-d)*g^2-16*d^4/e^2*ln(e*x-d)*
f*g-8*d^3/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.47, size = 175, normalized size = 1.24

$$-\frac{6\,e^{4}g^{2}x^{5}+15\,\left(e^{4}fg+2\,de^{3}g^{2}\right)x^{4}+10\,\left(e^{4}f^{2}+8\,de^{3}fg+7\,d^{2}e^{2}g^{2}\right)x^{3}+30\,\left(2\,de^{3}f^{2}+7\,d^{2}e^{2}fg+4\,d^{3}eg^{2}\right)x^{2}+30}{30\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*e^4*g^2*x^5 + 15*(e^4*f*g + 2*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 8*d*e
^3*f*g + 7*d^2*e^2*g^2)*x^3 + 30*(2*d*e^3*f^2 + 7*d^2*e^2*f*g + 4*d^3*e*g^2
```

```
)*x^2 + 30*(7*d^2*e^2*f^2 + 16*d^3*e*f*g + 8*d^4*g^2)*x)/e^2 - 8*(d^3*e^2*f
^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.11, size = 351, normalized size = 2.49

$$-x^2\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{2\,e}+\frac{d\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e}+\frac{d\left(e\,g\left(3\,d\,g+2\,e\,f\right)+d\,e\,g^2\right)}{e}\right)}{2\,e}\right)-x^3\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g}{3\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/(2*e) + (d*((e^3*f^2 + 3*d^2*e
*g^2 + 6*d*e^2*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/(2*e)) - x^
3*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/(3*e) + (d*(e*g*(3*d*g + 2*e*f) +
d*e*g^2))/(3*e)) - x^4*((e*g*(3*d*g + 2*e*f))/4 + (d*e*g^2)/4) - x*((d*((d^
3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e + (d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2
*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/e))/e + (d^2*f*(2*d*g + 3
*e*f))/e) - (log(e*x - d)*(8*d^5*g^2 + 8*d^3*e^2*f^2 + 16*d^4*e*f*g))/e^3 -
 (e^2*g^2*x^5)/5
```

sympy [A] time = 0.60, size = 150, normalized size = 1.06

$$-\frac{8d^3\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}-\frac{e^2g^2x^5}{5}-x^4\left(deg^2+\frac{e^2fg}{2}\right)-x^3\left(\frac{7d^2g^2}{3}+\frac{8defg}{3}+\frac{e^2f^2}{3}\right)-x^2\left(\frac{4d^3g^2}{e}+7d^2fg+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -8*d**3*(d*g + e*f)**2*log(-d + e*x)/e**3 - e**2*g**2*x**5/5 - x**4*(d*e*g*
*2 + e**2*f*g/2) - x**3*(7*d**2*g**2/3 + 8*d*e*f*g/3 + e**2*f**2/3) - x**2*
(4*d**3*g**2/e + 7*d**2*f*g + 2*d*e*f**2) - x*(8*d**4*g**2/e**2 + 16*d**3*f
*g/e + 7*d**2*f**2)
```

3.549 $\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

```
[Out] -d*(2*d*g+e*f)*(2*d*g+3*e*f)*x/e^2-1/2*(4*d^2*g^2+6*d*e*f*g+e^2*f^2)*x^2/e-
1/3*g*(3*d*g+2*e*f)*x^3-1/4*e*g^2*x^4-4*d^2*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2*
g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f +
 d*g)^2*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{d-ex}\,dx\\
&= \int \left(\frac{d(-3ef-2dg)(ef+2dg)}{e^2}-\frac{\left(e^2f^2+6defg+4d^2g^2\right)x}{e}-g(2ef+3dg)x^2-eg^2x^3\right.\\
&= -\frac{d(ef+2dg)(3ef+2dg)x}{e^2}-\frac{\left(e^2f^2+6defg+4d^2g^2\right)x^2}{2e}-\frac{1}{3}g(2ef+3dg)x^3-\frac{1}{4}eg^2x
\end{aligned}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.94

$$-\frac{48d^2(dg+ef)^2\log(d-ex)+ex\left(48d^3g^2+24d^2eg(4f+gx)+12de^2\left(3f^2+3fgx+g^2x^2\right)+e^3x\left(6f^2+8fgx+\right.\right.}{12e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/12*(e*x*(48*d^3*g^2 + 24*d^2*e*g*(4*f + g*x) + 12*d*e^2*(3*f^2 + 3*f*g*x
 + g^2*x^2) + e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2)) + 48*d^2*(e*f + d*g)^2*L
og[d - e*x])/e^3
```

fricas [A] time = 0.68, size = 139, normalized size = 1.28

$$-\frac{3\,e^4g^2x^4 + 4\left(2\,e^4fg + 3\,de^3g^2\right)x^3 + 6\left(e^4f^2 + 6\,de^3fg + 4\,d^2e^2g^2\right)x^2 + 12\left(3\,de^3f^2 + 8\,d^2e^2fg + 4\,d^3eg^2\right)x + \cdots}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^4*g^2*x^4 + 4*(2*e^4*f*g + 3*d*e^3*g^2)*x^3 + 6*(e^4*f^2 + 6*d*e
^3*f*g + 4*d^2*e^2*g^2)*x^2 + 12*(3*d*e^3*f^2 + 8*d^2*e^2*f*g + 4*d^3*e*g^2
)*x + 48*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d))/e^3
```

giac [B] time = 0.21, size = 211, normalized size = 1.94

$$-2\left(d^4g^2e^3 + 2\,d^3fge^4 + d^2f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{1}{12}\left(3\,g^2x^4e^9 + 12\,dg^2x^3e^8 + 24\,d^2g^2x^2e^7 + 48\,d^3g^2xe^6 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2
)) - 1/12*(3*g^2*x^4*e^9 + 12*d*g^2*x^3*e^8 + 24*d^2*g^2*x^2*e^7 + 48*d^3*g
^2*x*e^6 + 8*f*g*x^3*e^9 + 36*d*f*g*x^2*e^8 + 96*d^2*f*g*x*e^7 + 6*f^2*x^2*
e^9 + 36*d*f^2*x*e^8)*e^(-8) - 2*(d^5*g^2*e^2 + 2*d^4*f*g*e^3 + d^3*f^2*e^4
)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

maple [A] time = 0.00, size = 145, normalized size = 1.33

$$-\frac{e g^2x^4}{4} - d g^2x^3 - \frac{2efg x^3}{3} - \frac{2d^2g^2x^2}{e} - 3dfg x^2 - \frac{e f^2x^2}{2} - \frac{4d^4g^2\ln(ex-d)}{e^3} - \frac{8d^3fg\ln(ex-d)}{e^2} - \frac{4d^3g^2x}{e^2} - \frac{4d^2f^2\ln \cdots}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/4*e*g^2*x^4-x^3*d*g^2-2/3*e*x^3*f*g-2/e*x^2*d^2*g^2-3*x^2*d*f*g-1/2*e*x^
2*f^2-4/e^2*x*d^3*g^2-8/e*x*d^2*f*g-3*x*d*f^2-4*d^4/e^3*ln(e*x-d)*g^2-8*d^3
/e^2*ln(e*x-d)*f*g-4*d^2/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 138, normalized size = 1.27

$$-\frac{3\,e^3g^2x^4 + 4\left(2\,e^3fg + 3\,de^2g^2\right)x^3 + 6\left(e^3f^2 + 6\,de^2fg + 4\,d^2eg^2\right)x^2 + 12\left(3\,de^2f^2 + 8\,d^2efg + 4\,d^3g^2\right)x}{12\,e^2} - \frac{4\left(d \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^3*g^2*x^4 + 4*(2*e^3*f*g + 3*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 6*d*e
^2*f*g + 4*d^2*e*g^2)*x^2 + 12*(3*d*e^2*f^2 + 8*d^2*e*f*g + 4*d^3*g^2)*x)/e
^2 - 4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.59, size = 197, normalized size = 1.81

$$-x^3\left(\frac{2g\left(dg+ef\right)}{3} + \frac{dg^2}{3}\right) - x^2\left(\frac{d^2g^2 + 4defg + e^2f^2}{2e} + \frac{d\left(2g\left(dg+ef\right) + dg^2\right)}{2e}\right) - x\left(\frac{d\left(\frac{d^2g^2+4defg+e^2f \cdots}{e}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^3*((2*g*(d*g + e*f))/3 + (d*g^2)/3) - x^2*((d^2*g^2 + e^2*f^2 + 4*d*e*f
*g)/(2*e) + (d*(2*g*(d*g + e*f) + d*g^2))/(2*e)) - x*((d*((d^2*g^2 + e^2*f^
2 + 4*d*e*f*g)/e + (d*(2*g*(d*g + e*f) + d*g^2))/e))/e + (2*d*f*(d*g + e*f)
)/e) - (log(e*x - d)*(4*d^4*g^2 + 4*d^2*e^2*f^2 + 8*d^3*e*f*g))/e^3 - (e*g^
2*x^4)/4
```

sympy [A] time = 0.48, size = 109, normalized size = 1.00

$$-\frac{4d^2\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}-\frac{eg^2x^4}{4}-x^3\left(dg^2+\frac{2efg}{3}\right)-x^2\left(\frac{2d^2g^2}{e}+3dfg+\frac{ef^2}{2}\right)-x\left(\frac{4d^3g^2}{e^2}+\frac{8d^2fg}{e}+3df^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -4*d**2*(d*g + e*f)**2*log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 +
2*e*f*g/3) - x**2*(2*d**2*g**2/e + 3*d*f*g + e*f**2/2) - x*(4*d**3*g**2/e**
2 + 8*d**2*f*g/e + 3*d*f**2)
```

3.550 $\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{2dgx(dg+ef)}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

```
[Out] -2*d*g*(d*g+e*f)*x/e^2-d*(g*x+f)^2/e-1/3*(g*x+f)^3/g-2*d*(d*g+e*f)^2*ln(-e*
x+d)/e^3
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$-\frac{2dgx(dg+ef)}{e^2}-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] (-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(
e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)(f+gx)^2}{d-ex}\,dx\\
&= \int \left(-\frac{2dg(ef+dg)}{e^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)}-\frac{2dg(f+gx)}{e}-(f+gx)^2\right)dx\\
&= -\frac{2dg(ef+dg)x}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}-\frac{2d(ef+dg)^2\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.12

$$-\frac{ex\left(6d^2g^2+3deg(4f+gx)+e^2\left(3f^2+3fgx+g^2x^2\right)\right)+6d(dg+ef)^2\log(d-ex)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/3*(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^2
)) + 6*d*(e*f + d*g)^2*Log[d - e*x])/e^3
```

fricas [A] time = 0.98, size = 98, normalized size = 1.51

$$-\frac{e^3g^2x^3 + 3\left(e^3fg + de^2g^2\right)x^2 + 3\left(e^3f^2 + 4\,de^2fg + 2\,d^2eg^2\right)x + 6\left(de^2f^2 + 2\,d^2efg + d^3g^2\right)\log\left(ex - d\right)}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*g^2*x^3 + 3*(e^3*f*g + d*e^2*g^2)*x^2 + 3*(e^3*f^2 + 4*d*e^2*f*g
+ 2*d^2*e*g^2)*x + 6*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d))/e^3
```

giac [B] time = 0.16, size = 172, normalized size = 2.65

$$-\left(d^3g^2e + 2\,d^2fge^2 + df^2e^3\right)e^{(-4)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{1}{3}\left(g^2x^3e^6 + 3\,dg^2x^2e^5 + 6\,d^2g^2xe^4 + 3\,fgx^2e^6 + 12\,dfgxe^5 + 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3)*e^(-4)*log(abs(x^2*e^2 - d^2)) - 1
/3*(g^2*x^3*e^6 + 3*d*g^2*x^2*e^5 + 6*d^2*g^2*x*e^4 + 3*f*g*x^2*e^6 + 12*d*
f*g*x*e^5 + 3*f^2*x*e^6)*e^(-6) - (d^4*g^2*e^2 + 2*d^3*f*g*e^3 + d^2*f^2*e^
4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

maple [A] time = 0.00, size = 110, normalized size = 1.69

$$-\frac{g^2x^3}{3} - \frac{d\,g^2x^2}{e} - fg\,x^2 - \frac{2d^3g^2\ln\left(ex-d\right)}{e^3} - \frac{4d^2fg\ln\left(ex-d\right)}{e^2} - \frac{2d^2g^2x}{e^2} - \frac{2d\,f^2\ln\left(ex-d\right)}{e} - \frac{4dfgx}{e} - f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/3*g^2*x^3-1/e*x^2*d*g^2-x^2*f*g-2/e^2*x*d^2*g^2-4/e*x*d*f*g-x*f^2-2*d^3/
e^3*ln(e*x-d)*g^2-4*d^2/e^2*ln(e*x-d)*f*g-2*d/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 97, normalized size = 1.49

$$-\frac{e^2g^2x^3 + 3\left(e^2fg + deg^2\right)x^2 + 3\left(e^2f^2 + 4\,defg + 2\,d^2g^2\right)x}{3\,e^2} - \frac{2\left(de^2f^2 + 2\,d^2efg + d^3g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/3*(e^2*g^2*x^3 + 3*(e^2*f*g + d*e*g^2)*x^2 + 3*(e^2*f^2 + 4*d*e*f*g + 2*
d^2*g^2)*x)/e^2 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.07, size = 127, normalized size = 1.95

$$-x^2\left(\frac{d\,g^2+2\,e\,f\,g}{2\,e}+\frac{d\,g^2}{2\,e}\right)-x\left(\frac{e\,f^2+2\,d\,g\,f}{e}+\frac{d\left(\frac{d\,g^2+2\,e\,f\,g}{e}+\frac{d\,g^2}{e}\right)}{e}\right)-\frac{g^2\,x^3}{3}-\frac{\ln\left(e\,x-d\right)\,\left(2\,d^3\,g^2+4\,d^2\,e\,f\,g+2\right.}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d*g^2 + 2*e*f*g)/(2*e) + (d*g^2)/(2*e)) - x*((e*f^2 + 2*d*f*g)/e +
(d*((d*g^2 + 2*e*f*g)/e + (d*g^2)/e))/e) - (g^2*x^3)/3 - (log(e*x - d)*(2*d
^3*g^2 + 2*d*e^2*f^2 + 4*d^2*e*f*g))/e^3
```

sympy [A] time = 0.38, size = 70, normalized size = 1.08

$$-\frac{2d\left(dg+ef\right)^{2}\log\left(-d+ex\right)}{e^{3}}-\frac{g^{2}x^{3}}{3}-x^{2}\left(\frac{dg^{2}}{e}+fg\right)-x\left(\frac{2d^{2}g^{2}}{e^{2}}+\frac{4dfg}{e}+f^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -2*d*(d*g + e*f)**2*log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2/e + f*g
) - x*(2*d**2*g**2/e**2 + 4*d*f*g/e + f**2)
```

3.551 $\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{gx(dg+ef)}{e^2}-\frac{(f+gx)^2}{2e}$$

```
[Out] -g*(d*g+e*f)*x/e^2-1/2*(g*x+f)^2/e-(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {799, 43}

$$-\frac{gx(dg+ef)}{e^2}-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{(f+gx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x])
/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{g(ef+dg)}{e^2}+\frac{(ef+dg)^2}{e^2(d-ex)}-\frac{g(f+gx)}{e}\right)dx\\ &= -\frac{g(ef+dg)x}{e^2}-\frac{(f+gx)^2}{2e}-\frac{(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.86

$$-\frac{egx(2dg+4ef+egx)+2(dg+ef)^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/2*(e*g*x*(4*e*f + 2*d*g + e*g*x) + 2*(e*f + d*g)^2*Log[d - e*x])/e^3
```

fricas [A] time = 0.90, size = 64, normalized size = 1.28

$$-\frac{e^2g^2x^2 + 2\left(2\,e^2fg + deg^2\right)x + 2\left(e^2f^2 + 2\,defg + d^2g^2\right)\log\left(ex - d\right)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(e^2*g^2*x^2 + 2*(2*e^2*f*g + d*e*g^2)*x + 2*(e^2*f^2 + 2*d*e*f*g + d^
2*g^2)*log(e*x - d))/e^3
```

giac [B] time = 0.17, size = 134, normalized size = 2.68

$$-\frac{1}{2}\left(d^2g^2e + 2\,dfge^2 + f^2e^3\right)e^{(-4)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{1}{2}\left(g^2x^2e^3 + 2\,dg^2xe^2 + 4\,fgxe^3\right)e^{(-4)} - \frac{\left(d^3g^2 + 2\,d^2fge + d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(d^2*g^2*e + 2*d*f*g*e^2 + f^2*e^3)*e^(-4)*log(abs(x^2*e^2 - d^2)) - 1
/2*(g^2*x^2*e^3 + 2*d*g^2*x*e^2 + 4*f*g*x*e^3)*e^(-4) - 1/2*(d^3*g^2 + 2*d^
2*f*g*e + d*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*a
bs(d)*e))/abs(d)
```

maple [A] time = 0.00, size = 82, normalized size = 1.64

$$-\frac{g^2x^2}{2e} - \frac{d^2g^2\ln\left(ex - d\right)}{e^3} - \frac{2dfg\ln\left(ex - d\right)}{e^2} - \frac{d\,g^2x}{e^2} - \frac{f^2\ln\left(ex - d\right)}{e} - \frac{2fgx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/2*g^2*x^2/e-g^2/e^2*d*x-2*g/e*f*x-1/e^3*ln(e*x-d)*d^2*g^2-2/e^2*ln(e*x-d
)*d*f*g-1/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.44, size = 63, normalized size = 1.26

$$-\frac{eg^2x^2 + 2\left(2\,efg + dg^2\right)x}{2\,e^2} - \frac{\left(e^2f^2 + 2\,defg + d^2g^2\right)\log\left(ex - d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^2 + 2*(2*e*f*g + d*g^2)*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g
^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.61, size = 65, normalized size = 1.30

$$-x\left(\frac{d\,g^2}{e^2} + \frac{2\,f\,g}{e}\right) - \frac{\ln\left(e\,x - d\right)\,\left(d^2\,g^2 + 2\,d\,e\,f\,g + e^2\,f^2\right)}{e^3} - \frac{g^2\,x^2}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2),x)
```

```
[Out] - x*((d*g^2)/e^2 + (2*f*g)/e) - (log(e*x - d)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*
g))/e^3 - (g^2*x^2)/(2*e)
```

sympy [A] time = 0.29, size = 46, normalized size = 0.92

$$-x\left(\frac{dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{g^2x^2}{2e}-\frac{\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -x*(d*g**2/e**2 + 2*f*g/e) - g**2*x**2/(2*e) - (d*g + e*f)**2*log(-d + e*x)
/e**3
```

3.552 $\int \frac{(f+gx)^2}{d^2-e^2x^2} dx$

Optimal. Leaf size=62

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

```
[Out] -g^2*x/e^2-1/2*(d*g+e*f)^2*ln(-e*x+d)/d/e^3+1/2*(-d*g+e*f)^2*ln(e*x+d)/d/e^
3
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {702, 633, 31}

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] -((g^2*x)/e^2) - ((e*f + d*g)^2*Log[d - e*x])/(2*d*e^3) + ((e*f - d*g)^2*Lo
g[d + e*x])/(2*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{d^2-e^2x^2} dx &= \int \left(-\frac{g^2}{e^2}+\frac{e^2f^2+d^2g^2+2e^2fgx}{e^2\left(d^2-e^2x^2\right)}\right) dx \\
&= -\frac{g^2x}{e^2}+\frac{\int \frac{e^2f^2+d^2g^2+2e^2fgx}{d^2-e^2x^2} dx}{e^2} \\
&= -\frac{g^2x}{e^2}-\frac{(ef-dg)^2\int \frac{1}{-de-e^2x} dx}{2de}+\frac{(ef+dg)^2\int \frac{1}{de-e^2x} dx}{2de} \\
&= -\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.89

$$\frac{\left(d^2g^2+e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)-deg\left(f\log\left(d^2-e^2x^2\right)+gx\right)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] ((e^2*f^2 + d^2*g^2)*ArcTanh[(e*x)/d] - d*e*g*(g*x + f*Log[d^2 - e^2*x^2]))
/(d*e^3)
```

fricas [A] time = 0.91, size = 76, normalized size = 1.23

$$-\frac{2\,deg^2x-\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)+\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{2\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*e*g^2*x - (e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d) + (e^2*f^2
 + 2*d*e*f*g + d^2*g^2)*log(e*x - d))/(d*e^3)
```

giac [A] time = 0.15, size = 81, normalized size = 1.31

$$-g^2xe^{(-2)}-fge^{(-2)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{\left(d^2g^2+f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{2\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - f*g*e^(-2)*log(abs(x^2*e^2 - d^2)) - 1/2*(d^2*g^2 + f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

maple [A] time = 0.01, size = 107, normalized size = 1.73

$$-\frac{d\,g^2\ln\left(ex-d\right)}{2e^3}+\frac{d\,g^2\ln\left(ex+d\right)}{2e^3}-\frac{f^2\ln\left(ex-d\right)}{2de}+\frac{f^2\ln\left(ex+d\right)}{2de}-\frac{fg\ln\left(ex-d\right)}{e^2}-\frac{fg\ln\left(ex+d\right)}{e^2}-\frac{g^2x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -g^2*x/e^2-1/2/e^3*d*ln(e*x-d)*g^2-1/e^2*ln(e*x-d)*f*g-1/2/e/d*ln(e*x-d)*f^
2+1/2/e^3*d*ln(e*x+d)*g^2-1/e^2*ln(e*x+d)*f*g+1/2/e/d*ln(e*x+d)*f^2
```

maxima [A] time = 0.44, size = 82, normalized size = 1.32

$$-\frac{g^2x}{e^2}+\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)}{2\,de^3}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{2\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d*e^3) - 1/2
*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d*e^3)
```

mupad [B] time = 0.15, size = 81, normalized size = 1.31

$$\frac{\ln\left(d+e\,x\right)\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}{2\,d\,e^3}-\frac{g^2\,x}{e^2}-\frac{\ln\left(d-e\,x\right)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{2\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2),x)
```

```
[Out] (log(d + e*x)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))/(2*d*e^3) - (g^2*x)/e^2 - (l
og(d - e*x)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(2*d*e^3)
```

sympy [B] time = 0.64, size = 112, normalized size = 1.81

$$-\frac{g^2x}{e^2}+\frac{\left(dg-ef\right)^2\log\left(x+\frac{2d^2fg+\frac{d\left(dg-ef\right)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}-\frac{\left(dg+ef\right)^2\log\left(x+\frac{2d^2fg-\frac{d\left(dg+ef\right)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -g**2*x/e**2 + (d*g - e*f)**2*log(x + (2*d**2*f*g + d*(d*g - e*f)**2/e)/(d*
*2*g**2 + e**2*f**2))/(2*d*e**3) - (d*g + e*f)**2*log(x + (2*d**2*f*g - d*(
d*g + e*f)**2/e)/(d**2*g**2 + e**2*f**2))/(2*d*e**3)
```

3.553 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=86

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

```
[Out] -1/2*(-d*g+e*f)^2/d/e^3/(e*x+d)-1/4*(d*g+e*f)^2*ln(-e*x+d)/d^2/e^3+1/4*(-d*
g+e*f)*(3*d*g+e*f)*ln(e*x+d)/d^2/e^3
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(2*d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2*e^
3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^2}\,dx\\ &= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{2de^2(d+ex)^2}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)}\right)dx\\ &= -\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.95

$$\frac{(ef-dg)((d+ex)(3dg+ef)\log(d+ex)+2d(dg-ef))-(d+ex)(dg+ef)^2\log(d-ex)}{4d^2e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] (-((e*f + d*g)^2*(d + e*x)*Log[d - e*x]) + (e*f - d*g)*(2*d*(-(e*f) + d*g)
+ (e*f + 3*d*g)*(d + e*x)*Log[d + e*x]))/(4*d^2*e^3*(d + e*x))
```

fricas [B] time = 0.89, size = 165, normalized size = 1.92

$$-\frac{2\,de^2f^2 - 4\,d^2efg + 2\,d^3g^2 - \left(de^2f^2 + 2\,d^2efg - 3\,d^3g^2 + \left(e^3f^2 + 2\,de^2fg - 3\,d^2eg^2\right)x\right)\log\left(ex+d\right) + \left(de^2f^2\right.}{4\left(d^2e^4x + d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 4*d^2*e*f*g + 2*d^3*g^2 - (d*e^2*f^2 + 2*d^2*e*f*g - 3*
d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x + d) + (d*e^2*f^
2 + 2*d^2*e*f*g + d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x + d^3*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(d^2
*g^2-2*d*exp(1)*g*f+exp(1)^2*f^2)/(exp(2)*d^2*exp(1)-d^2*exp(1)^3)*ln(abs(x
*exp(1)+d))-(2*exp(2)*d*g*f-exp(2)*exp(1)*f^2-d^2*exp(1)*g^2)/(2*exp(2)^2*d
^2-2*exp(2)*d^2*exp(1)^2)*ln(abs(x^2*exp(2)-d^2))-(exp(2)*f^2+d^2*g^2-2*d*e
xp(1)*g*f)*1/2/(exp(2)*d-d*exp(1)^2)/exp(1)/abs(d)*ln(abs(2*x*exp(2)-2*exp(
1)*abs(d))/abs(2*x*exp(2)+2*exp(1)*abs(d)))
```

maple [A] time = 0.01, size = 149, normalized size = 1.73

$$-\frac{d\,g^2}{2\left(ex+d\right)e^3}-\frac{f^2}{2\left(ex+d\right)de}-\frac{fg\ln\left(ex-d\right)}{2d\,e^2}+\frac{fg\ln\left(ex+d\right)}{2d\,e^2}-\frac{f^2\ln\left(ex-d\right)}{4d^2e}+\frac{f^2\ln\left(ex+d\right)}{4d^2e}+\frac{fg}{\left(ex+d\right)e^2}-\frac{g^2\ln\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x)
```

```
[Out] -1/4/e^3*ln(e*x-d)*g^2-1/2/e^2/d*ln(e*x-d)*f*g-1/4/e/d^2*ln(e*x-d)*f^2-3/4/
e^3*ln(e*x+d)*g^2+1/2/e^2/d*ln(e*x+d)*f*g+1/4/e/d^2*ln(e*x+d)*f^2-1/2/e^3*d
/(e*x+d)*g^2+1/e^2/(e*x+d)*f*g-1/2/e/d/(e*x+d)*f^2
```

maxima [A] time = 0.46, size = 113, normalized size = 1.31

$$-\frac{e^2f^2 - 2\,defg + d^2g^2}{2\left(de^4x + d^2e^3\right)} + \frac{\left(e^2f^2 + 2\,defg - 3\,d^2g^2\right)\log\left(ex+d\right)}{4\,d^2e^3} - \frac{\left(e^2f^2 + 2\,defg + d^2g^2\right)\log\left(ex-d\right)}{4\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(d*e^4*x + d^2*e^3) + 1/4*(e^2*f^2 + 2
*d*e*f*g - 3*d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 + 2*d*e*f*g + d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

mupad [B] time = 2.70, size = 109, normalized size = 1.27

$$\frac{\ln\left(d+e\,x\right)\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}-\frac{\ln\left(d-e\,x\right)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}-\frac{d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d^2*e^3) - (log(d - e*x
)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^3) - (d^2*g^2 + e^2*f^2 - 2*d*e
*f*g)/(2*d*e^3*(d + e*x))
```

sympy [B] time = 1.00, size = 182, normalized size = 2.12

$$-\frac{d^2g^2 - 2defg + e^2f^2}{2d^2e^3 + 2de^4x} - \frac{(dg - ef)(3dg + ef)\log\left(x + \frac{-2d^3g^2 + d(dg-ef)(3dg+ef)}{d^2eg^2 - 2de^2fg - e^3f^2}\right)}{4d^2e^3} - \frac{(dg + ef)^2\log\left(x + \frac{-2d^3g^2 + d(dg+ef)^2}{d^2eg^2 - 2de^2fg - e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)
```

```
[Out] -(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) - (d*g - e*
f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*(3*d*g + e*f))/(d**2
*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) - (d*g + e*f)**2*log(x +
 (-2*d**3*g**2 + d*(d*g + e*f)**2)/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)
)/(4*d**2*e**3)
```

3.554 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=87

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

```
[Out] -1/4*(-d*g+e*f)^2/d/e^3/(e*x+d)^2-1/4*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+d
)+1/4*(d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(4*d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^2*e
^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^3}\,dx\\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^3}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^2}+\frac{(ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}\\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 1.00

$$\frac{\frac{2d(dg-ef)\left(2d^2g+de(2f+3gx)+e^2fx\right)}{(d+ex)^2}+(dg+ef)^2(-\log(d-ex))+(dg+ef)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] ((2*d*(-(e*f) + d*g)*(2*d^2*g + e^2*f*x + d*e*(2*f + 3*g*x)))/(d + e*x)^2 -
 (e*f + d*g)^2*Log[d - e*x] + (e*f + d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

fricas [B] time = 0.77, size = 271, normalized size = 3.11

$$-\frac{4\,d^2e^2f^2-4\,d^4g^2+2\left(de^3f^2+2\,d^2e^2fg-3\,d^3eg^2\right)x-\left(d^2e^2f^2+2\,d^3efg+d^4g^2+\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g - 3*d^3*e*g^
2)*x - (d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*
e^2*g^2)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) +
(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2
)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5
*x^2 + 2*d^4*e^4*x + d^5*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(-g^
2*d^2*exp(1)+g*d*exp(1)^2*f+g*d*f*exp(2)-exp(1)*f^2*exp(2))/(d^3*exp(1)^4-2
*d^3*exp(1)^2*exp(2)+d^3*exp(2)^2)*ln(abs(-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*
exp(1)^4+(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^2*exp(2)-2*(exp(1)*x+d)^-1/
exp(1)*d*exp(1)*exp(2)+exp(2)))-(g^2*d^2*exp(1)^4+g^2*d^2*exp(1)^2*exp(2)-4
*g*d*exp(1)^3*f*exp(2)+exp(1)^4*f^2*exp(2)+exp(1)^2*f^2*exp(2)^2)/2/(d^2*ex
p(1)^4-2*d^2*exp(1)^2*exp(2)+d^2*exp(2)^2)/exp(1)/abs(d)/exp(1)^2*ln(abs(2*
(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^2*e
xp(2)+2*d*exp(1)*exp(2)-2*exp(1)*abs(d)*exp(1)^2)/abs(2*(exp(1)*x+d)^-1/exp
(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^2*exp(2)+2*d*exp(1)*ex
p(2)+2*exp(1)*abs(d)*exp(1)^2))-((exp(1)*x+d)^-1/exp(1)*g^2*d^2*exp(1)^2-2*
(exp(1)*x+d)^-1/exp(1)*g*d*exp(1)^3*f+(exp(1)*x+d)^-1/exp(1)*exp(1)^4*f^2)/
(d^2*exp(1)^4-d^2*exp(1)^2*exp(2))
```

maple [B] time = 0.01, size = 206, normalized size = 2.37

$$-\frac{d\,g^2}{4\,(ex+d)^2\,e^3}-\frac{f^2}{4\,(ex+d)^2\,de}+\frac{fg}{2\,(ex+d)^2\,e^2}-\frac{fg}{2\,(ex+d)\,d\,e^2}-\frac{g^2\ln(ex-d)}{8d\,e^3}+\frac{g^2\ln(ex+d)}{8d\,e^3}-\frac{f^2}{4\,(ex+d)\,d^2e}-\frac{fg\ln}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/8/e^3/d*ln(e*x-d)*g^2-1/4/e^2/d^2*ln(e*x-d)*f*g-1/8/e/d^3*ln(e*x-d)*f^2+
3/4/e^3/(e*x+d)*g^2-1/2/d/e^2/(e*x+d)*f*g-1/4/d^2/e/(e*x+d)*f^2-1/4/e^3*d/(
```

```
e*x+d)^2*g^2+1/2/e^2/(e*x+d)^2*f*g-1/4/e/d/(e*x+d)^2*f^2+1/8/e^3/d*ln(e*x+d
)*g^2+1/4/e^2/d^2*ln(e*x+d)*f*g+1/8/e/d^3*ln(e*x+d)*f^2
```

maxima [A] time = 0.45, size = 149, normalized size = 1.71

$$-\frac{2\,de^2f^2-2\,d^3g^2+\left(e^3f^2+2\,de^2fg-3\,d^2eg^2\right)x}{4\left(d^2e^5x^2+2\,d^3e^4x+d^4e^3\right)}+\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex+d\right)}{8\,d^3e^3}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right.}{8\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 2*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d
^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d
^3*e^3)
```

mupad [B] time = 0.13, size = 100, normalized size = 1.15

$$\frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{4\,d^3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2)/(2*d*e^3) - (x*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d
^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*x) + (atanh((e*x)/d)*(d*g + e*f)^2)/(4*d^3*
e^3)
```

sympy [B] time = 1.03, size = 185, normalized size = 2.13

$$-\frac{-2d^3g^2+2de^2f^2+x\left(-3d^2eg^2+2de^2fg+e^3f^2\right)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2}-\frac{\left(dg+ef\right)^2\log\left(-\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)}+x\right)}{8d^3e^3}+\frac{\left(dg+ef\right)^2\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**3*g**2 + 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 + 2*d*e**2*f*g + e**3*f*
*2))/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g + e*f)**2*log(
-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3
) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f*
*2)) + x)/(8*d**3*e**3)
```

3.555 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

```
[Out] -1/6*(-d*g+e*f)^2/d/e^3/(e*x+d)^3-1/8*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+d
)^2-1/8*(d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(d*g+e*f)^2*arctanh(e*x/d)/d^4/e^3
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(6*d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^2*e
^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcT
anh[(e*x)/d])/(8*d^4*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^4}\,dx\\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^4}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^3}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^2}+\frac{(ef+dg)^2}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\int\frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}\\
&= -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 1.08

$$\frac{-\frac{8d^3(ef-dg)^2}{(d+ex)^3}+\frac{6d^2\left(3d^2g^2-2defg-e^2f^2\right)}{(d+ex)^2}-\frac{6d(dg+ef)^2}{d+ex}-3(dg+ef)^2\log(d-ex)+3(dg+ef)^2\log(d+ex)}{48d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] ((-8*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d*e*f*g + 3*d^
2*g^2))/(d + e*x)^2 - (6*d*(e*f + d*g)^2)/(d + e*x) - 3*(e*f + d*g)^2*Log[d
 - e*x] + 3*(e*f + d*g)^2*Log[d + e*x])/(48*d^4*e^3)
```

fricas [B] time = 0.82, size = 400, normalized size = 3.54

$$-\frac{20\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2+6\left(de^4f^2+2\,d^2e^3fg+d^3e^2g^2\right)x^2+6\left(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2\right)x-3\left(d^3e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/48*(20*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 + 6*(d*e^4*f^2 + 2*d^2*e^3*
f*g + d^3*e^2*g^2)*x^2 + 6*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x -
3*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g
^2)*x^3 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2
+ 2*d^3*e^2*f*g + d^4*e*g^2)*x)*log(e*x + d) + 3*(d^3*e^2*f^2 + 2*d^4*e*f*g
 + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g^2)*x^3 + 3*(d*e^4*f^2 + 2*d
^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2 + 2*d^3*e^2*f*g + d^4*e*g^2)
*x)*log(e*x - d))/(d^4*e^6*x^3 + 3*d^5*e^5*x^2 + 3*d^6*e^4*x + d^7*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(2*e
xp(2)^2*d*g*f-3*exp(2)^2*exp(1)*f^2-3*exp(2)*d^2*exp(1)*g^2+6*exp(2)*d*exp(
1)^2*g*f-exp(2)*exp(1)^3*f^2-d^2*exp(1)^3*g^2)/(2*exp(2)^3*d^4-6*exp(2)^2*d
^4*exp(1)^2+6*exp(2)*d^4*exp(1)^4-2*d^4*exp(1)^6)*ln(abs(-x^2*exp(2)+d^2))-
(-exp(2)^3*f^2-exp(2)^2*d^2*g^2+6*exp(2)^2*d*exp(1)*g*f-3*exp(2)^2*exp(1)^2
*f^2-3*exp(2)*d^2*exp(1)^2*g^2+2*exp(2)*d*exp(1)^3*g*f)*1/2/(exp(2)^3*d^3-3
*exp(2)^2*d^3*exp(1)^2+3*exp(2)*d^3*exp(1)^4-d^3*exp(1)^6)/exp(1)/abs(d)*ln
(abs(-2*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-(-2*exp
(2)^2*d*exp(1)*g*f+3*exp(2)^2*exp(1)^2*f^2+3*exp(2)*d^2*exp(1)^2*g^2-6*exp(
2)*d*exp(1)^3*g*f+exp(2)*exp(1)^4*f^2+d^2*exp(1)^4*g^2)/(exp(2)^3*d^4*exp(1
)-3*exp(2)^2*d^4*exp(1)^3+3*exp(2)*d^4*exp(1)^5-d^4*exp(1)^7)*ln(abs(x*exp(
1)+d))-(-exp(2)^2*d^4*g^2+6*exp(2)^2*d^3*exp(1)*g*f-5*exp(2)^2*d^2*exp(1)^2
*f^2-2*exp(2)*d^4*exp(1)^2*g^2-4*exp(2)*d^3*exp(1)^3*g*f+6*exp(2)*d^2*exp(1
)^4*f^2+3*d^4*exp(1)^4*g^2-2*d^3*exp(1)^5*g*f-d^2*exp(1)^6*f^2+(4*exp(2)^2*
d^2*exp(1)^2*g*f-4*exp(2)^2*d*exp(1)^3*f^2-4*exp(2)*d^3*exp(1)^3*g^2+4*exp(
2)*d*exp(1)^5*f^2+4*d^3*exp(1)^5*g^2-4*d^2*exp(1)^6*g*f)*x)/2/d^4/exp(1)/(e
xp(2)-exp(1)^2)^3/(x*exp(1)+d)^2
```

maple [B] time = 0.01, size = 259, normalized size = 2.29

$$-\frac{d\,g^2}{6\,(ex+d)^3\,e^3}-\frac{f^2}{6\,(ex+d)^3\,de}+\frac{fg}{3\,(ex+d)^3\,e^2}-\frac{fg}{4\,(ex+d)^2\,d\,e^2}-\frac{f^2}{8\,(ex+d)^2\,d^2e}+\frac{3g^2}{8\,(ex+d)^2\,e^3}-\frac{g^2}{8\,(ex+d)\,d\,e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x)
```

```
[Out] -1/16/e^3/d^2*ln(e*x-d)*g^2-1/8/e^2/d^3*ln(e*x-d)*f*g-1/16/e/d^4*ln(e*x-d)*
f^2+3/8/e^3/(e*x+d)^2*g^2-1/4/d/e^2/(e*x+d)^2*f*g-1/8/d^2/e/(e*x+d)^2*f^2-1
/6/e^3*d/(e*x+d)^3*g^2+1/3/e^2/(e*x+d)^3*f*g-1/6/e/d/(e*x+d)^3*f^2+1/16/e^3
/d^2*ln(e*x+d)*g^2+1/8/e^2/d^3*ln(e*x+d)*f*g+1/16/e/d^4*ln(e*x+d)*f^2-1/8/d
/e^3/(e*x+d)*g^2-1/4/d^2/e^2/(e*x+d)*f*g-1/8/d^3/e/(e*x+d)*f^2
```

maxima [A] time = 0.48, size = 206, normalized size = 1.82

$$-\frac{10\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2+3\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^2+3\left(3\,de^3f^2+6\,d^2e^2fg-d^3eg^2\right)x}{24\left(d^3e^6x^3+3\,d^4e^5x^2+3\,d^5e^4x+d^6e^3\right)}+\frac{\left(e^2f^2+2\,def\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/24*(10*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 + 3*(e^4*f^2 + 2*d*e^3*f*g
+ d^2*e^2*g^2)*x^2 + 3*(3*d*e^3*f^2 + 6*d^2*e^2*f*g - d^3*e*g^2)*x)/(d^3*e^
6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3) + 1/16*(e^2*f^2 + 2*d*e*f*g
+ d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*lo
g(e*x - d)/(d^4*e^3)
```

mupad [B] time = 2.65, size = 152, normalized size = 1.35

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{8\,d^4\,e^3}-\frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2}{12\,d\,e^3}+\frac{x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{8\,d^3\,e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^3),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(8*d^4*e^3) - ((5*e^2*f^2 - d^2*g^2 + 2*d*e*
f*g)/(12*d*e^3) + (x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(8*d^2*e^2) + (x^2*
(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 +
3*d^2*e*x)
```

sympy [B] time = 1.42, size = 248, normalized size = 2.19

$$-\frac{-2d^4g^2+4d^3efg+10d^2e^2f^2+x^2\left(3d^2e^2g^2+6de^3fg+3e^4f^2\right)+x\left(-3d^3eg^2+18d^2e^2fg+9de^3f^2\right)}{24d^6e^3+72d^5e^4x+72d^4e^5x^2+24d^3e^6x^3}-\frac{\left(dg+ef\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**4*g**2 + 4*d**3*e*f*g + 10*d**2*e**2*f**2 + x**2*(3*d**2*e**2*g**2
+ 6*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 18*d**2*e**2*f*g + 9*d*
e**3*f**2))/(24*d**6*e**3 + 72*d**5*e**4*x + 72*d**4*e**5*x**2 + 24*d**3*e*
*6*x**3) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g +
 e**2*f**2)) + x)/(16*d**4*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(
d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(16*d**4*e**3)
```

3.556 $\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=139

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

```
[Out] -1/8*(-d*g+e*f)^2/d/e^3/(e*x+d)^4-1/12*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+
d)^3-1/16*(d*g+e*f)^2/d^3/e^3/(e*x+d)^2-1/16*(d*g+e*f)^2/d^4/e^3/(e*x+d)+1/
16*(d*g+e*f)^2*arctanh(e*x/d)/d^5/e^3
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(8*d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d^2*
e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)^2/(
16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^5}\,dx\\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^5}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^4}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^3}+\frac{(ef+dg)^2}{16d^4e^2(d+ex)^2}+\right.\\
&= -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(e}{}\\
&= -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(e}{}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 142, normalized size = 1.02

$$-\frac{\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{6d^2(dg+ef)^2}{(d+ex)^2}+\frac{8d^3(-3d^2g^2+2defg+e^2f^2)}{(d+ex)^3}+\frac{6d(dg+ef)^2}{d+ex}+3(dg+ef)^2\log(d-ex)-3(dg+ef)^2\log(d+ex)}{96d^5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/96*((12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(e^2*f^2 + 2*d*e*f*g - 3
*d^2*g^2))/(d + e*x)^3 + (6*d^2*(e*f + d*g)^2)/(d + e*x)^2 + (6*d*(e*f + d*
g)^2)/(d + e*x) + 3*(e*f + d*g)^2*Log[d - e*x] - 3*(e*f + d*g)^2*Log[d + e*
x])/(d^5*e^3)
```

fricas [B] time = 0.89, size = 511, normalized size = 3.68

$$-\frac{32\,d^4e^2f^2+16\,d^5efg+6\left(de^5f^2+2\,d^2e^4fg+d^3e^3g^2\right)x^3+24\left(d^2e^4f^2+2\,d^3e^3fg+d^4e^2g^2\right)x^2+2\left(19\,d^3e^3f^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/96*(32*d^4*e^2*f^2 + 16*d^5*e*f*g + 6*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e
^3*g^2)*x^3 + 24*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 2*(19*d^
3*e^3*f^2 + 38*d^4*e^2*f*g + 3*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + 2*d^5*e*f*g
+ d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^
2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2
)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*log(e*x + d) + 3*(d^
4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x
^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d
^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)
*x)*log(e*x - d))/(d^5*e^7*x^4 + 4*d^6*e^6*x^3 + 6*d^7*e^5*x^2 + 4*d^8*e^4*
x + d^9*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(exp
(2)^3*d*g*f-2*exp(2)^3*exp(1)*f^2-2*exp(2)^2*d^2*exp(1)*g^2+6*exp(2)^2*d*ex
p(1)^2*g*f-2*exp(2)^2*exp(1)^3*f^2-2*exp(2)*d^2*exp(1)^3*g^2+exp(2)*d*exp(1
)^4*g*f)/(exp(2)^4*d^5-4*exp(2)^3*d^5*exp(1)^2+6*exp(2)^2*d^5*exp(1)^4-4*ex
p(2)*d^5*exp(1)^6+d^5*exp(1)^8)*ln(abs(-x^2*exp(2)+d^2))-(-exp(2)^4*f^2-exp
(2)^3*d^2*g^2+8*exp(2)^3*d*exp(1)*g*f-6*exp(2)^3*exp(1)^2*f^2-6*exp(2)^2*d^
2*exp(1)^2*g^2+8*exp(2)^2*d*exp(1)^3*g*f-exp(2)^2*exp(1)^4*f^2-exp(2)*d^2*e
xp(1)^4*g^2)*1/2/(exp(2)^4*d^4-4*exp(2)^3*d^4*exp(1)^2+6*exp(2)^2*d^4*exp(1
)^4-4*exp(2)*d^4*exp(1)^6+d^4*exp(1)^8)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*
exp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-(-2*exp(2)^3*d*exp(1)*g*f+
4*exp(2)^3*exp(1)^2*f^2+4*exp(2)^2*d^2*exp(1)^2*g^2-12*exp(2)^2*d*exp(1)^3*
g*f+4*exp(2)^2*exp(1)^4*f^2+4*exp(2)*d^2*exp(1)^4*g^2-2*exp(2)*d*exp(1)^5*g
*f)/(exp(2)^4*d^5*exp(1)-4*exp(2)^3*d^5*exp(1)^3+6*exp(2)^2*d^5*exp(1)^5-4*
exp(2)*d^5*exp(1)^7+d^5*exp(1)^9)*ln(abs(x*exp(1)+d))-((6*exp(2)^3*d^2*exp(
1)^3*g*f-9*exp(2)^3*d*exp(1)^4*f^2-9*exp(2)^2*d^3*exp(1)^4*g^2+12*exp(2)^2*
d^2*exp(1)^5*g*f+6*exp(2)^2*d*exp(1)^6*f^2+6*exp(2)*d^3*exp(1)^6*g^2-18*exp
(2)*d^2*exp(1)^7*g*f+3*exp(2)*d*exp(1)^8*f^2+3*d^3*exp(1)^8*g^2)*x^2+(15*ex
```

```
p(2)^3*d^3*exp(1)^2*g*f-21*exp(2)^3*d^2*exp(1)^3*f^2-21*exp(2)^2*d^4*exp(1)
^3*g^2+21*exp(2)^2*d^3*exp(1)^4*g*f+18*exp(2)^2*d^2*exp(1)^5*f^2+18*exp(2)*
d^4*exp(1)^5*g^2-39*exp(2)*d^3*exp(1)^6*g*f+3*exp(2)*d^2*exp(1)^7*f^2+3*d^4
*exp(1)^7*g^2+3*d^3*exp(1)^8*g*f)*x-exp(2)^3*d^5*g^2+11*exp(2)^3*d^4*exp(1)
*g*f-13*exp(2)^3*d^3*exp(1)^2*f^2-9*exp(2)^2*d^5*exp(1)^2*g^2+3*exp(2)^2*d^
4*exp(1)^3*g*f+15*exp(2)^2*d^3*exp(1)^4*f^2+9*exp(2)*d^5*exp(1)^4*g^2-15*ex
p(2)*d^4*exp(1)^5*g*f-3*exp(2)*d^3*exp(1)^6*f^2+d^5*exp(1)^6*g^2+d^4*exp(1)
^7*g*f+d^3*exp(1)^8*f^2)/3/d^5/exp(1)/(exp(2)-exp(1)^2)^4/(x*exp(1)+d)^3
```

maple [B] time = 0.01, size = 312, normalized size = 2.24

$$-\frac{d\,g^2}{8\,(ex+d)^4\,e^3}-\frac{f^2}{8\,(ex+d)^4\,de}+\frac{fg}{4\,(ex+d)^4\,e^2}-\frac{fg}{6\,(ex+d)^3\,d\,e^2}-\frac{f^2}{12\,(ex+d)^3\,d^2e}+\frac{g^2}{4\,(ex+d)^3\,e^3}-\frac{g^2}{16\,(ex+d)^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x)
```

```
[Out] -1/32/e^3/d^3*ln(e*x-d)*g^2-1/16/e^2/d^4*ln(e*x-d)*f*g-1/32/e/d^5*ln(e*x-d)
*f^2+1/4/e^3/(e*x+d)^3*g^2-1/6/d/e^2/(e*x+d)^3*f*g-1/12/d^2/e/(e*x+d)^3*f^2
-1/8/e^3*d/(e*x+d)^4*g^2+1/4/e^2/(e*x+d)^4*f*g-1/8/e/d/(e*x+d)^4*f^2+1/32/e
^3/d^3*ln(e*x+d)*g^2+1/16/e^2/d^4*ln(e*x+d)*f*g+1/32/e/d^5*ln(e*x+d)*f^2-1/
16/d^2/e^3/(e*x+d)*g^2-1/8/d^3/e^2/(e*x+d)*f*g-1/16/d^4/e/(e*x+d)*f^2-1/16/
d/e^3/(e*x+d)^2*g^2-1/8/d^2/e^2/(e*x+d)^2*f*g-1/16/d^3/e/(e*x+d)^2*f^2
```

maxima [A] time = 0.49, size = 236, normalized size = 1.70

$$-\frac{16\,d^3ef^2+8\,d^4fg+3\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^3+12\left(de^3f^2+2\,d^2e^2fg+d^3eg^2\right)x^2+\left(19\,d^2e^2f^2+38\,d^3e\right.}{48\left(d^4e^6x^4+4\,d^5e^5x^3+6\,d^6e^4x^2+4\,d^7e^3x+d^8e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/48*(16*d^3*e*f^2 + 8*d^4*f*g + 3*(e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2)*x
^3 + 12*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x^2 + (19*d^2*e^2*f^2 + 38*
d^3*e*f*g + 3*d^4*g^2)*x)/(d^4*e^6*x^4 + 4*d^5*e^5*x^3 + 6*d^6*e^4*x^2 + 4*
d^7*e^3*x + d^8*e^2) + 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d
^5*e^3) - 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

mupad [B] time = 0.15, size = 180, normalized size = 1.29

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)^2}{16\,d^5\,e^3}-\frac{\frac{x^3\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{16\,d^4}+\frac{2\,e\,f^2+d\,g\,f}{6\,d\,e^2}+\frac{x\,\left(3\,d^2\,g^2+38\,d\,e\,f\,g+19\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^3\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^4),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(16*d^5*e^3) - ((x^3*(d^2*g^2 + e^2*f^2 + 2*
d*e*f*g))/(16*d^4) + (2*e*f^2 + d*f*g)/(6*d*e^2) + (x*(3*d^2*g^2 + 19*e^2*f
^2 + 38*d*e*f*g))/(48*d^2*e^2) + (x^2*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d
^3*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

sympy [B] time = 1.93, size = 282, normalized size = 2.03

$$-\frac{8d^4fg+16d^3ef^2+x^3\left(3d^2e^2g^2+6de^3fg+3e^4f^2\right)+x^2\left(12d^3eg^2+24d^2e^2fg+12de^3f^2\right)+x\left(3d^4g^2+38d^3e\right.}{48d^8e^2+192d^7e^3x+288d^6e^4x^2+192d^5e^5x^3+48d^4e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2),x)
```

```
[Out] -(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*e
**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x*(
3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d**7*
e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) - (d*
g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) +
 x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*
d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)
```

3.557 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{32d^5(dg+ef)^2}{e^3(d-ex)}+\frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3}+\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46d\right.$$

```
[Out] d^3*(112*d^2*g^2+160*d*e*f*g+49*e^2*f^2)*x/e^2+1/2*d^2*(80*d^2*g^2+98*d*e*f
*g+23*e^2*f^2)*x^2/e+1/3*d*(49*d^2*g^2+46*d*e*f*g+7*e^2*f^2)*x^3+1/4*e*(23*
d^2*g^2+14*d*e*f*g+e^2*f^2)*x^4+1/5*e^2*g*(7*d*g+2*e*f)*x^5+1/6*e^3*g^2*x^6
+32*d^5*(d*g+e*f)^2/e^3/(-e*x+d)+16*d^4*(d*g+e*f)*(9*d*g+5*e*f)*ln(-e*x+d)/
e^3
```

Rubi [A] time = 0.28, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+23e^2f^2\right)}{2e}+\frac{d^3x\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) +
 (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^5(f+gx)^2}{(d-ex)^2}\,dx$$

$$=\int \left(\frac{d^3\left(49e^2f^2+160defg+112d^2g^2\right)}{e^2}+\frac{d^2\left(23e^2f^2+98defg+80d^2g^2\right)x}{e}+d\left(7e\right.\right.$$

$$=\frac{d^3\left(49e^2f^2+160defg+112d^2g^2\right)x}{e^2}+\frac{d^2\left(23e^2f^2+98defg+80d^2g^2\right)x^2}{2e}+\frac{1}{3}d\left(7e\right.$$

Mathematica [A] time = 0.12, size = 226, normalized size = 1.04

$$-\frac{32d^5(dg+ef)^2}{e^3(ex-d)}+\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 - (32*d^5*(e*f + d*g)^2)/(e^3*(-d + e*x))
+ (16*d^4*(5*e^2*f^2 + 14*d*e*f*g + 9*d^2*g^2)*Log[d - e*x])/e^3
```

fricas [A] time = 0.82, size = 328, normalized size = 1.50

$$\frac{10\,e^7g^2x^7-1920\,d^5e^2f^2-3840\,d^6efg-1920\,d^7g^2+2\left(12\,e^7fg+37\,de^6g^2\right)x^6+3\left(5\,e^7f^2+62\,de^6fg+87\,d^2e^5g^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*e^7*g^2*x^7 - 1920*d^5*e^2*f^2 - 3840*d^6*e*f*g - 1920*d^7*g^2 + 2
*(12*e^7*f*g + 37*d*e^6*g^2)*x^6 + 3*(5*e^7*f^2 + 62*d*e^6*f*g + 87*d^2*e^5
*g^2)*x^5 + 5*(25*d*e^6*f^2 + 142*d^2*e^5*f*g + 127*d^3*e^4*g^2)*x^4 + 10*(
55*d^2*e^5*f^2 + 202*d^3*e^4*f*g + 142*d^4*e^3*g^2)*x^3 + 90*(25*d^3*e^4*f^
2 + 74*d^4*e^3*f*g + 48*d^5*e^2*g^2)*x^2 - 60*(49*d^4*e^3*f^2 + 160*d^5*e^2
*f*g + 112*d^6*e*g^2)*x - 960*(5*d^5*e^2*f^2 + 14*d^6*e*f*g + 9*d^7*g^2 - (
5*d^4*e^3*f^2 + 14*d^5*e^2*f*g + 9*d^6*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e
^3)
```

giac [A] time = 0.18, size = 367, normalized size = 1.68

$$8\left(9\,d^6g^2e^7+14\,d^5fge^8+5\,d^4f^2e^9\right)e^{(-10)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{60}\left(10\,g^2x^6e^{27}+84\,dg^2x^5e^{26}+345\,d^2g^2x^4e^{25}+980\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 8*(9*d^6*g^2*e^7 + 14*d^5*f*g*e^8 + 5*d^4*f^2*e^9)*e^(-10)*log(abs(x^2*e^2
- d^2)) + 1/60*(10*g^2*x^6*e^27 + 84*d*g^2*x^5*e^26 + 345*d^2*g^2*x^4*e^25
+ 980*d^3*g^2*x^3*e^24 + 2400*d^4*g^2*x^2*e^23 + 6720*d^5*g^2*x*e^22 + 24*f
*g*x^5*e^27 + 210*d*f*g*x^4*e^26 + 920*d^2*f*g*x^3*e^25 + 2940*d^3*f*g*x^2*
e^24 + 9600*d^4*f*g*x*e^23 + 15*f^2*x^4*e^27 + 140*d*f^2*x^3*e^26 + 690*d^2
*f^2*x^2*e^25 + 2940*d^3*f^2*x*e^24)*e^(-24) + 8*(9*d^7*g^2*e^6 + 14*d^6*f*
g*e^7 + 5*d^5*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2
*abs(d)*e))/abs(d) - 32*(d^8*g^2*e^7 + 2*d^7*f*g*e^8 + d^6*f^2*e^9 + (d^7*g
^2*e^8 + 2*d^6*f*g*e^9 + d^5*f^2*e^10)*x)*e^(-10)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 286, normalized size = 1.31

$$\frac{e^3g^2x^6}{6}+\frac{7d\,e^2g^2x^5}{5}+\frac{2e^3fg\,x^5}{5}+\frac{23d^2e\,g^2x^4}{4}+\frac{7d\,e^2fg\,x^4}{2}+\frac{e^3f^2x^4}{4}+\frac{49d^3g^2x^3}{3}+\frac{46d^2efg\,x^3}{3}+\frac{7d\,e^2f^2x^3}{3}+\frac{40d^4g^2x^2}{e}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/6*e^3*g^2*x^6+7/5*e^2*x^5*d*g^2+2/5*e^3*x^5*f*g+23/4*e*x^4*d^2*g^2+7/2*e^
2*x^4*d*f*g+1/4*e^3*x^4*f^2+49/3*x^3*d^3*g^2+46/3*e*x^3*d^2*f*g+7/3*e^2*x^3
*d*f^2+40/e*x^2*d^4*g^2+49*x^2*d^3*f*g+23/2*e*x^2*d^2*f^2+112/e^2*d^5*g^2*x
+160/e*d^4*f*g*x+49*d^3*f^2*x+144*d^6/e^3*ln(e*x-d)*g^2+224*d^5/e^2*ln(e*x-
d)*f*g+80*d^4/e*ln(e*x-d)*f^2-32*d^7/e^3/(e*x-d)*g^2-64*d^6/e^2/(e*x-d)*f*g
-32*d^5/e/(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 258, normalized size = 1.18

$$-\frac{32\left(d^5e^2f^2+2\,d^6efg+d^7g^2\right)}{e^4x-de^3}+\frac{10\,e^5g^2x^6+12\left(2\,e^5fg+7\,de^4g^2\right)x^5+15\left(e^5f^2+14\,de^4fg+23\,d^2e^3g^2\right)x^4+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -32*(d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/(e^4*x - d*e^3) + 1/60*(10*e^5*g^
2*x^6 + 12*(2*e^5*f*g + 7*d*e^4*g^2)*x^5 + 15*(e^5*f^2 + 14*d*e^4*f*g + 23*
d^2*e^3*g^2)*x^4 + 20*(7*d*e^4*f^2 + 46*d^2*e^3*f*g + 49*d^3*e^2*g^2)*x^3 +
 30*(23*d^2*e^3*f^2 + 98*d^3*e^2*f*g + 80*d^4*e*g^2)*x^2 + 60*(49*d^3*e^2*f
^2 + 160*d^4*e*f*g + 112*d^5*g^2)*x)/e^2 + 16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g
 + 9*d^6*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.64, size = 1029, normalized size = 4.72

$$x^5\left(\frac{e^2\,g\,\left(5\,d\,g+2\,e\,f\right)}{5}+\frac{2\,d\,e^2\,g^2}{5}\right)+x^3\left(\frac{5\,d\,\left(2\,d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2\right)}{3}+\frac{2\,d\,\left(\frac{10\,d^2\,e^3\,g^2+10\,d\,e^4\,f\,g+e^5\,f^2}{e^2}-d^2\,e\,g^2\right.}{3\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^5*((e^2*g*(5*d*g + 2*e*f))/5 + (2*d*e^2*g^2)/5) + x^3*((5*d*(2*d^2*g^2 +
e^2*f^2 + 4*d*e*f*g))/3 + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e
^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(3*e) - (d
^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/(3*e^2)) + x^4*((e^5*f^2 + 10*d^2
*e^3*g^2 + 10*d*e^4*f*g)/(4*e^2) - (d^2*e*g^2)/4 + (d*(e^2*g*(5*d*g + 2*e*f
) + 2*d*e^2*g^2))/(2*e)) + x^2*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/(
2*e) - (d^2*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2
*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(2*e^2) + (d*(5*d*(2*d^2*g^2
+ e^2*f^2 + 4*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^
2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e
^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e^2))/e) + x*((d^5*g^2 + 10*d^3*e^2*f^
2 + 10*d^4*e*f*g)/e^2 - (d^2*(5*d*(2*d^2*g^2 + e^2*f^2 + 4*d*e*f*g) + (2*d*
((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5
*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2
*g^2))/e^2))/e^2 + (2*d*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/e - (d^2
*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(
5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e^2 + (2*d*(5*d*(2*d^2*g^2 + e^2*f^2 + 4
*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2
```

```
 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g +
 2*e*f) + 2*d*e^2*g^2))/e^2))/e))/e) + (log(e*x - d)*(144*d^6*g^2 + 80*d^4*
e^2*f^2 + 224*d^5*e*f*g))/e^3 + (32*(d^7*g^2 + d^5*e^2*f^2 + 2*d^6*e*f*g))/
(e*(d*e^2 - e^3*x)) + (e^3*g^2*x^6)/6
```

sympy [A] time = 1.20, size = 250, normalized size = 1.15

$$\frac{16d^4\left(dg+ef\right)\left(9dg+5ef\right)\log\left(-d+ex\right)}{e^3}+\frac{e^3g^2x^6}{6}+x^5\left(\frac{7de^2g^2}{5}+\frac{2e^3fg}{5}\right)+x^4\left(\frac{23d^2eg^2}{4}+\frac{7de^2fg}{2}+\frac{e^3f^2}{4}\right)+x^3\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*log(-d + e*x)/e**3 + e**3*g**2*x**6/6 +
 x**5*(7*d*e**2*g**2/5 + 2*e**3*f*g/5) + x**4*(23*d**2*e*g**2/4 + 7*d*e**2*
f*g/2 + e**3*f**2/4) + x**3*(49*d**3*g**2/3 + 46*d**2*e*f*g/3 + 7*d*e**2*f*
*2/3) + x**2*(40*d**4*g**2/e + 49*d**3*f*g + 23*d**2*e*f**2/2) + x*(112*d**
5*g**2/e**2 + 160*d**4*f*g/e + 49*d**3*f**2) + (-32*d**7*g**2 - 64*d**6*e*f
*g - 32*d**5*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.558 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=177

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)}+\frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3}+\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg\right.}{e}$$

```
[Out] d^2*(48*d^2*g^2+64*d*e*f*g+17*e^2*f^2)*x/e^2+d*(16*d^2*g^2+17*d*e*f*g+3*e^2
*f^2)*x^2/e+1/3*(17*d^2*g^2+12*d*e*f*g+e^2*f^2)*x^3+1/2*e*g*(3*d*g+e*f)*x^4
+1/5*e^2*g^2*x^5+16*d^4*(d*g+e*f)^2/e^3/(-e*x+d)+32*d^3*(d*g+e*f)*(2*d*g+e*
f)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.23, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{16d^4(dg}{e^3(d-}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d
 - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^2}\,dx$$

$$=\int \left(\frac{d^2\left(17e^2f^2+64defg+48d^2g^2\right)}{e^2}+\frac{2d\left(3e^2f^2+17defg+16d^2g^2\right)x}{e}+\left(e^2f^2+\right.\right.$$

$$=\frac{d^2\left(17e^2f^2+64defg+48d^2g^2\right)x}{e^2}+\frac{d\left(3e^2f^2+17defg+16d^2g^2\right)x^2}{e}+\frac{1}{3}\left(e^2f^2+1\right.$$

Mathematica [A] time = 0.12, size = 185, normalized size = 1.05

$$-\frac{16d^4(dg+ef)^2}{e^3(ex-d)}+\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+1\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 - (16*d^4*(e*f + d*g)^2)/(e^3*(-
d + e*x)) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2*g^2)*Log[d - e*x])/e^3
```

fricas [A] time = 0.90, size = 288, normalized size = 1.63

$$\frac{6 e^6 g^2 x^6 - 480 d^4 e^2 f^2 - 960 d^5 e f g - 480 d^6 g^2 + 3 \left(5 e^6 f g + 13 d e^5 g^2\right) x^5 + 5 \left(2 e^6 f^2 + 21 d e^5 f g + 25 d^2 e^4 g^2\right) x^4 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/30*(6*e^6*g^2*x^6 - 480*d^4*e^2*f^2 - 960*d^5*e*f*g - 480*d^6*g^2 + 3*(5*
e^6*f*g + 13*d*e^5*g^2)*x^5 + 5*(2*e^6*f^2 + 21*d*e^5*f*g + 25*d^2*e^4*g^2)
*x^4 + 10*(8*d*e^5*f^2 + 39*d^2*e^4*f*g + 31*d^3*e^3*g^2)*x^3 + 30*(14*d^2*
e^4*f^2 + 47*d^3*e^3*f*g + 32*d^4*e^2*g^2)*x^2 - 30*(17*d^3*e^3*f^2 + 64*d^
4*e^2*f*g + 48*d^5*e*g^2)*x - 960*(d^4*e^2*f^2 + 3*d^5*e*f*g + 2*d^6*g^2 -
(d^3*e^3*f^2 + 3*d^4*e^2*f*g + 2*d^5*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3
)
```

giac [A] time = 0.18, size = 327, normalized size = 1.85

$$16 \left(2 d^5 g^2 e^5 + 3 d^4 f g e^6 + d^3 f^2 e^7\right) e^{(-8)} \log\left(\left|x^2 e^2 - d^2\right|\right) + \frac{1}{30} \left(6 g^2 x^5 e^{22} + 45 d g^2 x^4 e^{21} + 170 d^2 g^2 x^3 e^{20} + 480 d^3 g^2 x \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 16*(2*d^5*g^2*e^5 + 3*d^4*f*g*e^6 + d^3*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d
^2)) + 1/30*(6*g^2*x^5*e^22 + 45*d*g^2*x^4*e^21 + 170*d^2*g^2*x^3*e^20 + 48
0*d^3*g^2*x^2*e^19 + 1440*d^4*g^2*x*e^18 + 15*f*g*x^4*e^22 + 120*d*f*g*x^3*
e^21 + 510*d^2*f*g*x^2*e^20 + 1920*d^3*f*g*x*e^19 + 10*f^2*x^3*e^22 + 90*d*
f^2*x^2*e^21 + 510*d^2*f^2*x*e^20)*e^(-20) + 16*(2*d^6*g^2*e^6 + 3*d^5*f*g*
e^7 + d^4*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs
(d)*e))/abs(d) - 16*(d^7*g^2*e^5 + 2*d^6*f*g*e^6 + d^5*f^2*e^7 + (d^6*g^2*e
^6 + 2*d^5*f*g*e^7 + d^4*f^2*e^8)*x)*e^(-8)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 245, normalized size = 1.38

$$\frac{e^2 g^2 x^5}{5} + \frac{3 d e g^2 x^4}{2} + \frac{e^2 f g x^4}{2} + \frac{17 d^2 g^2 x^3}{3} + 4 d e f g x^3 + \frac{e^2 f^2 x^3}{3} + \frac{16 d^3 g^2 x^2}{e} + 17 d^2 f g x^2 + 3 d e f^2 x^2 - \frac{16 d^6 g^2}{(e x - d) e^3} - \frac{32 d^5 f g}{(e x - d) \cdots} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/5*e^2*g^2*x^5+3/2*d*e*g^2*x^4+1/2*e^2*f*g*x^4+17/3*d^2*g^2*x^3+4*d*e*f*g*
x^3+1/3*e^2*f^2*x^3+16*d^3/e*g^2*x^2+17*d^2*f*g*x^2+3*d*e*f^2*x^2+48*d^4/e^
2*g^2*x+64*d^3/e*f*g*x+17*d^2*f^2*x+64*d^5/e^3*g^2*ln(e*x-d)+96*d^4/e^2*f*g
*ln(e*x-d)+32*d^3/e*f^2*ln(e*x-d)-16*d^6/e^3/(e*x-d)*g^2-32*d^5/e^2/(e*x-d)
*f*g-16*d^4/e/(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 218, normalized size = 1.23

$$-\frac{16 \left(d^4 e^2 f^2 + 2 d^5 e f g + d^6 g^2\right)}{e^4 x - d e^3} + \frac{6 e^4 g^2 x^5 + 15 \left(e^4 f g + 3 d e^3 g^2\right) x^4 + 10 \left(e^4 f^2 + 12 d e^3 f g + 17 d^2 e^2 g^2\right) x^3 + 30 \left(3 d \cdots\right.}{30 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -16*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/(e^4*x - d*e^3) + 1/30*(6*e^4*g^2
*x^5 + 15*(e^4*f*g + 3*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 12*d*e^3*f*g + 17*d^2
*e^2*g^2)*x^3 + 30*(3*d*e^3*f^2 + 17*d^2*e^2*f*g + 16*d^3*e*g^2)*x^2 + 30*(
17*d^2*e^2*f^2 + 64*d^3*e*f*g + 48*d^4*g^2)*x)/e^2 + 32*(d^3*e^2*f^2 + 3*d^
4*e*f*g + 2*d^5*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.61, size = 565, normalized size = 3.19

$$x^2\left(\frac{2\,d\,\left(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2\right)}{e}-\frac{d^2\,\left(2\,e\,g\,\left(2\,d\,g+e\,f\right)+2\,d\,e\,g^2\right)}{2\,e^2}+\frac{d\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^2}-d^2\,g^2+\frac{2\,d\,(2}{}\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((2*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e - (d^2*(2*e*g*(2*d*g + e*f) +
2*d*e*g^2))/(2*e^2) + (d*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2
*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e))/e) + x^4*((e*g*(2*d*g +
e*f))/2 + (d*e*g^2)/2) + x*((d^4*g^2 + 6*d^2*e^2*f^2 + 8*d^3*e*f*g)/e^2 - (
d^2*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2
*d*g + e*f) + 2*d*e*g^2))/e))/e^2 + (2*d*((4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f
*g))/e - (d^2*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e^2 + (2*d*((e^4*f^2 + 6*d
^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e
*g^2))/e))/e))/e) + x^3*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(3*e^2) -
(d^2*g^2)/3 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/(3*e)) + (log(e*x - d
)*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g))/e^3 + (16*(d^6*g^2 + d^4*e^
2*f^2 + 2*d^5*e*f*g))/(e*(d*e^2 - e^3*x)) + (e^2*g^2*x^5)/5
```

sympy [A] time = 1.01, size = 199, normalized size = 1.12

$$\frac{32d^3\left(dg+ef\right)\left(2dg+ef\right)\log\left(-d+ex\right)}{e^3}+\frac{e^2g^2x^5}{5}+x^4\left(\frac{3deg^2}{2}+\frac{e^2fg}{2}\right)+x^3\left(\frac{17d^2g^2}{3}+4defg+\frac{e^2f^2}{3}\right)+x^2\left(\frac{16d^3}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 32*d**3*(d*g + e*f)*(2*d*g + e*f)*log(-d + e*x)/e**3 + e**2*g**2*x**5/5 + x
**4*(3*d*e*g**2/2 + e**2*f*g/2) + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2*f
**2/3) + x**2*(16*d**3*g**2/e + 17*d**2*f*g + 3*d*e*f**2) + x*(48*d**4*g**2
/e**2 + 64*d**3*f*g/e + 17*d**2*f**2) + (-16*d**6*g**2 - 32*d**5*e*f*g - 16
*d**4*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.559 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=146

$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}+\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2\right.}{e^2}$$

[Out] d*(20*d^2*g^2+24*d*e*f*g+5*e^2*f^2)*x/e^2+1/2*(12*d^2*g^2+10*d*e*f*g+e^2*f^2)*x^2/e+1/3*g*(5*d*g+2*e*f)*x^3+1/4*e*g^2*x^4+8*d^3*(d*g+e*f)^2/e^3/(-e*x+d)+4*d^2*(d*g+e*f)*(7*d*g+3*e*f)*ln(-e*x+d)/e^3

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {848, 88}

$$\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-}{e^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]

[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g + 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log[d - e*x])/e^3

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx &= \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^2} dx \\
&= \int \left(\frac{d\left(5e^2f^2+24defg+20d^2g^2\right)}{e^2}+\frac{\left(e^2f^2+10defg+12d^2g^2\right)x}{e}+g(2ef+5dg)x^2+\right. \\
&= \frac{d\left(5e^2f^2+24defg+20d^2g^2\right)x}{e^2}+\frac{\left(e^2f^2+10defg+12d^2g^2\right)x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3+
\end{aligned}$$

Mathematica [A] time = 0.09, size = 154, normalized size = 1.05

$$-\frac{8d^3(dg+ef)^2}{e^3(ex-d)}+\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{4d^2\left(7d^2g^2+10defg+3e^2f^2\right)l}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 - (8*d^3
*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^2*g
^2)*Log[d - e*x])/e^3
```

fricas [A] time = 0.83, size = 251, normalized size = 1.72

$$\frac{3\,e^{5}g^{2}x^{5}-96\,d^{3}e^{2}f^{2}-192\,d^{4}efg-96\,d^{5}g^{2}+\left(8\,e^{5}fg+17\,de^{4}g^{2}\right)x^{4}+2\left(3\,e^{5}f^{2}+26\,de^{4}fg+26\,d^{2}e^{3}g^{2}\right)x^{3}+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*e^5*g^2*x^5 - 96*d^3*e^2*f^2 - 192*d^4*e*f*g - 96*d^5*g^2 + (8*e^5*
f*g + 17*d*e^4*g^2)*x^4 + 2*(3*e^5*f^2 + 26*d*e^4*f*g + 26*d^2*e^3*g^2)*x^3
 + 6*(9*d*e^4*f^2 + 38*d^2*e^3*f*g + 28*d^3*e^2*g^2)*x^2 - 12*(5*d^2*e^3*f^
2 + 24*d^3*e^2*f*g + 20*d^4*e*g^2)*x - 48*(3*d^3*e^2*f^2 + 10*d^4*e*f*g + 7
*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*d^4*e*g^2)*x)*log(e*x - d))/
(e^4*x - d*e^3)
```

giac [B] time = 0.19, size = 291, normalized size = 1.99

$$2\left(7\,d^{4}g^{2}e^{5}+10\,d^{3}fge^{6}+3\,d^{2}f^{2}e^{7}\right)e^{(-8)}\log\left(\left|x^{2}e^{2}-d^{2}\right|\right)+\frac{1}{12}\left(3\,g^{2}x^{4}e^{17}+20\,dg^{2}x^{3}e^{16}+72\,d^{2}g^{2}x^{2}e^{15}+240\,d^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 2*(7*d^4*g^2*e^5 + 10*d^3*f*g*e^6 + 3*d^2*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 -
 d^2)) + 1/12*(3*g^2*x^4*e^17 + 20*d*g^2*x^3*e^16 + 72*d^2*g^2*x^2*e^15 + 2
40*d^3*g^2*x*e^14 + 8*f*g*x^3*e^17 + 60*d*f*g*x^2*e^16 + 288*d^2*f*g*x*e^15
 + 6*f^2*x^2*e^17 + 60*d*f^2*x*e^16)*e^(-16) + 2*(7*d^5*g^2*e^4 + 10*d^4*f*
g*e^5 + 3*d^3*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2
*abs(d)*e))/abs(d) - 8*(d^6*g^2*e^5 + 2*d^5*f*g*e^6 + d^4*f^2*e^7 + (d^5*g^
2*e^6 + 2*d^4*f*g*e^7 + d^3*f^2*e^8)*x)*e^(-8)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 204, normalized size = 1.40

$$\frac{e\,g^{2}x^{4}}{4}+\frac{5d\,g^{2}x^{3}}{3}+\frac{2efg\,x^{3}}{3}+\frac{6d^{2}g^{2}x^{2}}{e}+5dfg\,x^{2}+\frac{e\,f^{2}x^{2}}{2}-\frac{8d^{5}g^{2}}{(ex-d)\,e^{3}}-\frac{16d^{4}fg}{(ex-d)\,e^{2}}+\frac{28d^{4}g^{2}\ln(ex-d)}{e^{3}}-\frac{8d^{3}f^{2}}{(ex-d)\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/4*e*g^2*x^4+5/3*d*g^2*x^3+2/3*e*f*g*x^3+6*d^2/e*g^2*x^2+5*d*f*g*x^2+1/2*e
*f^2*x^2+20*d^3/e^2*g^2*x+24*d^2/e*f*g*x+5*d*f^2*x+28*d^4/e^3*g^2*ln(e*x-d)
+40*d^3/e^2*f*g*ln(e*x-d)+12*d^2/e*f^2*ln(e*x-d)-8*d^5/e^3/(e*x-d)*g^2-16*d
^4/e^2/(e*x-d)*f*g-8*d^3/e/(e*x-d)*f^2
```

maxima [A] time = 0.46, size = 182, normalized size = 1.25

$$-\frac{8\left(d^{3}e^{2}f^{2}+2\,d^{4}efg+d^{5}g^{2}\right)}{e^{4}x-de^{3}}+\frac{3\,e^{3}g^{2}x^{4}+4\left(2\,e^{3}fg+5\,de^{2}g^{2}\right)x^{3}+6\left(e^{3}f^{2}+10\,de^{2}fg+12\,d^{2}eg^{2}\right)x^{2}+12\left(5\,d\right.}{12\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/(e^4*x - d*e^3) + 1/12*(3*e^3*g^2*
x^4 + 4*(2*e^3*f*g + 5*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 10*d*e^2*f*g + 12*d^2*
e*g^2)*x^2 + 12*(5*d*e^2*f^2 + 24*d^2*e*f*g + 20*d^3*g^2)*x)/e^2 + 4*(3*d^2
*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.09, size = 316, normalized size = 2.16

$$x\left(\frac{d^3\,g^2+6\,d^2\,e\,f\,g+3\,d\,e^2\,f^2}{e^2}-\frac{d^2\,\left(g\,\left(3\,d\,g+2\,e\,f\right)+2\,d\,g^2\right)}{e^2}+\frac{2\,d\,\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^2}-\frac{d^2\,g^2}{e}+\frac{2\,d\,\left(g\,\left(3\,d\,g+2\,e\,f\right.\right.}{e}\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e^2 - (d^2*(g*(3*d*g + 2*e*f) + 2*
d*g^2))/e^2 + (2*d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/e^2 - (d^2*g^2)/e
 + (2*d*(g*(3*d*g + 2*e*f) + 2*d*g^2))/e))/e) + x^2*((e^3*f^2 + 3*d^2*e*g^2
 + 6*d*e^2*f*g)/(2*e^2) - (d^2*g^2)/(2*e) + (d*(g*(3*d*g + 2*e*f) + 2*d*g^2
))/e) + x^3*((g*(3*d*g + 2*e*f))/3 + (2*d*g^2)/3) + (log(e*x - d)*(28*d^4*g
^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g))/e^3 + (8*(d^5*g^2 + d^3*e^2*f^2 + 2*d^
4*e*f*g))/(e*(d*e^2 - e^3*x)) + (e*g^2*x^4)/4
```

sympy [A] time = 0.85, size = 162, normalized size = 1.11

$$\frac{4d^2\left(dg+ef\right)\left(7dg+3ef\right)\log\left(-d+ex\right)}{e^3}+\frac{eg^2x^4}{4}+x^3\left(\frac{5dg^2}{3}+\frac{2efg}{3}\right)+x^2\left(\frac{6d^2g^2}{e}+5dfg+\frac{ef^2}{2}\right)+x\left(\frac{20d^3g^2}{e^2}+\frac{24}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*log(-d + e*x)/e**3 + e*g**2*x**4/4 + x**
3*(5*d*g**2/3 + 2*e*f*g/3) + x**2*(6*d**2*g**2/e + 5*d*f*g + e*f**2/2) + x*
(20*d**3*g**2/e**2 + 24*d**2*f*g/e + 5*d*f**2) + (-8*d**5*g**2 - 16*d**4*e*
f*g - 8*d**3*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.560 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=107

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

```
[Out] (8*d^2*g^2+8*d*e*f*g+e^2*f^2)*x/e^2+g*(2*d*g+e*f)*x^2/e+1/3*g^2*x^3+4*d^2*(
d*g+e*f)^2/e^3/(-e*x+d)+4*d*(d*g+e*f)*(3*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f + 3*
d*g)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{e^2f^2+8defg+8d^2g^2}{e^2}+\frac{2g(ef+2dg)x}{e}+g^2x^2+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)}+\right.\\ &= \frac{\left(e^2f^2+8defg+8d^2g^2\right)x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3}+\frac{4d^2(ef+dg)^2}{e^3(d-ex)}+\frac{4d(ef+dg}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 115, normalized size = 1.07

$$-\frac{4d^2(dg+ef)^2}{e^3(ex-d)}+\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d\left(3d^2g^2+4defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 - (4*d^2*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d*(e^2*f^2 + 4*d*e*f*g
 + 3*d^2*g^2)*Log[d - e*x])/e^3
```

fricas [A] time = 0.75, size = 206, normalized size = 1.93

$$\frac{e^4g^2x^4 - 12\,d^2e^2f^2 - 24\,d^3efg - 12\,d^4g^2 + \left(3\,e^4fg + 5\,de^3g^2\right)x^3 + 3\left(e^4f^2 + 7\,de^3fg + 6\,d^2e^2g^2\right)x^2 - 3\left(de^3f^2 + 8\right.}{3\left(e^4x - de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(e^4*g^2*x^4 - 12*d^2*e^2*f^2 - 24*d^3*e*f*g - 12*d^4*g^2 + (3*e^4*f*g
+ 5*d*e^3*g^2)*x^3 + 3*(e^4*f^2 + 7*d*e^3*f*g + 6*d^2*e^2*g^2)*x^2 - 3*(d*e
^3*f^2 + 8*d^2*e^2*f*g + 8*d^3*e*g^2)*x - 12*(d^2*e^2*f^2 + 4*d^3*e*f*g + 3
*d^4*g^2 - (d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*x)*log(e*x - d))/(e^4*
x - d*e^3)
```

giac [B] time = 0.17, size = 250, normalized size = 2.34

$$2\left(3\,d^3g^2e^3 + 4\,d^2fge^4 + df^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2 - d^2\right|\right) + \frac{1}{3}\left(g^2x^3e^{12} + 6\,dg^2x^2e^{11} + 24\,d^2g^2xe^{10} + 3\,fgx^2e^{12} + 24\,df\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 2*(3*d^3*g^2*e^3 + 4*d^2*f*g*e^4 + d*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2)
) + 1/3*(g^2*x^3*e^12 + 6*d*g^2*x^2*e^11 + 24*d^2*g^2*x*e^10 + 3*f*g*x^2*e^
12 + 24*d*f*g*x*e^11 + 3*f^2*x*e^12)*e^(-12) + 2*(3*d^4*g^2*e^4 + 4*d^3*f*g
*e^5 + d^2*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*ab
s(d)*e))/abs(d) - 4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5 + (d^4*g^2*e
^4 + 2*d^3*f*g*e^5 + d^2*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 167, normalized size = 1.56

$$\frac{g^2x^3}{3} + \frac{2d\,g^2x^2}{e} + fg\,x^2 - \frac{4d^4g^2}{(ex-d)\,e^3} - \frac{8d^3fg}{(ex-d)\,e^2} + \frac{12d^3g^2\ln(ex-d)}{e^3} - \frac{4d^2f^2}{(ex-d)\,e} + \frac{16d^2fg\ln(ex-d)}{e^2} + \frac{8d^2g^2x}{e^2} + \frac{4d\,f^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/3*g^2*x^3+2*d/e*g^2*x^2+f*g*x^2+8*d^2/e^2*g^2*x+8*d/e*f*g*x+f^2*x+12*d^3/
e^3*g^2*ln(e*x-d)+16*d^2/e^2*f*g*ln(e*x-d)+4*d/e*f^2*ln(e*x-d)-4*d^4/e^3/(e
*x-d)*g^2-8*d^3/e^2/(e*x-d)*f*g-4*d^2/e/(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 141, normalized size = 1.32

$$-\frac{4\left(d^2e^2f^2 + 2\,d^3efg + d^4g^2\right)}{e^4x - de^3} + \frac{e^2g^2x^3 + 3\left(e^2fg + 2\,deg^2\right)x^2 + 3\left(e^2f^2 + 8\,defg + 8\,d^2g^2\right)x}{3\,e^2} + \frac{4\left(de^2f^2 + 4\,d^2efg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)/(e^4*x - d*e^3) + 1/3*(e^2*g^2*x^3
 + 3*(e^2*f*g + 2*d*e*g^2)*x^2 + 3*(e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2
 + 4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.07, size = 185, normalized size = 1.73

$$x^2\left(\frac{g\left(d\,g+e\,f\right)}{e}+\frac{d\,g^2}{e}\right)+x\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{e^2}+\frac{2\,d\,\left(\frac{2\,g\,\left(d\,g+e\,f\right)}{e}+\frac{2\,d\,g^2}{e}\right)}{e}-\frac{d^2\,g^2}{e^2}\right)+\frac{g^2\,x^3}{3}+\frac{4\,\left(d^4\,g^2+2\right.}{e\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((g*(d*g + e*f))/e + (d*g^2)/e) + x*((d^2*g^2 + e^2*f^2 + 4*d*e*f*g)/e^
2 + (2*d*((2*g*(d*g + e*f))/e + (2*d*g^2)/e))/e - (d^2*g^2)/e^2) + (g^2*x^3
)/3 + (4*(d^4*g^2 + d^2*e^2*f^2 + 2*d^3*e*f*g))/(e*(d*e^2 - e^3*x)) + (log(
e*x - d)*(12*d^3*g^2 + 4*d*e^2*f^2 + 16*d^2*e*f*g))/e^3
```

sympy [A] time = 0.74, size = 119, normalized size = 1.11

$$\frac{4d\left(dg+ef\right)\left(3dg+ef\right)\log\left(-d+ex\right)}{e^3}+\frac{g^2x^3}{3}+x^2\left(\frac{2dg^2}{e}+fg\right)+x\left(\frac{8d^2g^2}{e^2}+\frac{8dfg}{e}+f^2\right)+\frac{-4d^4g^2-8d^3efg-4}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d*(d*g + e*f)*(3*d*g + e*f)*log(-d + e*x)/e**3 + g**2*x**3/3 + x**2*(2*d*
g**2/e + f*g) + x*(8*d**2*g**2/e**2 + 8*d*f*g/e + f**2) + (-4*d**4*g**2 - 8
*d**3*e*f*g - 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.561 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{gx(3dg+2ef)}{e^2}+\frac{g^2x^2}{2e}$$

```
[Out] g*(3*d*g+2*e*f)*x/e^2+1/2*g^2*x^2/e+2*d*(d*g+e*f)^2/e^3/(-e*x+d)+(d*g+e*f)*
(5*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {848, 77}

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{gx(3dg+2ef)}{e^2}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d -
 e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx&=\int \frac{(d+ex)(f+gx)^2}{(d-ex)^2}\,dx\\&=\int \left(\frac{g(2ef+3dg)}{e^2}+\frac{g^2x}{e}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)}+\frac{2d(ef+dg)^2}{e^2(-d+ex)^2}\right)dx\\&=\frac{g(2ef+3dg)x}{e^2}+\frac{g^2x^2}{2e}+\frac{2d(ef+dg)^2}{e^3(d-ex)}+\frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 1.06

$$\frac{2\left(5d^2g^2+6defg+e^2f^2\right)\log(d-ex)+\frac{4d(dg+ef)^2}{d-ex}+2egx(3dg+2ef)+e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*e*g*(2*e*f + 3*d*g)*x + e^2*g^2*x^2 + (4*d*(e*f + d*g)^2)/(d - e*x) + 2*
(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

fricas [B] time = 0.72, size = 157, normalized size = 2.01

$$\frac{e^3g^2x^3 - 4\,de^2f^2 - 8\,d^2efg - 4\,d^3g^2 + \left(4\,e^3fg + 5\,de^2g^2\right)x^2 - 2\left(2\,de^2fg + 3\,d^2eg^2\right)x - 2\left(de^2f^2 + 6\,d^2efg + 5\right.}{2\left(e^4x - de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(e^3*g^2*x^3 - 4*d*e^2*f^2 - 8*d^2*e*f*g - 4*d^3*g^2 + (4*e^3*f*g + 5*d
*e^2*g^2)*x^2 - 2*(2*d*e^2*f*g + 3*d^2*e*g^2)*x - 2*(d*e^2*f^2 + 6*d^2*e*f*
g + 5*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)*log(e*x - d))/(e^4
*x - d*e^3)
```

giac [B] time = 0.18, size = 212, normalized size = 2.72

$$\frac{1}{2}\left(5\,d^2g^2e^3 + 6\,dfge^4 + f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2 - d^2\right|\right) + \frac{1}{2}\left(g^2x^2e^7 + 6\,dg^2xe^6 + 4\,fgxe^7\right)e^{(-8)} + \frac{\left(5\,d^3g^2e^2 + 6\,d^2fg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*d^2*g^2*e^3 + 6*d*f*g*e^4 + f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2))
+ 1/2*(g^2*x^2*e^7 + 6*d*g^2*x*e^6 + 4*f*g*x*e^7)*e^(-8) + 1/2*(5*d^3*g^2*e
^2 + 6*d^2*f*g*e^3 + d*f^2*e^4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*
x*e^2 + 2*abs(d)*e))/abs(d) - 2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5
+ (d^3*g^2*e^4 + 2*d^2*f*g*e^5 + d*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 138, normalized size = 1.77

$$\frac{g^2x^2}{2e} - \frac{2d^3g^2}{(ex-d)\,e^3} - \frac{4d^2fg}{(ex-d)\,e^2} + \frac{5d^2g^2\ln(ex-d)}{e^3} - \frac{2d\,f^2}{(ex-d)\,e} + \frac{6dfg\ln(ex-d)}{e^2} + \frac{3d\,g^2x}{e^2} + \frac{f^2\ln(ex-d)}{e} + \frac{2fgx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/2/e*g^2*x^2+3*d/e^2*g^2*x+2/e*f*g*x+5*d^2/e^3*g^2*ln(e*x-d)+6*d/e^2*f*g*l
n(e*x-d)+1/e*f^2*ln(e*x-d)-2*d^3/e^3/(e*x-d)*g^2-4*d^2/e^2/(e*x-d)*f*g-2*d/
e/(e*x-d)*f^2
```

maxima [A] time = 0.44, size = 104, normalized size = 1.33

$$-\frac{2\left(de^2f^2 + 2\,d^2efg + d^3g^2\right)}{e^4x - de^3} + \frac{eg^2x^2 + 2\left(2\,efg + 3\,dg^2\right)x}{2\,e^2} + \frac{\left(e^2f^2 + 6\,defg + 5\,d^2g^2\right)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)/(e^4*x - d*e^3) + 1/2*(e*g^2*x^2 + 2
*(2*e*f*g + 3*d*g^2)*x)/e^2 + (e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(e*x - d
)/e^3
```

mupad [B] time = 2.53, size = 116, normalized size = 1.49

$$x\left(\frac{d\,g^2 + 2\,e\,f\,g}{e^2} + \frac{2\,d\,g^2}{e^2}\right) + \frac{\ln(e\,x-d)\,\left(5\,d^2\,g^2 + 6\,d\,e\,f\,g + e^2\,f^2\right)}{e^3} + \frac{g^2\,x^2}{2\,e} + \frac{2\,\left(d^3\,g^2 + 2\,d^2\,e\,f\,g + d\,e^2\,f^2\right)}{e\,\left(d\,e^2 - e^3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d*g^2 + 2*e*f*g)/e^2 + (2*d*g^2)/e^2) + (log(e*x - d)*(5*d^2*g^2 + e^2*
f^2 + 6*d*e*f*g))/e^3 + (g^2*x^2)/(2*e) + (2*(d^3*g^2 + d*e^2*f^2 + 2*d^2*e
*f*g))/(e*(d*e^2 - e^3*x))
```

sympy [A] time = 0.59, size = 94, normalized size = 1.21

$$x\left(\frac{3dg^2}{e^2}+\frac{2fg}{e}\right)+\frac{-2d^3g^2-4d^2efg-2de^2f^2}{-de^3+e^4x}+\frac{g^2x^2}{2e}+\frac{(dg+ef)(5dg+ef)\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] x*(3*d*g**2/e**2 + 2*f*g/e) + (-2*d**3*g**2 - 4*d**2*e*f*g - 2*d*e**2*f**2)
/(-d*e**3 + e**4*x) + g**2*x**2/(2*e) + (d*g + e*f)*(5*d*g + e*f)*log(-d +
e*x)/e**3
```

3.562 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=50

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

```
[Out] g^2*x/e^2+(d*g+e*f)^2/e^3/(-e*x+d)+2*g*(d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 43}

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x]
)/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2}\,dx\\
&= \int \left(\frac{g^2}{e^2}+\frac{(ef+dg)^2}{e^2(-d+ex)^2}+\frac{2g(ef+dg)}{e^2(-d+ex)}\right)dx\\
&= \frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.92

$$\frac{\frac{(dg+ef)^2}{d-ex}+2g(dg+ef)\log(d-ex)+eg^2x}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

[Out] (e*g^2*x + (e*f + d*g)^2/(d - e*x) + 2*g*(e*f + d*g)*Log[d - e*x])/e^3

fricas [A] time = 0.74, size = 95, normalized size = 1.90

$$\frac{e^2g^2x^2 - deg^2x - e^2f^2 - 2\,defg - d^2g^2 - 2\left(defg + d^2g^2 - \left(e^2fg + deg^2\right)x\right)\log\left(ex - d\right)}{e^4x - de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")

[Out] (e^2*g^2*x^2 - d*e*g^2*x - e^2*f^2 - 2*d*e*f*g - d^2*g^2 - 2*(d*e*f*g + d^2*g^2 - (e^2*f*g + d*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)

giac [B] time = 0.17, size = 160, normalized size = 3.20

$$g^2xe^{(-2)}+\left(dg^2e + fge^2\right)e^{(-4)}\log\left(\left|x^2e^2 - d^2\right|\right)+\frac{\left(d^2g^2e^2 + dfge^3\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{|d|}-\frac{\left(d^3g^2e + 2\,d^2fge^2 + df^2e^3\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")

[Out] g^2*x*e^(-2) + (d*g^2*e + f*g*e^2)*e^(-4)*log(abs(x^2*e^2 - d^2)) + (d^2*g^2*e^2 + d*f*g*e^3)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - (d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3 + (d^2*g^2*e^2 + 2*d*f*g*e^3 + f^2*e^4)*x)*e^(-4)/(x^2*e^2 - d^2)

maple [A] time = 0.01, size = 96, normalized size = 1.92

$$-\frac{d^2g^2}{(ex-d)\,e^3} - \frac{2dfg}{(ex-d)\,e^2} + \frac{2d\,g^2\ln\left(ex-d\right)}{e^3} - \frac{f^2}{(ex-d)\,e} + \frac{2fg\ln\left(ex-d\right)}{e^2} + \frac{g^2x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)

[Out] 1/e^2*g^2*x+2*d/e^3*g^2*ln(e*x-d)+2/e^2*f*g*ln(e*x-d)-1/e^3/(e*x-d)*d^2*g^2-2/e^2/(e*x-d)*d*f*g-1/e/(e*x-d)*f^2

maxima [A] time = 0.44, size = 69, normalized size = 1.38

$$\frac{g^2x}{e^2} - \frac{e^2f^2 + 2\,defg + d^2g^2}{e^4x - de^3} + \frac{2\left(efg + dg^2\right)\log\left(ex - d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")

[Out] g^2*x/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(e^4*x - d*e^3) + 2*(e*f*g + d*g^2)*log(e*x - d)/e^3

mupad [B] time = 2.56, size = 72, normalized size = 1.44

$$\frac{d^2\,g^2 + 2\,d\,e\,f\,g + e^2\,f^2}{e\,\left(d\,e^2 - e^3\,x\right)} + \frac{g^2\,x}{e^2} + \frac{\ln\left(e\,x - d\right)\,\left(2\,d\,g^2 + 2\,e\,f\,g\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^2,x)

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(e*(d*e^2 - e^3*x)) + (g^2*x)/e^2 + (log(e*
x - d)*(2*d*g^2 + 2*e*f*g))/e^3
```

sympy [A] time = 0.40, size = 61, normalized size = 1.22

$$\frac{-d^2g^2 - 2defg - e^2f^2}{-de^3 + e^4x} + \frac{g^2x}{e^2} + \frac{2g\left(dg + ef\right)\log\left(-d + ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*g
*(d*g + e*f)*log(-d + e*x)/e**3
```

3.563 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=86

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

```
[Out] 1/2*(d*g+e*f)^2/d/e^3/(-e*x+d)-1/4*(-3*d*g+e*f)*(d*g+e*f)*ln(-e*x+d)/d^2/e^
3+1/4*(-d*g+e*f)^2*ln(e*x+d)/d^2/e^3
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {799, 88}

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x]
)/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)}\right)dx\\
&= \frac{(ef+dg)^2}{2de^3(d-ex)}-\frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 1.06

$$\frac{(d-ex)\left(3d^2g^2+2defg-e^2f^2\right)\log(d-ex)+(d-ex)(ef-dg)^2\log(d+ex)+2d(dg+ef)^2}{4d^2e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*d*(e*f + d*g)^2 + (-(e^2*f^2) + 2*d*e*f*g + 3*d^2*g^2)*(d - e*x)*Log[d -
 e*x] + (e*f - d*g)^2*(d - e*x)*Log[d + e*x])/(4*d^2*e^3*(d - e*x))
```

fricas [B] time = 0.55, size = 168, normalized size = 1.95

$$-\frac{2\,de^2f^2+4\,d^2efg+2\,d^3g^2+\left(de^2f^2-2\,d^2efg+d^3g^2-\left(e^3f^2-2\,de^2fg+d^2eg^2\right)x\right)\log\left(ex+d\right)-\left(de^2f^2-2\right.}{4\left(d^2e^4x-d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*e^2*f^2 + 4*d^2*e*f*g + 2*d^3*g^2 + (d*e^2*f^2 - 2*d^2*e*f*g + d^
3*g^2 - (e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x + d) - (d*e^2*f^2 -
2*d^2*e*f*g - 3*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x - d^3*e^3)
```

giac [A] time = 0.19, size = 159, normalized size = 1.85

$$\frac{1}{2}\,g^2e^{(-3)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{\left(d^2g^2+2\,dfge-f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{4\,d|d|}-\frac{\left(\left(d^2g^2+2\,dfge+f^2e^2\right)x+\left(d^3g^2e+\right.\right.}{2\left(x^2e^2-d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*g^2*e^(-3)*log(abs(x^2*e^2 - d^2)) + 1/4*(d^2*g^2 + 2*d*f*g*e - f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/(d*abs(d)
) - 1/2*((d^2*g^2 + 2*d*f*g*e + f^2*e^2)*x + (d^3*g^2*e + 2*d^2*f*g*e^2 + d
*f^2*e^3)*e^(-2))*e^(-2)/((x^2*e^2 - d^2)*d)
```

maple [A] time = 0.01, size = 156, normalized size = 1.81

$$-\frac{d\,g^2}{2\,(ex-d)\,e^3}-\frac{f^2}{2\,(ex-d)\,de}+\frac{fg\ln\left(ex-d\right)}{2d\,e^2}-\frac{fg\ln\left(ex+d\right)}{2d\,e^2}-\frac{f^2\ln\left(ex-d\right)}{4d^2e}+\frac{f^2\ln\left(ex+d\right)}{4d^2e}-\frac{fg}{(ex-d)\,e^2}+\frac{3g^2\ln}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/2/e^3*d/(e*x-d)*g^2-1/e^2/(e*x-d)*f*g-1/2/e/d/(e*x-d)*f^2+3/4/e^3*g^2*ln
(e*x-d)+1/2/d/e^2*f*g*ln(e*x-d)-1/4/d^2/e*f^2*ln(e*x-d)+1/4/e^3*g^2*ln(e*x+
d)-1/2/d/e^2*f*g*ln(e*x+d)+1/4/d^2/e*f^2*ln(e*x+d)
```

maxima [A] time = 0.45, size = 114, normalized size = 1.33

$$-\frac{e^2f^2+2\,defg+d^2g^2}{2\left(de^4x-d^2e^3\right)}+\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)}{4\,d^2e^3}-\frac{\left(e^2f^2-2\,defg-3\,d^2g^2\right)\log\left(ex-d\right)}{4\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(d*e^4*x - d^2*e^3) + 1/4*(e^2*f^2 - 2
*d*e*f*g + d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 - 2*d*e*f*g - 3*d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

mupad [B] time = 2.64, size = 111, normalized size = 1.29

$$\frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,\left(d-e\,x\right)}+\frac{\ln\left(d+e\,x\right)\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}+\frac{\ln\left(d-e\,x\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2\right)}{4\,d^2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(2*d*e^3*(d - e*x)) + (log(d + e*x)*(d^2*g^
2 + e^2*f^2 - 2*d*e*f*g))/(4*d^2*e^3) + (log(d - e*x)*(3*d^2*g^2 - e^2*f^2
+ 2*d*e*f*g))/(4*d^2*e^3)
```

sympy [B] time = 1.04, size = 182, normalized size = 2.12

$$\frac{-d^2g^2-2defg-e^2f^2}{-2d^2e^3+2de^4x}+\frac{(dg-ef)^2\log\left(x+\frac{2d^3g^2-d(dg-ef)^2}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}+\frac{(dg+ef)(3dg-ef)\log\left(x+\frac{2d^3g^2-d(dg+ef)(3dg-ef)}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x) + (d*g - e
*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e*g**2 + 2*d*e**2*f*g
 - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3*d*g - e*f)*log(x + (2*d**3*g*
*2 - d*(d*g + e*f)*(3*d*g - e*f))/(d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))
/(4*d**2*e**3)
```

3.564 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}+\frac{(f+gx)\left(d^2g+e^2fx\right)}{2d^2e^2\left(d^2-e^2x^2\right)}$$

```
[Out] 1/2*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)+1/2*(-d*g+e*f)*(d*g+e*f)
*arctanh(e*x/d)/d^3/e^3
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {723, 208}

$$\frac{(f+gx)\left(d^2g+e^2fx\right)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(2*d^2*e^2*(d^2 - e^2*x^2)) + ((e*f - d*g)*(e
*f + d*g)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}-\frac{1}{2}\left(-\frac{f^2}{d^2}+\frac{g^2}{e^2}\right)\int \frac{1}{d^2-e^2x^2}\,dx\\ &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.15

$$\frac{-2d^2fg-d^2g^2x-e^2f^2x}{2d^2e^2\left(e^2x^2-d^2\right)}-\frac{\left(d^2g^2-e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (-2*d^2*f*g - e^2*f^2*x - d^2*g^2*x)/(2*d^2*e^2*(-d^2 + e^2*x^2)) - ((-(e^2
*f^2) + d^2*g^2)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

fricas [B] time = 0.88, size = 155, normalized size = 2.09

$$-\frac{4\,d^3efg+2\left(de^3f^2+d^3eg^2\right)x+\left(d^2e^2f^2-d^4g^2-\left(e^4f^2-d^2e^2g^2\right)x^2\right)\log\left(ex+d\right)-\left(d^2e^2f^2-d^4g^2-\left(e^4f^2-d^2\right.\right.}{4\left(d^3e^5x^2-d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^3*e*f*g + 2*(d*e^3*f^2 + d^3*e*g^2)*x + (d^2*e^2*f^2 - d^4*g^2 -
(e^4*f^2 - d^2*e^2*g^2)*x^2)*log(e*x + d) - (d^2*e^2*f^2 - d^4*g^2 - (e^4*f
^2 - d^2*e^2*g^2)*x^2)*log(e*x - d))/(d^3*e^5*x^2 - d^5*e^3)
```

giac [A] time = 0.16, size = 101, normalized size = 1.36

$$\frac{\left(d^2g^2-f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{4\,d^2|d|}-\frac{\left(d^2g^2x+2\,d^2fg+f^2xe^2\right)e^{(-2)}}{2\left(x^2e^2-d^2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^2*g^2 - f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
2*abs(d)*e))/(d^2*abs(d)) - 1/2*(d^2*g^2*x + 2*d^2*f*g + f^2*x*e^2)*e^(-2)/
((x^2*e^2 - d^2)*d^2)
```

maple [B] time = 0.01, size = 180, normalized size = 2.43

$$-\frac{fg}{2\left(ex-d\right)d\,e^2}+\frac{fg}{2\left(ex+d\right)d\,e^2}+\frac{g^2\ln\left(ex-d\right)}{4d\,e^3}-\frac{g^2\ln\left(ex+d\right)}{4d\,e^3}-\frac{f^2}{4\left(ex-d\right)d^2e}-\frac{f^2}{4\left(ex+d\right)d^2e}-\frac{f^2\ln\left(ex-d\right)}{4d^3e}+\frac{f^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/4/d/e^3*g^2*ln(e*x-d)-1/4/d^3/e*f^2*ln(e*x-d)-1/4/e^3/(e*x-d)*g^2-1/2/e^2
/d/(e*x-d)*f*g-1/4/e/d^2/(e*x-d)*f^2-1/4/d/e^3*g^2*ln(e*x+d)+1/4/d^3/e*f^2*
ln(e*x+d)-1/4/(e*x+d)/e^3*g^2+1/2/(e*x+d)/d/e^2*f*g-1/4/(e*x+d)/d^2/e*f^2
```

maxima [A] time = 0.44, size = 111, normalized size = 1.50

$$-\frac{2\,d^2fg+\left(e^2f^2+d^2g^2\right)x}{2\left(d^2e^4x^2-d^4e^2\right)}+\frac{\left(e^2f^2-d^2g^2\right)\log\left(ex+d\right)}{4\,d^3e^3}-\frac{\left(e^2f^2-d^2g^2\right)\log\left(ex-d\right)}{4\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^2*f*g + (e^2*f^2 + d^2*g^2)*x)/(d^2*e^4*x^2 - d^4*e^2) + 1/4*(e^2
*f^2 - d^2*g^2)*log(e*x + d)/(d^3*e^3) - 1/4*(e^2*f^2 - d^2*g^2)*log(e*x -
d)/(d^3*e^3)
```

mupad [B] time = 2.61, size = 115, normalized size = 1.55

$$\frac{\frac{f\,g}{e^2}+\frac{x\,\left(d^2\,g^2+e^2\,f^2\right)}{2\,d^2\,e^2}}{d^2-e^2\,x^2}-\frac{2\,\mathrm{atanh}\left(\frac{4\,e\,x\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d\,\left(d^2\,g^2-e^2\,f^2\right)}\right)\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d^3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^2,x)
```

```
[Out] ((f*g)/e^2 + (x*(d^2*g^2 + e^2*f^2))/(2*d^2*e^2))/(d^2 - e^2*x^2) - (2*atan
h((4*e*x*((d^2*g^2)/4 - (e^2*f^2)/4))/(d*(d^2*g^2 - e^2*f^2)))*((d^2*g^2)/4
 - (e^2*f^2)/4))/(d^3*e^3)
```

sympy [B] time = 0.71, size = 156, normalized size = 2.11

$$\frac{-2d^2fg+x\left(-d^2g^2-e^2f^2\right)}{-2d^4e^2+2d^2e^4x^2}+\frac{\left(dg-ef\right)\left(dg+ef\right)\log\left(-\frac{d\left(dg-ef\right)\left(dg+ef\right)}{e\left(d^2g^2-e^2f^2\right)}+x\right)}{4d^3e^3}-\frac{\left(dg-ef\right)\left(dg+ef\right)\log\left(\frac{d\left(dg-ef\right)\left(\right.}{e\left(d^2g^2-\right.}\right.}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**2*f*g + x*(-d**2*g**2 - e**2*f**2))/(-2*d**4*e**2 + 2*d**2*e**4*x**2
) + (d*g - e*f)*(d*g + e*f)*log(-d*(d*g - e*f)*(d*g + e*f)/(e*(d**2*g**2 -
e**2*f**2)) + x)/(4*d**3*e**3) - (d*g - e*f)*(d*g + e*f)*log(d*(d*g - e*f)*
(d*g + e*f)/(e*(d**2*g**2 - e**2*f**2)) + x)/(4*d**3*e**3)
```

3.565 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=121

$$\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

```
[Out] 1/8*(d*g+e*f)^2/d^3/e^3/(-e*x+d)-1/8*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^2+1/4*(d^
2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)+1/8*(-d*g+3*e*f)*(d*g+e*f)*arctanh(e*x/d)/d^
4/e^3
```

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}+\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2)
 - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^3}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{8d^3e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^2}+\frac{(3ef-dg)(ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{((3ef-dg)(ef+dg))\int \frac{1}{d^2-e^2x^2}\,d}{8d^3e^2}\\
&= \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{(3ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 1.15

$$\frac{\frac{4d(d^2g^2-e^2f^2)}{d+ex}+\left(d^2g^2-2defg-3e^2f^2\right)\log(d-ex)+\left(-d^2g^2+2defg+3e^2f^2\right)\log(d+ex)-\frac{2d^2(ef-dg)^2}{(d+ex)^2}+\frac{2d(dg}{d-}}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((2*d*(e*f + d*g)^2)/(d - e*x) - (2*d^2*(e*f - d*g)^2)/(d + e*x)^2 + (4*d*(
-(e^2*f^2) + d^2*g^2))/(d + e*x) + (-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Log[d
 - e*x] + (3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

fricas [B] time = 0.80, size = 417, normalized size = 3.45

$$\frac{4\,d^3e^2f^2-8\,d^4efg-4\,d^5g^2-2\left(3\,de^4f^2+2\,d^2e^3fg-d^3e^2g^2\right)x^2-2\left(3\,d^2e^3f^2+2\,d^3e^2fg+3\,d^4eg^2\right)x-\left(3\,d^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 - 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 + 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
- (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*
f^2 + 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) + (3*d^3*e^2*f^2 + 2*d^4*e
*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 + d^5*e^5*x^2 - d^6*e^4*x - d^7*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (d^2*
exp(1)^2*g^2-2*d*exp(1)^3*g*f+exp(1)^4*f^2)/(exp(2)^2*d^4*exp(1)-2*exp(2)*d
^4*exp(1)^3+d^4*exp(1)^5)*ln(abs(x*exp(1)+d))+(-d^2*exp(1)*g^2+2*d*exp(1)^2
*g*f-exp(1)^3*f^2)/(2*exp(2)^2*d^4-4*exp(2)*d^4*exp(1)^2+2*d^4*exp(1)^4)*ln
(abs(-x^2*exp(2)+d^2))+(exp(2)^2*f^2-exp(2)*d^2*g^2+2*exp(2)*d*exp(1)*g*f-3
*exp(2)*exp(1)^2*f^2-d^2*exp(1)^2*g^2+2*d*exp(1)^3*g*f)*1/2/(2*exp(2)^2*d^3
-4*exp(2)*d^3*exp(1)^2+2*d^3*exp(1)^4)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*e
xp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-(-2*exp(2)^2*d^3*g*f+exp(2)
^2*d^2*exp(1)*f^2+exp(2)*d^4*exp(1)*g^2+2*exp(2)*d^3*exp(1)^2*g*f-exp(2)*d^
2*exp(1)^3*f^2-d^4*exp(1)^3*g^2+(-exp(2)^3*d*f^2-exp(2)^2*d^3*g^2+2*exp(2)^
2*d^2*exp(1)*g*f+exp(2)^2*d*exp(1)^2*f^2+exp(2)*d^3*exp(1)^2*g^2-2*exp(2)*d
^2*exp(1)^3*g*f)*x)/2/d^4/exp(2)/(exp(2)-exp(1)^2)^2/(-x^2*exp(2)+d^2)
```

maple [B] time = 0.01, size = 253, normalized size = 2.09

$$\frac{fg}{4\left(ex+d\right)^2de^2}-\frac{f^2}{8\left(ex+d\right)^2d^2e}-\frac{g^2}{8\left(ex+d\right)^2e^3}-\frac{g^2}{8\left(ex-d\right)de^3}+\frac{g^2}{4\left(ex+d\right)de^3}-\frac{fg}{4\left(ex-d\right)d^2e^2}+\frac{g^2\ln\left(ex-d\right)}{16d^2e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/16/d^2/e^3*g^2*ln(e*x-d)-1/8/d^3/e^2*f*g*ln(e*x-d)-3/16/d^4/e*f^2*ln(e*x-
d)-1/8/e^3/d/(e*x-d)*g^2-1/4/e^2/d^2/(e*x-d)*f*g-1/8/e/d^3/(e*x-d)*f^2+1/4/
(e*x+d)/d/e^3*g^2-1/4/(e*x+d)/d^3/e*f^2-1/16/d^2/e^3*g^2*ln(e*x+d)+1/8/d^3/
e^2*f*g*ln(e*x+d)+3/16/d^4/e*f^2*ln(e*x+d)-1/8/(e*x+d)^2/e^3*g^2+1/4/(e*x+d
)^2/d/e^2*f*g-1/8/(e*x+d)^2/d^2/e*f^2
```

maxima [A] time = 0.47, size = 212, normalized size = 1.75

$$\frac{2\,d^2e^2f^2-4\,d^3efg-2\,d^4g^2-\left(3\,e^4f^2+2\,de^3fg-d^2e^2g^2\right)x^2-\left(3\,de^3f^2+2\,d^2e^2fg+3\,d^3eg^2\right)x}{8\left(d^3e^6x^3+d^4e^5x^2-d^5e^4x-d^6e^3\right)}+\frac{\left(3\,e^2f^2+2\,defg\right.}{1\!\!}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 - 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 + 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 - (3*d*e^3*f^2 + 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3) + 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

mupad [B] time = 0.15, size = 198, normalized size = 1.64

$$\frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3+d^2\,e\,x-d\,e^2\,x^2-e^3\,x^3}+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g-3\,e\,f\right)}{d\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g\right.}{8\,d^4\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 + 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 - e^3*x^3 - d*e^2*x^2 + d^2*e*x) + (atanh((e*x*(d*g + e*f)*(d*g - 3*e*
f))/(d*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g)))*(d*g + e*f)*(d*g - 3*e*f))/(8*d^
4*e^3)
```

sympy [B] time = 1.26, size = 279, normalized size = 2.31

$$\frac{-2d^4g^2-4d^3efg+2d^2e^2f^2+x^2\left(d^2e^2g^2-2de^3fg-3e^4f^2\right)+x\left(-3d^3eg^2-2d^2e^2fg-3de^3f^2\right)}{-8d^6e^3-8d^5e^4x+8d^4e^5x^2+8d^3e^6x^3}+\frac{\left(dg-3ef\right)\left(dg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2*
d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**3*
f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - 3*e*f)*(d*g + e*f)*log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2
 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g + e*f
)*log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

3.566 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=146

$$\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

```
[Out] 1/16*(d*g+e*f)^2/d^4/e^3/(-e*x+d)-1/12*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^3+1/8*(
d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^2-1/16*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*x
+d)+1/4*f*(d*g+e*f)*arctanh(e*x/d)/d^5/e^2
```

Rubi [A] time = 0.16, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}+\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)^
3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f + d*
g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^4}dx \\
&= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^4}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^3}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^2}\right. \\
&= \frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)}+\frac{(f}{} \\
&= \frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)}+\frac{f(}{}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 171, normalized size = 1.17

$$\frac{2d\left(2d^5g^2+2d^4eg(f+2gx)+d^3e^2f(gx-4f)+d^2e^3fx(f+6gx)+3de^4fx^2(2f+gx)+3e^5f^2x^3\right)+3ef(ex-d)(}{24d^5e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (2*d*(2*d^5*g^2 + 3*e^5*f^2*x^3 + d^3*e^2*f*(-4*f + g*x) + 3*d*e^4*f*x^2*(2
*f + g*x) + 2*d^4*e*g*(f + 2*g*x) + d^2*e^3*f*x*(f + 6*g*x)) + 3*e*f*(e*f +
 d*g)*(-d + e*x)*(d + e*x)^3*Log[d - e*x] + 3*e*f*(e*f + d*g)*(d - e*x)*(d
+ e*x)^3*Log[d + e*x])/(24*d^5*e^3*(d - e*x)*(d + e*x)^3)
```

fricas [B] time = 0.84, size = 337, normalized size = 2.31

$$8\,d^4e^2f^2-4\,d^5efg-4\,d^6g^2-6\left(de^5f^2+d^2e^4fg\right)x^3-12\left(d^2e^4f^2+d^3e^3fg\right)x^2-2\left(d^3e^3f^2+d^4e^2fg+4\,d^5eg^2\right)x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(8*d^4*e^2*f^2 - 4*d^5*e*f*g - 4*d^6*g^2 - 6*(d*e^5*f^2 + d^2*e^4*f*g)
*x^3 - 12*(d^2*e^4*f^2 + d^3*e^3*f*g)*x^2 - 2*(d^3*e^3*f^2 + d^4*e^2*f*g +
4*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2
*(d*e^5*f^2 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x +
 d) + 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2*(d*e^5*f^2
 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x - d))/(d^5*e
^7*x^4 + 2*d^6*e^6*x^3 - 2*d^8*e^4*x - d^9*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-(ex
p(1)*x+d)^-1/exp(1)*g^2*d^2*exp(1)^6+2*(exp(1)*x+d)^-1/exp(1)*g*d*exp(1)^7*
f-(exp(1)*x+d)^-1/exp(1)*exp(1)^8*f^2)/(d^4*exp(1)^8-2*d^4*exp(1)^6*exp(2)+
d^4*exp(1)^4*exp(2)^2)-(-(-g^2*d^3*exp(1)^6-6*g^2*d^3*exp(1)^4*exp(2)-g^2*d
^3*exp(1)^2*exp(2)^2+8*g*d^2*exp(1)^5*exp(2)*f+8*g*d^2*exp(1)^3*exp(2)^2*f-
d*exp(1)^6*exp(2)*f^2-6*d*exp(1)^4*exp(2)^2*f^2-d*exp(1)^2*exp(2)^3*f^2)/(e
xp(1)^2-exp(2))*(exp(1)*x+d)^-1/exp(1)+(-3*g^2*d^2*exp(1)^3*exp(2)-g^2*d^2*
exp(1)*exp(2)^2+2*g*d*exp(1)^4*exp(2)*f+6*g*d*exp(1)^2*exp(2)^2*f-3*exp(1)^
3*exp(2)^2*f^2-exp(1)*exp(2)^3*f^2)/(exp(1)^2-exp(2)))/2/d^5/(exp(2)-exp(1)
^2)^2/(-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4+(-(exp(1)*x+d)^-1/exp(1))^
2*d^2*exp(1)^2*exp(2)-2*(exp(1)*x+d)^-1/exp(1)*d*exp(1)*exp(2)+exp(2))+(g^2
*d^2*exp(1)^3+g^2*d^2*exp(1)*exp(2)-g*d*exp(1)^4*f-3*g*d*exp(1)^2*f*exp(2)+
2*exp(1)^3*f^2*exp(2))/(d^5*exp(1)^6-3*d^5*exp(1)^4*exp(2)+3*d^5*exp(1)^2*e
xp(2)^2-d^5*exp(2)^3)*ln(abs(-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4+(-(e
xp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^2*exp(2)-2*(exp(1)*x+d)^-1/exp(1)*d*exp(
1)*exp(2)+exp(2)))+(-g^2*d^2*exp(1)^6-6*g^2*d^2*exp(1)^4*exp(2)-g^2*d^2*exp
(1)^2*exp(2)^2+12*g*d*exp(1)^5*f*exp(2)+4*g*d*exp(1)^3*f*exp(2)^2-3*exp(1)^
6*f^2*exp(2)-6*exp(1)^4*f^2*exp(2)^2+exp(1)^2*f^2*exp(2)^3)/2/(2*d^4*exp(1)
^6-6*d^4*exp(1)^4*exp(2)+6*d^4*exp(1)^2*exp(2)^2-2*d^4*exp(2)^3)/exp(1)/abs
(d)/exp(1)^2*ln(abs(2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1
/exp(1)*d^2*exp(1)^2*exp(2)+2*d*exp(1)*exp(2)-2*exp(1)*abs(d)*exp(1)^2)/abs
```

```
(2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^
2*exp(2)+2*d*exp(1)*exp(2)+2*exp(1)*abs(d)*exp(1)^2))
```

maple [A] time = 0.02, size = 270, normalized size = 1.85

$$\frac{fg}{6\left(ex+d\right)^{3}d\,e^{2}}-\frac{f^{2}}{12\left(ex+d\right)^{3}d^{2}e}-\frac{g^{2}}{12\left(ex+d\right)^{3}e^{3}}+\frac{g^{2}}{8\left(ex+d\right)^{2}d\,e^{3}}-\frac{f^{2}}{8\left(ex+d\right)^{2}d^{3}e}-\frac{g^{2}}{16\left(ex-d\right)d^{2}e^{3}}+\frac{g^{2}}{16\left(ex+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/16/e^3/d^2/(e*x-d)*g^2-1/8/e^2/d^3/(e*x-d)*f*g-1/16/e/d^4/(e*x-d)*f^2-1/
8/d^4/e^2*f*g*ln(e*x-d)-1/8/d^5/e*f^2*ln(e*x-d)+1/8/(e*x+d)^2/d/e^3*g^2-1/8
/(e*x+d)^2/d^3/e*f^2+1/16/(e*x+d)/d^2/e^3*g^2-1/8/(e*x+d)/d^3/e^2*f*g-3/16/
(e*x+d)/d^4/e*f^2-1/12/(e*x+d)^3/e^3*g^2+1/6/(e*x+d)^3/d/e^2*f*g-1/12/(e*x+
d)^3/d^2/e*f^2+1/8/d^4/e^2*f*g*ln(e*x+d)+1/8/d^5/e*f^2*ln(e*x+d)
```

maxima [A] time = 0.48, size = 197, normalized size = 1.35

$$\frac{4\,d^{3}e^{2}f^{2}-2\,d^{4}efg-2\,d^{5}g^{2}-3\left(e^{5}f^{2}+de^{4}fg\right)x^{3}-6\left(de^{4}f^{2}+d^{2}e^{3}fg\right)x^{2}-\left(d^{2}e^{3}f^{2}+d^{3}e^{2}fg+4\,d^{4}eg^{2}\right)x}{12\left(d^{4}e^{7}x^{4}+2\,d^{5}e^{6}x^{3}-2\,d^{7}e^{4}x-d^{8}e^{3}\right)}+\frac{(e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*d^3*e^2*f^2 - 2*d^4*e*f*g - 2*d^5*g^2 - 3*(e^5*f^2 + d*e^4*f*g)*x^3
 - 6*(d*e^4*f^2 + d^2*e^3*f*g)*x^2 - (d^2*e^3*f^2 + d^3*e^2*f*g + 4*d^4*e*g
^2)*x)/(d^4*e^7*x^4 + 2*d^5*e^6*x^3 - 2*d^7*e^4*x - d^8*e^3) + 1/8*(e*f^2 +
 d*f*g)*log(e*x + d)/(d^5*e^2) - 1/8*(e*f^2 + d*f*g)*log(e*x - d)/(d^5*e^2)
```

mupad [B] time = 2.63, size = 148, normalized size = 1.01

$$\frac{\frac{d^2\,g^2+d\,e\,f\,g-2\,e^2\,f^2}{6\,d\,e^3}+\frac{f\,x^2\,\left(d\,g+e\,f\right)}{2\,d^3}+\frac{x\,\left(4\,d^2\,g^2+d\,e\,f\,g+e^2\,f^2\right)}{12\,d^2\,e^2}+\frac{e\,f\,x^3\,\left(d\,g+e\,f\right)}{4\,d^4}}{d^4+2\,d^3\,e\,x-2\,d\,e^3\,x^3-e^4\,x^4}+\frac{f\,\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)}{4\,d^5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + d*e*f*g)/(6*d*e^3) + (f*x^2*(d*g + e*f))/(2*d^3) +
(x*(4*d^2*g^2 + e^2*f^2 + d*e*f*g))/(12*d^2*e^2) + (e*f*x^3*(d*g + e*f))/(4
*d^4))/(d^4 - e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x) + (f*atanh((e*x)/d)*(d*g +
 e*f))/(4*d^5*e^2)
```

sympy [A] time = 1.36, size = 241, normalized size = 1.65

$$\frac{-2d^{5}g^{2}-2d^{4}efg+4d^{3}e^{2}f^{2}+x^{3}\left(-3de^{4}fg-3e^{5}f^{2}\right)+x^{2}\left(-6d^{2}e^{3}fg-6de^{4}f^{2}\right)+x\left(-4d^{4}eg^{2}-d^{3}e^{2}fg-d^{2}e^{3}\right.}{-12d^{8}e^{3}-24d^{7}e^{4}x+24d^{5}e^{6}x^{3}+12d^{4}e^{7}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**5*g**2 - 2*d**4*e*f*g + 4*d**3*e**2*f**2 + x**3*(-3*d*e**4*f*g - 3*e
**5*f**2) + x**2*(-6*d**2*e**3*f*g - 6*d*e**4*f**2) + x*(-4*d**4*e*g**2 - d
**3*e**2*f*g - d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**5*e
**6*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*log(-d*f*(d*g + e*f)/(e*(d*f*
g + e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*log(d*f*(d*g + e*f)/(e*(d*f
*g + e*f**2)) + x)/(8*d**5*e**2)
```

3.567 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=178

$$\frac{(dg+ef)(dg+5ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-}{12d^3e^3(}$$

```
[Out] 1/32*(d*g+e*f)^2/d^5/e^3/(-e*x+d)-1/16*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^4+1/12*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^3-1/32*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*
x+d)^2-1/8*f*(d*g+e*f)/d^5/e^2/(e*x+d)+1/32*(d*g+e*f)*(d*g+5*e*f)*arctanh(e
*x/d)/d^6/e^3
```

Rubi [A] time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)(dg+5ef)\,\mathrm{ta}}{32d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)^
4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f + d
*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) + ((e
*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^5}\,dx$$
$$=\int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^5}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^4}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^3}\right.$$
$$=\frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2}-$$
$$=\frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2}-$$

Mathematica [A] time = 0.14, size = 195, normalized size = 1.10

$$\frac{-\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{6d^2\left(d^2g^2-2defg-3e^2f^2\right)}{(d+ex)^2}-3\left(d^2g^2+6defg+5e^2f^2\right)\log(d-ex)+3\left(d^2g^2+6defg+5e^2f^2\right)\log(d+\right.}{192d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*d
^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g + d
^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 + 6*
d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*Log[d
 + e*x])/(192*d^6*e^3)
```

fricas [B] time = 0.85, size = 648, normalized size = 3.64

$$64\,d^5e^2f^2-16\,d^7g^2-6\left(5\,de^6f^2+6\,d^2e^5fg+d^3e^4g^2\right)x^4-18\left(5\,d^2e^5f^2+6\,d^3e^4fg+d^4e^3g^2\right)x^3-14\left(5\,d^3e^4f^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(64*d^5*e^2*f^2 - 16*d^7*g^2 - 6*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*e
^4*g^2)*x^4 - 18*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 14*(5*
d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 6*(5*d^4*e^3*f^2 + 6*d^5*e
^2*f*g - 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7*g^2 - (5*e^7
*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^
3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 + 2*(5
*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 + 6*d^5*
e^2*f*g + d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7
*g^2 - (5*e^7*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2
*e^5*f*g + d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^
2)*x^3 + 2*(5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3
*f^2 + 6*d^5*e^2*f*g + d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + 3*d^7*e^7
*x^4 + 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 - 3*d^10*e^4*x - d^11*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-3*e
xp(2)^2*d^2*exp(1)*g^2+12*exp(2)^2*d*exp(1)^2*g*f-10*exp(2)^2*exp(1)^3*f^2-
8*exp(2)*d^2*exp(1)^3*g^2+12*exp(2)*d*exp(1)^4*g*f-2*exp(2)*exp(1)^5*f^2-d^
2*exp(1)^5*g^2)/(2*exp(2)^4*d^6-8*exp(2)^3*d^6*exp(1)^2+12*exp(2)^2*d^6*exp
(1)^4-8*exp(2)*d^6*exp(1)^6+2*d^6*exp(1)^8)*ln(abs(-x^2*exp(2)+d^2))+(exp(2
)^4*f^2-exp(2)^3*d^2*g^2+6*exp(2)^3*d*exp(1)*g*f-10*exp(2)^3*exp(1)^2*f^2-1
4*exp(2)^2*d^2*exp(1)^2*g^2+36*exp(2)^2*d*exp(1)^3*g*f-15*exp(2)^2*exp(1)^4
*f^2-9*exp(2)*d^2*exp(1)^4*g^2+6*exp(2)*d*exp(1)^5*g*f)*1/2/(2*exp(2)^4*d^5
-8*exp(2)^3*d^5*exp(1)^2+12*exp(2)^2*d^5*exp(1)^4-8*exp(2)*d^5*exp(1)^6+2*d
^5*exp(1)^8)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp
(2)+2*exp(1)*abs(d)))+(3*exp(2)^2*d^2*exp(1)^2*g^2-12*exp(2)^2*d*exp(1)^3*g
*f+10*exp(2)^2*exp(1)^4*f^2+8*exp(2)*d^2*exp(1)^4*g^2-12*exp(2)*d*exp(1)^5*
g*f+2*exp(2)*exp(1)^6*f^2+d^2*exp(1)^6*g^2)/(exp(2)^4*d^6*exp(1)-4*exp(2)^3
*d^6*exp(1)^3+6*exp(2)^2*d^6*exp(1)^5-4*exp(2)*d^6*exp(1)^7+d^6*exp(1)^9)*l
n(abs(x*exp(1)+d))-((-exp(2)^4*d*exp(1)^2*f^2-5*exp(2)^3*d^3*exp(1)^2*g^2+1
8*exp(2)^3*d^2*exp(1)^3*g*f-10*exp(2)^3*d*exp(1)^4*f^2-2*exp(2)^2*d^3*exp(1
)^4*g^2-12*exp(2)^2*d^2*exp(1)^5*g*f+11*exp(2)^2*d*exp(1)^6*f^2+7*exp(2)*d^
3*exp(1)^6*g^2-6*exp(2)*d^2*exp(1)^7*g*f)*x^3+(-2*exp(2)^4*d^2*exp(1)*f^2-7
*exp(2)^3*d^4*exp(1)*g^2+24*exp(2)^3*d^3*exp(1)^2*g*f-10*exp(2)^3*d^2*exp(1
)^3*f^2+exp(2)^2*d^4*exp(1)^3*g^2-24*exp(2)^2*d^3*exp(1)^4*g*f+14*exp(2)^2*
d^2*exp(1)^5*f^2+7*exp(2)*d^4*exp(1)^5*g^2-2*exp(2)*d^2*exp(1)^7*f^2-d^4*ex
p(1)^7*g^2)*x^2+(-exp(2)^4*d^3*f^2-exp(2)^3*d^5*g^2+2*exp(2)^3*d^4*exp(1)*g
*f+4*exp(2)^3*d^3*exp(1)^2*f^2+8*exp(2)^2*d^5*exp(1)^2*g^2-24*exp(2)^2*d^4*
exp(1)^3*g*f+7*exp(2)^2*d^3*exp(1)^4*f^2-exp(2)*d^5*exp(1)^4*g^2+18*exp(2)*
d^4*exp(1)^5*g*f-10*exp(2)*d^3*exp(1)^6*f^2-6*d^5*exp(1)^6*g^2+4*d^4*exp(1)
^7*g*f)*x-2*exp(2)^3*d^5*g*f+3*exp(2)^3*d^4*exp(1)*f^2+8*exp(2)^2*d^6*exp(1
)*g^2-18*exp(2)^2*d^5*exp(1)^2*g*f+7*exp(2)^2*d^4*exp(1)^3*f^2-4*exp(2)*d^6
*exp(1)^3*g^2+18*exp(2)*d^5*exp(1)^4*g*f-11*exp(2)*d^4*exp(1)^5*f^2-4*d^6*e
xp(1)^5*g^2+2*d^5*exp(1)^6*g*f+d^4*exp(1)^7*f^2)/2/d^6/(exp(2)-exp(1)^2)^4/
(-x*exp(1)-d)^2/(-x^2*exp(2)+d^2)
```

maple [B] time = 0.02, size = 341, normalized size = 1.92

$$\frac{fg}{8\left(ex+d\right)^{4} d\, e^{2}}-\frac{f^{2}}{16\left(ex+d\right)^{4} d^{2} e}-\frac{g^{2}}{16\left(ex+d\right)^{4} e^{3}}+\frac{g^{2}}{12\left(ex+d\right)^{3} d\, e^{3}}-\frac{f^{2}}{12\left(ex+d\right)^{3} d^{3} e}+\frac{g^{2}}{32\left(ex+d\right)^{2} d^{2} e^{3}}-\frac{f}{16\left(ex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/64/e^3/d^4*ln(e*x-d)*g^2-3/32/e^2/d^5*ln(e*x-d)*f*g-5/64/e/d^6*ln(e*x-d)
*f^2-1/32/e^3/d^3/(e*x-d)*g^2-1/16/e^2/d^4/(e*x-d)*f*g-1/32/e/d^5/(e*x-d)*f
^2+1/64/e^3/d^4*ln(e*x+d)*g^2+3/32/e^2/d^5*ln(e*x+d)*f*g+5/64/e/d^6*ln(e*x+
d)*f^2+1/12/e^3/d/(e*x+d)^3*g^2-1/12/e/d^3/(e*x+d)^3*f^2+1/32/e^3/d^2/(e*x+
d)^2*g^2-1/16/e^2/d^3/(e*x+d)^2*f*g-3/32/e/d^4/(e*x+d)^2*f^2-1/16/e^3/(e*x+
d)^4*g^2+1/8/e^2/d/(e*x+d)^4*f*g-1/16/e/d^2/(e*x+d)^4*f^2-1/8/e^2*f/d^4/(e*
x+d)*g-1/8/e*f^2/d^5/(e*x+d)
```

maxima [A] time = 0.50, size = 298, normalized size = 1.67

$$\frac{32\, d^{4} e^{2} f^{2}-8\, d^{6} g^{2}-3\left(5\, e^{6} f^{2}+6\, d e^{5} f g+d^{2} e^{4} g^{2}\right)x^{4}-9\left(5\, d e^{5} f^{2}+6\, d^{2} e^{4} f g+d^{3} e^{3} g^{2}\right)x^{3}-7\left(5\, d^{2} e^{4} f^{2}+6\, d^{3} e^{3} f\right.}{96\left(d^{5} e^{8} x^{5}+3\, d^{6} e^{7} x^{4}+2\, d^{7} e^{6} x^{3}-2\, d^{8} e^{5} x^{2}-3\, d^{9} e^{4} x-d^{10} e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(32*d^4*e^2*f^2 - 8*d^6*g^2 - 3*(5*e^6*f^2 + 6*d*e^5*f*g + d^2*e^4*g^2
)*x^4 - 9*(5*d*e^5*f^2 + 6*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 - 7*(5*d^2*e^4*f^
2 + 6*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 3*(5*d^3*e^3*f^2 + 6*d^4*e^2*f*g - 7
*d^5*e*g^2)*x)/(d^5*e^8*x^5 + 3*d^6*e^7*x^4 + 2*d^7*e^6*x^3 - 2*d^8*e^5*x^2
```

```
 - 3*d^9*e^4*x - d^10*e^3) + 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x
 + d)/(d^6*e^3) - 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^6*
e^3)
```

mupad [B] time = 2.70, size = 274, normalized size = 1.54

$$\frac{\frac{d^2\,g^2-4\,e^2\,f^2}{12\,d\,e^3}+\frac{3\,x^3\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^4}+\frac{e\,x^4\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^5}-\frac{x\,\left(-7\,d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^2\,e^2}+\frac{7\,x^2\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{96\,d^3\,e}}{d^5+3\,d^4\,e\,x+2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3-3\,d\,e^4\,x^4-e^5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^3),x)
```

```
[Out] ((d^2*g^2 - 4*e^2*f^2)/(12*d*e^3) + (3*x^3*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g
))/(32*d^4) + (e*x^4*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g))/(32*d^5) - (x*(5*e^
2*f^2 - 7*d^2*g^2 + 6*d*e*f*g))/(32*d^2*e^2) + (7*x^2*(d^2*g^2 + 5*e^2*f^2
+ 6*d*e*f*g))/(96*d^3*e))/(d^5 - e^5*x^5 - 3*d*e^4*x^4 + 2*d^3*e^2*x^2 - 2*
d^2*e^3*x^3 + 3*d^4*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 5*e*f))/(d*(d^2*g
^2 + 5*e^2*f^2 + 6*d*e*f*g)))*(d*g + e*f)*(d*g + 5*e*f))/(32*d^6*e^3)
```

sympy [B] time = 1.93, size = 376, normalized size = 2.11

$$\frac{-8d^6g^2+32d^4e^2f^2+x^4\left(-3d^2e^4g^2-18de^5fg-15e^6f^2\right)+x^3\left(-9d^3e^3g^2-54d^2e^4fg-45de^5f^2\right)+x^2\left(-7d^4e^2g\right.}{-96d^{10}e^3-288d^9e^4x-192d^8e^5x^2+192d^7e^6x^3+288d^6e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-8*d**6*g**2 + 32*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2 - 18*d*e**5*f*g
 - 15*e**6*f**2) + x**3*(-9*d**3*e**3*g**2 - 54*d**2*e**4*f*g - 45*d*e**5*f
**2) + x**2*(-7*d**4*e**2*g**2 - 42*d**3*e**3*f*g - 35*d**2*e**4*f**2) + x*
(-21*d**5*e*g**2 + 18*d**4*e**2*f*g + 15*d**3*e**3*f**2))/(-96*d**10*e**3 -
 288*d**9*e**4*x - 192*d**8*e**5*x**2 + 192*d**7*e**6*x**3 + 288*d**6*e**7*
x**4 + 96*d**5*e**8*x**5) - (d*g + e*f)*(d*g + 5*e*f)*log(-d*(d*g + e*f)*(d
*g + 5*e*f)/(e*(d**2*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3) +
 (d*g + e*f)*(d*g + 5*e*f)*log(d*(d*g + e*f)*(d*g + 5*e*f)/(e*(d**2*g**2 +
6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3)
```

3.568 $\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{(dg+ef)(dg+3ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}-$$

```
[Out] 1/64*(d*g+e*f)^2/d^6/e^3/(-e*x+d)-1/20*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^5+1/16*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^4-1/48*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*
x+d)^3-1/16*f*(d*g+e*f)/d^5/e^2/(e*x+d)^2-1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^
3/(e*x+d)+1/32*(d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/d^7/e^3
```

Rubi [A] time = 0.24, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)^
5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f + d
*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2) -
((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*f +
d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^6}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{64d^6e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^6} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^5} + \frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^4}\right.$$
$$= \frac{(ef+dg)^2}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4} - \frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} - 1$$
$$= \frac{(ef+dg)^2}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4} - \frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} - 1$$

Mathematica [A] time = 0.18, size = 229, normalized size = 1.09

$$\frac{-\frac{48d^5(ef-dg)^2}{(d+ex)^5} - \frac{15d\left(d^2g^2+6defg+5e^2f^2\right)}{d+ex} - 15\left(d^2g^2+4defg+3e^2f^2\right)\log(d-ex) + 15\left(d^2g^2+4defg+3e^2f^2\right)\log(}{960d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((15*d*(e*f + d*g)^2)/(d - e*x) - (48*d^5*(e*f - d*g)^2)/(d + e*x)^5 + (60*
d^4*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^4 + (20*d^3*(-3*e^2*f^2 - 2*d*e*f*g +
 d^2*g^2))/(d + e*x)^3 - (60*d^2*e*f*(e*f + d*g))/(d + e*x)^2 - (15*d*(5*e^
2*f^2 + 6*d*e*f*g + d^2*g^2))/(d + e*x) - 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g
^2)*Log[d - e*x] + 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*Log[d + e*x])/(960*
d^7*e^3)
```

fricas [B] time = 0.92, size = 693, normalized size = 3.30

$$\frac{288\,d^6e^2f^2 + 64\,d^7efg - 32\,d^8g^2 - 30\left(3\,de^7f^2 + 4\,d^2e^6fg + d^3e^5g^2\right)x^5 - 120\left(3\,d^2e^6f^2 + 4\,d^3e^5fg + d^4e^4g^2\right)x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/960*(288*d^6*e^2*f^2 + 64*d^7*e*f*g - 32*d^8*g^2 - 30*(3*d*e^7*f^2 + 4*d^
2*e^6*f*g + d^3*e^5*g^2)*x^5 - 120*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4
*g^2)*x^4 - 160*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 40*(3*d
^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 2*(141*d^5*e^3*f^2 + 188*d^
6*e^2*f*g - 49*d^7*e*g^2)*x - 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g + d^8*g^2 - (
3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 + 4*d^2*e^6*f*g
 + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2)*x^4 +
 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d^5*e^3*f^2 + 4
*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x + d) + 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g
 + d^8*g^2 - (3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 +
 4*d^2*e^6*f*g + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*
e^4*g^2)*x^4 + 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d
^5*e^3*f^2 + 4*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 4*d
^8*e^8*x^5 + 5*d^9*e^7*x^4 - 5*d^11*e^5*x^2 - 4*d^12*e^4*x - d^13*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-2*e
xp(2)^3*d^2*exp(1)*g^2+10*exp(2)^3*d*exp(1)^2*g*f-10*exp(2)^3*exp(1)^3*f^2-
10*exp(2)^2*d^2*exp(1)^3*g^2+20*exp(2)^2*d*exp(1)^4*g*f-6*exp(2)^2*exp(1)^5
*f^2-4*exp(2)*d^2*exp(1)^5*g^2+2*exp(2)*d*exp(1)^6*g*f)/(exp(2)^5*d^7-5*exp
(2)^4*d^7*exp(1)^2+10*exp(2)^3*d^7*exp(1)^4-10*exp(2)^2*d^7*exp(1)^6+5*exp(
2)*d^7*exp(1)^8-d^7*exp(1)^10)*ln(abs(-x^2*exp(2)+d^2))+(exp(2)^5*f^2-exp(2
)^4*d^2*g^2+8*exp(2)^4*d*exp(1)*g*f-15*exp(2)^4*exp(1)^2*f^2-25*exp(2)^3*d^
2*exp(1)^2*g^2+80*exp(2)^3*d*exp(1)^3*g*f-45*exp(2)^3*exp(1)^4*f^2-35*exp(2
)^2*d^2*exp(1)^4*g^2+40*exp(2)^2*d*exp(1)^5*g*f-5*exp(2)^2*exp(1)^6*f^2-3*e
xp(2)*d^2*exp(1)^6*g^2)*1/2/(2*exp(2)^5*d^6-10*exp(2)^4*d^6*exp(1)^2+20*exp
(2)^3*d^6*exp(1)^4-20*exp(2)^2*d^6*exp(1)^6+10*exp(2)*d^6*exp(1)^8-2*d^6*ex
p(1)^10)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp(2)+
2*exp(1)*abs(d)))+(4*exp(2)^3*d^2*exp(1)^2*g^2-20*exp(2)^3*d*exp(1)^3*g*f+2
0*exp(2)^3*exp(1)^4*f^2+20*exp(2)^2*d^2*exp(1)^4*g^2-40*exp(2)^2*d*exp(1)^5
*g*f+12*exp(2)^2*exp(1)^6*f^2+8*exp(2)*d^2*exp(1)^6*g^2-4*exp(2)*d*exp(1)^7
*g*f)/(exp(2)^5*d^7*exp(1)-5*exp(2)^4*d^7*exp(1)^3+10*exp(2)^3*d^7*exp(1)^5
-10*exp(2)^2*d^7*exp(1)^7+5*exp(2)*d^7*exp(1)^9-d^7*exp(1)^11)*ln(abs(x*exp
(1)+d))-((-3*exp(2)^5*d*exp(1)^3*f^2-21*exp(2)^4*d^3*exp(1)^3*g^2+96*exp(2)
^4*d^2*exp(1)^4*g*f-75*exp(2)^4*d*exp(1)^5*f^2-45*exp(2)^3*d^3*exp(1)^5*g^2
+63*exp(2)^3*d*exp(1)^7*f^2+57*exp(2)^2*d^3*exp(1)^7*g^2-96*exp(2)^2*d^2*ex
p(1)^8*g*f+15*exp(2)^2*d*exp(1)^9*f^2+9*exp(2)*d^3*exp(1)^9*g^2)*x^4+(-9*ex
p(2)^5*d^2*exp(1)^2*f^2-51*exp(2)^4*d^4*exp(1)^2*g^2+228*exp(2)^4*d^3*exp(1
)^3*g*f-165*exp(2)^4*d^2*exp(1)^4*f^2-87*exp(2)^3*d^4*exp(1)^4*g^2-60*exp(2
)^3*d^3*exp(1)^5*g*f+165*exp(2)^3*d^2*exp(1)^6*f^2+135*exp(2)^2*d^4*exp(1)^
6*g^2-180*exp(2)^2*d^3*exp(1)^7*g*f+9*exp(2)^2*d^2*exp(1)^8*f^2+3*exp(2)*d^
4*exp(1)^8*g^2+12*exp(2)*d^3*exp(1)^9*g*f)*x^3+(-9*exp(2)^5*d^3*exp(1)*f^2-
35*exp(2)^4*d^5*exp(1)*g^2+148*exp(2)^4*d^4*exp(1)^2*g*f-83*exp(2)^4*d^3*ex
p(1)^3*f^2-9*exp(2)^3*d^5*exp(1)^3*g^2-204*exp(2)^3*d^4*exp(1)^4*g*f+183*ex
p(2)^3*d^3*exp(1)^5*f^2+117*exp(2)^2*d^5*exp(1)^5*g^2-36*exp(2)^2*d^4*exp(1
)^6*g*f-81*exp(2)^2*d^3*exp(1)^7*f^2-67*exp(2)*d^5*exp(1)^7*g^2+92*exp(2)*d
^4*exp(1)^8*g*f-10*exp(2)*d^3*exp(1)^9*f^2-6*d^5*exp(1)^9*g^2)*x^2+(-3*exp(
2)^5*d^4*f^2-3*exp(2)^4*d^6*g^2+6*exp(2)^4*d^5*exp(1)*g*f+21*exp(2)^4*d^4*e
xp(1)^2*f^2+63*exp(2)^3*d^6*exp(1)^2*g^2-252*exp(2)^3*d^5*exp(1)^3*g*f+147*
exp(2)^3*d^4*exp(1)^4*f^2+69*exp(2)^2*d^6*exp(1)^4*g^2+96*exp(2)^2*d^5*exp(
1)^5*g*f-153*exp(2)^2*d^4*exp(1)^6*f^2-123*exp(2)*d^6*exp(1)^6*g^2+156*exp(
2)*d^5*exp(1)^7*g*f-12*exp(2)*d^4*exp(1)^8*f^2-6*d^6*exp(1)^8*g^2-6*d^5*exp
(1)^9*g*f)*x-6*exp(2)^4*d^6*g*f+12*exp(2)^4*d^5*exp(1)*f^2+38*exp(2)^3*d^7*
exp(1)*g^2-124*exp(2)^3*d^6*exp(1)^2*g*f+74*exp(2)^3*d^5*exp(1)^3*f^2+18*ex
p(2)^2*d^7*exp(1)^3*g^2+72*exp(2)^2*d^6*exp(1)^4*g*f-90*exp(2)^2*d^5*exp(1)
^5*f^2-54*exp(2)*d^7*exp(1)^5*g^2+60*exp(2)*d^6*exp(1)^6*g*f+6*exp(2)*d^5*e
xp(1)^7*f^2-2*d^7*exp(1)^7*g^2-2*d^6*exp(1)^8*g*f-2*d^5*exp(1)^9*f^2)/6/d^7
/(exp(2)-exp(1)^2)^5/(-x*exp(1)-d)^3/(x^2*exp(2)-d^2)
```

maple [B] time = 0.02, size = 394, normalized size = 1.88

$$\frac{fg}{10\left(ex+d\right)^{5} d\,e^{2}}-\frac{f^{2}}{20\left(ex+d\right)^{5} d^{2} e}-\frac{g^{2}}{20\left(ex+d\right)^{5} e^{3}}+\frac{g^{2}}{16\left(ex+d\right)^{4} d\,e^{3}}-\frac{f^{2}}{16\left(ex+d\right)^{4} d^{3} e}+\frac{g^{2}}{48\left(ex+d\right)^{3} d^{2} e^{3}}-\frac{}{24\left(ex\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/64/e^3/d^5*ln(e*x-d)*g^2-1/16/e^2/d^6*ln(e*x-d)*f*g-3/64/e/d^7*ln(e*x-d)
*f^2-1/64/e^3/d^4/(e*x-d)*g^2-1/32/e^2/d^5/(e*x-d)*f*g-1/64/e/d^6/(e*x-d)*f
^2+1/64/e^3/d^5*ln(e*x+d)*g^2+1/16/e^2/d^6*ln(e*x+d)*f*g+3/64/e/d^7*ln(e*x+
d)*f^2-1/64/e^3/d^4/(e*x+d)*g^2-3/32/e^2/d^5/(e*x+d)*f*g-5/64/e/d^6/(e*x+d)
*f^2+1/16/e^3/d/(e*x+d)^4*g^2-1/16/e/d^3/(e*x+d)^4*f^2+1/48/e^3/d^2/(e*x+d)
^3*g^2-1/24/e^2/d^3/(e*x+d)^3*f*g-1/16/e/d^4/(e*x+d)^3*f^2-1/20/e^3/(e*x+d)
```

```
^5*g^2+1/10/e^2/d/(e*x+d)^5*f*g-1/20/e/d^2/(e*x+d)^5*f^2-1/16/e^2*f/d^4/(e*
x+d)^2*g-1/16/e*f^2/d^5/(e*x+d)^2
```

maxima [A] time = 0.53, size = 342, normalized size = 1.63

$$\frac{144\,d^5e^2f^2+32\,d^6efg-16\,d^7g^2-15\left(3\,e^7f^2+4\,de^6fg+d^2e^5g^2\right)x^5-60\left(3\,de^6f^2+4\,d^2e^5fg+d^3e^4g^2\right)x^4-80}{480\left(d^6e^9x^6+4\,d^7e^8x^5+5\,d^8e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(144*d^5*e^2*f^2 + 32*d^6*e*f*g - 16*d^7*g^2 - 15*(3*e^7*f^2 + 4*d*e^
6*f*g + d^2*e^5*g^2)*x^5 - 60*(3*d*e^6*f^2 + 4*d^2*e^5*f*g + d^3*e^4*g^2)*x
^4 - 80*(3*d^2*e^5*f^2 + 4*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 20*(3*d^3*e^4*f
^2 + 4*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + (141*d^4*e^3*f^2 + 188*d^5*e^2*f*g
- 49*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 4*d^7*e^8*x^5 + 5*d^8*e^7*x^4 - 5*d^10*e^
5*x^2 - 4*d^11*e^4*x - d^12*e^3) + 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^7*e^3) - 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(e*x - d)
/(d^7*e^3)
```

mupad [B] time = 2.72, size = 314, normalized size = 1.50

$$\frac{\frac{x^3\left(d^2g^2+4\,d\,e\,f\,g+3\,e^2f^2\right)}{6\,d^4}-\frac{-d^2g^2+2\,d\,e\,f\,g+9\,e^2f^2}{30\,d\,e^3}+\frac{e\,x^4\left(d^2g^2+4\,d\,e\,f\,g+3\,e^2f^2\right)}{8\,d^5}-\frac{x\left(-49\,d^2g^2+188\,d\,e\,f\,g+141\,e^2f^2\right)}{480\,d^2e^2}+\frac{x^2\left(d^2g^2+4\,d\,e\right.}{24\,d}}{d^6+4\,d^5\,e\,x+5\,d^4\,e^2\,x^2-5\,d^2\,e^4\,x^4-4\,d\,e^5\,x^5-e^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^4),x)
```

```
[Out] ((x^3*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(6*d^4) - (9*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g)/(30*d*e^3) + (e*x^4*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(8*d^5) -
(x*(141*e^2*f^2 - 49*d^2*g^2 + 188*d*e*f*g))/(480*d^2*e^2) + (x^2*(d^2*g^2
+ 3*e^2*f^2 + 4*d*e*f*g))/(24*d^3*e) + (e^2*x^5*(d^2*g^2 + 3*e^2*f^2 + 4*d*
e*f*g))/(32*d^6))/(d^6 - e^6*x^6 - 4*d*e^5*x^5 + 5*d^4*e^2*x^2 - 5*d^2*e^4*
x^4 + 4*d^5*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 3*e*f))/(d*(d^2*g^2 + 3*e
^2*f^2 + 4*d*e*f*g)))*(d*g + e*f)*(d*g + 3*e*f))/(32*d^7*e^3)
```

sympy [B] time = 2.15, size = 427, normalized size = 2.03

$$\frac{-16d^7g^2+32d^6efg+144d^5e^2f^2+x^5\left(-15d^2e^5g^2-60de^6fg-45e^7f^2\right)+x^4\left(-60d^3e^4g^2-240d^2e^5fg-180de\right.}{-480d^{12}e^3-1920d^{11}e^4x-24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-16*d**7*g**2 + 32*d**6*e*f*g + 144*d**5*e**2*f**2 + x**5*(-15*d**2*e**5*g
**2 - 60*d*e**6*f*g - 45*e**7*f**2) + x**4*(-60*d**3*e**4*g**2 - 240*d**2*e
**5*f*g - 180*d*e**6*f**2) + x**3*(-80*d**4*e**3*g**2 - 320*d**3*e**4*f*g -
 240*d**2*e**5*f**2) + x**2*(-20*d**5*e**2*g**2 - 80*d**4*e**3*f*g - 60*d**
3*e**4*f**2) + x*(-49*d**6*e*g**2 + 188*d**5*e**2*f*g + 141*d**4*e**3*f**2)
)/(-480*d**12*e**3 - 1920*d**11*e**4*x - 2400*d**10*e**5*x**2 + 2400*d**8*e
**7*x**4 + 1920*d**7*e**8*x**5 + 480*d**6*e**9*x**6) - (d*g + e*f)*(d*g + 3
*e*f)*log(-d*(d*g + e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f
**2)) + x)/(64*d**7*e**3) + (d*g + e*f)*(d*g + 3*e*f)*log(d*(d*g + e*f)*(d*
g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2)) + x)/(64*d**7*e**3)
```

3.569 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=179

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)}-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d}{e^3}$$

```
[Out] -d*(56*d^2*g^2+48*d*e*f*g+7*e^2*f^2)*x/e^2-1/2*(2*d*g+e*f)*(12*d*g+e*f)*x^2
/e-1/3*g*(7*d*g+2*e*f)*x^3-1/4*e*g^2*x^4+8*d^4*(d*g+e*f)^2/e^3/(-e*x+d)^2-3
2*d^3*(d*g+e*f)*(2*d*g+e*f)/e^3/(-e*x+d)-8*d^2*(13*d^2*g^2+14*d*e*f*g+3*e^2
*f^2)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {848, 88}

$$-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2d}{e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(e*
f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*(d
- e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(-\frac{d\left(7e^2f^2+48defg+56d^2g^2\right)}{e^2}+\frac{(-ef-12dg)(ef+2dg)x}{e}-g(2ef+7dg)x^2-e\right.\\ &= -\frac{d\left(7e^2f^2+48defg+56d^2g^2\right)x}{e^2}-\frac{(ef+2dg)(ef+12dg)x^2}{2e}-\frac{1}{3}g(2ef+7dg)x^3-\frac{1}{4}e\end{aligned}$$

Mathematica [A] time = 0.10, size = 193, normalized size = 1.08

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{x^2\left(24d^2g^2+14defg+e^2f^2\right)}{2e}-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e^2*f^2 + 14*d*e*f*g
 + 24*d^2*g^2)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*
d^4*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2
*g^2))/(e^3*(-d + e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[
d - e*x])/e^3
```

fricas [A] time = 0.83, size = 336, normalized size = 1.88

$$-\frac{3 e^6 g^2 x^6 + 288 d^4 e^2 f^2 + 960 d^5 e f g + 672 d^6 g^2 + 2 \left(4 e^6 f g + 11 d e^5 g^2\right) x^5 + \left(6 e^6 f^2 + 68 d e^5 f g + 91 d^2 e^4 g^2\right) x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^6*g^2*x^6 + 288*d^4*e^2*f^2 + 960*d^5*e*f*g + 672*d^6*g^2 + 2*(4
*e^6*f*g + 11*d*e^5*g^2)*x^5 + (6*e^6*f^2 + 68*d*e^5*f*g + 91*d^2*e^4*g^2)*
x^4 + 4*(18*d*e^5*f^2 + 104*d^2*e^4*f*g + 103*d^3*e^3*g^2)*x^3 - 6*(27*d^2*
e^4*f^2 + 178*d^3*e^3*f*g + 200*d^4*e^2*g^2)*x^2 - 12*(25*d^3*e^3*f^2 + 48*
d^4*e^2*f*g + 8*d^5*e*g^2)*x + 96*(3*d^4*e^2*f^2 + 14*d^5*e*f*g + 13*d^6*g^
2 + (3*d^2*e^4*f^2 + 14*d^3*e^3*f*g + 13*d^4*e^2*g^2)*x^2 - 2*(3*d^3*e^3*f^
2 + 14*d^4*e^2*f*g + 13*d^5*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x +
d^2*e^3)
```

giac [B] time = 0.19, size = 364, normalized size = 2.03

$$-4 \left(13 d^4 g^2 e^7 + 14 d^3 f g e^8 + 3 d^2 f^2 e^9\right) e^{(-10)} \log\left(\left|x^2 e^2 - d^2\right|\right) - \frac{1}{12} \left(3 g^2 x^4 e^{25} + 28 d g^2 x^3 e^{24} + 144 d^2 g^2 x^2 e^{23} + 6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -4*(13*d^4*g^2*e^7 + 14*d^3*f*g*e^8 + 3*d^2*f^2*e^9)*e^(-10)*log(abs(x^2*e^
2 - d^2)) - 1/12*(3*g^2*x^4*e^25 + 28*d*g^2*x^3*e^24 + 144*d^2*g^2*x^2*e^23
 + 672*d^3*g^2*x*e^22 + 8*f*g*x^3*e^25 + 84*d*f*g*x^2*e^24 + 576*d^2*f*g*x*
e^23 + 6*f^2*x^2*e^25 + 84*d*f^2*x*e^24)*e^(-24) - 4*(13*d^5*g^2*e^6 + 14*d
^4*f*g*e^7 + 3*d^3*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^
2 + 2*abs(d)*e))/abs(d) - 8*(7*d^8*g^2*e^7 + 10*d^7*f*g*e^8 + 3*d^6*f^2*e^9
 - 4*(2*d^5*g^2*e^10 + 3*d^4*f*g*e^11 + d^3*f^2*e^12)*x^3 - (9*d^6*g^2*e^9
+ 14*d^5*f*g*e^10 + 5*d^4*f^2*e^11)*x^2 + 2*(3*d^7*g^2*e^8 + 4*d^6*f*g*e^9
+ d^5*f^2*e^10)*x)*e^(-10)/(x^2*e^2 - d^2)^2
```

maple [A] time = 0.01, size = 263, normalized size = 1.47

$$-\frac{e g^2 x^4}{4} - \frac{7 d g^2 x^3}{3} - \frac{2 e f g x^3}{3} + \frac{8 d^6 g^2}{\left(e x - d\right)^2 e^3} + \frac{16 d^5 f g}{\left(e x - d\right)^2 e^2} + \frac{8 d^4 f^2}{\left(e x - d\right)^2 e} - \frac{12 d^2 g^2 x^2}{e} - 7 d f g x^2 - \frac{e f^2 x^2}{2} + \frac{64 d^5 g^2}{\left(e x - d\right) e^3} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/4*e*g^2*x^4-7/3*d*g^2*x^3-2/3*e*f*g*x^3-12*d^2/e*g^2*x^2-7*d*f*g*x^2-1/2
*e*f^2*x^2-56*d^3/e^2*g^2*x-48*d^2/e*f*g*x-7*d*f^2*x-104*d^4/e^3*g^2*ln(e*x
-d)-112*d^3/e^2*f*g*ln(e*x-d)-24*d^2/e*f^2*ln(e*x-d)+8*d^6/e^3/(e*x-d)^2*g^
2+16*d^5/e^2/(e*x-d)^2*f*g+8*d^4/e/(e*x-d)^2*f^2+64/(e*x-d)*d^5/e^3*g^2+96/
(e*x-d)*d^4/e^2*f*g+32/(e*x-d)*d^3/e*f^2
```

maxima [A] time = 0.47, size = 227, normalized size = 1.27

$$-\frac{8\left(3\,d^4e^2f^2+10\,d^5efg+7\,d^6g^2-4\left(d^3e^3f^2+3\,d^4e^2fg+2\,d^5eg^2\right)x\right)}{e^5x^2-2\,de^4x+d^2e^3}-\frac{3\,e^3g^2x^4+4\left(2\,e^3fg+7\,de^2g^2\right)x^3+6\left(e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -8*(3*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*f
*g + 2*d^5*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/12*(3*e^3*g^2*x^4
+ 4*(2*e^3*f*g + 7*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 14*d*e^2*f*g + 24*d^2*e*g^
2)*x^2 + 12*(7*d*e^2*f^2 + 48*d^2*e*f*g + 56*d^3*g^2)*x)/e^2 - 8*(3*d^2*e^2
*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.14, size = 375, normalized size = 2.09

$$\frac{x\,\left(64\,d^5\,g^2+96\,d^4\,e\,f\,g+32\,d^3\,e^2\,f^2\right)-\frac{8\,\left(7\,d^6\,g^2+10\,d^5\,e\,f\,g+3\,d^4\,e^2\,f^2\right)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x^2\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{2\,e^3}-\frac{3\,d^2\,g^2}{2\,e}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g) - (8*(7*d^6*g^2 + 3*d^4*e^2
*f^2 + 10*d^5*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x^2*((e^4*f^2 +
6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(2*e^3) - (3*d^2*g^2)/(2*e) + (3*d*(2*g*(2*d*g
 + e*f) + 3*d*g^2))/(2*e)) - x*((d^3*g^2)/e^2 - (3*d^2*(2*g*(2*d*g + e*f) +
 3*d*g^2))/e^2 + (4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e^2 + (3*d*((e^4*f^2
 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^3 - (3*d^2*g^2)/e + (3*d*(2*g*(2*d*g + e*
f) + 3*d*g^2))/e))/e) - x^3*((2*g*(2*d*g + e*f))/3 + d*g^2) - (log(e*x - d)
*(104*d^4*g^2 + 24*d^2*e^2*f^2 + 112*d^3*e*f*g))/e^3 - (e*g^2*x^4)/4
```

sympy [A] time = 1.55, size = 219, normalized size = 1.22

$$-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log\left(-d+ex\right)}{e^3}-\frac{eg^2x^4}{4}-x^3\left(\frac{7dg^2}{3}+\frac{2efg}{3}\right)-x^2\left(\frac{12d^2g^2}{e}+7dfg+\frac{ef^2}{2}\right)-x\left(\frac{56d^3g}{e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -8*d**2*(13*d**2*g**2 + 14*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - e*g*
*2*x**4/4 - x**3*(7*d*g**2/3 + 2*e*f*g/3) - x**2*(12*d**2*g**2/e + 7*d*f*g
+ e*f**2/2) - x*(56*d**3*g**2/e**2 + 48*d**2*f*g/e + 7*d*f**2) - (56*d**6*g
**2 + 80*d**5*e*f*g + 24*d**4*e**2*f**2 + x*(-64*d**5*e*g**2 - 96*d**4*e**2
*f*g - 32*d**3*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

3.570 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^3} dx$

Optimal. Leaf size=149

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)}-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-}{e^3}$$

```
[Out] -(18*d^2*g^2+12*d*e*f*g+e^2*f^2)*x/e^2-g*(3*d*g+e*f)*x^2/e-1/3*g^2*x^3+4*d^
3*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d^2*(d*g+e*f)*(7*d*g+3*e*f)/e^3/(-e*x+d)-2*d
*(19*d^2*g^2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg}{e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g)*
(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^
2)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^3} dx &= \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^3} dx\\ &= \int \left(\frac{-e^2f^2-12defg-18d^2g^2}{e^2}-\frac{2g(ef+3dg)x}{e}-g^2x^2+\frac{4d^2(-3ef-7dg)(ef+dg}{e^2(d-ex)^2}\right.\\ &= -\frac{\left(e^2f^2+12defg+18d^2g^2\right)x}{e^2}-\frac{g(ef+3dg)x^2}{e}-\frac{g^2x^3}{3}+\frac{4d^3(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d^2(ef}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 157, normalized size = 1.05

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}+\frac{4d^2\left(7d^2g^2+10defg+3e^2f^2\right)}{e^3(ex-d)}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\text{l}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (4*d^2*(3*e^2*f^2 +
 10*d*e*f*g + 7*d^2*g^2))/(e^3*(-d + e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g +
 19*d^2*g^2)*Log[d - e*x])/e^3
```

fricas [A] time = 0.85, size = 294, normalized size = 1.97

$$-\frac{e^5g^2x^5 + 24\,d^3e^2f^2 + 96\,d^4efg + 72\,d^5g^2 + \left(3\,e^5fg + 7\,de^4g^2\right)x^4 + \left(3\,e^5f^2 + 30\,de^4fg + 37\,d^2e^3g^2\right)x^3 - 3\,\left(2\,de^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(e^5*g^2*x^5 + 24*d^3*e^2*f^2 + 96*d^4*e*f*g + 72*d^5*g^2 + (3*e^5*f*g
 + 7*d*e^4*g^2)*x^4 + (3*e^5*f^2 + 30*d*e^4*f*g + 37*d^2*e^3*g^2)*x^3 - 3*(
2*d*e^4*f^2 + 23*d^2*e^3*f*g + 33*d^3*e^2*g^2)*x^2 - 3*(11*d^2*e^3*f^2 + 28
*d^3*e^2*f*g + 10*d^4*e*g^2)*x + 6*(3*d^3*e^2*f^2 + 18*d^4*e*f*g + 19*d^5*g
^2 + (3*d*e^4*f^2 + 18*d^2*e^3*f*g + 19*d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2
 + 18*d^3*e^2*f*g + 19*d^4*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d
^2*e^3)
```

giac [B] time = 0.20, size = 324, normalized size = 2.17

$$-\left(19\,d^3g^2e^5 + 18\,d^2fge^6 + 3\,df^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{1}{3}\left(g^2x^3e^{18} + 9\,dg^2x^2e^{17} + 54\,d^2g^2xe^{16} + 3\,fgx^2e^{18} + 36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -(19*d^3*g^2*e^5 + 18*d^2*f*g*e^6 + 3*d*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d
^2)) - 1/3*(g^2*x^3*e^18 + 9*d*g^2*x^2*e^17 + 54*d^2*g^2*x*e^16 + 3*f*g*x^2
*e^18 + 36*d*f*g*x*e^17 + 3*f^2*x*e^18)*e^(-18) - (19*d^4*g^2*e^6 + 18*d^3*
f*g*e^7 + 3*d^2*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
 2*abs(d)*e))/abs(d) - 4*(6*d^7*g^2*e^5 + 8*d^6*f*g*e^6 + 2*d^5*f^2*e^7 - (
7*d^4*g^2*e^8 + 10*d^3*f*g*e^9 + 3*d^2*f^2*e^10)*x^3 - 4*(2*d^5*g^2*e^7 + 3
*d^4*f*g*e^8 + d^3*f^2*e^9)*x^2 + (5*d^6*g^2*e^6 + 6*d^5*f*g*e^7 + d^4*f^2*
e^8)*x)*e^(-8)/(x^2*e^2 - d^2)^2
```

maple [A] time = 0.01, size = 228, normalized size = 1.53

$$-\frac{g^2x^3}{3} + \frac{4d^5g^2}{\left(ex-d\right)^2e^3} + \frac{8d^4fg}{\left(ex-d\right)^2e^2} + \frac{4d^3f^2}{\left(ex-d\right)^2e} - \frac{3d\,g^2x^2}{e} - fg\,x^2 + \frac{28d^4g^2}{\left(ex-d\right)e^3} + \frac{40d^3fg}{\left(ex-d\right)e^2} - \frac{38d^3g^2\ln\left(ex-d\right)}{e^3} + \frac{12d}{\left(ex-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/3*g^2*x^3-3*d/e*g^2*x^2-f*g*x^2-18*d^2/e^2*g^2*x-12*d/e*f*g*x-f^2*x-38*d
^3/e^3*g^2*ln(e*x-d)-36*d^2/e^2*f*g*ln(e*x-d)-6*d/e*f^2*ln(e*x-d)+4*d^5/e^3
/(e*x-d)^2*g^2+8*d^4/e^2/(e*x-d)^2*f*g+4*d^3/e/(e*x-d)^2*f^2+28/(e*x-d)*d^4
/e^3*g^2+40/(e*x-d)*d^3/e^2*f*g+12/(e*x-d)*d^2/e*f^2
```

maxima [A] time = 0.46, size = 188, normalized size = 1.26

$$-\frac{4\left(2\,d^3e^2f^2 + 8\,d^4efg + 6\,d^5g^2 - \left(3\,d^2e^3f^2 + 10\,d^3e^2fg + 7\,d^4eg^2\right)x\right)}{e^5x^2 - 2\,de^4x + d^2e^3} - \frac{e^2g^2x^3 + 3\left(e^2fg + 3\,deg^2\right)x^2 + 3\left(e^2f^2 + \right.}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -4*(2*d^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f
*g + 7*d^4*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/3*(e^2*g^2*x^3 + 3
*(e^2*f*g + 3*d*e*g^2)*x^2 + 3*(e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2 -
 2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.10, size = 240, normalized size = 1.61

$$\frac{x\left(28\,d^4\,g^2+40\,d^3\,e\,f\,g+12\,d^2\,e^2\,f^2\right)-\frac{8\left(3\,d^5\,g^2+4\,d^4\,e\,f\,g+d^3\,e^2\,f^2\right)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^3}+\frac{3\,d\left(\frac{g\left(3\,d\,g\right.}{e}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(28*d^4*g^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g) - (8*(3*d^5*g^2 + d^3*e^2*f
^2 + 4*d^4*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((e^3*f^2 + 3*d^2
*e*g^2 + 6*d*e^2*f*g)/e^3 + (3*d*((g*(3*d*g + 2*e*f))/e + (3*d*g^2)/e))/e -
 (3*d^2*g^2)/e^2) - x^2*((g*(3*d*g + 2*e*f))/(2*e) + (3*d*g^2)/(2*e)) - (g^
2*x^3)/3 - (log(e*x - d)*(38*d^3*g^2 + 6*d*e^2*f^2 + 36*d^2*e*f*g))/e^3
```

sympy [A] time = 1.37, size = 178, normalized size = 1.19

$$-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log\left(-d+ex\right)}{e^3}-\frac{g^2x^3}{3}-x^2\left(\frac{3dg^2}{e}+fg\right)-x\left(\frac{18d^2g^2}{e^2}+\frac{12dfg}{e}+f^2\right)-\frac{24d^5g^2+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - g**2*x*
*3/3 - x**2*(3*d*g**2/e + f*g) - x*(18*d**2*g**2/e**2 + 12*d*f*g/e + f**2)
- (24*d**5*g**2 + 32*d**4*e*f*g + 8*d**3*e**2*f**2 + x*(-28*d**4*e*g**2 - 4
0*d**3*e**2*f*g - 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

3.571 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=118

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

```
[Out] -g*(5*d*g+2*e*f)*x/e^2-1/2*g^2*x^2/e+2*d^2*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d*(
d*g+e*f)*(3*d*g+e*f)/e^3/(-e*x+d)-(13*d^2*g^2+10*d*e*f*g+e^2*f^2)*ln(-e*x+d
)/e^3
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {848, 88}

$$-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^3
*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f^2
 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^3}\,dx$$
$$=\int \left(-\frac{g(2ef+5dg)}{e^2}-\frac{g^2x}{e}+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)^2}-\frac{4d^2(ef+dg)^2}{e^2(-d+ex)^3}+\frac{-e^2f^2-10d\ldots}{e^2(-d\ldots}\right.$$
$$=-\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)}-\frac{\left(e^2f^2+10defg\ldots\right.}{}$$

Mathematica [A] time = 0.09, size = 118, normalized size = 1.00

$$-\frac{\frac{8d\left(3d^2g^2+4defg+e^2f^2\right)}{d-ex}+2\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)-\frac{4d^2(dg+ef)^2}{(d-ex)^2}+2egx(5dg+2ef)+e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -1/2*(2*e*g*(2*e*f + 5*d*g)*x + e^2*g^2*x^2 - (4*d^2*(e*f + d*g)^2)/(d - e*
x)^2 + (8*d*(e^2*f^2 + 4*d*e*f*g + 3*d^2*g^2))/(d - e*x) + 2*(e^2*f^2 + 10*
d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

fricas [B] time = 0.88, size = 241, normalized size = 2.04

$$-\frac{e^4g^2x^4 + 4\,d^2e^2f^2 + 24\,d^3efg + 20\,d^4g^2 + 4\left(e^4fg + 2\,de^3g^2\right)x^3 - \left(8\,de^3fg + 19\,d^2e^2g^2\right)x^2 - 2\left(4\,de^3f^2 + 14\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(e^4*g^2*x^4 + 4*d^2*e^2*f^2 + 24*d^3*e*f*g + 20*d^4*g^2 + 4*(e^4*f*g
+ 2*d*e^3*g^2)*x^3 - (8*d*e^3*f*g + 19*d^2*e^2*g^2)*x^2 - 2*(4*d*e^3*f^2 +
14*d^2*e^2*f*g + 7*d^3*e*g^2)*x + 2*(d^2*e^2*f^2 + 10*d^3*e*f*g + 13*d^4*g^
2 + (e^4*f^2 + 10*d*e^3*f*g + 13*d^2*e^2*g^2)*x^2 - 2*(d*e^3*f^2 + 10*d^2*e
^2*f*g + 13*d^3*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)
```

giac [B] time = 0.38, size = 273, normalized size = 2.31

$$-\frac{1}{2}\left(13\,d^2g^2e^5 + 10\,dfge^6 + f^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{1}{2}\left(g^2x^2e^{11} + 10\,dg^2xe^{10} + 4\,fgxe^{11}\right)e^{(-12)} - \frac{\left(13\,d^3g^2e^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*(13*d^2*g^2*e^5 + 10*d*f*g*e^6 + f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d^2
)) - 1/2*(g^2*x^2*e^11 + 10*d*g^2*x*e^10 + 4*f*g*x*e^11)*e^(-12) - 1/2*(13*
d^3*g^2*e^4 + 10*d^2*f*g*e^5 + d*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)
*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - 2*(5*d^6*g^2*e^5 + 6*d^5*f*g*e^6 +
d^4*f^2*e^7 - 2*(3*d^3*g^2*e^8 + 4*d^2*f*g*e^9 + d*f^2*e^10)*x^3 - (7*d^4*g
^2*e^7 + 10*d^3*f*g*e^8 + 3*d^2*f^2*e^9)*x^2 + 4*(d^5*g^2*e^6 + d^4*f*g*e^7
)*x)*e^(-8)/(x^2*e^2 - d^2)^2
```

maple [A] time = 0.01, size = 198, normalized size = 1.68

$$\frac{2d^4g^2}{\left(ex-d\right)^2e^3}+\frac{4d^3fg}{\left(ex-d\right)^2e^2}+\frac{2d^2f^2}{\left(ex-d\right)^2e}-\frac{g^2x^2}{2e}+\frac{12d^3g^2}{\left(ex-d\right)e^3}+\frac{16d^2fg}{\left(ex-d\right)e^2}-\frac{13d^2g^2\ln\left(ex-d\right)}{e^3}+\frac{4df^2}{\left(ex-d\right)e}-\frac{10dfg\ln}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/2/e*g^2*x^2-5*d/e^2*g^2*x-2/e*f*g*x-13*d^2/e^3*g^2*ln(e*x-d)-10*d/e^2*f*
g*ln(e*x-d)-1/e*f^2*ln(e*x-d)+2*d^4/e^3/(e*x-d)^2*g^2+4*d^3/e^2/(e*x-d)^2*f
*g+2*d^2/e/(e*x-d)^2*f^2+12/(e*x-d)*d^3/e^3*g^2+16/(e*x-d)*d^2/e^2*f*g+4/(e
*x-d)*d/e*f^2
```

maxima [A] time = 0.46, size = 149, normalized size = 1.26

$$-\frac{2\left(d^2e^2f^2 + 6\,d^3efg + 5\,d^4g^2 - 2\left(de^3f^2 + 4\,d^2e^2fg + 3\,d^3eg^2\right)x\right)}{e^5x^2 - 2\,de^4x + d^2e^3} - \frac{eg^2x^2 + 2\left(2\,efg + 5\,dg^2\right)x}{2\,e^2} - \frac{\left(e^2f^2 + 10\,de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -2*(d^2*e^2*f^2 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g +
3*d^3*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/2*(e*g^2*x^2 + 2*(2*e*f
*g + 5*d*g^2)*x)/e^2 - (e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.60, size = 161, normalized size = 1.36

$$-\frac{\frac{2\left(5\,d^4\,g^2+6\,d^3\,e\,f\,g+d^2\,e^2\,f^2\right)}{e}-x\,\left(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2\right)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x\,\left(\frac{2\,g\,\left(d\,g+e\,f\right)}{e^2}+\frac{3\,d\,g^2}{e^2}\right)-\frac{\ln\left(e\,x-d\right)\,\left(13\,d^2\,g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((2*(5*d^4*g^2 + d^2*e^2*f^2 + 6*d^3*e*f*g))/e - x*(12*d^3*g^2 + 4*d*e^2*
f^2 + 16*d^2*e*f*g))/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((2*g*(d*g + e*f))
/e^2 + (3*d*g^2)/e^2) - (log(e*x - d)*(13*d^2*g^2 + e^2*f^2 + 10*d*e*f*g))/
e^3 - (g^2*x^2)/(2*e)
```

sympy [A] time = 1.21, size = 151, normalized size = 1.28

$$-x\left(\frac{5dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{10d^4g^2+12d^3efg+2d^2e^2f^2+x\left(-12d^3eg^2-16d^2e^2fg-4de^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x^2}{2e}-\frac{\left(13d^2g^2+10defg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -x*(5*d*g**2/e**2 + 2*f*g/e) - (10*d**4*g**2 + 12*d**3*e*f*g + 2*d**2*e**2*
f**2 + x*(-12*d**3*e*g**2 - 16*d**2*e**2*f*g - 4*d*e**3*f**2))/(d**2*e**3 -
 2*d*e**4*x + e**5*x**2) - g**2*x**2/(2*e) - (13*d**2*g**2 + 10*d*e*f*g + e
**2*f**2)*log(-d + e*x)/e**3
```

3.572 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

```
[Out] -g^2*x/e^2+d*(d*g+e*f)^2/e^3/(-e*x+d)^2-(d*g+e*f)*(5*d*g+e*f)/e^3/(-e*x+d)-
2*g*(2*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^3}\,dx\\
&= \int \left(-\frac{g^2}{e^2}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)^3}-\frac{2g(ef+2dg)}{e^2(-d+ex)}\right)dx\\
&= -\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)}-\frac{2g(ef+2dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 1.15

$$\frac{-4d^3g^2+4d^2eg(gx-f)+2de^2gx(3f+gx)-2g(d-ex)^2(2dg+ef)\log(d-ex)+e^3x\left(f^2-g^2x^2\right)}{e^3(d-ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]

[Out] (-4*d^3*g^2 + 4*d^2*e*g*(-f + g*x) + 2*d*e^2*g*x*(3*f + g*x) + e^3*x*(f^2 - g^2*x^2) - 2*g*(e*f + 2*d*g)*(d - e*x)^2*Log[d - e*x])/(e^3*(d - e*x)^2)

fricas [A] time = 1.03, size = 159, normalized size = 1.96

$$-\frac{e^3g^2x^3 - 2\,de^2g^2x^2 + 4\,d^2efg + 4\,d^3g^2 - \left(e^3f^2 + 6\,de^2fg + 4\,d^2eg^2\right)x + 2\left(d^2efg + 2\,d^3g^2 + \left(e^3fg + 2\,de^2g^2\right)x^2\right.}{e^5x^2 - 2\,de^4x + d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")

[Out] -(e^3*g^2*x^3 - 2*d*e^2*g^2*x^2 + 4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 4*d^2*e*g^2)*x + 2*(d^2*e*f*g + 2*d^3*g^2 + (e^3*f*g + 2*d*e^2*g^2)*x^2 - 2*(d*e^2*f*g + 2*d^2*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)

giac [B] time = 0.21, size = 227, normalized size = 2.80

$$-g^2xe^{(-2)} - \left(2\,dg^2e^3 + fge^4\right)e^{(-6)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{\left(2\,d^2g^2e^4 + dfge^5\right)e^{(-7)}\log\left(\frac{\left|2\,xe^2 - 2\,|d|e\right|}{\left|2\,xe^2 + 2\,|d|e\right|}\right)}{|d|} - \frac{\left(4\,d^5g^2e^3 + 4\,d^4fge^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")

[Out] -g^2*x*e^(-2) - (2*d*g^2*e^3 + f*g*e^4)*e^(-6)*log(abs(x^2*e^2 - d^2)) - (2*d^2*g^2*e^4 + d*f*g*e^5)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - (4*d^5*g^2*e^3 + 4*d^4*f*g*e^4 - (5*d^2*g^2*e^6 + 6*d*f*g*e^7 + f^2*e^8)*x^3 - 2*(3*d^3*g^2*e^5 + 4*d^2*f*g*e^6 + d*f^2*e^7)*x^2 + (3*d^4*g^2*e^4 + 2*d^3*f*g*e^5 - d^2*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)^2

maple [A] time = 0.01, size = 151, normalized size = 1.86

$$\frac{d^3g^2}{(ex-d)^2\,e^3} + \frac{2d^2fg}{(ex-d)^2\,e^2} + \frac{d\,f^2}{(ex-d)^2\,e} + \frac{5d^2g^2}{(ex-d)\,e^3} + \frac{6dfg}{(ex-d)\,e^2} - \frac{4d\,g^2\ln(ex-d)}{e^3} + \frac{f^2}{(ex-d)\,e} - \frac{2fg\ln(ex-d)}{e^2} - \frac{g^2x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)

[Out] -1/e^2*g^2*x-4*d/e^3*g^2*ln(e*x-d)-2/e^2*f*g*ln(e*x-d)+d^3/e^3/(e*x-d)^2*g^2+2*d^2/e^2/(e*x-d)^2*f*g+d/e/(e*x-d)^2*f^2+5/(e*x-d)*d^2/e^3*g^2+6/(e*x-d)*d/e^2*f*g+1/(e*x-d)/e*f^2

maxima [A] time = 0.45, size = 105, normalized size = 1.30

$$-\frac{g^2x}{e^2} - \frac{4\,d^2efg + 4\,d^3g^2 - \left(e^3f^2 + 6\,de^2fg + 5\,d^2eg^2\right)x}{e^5x^2 - 2\,de^4x + d^2e^3} - \frac{2\left(efg + 2\,dg^2\right)\log\left(ex - d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")

[Out] -g^2*x/e^2 - (4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 2*(e*f*g + 2*d*g^2)*log(e*x - d)/e^3

mupad [B] time = 2.60, size = 107, normalized size = 1.32

$$-\frac{\frac{4\left(d^3\,g^2+e\,f\,d^2\,g\right)}{e}-x\,\left(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2\right)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-\frac{g^2\,x}{e^2}-\frac{\ln\left(e\,x-d\right)\,\left(4\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((4*(d^3*g^2 + d^2*e*f*g))/e - x*(5*d^2*g^2 + e^2*f^2 + 6*d*e*f*g))/(d^2*
e^2 + e^4*x^2 - 2*d*e^3*x) - (g^2*x)/e^2 - (log(e*x - d)*(4*d*g^2 + 2*e*f*g
))/e^3
```

sympy [A] time = 0.87, size = 102, normalized size = 1.26

$$-\frac{4d^3g^2+4d^2efg+x\left(-5d^2eg^2-6de^2fg-e^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x}{e^2}-\frac{2g\left(2dg+ef\right)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(4*d**3*g**2 + 4*d**2*e*f*g + x*(-5*d**2*e*g**2 - 6*d*e**2*f*g - e**3*f**2
))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)*l
og(-d + e*x)/e**3
```

3.573 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=61

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

```
[Out] 1/2*(d*g+e*f)^2/e^3/(-e*x+d)^2-2*g*(d*g+e*f)/e^3/(-e*x+d)-g^2*ln(-e*x+d)/e^
3
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 43}

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g^
2*Log[d - e*x])/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{e^2(d-ex)^3}-\frac{2g(ef+dg)}{e^2(d-ex)^2}+\frac{g^2}{e^2(d-ex)}\right)dx\\
&= \frac{(ef+dg)^2}{2e^3(d-ex)^2}-\frac{2g(ef+dg)}{e^3(d-ex)}-\frac{g^2\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.80

$$\frac{\frac{(dg+ef)(e(f+4gx)-3dg)}{(d-ex)^2}-2g^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (((e*f + d*g)*(-3*d*g + e*(f + 4*g*x)))/(d - e*x)^2 - 2*g^2*Log[d - e*x])/(
2*e^3)
```

fricas [A] time = 1.19, size = 100, normalized size = 1.64

$$\frac{e^2f^2 - 2\,defg - 3\,d^2g^2 + 4\left(e^2fg + deg^2\right)x - 2\left(e^2g^2x^2 - 2\,deg^2x + d^2g^2\right)\log\left(ex - d\right)}{2\left(e^5x^2 - 2\,de^4x + d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x - 2*(e^2*g^2
*x^2 - 2*d*e*g^2*x + d^2*g^2)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)
```

giac [B] time = 0.20, size = 195, normalized size = 3.20

$$-\frac{dg^2e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d|e|}{|2\,xe^2+2\,|d|e|}\right)}{2\,|d|}-\frac{1}{2}\,g^2e^{(-3)}\log\left(|x^2e^2-d^2|\right)+\frac{\left(4\left(d^2g^2e^4+dfge^5\right)x^3+\left(5\,d^3g^2e^3+6\,d^2fge^4+df^2e^5\right)\ldots}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*d*g^2*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/
abs(d) - 1/2*g^2*e^(-3)*log(abs(x^2*e^2 - d^2)) + 1/2*(4*(d^2*g^2*e^4 + d*f
*g*e^5)*x^3 + (5*d^3*g^2*e^3 + 6*d^2*f*g*e^4 + d*f^2*e^5)*x^2 - 2*(d^4*g^2*
e^2 - d^2*f^2*e^4)*x - (3*d^5*g^2*e^3 + 2*d^4*f*g*e^4 - d^3*f^2*e^5)*e^(-2)
)*e^(-4)/((x^2*e^2 - d^2)^2*d)
```

maple [A] time = 0.01, size = 105, normalized size = 1.72

$$\frac{d^2g^2}{2\left(ex-d\right)^2e^3}+\frac{dfg}{\left(ex-d\right)^2e^2}+\frac{f^2}{2\left(ex-d\right)^2e}+\frac{2dg^2}{\left(ex-d\right)e^3}+\frac{2fg}{\left(ex-d\right)e^2}-\frac{g^2\ln\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/e^3*g^2*ln(e*x-d)+1/2/e^3/(e*x-d)^2*d^2*g^2+1/e^2/(e*x-d)^2*d*f*g+1/2/e/
(e*x-d)^2*f^2+2/(e*x-d)*d/e^3*g^2+2/(e*x-d)/e^2*f*g
```

maxima [A] time = 0.44, size = 81, normalized size = 1.33

$$\frac{e^2f^2 - 2\,defg - 3\,d^2g^2 + 4\left(e^2fg + deg^2\right)x}{2\left(e^5x^2 - 2\,de^4x + d^2e^3\right)} - \frac{g^2\log\left(ex - d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x)/(e^5*x^2 -
2*d*e^4*x + d^2*e^3) - g^2*log(e*x - d)/e^3
```

mupad [B] time = 0.07, size = 80, normalized size = 1.31

$$-\frac{\frac{3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{2\,e^3}-\frac{2\,g\,x\,\left(d\,g+e\,f\right)}{e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}-\frac{g^2\,\ln\left(e\,x-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(2*e^3) - (2*g*x*(d*g + e*f))/e^2)/(d^
2 + e^2*x^2 - 2*d*e*x) - (g^2*log(e*x - d))/e^3
```

sympy [**A**] time = 0.54, size = 83, normalized size = 1.36

$$-\frac{3d^2g^2 + 2defg - e^2f^2 + x\left(-4deg^2 - 4e^2fg\right)}{2d^2e^3 - 4de^4x + 2e^5x^2} - \frac{g^2\log\left(-d + ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(3*d**2*g**2 + 2*d*e*f*g - e**2*f**2 + x*(-4*d*e*g**2 - 4*e**2*f*g))/(2*d*
*2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x)/e**3
```

3.574 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=88

$$\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

```
[Out] 1/4*(d*g+e*f)^2/d/e^3/(-e*x+d)^2+1/4*(-3*d*g+e*f)*(d*g+e*f)/d^2/e^3/(-e*x+d
)+1/4*(-d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e^
3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^3}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= \frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}\\
&= \frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 1.02

$$\frac{-\frac{2d(dg+ef)\left(2d^2g-de(2f+3gx)+e^2fx\right)}{(d-ex)^2}+(ef-dg)^2(-\log(d-ex))+(ef-dg)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((-2*d*(e*f + d*g)*(2*d^2*g + e^2*f*x - d*e*(2*f + 3*g*x)))/(d - e*x)^2 - (
e*f - d*g)^2*Log[d - e*x] + (e*f - d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

fricas [B] time = 0.83, size = 271, normalized size = 3.08

$$4\,d^2e^2f^2-4\,d^4g^2-2\left(de^3f^2-2\,d^2e^2fg-3\,d^3eg^2\right)x+\left(d^2e^2f^2-2\,d^3efg+d^4g^2+\left(e^4f^2-2\,de^3fg+d^2e^2g^2\right)x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^2
)*x + (d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e
^2*g^2)*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) - (
d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)
*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5*
x^2 - 2*d^4*e^4*x + d^5*e^3)
```

giac [B] time = 0.17, size = 197, normalized size = 2.24

$$-\frac{\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{8\,d^2|d|}+\frac{\left(3\,d^2g^2x^3e^4+4\,d^3g^2x^2e^3-d^4g^2xe^2-2\,d^5g^2e+2\,dfgx^3e^5+4\,d\right.}{4\left(x^2e^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d
)*e)/abs(2*x*e^2 + 2*abs(d)*e))/(d^2*abs(d)) + 1/4*(3*d^2*g^2*x^3*e^4 + 4*d
^3*g^2*x^2*e^3 - d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 4*d^2*f*g*
x^2*e^4 + 2*d^3*f*g*x*e^3 - f^2*x^3*e^6 + 3*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*
e^(-4)/((x^2*e^2 - d^2)^2*d^2)
```

maple [B] time = 0.01, size = 218, normalized size = 2.48

$$\frac{d\,g^2}{4\,(ex-d)^2\,e^3}+\frac{f^2}{4\,(ex-d)^2\,de}+\frac{fg}{2\,(ex-d)^2\,e^2}+\frac{fg}{2\,(ex-d)\,d\,e^2}-\frac{g^2\ln(ex-d)}{8d\,e^3}+\frac{g^2\ln(ex+d)}{8d\,e^3}-\frac{f^2}{4\,(ex-d)\,d^2e}+\frac{fg\ln(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 3/4/(e*x-d)/e^3*g^2+1/2/(e*x-d)/d/e^2*f*g-1/4/(e*x-d)/d^2/e*f^2+1/4/e^3*d/(
e*x-d)^2*g^2+1/2/e^2/(e*x-d)^2*f*g+1/4/e/d/(e*x-d)^2*f^2-1/8/d/e^3*g^2*ln(e
*x-d)+1/4/d^2/e^2*f*g*ln(e*x-d)-1/8/d^3/e*f^2*ln(e*x-d)+1/8/d/e^3*g^2*ln(e*
x+d)-1/4/d^2/e^2*f*g*ln(e*x+d)+1/8/d^3/e*f^2*ln(e*x+d)
```

maxima [A] time = 0.46, size = 150, normalized size = 1.70

$$\frac{2\,de^2f^2-2\,d^3g^2-\left(e^3f^2-2\,de^2fg-3\,d^2eg^2\right)x}{4\left(d^2e^5x^2-2\,d^3e^4x+d^4e^3\right)}+\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log(ex+d)}{8\,d^3e^3}-\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log}{8\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*e^2*f^2 - 2*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d^
2*e^5*x^2 - 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*lo
g(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^
3*e^3)
```

mupad [B] time = 0.13, size = 103, normalized size = 1.17

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g-e\,f\right)^2}{4\,d^3\,e^3}-\frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (atanh((e*x)/d)*(d*g - e*f)^2)/(4*d^3*e^3) - ((d^2*g^2 - e^2*f^2)/(2*d*e^3)
 - (x*(3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^2))/(d^2 + e^2*x^2 - 2*d*
e*x)
```

sympy [B] time = 1.01, size = 185, normalized size = 2.10

$$-\frac{2d^3g^2-2de^2f^2+x\left(-3d^2eg^2-2de^2fg+e^3f^2\right)}{4d^4e^3-8d^3e^4x+4d^2e^5x^2}-\frac{\left(dg-ef\right)^2\log\left(-\frac{d\left(dg-ef\right)^2}{e\left(d^2g^2-2defg+e^2f^2\right)}+x\right)}{8d^3e^3}+\frac{\left(dg-ef\right)^2\log\left(\frac{\ldots}{e(d\ldots}\right.}{8d\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**3*g**2 - 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 - 2*d*e**2*f*g + e**3*f**
2))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*log(-
d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3)
 + (d*g - e*f)**2*log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**
2)) + x)/(8*d**3*e**3)
```

3.575 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3} dx$

Optimal. Leaf size=122

$$\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

```
[Out] 1/8*(d*g+e*f)^2/d^2/e^3/(-e*x+d)^2+1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)-
1/8*(-d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(-d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/
d^4/e^3
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {799, 88, 208}

$$\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d -
 e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3} dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^2} dx \\
&= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^2}+\frac{(ef-dg)(3ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right) dx \\
&= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{((ef-dg)(3ef+dg))\int \frac{1}{d^2-e^2x^2} dx}{8d^3e^2} \\
&= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(ef-dg)(3ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 140, normalized size = 1.15

$$\frac{\frac{4de^2f^2-4d^3g^2}{d-ex}+\left(d^2g^2+2defg-3e^2f^2\right)\log(d-ex)+\left(-d^2g^2-2defg+3e^2f^2\right)\log(d+ex)+\frac{2d^2(dg+ef)^2}{(d-ex)^2}-\frac{2d(ef-}{d+}}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((2*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (4*d*e^2*f^2 - 4*d^3*g^2)/(d - e*x) -
(2*d*(e*f - d*g)^2)/(d + e*x) + (-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2)*Log[d -
e*x] + (3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

fricas [B] time = 1.07, size = 417, normalized size = 3.42

$$4\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2-2\left(3\,de^4f^2-2\,d^2e^3fg-d^3e^2g^2\right)x^2+2\left(3\,d^2e^3f^2-2\,d^3e^2fg+3\,d^4eg^2\right)x+\left(3\,d^3e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 - 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 + 2*(3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
+ (3*d^3*e^2*f^2 - 2*d^4*e*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*
f^2 - 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) - (3*d^3*e^2*f^2 - 2*d^4*e
*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 - d^5*e^5*x^2 - d^6*e^4*x + d^7*e^3)
```

giac [A] time = 0.17, size = 191, normalized size = 1.57

$$\frac{\left(d^2g^2+2\,dfge-3\,f^2e^2\right)e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d||e||}{|2\,xe^2+2\,|d||e||}\right)}{16\,d^3|d|}+\frac{\left(d^2g^2x^3e^4+4\,d^3g^2x^2e^3+d^4g^2xe^2-2\,d^5g^2e+2\,dfgx^3e^5+2\,d^3\right.}{8\left(x^2e^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)
/abs(2*x*e^2 + 2*abs(d)*e))/(d^3*abs(d)) + 1/8*(d^2*g^2*x^3*e^4 + 4*d^3*g^2
*x^2*e^3 + d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 2*d^3*f*g*x*e^3
+ 4*d^4*f*g*e^2 - 3*f^2*x^3*e^6 + 5*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*e^(-4)/(
(x^2*e^2 - d^2)^2*d^3)
```

maple [B] time = 0.03, size = 257, normalized size = 2.11

$$\frac{fg}{4\left(ex-d\right)^2de^2}+\frac{f^2}{8\left(ex-d\right)^2d^2e}+\frac{g^2}{8\left(ex-d\right)^2e^3}+\frac{g^2}{4\left(ex-d\right)de^3}-\frac{g^2}{8\left(ex+d\right)de^3}+\frac{fg}{4\left(ex+d\right)d^2e^2}+\frac{g^2\ln\left(ex-d\right)}{16d^2e^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/4/(e*x-d)/d/e^3*g^2-1/4/(e*x-d)/d^3/e*f^2+1/8/e^3/(e*x-d)^2*g^2+1/4/e^2/d
/(e*x-d)^2*f*g+1/8/e/d^2/(e*x-d)^2*f^2+1/16/d^2/e^3*g^2*ln(e*x-d)+1/8/d^3/e
^2*f*g*ln(e*x-d)-3/16/d^4/e*f^2*ln(e*x-d)-1/16/d^2/e^3*g^2*ln(e*x+d)-1/8/d^
3/e^2*f*g*ln(e*x+d)+3/16/d^4/e*f^2*ln(e*x+d)-1/8/(e*x+d)/d/e^3*g^2+1/4/(e*x
+d)/d^2/e^2*f*g-1/8/(e*x+d)/d^3/e*f^2
```

maxima [A] time = 0.47, size = 211, normalized size = 1.73

$$\frac{2\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2-\left(3\,e^4f^2-2\,de^3fg-d^2e^2g^2\right)x^2+\left(3\,de^3f^2-2\,d^2e^2fg+3\,d^3eg^2\right)x}{8\left(d^3e^6x^3-d^4e^5x^2-d^5e^4x+d^6e^3\right)}+\frac{\left(3\,e^2f^2-2\,defg\right.}{1\!\!\!\!}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 - 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 + (3*d*e^3*f^2 - 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
^3 - d^4*e^5*x^2 - d^5*e^4*x + d^6*e^3) + 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

mupad [B] time = 2.64, size = 198, normalized size = 1.62

$$\frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2-2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3-d^2\,e\,x-d\,e^2\,x^2+e^3\,x^3}-\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}\right)\,\left(d\,g-e\,f\right)\,\left(d\,g+\right.}{8\,d^4\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((e^2*f^2 - d^2*g^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 - 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 + e^3*x^3 - d*e^2*x^2 - d^2*e*x) - (atanh((e*x*(d*g - e*f)*(d*g + 3*e*
f))/(d*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)))*(d*g - e*f)*(d*g + 3*e*f))/(8*d^
4*e^3)
```

sympy [B] time = 1.32, size = 277, normalized size = 2.27

$$-\frac{2d^4g^2-4d^3efg-2d^2e^2f^2+x^2\left(-d^2e^2g^2-2de^3fg+3e^4f^2\right)+x\left(-3d^3eg^2+2d^2e^2fg-3de^3f^2\right)}{8d^6e^3-8d^5e^4x-8d^4e^5x^2+8d^3e^6x^3}+\frac{\left(dg-ef\right)\left(dg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**4*g**2 - 4*d**3*e*f*g - 2*d**2*e**2*f**2 + x**2*(-d**2*e**2*g**2 - 2
*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 2*d**2*e**2*f*g - 3*d*e**3
*f**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - e*f)*(d*g + 3*e*f)*log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2
 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g + 3*e*f
)*log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

3.576 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3} \, dx$

Optimal. Leaf size=127

$$\frac{(f+gx)\left(d^2g+e^2fx\right)}{4d^2e^2\left(d^2-e^2x^2\right)^2}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{x\left(3e^2f^2-d^2g^2\right)+2d^2fg}{8d^4e^2\left(d^2-e^2x^2\right)}$$

```
[Out] 1/4*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)^2+1/8*(2*d^2*f*g+(-d^2*g
^2+3*e^2*f^2)*x)/d^4/e^2/(-e^2*x^2+d^2)+1/8*(-d^2*g^2+3*e^2*f^2)*arctanh(e*
x/d)/d^5/e^3
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {739, 639, 208}

$$\frac{x\left(3e^2f^2-d^2g^2\right)+2d^2fg}{8d^4e^2\left(d^2-e^2x^2\right)}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{(f+gx)\left(d^2g+e^2fx\right)}{4d^2e^2\left(d^2-e^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(4*d^2*e^2*(d^2 - e^2*x^2)^2) + (2*d^2*f*g +
(3*e^2*f^2 - d^2*g^2)*x)/(8*d^4*e^2*(d^2 - e^2*x^2)) + ((3*e^2*f^2 - d^2*g^
2)*ArcTanh[(e*x)/d])/(8*d^5*e^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx = \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2} - \frac{\int \frac{-3e^2f^2+d^2g^2-2e^2fgx}{\left(d^2-e^2x^2\right)^2}\,dx}{4d^2e^2}$$

$$= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2} + \frac{2d^2fg+\left(3e^2f^2-d^2g^2\right)x}{8d^4e^2\left(d^2-e^2x^2\right)} - \frac{\left(-\frac{3e^2f^2}{d^2}+g^2\right)\int \frac{1}{d^2-e^2x^2}\,dx}{8d^2e^2}$$

$$= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2} + \frac{2d^2fg+\left(3e^2f^2-d^2g^2\right)x}{8d^4e^2\left(d^2-e^2x^2\right)} + \frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

Mathematica [A] time = 0.04, size = 110, normalized size = 0.87

$$\frac{d^5eg(4f+gx)+d^3e^3x\left(5f^2+g^2x^2\right)+\left(d^2-e^2x^2\right)^2\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)-3de^5f^2x^3}{8d^5e^3\left(d^2-e^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-3*d*e^5*f^2*x^3 + d^5*e*g*(4*f + g*x) + d^3*e^3*x*(5*f^2 + g^2*x^2) + (3*
e^2*f^2 - d^2*g^2)*(d^2 - e^2*x^2)^2*ArcTanh[(e*x)/d])/(8*d^5*e^3*(d^2 - e^
2*x^2)^2)
```

fricas [B] time = 1.26, size = 252, normalized size = 1.98

$$\frac{8\,d^5efg-2\left(3\,de^5f^2-d^3e^3g^2\right)x^3+2\left(5\,d^3e^3f^2+d^5eg^2\right)x+\left(3\,d^4e^2f^2-d^6g^2+\left(3\,e^6f^2-d^2e^4g^2\right)x^4-2\left(3\,d^2e^4f^2\right.\right.}{16\left(d^5e^7x^4-2\,d^7e^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^5*e*f*g - 2*(3*d*e^5*f^2 - d^3*e^3*g^2)*x^3 + 2*(5*d^3*e^3*f^2 +
d^5*e*g^2)*x + (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2
*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log(e*x + d) - (3*d^4*e^2*f^2 - d^6*g^2
 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log
(e*x - d))/(d^5*e^7*x^4 - 2*d^7*e^5*x^2 + d^9*e^3)
```

giac [A] time = 0.17, size = 127, normalized size = 1.00

$$\frac{\left(d^2g^2-3\,f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{16\,d^4|d|}+\frac{\left(d^2g^2x^3e^2+d^4g^2x+4\,d^4fg-3\,f^2x^3e^4+5\,d^2f^2xe^2\right)e^{(-2)}}{8\left(x^2e^2-d^2\right)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2
 + 2*abs(d)*e))/(d^4*abs(d)) + 1/8*(d^2*g^2*x^3*e^2 + d^4*g^2*x + 4*d^4*f*g
 - 3*f^2*x^3*e^4 + 5*d^2*f^2*x*e^2)*e^(-2)/((x^2*e^2 - d^2)^2*d^4)
```

maple [B] time = 0.02, size = 298, normalized size = 2.35

$$\frac{g^2}{16\,(ex-d)^2\,d\,e^3}-\frac{g^2}{16\,(ex+d)^2\,d\,e^3}+\frac{fg}{8\,(ex-d)^2\,d^2e^2}+\frac{fg}{8\,(ex+d)^2\,d^2e^2}+\frac{f^2}{16\,(ex-d)^2\,d^3e}-\frac{f^2}{16\,(ex+d)^2\,d^3e}+\frac{g}{16\,(ex-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/16/d^3/e^3*g^2*ln(e*x-d)-3/16/d^5/e*f^2*ln(e*x-d)+1/16/e^3/d/(e*x-d)^2*g^
2+1/8/e^2/d^2/(e*x-d)^2*f*g+1/16/e/d^3/(e*x-d)^2*f^2+1/16/(e*x-d)/d^2/e^3*g
^2-1/8/(e*x-d)/d^3/e^2*f*g-3/16/(e*x-d)/d^4/e*f^2-1/16/d^3/e^3*g^2*ln(e*x+d
)+3/16/d^5/e*f^2*ln(e*x+d)+1/16/(e*x+d)/d^2/e^3*g^2+1/8/(e*x+d)/d^3/e^2*f*g
-3/16/(e*x+d)/d^4/e*f^2-1/16/(e*x+d)^2/d/e^3*g^2+1/8/(e*x+d)^2/d^2/e^2*f*g-
1/16/(e*x+d)^2/d^3/e*f^2
```

maxima [A] time = 0.45, size = 152, normalized size = 1.20

$$\frac{4\,d^4fg - \left(3\,e^4f^2 - d^2e^2g^2\right)x^3 + \left(5\,d^2e^2f^2 + d^4g^2\right)x}{8\left(d^4e^6x^4 - 2\,d^6e^4x^2 + d^8e^2\right)} + \frac{\left(3\,e^2f^2 - d^2g^2\right)\log\left(ex+d\right)}{16\,d^5e^3} - \frac{\left(3\,e^2f^2 - d^2g^2\right)\log\left(ex-d\right)}{16\,d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*d^4*f*g - (3*e^4*f^2 - d^2*e^2*g^2)*x^3 + (5*d^2*e^2*f^2 + d^4*g^2)*
x)/(d^4*e^6*x^4 - 2*d^6*e^4*x^2 + d^8*e^2) + 1/16*(3*e^2*f^2 - d^2*g^2)*log
(e*x + d)/(d^5*e^3) - 1/16*(3*e^2*f^2 - d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

mupad [B] time = 0.10, size = 114, normalized size = 0.90

$$\frac{\frac{x^3\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^4} + \frac{f\,g}{2\,e^2} + \frac{x\left(d^2\,g^2+5\,e^2\,f^2\right)}{8\,d^2\,e^2}}{d^4 - 2\,d^2\,e^2\,x^2 + e^4\,x^4} - \frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\left(d^2\,g^2 - 3\,e^2\,f^2\right)}{8\,d^5\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((x^3*(d^2*g^2 - 3*e^2*f^2))/(8*d^4) + (f*g)/(2*e^2) + (x*(d^2*g^2 + 5*e^2*
f^2))/(8*d^2*e^2))/(d^4 + e^4*x^4 - 2*d^2*e^2*x^2) - (atanh((e*x)/d)*(d^2*g
^2 - 3*e^2*f^2))/(8*d^5*e^3)
```

sympy [A] time = 1.00, size = 144, normalized size = 1.13

$$-\frac{-4d^4fg + x^3\left(-d^2e^2g^2 + 3e^4f^2\right) + x\left(-d^4g^2 - 5d^2e^2f^2\right)}{8d^8e^2 - 16d^6e^4x^2 + 8d^4e^6x^4} + \frac{\left(d^2g^2 - 3e^2f^2\right)\log\left(-\frac{d}{e} + x\right)}{16d^5e^3} - \frac{\left(d^2g^2 - 3e^2f^2\right)\log\left(\frac{d}{e} + x\right)}{16d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**4*f*g + x**3*(-d**2*e**2*g**2 + 3*e**4*f**2) + x*(-d**4*g**2 - 5*d*
*2*e**2*f**2))/(8*d**8*e**2 - 16*d**6*e**4*x**2 + 8*d**4*e**6*x**4) + (d**2
*g**2 - 3*e**2*f**2)*log(-d/e + x)/(16*d**5*e**3) - (d**2*g**2 - 3*e**2*f**
2)*log(d/e + x)/(16*d**5*e**3)
```

3.577 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3} dx$

Optimal. Leaf size=188

$$\frac{f(dg+ef)}{8d^5e^2(d-ex)}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)^2}{32d^4e^3(d-ex)^2}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-\cdots$$

```
[Out] 1/32*(d*g+e*f)^2/d^4/e^3/(-e*x+d)^2+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)-1/24*(
-d*g+e*f)^2/d^3/e^3/(e*x+d)^3-1/32*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^3/(e*x+d)^2
+1/16*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)+1/16*(-d^2*g^2+2*d*e*f*g+5*e^2*f^
2)*arctanh(e*x/d)/d^6/e^3
```

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{f(dg+ef)}{8d^5e^2(d-ex)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e*
x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g))/
(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x)) + (
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^4}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^3} + \frac{f(ef+dg)}{8d^5e(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^4} + \frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^3} + \right.$$
$$= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2} + \frac{f(ef+dg)}{8d^5e^2(d-ex)} - \frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} - \frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} - \frac{3e}{16d}$$
$$= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2} + \frac{f(ef+dg)}{8d^5e^2(d-ex)} - \frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} - \frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} - \frac{3e}{16d}$$

Mathematica [A] time = 0.15, size = 197, normalized size = 1.05

$$\frac{-\frac{4d^3(ef-dg)^2}{(d+ex)^3} + \frac{3d^2\left(d^2g^2+2defg-3e^2f^2\right)}{(d+ex)^2} + \frac{6d\left(d^2g^2-3e^2f^2\right)}{d+ex} + 3\left(d^2g^2-2defg-5e^2f^2\right)\log(d-ex) + 3\left(-d^2g^2+2defg\right.}{96d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((3*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (12*d*e*f*(e*f + d*g))/(d - e*x) - (4*
d^3*(e*f - d*g)^2)/(d + e*x)^3 + (3*d^2*(-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))
/(d + e*x)^2 + (6*d*(-3*e^2*f^2 + d^2*g^2))/(d + e*x) + 3*(-5*e^2*f^2 - 2*d
*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d
+ e*x])/(96*d^6*e^3)
```

fricas [B] time = 0.68, size = 662, normalized size = 3.52

$$-\frac{16\,d^5e^2f^2 - 32\,d^6efg - 8\,d^7g^2 + 6\left(5\,de^6f^2 + 2\,d^2e^5fg - d^3e^4g^2\right)x^4 + 6\left(5\,d^2e^5f^2 + 2\,d^3e^4fg - d^4e^3g^2\right)x^3 - 10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(16*d^5*e^2*f^2 - 32*d^6*e*f*g - 8*d^7*g^2 + 6*(5*d*e^6*f^2 + 2*d^2*e
^5*f*g - d^3*e^4*g^2)*x^4 + 6*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)
*x^3 - 10*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - 2*(25*d^4*e^3
*f^2 + 10*d^5*e^2*f*g + 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g - d
^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 + 2*d^2
*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^
2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^4*e^3*f
^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 2*d^6*
e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^
2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d
^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*
d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + d^
7*e^7*x^4 - 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 + d^10*e^4*x + d^11*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(d^2
*exp(1)^4*g^2-2*d*exp(1)^5*g*f+exp(1)^6*f^2)/(exp(2)^3*d^6*exp(1)-3*exp(2)^
2*d^6*exp(1)^3+3*exp(2)*d^6*exp(1)^5-d^6*exp(1)^7)*ln(abs(x*exp(1)+d))-(-d^
2*exp(1)^3*g^2+2*d*exp(1)^4*g*f-exp(1)^5*f^2)/(2*exp(2)^3*d^6-6*exp(2)^2*d^
6*exp(1)^2+6*exp(2)*d^6*exp(1)^4-2*d^6*exp(1)^6)*ln(abs(-x^2*exp(2)+d^2))-(
-3*exp(2)^3*f^2+exp(2)^2*d^2*g^2-2*exp(2)^2*d*exp(1)*g*f+10*exp(2)^2*exp(1)
^2*f^2-6*exp(2)*d^2*exp(1)^2*g^2+12*exp(2)*d*exp(1)^3*g*f-15*exp(2)*exp(1)^
4*f^2-3*d^2*exp(1)^4*g^2+6*d*exp(1)^5*g*f)*1/2/(8*exp(2)^3*d^5-24*exp(2)^2*
d^5*exp(1)^2+24*exp(2)*d^5*exp(1)^4-8*d^5*exp(1)^6)/exp(1)/abs(d)*ln(abs(-2
*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-((3*exp(2)^5*d
*f^2-exp(2)^4*d^3*g^2+2*exp(2)^4*d^2*exp(1)*g*f-10*exp(2)^4*d*exp(1)^2*f^2-
2*exp(2)^3*d^3*exp(1)^2*g^2+4*exp(2)^3*d^2*exp(1)^3*g*f+7*exp(2)^3*d*exp(1)
^4*f^2+3*exp(2)^2*d^3*exp(1)^4*g^2-6*exp(2)^2*d^2*exp(1)^5*g*f)*x^3+(4*exp(
2)^3*d^4*exp(1)*g^2-8*exp(2)^3*d^3*exp(1)^2*g*f+4*exp(2)^3*d^2*exp(1)^3*f^2
-4*exp(2)^2*d^4*exp(1)^3*g^2+8*exp(2)^2*d^3*exp(1)^4*g*f-4*exp(2)^2*d^2*exp
(1)^5*f^2)*x^2+(-5*exp(2)^4*d^3*f^2-exp(2)^3*d^5*g^2+2*exp(2)^3*d^4*exp(1)*
g*f+14*exp(2)^3*d^3*exp(1)^2*f^2+6*exp(2)^2*d^5*exp(1)^2*g^2-12*exp(2)^2*d^
4*exp(1)^3*g*f-9*exp(2)^2*d^3*exp(1)^4*f^2-5*exp(2)*d^5*exp(1)^4*g^2+10*exp
(2)*d^4*exp(1)^5*g*f)*x-4*exp(2)^3*d^5*g*f+2*exp(2)^3*d^4*exp(1)*f^2-2*exp(
2)^2*d^6*exp(1)*g^2+16*exp(2)^2*d^5*exp(1)^2*g*f-8*exp(2)^2*d^4*exp(1)^3*f^
2-12*exp(2)*d^5*exp(1)^4*g*f+6*exp(2)*d^4*exp(1)^5*f^2+2*d^6*exp(1)^5*g^2)/
8/d^6/exp(2)/(exp(2)-exp(1)^2)^3/(-x^2*exp(2)+d^2)^2
```

maple [A] time = 0.02, size = 348, normalized size = 1.85

$$-\frac{g^2}{24\left(ex+d\right)^3 d\,e^3}+\frac{fg}{12\left(ex+d\right)^3 d^2e^2}-\frac{f^2}{24\left(ex+d\right)^3 d^3e}+\frac{g^2}{32\left(ex-d\right)^2 d^2e^3}+\frac{g^2}{32\left(ex+d\right)^2 d^2e^3}+\frac{fg}{16\left(ex-d\right)^2 d^3e^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/32/e^3/d^2/(e*x-d)^2*g^2+1/16/e^2/d^3/(e*x-d)^2*f*g+1/32/e/d^4/(e*x-d)^2*
f^2+1/32/d^4/e^3*g^2*ln(e*x-d)-1/16/d^5/e^2*f*g*ln(e*x-d)-5/32/d^6/e*f^2*ln
(e*x-d)-1/8/(e*x-d)/d^4/e^2*f*g-1/8/(e*x-d)/d^5/e*f^2+1/16/e^3/d^3/(e*x+d)*
g^2-3/16/(e*x+d)/d^5/e*f^2+1/32/(e*x+d)^2/d^2/e^3*g^2+1/16/(e*x+d)^2/d^3/e^
2*f*g-3/32/(e*x+d)^2/d^4/e*f^2-1/32/d^4/e^3*g^2*ln(e*x+d)+1/16/d^5/e^2*f*g*
ln(e*x+d)+5/32/d^6/e*f^2*ln(e*x+d)-1/24/(e*x+d)^3/d/e^3*g^2+1/12/e^2/d^2/(e
*x+d)^3*f*g-1/24/(e*x+d)^3/d^3/e*f^2
```

maxima [A] time = 0.50, size = 308, normalized size = 1.64

$$-\frac{8\,d^4e^2f^2-16\,d^5efg-4\,d^6g^2+3\left(5\,e^6f^2+2\,de^5fg-d^2e^4g^2\right)x^4+3\left(5\,de^5f^2+2\,d^2e^4fg-d^3e^3g^2\right)x^3-5\left(5\,d^2e^4 \cdots\right.}{48\left(d^5e^8x^5+d^6e^7x^4-2\,d^7e^6x^3-2\,d^8e^5x^2+d^9e^4x+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(8*d^4*e^2*f^2 - 16*d^5*e*f*g - 4*d^6*g^2 + 3*(5*e^6*f^2 + 2*d*e^5*f*
g - d^2*e^4*g^2)*x^4 + 3*(5*d*e^5*f^2 + 2*d^2*e^4*f*g - d^3*e^3*g^2)*x^3 -
5*(5*d^2*e^4*f^2 + 2*d^3*e^3*f*g - d^4*e^2*g^2)*x^2 - (25*d^3*e^3*f^2 + 10*
d^4*e^2*f*g + 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + d^6*e^7*x^4 - 2*d^7*e^6*x^3 -
2*d^8*e^5*x^2 + d^9*e^4*x + d^10*e^3) + 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g
^2)*log(e*x + d)/(d^6*e^3) - 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e*x
 - d)/(d^6*e^3)
```

mupad [B] time = 2.68, size = 249, normalized size = 1.32

$$\frac{\frac{d^2\,g^2+4\,d\,e\,f\,g-2\,e^2\,f^2}{12\,d\,e^3}-\frac{x^3\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^5}+\frac{x\,\left(7\,d^2\,g^2+10\,d\,e\,f\,g+25\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+\cdots\right)}{48\,d^3\,e}}{d^5+d^4\,e\,x-2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3+d\,e^4\,x^4+e^5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + 4*d*e*f*g)/(12*d*e^3) - (x^3*(5*e^2*f^2 - d^2*g^2 +
 2*d*e*f*g))/(16*d^4) - (e*x^4*(5*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(16*d^5)
+ (x*(7*d^2*g^2 + 25*e^2*f^2 + 10*d*e*f*g))/(48*d^2*e^2) + (5*x^2*(5*e^2*f^
2 - d^2*g^2 + 2*d*e*f*g))/(48*d^3*e))/(d^5 + e^5*x^5 + d*e^4*x^4 - 2*d^3*e^
2*x^2 - 2*d^2*e^3*x^3 + d^4*e*x) + (atanh((e*x)/d)*(5*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g))/(16*d^6*e^3)
```

sympy [A] time = 1.83, size = 321, normalized size = 1.71

$$-\frac{-4d^6g^2 - 16d^5efg + 8d^4e^2f^2 + x^4\left(-3d^2e^4g^2 + 6de^5fg + 15e^6f^2\right) + x^3\left(-3d^3e^3g^2 + 6d^2e^4fg + 15de^5f^2\right) + x^2}{48d^{10}e^3 + 48d^9e^4x - 96d^8e^5x^2 - 96d^7e^6x^3 + 48d^6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**6*g**2 - 16*d**5*e*f*g + 8*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2
 + 6*d*e**5*f*g + 15*e**6*f**2) + x**3*(-3*d**3*e**3*g**2 + 6*d**2*e**4*f*g
 + 15*d*e**5*f**2) + x**2*(5*d**4*e**2*g**2 - 10*d**3*e**3*f*g - 25*d**2*e*
*4*f**2) + x*(-7*d**5*e*g**2 - 10*d**4*e**2*f*g - 25*d**3*e**3*f**2))/(48*d
**10*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 48*d**
6*e**7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*lo
g(-d/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(d/e
+ x)/(32*d**6*e**3)
```

3.578 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3} dx$

Optimal. Leaf size=235

$$\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)}+\frac{(dg+ef)^2}{64d^5e^3(d-ex)^2}-\frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}+\frac{\left(-d^2g^2+10defg+15e^2f^2\right)\tanh^{-1}}{64d^7e^3}$$

```
[Out] 1/64*(d*g+e*f)^2/d^5/e^3/(-e*x+d)^2+1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^3/(-e*
x+d)-1/32*(-d*g+e*f)^2/d^3/e^3/(e*x+d)^4-1/48*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^
3/(e*x+d)^3+1/32*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)^2+1/32*(d^2*g^2-2*d*e*
f*g-5*e^2*f^2)/d^6/e^3/(e*x+d)+1/64*(-d^2*g^2+10*d*e*f*g+15*e^2*f^2)*arctan
h(e*x/d)/d^7/e^3
```

Rubi [A] time = 0.27, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}+\frac{\left(-d^2g^2+10defg+15e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(dg+3e}{48d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d^
6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*(3
*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^3*(
d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x)) + ((
15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^5}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^3} + \frac{(ef+dg)(5ef+dg)}{64d^6e^2(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^5} + \frac{(ef-dg)(3ef+}{16d^4e^2(d+ex}\right.$$
$$= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef+dg}{48d^4e^3(d+ex)^3}$$
$$= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef+dg}{48d^4e^3(d+ex)^3}$$

Mathematica [**A**] time = 0.17, size = 244, normalized size = 1.04

$$\frac{-\frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{12d^2\left(d^2g^2-3e^2f^2\right)}{(d+ex)^2} + \frac{6d\left(d^2g^2+6defg+5e^2f^2\right)}{d-ex} + \frac{12d\left(d^2g^2-2defg-5e^2f^2\right)}{d+ex} + 3\left(d^2g^2-10defg-15e^2f^2\right)\log(d}{384d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((6*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (6*d*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)
)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(-3*e^2*f^2 + 2*d
*e*f*g + d^2*g^2))/(d + e*x)^3 + (12*d^2*(-3*e^2*f^2 + d^2*g^2))/(d + e*x)^
2 + (12*d*(-5*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x) + 3*(-15*e^2*f^2 -
10*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*
Log[d + e*x])/(384*d^7*e^3)
```

fricas [**B**] time = 0.91, size = 793, normalized size = 3.37

$$-\frac{96\,d^6e^2f^2 - 64\,d^7efg - 32\,d^8g^2 + 6\left(15\,de^7f^2 + 10\,d^2e^6fg - d^3e^5g^2\right)x^5 + 12\left(15\,d^2e^6f^2 + 10\,d^3e^5fg - d^4e^4g^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/384*(96*d^6*e^2*f^2 - 64*d^7*e*f*g - 32*d^8*g^2 + 6*(15*d*e^7*f^2 + 10*d
^2*e^6*f*g - d^3*e^5*g^2)*x^5 + 12*(15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e
^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - 20*(1
5*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 - 2*(51*d^5*e^3*f^2 + 34*
d^6*e^2*f*g + 35*d^7*e*g^2)*x - 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^2
+ (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^2*
e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2
)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e^4
*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2*f
*g - d^7*e*g^2)*x)*log(e*x + d) + 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^
2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^
2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g
^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e
^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2
*f*g - d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 2*d^8*e^8*x^5 - d^9*e^7*x
^4 - 4*d^10*e^6*x^3 - d^11*e^5*x^2 + 2*d^12*e^4*x + d^13*e^3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -((exp(1)*x+d)^-1/exp(1)*g^2*d^2*exp(1)^10-2*(exp(1)*x+d)^-1/exp(1)*g*d*exp(1)^11*f+(exp(1)*x+d)^-1/exp(1)*exp(1)^12*f^2)/(d^6*exp(1)^12-3*d^6*exp(1)^10*exp(2)+3*d^6*exp(1)^8*exp(2)^2-d^6*exp(1)^6*exp(2)^3)-((5*g^2*d^5*exp(1)^12+50*g^2*d^5*exp(1)^10*exp(2)-20*g^2*d^5*exp(1)^8*exp(2)^2-34*g^2*d^5*exp(1)^6*exp(2)^3-g^2*d^5*exp(1)^4*exp(2)^4-68*g*d^4*exp(1)^11*exp(2)*f-52*g*d^4*exp(1)^9*exp(2)^2*f+116*g*d^4*exp(1)^7*exp(2)^3*f+4*g*d^4*exp(1)^5*exp(2)^4*f+9*d^3*exp(1)^12*exp(2)*f^2+66*d^3*exp(1)^10*exp(2)^2*f^2-60*d^3*exp(1)^8*exp(2)^3*f^2-18*d^3*exp(1)^6*exp(2)^4*f^2+3*d^3*exp(1)^4*exp(2)^5*f^2)*(-(exp(1)*x+d)^-1/exp(1))^3+(17*g^2*d^4*exp(1)^9*exp(2)-85*g^2*d^4*exp(1)^7*exp(2)^2-89*g^2*d^4*exp(1)^5*exp(2)^3-3*g^2*d^4*exp(1)^3*exp(2)^4-16*g*d^3*exp(1)^10*exp(2)*f+44*g*d^3*exp(1)^8*exp(2)^2*f+280*g*d^3*exp(1)^6*exp(2)^3*f+12*g*d^3*exp(1)^4*exp(2)^4*f+21*d^2*exp(1)^9*exp(2)^2*f^2-145*d^2*exp(1)^7*exp(2)^3*f^2-45*d^2*exp(1)^5*exp(2)^4*f^2+9*d^2*exp(1)^3*exp(2)^5*f^2)*(-(exp(1)*x+d)^-1/exp(1))^2-(-3*g^2*d^3*exp(1)^8*exp(2)-77*g^2*d^3*exp(1)^6*exp(2)^2-77*g^2*d^3*exp(1)^4*exp(2)^3-3*g^2*d^3*exp(1)^2*exp(2)^4+76*g*d^2*exp(1)^7*exp(2)^2*f+232*g*d^2*exp(1)^5*exp(2)^3*f+12*g*d^2*exp(1)^3*exp(2)^4*f-7*d*exp(1)^8*exp(2)^2*f^2-121*d*exp(1)^6*exp(2)^3*f^2-41*d*exp(1)^4*exp(2)^4*f^2+9*d*exp(1)^2*exp(2)^5*f^2)*(exp(1)*x+d)^-1/exp(1)-17*g^2*d^2*exp(1)^5*exp(2)^2-22*g^2*d^2*exp(1)^3*exp(2)^3-g^2*d^2*exp(1)*exp(2)^4+12*g*d*exp(1)^6*exp(2)^2*f+64*g*d*exp(1)^4*exp(2)^3*f+4*g*d*exp(1)^2*exp(2)^4*f-29*exp(1)^5*exp(2)^3*f^2-14*exp(1)^3*exp(2)^4*f^2+3*exp(1)*exp(2)^5*f^2)/8/d^7/(exp(2)-exp(1)^2)^4/((-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^2*exp(2)+2*(exp(1)*x+d)^-1/exp(1)*d*exp(1)*exp(2)-exp(2))^2-(g^2*d^2*exp(1)^5+2*g^2*d^2*exp(1)^3*exp(2)-g*d*exp(1)^6*f-5*g*d*exp(1)^4*f*exp(2)+3*exp(1)^5*f^2*exp(2))/(-d^7*exp(1)^8+4*d^7*exp(1)^6*exp(2)-6*d^7*exp(1)^4*exp(2)^2+4*d^7*exp(1)^2*exp(2)^3-d^7*exp(2)^4)*ln(abs((-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^2*exp(2)+2*(exp(1)*x+d)^-1/exp(1)*d*exp(1)*exp(2)-exp(2)))-(3*g^2*d^2*exp(1)^8+33*g^2*d^2*exp(1)^6*exp(2)+13*g^2*d^2*exp(1)^4*exp(2)^2-g^2*d^2*exp(1)^2*exp(2)^3-60*g*d*exp(1)^7*f*exp(2)-40*g*d*exp(1)^5*f*exp(2)^2+4*g*d*exp(1)^3*f*exp(2)^3+15*exp(1)^8*f^2*exp(2)+45*exp(1)^6*f^2*exp(2)^2-15*exp(1)^4*f^2*exp(2)^3+3*exp(1)^2*f^2*exp(2)^4)/2/(-8*d^6*exp(1)^8+32*d^6*exp(1)^6*exp(2)-48*d^6*exp(1)^4*exp(2)^2+32*d^6*exp(1)^2*exp(2)^3-8*d^6*exp(2)^4)/exp(1)/abs(d)/exp(1)^2*ln(abs(-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4+2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^2*exp(2)-2*d*exp(1)*exp(2)-2*exp(1)*abs(d)*exp(1)^2)/abs(-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4+2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^2*exp(2)-2*d*exp(1)*exp(2)+2*exp(1)*abs(d)*exp(1)^2))

maple [A] time = 0.02, size = 421, normalized size = 1.79

$$-\frac{g^2}{32\,(ex+d)^4\,d\,e^3}+\frac{fg}{16\,(ex+d)^4\,d^2e^2}-\frac{f^2}{32\,(ex+d)^4\,d^3e}+\frac{g^2}{48\,(ex+d)^3\,d^2e^3}+\frac{fg}{24\,(ex+d)^3\,d^3e^2}-\frac{f^2}{16\,(ex+d)^3\,d^4e}+\frac{}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x)

[Out] -1/64/(e*x-d)/d^4/e^3*g^2-3/32/(e*x-d)/d^5/e^2*f*g-5/64/(e*x-d)/d^6/e*f^2+1/64/e^3/d^3/(e*x-d)^2*g^2+1/32/e^2/d^4/(e*x-d)^2*f*g+1/64/e/d^5/(e*x-d)^2*f^2+1/128/d^5/e^3*g^2*ln(e*x-d)-5/64/d^6/e^2*f*g*ln(e*x-d)-15/128/d^7/e*f^2*ln(e*x-d)+1/32/e^3/d^3/(e*x+d)^2*g^2-3/32/(e*x+d)^2/d^5/e*f^2+1/48/(e*x+d)^3/d^2/e^3*g^2+1/24/(e*x+d)^3/d^3/e^2*f*g-1/16/(e*x+d)^3/d^4/e*f^2+1/32/(e*x+d)/d^4/e^3*g^2-1/16/(e*x+d)/d^5/e^2*f*g-5/32/(e*x+d)/d^6/e*f^2-1/128/d^5/e^3*g^2*ln(e*x+d)+5/64/d^6/e^2*f*g*ln(e*x+d)+15/128/d^7/e*f^2*ln(e*x+d)-1/32/(e*x+d)^4/d/e^3*g^2+1/16/e^2/d^2/(e*x+d)^4*f*g-1/32/(e*x+d)^4/d^3/e*f^2

maxima [A] time = 0.51, size = 359, normalized size = 1.53

$$-\frac{48\,d^5e^2f^2-32\,d^6efg-16\,d^7g^2+3\left(15\,e^7f^2+10\,de^6fg-d^2e^5g^2\right)x^5+6\left(15\,de^6f^2+10\,d^2e^5fg-d^3e^4g^2\right)x^4-}{192\left(d^6e^9x^6+2\,d^7e^8x^5-d^8e^7x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(48*d^5*e^2*f^2 - 32*d^6*e*f*g - 16*d^7*g^2 + 3*(15*e^7*f^2 + 10*d*e
^6*f*g - d^2*e^5*g^2)*x^5 + 6*(15*d*e^6*f^2 + 10*d^2*e^5*f*g - d^3*e^4*g^2)
*x^4 - 2*(15*d^2*e^5*f^2 + 10*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(15*d^3*e
^4*f^2 + 10*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (51*d^4*e^3*f^2 + 34*d^5*e^2*f
*g + 35*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 2*d^7*e^8*x^5 - d^8*e^7*x^4 - 4*d^9*e^
6*x^3 - d^10*e^5*x^2 + 2*d^11*e^4*x + d^12*e^3) + 1/128*(15*e^2*f^2 + 10*d*
e*f*g - d^2*g^2)*log(e*x + d)/(d^7*e^3) - 1/128*(15*e^2*f^2 + 10*d*e*f*g -
d^2*g^2)*log(e*x - d)/(d^7*e^3)
```

mupad [B] time = 2.64, size = 296, normalized size = 1.26

$$\frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2}{12\,d\,e^3}+\frac{x^3\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{96\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{32\,d^5}+\frac{x\,\left(35\,d^2\,g^2+34\,d\,e\,f\,g+51\,e^2\,f^2\right)}{192\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2\right.}{}}{d^6+2\,d^5\,e\,x-d^4\,e^2\,x^2-4\,d^3\,e^3\,x^3-d^2\,e^4\,x^4+2\,d\,e^5\,x^5+e^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)/(12*d*e^3) + (x^3*(15*e^2*f^2 - d^2*g^2
+ 10*d*e*f*g))/(96*d^4) - (e*x^4*(15*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(32*d
^5) + (x*(35*d^2*g^2 + 51*e^2*f^2 + 34*d*e*f*g))/(192*d^2*e^2) + (5*x^2*(15
*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(96*d^3*e) - (e^2*x^5*(15*e^2*f^2 - d^2*g
^2 + 10*d*e*f*g))/(64*d^6))/(d^6 + e^6*x^6 + 2*d*e^5*x^5 - d^4*e^2*x^2 - 4*
d^3*e^3*x^3 - d^2*e^4*x^4 + 2*d^5*e*x) + (atanh((e*x)/d)*(15*e^2*f^2 - d^2*
g^2 + 10*d*e*f*g))/(64*d^7*e^3)
```

sympy [A] time = 2.15, size = 372, normalized size = 1.58

$$-\frac{-16d^7g^2-32d^6efg+48d^5e^2f^2+x^5\left(-3d^2e^5g^2+30de^6fg+45e^7f^2\right)+x^4\left(-6d^3e^4g^2+60d^2e^5fg+90de^6f^2\right)}{192d^{12}e^3+384d^{11}e^4x-192d^{10}e^5x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-16*d**7*g**2 - 32*d**6*e*f*g + 48*d**5*e**2*f**2 + x**5*(-3*d**2*e**5*g*
*2 + 30*d*e**6*f*g + 45*e**7*f**2) + x**4*(-6*d**3*e**4*g**2 + 60*d**2*e**5
*f*g + 90*d*e**6*f**2) + x**3*(2*d**4*e**3*g**2 - 20*d**3*e**4*f*g - 30*d**
2*e**5*f**2) + x**2*(10*d**5*e**2*g**2 - 100*d**4*e**3*f*g - 150*d**3*e**4*
f**2) + x*(-35*d**6*e*g**2 - 34*d**5*e**2*f*g - 51*d**4*e**3*f**2))/(192*d*
*12*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**3 - 19
2*d**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*g**2 -
10*d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g**2 - 10*
d*e*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

3.579 $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}\,(3dg+5ef)}{e^6}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] 1/5*(d*g+e*f)^5*(e*x+d)^3/d/e^6/(-e^2*x^2+d^2)^(5/2)+1/15*(-23*d*g+2*e*f)*(
d*g+e*f)^4*(e*x+d)^2/d^2/e^6/(-e^2*x^2+d^2)^(3/2)-1/2*g^3*(13*d^2*g^2+30*d*
e*f*g+20*e^2*f^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(d*g+e*f)^3*(12
7*d^2*g^2-21*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^3/e^6/(-e^2*x^2+d^2)^(1/2)+g^4*(3
*d*g+5*e*f)*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*g^5*x*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.97, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1635, 1815, 641, 217, 203}

$$\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{15d^3e^6\sqrt{d^2-e^2x^2}}-\frac{g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 23*
d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*Sqrt
[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e^6 + (g^5*x*S
qrt[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
```

```
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^5f^5-15de^4f^4g-30d^2e^3f^3g^2-30d^3e^2f^2g^3-15d^4efg^4-3d^5g^5}{e^5}+\frac{5dg^2\left(10e^3f^3 \cdots\right.}{\cdots}\right.}{\left(d^2-\cdots\right.}}{\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^5f^5-15de^4f^4g+70d^2e^{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d\cdots\right.}{15d^3e^6\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d\cdots\right.}{15d^3e^6\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d\cdots\right.}{15d^3e^6\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d\cdots\right.}{15d^3e^6\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d\cdots\right.}{15d^3e^6\sqrt{\cdots}}$$

Mathematica [A] time = 0.97, size = 193, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2(2ef-23dg)(dg+ef)^4}{d^2(d-ex)^2}+\frac{2(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{d^3(d-ex)}+30g^4(3dg+5ef)+\frac{6(dg+ef)^5}{d(d-ex)^3}+15eg^5x\right)-15g^3\left(\cdots\right.}{30e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(30*g^4*(5*e*f + 3*d*g) + 15*e*g^5*x + (6*(e*f + d*g)^
5)/(d*(d - e*x)^3) + (2*(2*e*f - 23*d*g)*(e*f + d*g)^4)/(d^2*(d - e*x)^2) +
 (2*(e*f + d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2))/(d^3*(d - e*x)))
- 15*g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(30*e^6)
```

fricas [B] time = 1.07, size = 807, normalized size = 3.00

$$-\frac{14\,d^3e^5f^5 - 30\,d^4e^4f^4g + 40\,d^5e^3f^3g^2 + 440\,d^6e^2f^2g^3 + 720\,d^7efg^4 + 304\,d^8g^5 - 2\left(7\,e^8f^5 - 15\,de^7f^4g + 20\,d^2e\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(14*d^3*e^5*f^5 - 30*d^4*e^4*f^4*g + 40*d^5*e^3*f^3*g^2 + 440*d^6*e^2
*f^2*g^3 + 720*d^7*e*f*g^4 + 304*d^8*g^5 - 2*(7*e^8*f^5 - 15*d*e^7*f^4*g +
20*d^2*e^6*f^3*g^2 + 220*d^3*e^5*f^2*g^3 + 360*d^4*e^4*f*g^4 + 152*d^5*e^3*
g^5)*x^3 + 6*(7*d*e^7*f^5 - 15*d^2*e^6*f^4*g + 20*d^3*e^5*f^3*g^2 + 220*d^4
*e^4*f^2*g^3 + 360*d^5*e^3*f*g^4 + 152*d^6*e^2*g^5)*x^2 - 6*(7*d^2*e^6*f^5
- 15*d^3*e^5*f^4*g + 20*d^4*e^4*f^3*g^2 + 220*d^5*e^3*f^2*g^3 + 360*d^6*e^2
*f*g^4 + 152*d^7*e*g^5)*x + 30*(20*d^6*e^2*f^2*g^3 + 30*d^7*e*f*g^4 + 13*d^
8*g^5 - (20*d^3*e^5*f^2*g^3 + 30*d^4*e^4*f*g^4 + 13*d^5*e^3*g^5)*x^3 + 3*(2
0*d^4*e^4*f^2*g^3 + 30*d^5*e^3*f*g^4 + 13*d^6*e^2*g^5)*x^2 - 3*(20*d^5*e^3*
f^2*g^3 + 30*d^6*e^2*f*g^4 + 13*d^7*e*g^5)*x)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*d^3*e^4*g^5*x^4 - 14*d^2*e^5*f^5 + 30*d^3*e^4*f^4*g - 40
*d^4*e^3*f^3*g^2 - 440*d^5*e^2*f^2*g^3 - 720*d^6*e*f*g^4 - 304*d^7*g^5 + 15
*(10*d^3*e^4*f*g^4 + 3*d^4*e^3*g^5)*x^3 - (4*e^7*f^5 - 30*d*e^6*f^4*g + 140
*d^2*e^5*f^3*g^2 + 640*d^3*e^4*f^2*g^3 + 1170*d^4*e^3*f*g^4 + 479*d^5*e^2*g
^5)*x^2 + 3*(4*d*e^6*f^5 - 30*d^2*e^5*f^4*g + 40*d^3*e^4*f^3*g^2 + 340*d^4*
e^3*f^2*g^3 + 570*d^5*e^2*f*g^4 + 239*d^6*e*g^5)*x)*sqrt(-e^2*x^2 + d^2))/(
d^3*e^9*x^3 - 3*d^4*e^8*x^2 + 3*d^5*e^7*x - d^6*e^6)
```

giac [B] time = 0.66, size = 537, normalized size = 2.00

$$-\frac{1}{2}\left(13\,d^2g^5 + 30\,dfg^4e + 20\,f^2g^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d) + \frac{\sqrt{-x^2e^2 + d^2}\left(\left(\left(\left(\left(\left(15\left(g^5xe + \frac{2\left(3\,d^5g^5e^{12} + 5\,d^4fg^4e^{13}\right)\cdots}{d^4}\right.\right.\right.\right.\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*(13*d^2*g^5 + 30*d*f*g^4*e + 20*f^2*g^3*e^2)*arcsin(x*e/d)*e^(-6)*sgn(
d) + 1/30*sqrt(-x^2*e^2 + d^2)*((((((15*(g^5*x*e + 2*(3*d^5*g^5*e^12 + 5*d^
4*f*g^4*e^13)*e^(-12)/d^4)*x - (299*d^6*g^5*e^11 + 720*d^5*f*g^4*e^12 + 640
*d^4*f^2*g^3*e^13 + 140*d^3*f^3*g^2*e^14 - 30*d^2*f^4*g*e^15 + 4*d*f^5*e^16
)*e^(-12)/d^4)*x - 30*(19*d^7*g^5*e^10 + 45*d^6*f*g^4*e^11 + 30*d^5*f^2*g^3
*e^12 + 10*d^4*f^3*g^2*e^13)*e^(-12)/d^4)*x + 5*(91*d^8*g^5*e^9 + 210*d^7*f
*g^4*e^10 + 140*d^6*f^2*g^3*e^11 - 20*d^5*f^3*g^2*e^12 - 30*d^4*f^4*g*e^13
+ 2*d^3*f^5*e^14)*e^(-12)/d^4)*x + 10*(76*d^9*g^5*e^8 + 180*d^8*f*g^4*e^9 +
 110*d^7*f^2*g^3*e^10 + 10*d^6*f^3*g^2*e^11 - 15*d^5*f^4*g*e^12 - d^4*f^5*e
^13)*e^(-12)/d^4)*x - 15*(13*d^10*g^5*e^7 + 30*d^9*f*g^4*e^8 + 20*d^8*f^2*g
^3*e^9 + 2*d^5*f^5*e^12)*e^(-12)/d^4)*x - 2*(152*d^11*g^5*e^6 + 360*d^10*f*
g^4*e^7 + 220*d^9*f^2*g^3*e^8 + 20*d^8*f^3*g^2*e^9 - 15*d^7*f^4*g*e^10 + 7*
d^6*f^5*e^11)*e^(-12)/d^4)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.06, size = 1308, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 3/2*d^2/e*x/(-e^2*x^2+d^2)^(5/2)*f^4*g+1/2/e^4*x/(-e^2*x^2+d^2)^(3/2)*d^3*f
*g^4+3/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*f^2*g^3+7/3/e^2*x/(-e^2*x^2+d^2)^(3/2
)*d*f^3*g^2-5/e^2*x^3/(-e^2*x^2+d^2)^(3/2)*d*f*g^4+16/e^4*x/(-e^2*x^2+d^2)^
(1/2)*d*f*g^4-15/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
*d*f*g^4-110/3*d^3/e^2*x^2/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-10/3*d^2/e*x^2/(-e^
2*x^2+d^2)^(5/2)*f^3*g^2+5/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)*d^3*f*g^4+15*x^3/
e/(-e^2*x^2+d^2)^(5/2)*d^2*f^2*g^3-3/2*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)*f*g^4
+45*d^2/e*x^4/(-e^2*x^2+d^2)^(5/2)*f*g^4-60*d^4/e^3*x^2/(-e^2*x^2+d^2)^(5/2
)*f*g^4+14/3/d/e^2*x/(-e^2*x^2+d^2)^(1/2)*f^3*g^2-1/d^2/e*x/(-e^2*x^2+d^2)^
(1/2)*f^4*g-9*d^4/e^3*x/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-7*d^3/e^2*x/(-e^2*x^2+
d^2)^(5/2)*f^3*g^2-1/2*e*g^5*x^7/(-e^2*x^2+d^2)^(5/2)+7/15*d^2/e/(-e^2*x^2+
d^2)^(5/2)*f^5+4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^5+1/15/d*x/(-e^2*x^2+d^2)^(3/
2)*f^5+2/15/d^3*x/(-e^2*x^2+d^2)^(1/2)*f^5+1/3*x^2*e/(-e^2*x^2+d^2)^(5/2)*f
^5-3*x^6/(-e^2*x^2+d^2)^(5/2)*d*g^5+152/15*d^7/e^6/(-e^2*x^2+d^2)^(5/2)*g^5
+5/2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^4*g-1/2/e*x/(-e^2*x^2+d^2)^(3/2)*f^4*g+5*
x^2/(-e^2*x^2+d^2)^(5/2)*d*f^4*g-d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^4*g-5*x^6*e
/(-e^2*x^2+d^2)^(5/2)*f*g^4+19*d^3/e^2*x^4/(-e^2*x^2+d^2)^(5/2)*g^5-76/3*d^
5/e^4*x^2/(-e^2*x^2+d^2)^(5/2)*g^5+24*d^6/e^5/(-e^2*x^2+d^2)^(5/2)*f*g^4+3*
x^5/(-e^2*x^2+d^2)^(5/2)*d*f*g^4+2*x^5*e/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-10/3/
e*x^3/(-e^2*x^2+d^2)^(3/2)*f^2*g^3+16/e^3*x/(-e^2*x^2+d^2)^(1/2)*f^2*g^3-10
/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*f^2*g^3+30*x^4/
(-e^2*x^2+d^2)^(5/2)*d*f^2*g^3+10*x^4*e/(-e^2*x^2+d^2)^(5/2)*f^3*g^2+13/10/
e*g^5*d^2*x^5/(-e^2*x^2+d^2)^(5/2)-13/6/e^3*g^5*d^2*x^3/(-e^2*x^2+d^2)^(3/2
)+13/2/e^5*g^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-13/2/e^5*g^5*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+44/3*d^5/e^4/(-e^2*x^2+d^2)^(5/2)*f^
2*g^3+4/3*d^4/e^3/(-e^2*x^2+d^2)^(5/2)*f^3*g^2+15*x^3/(-e^2*x^2+d^2)^(5/2)*
d*f^3*g^2
```

maxima [B] time = 1.05, size = 1579, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e*g^5*x^7/(-e^2*x^2 + d^2)^(5/2) + 7/30*d^2*e*g^5*x*(15*x^4/((-e^2*x^2
 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2
*x^2 + d^2)^(5/2)*e^6)) - 7/6*d^2*g^5*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2)
 - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 1/5*d*f^5*x/(-e^2*x^2 + d^2)^(5/
2) + 3/5*d^2*f^5/((-e^2*x^2 + d^2)^(5/2)*e) + d^3*f^4*g/((-e^2*x^2 + d^2)^(
5/2)*e^2) + 4/15*f^5*x/((-e^2*x^2 + d^2)^(3/2)*d) + 14/15*d^4*g^5*x/((-e^2*
x^2 + d^2)^(3/2)*e^5) + 1/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5
)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 8/15*f^5*x/(sqrt(-e^2*x^2 +
 d^2)*d^3) - 49/30*d^2*g^5*x/(sqrt(-e^2*x^2 + d^2)*e^5) - (5*e^3*f*g^4 + 3*
d*e^2*g^5)*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/2*d^2*g^5*arcsin(e*x/d)/e^6
 - 1/3*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*x*(3*x^2/((-e^2*x^2
+ d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 6*(5*e^3*f*g^
4 + 3*d*e^2*g^5)*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) + (10*e^3*f^3*g^2 + 3
0*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2
) + 5/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x^3/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 8*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d^4*x^2/((-e^2*x^
2 + d^2)^(5/2)*e^6) - 4/3*(10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g
^4 + d^3*g^5)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f^5 + 15*d*e^
2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e^
2) - 3/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*d^2*x/
((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*(3*d*e^2*f^5 + 15*d^2*e*f^4*g + 10*d^3*f
^3*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 16/5*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d
^6/((-e^2*x^2 + d^2)^(5/2)*e^8) + 8/15*(10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 +
 15*d^2*e*f*g^4 + d^3*g^5)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^5
```

```
 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2*g^3)*d^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 4/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*d^2*x/(
(-e^2*x^2 + d^2)^(3/2)*e^6) + 1/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^
2*g^3 + d^3*f*g^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/15*(3*d*e^2*f^5 + 15*
d^2*e*f^4*g + 10*d^3*f^3*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(10
*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*x/(sqrt(-e^2*x^2 + d^2)*e^6) +
 (e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x/(sqrt(-e^2*x
^2 + d^2)*d^2*e^4) - 2/15*(3*d*e^2*f^5 + 15*d^2*e*f^4*g + 10*d^3*f^3*g^2)*x
/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*
e*g^5)*arcsin(e*x/d)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^5\,(d+e\,x)^3}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)
[Out] Timed out
```

3.580 $\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}+\frac{2(d}{}$$

```
[Out] 1/5*(d*g+e*f)^4*(e*x+d)^3/d/e^5/(-e^2*x^2+d^2)^(5/2)+2/15*(-9*d*g+e*f)*(d*g
+e*f)^3*(e*x+d)^2/d^2/e^5/(-e^2*x^2+d^2)^(3/2)-g^3*(3*d*g+4*e*f)*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(d*g+e*f)^2*(36*d^2*g^2-8*d*e*f*g+e^2*f^2)*
(e*x+d)/d^3/e^5/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A] time = 0.67, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1635, 641, 217, 203}

$$\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{15d^3e^5\sqrt{d^2-e^2x^2}}-\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 9*d
*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(e*f
 + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*Sqrt[d^
2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^4f^4-12de^3f^3g-18d^2e^2f^2g^2-12d^3efg^3-3d^4g^4}{e^4}+\frac{5dg^2\left(6e^2f^2+4defg+d^2g^2\right)x}{e^3}+\right.}{\left(d^2-e^2x^2\right)^{5/2}}}{5d}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^4f^4-12de^3f^3g+42d^2e^2f^2g^2}{e^4}\right.}{(}}{}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\right.}{15d^3e^5\sqrt{d^2-e}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\right.}{15d^3e^5\sqrt{d^2-e}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\right.}{15d^3e^5\sqrt{d^2-e}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\right.}{15d^3e^5\sqrt{d^2-e}}$$

Mathematica [A] time = 0.74, size = 168, normalized size = 0.78

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(15d^3g^4(d-ex)^3+2(d-ex)^2(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)+3d^2(dg+ef)^4+2d(d-ex)(ef-9dg)(dg+ef)^3\right)}{d^3(d-ex)^3} - 15g^3(3dg+4ef)\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{15e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^2*(e*f + d*g)^4 + 2*d*(e*f - 9*d*g)*(e*f + d*g)^
3*(d - e*x) + 2*(e*f + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d - e*x)^
2 + 15*d^3*g^4*(d - e*x)^3))/(d^3*(d - e*x)^3) - 15*g^3*(4*e*f + 3*d*g)*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^5)
```

fricas [B] time = 1.26, size = 624, normalized size = 2.90

$$-\frac{7\,d^3e^4f^4-12\,d^4e^3f^3g+12\,d^5e^2f^2g^2+88\,d^6efg^3+72\,d^7g^4-\left(7\,e^7f^4-12\,de^6f^3g+12\,d^2e^5f^2g^2+88\,d^3e^4fg^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^4*f^4 - 12*d^4*e^3*f^3*g + 12*d^5*e^2*f^2*g^2 + 88*d^6*e*f*g
^3 + 72*d^7*g^4 - (7*e^7*f^4 - 12*d*e^6*f^3*g + 12*d^2*e^5*f^2*g^2 + 88*d^3
*e^4*f*g^3 + 72*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 - 12*d^2*e^5*f^3*g + 12*d
^3*e^4*f^2*g^2 + 88*d^4*e^3*f*g^3 + 72*d^5*e^2*g^4)*x^2 - 3*(7*d^2*e^5*f^4
- 12*d^3*e^4*f^3*g + 12*d^4*e^3*f^2*g^2 + 88*d^5*e^2*f*g^3 + 72*d^6*e*g^4)*
```

```
x + 30*(4*d^6*e*f*g^3 + 3*d^7*g^4 - (4*d^3*e^4*f*g^3 + 3*d^4*e^3*g^4)*x^3 +
 3*(4*d^4*e^3*f*g^3 + 3*d^5*e^2*g^4)*x^2 - 3*(4*d^5*e^2*f*g^3 + 3*d^6*e*g^4
)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15*d^3*e^3*g^4*x^3 - 7*d^
2*e^4*f^4 + 12*d^3*e^3*f^3*g - 12*d^4*e^2*f^2*g^2 - 88*d^5*e*f*g^3 - 72*d^6
*g^4 - (2*e^6*f^4 - 12*d*e^5*f^3*g + 42*d^2*e^4*f^2*g^2 + 128*d^3*e^3*f*g^3
 + 117*d^4*e^2*g^4)*x^2 + 3*(2*d*e^5*f^4 - 12*d^2*e^4*f^3*g + 12*d^3*e^3*f^
2*g^2 + 68*d^4*e^2*f*g^3 + 57*d^5*e*g^4)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*
x^3 - 3*d^4*e^7*x^2 + 3*d^5*e^6*x - d^6*e^5)
```

giac [B] time = 0.40, size = 411, normalized size = 1.91

$$-\left(3\,dg^4+4\,fg^3e\right)\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(\left(\left(15\,g^4xe-\frac{2\left(36\,d^5g^4e^{10}+64\,d^4fg^3e^{11}+21\,d^3f^2g^2e^{12}-6\,d^2f^3\right.}{d^4}\right.\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(3*d*g^4 + 4*f*g^3*e)*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/15*sqrt(-x^2*e^2 + d
^2)*((((((15*g^4*x*e - 2*(36*d^5*g^4*e^10 + 64*d^4*f*g^3*e^11 + 21*d^3*f^2*
g^2*e^12 - 6*d^2*f^3*g*e^13 + d*f^4*e^14)*e^(-10)/d^4)*x - 45*(3*d^6*g^4*e^
9 + 4*d^5*f*g^3*e^10 + 2*d^4*f^2*g^2*e^11)*e^(-10)/d^4)*x + 5*(21*d^7*g^4*e
^8 + 28*d^6*f*g^3*e^9 - 6*d^5*f^2*g^2*e^10 - 12*d^4*f^3*g*e^11 + d^3*f^4*e^
12)*e^(-10)/d^4)*x + 5*(36*d^8*g^4*e^7 + 44*d^7*f*g^3*e^8 + 6*d^6*f^2*g^2*e
^9 - 12*d^5*f^3*g*e^10 - d^4*f^4*e^11)*e^(-10)/d^4)*x - 15*(3*d^9*g^4*e^6 +
 4*d^8*f*g^3*e^7 + d^5*f^4*e^10)*e^(-10)/d^4)*x - (72*d^10*g^4*e^5 + 88*d^9
*f*g^3*e^6 + 12*d^8*f^2*g^2*e^7 - 12*d^7*f^3*g*e^8 + 7*d^6*f^4*e^9)*e^(-10)
/d^4)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 1030, normalized size = 4.79

$$-\frac{e\,g^4x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3d\,g^4x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4ef\,g^3x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{9d^2g^4x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{12df\,g^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{6e\,f^2g^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{}{2\left(-e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^4+1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^4+2/15/d^3
*x/(-e^2*x^2+d^2)^(1/2)*f^4+1/3*x^2*e/(-e^2*x^2+d^2)^(5/2)*f^4+7/15*d^2/e/(
-e^2*x^2+d^2)^(5/2)*f^4+3/5*x^5/(-e^2*x^2+d^2)^(5/2)*d*g^4+24/5/e^5*g^4*d^6
/(-e^2*x^2+d^2)^(5/2)-e*g^4*x^6/(-e^2*x^2+d^2)^(5/2)+6*x^3/e/(-e^2*x^2+d^2)
^(5/2)*d^2*f*g^3-18/5*d^4/e^3*x/(-e^2*x^2+d^2)^(5/2)*f*g^3-44/3*d^3/e^2*x^2
/(-e^2*x^2+d^2)^(5/2)*f*g^3-2*d^2/e*x^2/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+14/5/d
/e^2*x/(-e^2*x^2+d^2)^(1/2)*f^2*g^2-4/5/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*f^3*g-
21/5*d^3/e^2*x/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+6/5*d^2/e*x/(-e^2*x^2+d^2)^(5/2
)*f^3*g+6/5/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*f*g^3+7/5/e^2*x/(-e^2*x^2+d^2)^(
3/2)*d*f^2*g^2+6*x^4*e/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+88/15*d^5/e^4/(-e^2*x^2
+d^2)^(5/2)*f*g^3+4/5*d^4/e^3/(-e^2*x^2+d^2)^(5/2)*f^2*g^2-1/e^2*x^3/(-e^2*
x^2+d^2)^(3/2)*d*g^4-4/3/e*x^3/(-e^2*x^2+d^2)^(3/2)*f*g^3+16/5/e^4*x/(-e^2*
x^2+d^2)^(1/2)*d*g^4+32/5/e^3*x/(-e^2*x^2+d^2)^(1/2)*f*g^3-3/e^4/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*d*g^4-4/e^3/(e^2)^(1/2)*arctan
((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*f*g^3+4/5*x^5*e/(-e^2*x^2+d^2)^(5/2)*f
*g^3+2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^3*g-3/10*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)
*g^4+12*x^4/(-e^2*x^2+d^2)^(5/2)*d*f*g^3+9*x^3/(-e^2*x^2+d^2)^(5/2)*d*f^2*g
^2+1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)*d^3*g^4-12/e^3*g^4*d^4*x^2/(-e^2*x^2+d^
2)^(5/2)+9/e*g^4*d^2*x^4/(-e^2*x^2+d^2)^(5/2)+1/10/e^4*x/(-e^2*x^2+d^2)^(3/
```

```
2)*d^3*g^4-2/5/e*x/(-e^2*x^2+d^2)^(3/2)*f^3*g+4*x^2/(-e^2*x^2+d^2)^(5/2)*d*
f^3*g-4/5*d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^3*g
```

maxima [B] time = 1.04, size = 1178, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -e*g^4*x^6/(-e^2*x^2 + d^2)^(5/2) + 6*d^2*g^4*x^4/((-e^2*x^2 + d^2)^(5/2)*e
) - 8*d^4*g^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) + 1/5*d*f^4*x/(-e^2*x^2 + d^
2)^(5/2) + 1/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)
^(5/2)*e^6)) + 3/5*d^2*f^4/((-e^2*x^2 + d^2)^(5/2)*e) + 4/5*d^3*f^3*g/((-e^
2*x^2 + d^2)^(5/2)*e^2) + 16/5*d^6*g^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 4/15*
f^4*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^4*x/(sqrt(-e^2*x^2 + d^2)*d^3) -
1/3*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d
^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 3*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d
^2*e*g^4)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 1/2*(4*e^3*f^3*g + 18*d*e^2*f^
2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4*(2*e
^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4
) + 1/3*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e
*f*g^3 + d^3*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^4 + 4*d
^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(2*e^3*f^2
*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(
e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*d^2/((-e^2*x^2 +
 d^2)^(5/2)*e^4) + 4/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*d^2*x/((-e^2*x^2 + d^2)
^(3/2)*e^6) + 1/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g
^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3
*f^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(4*e^3*f*g^3 + 3*d*e^2*
g^4)*x/(sqrt(-e^2*x^2 + d^2)*e^6) + 1/5*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 1
2*d^2*e*f*g^3 + d^3*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^4
+ 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (4*e^3*
f*g^3 + 3*d*e^2*g^4)*arcsin(e*x/d)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^4\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

3.581 $\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{g^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{(d+ex)(dg+ef)\left(32d^2g^2-11de\right.}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)^3*(e*x+d)^3/d/e^4/(-e^2*x^2+d^2)^(5/2)+1/15*(-13*d*g+2*e*f)*(
d*g+e*f)^2*(e*x+d)^2/d^2/e^4/(-e^2*x^2+d^2)^(3/2)-g^3*arctan(e*x/(-e^2*x^2+
d^2)^(1/2))/e^4+1/15*(d*g+e*f)*(32*d^2*g^2-11*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^
3/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1635, 778, 217, 203}

$$\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{15d^3e^4\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{g^3\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 13*
d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*Sqrt[d^
2 - e^2*x^2]) - (g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^3f^3-9de^2f^2g-9d^2efg^2-3d^3g^3}{e^3}+\frac{5dg^2(3ef+dg)x}{e^2}+\frac{5dg^3x^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^3f^3-9de^2f^2g+21d^2efg^2+1}{e^3}\right.}{\left(d^2-e^2x^2\right)^{3/2}}}{15d^2}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg\right.}{15d^3e^4\sqrt{d^2-}}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg\right.}{15d^3e^4\sqrt{d^2-}}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg\right.}{15d^3e^4\sqrt{d^2-}}$$

Mathematica [A] time = 0.81, size = 182, normalized size = 0.99

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\,(dg+ef)\left(22d^4g^2-d^3eg(16f+51gx)+d^2e^2\left(7f^2+33fgx+32g^2x^2\right)-de^3fx(6f+11gx)+\right.\right.}{15d^3e^4(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*((e*f + d*g)*Sqrt[1 - (e^2*x^2)/d^2]*(22*d^4*g^2 + 2*e^4*f^2*x^2
 - d*e^3*f*x*(6*f + 11*g*x) - d^3*e*g*(16*f + 51*g*x) + d^2*e^2*(7*f^2 + 33
*f*g*x + 32*g^2*x^2)) - 15*d^2*g^3*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d^3*e^
4*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [B] time = 1.00, size = 454, normalized size = 2.48

$$-\frac{7\,d^3e^3f^3-9\,d^4e^2f^2g+6\,d^5efg^2+22\,d^6g^3-\left(7\,e^6f^3-9\,de^5f^2g+6\,d^2e^4fg^2+22\,d^3e^3g^3\right)x^3+3\left(7\,de^5f^3-9\,d^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^3*f^3 - 9*d^4*e^2*f^2*g + 6*d^5*e*f*g^2 + 22*d^6*g^3 - (7*e^
6*f^3 - 9*d*e^5*f^2*g + 6*d^2*e^4*f*g^2 + 22*d^3*e^3*g^3)*x^3 + 3*(7*d*e^5*
f^3 - 9*d^2*e^4*f^2*g + 6*d^3*e^3*f*g^2 + 22*d^4*e^2*g^3)*x^2 - 3*(7*d^2*e^
4*f^3 - 9*d^3*e^3*f^2*g + 6*d^4*e^2*f*g^2 + 22*d^5*e*g^3)*x - 30*(d^3*e^3*g
^3*x^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*arctan(-(d - sqrt(-e^
2*x^2 + d^2))/(e*x)) + (7*d^2*e^3*f^3 - 9*d^3*e^2*f^2*g + 6*d^4*e*f*g^2 + 2
2*d^5*g^3 + (2*e^5*f^3 - 9*d*e^4*f^2*g + 21*d^2*e^3*f*g^2 + 32*d^3*e^2*g^3)
*x^2 - 3*(2*d*e^4*f^3 - 9*d^2*e^3*f^2*g + 6*d^3*e^2*f*g^2 + 17*d^4*e*g^3)*x
```

```
)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^3 - 3*d^4*e^6*x^2 + 3*d^5*e^5*x - d^6*e^
4)
```

giac [A] time = 0.38, size = 309, normalized size = 1.69

$$-g^3 \arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d) - \frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(\left(x\left(\frac{\left(32\,d^4g^3e^8+21\,d^3fg^2e^9-9\,d^2f^2ge^{10}+2\,df^3e^{11}\right)xe^{(-7)}}{d^4}+\frac{45\left(d^5g^3e^7+d^4fg^2e^8\right)e^{(-7)}}{d^4}\right.\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -g^3*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/15*sqrt(-x^2*e^2 + d^2)*((((x*((32*d^4
*g^3*e^8 + 21*d^3*f*g^2*e^9 - 9*d^2*f^2*g*e^10 + 2*d*f^3*e^11)*x*e^(-7)/d^4
 + 45*(d^5*g^3*e^7 + d^4*f*g^2*e^8)*e^(-7)/d^4) - 5*(7*d^6*g^3*e^6 - 3*d^5*
f*g^2*e^7 - 9*d^4*f^2*g*e^8 + d^3*f^3*e^9)*e^(-7)/d^4)*x - 5*(11*d^7*g^3*e^
5 + 3*d^6*f*g^2*e^6 - 9*d^5*f^2*g*e^7 - d^4*f^3*e^8)*e^(-7)/d^4)*x + 15*(d^
8*g^3*e^4 + d^5*f^3*e^7)*e^(-7)/d^4)*x + (22*d^9*g^3*e^3 + 6*d^8*f*g^2*e^4
- 9*d^7*f^2*g*e^5 + 7*d^6*f^3*e^6)*e^(-7)/d^4)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 713, normalized size = 3.90

$$\frac{e g^3x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3d g^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3ef g^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3d^2g^3x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{9df g^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3e f^2g x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^3+1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^3+2/15/d^3
*x/(-e^2*x^2+d^2)^(1/2)*f^3+7/15*d^2/e/(-e^2*x^2+d^2)^(5/2)*f^3+1/3*x^2*e/(
-e^2*x^2+d^2)^(5/2)*f^3+3*x^4/(-e^2*x^2+d^2)^(5/2)*d*g^3+22/15*d^5/e^4/(-e^
2*x^2+d^2)^(5/2)*g^3+8/5/e^3*g^3*x/(-e^2*x^2+d^2)^(1/2)-1/e^3*g^3/(e^2)^(1/
2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*e*g^3*x^5/(-e^2*x^2+d^2)^
(5/2)-1/3/e*g^3*x^3/(-e^2*x^2+d^2)^(3/2)-21/10*d^3/e^2*x/(-e^2*x^2+d^2)^(5/
2)*f*g^2-d^2/e*x^2/(-e^2*x^2+d^2)^(5/2)*f*g^2+9/10*d^2/e*x/(-e^2*x^2+d^2)^(
5/2)*f^2*g+7/10/e^2*x/(-e^2*x^2+d^2)^(3/2)*d*f*g^2+7/5/d/e^2*x/(-e^2*x^2+d^
2)^(1/2)*f*g^2-3/5/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*f^2*g-9/10*d^4/e^3*x/(-e^2*
x^2+d^2)^(5/2)*g^3+3/10/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*g^3-3/10/e*x/(-e^2*x
^2+d^2)^(3/2)*f^2*g+3/2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^2*g+3*x^2/(-e^2*x^2+d^
2)^(5/2)*d*f^2*g-3/5*d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^2*g-11/3*d^3/e^2*x^2/(-
e^2*x^2+d^2)^(5/2)*g^3+2/5*d^4/e^3/(-e^2*x^2+d^2)^(5/2)*f*g^2+3/2*x^3/e/(-e
^2*x^2+d^2)^(5/2)*d^2*g^3+9/2*x^3/(-e^2*x^2+d^2)^(5/2)*d*f*g^2+3*x^4*e/(-e^
2*x^2+d^2)^(5/2)*f*g^2
```

maxima [B] time = 1.03, size = 891, normalized size = 4.87

$$\frac{1}{15}e^3g^3x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{1}{3}eg^3x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*g^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2
 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*g^3*x*(3*x
^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 1/5
```

```
*d*f^3*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^3/((-e^2*x^2 + d^2)^(5/2)*e) +
3/5*d^3*f^2*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f^3*x/((-e^2*x^2 + d^2)^(
3/2)*d) + 4/15*d^2*g^3*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 8/15*f^3*x/(sqrt(-e
^2*x^2 + d^2)*d^3) - 7/15*g^3*x/(sqrt(-e^2*x^2 + d^2)*e^3) + 3*(e^3*f*g^2 +
 d*e^2*g^3)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - g^3*arcsin(e*x/d)/e^4 + 3/2*
(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) -
4*(e^3*f*g^2 + d*e^2*g^3)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f
^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e
^2) - 9/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*d^2*x/((-e^2*x^2 + d^2)^
(5/2)*e^4) + 3/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2
)^(5/2)*e^2) + 8/5*(e^3*f*g^2 + d*e^2*g^3)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)
 - 2/15*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*d^2/((-e^2*x^2
+ d^2)^(5/2)*e^4) + 3/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/((-e^2*x
^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2
*x^2 + d^2)^(3/2)*d^2*e^2) + 3/5*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/
(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2
)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)^3\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\left(f+gx\right)^3}{\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.582 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)^2*(e*x+d)^3/d/e^3/(-e^2*x^2+d^2)^(5/2)+2/15*(-4*d*g+e*f)*(d*g
+e*f)*(e*x+d)^2/d^2/e^3/(-e^2*x^2+d^2)^(3/2)+1/15*(7*d^2*g^2-6*d*e*f*g+2*e^
2*f^2)*(e*x+d)/d^3/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1635, 789, 637}

$$\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 4*d
*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*f
^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)^2(d+ex)^3}{5de^3(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-2f^2+\frac{6dfg}{e}+\frac{3d^2g^2}{e^2}+\frac{5dg^2x}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3(d^2-e^2x^2)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3(d^2-e^2x^2)^{3/2}} + \frac{(2e^2f^2-6defg+7d^2g^2)\int -}{15d^2e^2}$$

$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3(d^2-e^2x^2)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3(d^2-e^2x^2)^{3/2}} + \frac{(2e^2f^2-6defg+7d^2g^2)(d+}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.39, size = 110, normalized size = 0.76

$$\frac{(d+ex)\left(2d^4g^2-6d^3eg(f+gx)+d^2e^2\left(7f^2+18fgx+7g^2x^2\right)-6de^3fx(f+gx)+2e^4f^2x^2\right)}{15d^3e^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d^4*g^2 + 2*e^4*f^2*x^2 - 6*d^3*e*g*(f + g*x) - 6*d*e^3*f*x*(
f + g*x) + d^2*e^2*(7*f^2 + 18*f*g*x + 7*g^2*x^2)))/(15*d^3*e^3*(d - e*x)^2
*Sqrt[d^2 - e^2*x^2])
```

fricas [B] time = 0.96, size = 279, normalized size = 1.92

$$-\frac{7\,d^3e^2f^2-6\,d^4efg+2\,d^5g^2-\left(7\,e^5f^2-6\,de^4fg+2\,d^2e^3g^2\right)x^3+3\left(7\,de^4f^2-6\,d^2e^3fg+2\,d^3e^2g^2\right)x^2-3\left(7\,d^2e^3\right.}{15\left(d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^2*f^2 - 6*d^4*e*f*g + 2*d^5*g^2 - (7*e^5*f^2 - 6*d*e^4*f*g +
 2*d^2*e^3*g^2)*x^3 + 3*(7*d*e^4*f^2 - 6*d^2*e^3*f*g + 2*d^3*e^2*g^2)*x^2 -
 3*(7*d^2*e^3*f^2 - 6*d^3*e^2*f*g + 2*d^4*e*g^2)*x + (7*d^2*e^2*f^2 - 6*d^3
*e*f*g + 2*d^4*g^2 + (2*e^4*f^2 - 6*d*e^3*f*g + 7*d^2*e^2*g^2)*x^2 - 6*(d*e
^3*f^2 - 3*d^2*e^2*f*g + d^3*e*g^2)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 -
 3*d^4*e^5*x^2 + 3*d^5*e^4*x - d^6*e^3)
```

giac [A] time = 0.34, size = 198, normalized size = 1.37

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(15\,df^2+\left(\left(\left(15\,g^2e+\frac{\left(7\,d^3g^2e^6-6\,d^2fge^7+2\,df^2e^8\right)xe^{(-4)}}{d^4}\right)x+\frac{5\left(d^5g^2e^4+6\,d^4fge^5-d^3f^2e^6\right)e^{(-4)}}{d^4}\right)x-\frac{5\left(d^6g^2e^3-6\,d^5f\right.}{}\right.\right.\right.}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((15*d*f^2 + (((15*g^2*e + (7*d^3*g^2*e^6 - 6*d^
2*f*g*e^7 + 2*d*f^2*e^8)*x*e^(-4)/d^4)*x + 5*(d^5*g^2*e^4 + 6*d^4*f*g*e^5 -
 d^3*f^2*e^6)*e^(-4)/d^4)*x - 5*(d^6*g^2*e^3 - 6*d^5*f*g*e^4 - d^4*f^2*e^5)
*e^(-4)/d^4)*x)*x + (2*d^8*g^2*e - 6*d^7*f*g*e^2 + 7*d^6*f^2*e^3)*e^(-4)/d^
4)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 131, normalized size = 0.90

$$\frac{(-ex+d)(ex+d)^4\left(7d^2e^2g^2x^2-6de^3fgx^2+2e^4f^2x^2-6d^3eg^2x+18d^2e^2fgx-6de^3f^2x+2d^4g^2-6d^3efg+\right.}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(7*d^2*e^2*g^2*x^2-6*d*e^3*f*g*x^2+2*e^4*f^2*x^2-6*
d^3*e*g^2*x+18*d^2*e^2*f*g*x-6*d*e^3*f^2*x+2*d^4*g^2-6*d^3*e*f*g+7*d^2*e^2*
f^2)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.47, size = 583, normalized size = 4.02

$$\frac{eg^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4\,d^2g^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{df^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,d^2f^2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{2\,d^3fg}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{8\,d^4g^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*g^2*x^4/(-e^2*x^2 + d^2)^(5/2) - 4/3*d^2*g^2*x^2/((-e^2*x^2 + d^2)^(5/2)*
e) + 1/5*d*f^2*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^2/((-e^2*x^2 + d^2)^(5/
2)*e) + 2/5*d^3*f*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/15*d^4*g^2/((-e^2*x^2
+ d^2)^(5/2)*e^3) + 4/15*f^2*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^2*x/(sqr
t(-e^2*x^2 + d^2)*d^3) + 1/2*(2*e^3*f*g + 3*d*e^2*g^2)*x^3/((-e^2*x^2 + d^2
)^(5/2)*e^2) + 1/3*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^2) - 3/10*(2*e^3*f*g + 3*d*e^2*g^2)*d^2*x/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 2/15*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*d^2/((-e^2*x^2 + d^2)^
(5/2)*e^4) + 1/10*(2*e^3*f*g + 3*d*e^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*e^4)
- 1/15*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*
e^2) + 1/5*(2*e^3*f*g + 3*d*e^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/1
5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

mupad [B] time = 2.87, size = 125, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4\,g^2-6\,d^3\,e\,f\,g-6\,d^3\,e\,g^2\,x+7\,d^2\,e^2\,f^2+18\,d^2\,e^2\,f\,g\,x+7\,d^2\,e^2\,g^2\,x^2-6\,d\,e^3\,f^2\,x-6\,d\,e^3\,f\,g\right.}{15\,d^3\,e^3\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4*g^2 + 7*d^2*e^2*f^2 + 2*e^4*f^2*x^2 - 6*d^3*e
*f*g + 7*d^2*e^2*g^2*x^2 - 6*d*e^3*f^2*x - 6*d^3*e*g^2*x + 18*d^2*e^2*f*g*x
 - 6*d*e^3*f*g*x^2))/(15*d^3*e^3*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\left(f+gx\right)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.583 $\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)*(e*x+d)^3/d/e^2/(-e^2*x^2+d^2)^(5/2)+2/15*(-3*d*g+2*e*f)*(e*x
+d)/d/e^2/(-e^2*x^2+d^2)^(3/2)+1/15*(-3*d*g+2*e*f)*x/d^3/e/(-e^2*x^2+d^2)^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {789, 653, 191}

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + (2*(2*e*f - 3*d
*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) + ((2*e*f - 3*d*g)*x)/(15*d
^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{(-5ef+3(ef+dg))\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}$$

$$= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{(-5ef+3(ef+dg))\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}$$

$$= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.24, size = 83, normalized size = 0.71

$$-\frac{(d+ex)\left(3d^3g-d^2e(7f+9gx)+3de^2x(2f+gx)-2e^3fx^2\right)}{15d^3e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*((d + e*x)*(3*d^3*g - 2*e^3*f*x^2 + 3*d*e^2*x*(2*f + g*x) - d^2*e*(7*
f + 9*g*x)))/(d^3*e^2*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

fricas [A] time = 0.58, size = 183, normalized size = 1.56

$$-\frac{7\,d^3ef-3\,d^4g-\left(7\,e^4f-3\,de^3g\right)x^3+3\left(7\,de^3f-3\,d^2e^2g\right)x^2-3\left(7\,d^2e^2f-3\,d^3eg\right)x+\left(7\,d^2ef-3\,d^3g+\left(2\,e^{3}\right.\right.}{15\left(d^3e^5x^3-3\,d^4e^4x^2+3\,d^5e^3x-d^6e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e*f - 3*d^4*g - (7*e^4*f - 3*d*e^3*g)*x^3 + 3*(7*d*e^3*f - 3*d
^2*e^2*g)*x^2 - 3*(7*d^2*e^2*f - 3*d^3*e*g)*x + (7*d^2*e*f - 3*d^3*g + (2*e
^3*f - 3*d*e^2*g)*x^2 - 3*(2*d*e^2*f - 3*d^2*e*g)*x)*sqrt(-e^2*x^2 + d^2))/
(d^3*e^5*x^3 - 3*d^4*e^4*x^2 + 3*d^5*e^3*x - d^6*e^2)
```

giac [A] time = 0.33, size = 139, normalized size = 1.19

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(15\,df-\left(x\left(\frac{\left(3\,d^2ge^7-2\,dfe^8\right)x^2e^{(-4)}}{d^4}-\frac{5\left(3\,d^4ge^5-d^3fe^6\right)e^{(-4)}}{d^4}\right)-\frac{5\left(3\,d^5ge^4+d^4fe^5\right)e^{(-4)}}{d^4}\right)x\right)x-\frac{\left(3\,d^7ge^2-7\,d^6fe^3\right)e}{d^4}\right.\right.}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((15*d*f - (x*((3*d^2*g*e^7 - 2*d*f*e^8)*x^2*e^(
-4)/d^4 - 5*(3*d^4*g*e^5 - d^3*f*e^6)*e^(-4)/d^4) - 5*(3*d^5*g*e^4 + d^4*f*
e^5)*e^(-4)/d^4)*x)*x - (3*d^7*g*e^2 - 7*d^6*f*e^3)*e^(-4)/d^4)/(x^2*e^2 -
d^2)^3
```

maple [A] time = 0.01, size = 85, normalized size = 0.73

$$-\frac{(-ex+d)\,(ex+d)^4\left(3d\,e^2g\,x^2-2e^3f\,x^2-9d^2egx+6d\,e^2fx+3d^3g-7d^2ef\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^4*(3*d*e^2*g*x^2-2*e^3*f*x^2-9*d^2*e*g*x+6*d*e^2*f*x
+3*d^3*g-7*d^2*e*f)/d^3/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.46, size = 373, normalized size = 3.19

$$\frac{egx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{dfx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{3\,d^2gx}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{3\,d^2f}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d^3g}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{4\,fx}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*g*x^3/(-e^2*x^2 + d^2)^(5/2) + 1/5*d*f*x/(-e^2*x^2 + d^2)^(5/2) - 3/1
0*d^2*g*x/((-e^2*x^2 + d^2)^(5/2)*e) + 3/5*d^2*f/((-e^2*x^2 + d^2)^(5/2)*e)
 + 1/5*d^3*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f*x/((-e^2*x^2 + d^2)^(3/2
)*d) + 1/10*g*x/((-e^2*x^2 + d^2)^(3/2)*e) + 8/15*f*x/(sqrt(-e^2*x^2 + d^2)
*d^3) + 1/5*g*x/(sqrt(-e^2*x^2 + d^2)*d^2*e) + 1/3*(e^3*f + 3*d*e^2*g)*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^2) + 3/5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 2/15*(e^3*f + 3*d*e^2*g)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/
5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 2/5*(d*e^2*f + d
^2*e*g)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

mupad [B] time = 2.79, size = 79, normalized size = 0.68

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,g\,d^3-9\,g\,d^2\,e\,x-7\,f\,d^2\,e+3\,g\,d\,e^2\,x^2+6\,f\,d\,e^2\,x-2\,f\,e^3\,x^2\right)}{15\,d^3\,e^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^3*g - 2*e^3*f*x^2 - 7*d^2*e*f + 6*d*e^2*f*x -
9*d^2*e*g*x + 3*d*e^2*g*x^2))/(15*d^3*e^2*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\left(f+gx\right)}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.584 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-e
*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

fricas [A] time = 0.65, size = 106, normalized size = 1.03

$$\frac{7\,e^3x^3 - 21\,de^2x^2 + 21\,d^2ex - 7\,d^3 - \left(2\,e^2x^2 - 6\,dex + 7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^4x^3 - 3\,d^4e^3x^2 + 3\,d^5e^2x - d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

giac [A] time = 0.30, size = 70, normalized size = 0.68

$$-\frac{\sqrt{-x^2e^2+d^2}\left(7\,d^2e^{(-1)} + \left(\left(x\left(\frac{2\,x^2e^4}{d^3} - \frac{5\,e^2}{d}\right) + 5\,e\right)x + 15\,d\right)x\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.53

$$\frac{(-ex+d)\,(ex+d)^4\left(2e^2x^2-6dex+7d^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

maxima [A] time = 0.44, size = 101, normalized size = 0.98

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{2\,x}{15\sqrt{-e^2x^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

mupad [B] time = 2.70, size = 49, normalized size = 0.48

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.585 $\int \frac{(d+ex)^3}{(f+gx)(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=242

$$\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}+\frac{15d^3g^2+ex\left(22d^2g^2+9\cdots\right.}{15d^3\sqrt{d^2-e^2x^2}\,(\cdots}$$

```
[Out] 4/5*d*(e*x+d)/(d*g+e*f)/(-e^2*x^2+d^2)^(5/2)+1/15*(-5*d*(-d*g+e*f)+e*(11*d*
g+e*f)*x)/d/(d*g+e*f)^2/(-e^2*x^2+d^2)^(3/2)+g^3*arctan((e^2*f*x+d^2*g)/(-d
^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(d*g+e*f)^3/(-d^2*g^2+e^2*f^2)^
(1/2)+1/15*(15*d^3*g^2+e*(22*d^2*g^2+9*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^
3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 823, 12, 725, 204}

$$\frac{ex\left(22d^2g^2+9defg+2e^2f^2\right)+15d^3g^2}{15d^3\sqrt{d^2-e^2x^2}\,(dg+ef)^3}+\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(\cdots}{5\left(d^2-e^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) - (5*d*(e*f - d*g) -
e*(e*f + 11*d*g)*x)/(15*d*(e*f + d*g)^2*(d^2 - e^2*x^2)^(3/2)) + (15*d^3*g^
2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f + d*g)^3*Sqrt[d^
2 - e^2*x^2]) + (g^3*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/((e*f + d*g)^3*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2(ef+5dg)}{ef+dg}-\frac{d^2e^3(5ef-11dg)x}{ef+dg}}{(f+gx)\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-\frac{d^3e^4(ef-dg)\left(2e^2f^2+7de\right.}{ef+dg}}{(f+}}{15d^4e}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2\right.}{15d^3(ef+dg}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2\right.}{15d^3(ef+dg}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2\right.}{15d^3(ef+dg}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2\right.}{15d^3(ef+dg}$$

Mathematica [A] time = 0.39, size = 225, normalized size = 0.93

$$\frac{\frac{(d+ex)\left(d^2g^2-e^2f^2\right)\left(32d^4g^2+3d^3eg(8f-17gx)+d^2e^2\left(7f^2-27fgx+22g^2x^2\right)+3de^3fx(3gx-2f)+2e^4f^2x^2\right)}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}} - 15g^3\sqrt{e^2f^2-d^2g^2}\,\tan^{-1}\left(\frac{}{\sqrt{d^2-}}\right.}{15(dg-ef)(dg+ef)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((-(e^2*f^2) + d^2*g^2)*(d + e*x)*(32*d^4*g^2 + 2*e^4*f^2*x^2 + 3*d^3*e*g*
(8*f - 17*g*x) + 3*d*e^3*f*x*(-2*f + 3*g*x) + d^2*e^2*(7*f^2 - 27*f*g*x + 2
2*g^2*x^2)))/(d^3*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) - 15*g^3*Sqrt[e^2*f^2 -
d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x
^2])])/(15*(-(e*f) + d*g)*(e*f + d*g)^4)
```

fricas [B] time = 1.14, size = 1767, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g
^3 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3
*e^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d
^3*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 + 15*(d^3*e^3*g^3*x
^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(-e^2*f^2 + d^2*g^2)*
log((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sq
rt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x +
f)) - 3*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2
*f*g^3 - 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f
^2*g^2 - 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*
e^4*f^2*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^
2*e^4*f^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(
-e^2*x^2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9
*e^2*f^2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g +
 2*d^5*e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3
 + 3*(d^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3
 - 3*d^8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g +
2*d^7*e^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x), 1
/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g^3
 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3*e
^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d^3
*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 - 30*(d^3*e^3*g^3*x^3
 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(e^2*f^2 - d^2*g^2)*arc
tan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - 3
*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2*f*g^3
- 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f^2*g^2
- 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*e^4*f^2
*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^2*e^4*f
^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(-e^2*x^
2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9*e^2*f^
2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g + 2*d^5*
e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3 + 3*(d
^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3 - 3*d^
8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g + 2*d^7*e
^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x)]
```

giac [B] time = 0.46, size = 2966, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*(d^3*g^6*e^2 - 3*d^2*f*g^5*e^3 + 3*d*f^2*g^4*e^4 - f^3*g^3*e^5)*arctan((
d*g*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f/x)/sqrt(-d^2*g^2*e^2 + f^2*e^4))/(
(d^6*g^6*e - 3*d^4*f^2*g^4*e^3 + 3*d^2*f^4*g^2*e^5 - f^6*e^7)*sqrt(-d^2*g^2
*e^2 + f^2*e^4)) - 1/15*sqrt(-x^2*e^2 + d^2)*((((((22*d^18*g^17*e^9 + 339*d
^17*f*g^16*e^10 + 2447*d^16*f^2*g^15*e^11 + 10985*d^15*f^3*g^14*e^12 + 3433
5*d^14*f^4*g^13*e^13 + 79261*d^13*f^5*g^12*e^14 + 139867*d^12*f^6*g^11*e^15
 + 192621*d^11*f^7*g^10*e^16 + 209495*d^10*f^8*g^9*e^17 + 180895*d^9*f^9*g^
8*e^18 + 123981*d^8*f^10*g^7*e^19 + 67067*d^7*f^11*g^6*e^20 + 28301*d^6*f^1
2*g^5*e^21 + 9135*d^5*f^13*g^4*e^22 + 2185*d^4*f^14*g^3*e^23 + 367*d^3*f^15
```

*g^2*e^24 + 39*d^2*f^16*g*e^25 + 2*d*f^17*e^26)*x/(d^22*g^18*e^4 + 18*d^21*
f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f^3*g^15*e^7 + 3060*d^18*f^4*
g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*f^6*g^12*e^10 + 31824*d^15*f
^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620*d^13*f^9*g^9*e^13 + 43758*d
^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 + 18564*d^10*f^12*g^6*e^16 + 8
568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18 + 816*d^7*f^15*g^3*e^19 + 15
3*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*f^18*e^22) + 15*(d^19*g^17*e
^8 + 15*d^18*f*g^16*e^9 + 105*d^17*f^2*g^15*e^10 + 455*d^16*f^3*g^14*e^11 +
1365*d^15*f^4*g^13*e^12 + 3003*d^14*f^5*g^12*e^13 + 5005*d^13*f^6*g^11*e^1
4 + 6435*d^12*f^7*g^10*e^15 + 6435*d^11*f^8*g^9*e^16 + 5005*d^10*f^9*g^8*e^
17 + 3003*d^9*f^10*g^7*e^18 + 1365*d^8*f^11*g^6*e^19 + 455*d^7*f^12*g^5*e^2
0 + 105*d^6*f^13*g^4*e^21 + 15*d^5*f^14*g^3*e^22 + d^4*f^15*g^2*e^23)/(d^22
*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f^3*g^15*
e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*f^6*g^12
*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620*d^13*f^
9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 + 18564*d^
10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18 + 816*d^
7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*f^18*e^2
2))*x - 5*(11*d^20*g^17*e^7 + 171*d^19*f*g^16*e^8 + 1246*d^18*f^2*g^15*e^9
+ 5650*d^17*f^3*g^14*e^10 + 17850*d^16*f^4*g^13*e^11 + 41678*d^15*f^5*g^12*
e^12 + 74438*d^14*f^6*g^11*e^13 + 103818*d^13*f^7*g^10*e^14 + 114400*d^12*f
^8*g^9*e^15 + 100100*d^11*f^9*g^8*e^16 + 69498*d^10*f^10*g^7*e^17 + 38038*d
^9*f^11*g^6*e^18 + 16198*d^8*f^12*g^5*e^19 + 5250*d^7*f^13*g^4*e^20 + 1250*
d^6*f^14*g^3*e^21 + 206*d^5*f^15*g^2*e^22 + 21*d^4*f^16*g*e^23 + d^3*f^17*e
^24)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19
*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^1
6*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 486
20*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15
+ 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^1
8 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^
4*f^18*e^22))*x - 5*(7*d^21*g^17*e^6 + 105*d^20*f*g^16*e^7 + 734*d^19*f^2*g
^15*e^8 + 3170*d^18*f^3*g^14*e^9 + 9450*d^17*f^4*g^13*e^10 + 20566*d^16*f^5
*g^12*e^11 + 33670*d^15*f^6*g^11*e^12 + 42042*d^14*f^7*g^10*e^13 + 40040*d^
13*f^8*g^9*e^14 + 28600*d^12*f^9*g^8*e^15 + 14586*d^11*f^10*g^7*e^16 + 4550
*d^10*f^11*g^6*e^17 + 182*d^9*f^12*g^5*e^18 - 630*d^8*f^13*g^4*e^19 - 350*d
^7*f^14*g^3*e^20 - 98*d^6*f^15*g^2*e^21 - 15*d^5*f^16*g*e^22 - d^4*f^17*e^2
3)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f
^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*
f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620
*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 +
18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18
+ 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*
f^18*e^22))*x + 15*(3*d^22*g^17*e^5 + 48*d^21*f*g^16*e^6 + 361*d^20*f^2*g^1
5*e^7 + 1695*d^19*f^3*g^14*e^8 + 5565*d^18*f^4*g^13*e^9 + 13559*d^17*f^5*g^
12*e^10 + 25389*d^16*f^6*g^11*e^11 + 37323*d^15*f^7*g^10*e^12 + 43615*d^14*
f^8*g^9*e^13 + 40755*d^13*f^9*g^8*e^14 + 30459*d^12*f^10*g^7*e^15 + 18109*d
^11*f^11*g^6*e^16 + 8463*d^10*f^12*g^5*e^17 + 3045*d^9*f^13*g^4*e^18 + 815*
d^8*f^14*g^3*e^19 + 153*d^7*f^15*g^2*e^20 + 18*d^6*f^16*g*e^21 + d^5*f^17*e
^22)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19
*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^1
6*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 486
20*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15
+ 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^1
8 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^
4*f^18*e^22))*x + (32*d^23*g^17*e^4 + 504*d^22*f*g^16*e^5 + 3727*d^21*f^2*g
^15*e^6 + 17185*d^20*f^3*g^14*e^7 + 55335*d^19*f^4*g^13*e^8 + 132041*d^18*f
^5*g^12*e^9 + 241787*d^17*f^6*g^11*e^10 + 347061*d^16*f^7*g^10*e^11 + 39539
5*d^15*f^8*g^9*e^12 + 359645*d^14*f^9*g^8*e^13 + 261261*d^13*f^10*g^7*e^14
+ 150787*d^12*f^11*g^6*e^15 + 68341*d^11*f^12*g^5*e^16 + 23835*d^10*f^13*g^

```
4*e^17 + 6185*d^9*f^14*g^3*e^18 + 1127*d^8*f^15*g^2*e^19 + 129*d^7*f^16*g*e
^20 + 7*d^6*f^17*e^21)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g
^16*e^6 + 816*d^19*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^
13*e^9 + 18564*d^16*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f
^8*g^10*e^12 + 48620*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d
^11*f^11*g^7*e^15 + 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 306
0*d^8*f^14*g^4*e^18 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^
5*f^17*g*e^21 + d^4*f^18*e^22))/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.05, size = 3961, normalized size = 16.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] g/(d^2*g^2-e^2*f^2)^3*f^4/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^
2*f^2)/g^2)^(1/2)*e^5*x+3*g^4/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(
1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2
)^(1/2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+
f/g))*d^2*e*f+1/3/g/(d^2*g^2-e^2*f^2)^2*e^5*f^4/d^2/(-(x+f/g)^2*e^2+2*e^2*f
/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+2/3/g/(d^2*g^2-e^2*f^2)^2*e^5*f^4
/d^4/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-g^4/(
d^2*g^2-e^2*f^2)^3*f*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/
g^2)^(1/2)*e^2*x-3*g^2/(d^2*g^2-e^2*f^2)^3*f^3/d/(-(x+f/g)^2*e^2+2*e^2*f/g*
(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*e^4*x-4/5*e^2/g^2/d^3*f*x/(-e^2*x^2+d^
2)^(3/2)+3/5/g*e^3*f^2/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(5/2)*x+8/15*e^3/g^3*f^2/d^6*x/(-e^2*x^2+d^2)^(1/2)-
3/5*e^2/g^2/d*f*x/(-e^2*x^2+d^2)^(5/2)-8/5*e^2/g^2/d^5*f*x/(-e^2*x^2+d^2)^(
1/2)+1/5*e^3/g^3*f^2*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/15*e^3/g^3*f^2/d^4*x/(-e^
2*x^2+d^2)^(3/2)+3/5/g/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d*e^2*f^2-g^2/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*
e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d^2*e*f-1/5*e^2*f/(d^2*g
^2-e^2*f^2)*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2
)*x-8/15*e^2*f/(d^2*g^2-e^2*f^2)/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2
*g^2-e^2*f^2)/g^2)^(1/2)*x-1/(d^2*g^2-e^2*f^2)^2*e^4*f^3/d/(-(x+f/g)^2*e^2+
2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x-2/(d^2*g^2-e^2*f^2)^2*e^4*
f^3/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-4/
15*e^2*f/(d^2*g^2-e^2*f^2)/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2
*f^2)/g^2)^(3/2)*x+g/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d*e^2*f^2-3*g^4/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^
2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*d^2*e*f+3*g^3/(d^2*g^2
-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*
d*e^2*f^2+g^2/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*
g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g
)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*e^3*f^3+3*
g^3/(d^2*g^2-e^2*f^2)^3*f^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*
f^2)/g^2)^(1/2)*e^3*x+g/(d^2*g^2-e^2*f^2)^2*e^3*f^2/(-(x+f/g)^2*e^2+2*e^2*f
/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+1/5*g/(d^2*g^2-e^2*f^2)/(-(x+f/g)
^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d^3+1/3*g^3/(d^2*g^2-
e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d
^3+g^5/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f
^2)/g^2)^(1/2)*d^3-1/3/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g
)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*e^3*f^3+4/5*e/g*x/(-e^2*x^2+d^2)^(5/2)-1/5*e
/g^2/(-e^2*x^2+d^2)^(5/2)*f+3/5/g/(-e^2*x^2+d^2)^(5/2)*d+8/15/g^3*e^5*f^4/(
d^2*g^2-e^2*f^2)/d^6/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^
2)^(1/2)*x-1/3*g^2/(d^2*g^2-e^2*f^2)^2*e^2*f*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x
+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+11/15*e/g/d^2*x/(-e^2*x^2+d^2)^(3/2)+2
2/15*e/g/d^4*x/(-e^2*x^2+d^2)^(1/2)-3/5/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2
*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d^2*e*f-1/5/g^2/(d^2*g^2-e^2*
```

```
f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*e^3*f^3
-g^2/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2
)/g^2)^(1/2)*e^3*f^3-g^5/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*
ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/
2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))
*d^3-3*g^3/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2
-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2
*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*d*e^2*f^2-3/5
/g^2*e^4*f^3/(d^2*g^2-e^2*f^2)/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2
-e^2*f^2)/g^2)^(5/2)*x+1/5/g^3*e^5*f^4/(d^2*g^2-e^2*f^2)/d^2/(-(x+f/g)^2*e^
2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*x+4/5/g*e^3*f^2/(d^2*g^2-e
^2*f^2)/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*
x-2/3*g^2/(d^2*g^2-e^2*f^2)^2*e^2*f/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^
2*g^2-e^2*f^2)/g^2)^(1/2)*x+2*g/(d^2*g^2-e^2*f^2)^2*e^3*f^2/d^2/(-(x+f/g)^2
*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-4/5/g^2*e^4*f^3/(d^2*
g^2-e^2*f^2)/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(
3/2)*x+4/15/g^3*e^5*f^4/(d^2*g^2-e^2*f^2)/d^4/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+
f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+8/5/g*e^3*f^2/(d^2*g^2-e^2*f^2)/d^4/(-(
x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-8/5/g^2*e^4*f
^3/(d^2*g^2-e^2*f^2)/d^5/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2
)/g^2)^(1/2)*x
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for mo
re details)Is (d*g-e*f)    *(d*g+e*f)    positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\left(f+g\,x\right)\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)), x)
```

3.586 $\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=311

$$\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}-\frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^3}+\frac{\ldots}{5\left(d^2\ldots\right.}$$

```
[Out] 4/5*d*e*(e*x+d)/(d*g+e*f)^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(5*d*(-3*d*g+e*f)-e
*(21*d*g+e*f)*x)/d/(d*g+e*f)^3/(-e^2*x^2+d^2)^(3/2)+e*g^3*(-3*d*g+4*e*f)*ar
ctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(-d*g+e
*f)/(d*g+e*f)^4/(-d^2*g^2+e^2*f^2)^(1/2)+1/15*e*(45*d^3*g^2+e*(57*d^2*g^2+1
4*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^4/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+
d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g*x+f)
```

Rubi [A] time = 1.26, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = $\{1647, 807, 725, 204\}$

$$\frac{e\left(ex\left(57d^2g^2+14defg+2e^2f^2\right)+45d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}+\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d*(e*f -
3*d*g) - e*(e*f + 21*d*g)*x))/(15*d*(e*f + d*g)^3*(d^2 - e^2*x^2)^(3/2)) +
(e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f +
 d*g)^4*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(e*f
+ d*g)^4*(f + g*x)) + (e*g^3*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt
[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*(e*f + d*g)^4*Sqrt[
e^2*f^2 - d^2*g^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^2f^2+10defg+5d^2g^2\right)}{(ef+dg)^2} - \frac{d^2e^3(ef-5dg)(5ef+3dg)x}{(ef+dg)^2} + \frac{16d^3e^4g^2x^2}{(ef+dg)^2}}{(f+gx)^2\left(d^2-e^2x^2\right)^{5/2}}\,d}{5d^2e^2}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2e^3f^3+12\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\ldots\right.}{15d^3\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\ldots\right.}{15d^3\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\ldots\right.}{15d^3\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\ldots\right.}{15d^3\ldots}$$

Mathematica [A] time = 0.61, size = 341, normalized size = 1.10

$$\frac{15eg^3(4ef-3dg)\sqrt{e^2f^2-d^2g^2}\,\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\,\sqrt{e^2f^2-d^2g^2}}\right) + \frac{(d+ex)\left(e^2f^2-d^2g^2\right)\left(15d^6g^4-9d^5eg^3(8f+13gx)+d^4e^2g^2\left(38f^2+16\ldots\right.\right.}{\ldots}}{15(ef-dg)^2\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((e^2*f^2 - d^2*g^2)*(d + e*x)*(15*d^6*g^4 + 2*e^6*f^3*x^2*(f + g*x) - 9*d
^5*e*g^3*(8*f + 13*g*x) + 6*d*e^5*f^2*x*(-f^2 + f*g*x + 2*g^2*x^2) + d^4*e^
2*g^2*(38*f^2 + 164*f*g*x + 171*g^2*x^2) - 3*d^3*e^3*g*(-9*f^3 + 19*f^2*g*x
 + 47*f*g^2*x^2 + 24*g^3*x^3) + d^2*e^4*f*(7*f^3 - 29*f^2*g*x + 7*f*g^2*x^2
 + 43*g^3*x^3)))/(d^3*(d - e*x)^2*(f + g*x)*Sqrt[d^2 - e^2*x^2]) + 15*e*g^3
*(4*e*f - 3*d*g)*Sqrt[e^2*f^2 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2
*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/(15*(e*f - d*g)^2*(e*f + d*g)^5)
```

fricas [B] time = 2.09, size = 3305, normalized size = 10.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*e^4*f^5*g^2 - 99*d^6*e^3*f
^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 - 15*d^9*f*g^6 - (7*e^9*f^6*
g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^3*e^6*f^3*g^4 - 23*d^4*e^5
*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4 - (7*e^9*f^7 + 6*d*e^8*f^
6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 + 274*d^4*e^5*f^3*g^4 + 141*
d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*g^7)*x^3 + 3*(7*d*e^8*f^7
+ 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*e^5*f^4*g^3 + 76*d^5*e^4*f
^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 + 15*d^8*e*g^7)*x^2 - 15*(4*
d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^2*g^4 - 3*d^4*e^4*f*g^5)*x
^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*d^5*e^3*f*g^5)*x^3 + 3*(4*
d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f*g^5)*x^2 - (12*d^5*e^3*f^
3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log
((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(
-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f))
 - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5*g^2 - 328*d^5*e^4*f^
4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117*d^8*e*f*g^6 + 15*d^9
*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e^4*f^5*g^2 - 99*d^5*e
^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 15*d^8*f*g^6 + (2*e^8*
f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3*e^5*f^3*g^4 - 43*d^4
*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*d*e^7*f^6*g + 5*d^2*e
^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4 + 141*d^5*e^3*f^2*g^
5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6*f^6*g + 51*d^3*e^5*f
^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 164*d^6*e^2*f^2*g^5 - 1
17*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9 + 3*d^7*e^6*f^8*g + d
^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^4 + d^11*e^2*f^4*g^5
+ 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g + 3*d^4*e^9*f^7*g^2 + d
^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5 + d^8*e^5*f^3*g^6 +
3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^9 - 8*d^5*e^8*f^7*g^2
 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5*f^4*g^5 - 8*d^10*e^3
*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*d^5*e^8*f^8*g - 2*d^6
*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 + 2*d^10*e^3*f^3*g^6 -
 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f^9 + 8*d^6*e^7*f^8*g
- 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3*f^4*g^5 + 8*d^11*e^2
*f^3*g^6 - d^13*f*g^8)*x), 1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*
e^4*f^5*g^2 - 99*d^6*e^3*f^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 -
15*d^9*f*g^6 - (7*e^9*f^6*g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^
3*e^6*f^3*g^4 - 23*d^4*e^5*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4
 - (7*e^9*f^7 + 6*d*e^8*f^6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 +
274*d^4*e^5*f^3*g^4 + 141*d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*
g^7)*x^3 + 3*(7*d*e^8*f^7 + 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*
e^5*f^4*g^3 + 76*d^5*e^4*f^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 +
15*d^8*e*g^7)*x^2 + 30*(4*d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^
2*g^4 - 3*d^4*e^4*f*g^5)*x^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*
d^5*e^3*f*g^5)*x^3 + 3*(4*d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f
*g^5)*x^2 - (12*d^5*e^3*f^3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sq
rt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e
^2*f^2 - d^2*g^2)*x)) - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5
*g^2 - 328*d^5*e^4*f^4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117
*d^8*e*f*g^6 + 15*d^9*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e
^4*f^5*g^2 - 99*d^5*e^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 1
5*d^8*f*g^6 + (2*e^8*f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3
*e^5*f^3*g^4 - 43*d^4*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*
d*e^7*f^6*g + 5*d^2*e^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4
 + 141*d^5*e^3*f^2*g^5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6
*f^6*g + 51*d^3*e^5*f^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 16
4*d^6*e^2*f^2*g^5 - 117*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9
```

```
+ 3*d^7*e^6*f^8*g + d^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^
4 + d^11*e^2*f^4*g^5 + 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g +
3*d^4*e^9*f^7*g^2 + d^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5
 + d^8*e^5*f^3*g^6 + 3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^
9 - 8*d^5*e^8*f^7*g^2 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5
*f^4*g^5 - 8*d^10*e^3*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*
d^5*e^8*f^8*g - 2*d^6*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 +
 2*d^10*e^3*f^3*g^6 - 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f
^9 + 8*d^6*e^7*f^8*g - 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3
*f^4*g^5 + 8*d^11*e^2*f^3*g^6 - d^13*f*g^8)*x)]
```

giac [C] time = 2.97, size = 4343, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(-45*I*d^9*g^12*e^6*log(d^2*g^4*e^2) - 75*I*d^8*f*g^11*e^7*log(d^
2*g^4*e^2) + 90*I*d^7*f^2*g^10*e^8*log(d^2*g^4*e^2) + 144*sqrt(d^2*g^2 - f^
2*e^2)*d^8*g^10*abs(g)*e^6 + 210*I*d^6*f^3*g^9*e^9*log(d^2*g^4*e^2) + 346*s
qrt(d^2*g^2 - f^2*e^2)*d^7*f*g^9*abs(g)*e^7 + 15*I*d^5*f^4*g^8*e^10*log(d^2
*g^4*e^2) + 6*sqrt(d^2*g^2 - f^2*e^2)*d^6*f^2*g^8*abs(g)*e^8 - 135*I*d^4*f^
5*g^7*e^11*log(d^2*g^4*e^2) - 536*sqrt(d^2*g^2 - f^2*e^2)*d^5*f^3*g^7*abs(g
)*e^9 - 60*I*d^3*f^6*g^6*e^12*log(d^2*g^4*e^2) - 320*sqrt(d^2*g^2 - f^2*e^2
)*d^4*f^4*g^6*abs(g)*e^10 + 154*sqrt(d^2*g^2 - f^2*e^2)*d^3*f^5*g^5*abs(g)*
e^11 + 166*sqrt(d^2*g^2 - f^2*e^2)*d^2*f^6*g^4*abs(g)*e^12 + 36*sqrt(d^2*g^
2 - f^2*e^2)*d*f^7*g^3*abs(g)*e^13 + 4*sqrt(d^2*g^2 - f^2*e^2)*f^8*g^2*abs(
g)*e^14)*sgn(1/(g*x + f))*sgn(g)/(30*I*sqrt(d^2*g^2 - f^2*e^2)*d^13*g^10*ab
s(g)*e^5 + 180*I*sqrt(d^2*g^2 - f^2*e^2)*d^12*f*g^9*abs(g)*e^6 + 390*I*sqrt
(d^2*g^2 - f^2*e^2)*d^11*f^2*g^8*abs(g)*e^7 + 240*I*sqrt(d^2*g^2 - f^2*e^2)
*d^10*f^3*g^7*abs(g)*e^8 - 420*I*sqrt(d^2*g^2 - f^2*e^2)*d^9*f^4*g^6*abs(g)
*e^9 - 840*I*sqrt(d^2*g^2 - f^2*e^2)*d^8*f^5*g^5*abs(g)*e^10 - 420*I*sqrt(d
^2*g^2 - f^2*e^2)*d^7*f^6*g^4*abs(g)*e^11 + 240*I*sqrt(d^2*g^2 - f^2*e^2)*d
^6*f^7*g^3*abs(g)*e^12 + 390*I*sqrt(d^2*g^2 - f^2*e^2)*d^5*f^8*g^2*abs(g)*e
^13 + 180*I*sqrt(d^2*g^2 - f^2*e^2)*d^4*f^9*g*abs(g)*e^14 + 30*I*sqrt(d^2*g
^2 - f^2*e^2)*d^3*f^10*abs(g)*e^15) + 15*(3*d*g^7*e - 4*f*g^6*e^2)*log(abs(
f*g*e^2 + sqrt(d^2*g^2 - f^2*e^2)*(sqrt(d^2*g^2/(g*x + f)^2 + 2*f*e^2/(g*x
+ f) - f^2*e^2/(g*x + f)^2 - e^2) + sqrt(d^2*g^4 - f^2*g^2*e^2)/((g*x + f)*
g))*abs(g)))/(sqrt(d^2*g^2 - f^2*e^2)*d^5*g^5*abs(g)*sgn(1/(g*x + f))*sgn(g
) + 3*sqrt(d^2*g^2 - f^2*e^2)*d^4*f*g^4*abs(g)*e*sgn(1/(g*x + f))*sgn(g) +
2*sqrt(d^2*g^2 - f^2*e^2)*d^3*f^2*g^3*abs(g)*e^2*sgn(1/(g*x + f))*sgn(g) -
2*sqrt(d^2*g^2 - f^2*e^2)*d^2*f^3*g^2*abs(g)*e^3*sgn(1/(g*x + f))*sgn(g) -
3*sqrt(d^2*g^2 - f^2*e^2)*d*f^4*g*abs(g)*e^4*sgn(1/(g*x + f))*sgn(g) - sqrt
(d^2*g^2 - f^2*e^2)*f^5*abs(g)*e^5*sgn(1/(g*x + f))*sgn(g)) - ((72*d^8*g^24
*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 - 187*d^7*f*g^23*e^11*sgn(1/(g*x + f))^3*
sgn(g)^3 + 146*d^6*f^2*g^22*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 - 21*d^5*f^3*g
^21*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 - 8*d^4*f^4*g^20*e^14*sgn(1/(g*x + f))
^3*sgn(g)^3 - 2*d^3*f^5*g^19*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e
^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^
4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*s
gn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4
 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1
/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) + (
5*(9*d^9*g^26*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 - 102*d^8*f*g^25*e^10*sgn(1/(
g*x + f))^3*sgn(g)^3 + 220*d^7*f^2*g^24*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 -
158*d^6*f^3*g^23*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 + 21*d^5*f^4*g^22*e^13*sg
n(1/(g*x + f))^3*sgn(g)^3 + 8*d^4*f^5*g^21*e^14*sgn(1/(g*x + f))^3*sgn(g)^3
 + 2*d^3*f^6*g^20*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g
*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*
```

f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) - (5*(36*d^10*g^28*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 - 53*d^9*f*g^27*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 - 206*d^8*f^2*g^26*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 + 512*d^7*f^3*g^25*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 - 350*d^6*f^4*g^24*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 + 41*d^5*f^5*g^23*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 + 16*d^4*f^6*g^22*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 + 4*d^3*f^7*g^21*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) + (5*(21*d^11*g^30*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 - 178*d^10*f*g^29*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 + 287*d^9*f^2*g^28*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 + 132*d^8*f^3*g^27*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 - 601*d^7*f^4*g^26*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 + 398*d^6*f^5*g^25*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 - 39*d^5*f^6*g^24*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 - 16*d^4*f^7*g^23*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 - 4*d^3*f^8*g^22*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) - (5*(27*d^12*g^32*e^6*sgn(1/(g*x + f))^3*sgn(g)^3 - 18*d^11*f*g^31*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 - 227*d^10*f^2*g^30*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 + 406*d^9*f^3*g^29*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 - 27*d^8*f^4*g^28*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 - 368*d^7*f^5*g^27*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 + 235*d^6*f^6*g^26*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 - 18*d^5*f^7*g^25*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 - 8*d^4*f^8*g^24*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 - 2*d^3*f^9*g^23*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) + (2*(36*d^13*g^34*e^5*sgn(1/(g*x + f))^3*sgn(g)^3 - 181*d^12*f*g^33*e^6*sgn(1/(g*x + f))^3*sgn(g)^3 + 203*d^11*f^2*g^32*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 + 217*d^10*f^3*g^31*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 - 504*d^9*f^4*g^30*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 + 113*d^8*f^5*g^29*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 + 256*d^7*f^6*g^28*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 - 153*d^6*f^7*g^27*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 + 8*d^5*f^8*g^26*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 + 4*d^4*f^9*g^25*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 + d^3*f^10*g^24*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) - 15*(d^14*g^36*e^4*sgn(1/(g*x + f))^3*sgn(g)^3 - 2*d^13*f*g^35*e^5*sgn(1/(g*x + f))^3*sgn(g)^3 - 2*d^12*f^2*g^34*e^6*sgn(1/(g*x + f))^3*sgn(g)^3 + 6*d^11*f^3*g^33*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 - 6*d^9*f^5*g^31*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 + 2*d^8*f^6*g^30*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 + 2*d^7*f^7*g^29*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 - d^6*f^8*g^28*e^12*sgn(1/(g*x + f))^3*sgn(g)^3)/((d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g

```
*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f
^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4)*(g*x + f)*g))/((g*x + f)*g))/((g*
x + f)*g))/((g*x + f)*g))/((g*x + f)*g))/((g*x + f)*g))/(d^2*g^2/(g*x + f)^
2 + 2*f*e^2/(g*x + f) - f^2*e^2/(g*x + f)^2 - e^2)^(5/2))/g^2
```

maple [B] time = 0.03, size = 6760, normalized size = 21.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for mo
re details)Is (d*g-e*f)    *(d*g+e*f)    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\left(f+g\,x\right)^2\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**2), x)
```

3.587 $\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=398

$$\frac{e^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\,\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}}+\frac{3eg^4\sqrt{d^2-e^2x^2}\,(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5}+\frac{g^4\sqrt{d^2-e}}{2(f+gx)^2(ef-d}$$

```
[Out] 4/5*d*e^2*(e*x+d)/(d*g+e*f)^3/(-e^2*x^2+d^2)^(5/2)-1/15*e^2*(5*d*(-5*d*g+e*
f)-e*(31*d*g+e*f)*x)/d/(d*g+e*f)^4/(-e^2*x^2+d^2)^(3/2)+1/2*e^2*g^3*(13*d^2
*g^2-30*d*e*f*g+20*e^2*f^2)*arctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2))/(-d*g+e*f)^2/(d*g+e*f)^5/(-d^2*g^2+e^2*f^2)^(1/2)+1/
15*e^2*(90*d^3*g^2+e*(107*d^2*g^2+19*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^5/
(-e^2*x^2+d^2)^(1/2)+1/2*g^4*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g
*x+f)^2+3/2*e*g^4*(-2*d*g+3*e*f)*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)^2/(d*g+e*f
)^5/(g*x+f)
```

Rubi [A] time = 2.57, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1647, 1651, 807, 725, 204}

$$\frac{e^2\left(ex\left(107d^2g^2+19defg+2e^2f^2\right)+90d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}\,(dg+ef)^5}+\frac{e^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\,\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}}+\frac{3}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) - (e^2*(5*d*(e*
f - 5*d*g) - e*(e*f + 31*d*g)*x))/(15*d*(e*f + d*g)^4*(d^2 - e^2*x^2)^(3/2)
) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3
*(e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/(2*(e*f - d
*g)*(e*f + d*g)^4*(f + g*x)^2) + (3*e*g^4*(3*e*f - 2*d*g)*Sqrt[d^2 - e^2*x^
2])/(2*(e*f - d*g)^2*(e*f + d*g)^5*(f + g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d
*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/(2*(e*f - d*g)^2*(e*f + d*g)^5*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^3f^3+15de^2f^2g+15d^2efg^2+5d^3g^3\right)}{(ef+dg)^3} - \frac{d^2e^3\left(5e^3f^3-33de^2f^2g-45d\cdots\right.}{(ef+dg)^3}}{(f+gx)^3\left(d^2-e^2x^2\cdots\right.}}{5d^2e^2}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2e^4f^4+1\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2\cdots\right.}{15\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2\cdots\right.}{15\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2\cdots\right.}{15\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2\cdots\right.}{15\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2\cdots\right.}{15\cdots}$$

Mathematica [C] time = 1.14, size = 387, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2e^2(dg+ef)(17dg+2ef)}{d^2(d-ex)^2}+\frac{2e^2\left(107d^2g^2+19defg+2e^2f^2\right)}{d^3(d-ex)}+\frac{6e^2(dg+ef)^2}{d(d-ex)^3}+\frac{45eg^4(3ef-2dg)}{(f+gx)(ef-dg)^2}+\frac{15g^4(dg+ef)}{(f+gx)^2(ef-dg)}\right)-\frac{15ie^2g^3\left(13d^2\cdots\right.}{\cdots}}{30(dg+ef)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((6*e^2*(e*f + d*g)^2)/(d*(d - e*x)^3) + (2*e^2*(e*f +
 d*g)*(2*e*f + 17*d*g))/(d^2*(d - e*x)^2) + (2*e^2*(2*e^2*f^2 + 19*d*e*f*g
+ 107*d^2*g^2))/(d^3*(d - e*x)) + (15*g^4*(e*f + d*g))/((e*f - d*g)*(f + g*
x)^2) + (45*e*g^4*(3*e*f - 2*d*g))/((e*f - d*g)^2*(f + g*x))) - ((15*I)*e^2
*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*Log[(4*(e*f - d*g)^2*(e*f + d*g
)^5*(I*d^2*g + I*e^2*f*x + Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2]))/(e
^2*g^2*Sqrt[e^2*f^2 - d^2*g^2]*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*(f +
g*x))])/((e*f - d*g)^2*Sqrt[e^2*f^2 - d^2*g^2]))/(30*(e*f + d*g)^5)
```

fricas [B] time = 7.15, size = 5361, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(14*d^3*e^8*f^10 + 60*d^4*e^7*f^9*g + 78*d^5*e^6*f^8*g^2 - 480*d^6*e^
5*f^7*g^3 + 312*d^7*e^4*f^6*g^4 + 330*d^8*e^3*f^5*g^5 - 419*d^9*e^2*f^4*g^6
 + 90*d^10*e*f^3*g^7 + 15*d^11*f^2*g^8 - (14*e^11*f^8*g^2 + 60*d*e^10*f^7*g
^3 + 78*d^2*e^9*f^6*g^4 - 480*d^3*e^8*f^5*g^5 + 312*d^4*e^7*f^4*g^6 + 330*d
^5*e^6*f^3*g^7 - 419*d^6*e^5*f^2*g^8 + 90*d^7*e^4*f*g^9 + 15*d^8*e^3*g^10)*
x^5 - (28*e^11*f^9*g + 78*d*e^10*f^8*g^2 - 24*d^2*e^9*f^7*g^3 - 1194*d^3*e^
8*f^6*g^4 + 2064*d^4*e^7*f^5*g^5 - 276*d^5*e^6*f^4*g^6 - 1828*d^6*e^5*f^3*g
^7 + 1437*d^7*e^4*f^2*g^8 - 240*d^8*e^3*f*g^9 - 45*d^9*e^2*g^10)*x^4 - (14*
e^11*f^10 - 24*d*e^10*f^9*g - 240*d^2*e^9*f^8*g^2 - 768*d^3*e^8*f^7*g^3 + 3
426*d^4*e^7*f^6*g^4 - 2982*d^5*e^6*f^5*g^5 - 1463*d^6*e^5*f^4*g^6 + 3594*d^
7*e^4*f^3*g^7 - 1782*d^8*e^3*f^2*g^8 + 180*d^9*e^2*f*g^9 + 45*d^10*e*g^10)*
x^3 + (42*d*e^10*f^10 + 96*d^2*e^9*f^9*g - 112*d^3*e^8*f^8*g^2 - 1848*d^4*e
^7*f^7*g^3 + 3894*d^5*e^6*f^6*g^4 - 1362*d^6*e^5*f^5*g^5 - 2925*d^7*e^4*f^4
*g^6 + 3114*d^8*e^3*f^3*g^7 - 914*d^9*e^2*f^2*g^8 + 15*d^11*g^10)*x^2 - 15*
(20*d^6*e^4*f^6*g^3 - 30*d^7*e^3*f^5*g^4 + 13*d^8*e^2*f^4*g^5 - (20*d^3*e^7
*f^4*g^5 - 30*d^4*e^6*f^3*g^6 + 13*d^5*e^5*f^2*g^7)*x^5 - (40*d^3*e^7*f^5*g
^4 - 120*d^4*e^6*f^4*g^5 + 116*d^5*e^5*f^3*g^6 - 39*d^6*e^4*f^2*g^7)*x^4 -
(20*d^3*e^7*f^6*g^3 - 150*d^4*e^6*f^5*g^4 + 253*d^5*e^5*f^4*g^5 - 168*d^6*e
^4*f^3*g^6 + 39*d^7*e^3*f^2*g^7)*x^3 + (60*d^4*e^6*f^6*g^3 - 210*d^5*e^5*f^
5*g^4 + 239*d^6*e^4*f^4*g^5 - 108*d^7*e^3*f^3*g^6 + 13*d^8*e^2*f^2*g^7)*x^2
 - (60*d^5*e^5*f^6*g^3 - 130*d^6*e^4*f^5*g^4 + 99*d^7*e^3*f^4*g^5 - 26*d^8*
e^2*f^3*g^6)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log((d*e^2*f*g*x + d^3*g^2 - sqrt(
-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2
 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f)) - (42*d^2*e^9*f^10 + 152*d^3*e
^8*f^9*g + 114*d^4*e^7*f^8*g^2 - 1596*d^5*e^6*f^7*g^3 + 1896*d^6*e^5*f^6*g^
4 + 366*d^7*e^4*f^5*g^5 - 1917*d^8*e^3*f^4*g^6 + 1108*d^9*e^2*f^3*g^7 - 135
*d^10*e*f^2*g^8 - 30*d^11*f*g^9)*x + (14*d^2*e^8*f^10 + 60*d^3*e^7*f^9*g +
78*d^4*e^6*f^8*g^2 - 480*d^5*e^5*f^7*g^3 + 312*d^6*e^4*f^6*g^4 + 330*d^7*e^
3*f^5*g^5 - 419*d^8*e^2*f^4*g^6 + 90*d^9*e*f^3*g^7 + 15*d^10*f^2*g^8 + (4*e
^10*f^8*g^2 + 30*d*e^9*f^7*g^3 + 138*d^2*e^8*f^6*g^4 - 555*d^3*e^7*f^5*g^5
+ 162*d^4*e^6*f^4*g^6 + 525*d^5*e^5*f^3*g^7 - 304*d^6*e^4*f^2*g^8)*x^4 + (8
*e^10*f^9*g + 48*d*e^9*f^8*g^2 + 186*d^2*e^8*f^7*g^3 - 1224*d^3*e^7*f^6*g^4
```

+ 1539*d^4*e^6*f^5*g^5 + 459*d^5*e^5*f^4*g^6 - 1733*d^6*e^4*f^3*g^7 + 717*
d^7*e^3*f^2*g^8)*x^3 + (4*e^10*f^10 + 6*d*e^9*f^9*g - 28*d^2*e^8*f^8*g^2 -
828*d^3*e^7*f^7*g^3 + 2400*d^4*e^6*f^6*g^4 - 1197*d^5*e^5*f^5*g^5 - 1897*d^
6*e^4*f^4*g^6 + 2019*d^7*e^3*f^3*g^7 - 479*d^8*e^2*f^2*g^8)*x^2 - (12*d*e^9
*f^10 + 62*d^2*e^8*f^9*g + 114*d^3*e^7*f^8*g^2 - 1056*d^4*e^6*f^7*g^3 + 162
6*d^5*e^5*f^6*g^4 + 81*d^6*e^4*f^5*g^5 - 1707*d^7*e^3*f^4*g^6 + 913*d^8*e^2
*f^3*g^7 - 45*d^9*e*f^2*g^8)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^9*f^13 + 3*d^7
*e^8*f^12*g - 8*d^9*e^6*f^10*g^3 - 6*d^10*e^5*f^9*g^4 + 6*d^11*e^4*f^8*g^5
+ 8*d^12*e^3*f^7*g^6 - 3*d^14*e*f^5*g^8 - d^15*f^4*g^9 - (d^3*e^12*f^11*g^2
+ 3*d^4*e^11*f^10*g^3 - 8*d^6*e^9*f^8*g^5 - 6*d^7*e^8*f^7*g^6 + 6*d^8*e^7*
f^6*g^7 + 8*d^9*e^6*f^5*g^8 - 3*d^11*e^4*f^3*g^10 - d^12*e^3*f^2*g^11)*x^5
- (2*d^3*e^12*f^12*g + 3*d^4*e^11*f^11*g^2 - 9*d^5*e^10*f^10*g^3 - 16*d^6*e
^9*f^9*g^4 + 12*d^7*e^8*f^8*g^5 + 30*d^8*e^7*f^7*g^6 - 2*d^9*e^6*f^6*g^7 -
24*d^10*e^5*f^5*g^8 - 6*d^11*e^4*f^4*g^9 + 7*d^12*e^3*f^3*g^10 + 3*d^13*e^2
*f^2*g^11)*x^4 - (d^3*e^12*f^13 - 3*d^4*e^11*f^12*g - 15*d^5*e^10*f^11*g^2
+ d^6*e^9*f^10*g^3 + 42*d^7*e^8*f^9*g^4 + 18*d^8*e^7*f^8*g^5 - 46*d^9*e^6*f
^7*g^6 - 30*d^10*e^5*f^6*g^7 + 21*d^11*e^4*f^5*g^8 + 17*d^12*e^3*f^4*g^9 -
3*d^13*e^2*f^3*g^10 - 3*d^14*e*f^2*g^11)*x^3 + (3*d^4*e^11*f^13 + 3*d^5*e^1
0*f^12*g - 17*d^6*e^9*f^11*g^2 - 21*d^7*e^8*f^10*g^3 + 30*d^8*e^7*f^9*g^4 +
46*d^9*e^6*f^8*g^5 - 18*d^10*e^5*f^7*g^6 - 42*d^11*e^4*f^6*g^7 - d^12*e^3*
f^5*g^8 + 15*d^13*e^2*f^4*g^9 + 3*d^14*e*f^3*g^10 - d^15*f^2*g^11)*x^2 - (3
*d^5*e^10*f^13 + 7*d^6*e^9*f^12*g - 6*d^7*e^8*f^11*g^2 - 24*d^8*e^7*f^10*g^
3 - 2*d^9*e^6*f^9*g^4 + 30*d^10*e^5*f^8*g^5 + 12*d^11*e^4*f^7*g^6 - 16*d^12
*e^3*f^6*g^7 - 9*d^13*e^2*f^5*g^8 + 3*d^14*e*f^4*g^9 + 2*d^15*f^3*g^10)*x),
1/30*(14*d^3*e^8*f^10 + 60*d^4*e^7*f^9*g + 78*d^5*e^6*f^8*g^2 - 480*d^6*e^
5*f^7*g^3 + 312*d^7*e^4*f^6*g^4 + 330*d^8*e^3*f^5*g^5 - 419*d^9*e^2*f^4*g^6
+ 90*d^10*e*f^3*g^7 + 15*d^11*f^2*g^8 - (14*e^11*f^8*g^2 + 60*d*e^10*f^7*g
^3 + 78*d^2*e^9*f^6*g^4 - 480*d^3*e^8*f^5*g^5 + 312*d^4*e^7*f^4*g^6 + 330*d
^5*e^6*f^3*g^7 - 419*d^6*e^5*f^2*g^8 + 90*d^7*e^4*f*g^9 + 15*d^8*e^3*g^10)*
x^5 - (28*e^11*f^9*g + 78*d*e^10*f^8*g^2 - 24*d^2*e^9*f^7*g^3 - 1194*d^3*e^
8*f^6*g^4 + 2064*d^4*e^7*f^5*g^5 - 276*d^5*e^6*f^4*g^6 - 1828*d^6*e^5*f^3*g
^7 + 1437*d^7*e^4*f^2*g^8 - 240*d^8*e^3*f*g^9 - 45*d^9*e^2*g^10)*x^4 - (14*
e^11*f^10 - 24*d*e^10*f^9*g - 240*d^2*e^9*f^8*g^2 - 768*d^3*e^8*f^7*g^3 + 3
426*d^4*e^7*f^6*g^4 - 2982*d^5*e^6*f^5*g^5 - 1463*d^6*e^5*f^4*g^6 + 3594*d^
7*e^4*f^3*g^7 - 1782*d^8*e^3*f^2*g^8 + 180*d^9*e^2*f*g^9 + 45*d^10*e*g^10)*
x^3 + (42*d*e^10*f^10 + 96*d^2*e^9*f^9*g - 112*d^3*e^8*f^8*g^2 - 1848*d^4*e
^7*f^7*g^3 + 3894*d^5*e^6*f^6*g^4 - 1362*d^6*e^5*f^5*g^5 - 2925*d^7*e^4*f^4
*g^6 + 3114*d^8*e^3*f^3*g^7 - 914*d^9*e^2*f^2*g^8 + 15*d^11*g^10)*x^2 + 30*
(20*d^6*e^4*f^6*g^3 - 30*d^7*e^3*f^5*g^4 + 13*d^8*e^2*f^4*g^5 - (20*d^3*e^7
*f^4*g^5 - 30*d^4*e^6*f^3*g^6 + 13*d^5*e^5*f^2*g^7)*x^5 - (40*d^3*e^7*f^5*g
^4 - 120*d^4*e^6*f^4*g^5 + 116*d^5*e^5*f^3*g^6 - 39*d^6*e^4*f^2*g^7)*x^4 -
(20*d^3*e^7*f^6*g^3 - 150*d^4*e^6*f^5*g^4 + 253*d^5*e^5*f^4*g^5 - 168*d^6*e
^4*f^3*g^6 + 39*d^7*e^3*f^2*g^7)*x^3 + (60*d^4*e^6*f^6*g^3 - 210*d^5*e^5*f^
5*g^4 + 239*d^6*e^4*f^4*g^5 - 108*d^7*e^3*f^3*g^6 + 13*d^8*e^2*f^2*g^7)*x^2
- (60*d^5*e^5*f^6*g^3 - 130*d^6*e^4*f^5*g^4 + 99*d^7*e^3*f^4*g^5 - 26*d^8*
e^2*f^3*g^6)*x)*sqrt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2
+ d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - (42*d^2*e^9*f^10 + 152*d^3*e^8*f^
9*g + 114*d^4*e^7*f^8*g^2 - 1596*d^5*e^6*f^7*g^3 + 1896*d^6*e^5*f^6*g^4 + 3
66*d^7*e^4*f^5*g^5 - 1917*d^8*e^3*f^4*g^6 + 1108*d^9*e^2*f^3*g^7 - 135*d^10
*e*f^2*g^8 - 30*d^11*f*g^9)*x + (14*d^2*e^8*f^10 + 60*d^3*e^7*f^9*g + 78*d^
4*e^6*f^8*g^2 - 480*d^5*e^5*f^7*g^3 + 312*d^6*e^4*f^6*g^4 + 330*d^7*e^3*f^5
*g^5 - 419*d^8*e^2*f^4*g^6 + 90*d^9*e*f^3*g^7 + 15*d^10*f^2*g^8 + (4*e^10*f
^8*g^2 + 30*d*e^9*f^7*g^3 + 138*d^2*e^8*f^6*g^4 - 555*d^3*e^7*f^5*g^5 + 162
*d^4*e^6*f^4*g^6 + 525*d^5*e^5*f^3*g^7 - 304*d^6*e^4*f^2*g^8)*x^4 + (8*e^10
*f^9*g + 48*d*e^9*f^8*g^2 + 186*d^2*e^8*f^7*g^3 - 1224*d^3*e^7*f^6*g^4 + 15
39*d^4*e^6*f^5*g^5 + 459*d^5*e^5*f^4*g^6 - 1733*d^6*e^4*f^3*g^7 + 717*d^7*e
^3*f^2*g^8)*x^3 + (4*e^10*f^10 + 6*d*e^9*f^9*g - 28*d^2*e^8*f^8*g^2 - 828*d
^3*e^7*f^7*g^3 + 2400*d^4*e^6*f^6*g^4 - 1197*d^5*e^5*f^5*g^5 - 1897*d^6*e^4
*f^4*g^6 + 2019*d^7*e^3*f^3*g^7 - 479*d^8*e^2*f^2*g^8)*x^2 - (12*d*e^9*f^10

```
 + 62*d^2*e^8*f^9*g + 114*d^3*e^7*f^8*g^2 - 1056*d^4*e^6*f^7*g^3 + 1626*d^5
*e^5*f^6*g^4 + 81*d^6*e^4*f^5*g^5 - 1707*d^7*e^3*f^4*g^6 + 913*d^8*e^2*f^3*
g^7 - 45*d^9*e*f^2*g^8)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^9*f^13 + 3*d^7*e^8*
f^12*g - 8*d^9*e^6*f^10*g^3 - 6*d^10*e^5*f^9*g^4 + 6*d^11*e^4*f^8*g^5 + 8*d
^12*e^3*f^7*g^6 - 3*d^14*e*f^5*g^8 - d^15*f^4*g^9 - (d^3*e^12*f^11*g^2 + 3*
d^4*e^11*f^10*g^3 - 8*d^6*e^9*f^8*g^5 - 6*d^7*e^8*f^7*g^6 + 6*d^8*e^7*f^6*g
^7 + 8*d^9*e^6*f^5*g^8 - 3*d^11*e^4*f^3*g^10 - d^12*e^3*f^2*g^11)*x^5 - (2*
d^3*e^12*f^12*g + 3*d^4*e^11*f^11*g^2 - 9*d^5*e^10*f^10*g^3 - 16*d^6*e^9*f^
9*g^4 + 12*d^7*e^8*f^8*g^5 + 30*d^8*e^7*f^7*g^6 - 2*d^9*e^6*f^6*g^7 - 24*d^
10*e^5*f^5*g^8 - 6*d^11*e^4*f^4*g^9 + 7*d^12*e^3*f^3*g^10 + 3*d^13*e^2*f^2*
g^11)*x^4 - (d^3*e^12*f^13 - 3*d^4*e^11*f^12*g - 15*d^5*e^10*f^11*g^2 + d^6
*e^9*f^10*g^3 + 42*d^7*e^8*f^9*g^4 + 18*d^8*e^7*f^8*g^5 - 46*d^9*e^6*f^7*g^
6 - 30*d^10*e^5*f^6*g^7 + 21*d^11*e^4*f^5*g^8 + 17*d^12*e^3*f^4*g^9 - 3*d^1
3*e^2*f^3*g^10 - 3*d^14*e*f^2*g^11)*x^3 + (3*d^4*e^11*f^13 + 3*d^5*e^10*f^1
2*g - 17*d^6*e^9*f^11*g^2 - 21*d^7*e^8*f^10*g^3 + 30*d^8*e^7*f^9*g^4 + 46*d
^9*e^6*f^8*g^5 - 18*d^10*e^5*f^7*g^6 - 42*d^11*e^4*f^6*g^7 - d^12*e^3*f^5*g
^8 + 15*d^13*e^2*f^4*g^9 + 3*d^14*e*f^3*g^10 - d^15*f^2*g^11)*x^2 - (3*d^5*
e^10*f^13 + 7*d^6*e^9*f^12*g - 6*d^7*e^8*f^11*g^2 - 24*d^8*e^7*f^10*g^3 - 2
*d^9*e^6*f^9*g^4 + 30*d^10*e^5*f^8*g^5 + 12*d^11*e^4*f^7*g^6 - 16*d^12*e^3*
f^6*g^7 - 9*d^13*e^2*f^5*g^8 + 3*d^14*e*f^4*g^9 + 2*d^15*f^3*g^10)*x)]
```

giac [B] time = 2.09, size = 6017, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(13*d^9*g^12*e^8 - 69*d^8*f*g^11*e^9 + 123*d^7*f^2*g^10*e^10 - 25*d^6*f^3*
g^9*e^11 - 195*d^5*f^4*g^8*e^12 + 237*d^4*f^5*g^7*e^13 - 31*d^3*f^6*g^6*e^1
4 - 123*d^2*f^7*g^5*e^15 + 90*d*f^8*g^4*e^16 - 20*f^9*g^3*e^17)*arctan((d*g
*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f/x)/sqrt(-d^2*g^2*e^2 + f^2*e^4))/((d^
14*g^14*e^5 - 7*d^12*f^2*g^12*e^7 + 21*d^10*f^4*g^10*e^9 - 35*d^8*f^6*g^8*e
^11 + 35*d^6*f^8*g^6*e^13 - 21*d^4*f^10*g^4*e^15 + 7*d^2*f^12*g^2*e^17 - f^
14*e^19)*sqrt(-d^2*g^2*e^2 + f^2*e^4)) - 1/15*sqrt(-x^2*e^2 + d^2)*((((((10
7*d^28*g^27*e^11 + 2694*d^27*f*g^26*e^12 + 32577*d^26*f^2*g^25*e^13 + 25185
0*d^25*f^3*g^24*e^14 + 1397850*d^24*f^4*g^23*e^15 + 5929860*d^23*f^5*g^22*e
^16 + 19984470*d^22*f^6*g^21*e^17 + 54906060*d^21*f^7*g^20*e^18 + 125216025
*d^20*f^8*g^19*e^19 + 240109650*d^19*f^9*g^18*e^20 + 390736995*d^18*f^10*g^
17*e^21 + 543134190*d^17*f^11*g^16*e^22 + 647660220*d^16*f^12*g^15*e^23 + 6
64152600*d^15*f^13*g^14*e^24 + 586148100*d^14*f^14*g^13*e^25 + 444848520*d^
13*f^15*g^12*e^26 + 289619565*d^12*f^16*g^11*e^27 + 161082570*d^11*f^17*g^1
0*e^28 + 76070775*d^10*f^18*g^9*e^29 + 30246150*d^9*f^19*g^8*e^30 + 1001121
0*d^8*f^20*g^7*e^31 + 2717220*d^7*f^21*g^6*e^32 + 592710*d^6*f^22*g^5*e^33
+ 101100*d^5*f^23*g^4*e^34 + 12975*d^4*f^24*g^3*e^35 + 1182*d^3*f^25*g^2*e^
36 + 69*d^2*f^26*g*e^37 + 2*d*f^27*e^38)*x/(d^34*g^30*e^4 + 30*d^33*f*g^29*
e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*
e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f
^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 3
0045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*
f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e
^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 11975
9850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^1
9*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 +
5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*
g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^
27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34) +
 90*(d^29*g^27*e^10 + 25*d^28*f*g^26*e^11 + 300*d^27*f^2*g^25*e^12 + 2300*d
^26*f^3*g^24*e^13 + 12650*d^25*f^4*g^23*e^14 + 53130*d^24*f^5*g^22*e^15 + 1
77100*d^23*f^6*g^21*e^16 + 480700*d^22*f^7*g^20*e^17 + 1081575*d^21*f^8*g^1
```

9*e^18 + 2042975*d^20*f^9*g^18*e^19 + 3268760*d^19*f^10*g^17*e^20 + 4457400
*d^18*f^11*g^16*e^21 + 5200300*d^17*f^12*g^15*e^22 + 5200300*d^16*f^13*g^14
*e^23 + 4457400*d^15*f^14*g^13*e^24 + 3268760*d^14*f^15*g^12*e^25 + 2042975
*d^13*f^16*g^11*e^26 + 1081575*d^12*f^17*g^10*e^27 + 480700*d^11*f^18*g^9*e
^28 + 177100*d^10*f^19*g^8*e^29 + 53130*d^9*f^20*g^7*e^30 + 12650*d^8*f^21*
g^6*e^31 + 2300*d^7*f^22*g^5*e^32 + 300*d^6*f^23*g^4*e^33 + 25*d^5*f^24*g^3
*e^34 + d^4*f^25*g^2*e^35)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f
^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^2
9*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5
852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^1
0*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 +
 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*
d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g
^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30
045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^2
2*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 14250
6*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 43
5*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x - 5*(49*d^30*g
^27*e^9 + 1239*d^29*f*g^26*e^10 + 15051*d^28*f^2*g^25*e^11 + 116925*d^27*f^
3*g^24*e^12 + 652350*d^26*f^4*g^23*e^13 + 2782770*d^25*f^5*g^22*e^14 + 9434
370*d^24*f^6*g^21*e^15 + 26086830*d^23*f^7*g^20*e^16 + 59904075*d^22*f^8*g^
19*e^17 + 115728525*d^21*f^9*g^18*e^18 + 189852465*d^20*f^10*g^17*e^19 + 26
6218215*d^19*f^11*g^16*e^20 + 320487060*d^18*f^12*g^15*e^21 + 332076300*d^1
7*f^13*g^14*e^22 + 296417100*d^16*f^14*g^13*e^23 + 227773140*d^15*f^15*g^12
*e^24 + 150325815*d^14*f^16*g^11*e^25 + 84867585*d^13*f^17*g^10*e^26 + 4073
9325*d^12*f^18*g^9*e^27 + 16489275*d^11*f^19*g^8*e^28 + 5563470*d^10*f^20*g
^7*e^29 + 1540770*d^9*f^21*g^6*e^30 + 342930*d^8*f^22*g^5*e^31 + 59550*d^7*
f^23*g^4*e^32 + 7725*d^6*f^24*g^3*e^33 + 699*d^5*f^25*g^2*e^34 + 39*d^4*f^2
6*g*e^35 + d^3*f^27*e^36)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^
2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29
*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 58
52925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10
*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 +
119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d
^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^
13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 300
45015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22
*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506
*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435
*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x - 5*(41*d^31*g^
27*e^8 + 1029*d^30*f*g^26*e^9 + 12399*d^29*f^2*g^25*e^10 + 95475*d^28*f^3*g
^24*e^11 + 527550*d^27*f^4*g^23*e^12 + 2226630*d^26*f^5*g^22*e^13 + 7460970
*d^25*f^6*g^21*e^14 + 20363970*d^24*f^7*g^20*e^15 + 46090275*d^23*f^8*g^19*
e^16 + 87607575*d^22*f^9*g^18*e^17 + 141109485*d^21*f^10*g^17*e^18 + 193785
465*d^20*f^11*g^16*e^19 + 227773140*d^19*f^12*g^15*e^20 + 229556100*d^18*f^
13*g^14*e^21 + 198354300*d^17*f^14*g^13*e^22 + 146648460*d^16*f^15*g^12*e^2
3 + 92379615*d^15*f^16*g^11*e^24 + 49247715*d^14*f^17*g^10*e^25 + 21992025*
d^13*f^18*g^9*e^26 + 8102325*d^12*f^19*g^8*e^27 + 2406030*d^11*f^20*g^7*e^2
8 + 554070*d^10*f^21*g^6*e^29 + 91770*d^9*f^22*g^5*e^30 + 8850*d^8*f^23*g^4
*e^31 - 75*d^7*f^24*g^3*e^32 - 159*d^6*f^25*g^2*e^33 - 21*d^5*f^26*g*e^34 -
 d^4*f^27*e^35)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6
 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*
e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26
*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14
 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*
d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g
^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 +
86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14
*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26

+ 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*
g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*
g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x + 15*(10*d^32*g^27*e^7 +
255*d^31*f*g^26*e^8 + 3126*d^30*f^2*g^25*e^9 + 24525*d^29*f^3*g^24*e^10 + 1
38300*d^28*f^4*g^23*e^11 + 596850*d^27*f^5*g^22*e^12 + 2049300*d^26*f^6*g^2
1*e^13 + 5745630*d^25*f^7*g^20*e^14 + 13396350*d^24*f^8*g^19*e^15 + 2631832
5*d^23*f^9*g^18*e^16 + 43984050*d^22*f^10*g^17*e^17 + 62960775*d^21*f^11*g^
16*e^18 + 77558760*d^20*f^12*g^15*e^19 + 82461900*d^19*f^13*g^14*e^20 + 757
75800*d^18*f^14*g^13*e^21 + 60174900*d^17*f^15*g^12*e^22 + 41230950*d^16*f^
16*g^11*e^23 + 24299385*d^15*f^17*g^10*e^24 + 12257850*d^14*f^18*g^9*e^25 +
 5256075*d^13*f^19*g^8*e^26 + 1897500*d^12*f^20*g^7*e^27 + 569250*d^11*f^21
*g^6*e^28 + 139380*d^10*f^22*g^5*e^29 + 27150*d^9*f^23*g^4*e^30 + 4050*d^8*
f^24*g^3*e^31 + 435*d^7*f^25*g^2*e^32 + 30*d^6*f^26*g*e^33 + d^5*f^27*e^34)
/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^
3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^
28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12
+ 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14 + 54627300*d^2
3*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*
e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g^15*e^19 + 1454
22675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 + 86493225*d^16*f
^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24
 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*
f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 2740
5*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d
^5*f^29*g*e^33 + d^4*f^30*e^34))*x + (127*d^33*g^27*e^6 + 3219*d^32*f*g^26*
e^7 + 39207*d^31*f^2*g^25*e^8 + 305475*d^30*f^3*g^24*e^9 + 1709850*d^29*f^4
*g^23*e^10 + 7320210*d^28*f^5*g^22*e^11 + 24917970*d^27*f^6*g^21*e^12 + 692
13210*d^26*f^7*g^20*e^13 + 159750525*d^25*f^8*g^19*e^14 + 310412025*d^24*f^
9*g^18*e^15 + 512594445*d^23*f^10*g^17*e^16 + 724216065*d^22*f^11*g^16*e^17
 + 879445020*d^21*f^12*g^15*e^18 + 920453100*d^20*f^13*g^14*e^19 + 83130510
0*d^19*f^14*g^13*e^20 + 647660220*d^18*f^15*g^12*e^21 + 434485065*d^17*f^16
*g^11*e^22 + 250132245*d^16*f^17*g^10*e^23 + 122939025*d^15*f^18*g^9*e^24 +
 51213525*d^14*f^19*g^8*e^25 + 17904810*d^13*f^20*g^7*e^26 + 5183970*d^12*f
^21*g^6*e^27 + 1220610*d^11*f^22*g^5*e^28 + 227850*d^10*f^23*g^4*e^29 + 324
75*d^9*f^24*g^3*e^30 + 3327*d^8*f^25*g^2*e^31 + 219*d^7*f^26*g*e^32 + 7*d^6
*f^27*e^33)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6 + 4
060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*e^9
+ 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26*f^8
*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14 + 5
4627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*d^21
*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g^15*
e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 + 8649
3225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14*f^2
0*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26 + 20
35800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*g^5*
e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*g^2*
e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))/(x^2*e^2 - d^2)^3 + (2*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*d^10*g^13*e^3/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^9*f*g^12*e^6/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9*f*g^12*e^4/x^2
 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*f*g^12*e^2/x^3 + d^8*f^2*g^11*e^9
 + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*f^2*g^11*e^7/x - 51*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^2*d^8*f^2*g^11*e^5/x^2 + 3*d^7*f^3*g^10*e^10 - 79*(d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^7*f^3*g^10*e^8/x + 91*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*d^7*f^3*g^10*e^6/x^2 - 25*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^7*f^3*g^1
0*e^4/x^3 - 26*d^6*f^4*g^9*e^11 + 127*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*f^
4*g^9*e^9/x - 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*f^4*g^9*e^7/x^2 + 49*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^6*f^4*g^9*e^5/x^3 + 44*d^5*f^5*g^8*e^12
- 28*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5*f^5*g^8*e^10/x - 30*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*d^5*f^5*g^8*e^8/x^2 - 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3

```
*d^5*f^5*g^8*e^6/x^3 - 11*d^4*f^6*g^7*e^13 - 110*(d*e + sqrt(-x^2*e^2 + d^2
)*e)*d^4*f^6*g^7*e^11/x + 61*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*f^6*g^7*e
^9/x^2 - 38*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*f^6*g^7*e^7/x^3 - 37*d^3*f
^7*g^6*e^14 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*f^7*g^6*e^12/x - 57*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*f^7*g^6*e^10/x^2 + 39*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^3*d^3*f^7*g^6*e^8/x^3 + 36*d^2*f^8*g^5*e^15 - 29*(d*e + sqrt(-
x^2*e^2 + d^2)*e)*d^2*f^8*g^5*e^13/x + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
d^2*f^8*g^5*e^11/x^2 - 11*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*f^8*g^5*e^9/
x^3 - 10*d*f^9*g^4*e^16 - 10*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f^9*g^4*e^1
2/x^2)/((d^12*f^2*g^12*e^5 - 6*d^10*f^4*g^10*e^7 + 15*d^8*f^6*g^8*e^9 - 20*
d^6*f^8*g^6*e^11 + 15*d^4*f^10*g^4*e^13 - 6*d^2*f^12*g^2*e^15 + f^14*e^17)*
(2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*g*e^(-1)/x + f*e^2 + (d*e + sqrt(-x^2*e
^2 + d^2)*e)^2*f*e^(-2)/x^2)^2)
```

maple [B] time = 0.03, size = 9593, normalized size = 24.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for mo
re details)Is (d*g-e*f)    *(d*g+e*f)    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^3}{{\left(f+g\,x\right)}^3\,{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + e x\right)^{3}}{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{\frac{7}{2}} \left(f + g x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**3), x)
```

3.588 $\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}\,(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^2g}$$

```
[Out] -2*(a*g^2+c*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/g^(3/2)/(-d
*g+e*f)^(3/2)-2*(a*e^2+c*d^2)/e^2/(-d*g+e*f)/(e*x+d)^(1/2)+2*c*(e*x+d)^(1/2
)/e^2/g
```

Rubi [A] time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1261, 205}

$$-\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}\,(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/
(e^2*g) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]
])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cd^2+ae^2}{e^2}-\frac{2cdx^2}{e^2}+\frac{cx^4}{e^2}}{x^2\left(\frac{ef-dg}{e}+\frac{gx^2}{e}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cd^2+ae^2}{e(ef-dg)x^2}-\frac{e(cf^2+ag^2)}{g(-ef+dg)(-ef+dg-gx^2)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= -\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}+\frac{(2(cf^2+ag^2))\,\text{Subst}\left(\int \frac{1}{-ef+dg-gx^2}\,dx, x, \sqrt{\right.}}{g(ef-dg)}$$

$$= -\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2(cf^2+ag^2)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Mathematica [C] time = 0.09, size = 91, normalized size = 0.81

$$\frac{2c(ef-dg)(2dg+e(f+gx))-2e^2\left(ag^2+cf^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{g(d+ex)}{dg-ef}\right)}{e^2g^2\sqrt{d+ex}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (2*c*(e*f - d*g)*(2*d*g + e*(f + g*x)) - 2*e^2*(c*f^2 + a*g^2)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (g*(d + e*x))/(-(e*f) + d*g)])/(e^2*g^2*(e*f - d*g)*Sqr
t[d + e*x])
```

fricas [B] time = 0.91, size = 499, normalized size = 4.46

$$\left[\frac{\left(cde^2f^2+ade^2g^2+\left(ce^3f^2+ae^3g^2\right)x\right)\sqrt{-efg+dg^2}\,\log\left(\frac{egx-ef+2\,dg-2\,\sqrt{-efg+dg^2}\,\sqrt{ex+d}}{gx+f}\right)+2\left(cde^2f^2g-\left(3\,cd^2e\right.\right.}{de^4f^2g^2-2\,d^2e^3fg^3+d^3e^2g^4+\left(e^5f^2g^2-2\,de^4f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] [((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*sqrt(-e*f*g + d*g
^2)*log((e*g*x - e*f + 2*d*g - 2*sqrt(-e*f*g + d*g^2)*sqrt(e*x + d))/(g*x +
 f)) + 2*(c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g
^3 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4
*f^2*g^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d
^2*e^3*g^4)*x), 2*((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*
sqrt(e*f*g - d*g^2)*arctan(sqrt(e*f*g - d*g^2)*sqrt(e*x + d)/(e*g*x + d*g))
 + (c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g^3 + (
c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4*f^2*g
^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d^2*e^3
*g^4)*x)]
```

giac [A] time = 0.19, size = 116, normalized size = 1.04

$$\frac{2\sqrt{xe+d}\,ce^{(-2)}}{g}+\frac{2\left(cf^2+ag^2\right)\arctan\left(\frac{\sqrt{xe+d}\,g}{\sqrt{-dg^2+fge}}\right)}{\left(dg^2-fge\right)\sqrt{-dg^2+fge}}+\frac{2\left(cd^2+ae^2\right)}{\left(dge^2-fe^3\right)\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-2)/g + 2*(c*f^2 + a*g^2)*arctan(sqrt(x*e + d)*g/sqrt(
-d*g^2 + f*g*e))/((d*g^2 - f*g*e)*sqrt(-d*g^2 + f*g*e)) + 2*(c*d^2 + a*e^2)
/((d*g*e^2 - f*e^3)*sqrt(x*e + d))
```

maple [A] time = 0.02, size = 114, normalized size = 1.02

$$\frac{-\frac{2\left(a\,g^{2}+c\,f^{2}\right)e^{2}\operatorname{arctanh}\left(\frac{\sqrt{e x+d}\,g}{\sqrt{\left(d g-e f\right)g}}\right)}{\left(d g-e f\right)\sqrt{\left(d g-e f\right)g}\,g}+\frac{2\sqrt{e x+d}\,c}{g}-\frac{2\left(-a\,e^{2}-c\,d^{2}\right)}{\left(d g-e f\right)\sqrt{e x+d}}}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x)
```

```
[Out] 2/e^2*(c/g*(e*x+d)^(1/2)-e^2*(a*g^2+c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^(1/2)*
arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))-(-a*e^2-c*d^2)/(d*g-e*f)/(e*x+
d)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g*(d*g-e*f)>0)', see `assume?` for
more details)Is g*(d*g-e*f)    positive or negative?
```

mupad [B] time = 0.23, size = 124, normalized size = 1.11

$$\frac{2\,c\,\sqrt{d+e\,x}}{e^2\,g}+\frac{2\,\left(c\,g\,d^2+a\,g\,e^2\right)}{e^2\,g\,\left(d\,g-e\,f\right)\,\sqrt{d+e\,x}}+\frac{\mathrm{atan}\left(\frac{d\,g^{3/2}\,\sqrt{d+e\,x}\,1{}\mathrm{i}-e\,f\,\sqrt{g}\,\sqrt{d+e\,x}\,1{}\mathrm{i}}{{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,f^2+a\,g^2\right)\,2{}\mathrm{i}}{g^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)*(d + e*x)^(3/2)),x)
```

```
[Out] (atan((d*g^(3/2)*(d + e*x)^(1/2)*1i - e*f*g^(1/2)*(d + e*x)^(1/2)*1i)/(d*g
- e*f)^(3/2))*(a*g^2 + c*f^2)*2i)/(g^(3/2)*(d*g - e*f)^(3/2)) + (2*c*(d + e
*x)^(1/2))/(e^2*g) + (2*(a*e^2*g + c*d^2*g))/(e^2*g*(d*g - e*f)*(d + e*x)^(
1/2))
```

sympy [A] time = 87.88, size = 107, normalized size = 0.96

$$\frac{2c\sqrt{d+ex}}{e^{2}g}+\frac{2\left(ag^{2}+cf^{2}\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^{2}\sqrt{-\frac{dg-ef}{g}}\left(dg-ef\right)}+\frac{2\left(ae^{2}+cd^{2}\right)}{e^{2}\sqrt{d+ex}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)
```

```
[Out] 2*c*sqrt(d + e*x)/(e**2*g) + 2*(a*g**2 + c*f**2)*atan(sqrt(d + e*x)/sqrt(-(
d*g - e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(d*g - e*f)) + 2*(a*e**2 + c*d**2
)/(e**2*sqrt(d + e*x)*(d*g - e*f))
```

3.589 $\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=240

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}$$

```
[Out] 2/3*(-d*g+e*f)^2*(3*a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(3/2)/g^6-2/5*(-d*g
+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6+2/7*
e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*c*e
^2*(-3*d*g+5*e*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e
*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^6
```

Rubi [A] time = 0.34, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1153}

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*
a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*(3
*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6)
 + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c*e^3*
(f + g*x)^(11/2))/(11*g^6)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^3\left(cf^2+ag^2\right)}{g^5}+\frac{(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)x^2}{g^5}+\frac{(ef-dg)\left(-3ae^2g^2-c\left(10e^2f^2-\right.\right.}{g^5}\right.\right.}{}$$

$$= -\frac{2(ef-dg)^3\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)(f+gx}{3g^6}$$

Mathematica [A] time = 0.24, size = 207, normalized size = 0.86

$$\frac{2\sqrt{f+gx}\left(495e(f+gx)^3\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)-693(f+gx)^2(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*(e*f - d*g)^3*(c*f^2 + a*g^2) + 1155*(e*f - d*g)^2*
(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x) - 693*(e*f - d*g)*(3*a*e^2*g^2
+ c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^2 + 495*e*(a*e^2*g^2 + c*
(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^3 - 385*c*e^2*(5*e*f - 3*d
*g)*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

fricas [A] time = 0.87, size = 324, normalized size = 1.35

$$\frac{2\left(315\,ce^3g^5x^5-1280\,ce^3f^5+4224\,cde^2f^4g-6930\,ad^2efg^4+3465\,ad^3g^5-1584\left(3\,cd^2e+ae^3\right)f^3g^2+1848\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g - 6930*a*d^
2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f^3*g^2 + 1848*(c*d^3
 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 - 33*c*d*e^2*g^5)*x^4 + 5*(80*c*
e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 99*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(160*
c*e^3*f^3*g^2 - 528*c*d*e^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 - 231*(
c*d^3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2 + 346
5*a*d^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d*e^2)*f
*g^4)*x)*sqrt(g*x + f)/g^6
```

giac [A] time = 0.18, size = 378, normalized size = 1.58

$$\frac{2\left(3465\,\sqrt{gx+f}\,ad^3+\frac{3465\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)ad^2e}{g}+\frac{231\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)cd^3}{g^2}+\frac{693\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
```

```
+ f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sq
rt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*
f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5*(g*x
 + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
 f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(
g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d*e
^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/
2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x
 + f)*f^5)*c*e^3/g^5)/g
```

maple [A] time = 0.01, size = 365, normalized size = 1.52

$$\frac{2\sqrt{gx+f}\,\left(315e^3c\,x^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+1155*c*d*e^2*g^5*x^4-350*c*e^3*f*g^
4*x^4+495*a*e^3*g^5*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e
^3*f^2*g^3*x^3+2079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+693*c*d^3*g^5*x^2-1
782*c*d^2*e*f*g^4*x^2+1584*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a
*d^2*e*g^5*x-2772*a*d*e^2*f*g^4*x+792*a*e^3*f^2*g^3*x-924*c*d^3*f*g^4*x+237
6*c*d^2*e*f^2*g^3*x-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5
-6930*a*d^2*e*f*g^4+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2+1848*c*d^3*f^2*
g^3-4752*c*d^2*e*f^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.45, size = 326, normalized size = 1.36

$$\frac{2\left(315\left(gx+f\right)^{\frac{11}{2}}ce^3-385\left(5\,ce^3f-3\,cde^2g\right)\left(gx+f\right)^{\frac{9}{2}}+495\left(10\,ce^3f^2-12\,cde^2fg+\left(3\,cd^2e+ae^3\right)g^2\right)\left(gx+f\right)^{\frac{7}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f
)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*
x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)
*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 12*
c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a
*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2
)*f^2*g^3)*sqrt(g*x + f))/g^6
```

mupad [B] time = 0.12, size = 222, normalized size = 0.92

$$\frac{\left(f+g\,x\right)^{7/2}\,\left(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2\right)}{7\,g^6}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^3}{g^6}+\frac{2\,c\,e^3\,\left(f+g\right.}{11\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(7*g^6) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/g^6 + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6) + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g
^2 + 5*c*e*f^2 - 2*c*d*f*g))/(3*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*
```

```
e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(5*g^6) + (2*c*e^2*(f +
g*x)^(9/2)*(3*d*g - 5*e*f))/(9*g^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.590 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=175

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2\right.}{3g^5}$$

```
[Out] -4/3*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+f)^(3/2)/g^5+2/5*(a*e^2*g^2
+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(5/2)/g^5-4/7*c*e*(-d*g+2*e*f)*(g
*x+f)^(7/2)/g^5+2/9*c*e^2*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2+c*f^2)*(g
*x+f)^(1/2)/g^5
```

Rubi [A] time = 0.24, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1153}

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2\right.}{3g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g
^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2
*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx=\frac{2\,\text{Subst}\left(\int\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$=\frac{2\,\text{Subst}\left(\int\left(\frac{(-ef+dg)^2\left(cf^2+ag^2\right)}{g^4}+\frac{2(ef-dg)\left(-aeg^2-cf(2ef-dg)\right)x^2}{g^4}+\frac{\left(ae^2g^2+c\left(6e^2f^2-6defg+d^2g^2\right)\right)x^4}{g^4}\right.\right.}{g}$$

$$=\frac{2(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)(f+gx)^{3/2}}{3g^5}+\frac{2}{}$$

Mathematica [A] time = 0.15, size = 149, normalized size = 0.85

$$\frac{2\sqrt{f+gx}\left(63(f+gx)^2\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)+315\left(ag^2+cf^2\right)(ef-dg)^2-210(f+gx)(ef-d\right.}{315g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*(e*f - d*g)^2*(c*f^2 + a*g^2) - 210*(e*f - d*g)*(a*e*
g^2 + c*f*(2*e*f - d*g))*(f + g*x) + 63*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f
*g + d^2*g^2))*(f + g*x)^2 - 90*c*e*(2*e*f - d*g)*(f + g*x)^3 + 35*c*e^2*(f
 + g*x)^4))/(315*g^5)
```

fricas [A] time = 0.80, size = 197, normalized size = 1.13

$$\frac{2\left(35\,ce^2g^4x^4+128\,ce^2f^4-288\,cdef^3g-420\,adefg^3+315\,ad^2g^4+168\left(cd^2+ae^2\right)f^2g^2-10\left(4\,ce^2fg^3-9\,cd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 - 288*c*d*e*f^3*g - 420*a*d*e*f*g^3
 + 315*a*d^2*g^4 + 168*(c*d^2 + a*e^2)*f^2*g^2 - 10*(4*c*e^2*f*g^3 - 9*c*d*
e*g^4)*x^3 + 3*(16*c*e^2*f^2*g^2 - 36*c*d*e*f*g^3 + 21*(c*d^2 + a*e^2)*g^4)
*x^2 - 2*(32*c*e^2*f^3*g - 72*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2 + a
*e^2)*f*g^3)*x)*sqrt(g*x + f)/g^5
```

giac [A] time = 0.17, size = 243, normalized size = 1.39

$$\frac{2\left(315\sqrt{gx+f}\,ad^2+\frac{210\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ade}{g}+\frac{21\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd^2}{g^2}+\frac{21\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f\right.}{g^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*
x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 +
 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

maple [A] time = 0.01, size = 215, normalized size = 1.23

$$\frac{2\sqrt{gx+f}\,\left(35e^2c\,x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+90*c*d*e*g^4*x^3-40*c*e^2*f*g^3*x^3+6
3*a*e^2*g^4*x^2+63*c*d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+2
10*a*d*e*g^4*x-84*a*e^2*f*g^3*x-84*c*d^2*f*g^3*x+144*c*d*e*f^2*g^2*x-64*c*e
```

^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+168*a*e^2*f^2*g^2+168*c*d^2*f^2*g^2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5

maxima [A] time = 0.45, size = 197, normalized size = 1.13

$$2\left(35\left(gx+f\right)^{\frac{9}{2}}ce^2-90\left(2\,ce^2f-cdeg\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(6\,ce^2f^2-6\,cdefg+\left(cd^2+ae^2\right)g^2\right)\left(gx+f\right)^{\frac{5}{2}}-210\left(2\,ce^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 90*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(7/2)
+ 63*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) - 21
0*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*(g*x +
f)^(3/2) + 315*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 + (c*
d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5
```

mupad [B] time = 2.58, size = 159, normalized size = 0.91

$$\frac{\left(f+g\,x\right)^{5/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{5\,g^5}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,\left(d\,g-e\,f\right)^2}{g^5}+\frac{4\,\left(f+g\,x\right)^{3/2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(5*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/g^5 + (4*(f +
g*x)^(3/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(3*g^5) + (2*c*e^2*
(f + g*x)^(9/2))/(9*g^5) + (4*c*e*(f + g*x)^(7/2)*(d*g - 2*e*f))/(7*g^5)
```

sympy [A] time = 108.50, size = 673, normalized size = 3.85

$$\begin{cases} \frac{-\frac{2ad^2f}{\sqrt{f+gx}}-2ad^2\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)-\frac{4adef\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}-\frac{4ade\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}-\frac{2ae^2f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2ae^2\left(-\frac{f^3}{\sqrt{f+gx}}-3f\right.}{}}{} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3(ae^2+cd^2)}{3}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**2*f/sqrt(f + g*x) - 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 - 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f +
g*x)**(3/2)/3)/g**2 - 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
+ f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 4*c*d*e*f*(-f**3/sqrt(f +
 g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**
3 - 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**
(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*e**2*f*(f**
4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f +
 g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*c*e**2*(-f**5/sqrt(f + g*x) -
```

```
 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2
) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g, Ne(g, 0)), ((a*d*
*2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3
)/sqrt(f), True))
```

3.591 $\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}} dx$

Optimal. Leaf size=113

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

```
[Out] 2/3*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(3/2)/g^4-2/5*c*(-d*g+3*e*f)*(g*x+
f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)^(
1/2)/g^4
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {772}

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e
*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2))
/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^3}+\frac{c(-3ef+dg)(f+g$$
$$= -\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4}-\frac{2c(3ef-$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.83

$$\frac{2\sqrt{f+gx}\left(35ag^2(3dg-2ef+egx)+7cdg\left(8f^2-4fgx+3g^2x^2\right)-3ce\left(16f^3-8f^2gx+6fg^2x^2-5g^3x^3\right)\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f*
g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))/(
105*g^4)
```

fricas [A] time = 0.91, size = 100, normalized size = 0.88

$$\frac{2\left(15\,ceg^3x^3-48\,cef^3+56\,cdf^2g-70\,aefg^2+105\,adg^3-3\left(6\,cefg^2-7\,cdg^3\right)x^2+\left(24\,cef^2g-28\,cdfg^2+35\,ae\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 56*c*d*f^2*g - 70*a*e*f*g^2 + 105*a*d*g^3 - 3*(6*c*e*f*g^2 - 7*c*d*g^3)*x^2 + (24*c*e*f^2*g - 28*c*d*f*g^2 + 35*a*e*g^3)*x)*sqrt(g*x + f)/g^4

giac [A] time = 0.18, size = 134, normalized size = 1.19

$$\frac{2\left(105\sqrt{gx+f}\,ad+\frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ae}{g}+\frac{7\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd}{g^2}+\frac{3\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(g\right.}{g^3}\right.}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")

[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g

maple [A] time = 0.00, size = 101, normalized size = 0.89

$$\frac{2\sqrt{gx+f}\,\left(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+5\right.}{105g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x)

[Out] 2/105*(g*x+f)^(1/2)*(15*c*e*g^3*x^3+21*c*d*g^3*x^2-18*c*e*f*g^2*x^2+35*a*e*g^3*x-28*c*d*f*g^2*x+24*c*e*f^2*g*x+105*a*d*g^3-70*a*e*f*g^2+56*c*d*f^2*g-48*c*e*f^3)/g^4

maxima [A] time = 0.44, size = 104, normalized size = 0.92

$$\frac{2\left(15(gx+f)^{\frac{7}{2}}ce-21(3cef-cdg)(gx+f)^{\frac{5}{2}}+35(3cef^2-2cdfg+aeg^2)(gx+f)^{\frac{3}{2}}-105(cef^3-cdf^2g+\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - c*d*g)*(g*x + f)^(5/2) + 35*(3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(3/2) - 105*(c*e*f^3 - c*d*f^2*g + a*e*f*g^2 - a*d*g^3)*sqrt(g*x + f))/g^4

mupad [B] time = 0.07, size = 100, normalized size = 0.88

$$\frac{(f+gx)^{3/2}\,(6cef^2-4cdfg+2aeg^2)}{3g^4}+\frac{2ce\,(f+gx)^{7/2}}{7g^4}+\frac{2c\,(f+gx)^{5/2}\,(dg-3ef)}{5g^4}+\frac{2\sqrt{f+gx}\,(cf^2+}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(1/2),x)

```
[Out] ((f + g*x)^(3/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(3*g^4) + (2*c*e*(f +
 g*x)^(7/2))/(7*g^4) + (2*c*(f + g*x)^(5/2)*(d*g - 3*e*f))/(5*g^4) + (2*(f
+ g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f))/g^4
```

sympy [A] time = 61.13, size = 374, normalized size = 3.31

$$\begin{cases} \frac{-\frac{2adf}{\sqrt{f+gx}} - 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2cdf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2cd\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f\cdots}\right.}{}}{g} \\ \frac{adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d*f/sqrt(f + g*x) - 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) - 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*c*d*f*(f**2/sqrt(f + g*x
) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d*(-f**3/sqrt(f + g*
x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 -
 2*c*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
 (f + g*x)**(5/2)/5)/g**3 - 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x
) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/
g**3)/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f)
, True))
```

3.592 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{f+gx}\,(ag^2+cf^2)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

```
[Out] -4/3*c*f*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2+c*f^2)*(g*x+f)^
(1/2)/g^3
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2\sqrt{f+gx}\,(ag^2+cf^2)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) + (
2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2\sqrt{f+gx}}-\frac{2cf\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.72

$$\frac{2\sqrt{f+gx}\left(15ag^2+c\left(8f^2-4fgx+3g^2x^2\right)\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

fricas [A] time = 0.59, size = 40, normalized size = 0.66

$$\frac{2\left(3\,cg^2x^2-4\,cfgx+8\,cf^2+15\,ag^2\right)\sqrt{gx+f}}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

giac [A] time = 0.15, size = 53, normalized size = 0.87

$$\frac{2\left(15\sqrt{gx+f}\,a+\frac{\left(3\left(gx+f\right)^{\frac{5}{2}}-10\left(gx+f\right)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

maple [A] time = 0.00, size = 41, normalized size = 0.67

$$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2-4cfxg+15a\,g^2+8c\,f^2\right)}{15g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(1/2),x)
[Out] 2/15*(g*x+f)^(1/2)*(3*c*g^2*x^2-4*c*f*g*x+15*a*g^2+8*c*f^2)/g^3
```

maxima [A] time = 0.43, size = 53, normalized size = 0.87

$$\frac{2\left(15\sqrt{gx+f}\,a+\frac{\left(3\left(gx+f\right)^{\frac{5}{2}}-10\left(gx+f\right)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

mupad [B] time = 2.56, size = 44, normalized size = 0.72

$$\frac{2\,\sqrt{f+g\,x}\,\left(3\,c\,{\left(f+g\,x\right)}^2+15\,a\,g^2+15\,c\,f^2-10\,c\,f\,\left(f+g\,x\right)\right)}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(f + g*x)^(1/2),x)
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 - 10*c*f*(f + g*x
)))/(15*g^3)
```

sympy [A] time = 13.10, size = 150, normalized size = 2.46

$$\begin{cases} \frac{-\frac{2af}{\sqrt{f+gx}}-2a\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)-\frac{2cf\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2c\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g \neq 0 \\ \frac{ax+\frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*f/sqrt(f + g*x) - 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) -
 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 -
 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f), True))
```

3.593 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)-2*c*(d*g+e*f)*(g*x+f)^(1/2)/e^2/g^2
```

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1153, 208}

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*c*(e*f + d*g)*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2
) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e
^(5/2)*Sqrt[e*f - d*g])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f\right.}}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2\left(cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] time = 0.16, size = 92, normalized size = 0.88

$$\frac{2c\sqrt{f+gx}(-3dg-2ef+egx)}{3e^2g^2}-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) - (2*(c*d^2 + a*e^
2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*
g])
```

fricas [A] time = 0.94, size = 297, normalized size = 2.86

$$\left[\frac{3\left(cd^2+ae^2\right)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\,\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^2+cde^2fg-3\,cd^2eg^2-\left(ce^3fg-cde^2\right.\right.}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f - d*g -
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 + c*d*e^2*
f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^
2 - d*e^3*g^3), 2/3*(3*(c*d^2 + a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt
(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*e^3*f^2 + c*d*e^2*f*g
- 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 -
d*e^3*g^3)]
```

giac [A] time = 0.20, size = 107, normalized size = 1.03

$$\frac{2\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\,\sqrt{gx+f}\,cdg^5e-\left(gx+f\right)^{\frac{3}{2}}cg^4e^2+3\,\sqrt{gx+f}\,cfg^4e^2\right)e^{(-3)}}{3\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2)/sqrt(d
*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*g^4*e^2
+ 3*sqrt(g*x + f)*c*f*g^4*e^2)*e^(-3)/g^6
```

maple [A] time = 0.02, size = 132, normalized size = 1.27

$$\frac{2a\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}+\frac{2c\,d^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}\,e^2}-\frac{2\sqrt{gx+f}\,cd}{e^2g}-\frac{2\sqrt{gx+f}\,cf}{e\,g^2}+\frac{2\left(gx+f\right)^{\frac{3}{2}}c}{3e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2/g*c/e^2*d*(g*x+f)^(1/2)-2/g^2*c/e*f*(g*x+f)^(1/
2)+2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)*e/((d*g-e*f)*e)^(1/2))*a+2/e^
2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)*e/((d*g-e*f)*e)^(1/2))*c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.11, size = 107, normalized size = 1.03

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\left(c\,d^2+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}}-\sqrt{f+g\,x}\,\left(\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2))/(e^(5
/2)*(d*g - e*f)^(1/2)) - (f + g*x)^(1/2)*((2*c*(d*g^3 - e*f*g^2))/(e^2*g^4)
 + (4*c*f)/(e*g^2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

sympy [A] time = 48.66, size = 100, normalized size = 0.96

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3eg^2}-\frac{2c\sqrt{f+gx}\left(dg+ef\right)}{e^2g^2}-\frac{2\left(ae^2+cd^2\right)\mathrm{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*c*sqrt(f + g*x)*(d*g + e*f)/(e**2*g**2)
 - 2*(a*e**2 + c*d**2)*atan(1/(sqrt(e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sq
rt(e/(d*g - e*f))*(d*g - e*f))
```

3.594 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

```
[Out] (a*e^2*g+c*d*(-3*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+c*d^2/e^2)*(g*x+f)^(1
/2)/(-d*g+e*f)/(e*x+d)
```

Rubi [A] time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1157, 388, 208}

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (c*d^2)/e^2)*Sqrt[f + g*x])/((e*f - d*g
)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g
*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cf^2}{g^2}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} - \frac{\left(a+\frac{cd(4ef-3dg)}{e^2g}\right)\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+g\right.}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.29, size = 171, normalized size = 1.40

$$\frac{\frac{(ae^2+cd^2)\left(\sqrt{e}\sqrt{f+gx}(dg-ef)+g(d+ex)\sqrt{dg-ef}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)\right)}{(d+ex)(ef-dg)^2}+\frac{4cd\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}+\frac{2c\sqrt{e}\sqrt{f+gx}}{g}}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] ((2*c*Sqrt[e]*Sqrt[f + g*x])/g + ((c*d^2 + a*e^2)*(Sqrt[e]*(-(e*f) + d*g)*S
qrt[f + g*x] + g*Sqrt[-(e*f) + d*g]*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[f + g*x]
)/Sqrt[-(e*f) + d*g]]))/((e*f - d*g)^2*(d + e*x)) + (4*c*d*ArcTanh[(Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])/e^(5/2)
```

fricas [B] time = 0.74, size = 539, normalized size = 4.42

$$\left[-\frac{\left(4\,cd^2efg-\left(3\,cd^3-ade^2\right)g^2+\left(4\,cde^2fg-\left(3\,cd^2e-ae^3\right)g^2\right)x\right)\sqrt{e^2f-deg}\,\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\,\sqrt{gx+f}}{ex+d}\right)-}{2\left(de^5f^2g-2\,d^2e^4fg^2+d^3e^3g^3+\left(e^6f^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2
*e - a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e
^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 +
 a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*
d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3
 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -((4*c*d^2*e*f*g - (3*c*d^
3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2*e - a*e^3)*g^2)*x)*sqrt(-e^2*f
 + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d
*e^3*f^2 - (5*c*d^2*e^2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4
*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^
2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

giac [A] time = 0.17, size = 148, normalized size = 1.21

$$\frac{2\sqrt{gx+f}\,ce^{(-2)}}{g}-\frac{\left(3\,cd^2g-4\,cdfe-age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge^2-fe^3\right)\sqrt{dge-fe^2}}+\frac{\sqrt{gx+f}\,cd^2g+\sqrt{gx+f}\,age^2}{\left(dge^2-fe^3\right)\left(dg+\left(gx+f\right)e-fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - a*g*e^2)*arctan(sqrt(
g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sqrt(d*g*e - f*e^2)) + (
sqrt(g*x + f)*c*d^2*g + sqrt(g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g
*x + f)*e - f*e))
```

maple [B] time = 0.02, size = 237, normalized size = 1.94

$$\frac{ag\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}-\frac{3c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e^2}+\frac{4cdf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}+\frac{\sqrt{gx+f}\,ag}{\left(dg-ef\right)\left(egx+dg\right)}+\frac{\sqrt{g}}{\left(dg-e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e^2/g+g/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*a+g/e^2/(d*g-
e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan
((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^(1/
2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2+4/e/(d*g-e*f)/((d*g-e*
f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 2.68, size = 128, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-3\,c\,g\,d^2+4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)}+\frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 3*c*d^2*g + 4
*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(1/2)*(a*e^2*g + c*d^2*
g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d*e^2*g)) + (2*c*(f + g*x)^(1/2))
/(e^2*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.595 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=178

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(d+ex)^2(ef-dg)}-\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}+\frac{\sqrt{f+gx}\left(3ae^2g+cd(8ef-\right.}{4e^2(d+ex)(ef-dg)^2}$$

```
[Out] -1/4*(3*a*e^2*g^2+c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+c*d^2/e^2)*(g*x+f
)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*(-5*d*g+8*e*f))*(g*x+f)^(1/
2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A] time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1157, 385, 208}

$$-\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\left(3ae^2g+cd(8ef-\right.}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -((a + (c*d^2)/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*
g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(4*e^2*(e*f - d*g)^2*(d + e*x)) - (
(3*a*e^2*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cf^2}{g^2}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(8ef-5dg)\right)\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g^2+c\left(8e^2f^2-8d\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(8ef-5dg)\right)\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{\left(3ae^2g^2+c\left(8e^2f^2-8d\ldots\right.\right.}{4e^{5/\ldots}}$$

Mathematica [C] time = 0.82, size = 207, normalized size = 1.16

$$\frac{2\left(\frac{\sqrt{e}g^2\sqrt{f+gx}\left(ae^2+cd^2\right){}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{e(f+gx)}{ef-dg}\right)}{(dg-ef)^3}-\frac{cd\left(\sqrt{e}\sqrt{f+gx}(dg-ef)+g(d+ex)\sqrt{dg-ef}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)\right)}{(d+ex)(ef-dg)^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (2*(-((c*d*(Sqrt[e]*(-(e*f) + d*g)*Sqrt[f + g*x] + g*Sqrt[-(e*f) + d*g]*(d
+ e*x)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]))/((e*f - d*g)^2*
(d + e*x))) - (c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f
 - d*g] + (Sqrt[e]*(c*d^2 + a*e^2)*g^2*Sqrt[f + g*x]*Hypergeometric2F1[1/2,
 3, 3/2, (e*(f + g*x))/(e*f - d*g)])/(-(e*f) + d*g)^3))/e^(5/2)
```

fricas [B] time = 1.07, size = 896, normalized size = 5.03

$$\left[\frac{\left(8\,cd^2e^2f^2-8\,cd^3efg+3\left(cd^4+ad^2e^2\right)g^2+\left(8\,ce^4f^2-8\,cde^3fg+3\left(cd^2e^2+ae^4\right)g^2\right)x^2+2\left(8\,cde^3f^2-8\,cd^2e^2f\ldots\right.}{8\left(d^2e^6f^3-3\,d^3e^5f\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*g^2 + (8*c*e
^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2
- 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((
e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(
2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e -
5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3
*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
```

```
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*
g^2 + (8*c*e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*
c*d*e^3*f^2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(-e^2*f +
 d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*(3*c*
d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e - 5*a*d^2
*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3*e^2 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

giac [A] time = 0.21, size = 278, normalized size = 1.56

$$\frac{\left(3\,cd^2g^2-8\,cdfge+8\,cf^2e^2+3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)\sqrt{dge-fe^2}}-\frac{3\,\sqrt{gx+f}\,cd^3g^3+5\,(gx+f)^{\frac{3}{2}}cd^2g^2e-11\,\sqrt{gx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + 8*c*f^2*e^2 + 3*a*g^2*e^2)*arctan(sqrt(g*x
 + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*sqrt(d*
g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 + 5*(g*x + f)^(3/2)*c*d^2*g^
2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sq
rt(g*x + f)*c*d*f^2*g*e^2 - 5*sqrt(g*x + f)*a*d*g^3*e^2 - 3*(g*x + f)^(3/2)
*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2
*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

maple [B] time = 0.02, size = 384, normalized size = 2.16

$$\frac{3a\,g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{(dg-ef)e}}+\frac{3c\,d^2g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{(dg-ef)e}\,e^2}-\frac{2cdfg\arctan\left(\frac{\sqrt{gx+}}{\sqrt{(dg-}}\right)}{\left(d^2g^2-2defg+e^2f^2\right)\sqrt{(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e*f*g+e^2*f^2)*(g*x
+f)^(3/2)+1/8*(5*a*e^2*g-3*c*d^2*g+8*c*d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2)
)/(e*(g*x+f)+d*g-e*f)^2+3/4/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)
*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a*g^2+3/4/(d^2*g^2-2*d*e*f*g+e
^2*f^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)
*c*d^2*g^2-2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d*f*g+2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g
-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 2.91, size = 224, normalized size = 1.26

$$\frac{\frac{\sqrt{f+g\,x}\,\left(-3\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+5\,a\,e^2\,g^2\right)}{4\,e^2\,\left(d\,g-e\,f\right)}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+8\,c\right.}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (((f + g*x)^(1/2)*(5*a*e^2*g^2 - 3*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e^2*(d*g -
e*f)) + ((f + g*x)^(3/2)*(3*a*e^2*g^2 - 5*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e*(d
*g - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 +
e^2*f^2 - 2*d*e*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(
3*a*e^2*g^2 + 3*c*d^2*g^2 + 8*c*e^2*f^2 - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g - e
*f)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.596 $\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}} dx$

Optimal. Leaf size=238

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2\right.\right.}{3g^6}$$

```
[Out] -2/3*(-d*g+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3/2
)/g^6+2/5*e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g
^6-2/7*c*e^2*(-3*d*g+5*e*f)*(g*x+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2
*(-d*g+e*f)^3*(a*g^2+c*f^2)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(3*a*e*g^2+c*f
*(-2*d*g+5*e*f))*(g*x+f)^(1/2)/g^6
```

Rubi [A] time = 0.27, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1261}

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2\right.\right.}{3g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*e^
2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) + (2
*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*(f +
g*x)^(9/2))/(9*g^6)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int\left(\frac{(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{g^5}+\frac{(-ef+dg)^3\left(cf^2+ag^2\right)}{g^5x^2}+\frac{(ef-dg)\left(-3ae^2g^2-c\left(10e^2f^2-8defg\right.\right.}{g^5}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^3\left(cf^2+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)\sqrt{f+gx}}{g^6}-\frac{2(ef-d}{}$$

Mathematica [A] time = 0.24, size = 207, normalized size = 0.87

$$\frac{2\left(63e(f+gx)^3\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)-105(f+gx)^2(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*(e*f - d*g)^3*(c*f^2 + a*g^2) + 315*(e*f - d*g)^2*(3*a*e*g^2 + c*f*
(5*e*f - 2*d*g))*(f + g*x) - 105*(e*f - d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 -
 8*d*e*f*g + d^2*g^2))*(f + g*x)^2 + 63*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d
*e*f*g + 3*d^2*g^2))*(f + g*x)^3 - 45*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^4 + 3
5*c*e^3*(f + g*x)^5))/(315*g^6*Sqrt[f + g*x])
```

fricas [A] time = 0.93, size = 333, normalized size = 1.40

$$\frac{2\left(35\,ce^3g^5x^5+1280\,ce^3f^5-3456\,cde^2f^4g+1890\,ad^2efg^4-315\,ad^3g^5+1008\left(3\,cd^2e+ae^3\right)f^3g^2-840\left(cd^3+3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 3456*c*d*e^2*f^4*g + 1890*a*d^2*
e*f*g^4 - 315*a*d^3*g^5 + 1008*(3*c*d^2*e + a*e^3)*f^3*g^2 - 840*(c*d^3 + 3
*a*d*e^2)*f^2*g^3 - 5*(10*c*e^3*f*g^4 - 27*c*d*e^2*g^5)*x^4 + (80*c*e^3*f^2
*g^3 - 216*c*d*e^2*f*g^4 + 63*(3*c*d^2*e + a*e^3)*g^5)*x^3 - (160*c*e^3*f^3
*g^2 - 432*c*d*e^2*f^2*g^3 + 126*(3*c*d^2*e + a*e^3)*f*g^4 - 105*(c*d^3 + 3
*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 1728*c*d*e^2*f^3*g^2 + 945*a*d^2*e*
g^5 + 504*(3*c*d^2*e + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*a*d*e^2)*f*g^4)*x)*s
qrt(g*x + f)/(g^7*x + f*g^6)
```

giac [B] time = 0.21, size = 453, normalized size = 1.90

$$-\frac{2\left(cd^3f^2g^3+ad^3g^5-3\,cd^2f^3g^2e-3\,ad^2fg^4e+3\,cdf^4ge^2+3\,adf^2g^3e^2-cf^5e^3-af^3g^2e^3\right)}{\sqrt{gx+f}\,g^6}+\frac{2\left(105\left(gx+f\right)^{\frac{3}{2}}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e - 3*a*d^2*f*g^4*e + 3*c*d
*f^4*g*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*
```

```
g^6) + 2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^
51 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x + f)^(3/2)*c*d^2*f*g^50*e
+ 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 945*sqrt(g*x + f)*a*d^2*g^52*e + 13
5*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*d*f*g^49*e^2 + 1890*
(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c*d*f^3*g^49*e^2 + 31
5*(g*x + f)^(3/2)*a*d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*
x + f)^(9/2)*c*g^48*e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^
(5/2)*c*f^2*g^48*e^3 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x
+ f)*c*f^4*g^48*e^3 + 63*(g*x + f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a
*f*g^50*e^3 + 945*sqrt(g*x + f)*a*f^2*g^50*e^3)/g^54
```

maple [A] time = 0.01, size = 365, normalized size = 1.53

$$-\frac{2\left(-35e^3c\,x^5g^5-135cd\,e^2g^5x^4+50c\,e^3f\,g^4x^4-63a\,e^3g^5x^3-189c\,d^2e\,g^5x^3+216cd\,e^2f\,g^4x^3-80c\,e^3f^2g^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/315/(g*x+f)^(1/2)*(-35*c*e^3*g^5*x^5-135*c*d*e^2*g^5*x^4+50*c*e^3*f*g^4*
x^4-63*a*e^3*g^5*x^3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2
*g^3*x^3-315*a*d*e^2*g^5*x^2+126*a*e^3*f*g^4*x^2-105*c*d^3*g^5*x^2+378*c*d^
2*e*f*g^4*x^2-432*c*d*e^2*f^2*g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5
*x+1260*a*d*e^2*f*g^4*x-504*a*e^3*f^2*g^3*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*
f^2*g^3*x+1728*c*d*e^2*f^3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2
*e*f*g^4+2520*a*d*e^2*f^2*g^3-1008*a*e^3*f^3*g^2+840*c*d^3*f^2*g^3-3024*c*d
^2*e*f^3*g^2+3456*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.46, size = 334, normalized size = 1.40

$$2\left(\frac{35\left(gx+f\right)^{\frac{9}{2}}ce^3-45\left(5ce^3f-3cde^2g\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(10ce^3f^2-12cde^2fg+\left(3cd^2e+ae^3\right)g^2\right)\left(gx+f\right)^{\frac{5}{2}}-105\left(10ce^3f^3-18cde^2f^2g+3\left(3cd^2e+ae^3\right)fg\right.}{g^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^(
7/2) + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x +
f)^(5/2) - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)*f*g
^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 12*c*d*e
^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*a*d
*e^2)*f*g^3)*sqrt(g*x + f))/g^5 + 315*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a*d^
2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2)*f
^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

mupad [B] time = 0.09, size = 292, normalized size = 1.23

$$\frac{\left(f+g\,x\right)^{5/2}\,\left(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2\right)}{5\,g^6}-\frac{2\,c\,d^3\,f^2\,g^3+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2-6\,a\,d^2\,e}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(5*g^6) - (2*a*d^3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g
^3 - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*c*d^2*e*f^3*
```

```
g^2)/(g^6*(f + g*x)^(1/2)) + (2*c*e^3*(f + g*x)^(9/2))/(9*g^6) + (2*(f + g*
x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/g^6 + (2*(f + g
*x)^(3/2)*(d*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g
))/(3*g^6) + (2*c*e^2*(f + g*x)^(7/2)*(3*d*g - 5*e*f))/(7*g^6)
```

sympy [A] time = 110.87, size = 328, normalized size = 1.38

$$\frac{2ce^3\left(f+gx\right)^{\frac{9}{2}}}{9g^6}+\frac{\left(f+gx\right)^{\frac{7}{2}}\left(6cde^2g-10ce^3f\right)}{7g^6}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(2ae^3g^2+6cd^2eg^2-24cde^2fg+20ce^3f^2\right)}{5g^6}+\frac{\left(f+gx\right)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(6*c*d*e**2*g - 10*c*
e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*c*d**2*e*g**2 - 24*c
*d*e**2*f*g + 20*c*e**3*f**2)/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3
- 6*a*e**3*f*g**2 + 2*c*d**3*g**3 - 18*c*d**2*e*f*g**2 + 36*c*d*e**2*f**2*g
 - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6*a*d**2*e*g**4 - 12*a*d*e**2*
f*g**3 + 6*a*e**3*f**2*g**2 - 4*c*d**3*f*g**3 + 18*c*d**2*e*f**2*g**2 - 24*
c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2*(a*g**2 + c*f**2)*(d*g - e*f)**3
/(g**6*sqrt(f + g*x))
```

3.597 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}\,(ef-dg)\left(aeg^2+cf(2e\right.}{g^5}$$

```
[Out] 2/3*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3/2)/g^5-4/5*c*e*(
-d*g+2*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a
*g^2+c*f^2)/g^5/(g*x+f)^(1/2)-4*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+
f)^(1/2)/g^5
```

Rubi [A] time = 0.20, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {898, 1261}

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}\,(ef-dg)\left(aeg^2+cf(2e\right.}{g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a*
e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^
2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g)*(
f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx,x,\sqrt{f+gx}\right)}{g}\\
&= \frac{2\,\text{Subst}\left(\int \left(\frac{2(ef-dg)\left(-aeg^2-cf(2ef-dg)\right)}{g^4}+\frac{(-ef+dg)^2\left(cf^2+ag^2\right)}{g^4x^2}+\frac{\left(ae^2g^2+c\left(6e^2f^2-6defg+d^2g^2\right)\right)x}{g^4}\right.\right.}{g}\\
&= -\frac{2(ef-dg)^2\left(cf^2+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)\sqrt{f+gx}}{g^5}+\frac{2\left(ae^2g^2\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 149, normalized size = 0.86

$$\frac{2\left(35(f+gx)^2\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)-105\left(ag^2+cf^2\right)(ef-dg)^2-210(f+gx)(ef-dg)\left(aeg^2+c\right.}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*(e*f - d*g)^2*(c*f^2 + a*g^2) - 210*(e*f - d*g)*(a*e*g^2 + c*f*(2*
e*f - d*g))*(f + g*x) + 35*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 42*c*e*(2*e*f - d*g)*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(
105*g^5*Sqrt[f + g*x])
```

fricas [A] time = 0.93, size = 206, normalized size = 1.19

$$\frac{2\left(15\,ce^2g^4x^4-384\,ce^2f^4+672\,cdef^3g+420\,adefg^3-105\,ad^2g^4-280\left(cd^2+ae^2\right)f^2g^2-6\left(4\,ce^2fg^3-7\,cdeg^4\right)\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^3
 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c*d*e
*g^4)*x^3 + (48*c*e^2*f^2*g^2 - 84*c*d*e*f*g^3 + 35*(c*d^2 + a*e^2)*g^4)*x^
2 - 2*(96*c*e^2*f^3*g - 168*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 70*(c*d^2 + a*e
^2)*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

giac [A] time = 0.33, size = 275, normalized size = 1.59

$$-\frac{2\left(cd^2f^2g^2+ad^2g^4-2\,cdf^3ge-2\,adfg^3e+cf^4e^2+af^2g^2e^2\right)}{\sqrt{gx+f}\,g^5}+\frac{2\left(35\left(gx+f\right)^{\frac{3}{2}}cd^2g^{32}-210\,\sqrt{gx+f}\,cd^2fg^{32}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 + a*d^2*g^4 - 2*c*d*f^3*g*e - 2*a*d*f*g^3*e + c*f^4*e^2 +
 a*f^2*g^2*e^2)/(sqrt(g*x + f)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32
- 210*sqrt(g*x + f)*c*d^2*f*g^32 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g*x
 + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 210*sqrt(g*x
+ f)*a*d*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^
30*e^2 + 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*
e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*a*f*g^32*e^2)/g^35
```

maple [A] time = 0.01, size = 215, normalized size = 1.24

$$-\frac{2\left(-15e^2c\,x^4g^4-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-35c\,d^2g^4x^2+84cdef\,g^3x^2-48c\,e^2f^2g^2x^2-210ade\,g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-42*c*d*e*g^4*x^3+24*c*e^2*f*g^3*x^3
-35*a*e^2*g^4*x^2-35*c*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*g^2*x^2-
210*a*d*e*g^4*x+140*a*e^2*f*g^3*x+140*c*d^2*f*g^3*x-336*c*d*e*f^2*g^2*x+192
*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^3+280*a*e^2*f^2*g^2+280*c*d^2*f^
2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 205, normalized size = 1.18

$$\frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2-42\,(2\,ce^2f-cdeg)(gx+f)^{\frac{5}{2}}+35\,(6\,ce^2f^2-6\,cdefg+(cd^2+ae^2)g^2)(gx+f)^{\frac{3}{2}}-210\,(2\,ce^2f^3-3\,cdef^2g-adeg^3+(cd^2+ae^2)fg^2)\sqrt{gx+f}}{g^4}\right.}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(5/2)
 + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) - 2
10*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*sqrt(g
*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 +
 (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

mupad [B] time = 2.66, size = 199, normalized size = 1.15

$$\frac{(f+g\,x)^{3/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{3\,g^5}-\frac{2\,c\,d^2\,f^2\,g^2+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g-4\,a\,d\,e\,f\,g^3+}{g^5\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(3*g^5) - (2*a*d^2*g^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 -
 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^(1/2)) + (4*(f + g*x)^(1/2)*
(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/g^5 + (2*c*e^2*(f + g*x)^(7/2)
)/(7*g^5) + (4*c*e*(f + g*x)^(5/2)*(d*g - 2*e*f))/(5*g^5)
```

sympy [A] time = 50.85, size = 204, normalized size = 1.18

$$\frac{2ce^2\left(f+gx\right)^{\frac{7}{2}}}{7g^5}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(4cdeg-8ce^2f\right)}{5g^5}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2ae^2g^2+2cd^2g^2-12cdefg+12ce^2f^2\right)}{3g^5}+\frac{\sqrt{f+gx}\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(4*c*d*e*g - 8*c*e**2
*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 2*c*d**2*g**2 - 12*c*d*e*f
*g + 12*c*e**2*f**2)/(3*g**5) + sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g*
*2 - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/g**5 - 2*(a*g**2 +
c*f**2)*(d*g - e*f)**2/(g**5*sqrt(f + g*x))
```

3.598 $\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(ag^2+cf^2\right)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[Out] -2/3*c*(-d*g+3*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(-d*g+e*f
)*(a*g^2+c*f^2)/g^4/(g*x+f)^(1/2)+2*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(1
/2)/g^4
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {772}

$$\frac{2\left(ag^2+cf^2\right)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3*g
^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}}+\frac{c(-3ef+dg)\sqrt{f+gx}}{g^3}+\frac{ce(f-\right.\\ &= \frac{2(ef-dg)\left(cf^2+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4}-\frac{2c(3ef-dg)(f+gx)^{3/}}{3g^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.83

$$\frac{30ag^2(-dg+2ef+egx)+10cdg\left(-8f^2-4fgx+g^2x^2\right)+6ce\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 6
*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

fricas [A] time = 1.03, size = 110, normalized size = 0.99

$$\frac{2\left(3\,ceg^3x^3+48\,cef^3-40\,cdf^2g+30\,aefg^2-15\,adg^3-\left(6\,cefg^2-5\,cdg^3\right)x^2+\left(24\,cef^2g-20\,cdfg^2+15\,aeg^3\right)x\right.}{15\left(g^5x+fg^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^3
 - (6*c*e*f*g^2 - 5*c*d*g^3)*x^2 + (24*c*e*f^2*g - 20*c*d*f*g^2 + 15*a*e*g^
3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

giac [A] time = 0.21, size = 143, normalized size = 1.29

$$-\frac{2\left(cdf^2g+adg^3-cf^3e-afg^2e\right)}{\sqrt{gx+f}\,g^4}+\frac{2\left(5\left(gx+f\right)^{\frac{3}{2}}cdg^{17}-30\sqrt{gx+f}\,cdfg^{17}+3\left(gx+f\right)^{\frac{5}{2}}cg^{16}e-15\left(gx+f\right.\right.}{15\,g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g + a*d*g^3 - c*f^3*e - a*f*g^2*e)/(sqrt(g*x + f)*g^4) + 2/15*(
5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 + 3*(g*x + f)^(5/2
)*c*g^16*e - 15*(g*x + f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e
+ 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

maple [A] time = 0.00, size = 101, normalized size = 0.91

$$-\frac{2\left(-3ce\,x^3g^3-5cd\,g^3x^2+6cef\,g^2x^2-15ae\,g^3x+20cdf\,g^2x-24ce\,f^2gx+15ad\,g^3-30aef\,g^2+40cd\,f^2g-\right.}{15\sqrt{gx+f}\;g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/15/(g*x+f)^(1/2)*(-3*c*e*g^3*x^3-5*c*d*g^3*x^2+6*c*e*f*g^2*x^2-15*a*e*g^
3*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x+15*a*d*g^3-30*a*e*f*g^2+40*c*d*f^2*g-48*c
*e*f^3)/g^4
```

maxima [A] time = 0.45, size = 112, normalized size = 1.01

$$\frac{2\left(\frac{3\left(gx+f\right)^{\frac{5}{2}}ce-5\left(3\,cef-cdg\right)\left(gx+f\right)^{\frac{3}{2}}+15\left(3\,cef^2-2\,cdfg+aeg^2\right)\sqrt{gx+f}}{g^3}+\frac{15\left(cef^3-cdf^2g+aefg^2-adg^3\right)}{\sqrt{gx+f}\,g^3}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^(3/2) + 15*(3*
c*e*f^2 - 2*c*d*f*g + a*e*g^2)*sqrt(g*x + f))/g^3 + 15*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^3))/g
```

mupad [B] time = 0.07, size = 111, normalized size = 1.00

$$\frac{\sqrt{f+g\,x}\,\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{g^4}-\frac{-2\,c\,e\,f^3+2\,c\,d\,f^2\,g-2\,a\,e\,f\,g^2+2\,a\,d\,g^3}{g^4\,\sqrt{f+g\,x}}+\frac{2\,c\,e\,{\left(f+g\,x\right)}^{5/2}}{5\,g^4}+\frac{2\,c\,(f\,\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(1/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/g^4 - (2*a*d*g^3 - 2*
c*e*f^3 - 2*a*e*f*g^2 + 2*c*d*f^2*g)/(g^4*(f + g*x)^(1/2)) + (2*c*e*(f + g*
x)^(5/2))/(5*g^4) + (2*c*(f + g*x)^(3/2)*(d*g - 3*e*f))/(3*g^4)
```

sympy [A] time = 25.28, size = 112, normalized size = 1.01

$$\frac{2ce\left(f+gx\right)^{\frac{5}{2}}}{5g^4}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2cdg-6cef\right)}{3g^4}+\frac{\sqrt{f+gx}\left(2aeg^2-4cdfg+6cef^2\right)}{g^4}-\frac{2\left(ag^2+cf^2\right)\left(dg-ef\right)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*c*d*g - 6*c*e*f)/(3*g
**4) + sqrt(f + g*x)*(2*a*e*g**2 - 4*c*d*f*g + 6*c*e*f**2)/g**4 - 2*(a*g**2
 + c*f**2)*(d*g - e*f)/(g**4*sqrt(f + g*x))
```

3.599 $\int \frac{a+cx^2}{(f+gx)^{3/2}} \, dx$

Optimal. Leaf size=59

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2+c*f^2)/g^3/(g*x+f)^(1/2)-4*c*f*(g*x+f)^(1/
2)/g^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*c
*(f + g*x)^(3/2))/(3*g^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2(f+gx)^{3/2}}-\frac{2cf}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\ &= -\frac{2\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.73

$$\frac{2\left(c\left(-8f^2-4fgx+g^2x^2\right)-3ag^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

fricas [A] time = 0.61, size = 49, normalized size = 0.83

$$\frac{2\left(cg^2x^2-4\,cfgx-8\,cf^2-3\,ag^2\right)\sqrt{gx+f}}{3\left(g^4x+fg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g^
3)
```

giac [A] time = 0.18, size = 56, normalized size = 0.95

$$-\frac{2\left(cf^2+ag^2\right)}{\sqrt{gx+f}\,g^3}+\frac{2\left(\left(gx+f\right)^{\frac{3}{2}}cg^6-6\sqrt{gx+f}\,cfg^6\right)}{3\,g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sqr
t(g*x + f)*c*f*g^6)/g^9
```

maple [A] time = 0.00, size = 41, normalized size = 0.69

$$-\frac{2\left(-c\,x^2g^2+4cfxg+3a\,g^2+8c\,f^2\right)}{3\sqrt{gx+f}\;g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/3/(g*x+f)^(1/2)*(-c*g^2*x^2+4*c*f*g*x+3*a*g^2+8*c*f^2)/g^3
```

maxima [A] time = 0.44, size = 54, normalized size = 0.92

$$\frac{2\left(\frac{\left(gx+f\right)^{\frac{3}{2}}c-6\sqrt{gx+f}\,cf}{g^2}-\frac{3\left(cf^2+ag^2\right)}{\sqrt{gx+f}\,g^2}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sqr
t(g*x + f)*g^2))/g
```

mupad [B] time = 0.05, size = 44, normalized size = 0.75

$$-\frac{6\,a\,g^2-2\,c\,{\left(f+g\,x\right)}^2+6\,c\,f^2+12\,c\,f\,\left(f+g\,x\right)}{3\,g^3\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] -(6*a*g^2 - 2*c*(f + g*x)^2 + 6*c*f^2 + 12*c*f*(f + g*x))/(3*g^3*(f + g*x)^
(1/2))
```

sympy [A] time = 10.14, size = 58, normalized size = 0.98

$$-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3g^3}-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] -4*c*f*sqrt(f + g*x)/g**3 + 2*c*(f + g*x)**(3/2)/(3*g**3) - 2*(a*g**2 + c*f
**2)/(g**3*sqrt(f + g*x))
```

3.600 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

```
[Out] -2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-
d*g+e*f)^(3/2)+2*(a*g^2+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c*(g*x+f)^(1/
2)/e/g^2
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {898, 1261, 208}

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f + g*x])/(
e*g^2) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{(2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\,dx, x, \sqrt{f\right.}}{e(ef-dg)}$$

$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [C] time = 0.07, size = 90, normalized size = 0.80

$$-\frac{2\left(g^2(ae^2+cd^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)(dg+2ef+egx)\right)}{e^2g^2\sqrt{f+gx}\,(dg-ef)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*(e*f - d*g)*(2*e*f + d*g + e*g*x) + (c*d^2 + a*e^2)*g^2*Hypergeometr
ic2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*g)]))/(e^2*g^2*(-(e*f) + d*g)*Sq
rt[f + g*x])
```

fricas [B] time = 0.68, size = 492, normalized size = 4.39

$$\left[-\frac{\left((cd^2+ae^2)g^3x+(cd^2+ae^2)fg^2\right)\sqrt{e^2f-deg}\,\log\left(\frac{egx+2ef-dg+2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^3-3cde^2f^2g-\right.}{e^4f^3g^2-2de^3f^2g^3+d^2e^2fg^4+\left(e^4f^2g^3-2de^3fg\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(e^2*f - d*e*g)*log(
(e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*
(2*c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c
*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2
- 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*
g^5)*x), 2*(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(-e^2*f + d
*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f
^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c*e^3*f^2*g
 - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*
f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

giac [A] time = 0.20, size = 101, normalized size = 0.90

$$-\frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{(dge-fe^2)^{\frac{3}{2}}}+\frac{2\sqrt{gx+f}\,ce^{(-1)}}{g^2}-\frac{2(cf^2+ag^2)}{(dg^3-fg^2e)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g*e - f*e
^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 + a*g^2)/((d*g^3 - f*g^
2*e)*sqrt(g*x + f))
```

maple [A] time = 0.01, size = 165, normalized size = 1.47

$$-\frac{2ae\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}-\frac{2c\,d^{2}\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}-\frac{2a}{\left(dg-ef\right)\sqrt{gx+f}}-\frac{2c\,f^{2}}{\left(dg-ef\right)\sqrt{gx+f}\,g^{2}}+\frac{2\sqrt{gx+f}\,c}{e\,g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e/g^2-2/(d*g-e*f)*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1
/2)/((d*g-e*f)*e)^(1/2)*e)*a-2/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2-2/(d*g-e*f)/(g*x+f)^(1/2)*a-2/g^2/(d*
g-e*f)/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.14, size = 141, normalized size = 1.26

$$\frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)\,\left(e^2\,f-d\,e\,g\right)}{\sqrt{e}\,\left(2\,c\,d^2+2\,a\,e^2\right)\,{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2}-\frac{2\,\left(c\,e\,f^2+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(e^2*f - d*e*g))/(e^(1/2)*(2*a*e
^2 + 2*c*d^2)*(d*g - e*f)^(3/2)))*(a*e^2 + c*d^2))/(e^(3/2)*(d*g - e*f)^(3/
2)) + (2*c*(f + g*x)^(1/2))/(e*g^2) - (2*(a*e*g^2 + c*e*f^2))/(e*g^2*(f + g
*x)^(1/2)*(d*g - e*f))
```

sympy [A] time = 41.28, size = 104, normalized size = 0.93

$$\frac{2c\sqrt{f+gx}}{eg^{2}}-\frac{2\left(ag^{2}+cf^{2}\right)}{g^{2}\sqrt{f+gx}\,\left(dg-ef\right)}-\frac{2\left(ae^{2}+cd^{2}\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^{2}\sqrt{\frac{dg-ef}{e}}\,\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 + c*f**2)/(g**2*sqrt(f + g*x)*(d*g -
 e*f)) - 2*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**2*
sqrt((d*g - e*f)/e)*(d*g - e*f))
```

3.601 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

```
[Out] (3*a*e^2*g+c*d*(-d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2+c*f^2)/g/(-d*g+e*f)^2/(g*x+f)^(1/2)-(a*
e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1259, 453, 208}

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqr
t[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2
))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
```

```
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(ae^2g^2-c\left(2e^2f^2-4defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, \ldots\right.}{e^2(ef-dg)^2}$$

$$= -\frac{2\left(cf^2+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{\left(3ae^2g+cd(4ef-dg)\right)\text{Subst}\left(\ldots\right.}{eg(ef-\ldots}$$

$$= -\frac{2\left(cf^2+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\ldots\right.}{e^{3/2}(ef-dg)^{5/2}}$$

Mathematica [C] time = 0.08, size = 118, normalized size = 0.82

$$-\frac{2\left(g^2\left(ae^2+cd^2\right){}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+2cdg(ef-dg)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\right)}{e^2g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*(e*f - d*g)^2 + 2*c*d*g*(e*f - d*g)*Hypergeometric2F1[-1/2, 1, 1/2,
(e*(f + g*x))/(e*f - d*g)] + (c*d^2 + a*e^2)*g^2*Hypergeometric2F1[-1/2, 2,
 1/2, (e*(f + g*x))/(e*f - d*g)]))/(e^2*g*(e*f - d*g)^2*Sqrt[f + g*x])
```

fricas [B] time = 0.67, size = 906, normalized size = 6.29

$$\left[\frac{\left(4\,cd^2ef^2g-\left(cd^3-3\,ade^2\right)fg^2+\left(4\,cde^2fg^2-\left(cd^2e-3\,ae^3\right)g^3\right)x^2+\left(4\,cde^2f^2g+3\left(cd^2e+ae^3\right)fg^2-\left(cd^3-\ldots\right.\right.\right.}{2\left(de^5f^4g-3\,d^2e^4f^3g^2+\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 - (c*
d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^2 -
(c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g +
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a*d^
2*e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^
3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*
g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^
3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g
```

```
^5)*x), -((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 -
 (c*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^
2 - (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d
*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*
d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*c*d*e
^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^3)*g^3)*x)*sqrt
(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f
*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
+ (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

giac [A] time = 0.19, size = 225, normalized size = 1.56

$$\frac{\left(cd^2g - 4\,cdfe - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge - fe^2}} - \frac{(gx+f)cd^2g^2 + 2\,cdf^2ge + 2\,adg^3e + 2\,(gx+f)cf^2e^2 - 2\,cf^3e^2 + 3}{\left(d^2g^3e - 2\,dfg^2e^2 + f^2ge^3\right)\left(\sqrt{gx+f}\,dg + (gx+f)^{\frac{3}{2}}e - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e - 3*a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2
))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e - f*e^2)) - ((g*x + f)*c
*d^2*g^2 + 2*c*d*f^2*g*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^
2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g^3*e - 2*d*f*g^2*e^2 + f^
2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e - sqrt(g*x + f)*f*e))
```

maple [B] time = 0.02, size = 269, normalized size = 1.87

$$-\frac{3aeg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}} + \frac{c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}\,e} - \frac{4cdf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}} - \frac{\sqrt{gx+f}\,aeg}{(dg-ef)^2(egx+dg)} - \frac{\sqrt{g}}{(dg-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

```
[Out] -g/(d*g-e*f)^2*e*(g*x+f)^(1/2)/(e*g*x+d*g)*a-g/(d*g-e*f)^2/e*(g*x+f)^(1/2)/
(e*g*x+d*g)*c*d^2-3*g/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2
)/((d*g-e*f)*e)^(1/2)*e)*a+g/(d*g-e*f)^2/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2-4/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*arc
tan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d*f-2*g/(d*g-e*f)^2/(g*x+f)^(1/2
)*a-2/g/(d*g-e*f)^2/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.29, size = 187, normalized size = 1.30

$$-\frac{\frac{2\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f} + \frac{(f+g\,x)\left(c\,d^2\,g^2+2\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{e\,\left(d\,g-e\,f\right)^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,\left(f+g\,x\right)^{3/2}} - \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{5/2}}\right)\left(-c\,g\,d^2+4\,c\,f\,d\,e+3\,a\,g\,e^2\right)}{e^{3/2}\,\left(d\,g-e\,f\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] - ((2*(a*g^2 + c*f^2))/(d*g - e*f) + ((f + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 +
2*c*e^2*f^2))/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f
+ g*x)^(3/2)) - (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))
/(e^(1/2)*(d*g - e*f)^(5/2)))*(3*a*e^2*g - c*d^2*g + 4*c*d*e*f))/(e^(3/2)*(
d*g - e*f)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.602 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{f+gx}\,(ae^2+cd^2)}{2e(d+ex)^2(ef-dg)^2}-\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}+\frac{\sqrt{f+gx}\,(7ae^2g+cd(8ef-\ldots}{4e(d+ex)(ef-dg)^3}$$

```
[Out] -1/4*(15*a*e^2*g^2+c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^2+c*f^2)/(-d*g+e*
f)^3/(g*x+f)^(1/2)-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)^2
+1/4*(7*a*e^2*g+c*d*(-d*g+8*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

Rubi [A] time = 0.50, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {898, 1259, 456, 453, 208}

$$-\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\,(ae^2+cd^2)}{2e(d+ex)^2(ef-dg)^2}+\frac{\sqrt{f+gx}\,(7ae^2g+cd(8ef-\ldots}{4e(d+ex)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqrt[f
 + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((7*a*e^2*g + c*d*(8*e*f - d*g))
*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d + e*x)) - ((15*a*e^2*g^2 + c*(8*e^2*f
^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(3/2)*(e*f - d*g)^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
```

```
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(3ae^2g^2-c\left(4e^2f^2-8defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx\right.}{2e^2(ef-dg)^2}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\,\text{Subst}\left(\int \frac{\frac{8e^2(cf^2+}{g^4}}{}\right.}{}$$

$$= \frac{2\left(cf^2+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}$$

$$= \frac{2\left(cf^2+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}$$

Mathematica [C] time = 0.10, size = 140, normalized size = 0.65

$$\frac{2\left(g\left(g\left(ae^2+cd^2\right){}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+2cd(ef-dg){}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)\right)+c(ef-dg)^2{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+}{ef-}\right.\right.}{e^2\sqrt{f+gx}\,(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*(e*f - d*g)^2*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*
g)] + g*(2*c*d*(e*f - d*g)*Hypergeometric2F1[-1/2, 2, 1/2, (e*(f + g*x))/(e
*f - d*g)] + (c*d^2 + a*e^2)*g*Hypergeometric2F1[-1/2, 3, 1/2, (e*(f + g*x)
)/(e*f - d*g)])))/(e^2*(e*f - d*g)^3*Sqrt[f + g*x])
```

fricas [B] time = 1.03, size = 1539, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 +
(8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e
^4*f^3 + 24*c*d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*
a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a
*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g
*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a
*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*d^3*e^2 - 11*a*d*e^4)*f^
2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 3*(3*c*d^2*e^3 - 5*a*e^5
)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c*d*e^4*f^3 - (19*c*d^2*e
^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g^2 - (c*d^4*e + 25*a*d^2
*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3
*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6
*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*
f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3
*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*
f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 8*c*d^
3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2
 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 24*c*d*e^3*f^2*g + 15*(
c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*
f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^
2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x +
f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*
d^3*e^2 - 11*a*d*e^4)*f^2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
3*(3*c*d^2*e^3 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c
*d*e^4*f^3 - (19*c*d^2*e^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g
^2 - (c*d^4*e + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e
^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4
*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x
^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d
^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*
e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x)]
```

giac [A] time = 0.22, size = 361, normalized size = 1.69

$$\frac{\left(cd^2g^2 - 8\,cdfge - 8\,cf^2e^2 - 15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4\right)\sqrt{dge-fe^2}} - \frac{2\left(cf^2+ag^2\right)}{\left(d^3g^3 - 3\,d^2fg^2e + 3\,df^2ge^2 - f^3e^3\right)\sqrt{gx+f}} - \frac{\sqrt{gx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e - 8*c*f^2*e^2 - 15*a*g^2*e^2)*arctan(sqrt(g*x
+ f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 -
 f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*
e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x + f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3
- (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt(g*x + f)*c*d^2*f*g^2*e + 8*(g*x + f)
^(3/2)*c*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 + 9*sqrt(g*x + f)*a*d*g^
3*e^2 + 7*(g*x + f)^(3/2)*a*g^2*e^3 - 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^
3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^
2)
```

maple [B] time = 0.02, size = 546, normalized size = 2.55

$$-\frac{9\sqrt{gx+f}\;ade\,g^3}{4\left(dg-ef\right)^3\left(egx+dg\right)^2} + \frac{9\sqrt{gx+f}\;a\,e^2f\,g^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2} - \frac{\sqrt{gx+f}\;c\,d^3g^3}{4\left(dg-ef\right)^3\left(egx+dg\right)^2e} - \frac{7\sqrt{gx+f}\;c\,d^2f\,g^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2} + \frac{2}{\left(dg\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

```
[Out] -7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*a*e^2*g^2+1/4/(d*g-e*f)^3/(e*g
*x+d*g)^2*(g*x+f)^(3/2)*c*d^2*g^2-2/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)
*c*d*e*f*g-9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3*e*(g*x+f)^(1/2)*a*d+9/4/(d*g-e
*f)^3/(e*g*x+d*g)^2*g^2*e^2*(g*x+f)^(1/2)*a*f-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2
*g^3/e*(g*x+f)^(1/2)*c*d^3-7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*(g*x+f)^(1/2)*
f*c*d^2+2/(d*g-e*f)^3/(e*g*x+d*g)^2*g*e*(g*x+f)^(1/2)*c*d*f^2-15/4/(d*g-e*f
)^3*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a*g^2
+1/4/(d*g-e*f)^3/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(
1/2)*e)*c*d^2*g^2-2/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((
d*g-e*f)*e)^(1/2)*e)*c*d*f*g-2/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arctan((g*
x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*f^2-2/(d*g-e*f)^3/(g*x+f)^(1/2)*a*g^2-2
/(d*g-e*f)^3/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.37, size = 310, normalized size = 1.45

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}}-\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{(f+g\,x}{e^2\,{\left(f+g\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 +
 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) - ((2*(a*g^2 + c*f^2))/(d*g -
e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 + 8*c*d*e*f*g))
/(4*(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 +
8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2) - (f + g*x)^(3/2)*(
2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.603 $\int \frac{a+cx^2}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$

Optimal. Leaf size=147

$$\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\,\sqrt{f+gx}\,(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

```
[Out] 1/4*(8*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2))*arctanh(g^(1/2)*(e*x+d)
^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)^(3/2)*(g*x+f)^(
1/2)/e^2/g-1/4*c*(5*d*g+3*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^2
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {952, 80, 63, 217, 206}

$$\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\,\sqrt{f+gx}\,(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -(c*(3*e*f + 5*d*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) + (c*(d + e*x)
^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*
e^(5/2)*g^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
```

```
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}ce(3ef+5dg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{1}{8}\left(8a + \frac{c\left(3e^2f^2+2def\right.}{e^2g^2}\right.$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8a + \frac{c\left(3e^2f^2+2defg+3d^2g^2\right.}{e^2g^2}\right.}{}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8a + \frac{c\left(3e^2f^2+2defg+3d^2g^2\right.}{e^2g^2}\right.}{}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8ae^2g^2 + c\left(3e^2f^2+2d\right.\right.}{}$$

Mathematica [A] time = 0.56, size = 155, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)+ce\sqrt{g}\sqrt{d+ex}(f+gx)(-3dg-}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (c*e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(-3*e*f - 3*d*g + 2*e*g*x) + Sqrt[e*f
- d*g]*(8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*Sqrt[(e*(f + g
*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(4*e^3*
g^(5/2)*Sqrt[f + g*x])
```

fricas [A] time = 1.05, size = 336, normalized size = 2.29

$$\left[\frac{\left(3\,ce^2f^2+2\,cdefg+\left(3\,cd^2+8\,ae^2\right)g^2\right)\sqrt{eg}\,\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\left(2\,egx+ef+dg\right)\sqrt{eg}\right.}{16\,e^3g^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(e*g)*log(
8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqr
```

```
t(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e^2*
g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3),
-1/8*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(-e*g)*arct
an(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^
2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g
- 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

giac [A] time = 0.27, size = 155, normalized size = 1.05

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\sqrt{xe+d}\left(\frac{2(xe+d)ce^{(-3)}}{g}-\frac{\left(5cdg^2e^5+3cfge^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3cd^2g^2+2cdfge+3cf^2e^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2*g^2 + 2*c*d*f
*g*e + 3*c*f^2*e^2 + 8*a*g^2*e^2)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e
^(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(5/2)
```

maple [B] time = 0.04, size = 306, normalized size = 2.08

$$\frac{\left(8a\,e^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3c\,d^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+2cdefg\ln\left(\frac{2egx+dg+ef+2\sqrt{}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)
^(1/2))*a*e^2*g^2+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g
*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+4*(e*g)^(1/2)*((e*
x+d)*(g*x+f))^(1/2)*x*c*e*g-6*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*d*g-6*(
(e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)
^(1/2)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 20.13, size = 569, normalized size = 3.87

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}-\frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g\right.}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) -
f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*g))/(2*e^(5/2)*g^(5/2)) - (4*a*
atan((e*((f + g*x)^(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/
2)))))/(-e*g)^(1/2) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*e^3*f^2)/2 + (3*c
*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*
x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))
/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^7*((3*c
*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f + g*x)^(1/2) - f^(
1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2
)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (d^(1/2)*f^(1/
2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/
2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x)^(1/2) - f^(1/2)
)^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f + g*x)^(1/2) - f
^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((f + g*x)^(1/2) -
f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*((f + g*x)^(1/2) -
 f^(1/2))^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + cx^2}{\sqrt{d + ex}\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.604 $\int \frac{-1+2x^2}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx$

Optimal. Leaf size=16

$$\sqrt{x-1}\,x\sqrt{x+1}$$

```
[Out] x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {384}

$$\sqrt{x-1}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rule 384

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2), x] /; FreeQ[{a1, b1, a2, b2, c, d, n
, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d - b1*b2*
c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx = \sqrt{-1+x}\,x\sqrt{1+x}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 4.12

$$\frac{\sqrt{x-1}\left(x\sqrt{1-x^2}-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{\sqrt{1-x}}+2\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[-1 + x]*(x*Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x
] + 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]]
```

fricas [A] time = 0.66, size = 12, normalized size = 0.75

$$\sqrt{x+1}\,\sqrt{x-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

giac [A] time = 0.17, size = 12, normalized size = 0.75

$$\sqrt{x+1}\,\sqrt{x-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\sqrt{x-1}\,\sqrt{x+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(x-1)^(1/2)/(x+1)^(1/2),x)
[Out] x*(x-1)^(1/2)*(x+1)^(1/2)
```

maxima [C] time = 0.43, size = 9, normalized size = 0.56

$$\sqrt{x^2-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*x
```

mupad [B] time = 2.80, size = 16, normalized size = 1.00

$$\frac{\left(x^2+x\right)\sqrt{x-1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 1)/((x - 1)^(1/2)*(x + 1)^(1/2)),x)
[Out] ((x + x^2)*(x - 1)^(1/2))/(x + 1)^(1/2)
```

sympy [C] time = 43.48, size = 129, normalized size = 8.06

$$-\begin{cases} 2\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{|x+1|}{2}>1 \\ -2i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases} + \frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix} \middle| \frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 \\ -\frac{5}{4},-\frac{3}{4} \end{matrix}\right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
[Out] -Piecewise((2*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (-2*I*asin(s
qrt(2)*sqrt(x + 1)/2), True)) + meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
 ((-1, -3/4, -1/2, -1/4, 0, 0), ()), x**(-2))/(2*pi**(3/2)) - I*meijerg(((-
3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_
polar(2*I*pi)/x**2)/(2*pi**(3/2))
```

3.605 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=411

$$\frac{\left(\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)+\frac{a(ae}{}\right.}{ac\sqrt{\sqrt{-a}e+\sqrt{}}}$$

```
[Out] (3*d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/c/
g^(1/2)+e*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2
)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(-(c*d^2*
f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/c^(1/2))/a/c/(-e*
(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)
^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)
)^(1/2))*((c*d^2*f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/
c^(1/2))/a/c/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 2.51, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {904, 80, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)+\frac{a(ae}{}\right.}{ac\sqrt{\sqrt{-a}e+\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x])/c + (Sqrt[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[g]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqrt[g]) + (((a*(a*e^2*g - c*d
*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcTanh[
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*
Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*f - a
*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 904

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[g/c, Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
- 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c, Int[(Simp[c*d*f^2 - 2*a*e*f*g -
a*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*(f + g*x)
^(n - 2))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2
+ a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cd^2f-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{c} + \frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\int \left(\frac{-\frac{a(-ae^2g+cd(2ef+dg))}{\sqrt{c}}+\sqrt{-a}\left(cd^2f-ae(ef+2dg)\right)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\frac{a(-ae^2g+cd(2ef+dg))}{\sqrt{c}}+\sqrt{-a}\,(\cdots}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+\cdots}} \cdots}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{c} + \frac{\left(\frac{a(ae^2g-cd(2e\cdots}{\sqrt{c}}\cdots\right.}{\cdots}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}+\cdots\right.}{\cdots}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\sqrt{e}\,(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}-\sqrt{-a}\,(\cdots\right.}{ac\sqrt{\cdots}}$$

Mathematica [A] time = 2.44, size = 410, normalized size = 1.00

$$\frac{-\frac{\left(\sqrt{-a}\,cd^2+2a\sqrt{c}\,de+(-a)^{3/2}e^2\right)\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{a\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}} + \frac{\left(\sqrt{-a}\,cd^2-2a\sqrt{c}\,de+(-a)^{3/2}e^2\right)\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g\cdots}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e\cdots}}\right)}{a\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}} \cdots}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[f + g*x] + (Sqrt[c]*Sqrt[e*f - d*g]*(e*f + 3*
d*g)*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e
*f - d*g]])/(Sqrt[g]*Sqrt[f + g*x]) - ((Sqrt[-a]*c*d^2 + 2*a*Sqrt[c]*d*e +
(-a)^(3/2)*e^2)*Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f)
+ Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(a*Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]) + ((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*
e + (-a)^(3/2)*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f +
Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(
a*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]))/c^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.11, size = 2497, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c
*d*f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(e*g)^(1/2
)+d*g+e*f)/(e*g)^(1/2))*c*d*e*g+(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*
f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*e^2*f+(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*
e*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*
c)^(1/2)))*a*e^2*g-(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-
c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))
*c*d^2*g-2*(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c
)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f
+(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*
g)^(1/2)*ln((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+
e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e^2*g-(-a*c)^(1
/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*l
n((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)
^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^
(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g-2*(-a*c)^(1/2)*(((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((x*c*d
*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(
-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*
g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d*e*f+2*a*c*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+
x*c*d*g+x*c*e*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*
f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*
f)/(c*x-(-a*c)^(1/2)))*d*e*g+a*c*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g
-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2))
)*e^2*f-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1
/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f-2*a*c*(((-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((x*c*d*g+
x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(
-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*d*e*g-a*c*(((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2
)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x
+(-a*c)^(1/2)))*e^2*f+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(
```

```
1/2)*(e*g)^(1/2)*ln((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^
2+d*g*x+e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/
c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f
+2*(-a*c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c
*d*f)/c)^(1/2)*(e*g)^(1/2)*c*e)/(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)/(-a*c)^(1/2
)/c^2/(e*g)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2
)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

3.606 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=342

$$\frac{\left(-\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right.}{\sqrt{-a}\,c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}}}$$

```
[Out] 2*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)*g^(1/2)/c+ar
ctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^
(1/2)+d*c^(1/2))^(1/2))*(c*d*f-a*e*g-(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(
1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctan
h((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+
d*c^(1/2))^(1/2))*(c*d*f-a*e*g+(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 2.09, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {906, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(-\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right.}{\sqrt{-a}\,c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])]
)/c + ((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 906

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(e*g)/c, Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[(Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)
^(m - 1)*(f + g*x)^(n - 1))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{c} + \frac{(eg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c} + \frac{(2g)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\right.}{c}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+e}}}{2\sqrt{-a}\,c}$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-}\right.}{\sqrt{-a}\,c}$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f}}\right.}{\sqrt{-a}\,c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$

Mathematica [A] time = 1.27, size = 339, normalized size = 0.99

$$\frac{\frac{\frac{\left(\sqrt{-a}\sqrt{c}d-ae\right)\sqrt{\sqrt{-a}g+\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}} - \frac{\left(\sqrt{-a}\sqrt{c}d+ae\right)\sqrt{\sqrt{-a}g-\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e-\sqrt{c}d}}}{a} + \frac{2\sqrt{g}\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\sqrt{g}\right.}{\sqrt{f+gx}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] ((2*Sqrt[g]*Sqrt[e*f - d*g]*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g
]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/Sqrt[f + g*x] + (-(((Sqrt[-a]*Sqrt[c]*d
+ a*e)*Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a
]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[
-(Sqrt[c]*d) + Sqrt[-a]*e]) + ((Sqrt[-a]*Sqrt[c]*d - a*e)*Sqrt[Sqrt[c]*f +
Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt
[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/a)/c
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_0 x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.03, size = 1569, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+d*g+e*f+2*(e*g*x^2+d*g*x
+e*f*x+d*f)^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f)/c)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f)/c)^(1/2)*(-a*c)^(1/2)*e*g-(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*((-a*e*g
+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*(
-a*c)^(1/2)*d*g-(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*((-a*e*g+c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*(-a*c)^(1/2)*e
*f+(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*l
n((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*e*g-(e*g)^(1/2)*(-(a*e*g-
c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d
*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+
e*f*x+d*f)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
*c)/(c*x-(-a*c)^(1/2)))*c*d*f-(e*g)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a
*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*
f)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x
+(-a*c)^(1/2)))*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*
(-a*c)^(1/2)*d*g-(e*g)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g
```

```
*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-(a
*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)
))*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)*
e*f-(e*g)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/
2)*d*g-(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-(a*e*g-c*d*f+(-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*((-a*e*g+c
*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*a*e*g+(e*g)^(1/2)*ln((c*d*
g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+
2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*c*d*f)/(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)/(-a*c)^(1/2
)/c/(e*g)^(1/2)/(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)/
((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{gx+f}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/(c*x^2 + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(a + c*x^2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

3.607 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2
)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(
1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(g*(-a)^(1/2)+f*c
^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {910, 93, 208}

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}\,f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\sqrt{-a}\,f+\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right)dx$$

$$=\frac{1}{2}\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{af}{(-a)^{3/2}}+\frac{g}{\sqrt{c}}\right)\int \frac{}{\left(\sqrt{-\right.}}$$

$$=\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+\left(\right.$$

$$=\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}}$$

Mathematica [**A**] time = 0.35, size = 229, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}}{\sqrt{-a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(Sqr
t[c]*d) + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

fricas [**B**] time = 24.21, size = 1921, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a
^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e +
a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^
2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a
*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c
*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*
f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^
2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*
c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*
e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d
```

```
^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) - 1/4*sqrt(-(c*d*f
 + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(
e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g + (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-
(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^
2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*(c^2*d^
3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*s
qrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a
*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*f - c*d^2*g + (a*
c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f +
a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^
3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*
f*g - d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f +
(c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^
4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
,0] was discarded and replaced randomly by 0=[62,91]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.Non regular value [0,0] was discarded and replaced randomly by 0=[44,-43]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.Non regular value [0,0] was discarded and replaced r
andomly by 0=[-18,-31]Precision problem choosing root in common_EXT, curren
t precision 14Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{-2,[0,1]%%
%},0,%%%{1,[2,2]%%%}+%%%{2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values
[-27,26]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{-2,[0,1]%%%},0,%
%%{1,[2,2]%%%}+%%%{2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values [-89,6
3]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.Non regular value [0,0] was discarded and replace
d randomly by 0=[-59,-77]Precision problem choosing root in common_EXT, cur
rent precision 14Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.Non regular value [0,0] was discar
ded and replaced randomly by 0=[-37,-94]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regular
 value [0,0] was discarded and replaced randomly by 0=[-32,97]Warning, choo
sing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,
[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-82.3579015951,0]Warning,
choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%
{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-29.292030761,22]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.Non regular value [0,0] was discarded and replaced rando
mly by 0=[2,-99]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.Non regular value [0,0] was discard
ed and replaced randomly by 0=[-13,69]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [0,0] was discarded and replaced randomly by 0=[-55,-78]Warning, choos
```

```
ing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[
2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-57.0371161718,0]Warning, c
hoosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{
-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-53.6704242053,49]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.Non regular value [0,0] was discarded and replaced rando
mly by 0=[-20,-31]Precision problem choosing root in common_EXT, current pr
ecision 14Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.Non regular value [0,0] was discarded and
 replaced randomly by 0=[-67,8]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
,0] was discarded and replaced randomly by 0=[-69,98]Warning, choosing root
 of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[2,1]%%%}
+%%%{1,[2,0]%%%}] at parameters values [-41.1343540126,0]Warning, choosing
root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[2,1]
%%%}+%%%{1,[2,0]%%%}] at parameters values [-46.2420096635,-70]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.Non regular value [0,0] was discarded and replaced randomly by
0=[-53,73]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.Non regular value [0,0] was discarded and
 replaced randomly by 0=[-78,50]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.Non regular value [
0,0] was discarded and replaced randomly by 0=[-61,27]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.Non regular value [0,0] was discarded and replaced randomly by 0=[-18,-4
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.Non regular value [0,0] was discarded and replaced
 randomly by 0=[15,-93]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.Non regular value [0,0] was
discarded and replaced randomly by 0=[97,57]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.Non reg
ular value [0,0] was discarded and replaced randomly by 0=[70,-37]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.Non regular value [0,0] was discarded and replaced randomly
by 0=[8,40]Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.Non regular value [0,0] was discarded an
d replaced randomly by 0=[10,9]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
,0] was discarded and replaced randomly by 0=[85,-92]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.Non regular value [0,0] was discarded and replaced randomly by 0=[-83,95]
Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2
]%%%}+%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-49.355685115
3,0]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.Non regular value [0,0] was discarded and repla
ced randomly by 0=[66,42]Warning, need to choose a branch for the root of a
 polynomial with parameters. This might be wrong.Non regular value [0,0] wa
s discarded and replaced randomly by 0=[20,-21]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0,0] was discarded and replaced randomly by 0=[13,-34]Warnin
g, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+
%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-90.5690937298,0]Wa
rning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%
%%}+%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-36.6004387327,
-85]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.Non regular value [0,0] was discarded and repla
ced randomly by 0=[99,-89]Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.Non regular value [0,0] w
as discarded and replaced randomly by 0=[2,-9]Warning, need to choose a bra
```

```
nch for the root of a polynomial with parameters. This might be wrong.Non r
egular value [0,0] was discarded and replaced randomly by 0=[-74,46]Warning
, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%
%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-4.22288109735,0]War
ning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%
%}+%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-6.87379696826,-
21]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.Non regular value [0,0] was discarded and replac
ed randomly by 0=[-35,-95]Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.Non regular value [0,0] w
as discarded and replaced randomly by 0=[-9,27]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0,0] was discarded and replaced randomly by 0=[-19,90]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.Non regular value [0,0] was discarded and replaced random
ly by 0=[2,-39]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.Non regular value [0,0] was discarde
d and replaced randomly by 0=[55,-73]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.Non regular va
lue [0,0] was discarded and replaced randomly by 0=[61,1]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.Non regular value [0,0] was discarded and replaced randomly by 0=[10,
40]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.Non regular value [0,0] was discarded and replac
ed randomly by 0=[83,49]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.Non regular value [0,0] was
 discarded and replaced randomly by 0=[1,-81]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.Non re
gular value [0,0] was discarded and replaced randomly by 0=[76,-13]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.Non regular value [0,0] was discarded and replaced randomly
 by 0=[45,19]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.Non regular value [0,0] was discarded
and replaced randomly by 0=[37,-12]Evaluation time: 4.14index.cc index_m op
erator + Error: Bad Argument Value
```

maple [B] time = 0.04, size = 1383, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x)
```

```
[Out] -1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*
a*c*e^2*f*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((2*
(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*
x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1
/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e^2*g*(-a*c)^(1/2)*(-(a*e*g-c*d*f+(-a*
c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*
e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+
c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1
/2)))*c^2*d^2*f*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+
ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)
/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d^2*g*(-a*c)^(1/2)*(-(a*e*g-c*d*
f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x-2*(-a*c)^
(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a
```

```
*c)^(1/2)))*a*c*e^2*f*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^
(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^
(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-a*c)^(
1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e^2*g*((-a*e*g+c*d*f+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x-2*(
-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c
*x+(-a*c)^(1/2)))*c^2*d^2*f*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*
x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-
a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g*((-a*e*g+c*d*f
+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2))/((e*x+d)*(g*x+f)
)^(1/2)/(c*d-(-a*c)^(1/2)*e)/(-a*c)^(1/2)/(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f)/c)^(1/2)/((-a*c)^(1/2)*e+c*d)/((-a*e*g+c*d*f+(-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\left(cx^2+a\right)\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\left(a+cx^2\right)\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

3.608 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=351

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(ae^2+cd^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+a\ldots\right.}{\sqrt{-a}\,\sqrt{\sqrt{-a}\,e+\sqrt{\ldots}}}$$

```
[Out] -2*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c
*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/
2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1
/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 2.15, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {908, 37, 6725, 93, 208}

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(ae^2+cd^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+a\ldots\right.}{\sqrt{-a}\,\sqrt{\sqrt{-a}\,e+\sqrt{\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((c*d*f + a*e*g + Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f +
a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\,\sqrt{f+gx}\,\left(a+cx^2\right)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{\int \left(\frac{a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{-a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right)dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{2\sqrt{-a}\,\left(cd^2+ae^2\right)}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\right.}\right.}{\sqrt{-a}\,\left(cd^2+ae^2\right)}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{\left(cdf+aeg+\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right.}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(cd^2+ae^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}$$

Mathematica [A] time = 0.70, size = 265, normalized size = 0.75

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,\left(ae^2+cd^2\right)}+\frac{a\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{(-a)^{3/2}\left(\sqrt{-a}\,e-\sqrt{c}\,d\right)^{3/2}}+\frac{a\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}}}\right.}{(-a)^{3/2}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (a*Sqrt[-(Sqrt[c]*f)
 + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((-a)^(3/2)*(-(Sqrt[c]*d) + S
qrt[-a]*e)^(3/2)) + (a*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f
 + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]
)/((-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2))
```

fricas [B] time = 129.49, size = 5816, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2
)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2
 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*
c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10
 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))
*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*
d^2*e^2)*g^2 + 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*
a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c
*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*
f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6
*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 1
5*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e
^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2
)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2
 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*
c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10
 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))
 + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x + (2*(c^3*d^
7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^
2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a
^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a
^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c
^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^
10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a
^6*c*d^2*e^10 + a^7*e^12)))/x) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sq
rt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3
*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c
^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d
*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12
+ 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2
*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 +
3*a^3*c*d^2*e^4 + a^4*e^6))*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a
*d*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 +
 a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e +
3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*
a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*
c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^
12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*
c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sq
rt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3
*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c
^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d
*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12
+ 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2
*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 +
3*a^3*c*d^2*e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a
*d*e^3)*g^2)*x + (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^
6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*
d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^
4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^
3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)
/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6
```

```
 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) + (c*d^3 + a*d*e^
2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e -
a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sq
rt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 1
0*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c
*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a
^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*log(((3*c*d^2*e^2 -
a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 + 2*((3*
c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d
*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)
*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e
- 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^
2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 2
0*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqr
t(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e -
a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sq
rt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 1
0*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c
*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a
^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 -
a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x - (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 +
 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d
^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*
e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*
c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*
e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d
^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e
^12)))/x) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c
*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*
a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^
6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6
- 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2
+ 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^
2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4
*e^6))*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4
- 3*a*d^2*e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^
5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3
*a^3*c*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c
*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d
^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e
^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c
*d^2*e^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c
*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*
a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^
6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6
- 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2
+ 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^
2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4
*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x - (2*(
c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e +
3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2
 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e
^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*
g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*
c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8
 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) + 8*sqrt(e*x + d)*sqrt(g*x + f)*e)/(c*
d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [**B**] time = 0.06, size = 5383, normalized size = 15.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

3.609 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx$

Optimal. Leaf size=613

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{}{3\sqrt{d+e}}$$

```
[Out] -2/3*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(3/2)+4/3*e*g*(g*x+f)^(1/2)/(a*e
^2+c*d^2)/(-d*g+e*f)/(e*x+d)^(1/2)+e*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(
1/2))*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(
1/2))/(e*x+d)^(1/2)-e*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(g*x+f)^(
1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^
(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*
c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-g*(-a)^
(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f*(-a)^(1/2)+a*
e*g*(-a)^(1/2)+a*(-d*g+e*f)*c^(1/2))/a/(a*e^2+c*d^2)/(e*(-a)^(1/2)+d*c^(1/2
))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 3.16, antiderivative size = 613, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {908, 45, 37, 6725, 96, 93, 208}

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{}{3\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*e*g*Sqrt[f +
g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (e*(c*d*f + a*e*g - S
qrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[-
a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d - Sq
rt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(S
qrt[-a]*c*d*f + Sqrt[-a]*a*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[
c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*
x])])/(a*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{\int \left(\frac{a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{-a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+g}\cdots}\right.}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\,\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{\cdots}\right.}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\cdots}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{\cdots}\right.}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\cdots}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{\cdots}\right.}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\cdots}$$

Mathematica [A] time = 2.86, size = 353, normalized size = 0.58

$$-\frac{2e\sqrt{f+gx}\,\left(ae^3(f+gx)+cd\left(-6d^2g+7def-5degx+6e^2fx\right)\right)}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)^2(ef-dg)} - \frac{\sqrt{c}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f\cdots}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d\cdots}}\right.}{\left(\sqrt{-a}\,e-\sqrt{c}\,d\right)^{3/2}\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x]*(a*e^3*(f + g*x) + c*d*(7*d*e*f - 6*d^2*g + 6*e^2*f*x -
 5*d*e*g*x)))/(3*(c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*x)^(3/2)) - (Sqrt[c]*
Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sq
rt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((-(Sqrt[c]*
d) + Sqrt[-a]*e)^(3/2)*(Sqrt[-a]*Sqrt[c]*d + a*e)) - (Sqrt[c]*Sqrt[Sqrt[c]*
f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[
Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(S
qrt[-a]*Sqrt[c]*d - a*e))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.09, size = 14861, normalized size = 24.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\left(cx^2+a\right)(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

3.610 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=337

$$\frac{\left(-2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\cdots$$

```
[Out] 2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)+ar
ctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^
(1/2)+d*c^(1/2))^(1/2))*(c*d^2-a*e^2-2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e
*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^
(1/2))^(1/2))*(c*d^2-a*e^2+2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(e*(-a)^(
1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 2.46, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {910, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(-2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((
c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*
c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{cd^2-ae^2+2cdex}{c\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\right) dx$$

$$= \frac{\int \frac{cd^2-ae^2+2cdex}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-2a\sqrt{c}\,de+\sqrt{-a}\,(cd^2-ae^2)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{2a\sqrt{c}\,de+\sqrt{-a}\,(cd^2-ae^2)}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right) dx}{c} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}\ldots}}\ldots\right)}{c}$$

$$= \frac{(2e)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cd^2-2\sqrt{-a}\,\sqrt{c}\,de-ae^2\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{\ldots}}\ldots}{2\sqrt{-a}\,c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{\left(cd^2-2\sqrt{-a}\,\sqrt{c}\,de-ae^2\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}\,d+\sqrt{-a}\,e-\left(-\sqrt{c}\,f+\ldots\right.}\ldots\right)}{\sqrt{-a}\,c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\,\sqrt{c}\,de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}} - \ldots$$

Mathematica [A] time = 1.09, size = 339, normalized size = 1.01

$$\frac{\frac{\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\left(\sqrt{-a}\,\sqrt{c}\,d-ae\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}} - \frac{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}}{a} + \frac{2(ef-dg)^{3/2}\left(\frac{e(f+gx)}{ef-dg}\right)^{3/2}\sinh^{-1}\left(\ldots\right.}{\sqrt{g}\,(f+gx)^{3/2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((2*(e*f - d*g)^(3/2)*((e*(f + g*x))/(e*f - d*g))^(3/2)*ArcSinh[(Sqrt[g]*Sq
rt[d + e*x])/Sqrt[e*f - d*g]])/(Sqrt[g]*(f + g*x)^(3/2)) + (-((Sqrt[-(Sqrt[
c]*d) + Sqrt[-a]*e]*(Sqrt[-a]*Sqrt[c]*d + a*e)*ArcTanh[(Sqrt[-(Sqrt[c]*f) +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])
])/Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]) + (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[-
a]*Sqrt[c]*d - a*e)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[Sqrt[c]*f + Sqrt[-a]*g])/
a)/c
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 2336, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+
f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*e^2*g^2*(-a*c)^(1/2)*(-(a*e*g-c*d*f+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f)/c)^(1/2)+2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1
/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*e^2*f^2*(-a*c)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(
e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c
*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2)*(-(a*e*g-c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(
-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/
2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c
)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f)/c)^(1/2)-2*ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*d*e*g^2*(-a*
c)^(1/2)*(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(
1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*
```

```
c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f^2*(e*g)^(1/2)*(-(a*
e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-2*ln((c*d*g*x+c*e*f*x
+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*
(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c
)/(c*x-(-a*c)^(1/2)))*c*d*e*f^2*(-a*c)^(1/2)*(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*
c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*
(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(
1/2)))*a^2*e^2*g^2*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2
)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(
1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x
+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*(
(e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)
^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-2*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c
)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(
-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1
/2)))*a*d*e*g^2*(-a*c)^(1/2)*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(
-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f
^2*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-2
*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(
1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*c*d*e*f^2*(-a*c)^(1/2)*(e*g)^(1/2
)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2))/((e*x+d)*(g*x
+f))^(1/2)/(c*f-g*(-a*c)^(1/2))/(-a*c)^(1/2)/(e*g)^(1/2)/(-(a*e*g-c*d*f+(-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)/(g*(-a*c)^(1/2)+c*f)/((-a*e*g+c*d*
f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

3.611 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2
)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(
1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(e*(-a)^(1/2)+d*c
^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {910, 93, 208}

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+cx^2\right)}dx = \int \left(\frac{\sqrt{-a}\,d-\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\sqrt{-a}\,d+\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right)dx$$
$$=\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}dx+\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}+\frac{e}{\sqrt{c}}\right)\int \frac{1}{(\sqrt{}}$$
$$=\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+\left((-\right.$$
$$=\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Mathematica [A] time = 0.33, size = 229, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}}{\sqrt{-a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(Sqr
t[c]*f) + Sqrt[-a]*g] - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)]])/Sqrt[Sqrt[c]*f + Sqrt[-a]*g])/(Sqrt[-a]*Sqrt[c])
```

fricas [B] time = 20.73, size = 1913, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a
^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g +
a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2
*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^
2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^
2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^
2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*
c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2
*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^
2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*
f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*
g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f
*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^
2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e
*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g
```

```
 + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) - 1/4*sqrt(-(c
*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2
)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*lo
g(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g - (a*c^2*f^2*g + a^2*c*g^3)*sq
rt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*
g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*
c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2
 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*c^2
*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3
)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 +
 a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(
-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4
)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*f^2 - c*d*f*g
 - (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f
^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)
/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2
*(e^2*f*g - d*e*g^2)*x + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*
f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 1387, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)
*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g
-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a
*c*d*g^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d
*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f
+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*e*g^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)
^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-
(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/
2)))*c^2*d*f^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-l
n((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/
2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*c*e*f^2*((-a*e*g+c*d*f+(-a*c)^(1/2)
*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*
(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1
/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*
c)^(1/2)))*a*c*d*g^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(
1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*e*g^2*(-(a*e*g-c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*
```

```
d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+
f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*
x-(-a*c)^(1/2)))*c^2*d*f^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*c*e*f^2*(-(a*e*g-c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2))/((e*x+d)*(g*x+f))
^(1/2)/(c*f-(-a*c)^(1/2)*g)/(-a*c)^(1/2)/(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)/(c*f+(-a*c)^(1/2)*g)/((-a*e*g+c*d*f+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

3.612 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=230

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1
/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1/2)+
d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {912, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt
[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+g}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{2\sqrt{-a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{}\right.}{\sqrt{}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [A] time = 0.19, size = 225, normalized size = 0.98

$$-\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e-\sqrt{c}d}\sqrt{\sqrt{-a}g-\sqrt{c}f}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (-(ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*
d) + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[-(Sq
rt[c]*f) + Sqrt[-a]*g])) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e
*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[Sqrt[c]*d + Sqrt[-
a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/Sqrt[-a]
```

fricas [B] time = 38.21, size = 4325, normalized size = 18.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3
*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*
c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c
*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^
2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g +
 d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e +
 a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)
*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2
*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d
*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*
(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a
^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*
(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d
```

```
*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*s
qrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2
+ a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^
2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f
- a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(
c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*
c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2
 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*
e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e + a^2*c*e^3)*f^3 +
(a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^
3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*s
qrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*
f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2
)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a
^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*
(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e
^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c
*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 +
2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4
 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 +
 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) - 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*
d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*
e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(
a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4
*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^
3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2
+ (c*d^2 - a*e^2)*f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c
*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3
)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e
^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2
*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 +
 a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*
a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a
^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^
2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2
)*x - (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*
d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*
f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f
^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)
/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e
^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((
e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*
f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a
^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2
 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*
f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqr
t(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 +
a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2
```

```
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)
*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d^3
+ a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f
^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 +
 a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 1415, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] -1/2*c^2*(ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(-(a*e*g-c*d
*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+
2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^
(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-
a*c)^(1/2)))*a*c*d^2*g^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/
c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-(a*e*g-c*d*
f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2
*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(
1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a
*c)^(1/2)))*c^2*d^2*f^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c
)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(
-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*((-a*e*g+c*d*f
+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*
(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1
/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*
c)^(1/2)))*a*c*d^2*g^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)
^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-
a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*((-a*e*g+c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(
-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/
2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c
)^(1/2)))*c^2*d^2*f^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^
(1/2))*(g*x+f)^(1/2)*(e*x+d)^(1/2)/((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f)/c)^(1/2)/(c*f+(-a*c)^(1/2)*g)/(c*d+(-a*c)^(1/2)*e)/(-(a*e*g-c*d*f
+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)/(-a*c)^(1/2)/(c*f-(-a*c)^(1/2)
*g)/(c*d-(-a*c)^(1/2)*e)/((e*x+d)*(g*x+f))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + a\right)\sqrt{ex + d}\,\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + cx^2\right)\sqrt{d + ex}\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.613 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=354

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-}}\right.}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}}}$$

```
[Out] -e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1
/2)+e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^
(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)
)^(3/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)
+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a
)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {912, 96, 93, 208}

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-}}\right.}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d +
 e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*
Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - S
qrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqr
t[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*
g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/}}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-}$$

Mathematica [A] time = 0.70, size = 287, normalized size = 0.81

$$\frac{\frac{2\sqrt{-a}\,e^2\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)(dg-ef)} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\left(\sqrt{-a}\,e-\sqrt{c}\,d\right)^{3/2}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[-a]*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*
x]) + (Sqrt[c]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/((-(Sqrt[c]*d) + Sqrt[-a]*e)^
(3/2)*Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/Sqrt[-a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.09, size = 10977, normalized size = 31.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

3.614 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)} dx$

Optimal. Leaf size=625

$$\frac{2\sqrt{d+ex}\,(ef-dg)}{\sqrt{f+gx}\,\left(ag^2+cf^2\right)}-\frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\,\left(ag^2+cf^2\right)}-\frac{\sqrt{e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d}}{\sqrt{e}\,\sqrt{f}}\right.}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(ag^2+cf^2\right)}$$

```
[Out] -2*(-d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/
(a*g^2+c*f^2)/g^(1/2)-arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*
(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2
)/c^(1/2)/g^(1/2)+arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*(c*d
*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^
(1/2)/g^(1/2)+2*(-d*g+e*f)*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctan
h((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2
)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(-e*(-a)^(1
/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(-g*(-a)^(1/2)+f*c^(1
/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/
2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/
2))*(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 2.53, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {908, 47, 63, 217, 206, 6725, 105, 93, 208}

$$\frac{2\sqrt{d+ex}\,(ef-dg)}{\sqrt{f+gx}\,\left(ag^2+cf^2\right)}-\frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\,\left(ag^2+cf^2\right)}-\frac{\sqrt{e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d}}{\sqrt{e}\,\sqrt{f}}\right.}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(ag^2+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[e]*
(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt
[g]*(c*f^2 + a*g^2)) - (Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqr
t[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*
(e*f - d*g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqr
t[-a]*Sqrt[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d
*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-
a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-
a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - (Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[
Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(a+cx^2\right)}dx = \frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}\left(a+cx^2\right)}dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}dx}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\int \left(\frac{\left(-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{f+gx}} + \frac{\left(a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf\right.}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{}}\right.}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(2(ef-dg))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}dx, x, \sqrt{d+ex}\right)}{cf^2+ag^2} - \frac{(cd}{}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(2(ef-dg))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{cf^2+ag^2} - \frac{\left(e\left(cdf+a\right.\right.}{}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} - \frac{\left(cdf+aeg-\sqrt{-a}\right.}{}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} + \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd}{}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} - \frac{\sqrt{e}\left(cdf+aeg-\sqrt{-}\right.}{\sqrt{-a}}$$

Mathematica [A] time = 0.73, size = 336, normalized size = 0.54

$$-\left(\left(\frac{d}{\sqrt{-a}}-\frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)} + \frac{\sqrt{\sqrt{-a}e-\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\left(\sqrt{-a}g-\sqrt{c}f\right)^{3/2}}\right)\right) - \left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -((d/Sqrt[-a] - e/Sqrt[c])*(Sqrt[d + e*x]/((Sqrt[c]*f - Sqrt[-a]*g)*Sqrt[f
+ g*x]) + (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqr
t[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/(
-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2))) - ((a*d)/(-a)^(3/2) - e/Sqrt[c])*(Sqrt[d
 + e*x]/((Sqrt[c]*f + Sqrt[-a]*g)*Sqrt[f + g*x]) - (Sqrt[Sqrt[c]*d + Sqrt[-
a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.07, size = 8264, normalized size = 13.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+a\right)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

3.615 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=351

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}+\frac{\left(-\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(ag^2+cf^2)}-\frac{\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+\ldots\right.}{\sqrt{-a}\sqrt{\sqrt{-a}e+\ldots}}$$

```
[Out] -2*g*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c
*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/
2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1
/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A] time = 1.81, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {908, 37, 6725, 93, 208}

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}+\frac{\left(-\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(ag^2+cf^2)}-\frac{\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+\ldots\right.}{\sqrt{-a}\sqrt{\sqrt{-a}e+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((c*d*f + a*e*g - Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f +
a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\int \left(\frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right) dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}}{2\sqrt{-a}\left(cf^2+ag^2\right)}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(-\sqrt{}}\right.}{\sqrt{-a}\left(cf^2+ag^2\right)}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+e}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+g}}\right.}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(cf^2+ag^2\right)}$$

Mathematica [A] time = 0.64, size = 265, normalized size = 0.75

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)} + \frac{a\sqrt{\sqrt{-a}e-\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{(-a)^{3/2}\left(\sqrt{-a}g-\sqrt{c}f\right)^{3/2}} + \frac{a\sqrt{\sqrt{-a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{}}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{}}}\right.}{(-a)^{3/2}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (a*Sqrt[-(Sqrt[c]*d)
 + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((-a)^(3/2)*(-(Sqrt[c]*f) + S
qrt[-a]*g)^(3/2)) + (a*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f
 + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]
)/((-a)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.06, size = 5383, normalized size = 15.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

3.616 $\int \frac{1}{\sqrt{d+ex}\,(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=354

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-}}\right.}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\right.}$$

```
[Out] g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/
2)-g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(
1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-
e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))
^(3/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+
f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)
^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {912, 96, 93, 208}

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-}}\right.}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g
*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*Sqr
t[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d -
Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c
]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,(\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{2\sqrt{-a}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-d}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-d}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-d}$$

Mathematica [A] time = 0.78, size = 287, normalized size = 0.81

$$\frac{\frac{2\sqrt{-a}\,g^2\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\left(\sqrt{-a}\,g-\sqrt{c}\,f\right)^{3/2}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)^{3/2}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[-a]*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])
 + (Sqrt[c]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-
(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]
*(-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f + Sq
rt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2)))/Sqrt[-a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.09, size = 10977, normalized size = 31.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)\sqrt{ex+d}\,\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)\sqrt{d+ex}\,\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

3.617 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=549

$$-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{\cdots}{\sqrt{-a}\sqrt{f\cdots}}$$

```
[Out] c*arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(
-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-g*
(-a)^(1/2)+f*c^(1/2))^(3/2)-c*arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2)
)^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1
/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)-e/(-d*g+e*f)/(-a)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+e/(-d*g+e*f)/(-a)^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1/2
)-(d*g+e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1/2)+(d*g+
e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)
)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)
```

Rubi [A] time = 1.32, antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {912, 104, 152, 12, 93, 208}

$$-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{\cdots}{a\sqrt{f+g\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt
[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*
x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(Sqrt[c
]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x]) + (
c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sq
rt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)*(Sqrt[c
]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d
 + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+e\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}}dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}}dx}{2\sqrt{-a}}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-}\right.}$$

Mathematica [A] time = 2.07, size = 521, normalized size = 0.95

$$\frac{\frac{e}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}\,e-\sqrt{c}\,d\right)} + \frac{e}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)} + \frac{g\sqrt{d+ex}\left(2\sqrt{-a}\,eg+\sqrt{c}\,(dg+ef)\right)}{\sqrt{f+gx}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)(ef-dg)} + \frac{\frac{g\sqrt{d+ex}\left(2\sqrt{-a}\,eg-\sqrt{c}\,(dg+ef)\right)}{\sqrt{f+gx}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)}+}{\sqrt{c}}}{\sqrt{-a}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (e/((-(Sqrt[c]*d) + Sqrt[-a]*e)*Sqrt[d + e*x]*Sqrt[f + g*x]) + e/((Sqrt[c]*
d + Sqrt[-a]*e)*Sqrt[d + e*x]*Sqrt[f + g*x]) + (g*(2*Sqrt[-a]*e*g + Sqrt[c]
*(e*f + d*g))*Sqrt[d + e*x])/((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a
]*g)*(e*f - d*g)*Sqrt[f + g*x]) + ((g*(2*Sqrt[-a]*e*g - Sqrt[c]*(e*f + d*g)
)*Sqrt[d + e*x])/((Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g*x]) + (c*
(e*f - d*g)*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-
(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]
*(-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2)))/(Sqrt[c]*d - Sqrt[-a]*e) + (c*(-(e*f)
+ d*g)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d
 + Sqrt[-a]*e]*Sqrt[f + g*x])])/((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f
+ Sqrt[-a]*g)^(3/2)))/(Sqrt[-a]*(e*f - d*g))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.23, size = 30656, normalized size = 55.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)^{3/2}\,(c\,x^2+a)\,(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

3.618 $\int \frac{\sqrt{x}}{\sqrt{1+x}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[Out] -1/2*(1-I)^(3/2)*arctanh((1-I)^(1/2)*x^(1/2)/(1+x)^(1/2))-1/2*(1+I)^(3/2)*a
rctanh((1+I)^(1/2)*x^(1/2)/(1+x)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {910, 93, 208}

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]])/2 - ((1 + I)^(3
/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{1+x}\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(i-x)\sqrt{x}\,\sqrt{1+x}}+\frac{1}{2\sqrt{x}\,(i+x)\sqrt{1+x}}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{1+x}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{1+x}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{i-(1+i)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+x}}\right)+\text{Subst}\left(\int \frac{1}{i+(1-i)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+x}}\right)\\
&= -\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{1+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.97

$$\frac{1}{2}\left(-(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-(1+i)^{3/2}\tanh^{-1}\left(\sqrt{1+i}\sqrt{\frac{x}{x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] (-((-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]]) - (1 + I)^(3/2)*Arc
Tanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]])/2
```

fricas [B] time = 0.75, size = 744, normalized size = 11.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) +
8*x^2 + 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x
+ 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/8*2^(1/4)*sqrt(
2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 - 2*(2^(1/4
)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*sqrt
(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*ar
ctan(1/7*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) - 1/
7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2)
+ 8*x^2 - 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(
x + 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqr
t(2) + 6) - (2^(3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt
(2) + 4) + 16*sqrt(2) + 8) - 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt
(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*
(3*x + 2) + 5*x + 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2
) + 4) - 4/7*x - 5/7) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) + 1/7*sqrt(2)*(sqrt(
2)*(5*x + 1) + 6*x + 4) + 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 + 2*(2^(
1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(
3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt(2) + 4) + 16*sq
rt(2) + 8) + 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt(2) + 5) + 2*2^(
1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*(3*x + 2) + 5*x
+ 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2) + 4) + 4/7*x +
 5/7)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.17, size = 305, normalized size = 4.69

$$\sqrt{\frac{(x+1)x}{\left(x+\sqrt{2}-1\right)^{2}}}\left(x+\sqrt{2}-1\right)\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{(x+1)x}{\left(x+\sqrt{2}-1\right)^{2}}}}{\sqrt{1+\sqrt{2}}}\right)-6\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{(x+1)x}{\left(x+\sqrt{2}-1\right)^{2}}}}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{-2+2\sqrt{2}}\sqrt{1+\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^2+1)/(x+1)^(1/2),x)

[Out] 1/4/x^(1/2)/(x+1)^(1/2)*(x*(x+1)/(2^(1/2)-1+x)^2)^(1/2)*(2^(1/2)-1+x)*((-2+2*2^(1/2))^(1/2)*arctan(1/4*((3*2^(1/2)-4)*x*(x+1)*(4+3*2^(1/2))/(2^(1/2)-1+x)^2)^(1/2)*(-2+2*2^(1/2))^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)*(2^(1/2)-1+x)/x/(x+1))*(1+2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*arctan(1/4*((3*2^(1/2)-4)*x*(x+1)*(4+3*2^(1/2))/(2^(1/2)-1+x)^2)^(1/2)*(-2+2*2^(1/2))^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)*(2^(1/2)-1+x)/x/(x+1))*(1+2^(1/2))^(1/2)+4*arctanh(2^(1/2)*(x*(x+1)/(2^(1/2)-1+x)^2)^(1/2)/(1+2^(1/2))^(1/2))*2^(1/2)-6*arctanh(2^(1/2)*(x*(x+1)/(2^(1/2)-1+x)^2)^(1/2)/(1+2^(1/2))^(1/2)))*2^(1/2)/(3*2^(1/2)-4)/(1+2^(1/2))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x^2+1\right)\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/((x^2 + 1)*sqrt(x + 1)), x)

mupad [B] time = 8.49, size = 1610, normalized size = 24.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/((x^2 + 1)*(x + 1)^(1/2)),x)

[Out] - atan(((((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) - ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) - (12079595520*x)/((x + 1)^(1/2) - 1)^2 + 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (13555990528*x)/((x + 1)^(1/2) - 1)^2 + 9529458688) + (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i - (((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*((13555990528*x)/((x + 1)^(1/2) - 1)^2 - ((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) + ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (12079595520*x)/((x + 1)^(1/2) - 1)^2 - 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + 9529458688) - (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i)/((((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) - ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) - (12079595520*x)/((x + 1)^(1/2) - 1)^2 + 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (13555990528*x)/((x + 1)^(1/2) - 1)^2 + 9529458688) + (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*((13555990528*x)/((x + 1)^(1/2) - 1)^2 - ((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^

```
(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) + ((531502202880*x)/((x
 + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/
16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)
) + (12079595520*x)/((x + 1)^(1/2) - 1)^2 - 68451041280))*((- 2^(1/2)/16 -
1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + 9529458688) - (3556769792*x^(1/2
))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2)) + (7549747200*x)/((x + 1)^(1/2) - 1)^2 + 503316480))*((- 2^(1/2)/16
- 1/16)^(1/2)*2i - (2^(1/2)/16 - 1/16)^(1/2)*2i) - atan(((x^(1/2)*(- 2^(1/2
)/16 - 1/16)^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2)*678
4i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(3/2)*6784i)/((x + 1
)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2)
- 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2) - 1) + (x^
(1/2)*(2^(1/2)/16 - 1/16)^2*(- 2^(1/2)/16 - 1/16)^(1/2)*134400i)/((x + 1)^(
1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^(1/2)/16 + 1/16)^2*134400
i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)*(- 2^(1/2)/16 - 1/16)
^(1/2)*20352i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^
(1/2)/16 + 1/16)*20352i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
*(- 2^(1/2)/16 - 1/16)^(3/2)*268800i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/
2)/16 - 1/16)^(3/2)*(2^(1/2)/16 + 1/16)*268800i)/((x + 1)^(1/2) - 1))/(4544
*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 65280*(2^(1/2)/16
- 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2) + 65280*(2^(1/2)/16 - 1/16)^(3/2)
*(- 2^(1/2)/16 - 1/16)^(1/2) + 345600*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/
16 - 1/16)^(5/2) + 1152000*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^
(3/2) + 345600*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + x/((
x + 1)^(1/2) - 1)^2 + (6464*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/1
6)^(1/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(
1/2)/16 - 1/16)^(3/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)
^(3/2)*(- 2^(1/2)/16 - 1/16)^(1/2))/((x + 1)^(1/2) - 1)^2 - (760320*x*(2^(1
/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2))/((x + 1)^(1/2) - 1)^2 - (
2534400*x*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^(3/2))/((x + 1)^(
1/2) - 1)^2 - (760320*x*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/
2))/((x + 1)^(1/2) - 1)^2 + 1))*((- 2^(1/2)/16 - 1/16)^(1/2)*2i + (2^(1/2)/
16 - 1/16)^(1/2)*2i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x+1}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/(sqrt(x + 1)*(x**2 + 1)), x)
```

3.619 $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$

Optimal. Leaf size=80

$$\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

```
[Out] 1/5*(f+g)^2*(1+x)^4/(-x^2+1)^(5/2)+1/15*(f-9*g)*(f+g)*(1+x)^3/(-x^2+1)^(3/2
)-g^2*arcsin(x)+2*g^2*(1+x)/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {853, 1635, 789, 653, 216}

$$\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] ((f + g)^2*(1 + x)^4)/(5*(1 - x^2)^(5/2)) + ((f - 9*g)*(f + g)*(1 + x)^3)/(
15*(1 - x^2)^(3/2)) + (2*g^2*(1 + x))/Sqrt[1 - x^2] - g^2*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx &= \int \frac{(1+x)^4(f+gx)^2}{\left(1-x^2\right)^{7/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} - \frac{1}{5}\int \frac{(1+x)^3\left(-f^2+8fg+4g^2+5g^2x\right)}{\left(1-x^2\right)^{5/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + g^2\int \frac{(1+x)^2}{\left(1-x^2\right)^{3/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\sin^{-1}(x)
\end{aligned}
$$

Mathematica [C] time = 0.14, size = 91, normalized size = 1.14

$$
\frac{\sqrt{1-x^2}\left((x+1)^{3/2}\left(f^2(x-4)+fg(2-8x)+g^2(x-4)\right)-20\sqrt{2}g^2(x-1)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)\right)}{15(x-1)^3\sqrt{x+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] (Sqrt[1 - x^2]*((f*g*(2 - 8*x) + f^2*(-4 + x) + g^2*(-4 + x))*(1 + x)^(3/2)
 - 20*Sqrt[2]*g^2*(-1 + x)*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - x)/2]))
/(15*(-1 + x)^3*Sqrt[1 + x])
```

fricas [B] time = 1.22, size = 193, normalized size = 2.41

$$
\frac{2\left(2f^2-fg+12g^2\right)x^3-6\left(2f^2-fg+12g^2\right)x^2-4f^2+2fg-24g^2+6\left(2f^2-fg+12g^2\right)x+30\left(g^2x^3-3g^2\right.}{15\left(x^3-\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(2*(2*f^2 - f*g + 12*g^2)*x^3 - 6*(2*f^2 - f*g + 12*g^2)*x^2 - 4*f^2 +
 2*f*g - 24*g^2 + 6*(2*f^2 - f*g + 12*g^2)*x + 30*(g^2*x^3 - 3*g^2*x^2 + 3*
g^2*x - g^2)*arctan((sqrt(-x^2 + 1) - 1)/x) + ((f^2 - 8*f*g - 39*g^2)*x^2 -
 4*f^2 + 2*f*g - 24*g^2 - 3*(f^2 + 2*f*g - 19*g^2)*x)*sqrt(-x^2 + 1))/(x^3
- 3*x^2 + 3*x - 1)
```

giac [B] time = 0.35, size = 266, normalized size = 3.32

$$
-g^2\arcsin(x)+\frac{2\left(4f^2-2fg+24g^2+\frac{5f^2\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{10fg\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{105g^2\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{25f^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\ldots\right.}{\ldots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

```
[Out] -g^2*arcsin(x) + 2/15*(4*f^2 - 2*f*g + 24*g^2 + 5*f^2*(sqrt(-x^2 + 1) - 1)/
x - 10*f*g*(sqrt(-x^2 + 1) - 1)/x + 105*g^2*(sqrt(-x^2 + 1) - 1)/x + 25*f^2
*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10*f*g*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165*g^2*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 15*f^2*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30*f*g*(sqr
t(-x^2 + 1) - 1)^3/x^3 + 75*g^2*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15*f^2*(sqrt(-
x^2 + 1) - 1)^4/x^4 + 15*g^2*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2 + 1) -
 1)/x + 1)^5
```

maple [A] time = 0.02, size = 125, normalized size = 1.56

$$\left(-\arcsin(x)+\frac{\left(-2x-(x-1)^2+2\right)^{\frac{3}{2}}}{(x-1)^2}+\sqrt{-2x-(x-1)^2+2}\right)g^2+\frac{2\left(f+g\right)\left(-2x-(x-1)^2+2\right)^{\frac{3}{2}}g}{3\,(x-1)^3}+\left(f^2+2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x)
```

```
[Out] 2/3*g*(f+g)/(x-1)^3*(-(x-1)^2-2*x+2)^(3/2)+g^2*(1/(x-1)^2*(-(x-1)^2-2*x+2)^
(3/2)+(-(x-1)^2-2*x+2)^(1/2)-arcsin(x))+(f^2+2*f*g+g^2)*(1/5/(x-1)^4*(-(x-1
)^2-2*x+2)^(3/2)-1/15/(x-1)^3*(-(x-1)^2-2*x+2)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(gx+f\right)^2\sqrt{-x^2+1}}{(x-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*sqrt(-x^2 + 1)/(x - 1)^4, x)
```

mupad [B] time = 2.96, size = 164, normalized size = 2.05

$$\sqrt{1-x^2}\left(\frac{\frac{f^2}{3}+2\,f\,g+\frac{5\,g^2}{3}}{x-1}-\frac{\frac{f^2}{3}+2\,f\,g+\frac{5\,g^2}{3}}{(x-1)^2}\right)-\sqrt{1-x^2}\left(\frac{\frac{2\,f^2}{5}+\frac{4\,f\,g}{5}+\frac{2\,g^2}{5}}{(x-1)^3}+\frac{\frac{4\,f^2}{15}+\frac{8\,f\,g}{15}+\frac{4\,g^2}{15}}{x-1}-\frac{\frac{4\,f^2}{15}+\frac{8\,f\,g}{15}}{(x-1)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(1 - x^2)^(1/2))/(x - 1)^4,x)
```

```
[Out] (1 - x^2)^(1/2)*((2*f*g + f^2/3 + (5*g^2)/3)/(x - 1) - (2*f*g + f^2/3 + (5*
g^2)/3)/(x - 1)^2) - (1 - x^2)^(1/2)*(((4*f*g)/5 + (2*f^2)/5 + (2*g^2)/5)/(
x - 1)^3 + ((8*f*g)/15 + (4*f^2)/15 + (4*g^2)/15)/(x - 1) - ((8*f*g)/15 + (
4*f^2)/15 + (4*g^2)/15)/(x - 1)^2) - g^2*asin(x) - ((1 - x^2)^(1/2)*(2*f*g
+ 4*g^2))/(x - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)}\,\left(f+gx\right)^2}{(x-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-x**2+1)**(1/2)/(1-x)**4,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*(f + g*x)**2/(x - 1)**4, x)
```

3.620 $\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

```
[Out] -(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2)
/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {853, 1654, 844, 216, 725, 204}

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^2(c+dx)}\,dx &= \int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\,\mathrm{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,\frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}$$

Mathematica [C] time = 0.30, size = 148, normalized size = 1.38

$$-\frac{\frac{i(d-ac)^2\log\left(\frac{2d^3\left(\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}+ia^2cx+id\right)}{(d-ac)^2\sqrt{a^2c^2-d^2}(c+dx)}\right)}{\sqrt{a^2c^2-d^2}}+d\sqrt{1-a^2x^2}+(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -((d*Sqrt[1 - a^2*x^2] + (a*c - 2*d)*ArcSin[a*x] + (I*(-(a*c) + d)^2*Log[(2
*d^3*(I*d + I*a^2*c*x + Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2]))/((-(a*c) +
d)^2*Sqrt[a^2*c^2 - d^2]*(c + d*x))])/Sqrt[a^2*c^2 - d^2])/d^2)
```

fricas [A] time = 1.47, size = 318, normalized size = 2.97

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-\left(a^2c^2-d^2\right)\sqrt{-a^2x^2+1}-\left(acd+d^2+\left(a^3c^2+a^2cd\right)x+\sqrt{-a^2x^2+1}\left(acd+d^2\right)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\,\mathrm{arc}}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
```

```
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

giac [B] time = 0.59, size = 208, normalized size = 1.94

$$-\left(\frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)}{ad}-\frac{2\left(ac\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)-2\,d\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)\right)\arctan\left(\sqrt{-\frac{2}{ax-1}-1}\right)}{ad^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sgn(1/(a*x - 1))*sgn(a)/(a*d) - 2*(a*c*s
gn(1/(a*x - 1))*sgn(a) - 2*d*sgn(1/(a*x - 1))*sgn(a))*arctan(sqrt(-2/(a*x -
 1) - 1))/(a*d^2) + 2*(a^2*c^2*sgn(1/(a*x - 1))*sgn(a) - 2*a*c*d*sgn(1/(a*x
 - 1))*sgn(a) + d^2*sgn(1/(a*x - 1))*sgn(a))*arctan((a*c*sqrt(-2/(a*x - 1)
- 1) + d*sqrt(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*
a*d^2))*abs(a)
```

maple [B] time = 0.04, size = 1178, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x)
```

```
[Out] -1/a^2/(a*c+d)/(x-1/a)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(5/2)-1/(a*c+d)*(-(x-
1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+3/2*a/(a*c+d)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1
/2)*x+3/2*a/(a*c+d)/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-(x-1/a)^2*a^2-2*(x-1/a
)*a)^(1/2)*x)-1/3*d/(a*c+d)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/2*d/(a*c
+d)^2*a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)*x+1/2*d/(a*c+d)^2*a/(a^2)^(1/2)*
arctan((a^2)^(1/2)/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)*x)+1/3*d/(a*c+d)^2*(-
(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(3/2)+1/2/(a*c+d)^2*a^2*
c*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2)*x+3/2/(a*c+d)^
2*a^2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-
(a^2*c^2-d^2)/d^2)^(1/2))-1/d/(a*c+d)^2*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(
a^2*c^2-d^2)/d^2)^(1/2)*a^2*c^2+d/(a*c+d)^2*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/
d)-(a^2*c^2-d^2)/d^2)^(1/2)-1/d^2/(a*c+d)^2*a^4*c^3/(a^2)^(1/2)*arctan((a^2
)^(1/2)*x/(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))-1/d^3
/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x
+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c
^2-d^2)/d^2)^(1/2))/(x+c/d))*a^4*c^4+2/d/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/
2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*
(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))*a^2*c^
2-d/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d
*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^
2*c^2-d^2)/d^2)^(1/2))/(x+c/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax-1)^2(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x)
```

mupad [B] time = 0.29, size = 148, normalized size = 1.38

$$-\frac{\sqrt{1-a^2 x^2}}{d}-\frac{\operatorname{asinh}\left(x \sqrt{-a^2}\right) \left(2 a \sqrt{-a^2}-\frac{a^2 c \sqrt{-a^2}}{d}\right)}{a^2 d}-\frac{\left(\ln\left(\sqrt{1-\frac{a^2 c^2}{d^2}} \sqrt{1-a^2 x^2}+\frac{a^2 c x}{d}+1\right)-\ln\left(c+d x\right)\right)}{d^3 \sqrt{1-\frac{a^2 c^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((a*x - 1)^2*(c + d*x)),x)
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(c+dx\right)\left(ax-1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((c + d*x)*(a*x - 1)**2), x)
```

3.621 $\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\,\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

```
[Out] -(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2)
/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 216, 725, 204}

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\,\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\,\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,\frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

Mathematica [A] time = 0.11, size = 120, normalized size = 1.12

$$\frac{(ac-d)\sqrt{a^2c^2-d^2}\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2(ac+d)}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-d)\sin^{-1}(ax)}{d^2}+\frac{\sin^{-1}(ax)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - d)*ArcSin[a*x])/d^2 + ArcSin[a*x]/d + ((a*
c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1
 - a^2*x^2])])/(d^2*(a*c + d))
```

fricas [A] time = 1.29, size = 318, normalized size = 2.97

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-\left(a^2c^2-d^2\right)\sqrt{-a^2x^2+1}-\left(acd+d^2+\left(a^3c^2+a^2cd\right)x+\sqrt{-a^2x^2+1}\left(acd+d^2\right)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\,\text{arc}}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

giac [A] time = 0.46, size = 131, normalized size = 1.22

$$-\frac{\left(a^2c-2\,ad\right)\arcsin\left(ax\right)\text{sgn}(a)}{d^2|a|}-\frac{\sqrt{-a^2x^2+1}}{d}-\frac{2\left(a^3c^2-2\,a^2cd+ad^2\right)\arctan\left(\frac{d+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c}{ax}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}\,d^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a^2*c - 2*a*d)*arcsin(a*x)*sgn(a)/(d^2*abs(a)) - sqrt(-a^2*x^2 + 1)/d - 2
*(a^3*c^2 - 2*a^2*c*d + a*d^2)*arctan((d + (sqrt(-a^2*x^2 + 1)*abs(a) + a)*
c/(a*x))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*d^2*abs(a))
```

maple [B] time = 0.02, size = 524, normalized size = 4.90

$$-\frac{a^2c^2\ln\left(\frac{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\frac{2\left(a^2c^2-d^2\right)}{d^2}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\left(x+\frac{c}{d}\right)^2a^2-\frac{a^2c^2-d^2}{d^2}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{a^2c^2-d^2}{d^2}}\,d^3}-\frac{a^2c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,d^2}+\frac{2ac\ln\left(\frac{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\frac{2\left(a^2c^2\right.}{d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/d-a^2/d^2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)
^(1/2)*x)+2*a/d/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(1/2)*x)-1/d^3/
(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((2*(x+c/d)*a^2*c/d-2*(a^2*c^2-d^2)/d^2+2*(-(a
^2*c^2-d^2)/d^2)^(1/2)*(2*(x+c/d)*a^2*c/d-(x+c/d)^2*a^2-(a^2*c^2-d^2)/d^2)^
(1/2))/(x+c/d))*a^2*c^2+2/d^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((2*(x+c/d)*a^2*
c/d-2*(a^2*c^2-d^2)/d^2+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(2*(x+c/d)*a^2*c/d-(x+
c/d)^2*a^2-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))*a*c-1/d/(-(a^2*c^2-d^2)/d^2)^
(1/2)*ln((2*(x+c/d)*a^2*c/d-2*(a^2*c^2-d^2)/d^2+2*(-(a^2*c^2-d^2)/d^2)^(1/2
)*(2*(x+c/d)*a^2*c/d-(x+c/d)^2*a^2-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a*c>0)', see `assume?` for more d
etails)Is d-a*c positive, negative or zero?
```

mupad [B] time = 0.12, size = 148, normalized size = 1.38

$$-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\mathrm{asinh}\left(x\sqrt{-a^2}\right)\left(2a\sqrt{-a^2}-\frac{a^2c\sqrt{-a^2}}{d}\right)}{a^2d}-\frac{\left(\ln\left(\sqrt{1-\frac{a^2c^2}{d^2}}\sqrt{1-a^2x^2}+\frac{a^2cx}{d}+1\right)-\ln\left(c+dx\right)\right)\left(a^2\right.}{d^3\sqrt{1-\frac{a^2c^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((1 - a^2*x^2)^(1/2)*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^2}{\sqrt{-(ax-1)(ax+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(d*x+c)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**2/(sqrt(-(a*x - 1)*(a*x + 1))*(c + d*x)), x)
```

3.622 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=851

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}\,(d+ex)^4}{11e}+\frac{4\sqrt{-a}\left(3a^2e^2(26ef+231dg)g^4-9ac\left(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3\right)g^2\right.}{346}$$

```
[Out] -2/3465*(2*a*e^2*g^2*(-231*d*g+74*e*f)-c*(-567*d^3*g^3+1107*d^2*e*f*g^2-843
*d*e^2*f^2*g+233*e^3*f^3))*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^4+2/693*e*(18*
a*e^2*g^2-c*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+a)^(1/
2)/c/g^4+2/99*e^2*(-3*d*g+e*f)*(g*x+f)^(7/2)*(c*x^2+a)^(1/2)/g^4-2/3465*(15
0*a^2*e^4*g^4-6*a*c*e^2*g^2*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+c^2*(315*d^4
*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+187*e^4*f^4))*(g*
x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+a)
^(1/2)/e+4/3465*(3*a^2*e^2*g^4*(231*d*g+26*e*f)-c^2*f^2*(-231*d^3*g^3+396*d
^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)-9*a*c*g^2*(77*d^3*g^3+88*d^2*e*f*g^2
-33*d*e^2*f^2*g+6*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^
(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*
(c*x^2/a+1)^(1/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2
)+f*c^(1/2)))^(1/2)-4/3465*(a*g^2+c*f^2)*(75*a^2*e^3*g^4-3*a*c*e*g^2*(165*d
^2*g^2-33*d*e*f*g+2*e^2*f^2)-c^2*f*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*
f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2
*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(5/2)/g^5/(g*x+f)^(1/2)/(c*x^
2+a)^(1/2)
```

Rubi [A] time = 2.70, antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {919, 1654, 844, 719, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}\,(d+ex)^4}{11e}+\frac{4\sqrt{-a}\left(3a^2e^2(26ef+231dg)g^4-9ac\left(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3\right)g^2\right.}{346}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2)
 + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*
g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d
 + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f -
 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g
^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(
693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(99*g^4)
 + (4*Sqrt[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 26
4*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*
d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(3465*c^(3/2)*g^5*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(75*a^2*e^
3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*
```

```
f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3
465*c^(5/2)*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 919

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(e*(2*m + 5)), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[3*a*e*f
- a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3*c*d*g)*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{\int \frac{(d+ex)^3\left(a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{11e}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4} + \frac{2\int \frac{-\frac{1}{2}a}{}}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{2e\left(18ae^2g^2-c\left(29e^2f^2-96defg+81d^2g^2\right)\right)}{693cg^4}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} - \frac{2\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843\right.\right.}{}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de\right.\right.}{3465c^2eg}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de\right.\right.}{3465c^2eg}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de\right.\right.}{3465c^2eg}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de\right.\right.}{3465c^2eg}$$

Mathematica [C] time = 10.47, size = 1034, normalized size = 1.22

$$2\sqrt{f+gx}\left(\frac{2\left(-3a^2e^2(26ef+231dg)g^4+9ac\left(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3\right)g^2+c^2f^2\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3\right)\right)\left(cx^2+a\right)g^2}{f+gx} - \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*((2*g^2*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e
^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^
3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*(a + c*x^2))/(f + g*
x) - g^2*(a + c*x^2)*(150*a^2*e^3*g^4 - 2*a*c*e*g^2*(495*d^2*g^2 + 33*d*e*g
*(4*f + 7*g*x) + e^2*(-23*f^2 + 16*f*g*x + 45*g^2*x^2)) + c^2*(-231*d^3*g^3
*(f + 3*g*x) - 99*d^2*e*g^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) - 33*d*e^2*g*(8
*f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3) + e^3*(64*f^4 - 48*f^3*g*x + 4
```

```
0*f^2*g^2*x^2 - 35*f*g^3*x^3 - 315*g^4*x^4))) + (2*Sqrt[c]*((-I)*Sqrt[c]*f
+ Sqrt[a]*g)*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e^3*f^3 - 264
*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^3*f^3 - 33*d
*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x
))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x
]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqr
t[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]] + (2*Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(75*a^2*e^3*g^4 - (3*I)*a^
(3/2)*Sqrt[c]*e^2*g^3*(e*f + 231*d*g) - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g
 + 165*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 264*d*e^2*f^2*g - 396*d^2*e*f*g^2 +
231*d^3*g^3) + (3*I)*Sqrt[a]*c^(3/2)*g*(16*e^3*f^3 - 66*d*e^2*f^2*g + 99*d^
2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3465*c
^2*g^6*Sqrt[a + c*x^2])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + a}\,\sqrt{gx + f}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)*sqrt(g*x
 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + a}\,(ex + d)^3\sqrt{gx + f}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

maple [B] time = 0.16, size = 6457, normalized size = 7.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + a}\,(ex + d)^3\sqrt{gx + f}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3,x)
```

```
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+cx^2}\,(d+ex)^3\,\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x), x)
```

3.623 $\int (d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=635

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(21a^2e^2g^4+3acg^2\left(-21d^2g^2-16defg+3e^2f^2\right)+c^2f^2\left(21d^2g^2-24defg+8e^2f^2\right)\right)\cdots}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[Out] 4/315*(7*a*e^2*g^2-c*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^
2+a)^(1/2)/c/g^3+2/63*e*(-3*d*g+e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/g^3-2/31
5*(6*a*e^2*g^2*(-10*d*g+e*f)-c*(-35*d^3*g^3+63*d^2*e*f*g^2-57*d*e^2*f^2*g+1
9*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e/g^3+2/9*(e*x+d)^3*(g*x+f)^(1/
2)*(c*x^2+a)^(1/2)/e+4/315*(21*a^2*e^2*g^4+3*a*c*g^2*(-21*d^2*g^2-16*d*e*f*
g+3*e^2*f^2)+c^2*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*
c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2
))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(3/2)/g^4/(c*x^2+a)^(1/2)/(
(g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/315*(a*g^2+c*f^2)*(3*a*e*
g^2*(-10*d*g+e*f)+c*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))
^(1/2)/c^(3/2)/g^4/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 1.62, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {919, 1654, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(21a^2e^2g^4+3acg^2\left(-21d^2g^2-16defg+3e^2f^2\right)+c^2f^2\left(21d^2g^2-24defg+8e^2f^2\right)\right)\cdots}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e
*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*e*g^3) + (2*(d
+ e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + (4*(7*a*e^2*g^2 - c*(8*e^2*
f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*c*g^3
) + (2*e*(e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^3) + (4*Sqrt[
-a]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2
*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^4*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(3*a*e*g^2*
(e*f - 10*d*g) + c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(315*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 919

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(e*(2*m + 5)), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[3*a*e*f
- a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3*c*d*g)*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9e} + \frac{\int \frac{(d+ex)^2\left(a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2\right)}{\sqrt{f+gx}\,\sqrt{a+cx^2}}dx}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9e} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{2\int -}{}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9e} + \frac{4\left(7ae^2g^2-c\left(8e^2f^2-24defg+21d^2g^2\right)\right)}{315cg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f}}{315ceg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f}}{315ceg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f}}{315ceg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f}}{315ceg^3}$$

Mathematica [C] time = 7.26, size = 809, normalized size = 1.27

$$\sqrt{f+gx}\left(\frac{2(cx^2+a)\left(2ae(4ef+30dg+7egx)g^2+c\left(\left(8f^3-6gxf^2+5g^2x^2f+35g^3x^3\right)e^2+6dg\left(-4f^2+3gxf+15g^2x^2\right)e+21d^2g^2(f+3gx)\right)\right)}{cg^3} - \frac{4\left(\sqrt{-f-}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e*g^2*(4*e*f + 30*d*g + 7*e*g*x) + c*(2
1*d^2*g^2*(f + 3*g*x) + 6*d*e*g*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + e^2*(8*f^
3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3))))/(c*g^3) - (4*(g^2*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21
*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2))*(a + c*x^2) -
 I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 -
 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2
))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt
[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
```

```
qrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*e^
2*g^3 - 3*a*Sqrt[c]*e*g^2*(e*f - 10*d*g) + c^(3/2)*f*(-8*e^2*f^2 + 24*d*e*f
*g - 21*d^2*g^2) - (3*I)*Sqrt[a]*c*g*(-2*e^2*f^2 + 6*d*e*f*g + 21*d^2*g^2))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c^2*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(315*Sqrt
[a + c*x^2])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^2 + 2\,dex + d^2\right)\sqrt{cx^2 + a}\,\sqrt{gx + f}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + a}\,(ex + d)^2\sqrt{gx + f}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

maple [B] time = 0.04, size = 4351, normalized size = 6.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] -2/315*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(24*a*c^2*d*e*f^3*g^3-60*a^2*c*d*e*f*g
^5-35*x^6*c^3*e^2*g^6-63*x^4*c^3*d^2*g^6+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1
/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^3*e^2*g^6-4
2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c
)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipti
cE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*
c)^(1/2)*g))^(1/2))*a^3*e^2*g^6-16*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*e^2*f^6-168*x^2*a*
c^2*d*e*f*g^5+6*x*a*c^2*d*e*f^2*g^4-90*x^5*c^3*d*e*g^6-60*(-(g*x+f)*c/((-a*
c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*(
(c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a
*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))
*a^2*c*e^2*f^2*g^4+84*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)
^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1
/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d^2*f^2*g^4-34*(-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2
)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/
2))*a*c^2*e^2*f^4*g^2+48*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a
```

*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)
^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d*e*f^5*g-60*(-(g*x+f)*c/((-
a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)
*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2
))*(-a*c)^(1/2)*a^2*d*e*g^6+6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((
-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a^2*e^2*f*g^5+
126*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a
*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellip
ticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-
a*c)^(1/2)*g))^(1/2))*a^2*c*d^2*g^6-42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*
g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d^2*f^4*g^2-40
*x^5*c^3*e^2*f*g^5-49*x^4*a*c^2*e^2*g^6+x^4*c^3*e^2*f^2*g^4-84*x^3*c^3*d^2*
f*g^5-2*x^3*c^3*e^2*f^3*g^3-14*x^2*a^2*c*e^2*g^6-63*x^2*a*c^2*d^2*g^6-21*x^
2*c^3*d^2*f^2*g^4-8*x^2*c^3*e^2*f^4*g^2-126*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1
/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*d^2*g^6
-108*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-
a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elli
pticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(
-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*d*e*f^2*g^4+42*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-
a*c)^(1/2)*c^2*d^2*f^3*g^3+16*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((
-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c^2*e^2*f^5*g+
54*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*
c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipt
icF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*a^2*c*e^2*f^2*g^4-126*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d^2*f^2*
g^4+12*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+
(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*El
lipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f
+(-a*c)^(1/2)*g))^(1/2))*a*c^2*e^2*f^4*g^2-62*x^3*a*c^2*e^2*f*g^5-8*a^2*c*e
^2*f^2*g^4-21*a*c^2*d^2*f^2*g^4-8*a*c^2*e^2*f^4*g^2+6*x^3*c^3*d*e*f^2*g^4-7
*x^2*a*c^2*e^2*f^2*g^4+24*x^2*c^3*d*e*f^3*g^3-60*x*a^2*c*d*e*g^6-22*x*a^2*c
*e^2*f*g^5-84*x*a*c^2*d^2*f*g^5-2*x*a*c^2*e^2*f^3*g^3-108*x^4*c^3*d*e*f*g^5
-150*x^3*a*c^2*d*e*g^6+96*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c
)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*d*e*f*g^5+144*(-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(
1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*
c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^
(1/2))*a*c^2*d*e*f^3*g^3+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a
*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*d^2*f*g^5+22

```
*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)
^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elliptic
F((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*e^2*f^3*g^3-48*(-(g*x+f)*c/((-a*c)^(1/2
)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-
a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/
2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)
^(1/2)*c^2*d*e*f^4*g^2-36*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c
)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*d*e*f*g^5-36*(-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1
/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(
1/2))*a*c^2*d*e*f^3*g^3)/c^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+a}\,(ex+d)^2\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2,x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+cx^2}\,\left(d+ex\right)^2\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)
```

3.624 $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=434

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5aeg^2+cf(4ef-7dg)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{4\sqrt{-a}\ \cdots}{\cdots}$$

```
[Out] 2/7*e*(c*x^2+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(5*a*e*g^2+c*f*(-7*d*g+4*e*f)-3
*c*g*(7*d*g+e*f)*x)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2-4/105*(c*f^2*(-7*d*
g+4*e*f)+a*g^2*(21*d*g+8*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/
2)*(c*x^2/a+1)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e*g^2+c*f*(-7*d*g+4*e*f))*E
llipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1
/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {833, 815, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5aeg^2+cf(4ef-7dg)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{4\sqrt{-a}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)
*Sqrt[a + c*x^2])/(105*c*g^2) + (2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c)
 - (4*Sqrt[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(c*f^2
+ a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^
3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2e\sqrt{f+gx}\left(a+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-aeg)+\frac{1}{2}c(ef+7dg)x\right)\sqrt{a+cx^2}}{\sqrt{f+gx}}dx}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f}}{}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f}}{}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f}}{}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f}}{}$$

Mathematica [C] time = 4.58, size = 610, normalized size = 1.41

$$\sqrt{f+gx}\left(\frac{2\left(a+cx^2\right)\left(10aeg^2+7cdg(f+3gx)+ce\left(-4f^2+3fgx+15g^2x^2\right)\right)}{cg^2} + \frac{4\left(g^2\left(a+cx^2\right)\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(ag^2(21dg+8ef)+cf^2(4ef-7dg)\right)+i\sqrt{c}(f+g\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e*g^2 + 7*c*d*g*(f + 3*g*x) + c*e*(-4*
f^2 + 3*f*g*x + 15*g^2*x^2)))/(c*g^2) + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*(a + c*x^2) + I*Sqrt
[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(c*f^2*(-4*e*f + 7*d*g) - a*g^2*(8*e*f + 21*d
*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*g*(I*Sqrt[c]*f - Sqrt[a]*g)*((5*I)*a*e*g^2 + I*c*f*(
4*e*f - 7*d*g) + 3*Sqrt[a]*Sqrt[c]*g*(e*f + 7*d*g))*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+a}\,(ex+d)\sqrt{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + a}\,(ex + d)\sqrt{gx + f}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

maple [B] time = 0.03, size = 2549, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] -2/105*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(4*a*c^2*e*f^3*g^2-10*a^2*c*e*f*g^4-7*
a*c^2*d*f^2*g^3-21*x^4*c^3*d*g^5-15*x^5*c^3*e*g^5-18*x^4*c^3*e*f*g^4+14*(-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/
2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1
/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*d*f*g^4-18*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a
*c*e*f^2*g^3+8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))
*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^
(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c
*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*e*f^5+24*(-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*e*f^3
*g^2+14*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f
+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*E
llipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*
f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c^2*d*f^3*g^2-8*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-
a*c)^(1/2)*c^2*e*f^4*g+16*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c
)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*e*f*g^4+28*(-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2
)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/
2))*a*c^2*d*f^2*g^3-6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)
^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1
/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*e*f*g^4-42*(-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*
c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*
```

```
a*c^2*d*f^2*g^3-6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/
2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f
))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*
g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*e*f^3*g^2-10*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-
a*c)^(1/2)*a^2*e*g^5-25*x^3*a*c^2*e*g^5-28*x^3*c^3*d*f*g^4+x^3*c^3*e*f^2*g^
3-21*x^2*a*c^2*d*g^5-7*x^2*c^3*d*f^2*g^3+4*x^2*c^3*e*f^3*g^2-10*x*a^2*c*e*g
^5+x*a*c^2*e*f^2*g^3-28*x^2*a*c^2*e*f*g^4-28*x*a*c^2*d*f*g^4+42*(-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(
1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*
c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^
(1/2))*a^2*c*d*g^5-14*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)
^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1
/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d*f^4*g-42*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(c*f+(-a*c)^(1/2)*g))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^
2*c*d*g^5)/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^4/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + a}\,(ex + d)\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f + g\,x}\,\sqrt{c\,x^2 + a}\,\left(d + e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x),x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + cx^2}\,(d + ex)\,\sqrt{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x), x)
```

3.625 $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=362

$$-\frac{4\sqrt{-a}\,f\sqrt{\frac{cx^2}{a}+1}\,\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15\sqrt{c}\,g^2\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{4\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,\left(cf^2-\right.}{15\sqrt{c}\,g^2\sqrt{a}}$$

```
[Out] 2/5*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g-4/15*f*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+
4/15*(-3*a*g^2+c*f^2)*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2
/a+1)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)-4/15*f*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(
1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/
a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g^2/c^(1/2)/(g*
x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {735, 833, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}\,f\sqrt{\frac{cx^2}{a}+1}\,\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15\sqrt{c}\,g^2\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{4\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,\left(cf^2-\right.}{15\sqrt{c}\,g^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-4*f*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*g) + (2*(f + g*x)^(3/2)*Sqrt[a + c
*x^2])/(5*g) + (4*Sqrt[-a]*(c*f^2 - 3*a*g^2)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqr
t[-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*f*(c*f^2 + a*g^2)*Sqrt[(Sqr
t[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
 a*g)])/(15*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
```

```
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{2\int \frac{(ag-cfx)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{5g}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{4\int \frac{2acfg-\frac{1}{2}c\left(cf^2-3ag^2\right)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15cg}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{1}{15}\left(2\left(3a-\frac{cf^2}{g^2}\right)\right)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{\left(4a\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+gx}\sqrt{1+\frac{c}{a}\cdots}\right.}{15\sqrt{-a}\cdots}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} - \frac{4\sqrt{-a}\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+gx}\sqrt{1+\cdots}}{15\sqrt{c}\sqrt{\cdots}}$$

Mathematica [C] time = 2.91, size = 536, normalized size = 1.48

$$\sqrt{f+gx}\left(\frac{2(a+cx^2)(f+3gx)}{g}-\frac{4\left(\sqrt{c}(f+gx)^{3/2}\left(-3a^{3/2}g^3+\sqrt{a}cf^2g+3ia\sqrt{c}fg^2-ic^{3/2}f^3\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-}{\sqrt{c}f-}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]]*(-3*a^2*g^2 + c^2*f^2*x^2 + a*c*(f^2 - 3*g^2*x^2)) + Sqrt[c]*
((-I)*c^(3/2)*f^3 + Sqrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] - Sqrt[a]*Sqrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(15*Sqrt[a
+ c*x^2])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+a}\sqrt{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

maple [B] time = 0.02, size = 1162, normalized size = 3.21

$$2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\left(3c^2g^4x^4+4c^2f\,g^3x^3+3ac\,g^4x^2+c^2f^2g^2x^2-6\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(6*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-(g*x+f
```

```
)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*
g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*a^2*g^4+6*Ellip
ticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+
(-a*c)^(1/2)*g))^(1/2))*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a
*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(
1/2)*g)*g)^(1/2)*a*c*f^2*g^2-2*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-(g*x+f)/(-c*f
+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2
)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*(-a*c)^(1/2)*a*f*g^3-2
*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c
*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-
a*c)^(1/2)*g)*g)^(1/2)*(-a*c)^(1/2)*c*f^3*g-6*EllipticE((-(g*x+f)/(-c*f+(-a
*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(
1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*a^2*g^4
-4*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*
g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*a*c*f^2*g^2+2*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-(g*x+
f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*c^2*f^4+3*x^4*
c^2*g^4+4*x^3*c^2*f*g^3+3*x^2*a*c*g^4+x^2*c^2*f^2*g^2+4*x*a*c*f*g^3+a*c*f^2
*g^2)/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + a}\,\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f + g\,x}\,\sqrt{c\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + cx^2}\,\sqrt{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

3.626 $\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=683

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,\left(ae^2+cd^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^3\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,(ef-$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e-2/3*(-3*d*g+e*f)*EllipticE(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*
(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/g/(c*x^2+a)^(1/2)/((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*(2*a*e^2*g-3*c*d*(-d*g+e
*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+2/3
*f*(-3*d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*
g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*
((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2
+a)^(1/2)-2*(a*e^2+c*d^2)*(-d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2)
)^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 2.12, antiderivative size = 683, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {919, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,\left(ae^2+cd^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^3\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,(ef-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) - (2*Sqrt[-a]*Sqrt[c]*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqr
t[c]*f*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[f + g*x]*Sqrt[a + c*x^2
]) - (2*Sqrt[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c
]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 919

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(e*(2*m + 5)), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[3*a*e*f
- a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3*c*d*g)*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{\int \frac{a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{\int \left(\frac{2ae^2g-3cd(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{c(ef-3dg)x}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3\left(cd^2+ae^2\right)(ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e^2} + \frac{\left(\left(cd^2+ae^2\right)(ef-dg)\right)\int \frac{}{(a}}{e^3}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3e^2g} - \frac{(cf(ef-3dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e^2g}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\left(2ae^2g-3cd(ef-dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\right.}{3\sqrt{c}\,e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\right.}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\right.}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 8.98, size = 1216, normalized size = 1.78

$$\left(\frac{2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3}{(f+gx)^2}-\frac{4ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2}{f+gx}-\frac{6cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2}{(f+gx)^2}+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f+\frac{12cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f}{f+gx}+\frac{2ae^2g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{(f+gx)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) + ((f + g*x)^(3/2)*(2*c*e^2*f*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - 6*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
(2*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*c*d*e*f^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (2*a*e^2*f*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]])/(f + g*x)^2 - (4*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
g*x) + (12*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (2*Sqrt[
c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - 3*d*g)*Sqrt[1 - f/(f + g*x) - (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c
]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]
 + (2*e*(3*Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[1 -
 f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I
*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[f + g*x] + ((6*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*I)*a*e^2*g^2*Sqrt[1 - f
/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]
*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(
3*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/
(f + g*x))^2)/g^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)

maple [B] time = 0.05, size = 2496, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x)

```
[Out] -2/3*(2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3+3*(-(g*x+f)/(-c*f+(-a*
c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c
*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(
-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2)
)*(-a*c)^(1/2)*c*d^2*g^3-(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-
a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^
(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*
f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g+3*(
-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1
/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Elliptic
F((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*a*c*d*e*g^3-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-3*(-
(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/
2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF
((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*
c)^(1/2)*g))^(1/2))*c^2*d^2*f*g^2+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2
)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-
c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(
1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*e*f^2*g-3*(
-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1
/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Elliptic
E((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*a*c*d*e*g^3+(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*(
(-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f
+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2
),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-3*(-(g
*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)
*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((
-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)
^(1/2)*g))^(1/2))*c^2*d*e*f^2*g+(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e^2*f^3-3*(-(g*x+
f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)*e/c/(d*g-e*f),(
-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-
3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)
^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Ellip
ticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)*e/c/(d
*g-e*f),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c
*d^2*g^3+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1
/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*
g)*e/c/(d*g-e*f),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e
```

```
^2*f*g^2+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1
/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*
g)*e/c/(d*g-e*f),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d
^2*f*g^2-x^3*c^2*e^2*g^3-x^2*c^2*e^2*f*g^2-x*a*c*e^2*g^3-a*c*e^2*f*g^2)*(g*
x+f)^(1/2)*(c*x^2+a)^(1/2)/e^3/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\,\sqrt{gx+f}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\sqrt{f+gx}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)
```

3.627 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=650

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,(2ef-3dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\left(ae^2g-c\right.}{e^3\sqrt{a}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)-3*EllipticE(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)
*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+3*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2
))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(
1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-3*d*g+2*e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-
a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1
/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2*g-c*d*(-3*d*g+2*e*f))*Elliptic
Pi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),
2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))
/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 1.66, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {917, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,(2ef-3dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\left(ae^2g-c\right.}{e^3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) - (3*Sqrt[-a]*Sqrt[c]*Sqrt
[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c]*f*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt
[c]*f - a*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e
*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/
(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*
g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]
*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 917

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)*Simp[a*g +
2*c*f*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{c(2ef-3dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2} + \frac{(c(2ef-3dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^3} +$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2} + \frac{\left(\left(ae^2g-cd(2e\right.\right.}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right.}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right.}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 7.03, size = 1331, normalized size = 2.05

$$\sqrt{f+gx}\left(-\frac{(cx^2+a)e^2}{d+ex}-\frac{-3ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+6ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+3cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-3ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-6cdeg\sqrt{-f-\cdots}\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-3*c*e^2*f^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]] + 3*c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a
*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]] + 6*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) -
6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 3*c*e^2*f*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x)^2 + 3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] + e*(Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g - I*Sqrt[c]*(2*e*f - 3
*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)] + (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f
 + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*
(f + g*x))))/(e^3*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\,\sqrt{gx+f}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

maple [B] time = 0.06, size = 6044, normalized size = 9.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}\,\sqrt{gx + f}}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + a}}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2,x)
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}\,\sqrt{f + gx}}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**2,x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

3.628 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=1205

$$\frac{\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)\left(ae^2g-cd(2ef-3dg)\right)^2}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}-\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}}}{4e^2}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)^2-1/4*(a*e^2*g-c*d*(-3*d*g+2*e
*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-1/4*(
a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^
(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f
)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-3/2*g*EllipticF(1/2*(1-x*c^(1/
2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-
a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*(a*e^2*g-c*d*(-3*d*g+2*e*f
))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1
/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1
/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c
^(1/2)))^(1/2)/e^3/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-c
*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+
d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e
+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*(a*e^2*g-c*d*(-3*d
*g+2*e*f))^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d
*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))
*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(a*
e^2+c*d^2)/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2
)
```

Rubi [A] time = 4.20, antiderivative size = 1205, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {917, 6742, 719, 419, 940, 844, 424, 933, 168, 538, 537}

$$\frac{\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)\left(ae^2g-cd(2ef-3dg)\right)^2}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}-\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*e*(d + e*x)^2) - ((a*e^2*g - c*d*(2*e*f
 - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)*
(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x
]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[
2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*
g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (3
*Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt
```

```
[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*
x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
 (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e
^2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*S
qrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^3*(c*d^2 +
 a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (c*(e*f - 3*d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))^2
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^3*((Sqrt[c]*d
)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
```

```
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 917

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)*Simp[a*g +
2*c*f*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{2c(ef-3dg)}{e^2(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}} \cdots\right.}{4e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4e^3} + \frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,d\cdots}{2e^3}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\left(ae^2g-cd(2ef-\cdots\right.}{8\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\cdots}{\sqrt{c}\cdots}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\cdots}{\sqrt{c}\cdots}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\cdots}{\sqrt{c}\cdots}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\cdots}{\sqrt{c}\cdots}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\left(ae^2g-\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\left(ae^2g-\cdots\right.}{\cdots}$$

Mathematica [C] time = 11.23, size = 2703, normalized size = 2.24

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-1/2*1/(e*(d + e*x)^2) + (2*c*d*e*f - 3*c*d^
2*g - a*e^2*g)/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x))) + ((f + g*x)^(3
/2)*(-2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^2*e^2*f*g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]] - 3*c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*c*d*e^3*g^2*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (2*c^2*d*e^3*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]])/(f + g*x)^2 + (5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
)/(f + g*x)^2 + (a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^
2 - (3*c^2*d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3
*a*c*d*e^3*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (5*a*c*d
^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (a^2*e^4*f*g^3
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*c*d^3*e*g^4*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]])/(f + g*x)^2 + (4*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
])/(f + g*x) - (10*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
 g*x) - (2*a*c*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (6*c
^2*d^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (2*a*c*d*e^3*f
*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (Sqrt[c]*e*((-I)*Sqrt[c]
*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[1 - f/(f
+ g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[
a]*g)/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/
Sqrt[f + g*x] + (e*(Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)
*(-(a*e^2*g) - (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + 3*d*g))*
Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g
*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)])/Sqrt[f + g*x] + ((4*I)*a*c*e^4*f^2*g*Sqrt[1 - f/(f + g*x) - (
I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqr
t[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]
*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
 - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((4*I)*c^2*d^3*
e*f*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 -
f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[f + g*x] - ((12*I)*a*c*d*e^3*f*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*
g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f +
g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c^2*d^4*g^3*Sqrt[1
 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) +
(I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]
 + ((6*I)*a*c*d^2*e^2*g^3*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f
+ g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Elliptic
Pi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*
f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - (I*a^2*e^4*g^3*Sqrt[1 - f/(f + g*x) - (I
*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt
[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*
```

```
g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
- I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(4*e^3*(c*d^2 +
a*e^2)*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + (c*(f + g*
x)^2*(-1 + f/(f + g*x))^2)/g^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\,\sqrt{gx+f}}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

maple [B] time = 0.10, size = 19181, normalized size = 15.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\,\sqrt{gx+f}}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
[Out] Timed out
```

3.629 $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=666

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(21a^2e^3g^4-3aceg^2\left(63d^2g^2-39defg+10e^2f^2\right)-c^2f\left(-105d^3g^3+252d^2efg^2-216\right.\right.}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[Out] 4/315*e*(7*a*e^2*g^2+c*(42*d^2*g^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(
c*x^2+a)^(1/2)/c/g^4-4/63*e^2*(-3*d*g+4*e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/
g^4-4/315*(9*a*e^2*g^2*(-5*d*g+2*e*f)+c*(-35*d^3*g^3+168*d^2*e*f*g^2-204*d*
e^2*f^2*g+76*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^4+2/9*(e*x+d)^3*(g
*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/315*(21*a^2*e^3*g^4-3*a*c*e*g^2*(63*d^2*g^2
-39*d*e*f*g+10*e^2*f^2)-c^2*f*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f^2*g
+64*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/
(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1
/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^
(1/2)-4/315*(a*g^2+c*f^2)*(9*a*e^2*g^2*(-5*d*g+2*e*f)-c*(-105*d^3*g^3+252*d
^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c
*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^
5/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 1.57, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {921, 1654, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(21a^2e^3g^4-3aceg^2\left(63d^2g^2-39defg+10e^2f^2\right)-c^2f\left(252d^2efg^2-105d^3g^3-216d\right.\right.}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2*g + 168*d^
2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*g^4) + (2*(d
 + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) + (4*e*(7*a*e^2*g^2 + c*(64*
e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*
c*g^4) - (4*e^2*(4*e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^4) +
 (4*Sqrt[-a]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^
3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^5
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*S
qrt[-a]*(c*f^2 + a*g^2)*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f^3 - 216*
d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqr
t[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)
*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 921

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)*Simp[2*a*(e
*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*x - (2*c*(d*g*m - e*f*(m + 1)))*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2\left(2a(3ef-4dg)+2(cdf-aeg)x+2c(4ef-3dg)x^2\right)}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9g} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} - \frac{2\int \frac{-acg^2(20e}{}}{}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9g} + \frac{4e\left(7ae^2g^2+c\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}}{315cg^4}$$

Mathematica [C] time = 7.88, size = 864, normalized size = 1.30

$$2\sqrt{f+gx}\left(-c\left(cx^2+a\right)\left(c\left(\left(64f^3-48gxf^2+40g^2x^2f-35g^3x^3\right)e^3-27dg\left(8f^2-6gxf+5g^2x^2\right)e^2+63d^2g^2\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-(c*g^2*(a + c*x^2)*(-2*a*e^2*g^2*(-11*e*f + 45*d*g + 7*e
*g*x) + c*(-105*d^3*g^3 + 63*d^2*e*g^2*(4*f - 3*g*x) - 27*d*e^2*g*(8*f^2 -
6*f*g*x + 5*g^2*x^2) + e^3*(64*f^3 - 48*f^2*g*x + 40*f*g^2*x^2 - 35*g^3*x^3
)))) - (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^3*g^4 - 3*a*c*e*g
^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*
f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))*(a + c*x^2) - Sqrt[c]*(I*Sqrt[c]*f
- Sqrt[a]*g)*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g
^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
```

```
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*
e^3*g^3 - 9*a*Sqrt[c]*e^2*g^2*(2*e*f - 5*d*g) - (3*I)*Sqrt[a]*c*e*g*(16*e^2
*f^2 - 54*d*e*f*g + 63*d^2*g^2) + c^(3/2)*(64*e^3*f^3 - 216*d*e^2*f^2*g + 2
52*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)
]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elliptic
F[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x))))/(315*c^2*g^6*Sqrt[a + c*x^2])
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)/sqrt(g*x
 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}\,(ex + d)^3}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

maple [B] time = 0.06, size = 5079, normalized size = 7.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}\,(ex + d)^3}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}\,\left(d + e\,x\right)^3}{\sqrt{f + g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}\,(d + ex)^3}{\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)
```

3.630 $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=508

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5ae^2g^2-c\left(35d^2g^2-56defg+24e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}\cdots}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] -4/35*e*(-2*d*g+3*e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g^3+4/105*(5*a*e^2*g^2
+c*(10*d^2*g^2-34*d*e*f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^3+
2/7*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/105*(a*e*g^2*(-42*d*g+13*e*
f)+c*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)
*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g^4/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e^2*g^2-c*(35*d
^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^4/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {921, 1654, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5ae^2g^2-c\left(35d^2g^2-56defg+24e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}\cdots}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (4*(10*d^2 + e^2*((5*a)/c + (21*f^2)/g^2) - (34*d*e*f)/g)*Sqrt[f + g*x]*Sqr
t[a + c*x^2])/(105*g) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*g)
 - (4*e*(3*e*f - 2*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*g^3) + (4*Sqrt
[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2
))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^4*
Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sq
rt[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g
^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]
*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-
a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 921

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)*Simp[2*a*(e
*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*x - (2*c*(d*g*m - e*f*(m + 1)))*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(2a(2ef-3dg)+2(cdf-aeg)x+2c(3ef-2dg)x^2\right)}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{2\int \frac{-acg^2\left(9e^2f^2-1\right.}{}}{}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}}{7g}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}}{7g}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}}{7g}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}}{7g}$$

Mathematica [C] time = 5.26, size = 712, normalized size = 1.40

$$2\sqrt{f+gx}\left(g^2\left(a+cx^2\right)\left(10ae^2g^2+c\left(35d^2g^2+14deg(3gx-4f)+3e^2\left(8f^2-6fgx+5g^2x^2\right)\right)\right) - \frac{2\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(a^2\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 + 14*d*e*g*
(-4*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2))) - (2*(g^2*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]*(a^2*e*g^2*(13*e*f - 42*d*g) + c^2*f*(24*e^2*f^2 - 5
6*d*e*f*g + 35*d^2*g^2)*x^2 + a*c*(35*d^2*f*g^2 - 14*d*e*g*(4*f^2 + 3*g^2*x
^2) + e^2*(24*f^3 + 13*f*g^2*x^2))) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(
a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(5*a*e^2*g^2 + (6*I)*Sqrt[a]*Sqr
t[c]*e*g*(3*e*f - 7*d*g) + c*(-24*e^2*f^2 + 56*d*e*f*g - 35*d^2*g^2))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
```

```
)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*c*g^5*Sqrt[a + c*x
^2])
```

fricas [F] time = 2.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+a}}{\sqrt{gx+f}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2+a}\,(ex+d)^2}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

maple [B] time = 0.04, size = 3278, normalized size = 6.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(35*a*c^2*d^2*f*g^4+10*a^2*c*e^2*f*g^4+
24*a*c^2*e^2*f^3*g^2+15*x^5*c^3*e^2*g^5+35*x^3*c^3*d^2*g^5-56*a*c^2*d*e*f^2
*g^3-3*x^4*c^3*e^2*f*g^4+25*x^3*a*c^2*e^2*g^5+42*x^4*c^3*d*e*g^5+84*(-(g*x+
f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g
*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1
/2)*g))^(1/2))*a^2*c*d*e*g^5-36*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*e^2*f*g^4-36*(-
(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/
2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF
((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*
c)^(1/2)*g))^(1/2))*a*c^2*e^2*f^3*g^2+26*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2
))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*e^2*f*
g^4+70*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(
-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*
EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d^2*f*g^4+74*(-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*
c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2
*e^2*f^3*g^2-112*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/
2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)
^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d*e*f^4*g+6*x^3*c^3*e^2*f^2*g^3+
```

```
35*x^2*c^3*d^2*f*g^4+70*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a
*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(
1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f
+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d^2*f^3*g^2+24*x^2*c^3*e^
2*f^3*g^2+10*x*a^2*c*e^2*g^5+35*x*a*c^2*d^2*g^5+112*(-(g*x+f)/(-c*f+(-a*c)^
(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+
(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*
c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(
-a*c)^(1/2)*a*c*d*e*f*g^4+48*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*
x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a
*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-
(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*e^2*f^5-14*x*a*c^2*d
*e*f*g^4-70*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(
c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(
1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2
)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*d^2*g^5-70*(-(g*x+f)/(-c
*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1
/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/
(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g)
)^(1/2))*(-a*c)^(1/2)*c^2*d^2*f^2*g^3-48*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2
))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)
*c^2*e^2*f^4*g-84*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1
/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g
)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c
)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*d*e*g^5+6*x*a*c^2*e^2*f^2*g^3
+112*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a
*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*El
lipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c
*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c^2*d*e*f^3*g^2+84*(-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c
*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(
1/2))*a*c^2*d*e*f^2*g^3-38*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+
(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c
)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-
c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*e^2*f^2*g
^3-196*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(
-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*
EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d*e*f^2*g^3+10*(-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-
a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a
*c)^(1/2)*a^2*e^2*g^5-14*x^3*c^3*d*e*f*g^4+42*x^2*a*c^2*d*e*g^5+7*x^2*a*c^2
*e^2*f*g^4-56*x^2*c^3*d*e*f^2*g^3)/g^5/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}\,(ex + d)^2}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)^2}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2/sqrt(f + g*x), x)
```

3.631 $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=364

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5}{15g^2}$$

```
[Out] -2/15*(-3*e*g*x-5*d*g+4*e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g^2-4/15*(3*a*e*
g^2+c*f*(-5*d*g+4*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2
),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x
^2/a+1)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)+4/15*(-5*d*g+4*e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2
)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a
)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/
g^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {815, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5}{15g^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) - (4*
Sqrt[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[
-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^
2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
```

```
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{4\int \frac{-\frac{1}{2}acg(ef-5dg)+\frac{1}{2}c\left(3aeg^2+cf(4ef-5dg)\right)x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{15cg^2}$$

$$= -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{\left(2(4ef-5dg)\left(cf^2+ag^2\right)\right)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+}}}{15g^3}$$

$$= -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{\left(4a\left(3aeg^2+cf(4ef-5dg)\right)\sqrt{f+gx}\,\sqrt{}\right.}{15\sqrt{-a}\,\sqrt{c}}$$

$$= -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{4\sqrt{-a}\left(3aeg^2+cf(4ef-5dg)\right)\sqrt{f+g}}{15\sqrt{c}\,g^3\sqrt{\frac{}{\sqrt{}}}}$$

Mathematica [C] time = 4.83, size = 545, normalized size = 1.50

$$\sqrt{f+gx}\left(\frac{2\left(a+cx^2\right)(5dg-4ef+3egx)}{g^2} + \frac{4\left(g^2\left(a+cx^2\right)\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(3aeg^2+cf(4ef-5dg)\right)-\sqrt{c}\,(f+gx)^{3/2}\left(-\sqrt{a}g+i\sqrt{c}f\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{}}{\sqrt{}}}{f+gx}}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x^2
) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*g + Sqr
t[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-
(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
 + g*x))))/(15*Sqrt[a + c*x^2])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\,(ex+d)}{\sqrt{gx+f}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\,(ex+d)}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

maple [B] time = 0.02, size = 1828, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/
(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*e*g^4+6*(-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2
)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF(
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*a*c*e*f^2*g^2-10*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2
)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-
c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(
1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*d*
g^4+8*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-
a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*E
llipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(
c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e*f*g^3-10*(-(g*x+f)/(-c*f+(-a*c
```

```
)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*
x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-
a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))
*(-a*c)^(1/2)*c*d*f^2*g^2+8*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x
+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*
c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(
-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*e*f^3*g-6*
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(
1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Ellipti
cE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-
a*c)^(1/2)*g))^(1/2))*a^2*e*g^4+10*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*f*g^3-14*(-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2
)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE(
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*a*c*e*f^2*g^2+10*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2
)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-
c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(
1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*f^3*g-8*(-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2
)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE(
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*c^2*e*f^4+3*x^4*c^2*e*g^4+5*x^3*c^2*d*g^4-x^3*c^2*e*f*g^
3+3*x^2*a*c*e*g^4+5*x^2*c^2*d*f*g^3-4*x^2*c^2*e*f^2*g^2+5*x*a*c*d*g^4-x*a*c
*e*f*g^3+5*a*c*d*f*g^3-4*a*c*e*f^2*g^2)/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}\,(ex+d)}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)
```

3.632 $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}} \, dx$

Optimal. Leaf size=322

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\mathrm{s}\right.}{3g^2\sqrt{a+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/3*f*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(
1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*
c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2
)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/3*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1
/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(
-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/g^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {735, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\mathrm{s}\right.}{3g^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) + (4*Sqrt[-a]*Sqrt[c]*f*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*g^2*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqr
t[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
```

```
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} + \frac{2\int \frac{ag-cfx}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} + \frac{1}{3}\left(2\left(a+\frac{cf^2}{g^2}\right)\right)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx - \frac{(2cf)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} - \frac{\left(4a\sqrt{c}\,f\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right.}{3\sqrt{-a}\,g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\,\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} + \frac{4\sqrt{-a}\,\sqrt{c}\,f\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\sqrt{a+cx^2}} - \cdots$$

Mathematica [C] time = 1.99, size = 456, normalized size = 1.42

$$\frac{2\sqrt{f+gx}\left(g^2\left(a+cx^2\right) - \frac{2\left(fg^2\left(a+cx^2\right)\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}} - \sqrt{a}g(f+gx)^{3/2}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\,\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\cdots\right.\right.}{(f+gx)\sqrt{-f\cdots}}\cdots}{3g^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2) - (2*(f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]*(a + c*x^2) + Sqrt[c]*f*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a
])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))
]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt
[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqrt[a]*
g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x))))/(3*g^3*Sqrt[a + c*x^2])
```

fricas [F] time = 1.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

maple [B] time = 0.02, size = 688, normalized size = 2.14

$$-\frac{2\sqrt{c\,x^2+a}\,\sqrt{gx+f}\,\left(-c^2g^3x^3-c^2f\,g^2x^2-2\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\,acf\,g^2\,\text{EllipticE}\left(\sqrt{-\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] -2/3*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/
(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*
g^3+2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-
a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*E
llipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(
c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*f^2*g-2*(-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(
-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c
)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*
c*f*g^2-2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*
f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/
2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*
g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*f^3-x^3*c^2*g^3-x^2*c^2*f*g^2-x*a*c*g^3
-a*c*f*g^2)/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}}{\sqrt{f + g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

3.633 $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=473

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,(dg+ef)\sqrt{\frac{\sqrt{c}}{\sqrt{-a}}}}{e^2g\sqrt{a}}$$

```
[Out] -2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)
/e/g/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*(d*
g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f
*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*
c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
-2*(a*e^2+c*d^2)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {923, 933, 168, 538, 537, 844, 719, 424, 419}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,(dg+ef)\sqrt{\frac{\sqrt{c}}{\sqrt{-a}}}}{e^2g\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])
/(e*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*Sqrt[c]*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt
[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*g*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e),
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f +
 Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 923

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Dist[(c*d^2 + a*e^2)/e^2, Int[1/((d + e*x)*Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sqrt
[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &
& NeQ[c*d^2 + a*e^2, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx - \frac{\int \frac{cd-cex}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{eg} - \frac{(c(ef+dg))\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e^2g} + \frac{\left(\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{}{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\,\sqrt{1+}}}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}\,g}{\sqrt{c}}-\frac{\sqrt{-a}\,gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\right)}{\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\,\sqrt{c}\,(ef+dg)}{}$$

$$= -\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\,\sqrt{c}\,(ef+dg)}{}$$

Mathematica [C] time = 4.31, size = 1096, normalized size = 2.32

$$-\frac{2\left(-ce^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,f^3+2ce^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,f^2-ce^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+gx)^2f-2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(-(c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]] - a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d
*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]*(f + g*x) - 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g
*x) - c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a
]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*
Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]
*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)] - I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
```

```
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g
*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellip
ticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[
c]*f + I*Sqrt[a]*g)]))/(e^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)\sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

maple [B] time = 0.03, size = 1216, normalized size = 2.57

$$2\left(-acde\,g^3\,\mathrm{EllipticE}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}, \sqrt{-\frac{-cf+\sqrt{-ac}\,g}{cf+\sqrt{-ac}\,g}}\right) + acde\,g^3\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}, \sqrt{-\frac{-cf+\sqrt{-ac}\,g}{cf+\sqrt{-ac}\,g}}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2*(EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*
g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3-EllipticF((-(g*x+f)/
(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g)
)^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g+EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*e*g^3-
EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-EllipticF((-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2
*d^2*f*g^2+EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c
)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*e*f^2*g-EllipticE((-(g*x+f)/(
-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))
^(1/2))*a*c*d*e*g^3+EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-
c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-EllipticE((-
(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^
(1/2)*g))^(1/2))*c^2*d*e*f^2*g+EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e^2*f^3-Ell
ipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g
-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)
*a*e^2*g^3-EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)
^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2)
)*(-a*c)^(1/2)*c*d^2*g^3+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2
```

```
),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(
1/2)*g))^(1/2))*a*c*e^2*f*g^2+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*c^2*d^2*f*g^2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*
g)*g)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*(-(g*x+f)/(-
c*f+(-a*c)^(1/2)*g)*c)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e^2/c/g^2/(d*g-e
*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

3.634 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=694

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\,(ef-dg)}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,(ae^2g+\ldots}{e^2\sqrt{a+c\ldots}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)-EllipticE(1/2*(1-x*c^(1/2
)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a
)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e/(-d*g+e*f)/(c*x^2+a)^(1/2
)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*
(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/
2)))^(1/2)/e/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-d*g+2*e*f)*Elliptic
F(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(
1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^
2*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2)
,2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 1.72, antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {925, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\,(ef-dg)}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,(ae^2g+\ldots}{e^2\sqrt{a+c\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - (Sqrt[-a]*Sqrt
[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[
-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g
)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) - (Sqrt[-a]*Sqrt[c]*(2*e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*(e*f - d*g)*Sqrt[
f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f
 + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((S
qrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2
*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*
f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)*Simp[a*g*(2*m + 3) + 2*(c*f)*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f
+ g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{cgx}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-ae^2g-cd(2ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right) d}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2(ef-dg)} -$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{\left(\left(ag + \frac{cd(2ef-dg)}{e^2}\right.\right.}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\,(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{c}f-a}\right.}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{c}f-a}\right.}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 6.41, size = 1336, normalized size = 1.93

$$\sqrt{f+gx}\left(\frac{cx^2+a}{d+ex}-\frac{ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3-2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2-cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2+ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f+2cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f-2i}{}\right.$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]

[Out] (Sqrt[f + g*x]*((a + c*x^2)/(d + e*x) - (c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]] - c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*e^2*f*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] -
2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*c*d*e*f*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 - c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I
*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f
+ I*Sqrt[a]*g)*(Sqrt[a]*e*g + I*Sqrt[c]*(2*e*f - d*g))*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f
 + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d*
e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/S
qrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/
(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/S
qrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*d
^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*
e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((-(e*f) + d*g
)*Sqrt[a + c*x^2])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

maple [B] time = 0.04, size = 6034, normalized size = 8.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)^2 \sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}}{\sqrt{f + g\,x}\,(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)^2 \sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

3.635 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1241

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)\left(3age^2+cd(2ef+dg)\right)}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}-\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\cdots}}{4\cdots}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*
(d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(e*x+
d)+1/4*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(
1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)
*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/
2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(1/2*(1-
x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1
/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*f*(3*a*e^2
*g+c*d*(d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-
2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1
/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(-d*g+
e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*d*g*(3*a*e^2*g+c*d*(d*g+2*e*f))*El
lipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/
2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(
g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)^2/(g*x+f)^(1/2)
/(c*x^2+a)^(1/2)-c*(d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*
2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1
/2)-1/4*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticPi
(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^
(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 4.24, antiderivative size = 1241, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {925, 6742, 719, 419, 940, 844, 424, 933, 168, 538, 537}

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)\left(3age^2+cd(2ef+dg)\right)}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}-\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\cdots}}{4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*g
+ c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)*(e*f
 - d*g)^2*(d + e*x)) + (Sqrt[-a]*Sqrt[c]*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sq
rt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(
e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*
x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*
```

```
g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^2*(e*f - d*g)*Sqrt[f + g*
x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*f*(3*a*e^2*g + c*d*(2*e*f + d*g))*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqr
t[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c
*x^2]) + (Sqrt[-a]*Sqrt[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
(c*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2
*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((
a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)
/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^2*((Sqrt[c]*d)/Sqrt[-a] +
e)*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
```

```
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)*Simp[a*g*(2*m + 3) + 2*(c*f)*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f
+ g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{-3ag+2cfx-cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int\Big(-\frac{cg}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{-3ae^2g-cd(2ef+dg)}{e^2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{2c(ef+dg)}{e^2(d+ex)\sqrt{f+gx}\,\cdots}\cdots}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(c(ef+dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\cdots}{2e^2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g+cd(\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\cdots}}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\cdots}}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\cdots}}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\cdots}}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\,\sqrt{c}\,\left(3a\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\,\sqrt{c}\,\left(3a\cdots\right.}{\cdots}$$

Mathematica [C] time = 10.64, size = 2197, normalized size = 1.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (c^2*d^2*f^3 - 3*a*c*e^2*f^3 - (2*c^2*d*e*f^4)/g + (c^2*d^3*f^2*g)/e + a*c*
d*e*f^2*g + a*c*d^2*f*g^2 - 3*a^2*e^2*f*g^2 + (a*c*d^3*g^3)/e + 3*a^2*d*e*g
^3 - 2*c^2*d^2*f^2*(f + g*x) + 6*a*c*e^2*f^2*(f + g*x) + (4*c^2*d*e*f^3*(f
+ g*x))/g - (2*c^2*d^3*f*g*(f + g*x))/e - 6*a*c*d*e*f*g*(f + g*x) + c^2*d^2
*f*(f + g*x)^2 - 3*a*c*e^2*f*(f + g*x)^2 - (2*c^2*d*e*f^2*(f + g*x)^2)/g +
(c^2*d^3*g*(f + g*x)^2)/e + 3*a*c*d*e*g*(f + g*x)^2 - ((e*f - d*g)*(f + g*x
)*(a + c*x^2)*(a*e^2*(2*e*f - 5*d*g - 3*e*g*x) - c*d*(3*d^2*g + 2*e^2*f*x +
 d*e*g*x)))/(d + e*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d
*g)*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f +
 g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]) + ((I*Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(-3*a*e^2*g
 - (6*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + d*g))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g
*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((4*I)*a*c*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
 + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)
), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
 ((4*I)*c^2*d^3*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]
*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[
a]*g)])/(e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) - ((4*I)*a*c*d*e*f*g*Sqrt[(g*(
(I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(
f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f +
 I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]] + ((6*I)*a*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*c^2*d^4*g^2*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c
]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]) + ((3*I)*a^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(
3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4*(c*d^2
 + a*e^2)*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

maple [B] time = 0.08, size = 19187, normalized size = 15.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
[Out] Timed out
```

3.636 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=531

$$-\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\left(25ae^2g^2-c\left(105d^2g^2-42defg+8e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{}{\sqrt{-}}\right.}{105c^{5/2}g^3\sqrt{a+cx^2}\,\sqrt{f+gx}}$$

```
[Out] 2/35*e^2*(11*d*g+e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^2-2/105*e*(25*a*e^2
*g^2+c*(-90*d^2*g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^
2/g^2+2/7*e*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c+2/105*(a*e^2*g^2*(189
*d*g+19*e*f)-c*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^2*f^2*g+8*e^3*f^3))*Elli
pticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)
*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(3/2)/g^3/(c
*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/105*e*(a*g
^2+c*f^2)*(25*a*e^2*g^2-c*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*EllipticF(1/2
*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2))
)^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1
/2)))^(1/2)/c^(5/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 1.12, antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {942, 1654, 844, 719, 424, 419}

$$-\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\left(25ae^2g^2-c\left(105d^2g^2-42defg+8e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{}{\sqrt{-}}\right.}{105c^{5/2}g^3\sqrt{a+cx^2}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(90*d^2 - e^2*((25*a)/c + (7*f^2)/g^2) - (12*d*e*f)/g)*Sqrt[f + g*x]*S
qrt[a + c*x^2])/(105*c) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
7*c) + (2*e^2*(e*f + 11*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*c*g^2) +
(2*Sqrt[-a]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g +
 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(105*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*e*(c*f^2 + a*g^2)*(25*a*e^2*g^2 - c*(8
*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(5/2)*g^3*Sqr
t[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 942

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x])/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+ae(4ef+dg)+\left(5ae^2g-cd(12ef+7dg)\right)x-ce(ef+11dg}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(35cd^3\right.}{}}{}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+}}{7c}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+}}{7c}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+}}{7c}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+}}{7c}$$

Mathematica [C] time = 6.16, size = 747, normalized size = 1.41

$$2\sqrt{f+gx}\left(\frac{g\sqrt{f+gx}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(25a^{3/2}e^3g^2+\sqrt{a}ce\left(-105d^2g^2+42defg-8e^2f^2\right)+3ia\sqrt{c}e^2g(2ef-63dg)+105ic^{3/2}d^3g^2\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-(g^2*(a + c*x^2)*(25*a*e^3*g^2 + c*e*(-105*d^2*g^2 - 21*
d*e*g*(f + 3*g*x) + e^2*(4*f^2 - 3*f*g*x - 15*g^2*x^2)))) + (g^2*(-(a^2*e^2
*g^2*(19*e*f + 189*d*g)) + c^2*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^
2 + 105*d^3*g^3)*x^2 + a*c*(105*d^2*e*f*g^2 + 105*d^3*g^3 - 21*d*e^2*g*(2*f
^2 + 9*g^2*x^2) + e^3*(8*f^3 - 19*f*g^2*x^2))))/(f + g*x) + I*c*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(-(a*e^2*g^2*(19*e*f + 189*d*g)) + c*(8*e^3*f^3 - 42*
d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (g*(Sqrt[c]*f + I*Sqr
t[a]*g)*((105*I)*c^(3/2)*d^3*g^2 + 25*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2
*g*(2*e*f - 63*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
```

```
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(105*c^2*g^4*Sqrt[a + c*x^2])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}\right)\sqrt{gx+f}}{\sqrt{cx^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(g*x + f)/sqrt(c*x^2
 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{3}\sqrt{gx+f}}{\sqrt{cx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

maple [B] time = 0.06, size = 3924, normalized size = 7.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-25*a^2*c*e^3*f*g^4-4*a*c^2*e^3*f^3*g^
2+15*x^5*c^3*e^3*g^5+84*x*a*c^2*d*e^2*f*g^4-8*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*
g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)
^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*e^3
*f^5-42*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*d*e^2*f*g^4-189*(-(g*x+f)/(-
c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(
1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)
/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g
))^(1/2))*a*c^2*d*e^2*f^2*g^3+105*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*
((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*
f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/
2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*d^
2*e*g^5-17*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1
/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)
*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*c*e^3*f^2*g^3-105*(-(g*x+f)
/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g
)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x
+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2
)*g))^(1/2))*a*c^2*d^2*e*f*g^4+231*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d*e^2*f^2*g^
```

3+105*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-
a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*E
llipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(
c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c^2*d^2*e*f^2*g^3-42*(-(g*x+f)/(-c
*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1
/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/
(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g)
)^(1/2))*(-a*c)^(1/2)*c^2*d*e^2*f^3*g^2-105*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
c)^(1/2)((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(
1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)
*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d^3
g^5+6(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(
-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*
EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*e^3*f^3*g^2+8*(-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a
*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^
(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*
c)^(1/2)*c^2*e^3*f^4*g+11*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(
-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)
^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c
*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*e^3*f^3*g^2-105*(-(g*
x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*
g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-
(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^
(1/2)*g))^(1/2))*c^3*d^2*e*f^3*g^2+42*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/
(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d*e^2*f^4*g
+19*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*
c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Ell
ipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*
f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*e^3*f*g^4+189*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*
g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)
^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*d
*e^2*g^5-189*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/
(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^
(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/
2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*c*d*e^2*g^5+6*(-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x
+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a
*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*
a^2*c*e^3*f*g^4+63*x^4*c^3*d*e^2*g^5+63*x^2*a*c^2*d*e^2*g^5-7*x^2*a*c^2*e^3
*f*g^4+105*x^2*c^3*d^2*e*f*g^4+21*x^2*c^3*d*e^2*f^2*g^3+105*x*a*c^2*d^2*e*g
^5-x*a*c^2*e^3*f^2*g^3+84*x^3*c^3*d*e^2*f*g^4-105*(-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-
a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3
*d^3*f^2*g^3+105*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/
2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)
^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c^2*d^3*g^5-25*(-(g*x+f)/(-c*f+(-a
*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((
c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2
))*(-a*c)^(1/2)*a^2*e^3*g^5+105*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)

```
,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^3*d^3*f^2*g^3+21*a*
c^2*d*e^2*f^2*g^3+105*a*c^2*d^2*e*f*g^4+18*x^4*c^3*e^3*f*g^4-10*x^3*a*c^2*e
^3*g^5+105*x^3*c^3*d^2*e*g^5-x^3*c^3*e^3*f^2*g^3-4*x^2*c^3*e^3*f^3*g^2-25*x
*a^2*c*e^3*g^5)/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^3/g^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.637 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=410

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(9ae^2g^2+c\left(-15d^2g^2-10defg+2e^2f^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{4\sqrt{-a}e\sqrt{}}{}$$

```
[Out] 2/15*e*(7*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/5*e*(e*x+d)*(g*x+f)^
(1/2)*(c*x^2+a)^(1/2)/c+2/15*(9*a*e^2*g^2+c*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f
^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(3/
2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/1
5*e*(-5*d*g+e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {942, 1654, 844, 719, 424, 419}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(9ae^2g^2+c\left(-15d^2g^2-10defg+2e^2f^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{4\sqrt{-a}e\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(e*f + 7*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g) + (2*e*(d + e*x)
*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + (2*Sqrt[-a]*(9*a*e^2*g^2 + c*(2*e^2
*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(15*c^(3/2)*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]
*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*e*(e*f - 5*d*g)*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(15*c^(3/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 942

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x])/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+ae(2ef+dg)+\left(3ae^2g-cd(8ef+5dg)\right)x-ce(ef+7dg)x^2}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{5c}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(15cd^2f-ae(7\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{1}{15}\left(-15d^2+e^2\left(\frac{9a\cdots}{c}\right.\right.$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{\left(2a\left(-15d^2+e^2\left(\frac{9a\cdots}{c}\right.\right.\right.}{\cdots}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{2\sqrt{-a}\left(15d^2-e^2\left(\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 4.30, size = 596, normalized size = 1.45

$$\frac{2\sqrt{f+gx}\left(\frac{g^2\left(-9a^2e^2g^2+ac\left(15d^2g^2+10defg-\left(e^2\left(2f^2+9g^2x^2\right)\right)\right)+c^2x^2\left(15d^2g^2+10defg-2e^2f^2\right)\right)}{f+gx} - ic\sqrt{f+gx}\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*g^2*(a + c*x^2)*(10*d*g + e*(f + 3*g*x)) + (g^2*(-9*a
^2*e^2*g^2 + c^2*(-2*e^2*f^2 + 10*d*e*f*g + 15*d^2*g^2)*x^2 + a*c*(10*d*e*f
*g + 15*d^2*g^2 - e^2*(2*f^2 + 9*g^2*x^2))))/(f + g*x) - I*c*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]*(9*a*e^2*g^2 + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)] + (Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((15*I)*c*d^2*g - (9*I)*a*e^2*
g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]))/(15*c^2*g^3*Sqrt[a + c*x^2])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)

maple [B] time = 0.05, size = 2470, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)

[Out] -2/15*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-10*a*c*d*e*f*g^3-3*x^4*c^2*e^2*g^4-15*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d^2*g^4-a*c*e^2*f^2*g^2+9*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*e^2*g^4-9*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*e^2*g^4-2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e^2*f^4+9*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f^2*g^2-11*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f^2*g^2+10*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*e*f^3*g-10*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*d*e*g^4+2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e^2*f*g^3-15*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*

```
f+(-a*c)^(1/2)*g))^(1/2))*c^2*d^2*f^2*g^2+15*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g
)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^
(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d^2*
g^4+15*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(
-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*
EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d^2*f^2*g^2+10*(-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*
c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d
*e*f*g^3-10*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(
c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(
1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2
)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d*e*f^2*g^2+2*(-(g*x+f)/(-
c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(
1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)
/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g
))^(1/2))*(-a*c)^(1/2)*c*e^2*f^3*g-10*x^3*c^2*d*e*g^4-4*x*a*c*e^2*f*g^3-10*
x*a*c*d*e*g^4-10*x^2*c^2*d*e*f*g^3-4*x^3*c^2*e^2*f*g^3-3*x^2*a*c*e^2*g^4-x^
2*c^2*e^2*f^2*g^2)/c^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,(d+e\,x)^2}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.638 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=331

$$\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(3dg+}{3\sqrt{c}\,g\sqrt{a}}$$

```
[Out] 2/3*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c-2/3*(3*d*g+e*f)*EllipticE(1/2*(1-x*c^
(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))
*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g/c^(1/2)/(c*x^2+a)^(1/2)/((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*e*(a*g^2+c*f^2)*EllipticF(1
/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2
)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)/c^(3/2)/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {833, 844, 719, 424, 419}

$$\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(3dg+}{3\sqrt{c}\,g\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*Sqrt[-a]*(e*f + 3*d*g)*Sqrt[
f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*e*(c*
f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*g*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
```

```
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx &= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-aeg)+\frac{1}{2}c(ef+3dg)x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3c} \\
&= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} + \frac{(ef+3dg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g} - \frac{\left(e\left(cf^2+ag^2\right)\right)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3cg} \\
&= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} + \frac{\left(2a(ef+3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,d\cdots\right.}{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\,\sqrt{a+cx^2}} \cdots \\
&= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} - \frac{2\sqrt{-a}\,(ef+3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{\cdots}{\sqrt{-a}}\cdots\right)}{3\sqrt{c}\,g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\sqrt{a+cx^2}} \cdots
\end{aligned}
$$

Mathematica [C] time = 3.35, size = 464, normalized size = 1.40

$$
\frac{2\sqrt{f+gx}\left(\frac{ic\sqrt{f+gx}\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(3dg+ef)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g^2}+\frac{\left(a+cx^2\right)(3dg+ef)}{f+gx}+\frac{i\sqrt{f+gx}\,(3\cdots}{\cdots}\cdots\right)}{3c\sqrt{a+cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(e*(a + c*x^2) + ((e*f + 3*d*g)*(a + c*x^2))/(f + g*x) + (
I*c*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f + 3*d*g)*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt
[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/g^2 + (I*(3*Sqrt[
c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]])))/(3*c*Sqrt[a + c*x^2])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

maple [B] time = 0.04, size = 1286, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e*g
^3+(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c
)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Elli
pticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f
+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*e*f^2*g+3*(-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-
a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c
*d*g^3+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f
+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2
)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g
)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*f^2*g-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g
)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^
(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*g^
3-(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)
```

```
^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Ellip
ticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+
(-a*c)^(1/2)*g))^(1/2))*a*c*e*f*g^2-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/
(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*f^2*g-(-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2
)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE(
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*c^2*e*f^3+x^3*c^2*e*g^3+x^2*c^2*e*f*g^2+x*a*c*e*g^3+a*c*
e*f*g^2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2/g^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\,(d+e\,x)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.639 $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}$$

```
[Out] -2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(1/2)
/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {719, 424}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.47, size = 294, normalized size = 2.16

$$\frac{2i\sqrt{f+gx}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(\sqrt{a}+i\sqrt{c}x\right)}{\sqrt{a}g-i\sqrt{c}f}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\right.\right.}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{g\left(\sqrt{c}x+i\sqrt{a}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] ((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqr
t[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(f
+ g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/
(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c
*x^2])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

maple [B] time = 0.04, size = 396, normalized size = 2.91

$$2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\left(cf-\sqrt{-ac}\,g\right)\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\,\left(-cf\,\mathrm{EllipticE}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(c*f-(-a*c)^(1/2)*g)*(-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x
+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((-a*c)^(1/2)*EllipticF((-(g*
x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/
2)*g))^(1/2))*g+f*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*
f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c-EllipticE((-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2
))*(-a*c)^(1/2)*g-EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*
f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c*f)/g/(c*g*x^3+c*f*x^2+a*g*
x+a*f)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/(a + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)/(a + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

3.640 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=319

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\right.}{\sqrt{c}\,e\sqrt{a+cx^2}\,\sqrt{f}}$$

```
[Out] -2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d
*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1
/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x
^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(e+d*c^(1/
2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {944, 719, 419, 933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\right.}{\sqrt{c}\,e\sqrt{a+cx^2}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
 (2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
 (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*
((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\,\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}\,(d+ex)\sqrt{f+gx}}\,dx}{e\sqrt{a+cx^2}} + \frac{\left(2ag\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Su}}{\sqrt{-a}\ \cdots}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)\sqrt{1+\cdots}\right.}{\cdots}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)\sqrt{\frac{\sqrt{\cdots}}{\sqrt{c}\cdots}}\right.}{\cdots}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{2(ef-dg)\sqrt{\frac{\sqrt{c}\cdots}{\sqrt{c}f\cdots}}}{\cdots}$$

Mathematica [C] time = 1.18, size = 300, normalized size = 0.94

$$-\frac{2i\sqrt{f+gx}\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{\sqrt{a}g-i\sqrt{c}f}}\left(F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\Pi\left(\frac{e\left(f-\frac{i\sqrt{a}g}{\sqrt{c}}\right)}{ef-dg};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-\cdots}{\sqrt{c}f+\cdots}\right.\right.}{e\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{g(\sqrt{c}x+i\sqrt{a})}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqrt
[f + g*x]*(EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sq
rt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Ellipti
cPi[(e*(f - (I*Sqrt[a]*g)/Sqrt[c]))/(e*f - d*g), I*ArcSinh[Sqrt[-((Sqrt[c]*
(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)]))/(e*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x)
)]*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

maple [A] time = 0.05, size = 439, normalized size = 1.38

$$2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\,\left(cf\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}},\sqrt{-\frac{-cf+\sqrt{-ac}\,g}{cf+\sqrt{-ac}\,g}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*(f*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c-(-a*c)^(1/2)*Ell
ipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*
f+(-a*c)^(1/2)*g))^(1/2))*g-EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(
1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*c*f+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),
(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/
2)*g))^(1/2))*(-a*c)^(1/2)*g)/c/e/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.641 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=698

$$-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)}-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\ldots}{e\sqrt{\ldots}}$$

```
[Out] -e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)-EllipticE(1/2*(1-x*c
^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2)
)*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+c*d^2)/(c*x^2+a
)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1
-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(
1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2
*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 1.93, antiderivative size = 698, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {946, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)}-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\ldots}{e\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (Sqrt[-a
]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 +
a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[
a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/(
(Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*
d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 946

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
```

```
(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^
2])/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1) - e*(a*g) + 2*c*(d*g*(m + 1) - e*f
*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-2cdf-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{cgx}{\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{-ae^2g-cd(2ef-dg)}{e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\right)dx}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2e\left(cd^2+ae^2\right)} + \frac{\left(ae^2g+\right.}{}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)} + \frac{\left(\left(ae^2g+cd(2ef-\right.\right.}{}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,dg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2}{\sqrt{-a}}\right.}{e\left(cd^2+ae^2\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right.}{\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right.}{\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

Mathematica [C] time = 6.01, size = 1330, normalized size = 1.91

$$\sqrt{f+gx}\left(-\frac{(cx^2+a)e^2}{d+ex}-\frac{-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-2cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\,(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-(c*e^2*f^3*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2
*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*
e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((
I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] +
e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a]
)/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]
*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[
f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d
*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*
d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a
*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((c*d^2*e + a*e^3
)*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}\,(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

maple [B] time = 0.06, size = 5743, normalized size = 8.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}\,(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.642 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=1246

$$-\frac{\left(age^2+cd(6ef-5dg)\right)\sqrt{f+gx}\,\sqrt{cx^2+a}\,e}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}-\frac{\sqrt{f+gx}\,\sqrt{cx^2+a}\,e}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\sqrt{-a}\,\sqrt{c}\,\left(age^2+cd(6ef-5dg)\right)\sqrt{f+g}}{4\left(cd^2+ae^2\right)^2(ef-}$$

```
[Out] -1/2*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^2-1/4*e*(a*e^2*g
+c*d*(-5*d*g+6*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*
f)/(e*x+d)-1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)
^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2
)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(c*x^2
+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*
(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2
^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^
2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)
^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g+c*d*(-5*d*g+6*
e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*
(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c
^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(-d*g+e*f)/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2
))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+
f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)+1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*(a*e^2*g-c*d*(-3*d*g+2*e*f))
*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a
)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(
-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 4.40, antiderivative size = 1246, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {946, 6742, 719, 419, 940, 844, 424, 933, 168, 538, 537}

$$-\frac{\left(age^2+cd(6ef-5dg)\right)\sqrt{f+gx}\,\sqrt{cx^2+a}\,e}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}-\frac{\sqrt{f+gx}\,\sqrt{cx^2+a}\,e}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\sqrt{-a}\,\sqrt{c}\,\left(age^2+cd(6ef-5dg)\right)\sqrt{f+g}}{4\left(cd^2+ae^2\right)^2(ef-}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (e*(a*
e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e
^2)^2*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g + c*d*(6*e*f - 5*
d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*
x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*
e^2)^2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
```

```
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g + c*d*(6*e*f - 5
*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[(
Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*
f - a*g)])/(4*e*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]
)/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^
2]) + ((a*e^2*g + c*d*(6*e*f - 5*d*g))*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elliptic
Pi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e*((Sqrt[c]*d)/Sqrt[
-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
```

```
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 946

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^
2])/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1) - e*(a*g) + 2*c*(d*g*(m + 1) - e*f
*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{-4cdf-aeg+2c(ef-2dg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{\int \left(\frac{cg}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-ae^2g-cd(6ef-5dg)}{e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef-3dg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+c}}\right.}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e\left(cd^2+ae^2\right)} - \frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}}{2e\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\left(ae^2g+cd(\right.}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}\,g}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}\,g}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}\,g}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}\,g}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\,\left(a\right.}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\,\left(a\right.}{}$$

Mathematica [C] time = 10.97, size = 2450, normalized size = 1.97

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (-11*c^2*d^2*e^2*f^3 + a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 5*c^2*d^3*e*f^2*
g + 5*a*c*d*e^3*f^2*g - 11*a*c*d^2*e^2*f*g^2 + a^2*e^4*f*g^2 + 5*a*c*d^3*e*
g^3 - a^2*d*e^3*g^3 + 22*c^2*d^2*e^2*f^2*(f + g*x) - 2*a*c*e^4*f^2*(f + g*x
) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 10*c^2*d^3*e*f*g*(f + g*x) + 2*a*c*d*e
^3*f*g*(f + g*x) - 11*c^2*d^2*e^2*f*(f + g*x)^2 + a*c*e^4*f*(f + g*x)^2 + (
6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 5*c^2*d^3*e*g*(f + g*x)^2 - a*c*d*e^3*g*(f
 + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*e^2)*(e*f
- d*g) + (a*e^2*g + c*d*(6*e*f - 5*d*g))*(d + e*x)))/(d + e*x)^2 + (Sqrt[c]
*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(6*e*f - 5*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + (e*(I*Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(a*e^2*g + (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g)
 + c*d*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt
[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I)
*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*S
qrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e
*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x
))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt
[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c
]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]] - ((12*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*
Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi
[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((12*I)*a*c*d*e^3*f*g*S
qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
- g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + ((3*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))
/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2
)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((10*I)*a*c
*d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f +
 g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)),
 I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4
*e*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

maple [**B**] time = 0.10, size = 20359, normalized size = 16.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)
[Out] Timed out
```

3.643 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=600

$$\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\left(ae^2g^2+c\left(-3d^2g^2+6defg-2e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{a\sqrt{c}\,x}{(-a)^{3/2}}+1}}{\sqrt{2}}\right)|\frac{2ag}{ag-\sqrt{-a}\sqrt{c}\,f}\right)}{3c^{3/2}e^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2(ef\cdots}{\cdots}$$

```
[Out] 2/3*g^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e-2/3*g*(-3*d*g+7*e*f)*EllipticE(1/
2*(1+a*x*c^(1/2)/(-a)^(3/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c
^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/c^(1/2)/(c*x
^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*g*(a*e^2*g
^2+c*(-3*d^2*g^2+6*d*e*f*g-2*e^2*f^2))*EllipticF(1/2*(1+a*x*c^(1/2)/(-a)^(3
/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/
2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/
2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d*g+e*f)^2*EllipticPi((g*x+f)^(1/2
)*(c/(c*f+g*(-a)^(1/2)*c^(1/2)))^(1/2),e*(f+g*(-a)^(1/2)/c^(1/2))/(-d*g+e*f
),((g*(-a)^(1/2)+f*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(g*((-a)^(1/2
)-x*c^(1/2))/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)*(-g*((-a)^(1/2)+x*c^(1/2))/(-g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(c*x^2+a)^(1/2)/(c/(c*f+g*(-a)^(1/2)*c^(1
/2)))^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 808, normalized size of antiderivative = 1.35, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {958, 719, 419, 933, 168, 538, 537, 424, 743, 844}

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)(ef-dg)^3}{e^3\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{cx^2+a}}-\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{\frac{cx^2}{a}+1}\,F\left(\cdots\right.}{\sqrt{c}\,e^3\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) - (8*Sqrt[-a]*f*g*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*e*Sqrt[(Sqrt[c]*(f +
 g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*g*(e*f - d*
g)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*g*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-
2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + (2*Sqrt[-a]*g*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*e*Sqrt[f +
 g*x]*Sqrt[a + c*x^2]) - (2*(e*f - d*g)^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a
] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqr
t[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a
 + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{g(ef-dg)\sqrt{f+gx}}{e^2\sqrt{a+cx^2}} + \cdots\right.$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e^2} + \frac{\left(g(ef-dg)^2\right)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(e\cdots}{\cdots}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}\left(3cf^2-ag^2\right)+2cfgx}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3ce} + \frac{\left((ef-dg)^3\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{\cdots}{\sqrt{1-\cdots}}}{e^3\sqrt{a+\cdots}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} - \frac{2\sqrt{-a}\,g(ef-dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\cdots\right.}{\sqrt{c}\,e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} - \frac{2\sqrt{-a}\,g(ef-dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\cdots\right.}{\sqrt{c}\,e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} - \frac{8\sqrt{-a}\,fg\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f\cdots}\right.}{3\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

Mathematica [C] time = 9.58, size = 1440, normalized size = 2.40

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}\,g^2}{3ce}+\frac{2(f+gx)^{3/2}\left(\frac{7ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\,f^3}{(f+gx)^2}+\frac{3ice^2\sqrt{-\frac{f}{f+gx}-\frac{i\sqrt{a}g}{\sqrt{c}(f+gx)}+1}\sqrt{-\frac{f}{f+gx}+\frac{i\sqrt{a}g}{\sqrt{c}(f+gx)}+1}\,\Pi\left(\frac{\sqrt{c}(ef-dg)}{e\left(\sqrt{c}f+i\sqrt{a}g\right)};i\sinh^{-1}\ldots\right)}{\sqrt{f+gx}}\ldots\right)}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (2*(f + g*x)^(3/2)*(7*c*e^2
*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]] + (7*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*c*d
*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (7*a*e^2*f*g^2*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]])/(f + g*x)^2 - (14*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])/(f + g*x) + (6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) +
(Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(7*e*f - 3*d*g)*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[
f + g*x] + (I*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(I*Sqrt[a]*e*g + Sqrt[c]*(6*e*f -
 3*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 -
 f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt
[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c*e^2*f^2*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sq
rt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt
[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((6*I)*c*d
*e*f*g*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f
/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f -
d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/
Sqrt[f + g*x] + ((3*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt
[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(3*c*e^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/(f + g*x))^2)/g^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

maple [B] time = 0.05, size = 3164, normalized size = 5.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/
(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*e*g^3-6*(
-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1
/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Elliptic
F((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e^2
*g^3-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(
-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*
EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d^2*f*g^2+9*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g
)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^
(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*e*
f^2*g-9*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e^2*f^3+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(
1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)
*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1
/2)*c*d^2*g^3-6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2
))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*
g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^
(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d*e*f*g^2+2*(-(g*x+f)/
(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)
^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+
f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)
*g))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(
1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))
/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c
)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*e*g^3+7*
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(
1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Ellipti
cE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-
a*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/
(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*e*f^2*g+7
*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^
(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Ellipt
icE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(
-a*c)^(1/2)*g))^(1/2))*c^2*e^2*f^3+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/
```

```
2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(
-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*c^2*d^2*f*g^2-6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/
2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(
-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*c^2*d*e*f^2*g+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/
2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(
-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*c^2*e^2*f^3-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(
1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3+6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(
1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(
c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d*e*f*g^2-3*(-(g*x+f)/(-c*f+(-a*
c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c
*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticPi((-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)
^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g-c^2*e^2*g^3
*x^3-c^2*e^2*f*g^2*x^2-a*c*e^2*g^3*x-a*c*e^2*f*g^2)/e^3/c^2/(c*g*x^3+c*f*x^
2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{5/2}}{\sqrt{c\,x^2+a}\,\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^{\frac{5}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral((f + g*x)**(5/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.644 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=469

$$-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e^2\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,\ldots}}}{e^2\sqrt{a+cx^2}}$$

```
[Out] -2*g*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e/c^(
1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2*g*(
-d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*
g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2
)-2*(-d*g+e*f)^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {958, 719, 419, 933, 168, 538, 537, 424}

$$-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e^2\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,\ldots}}}{e^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
 (2*Sqrt[-a]*g*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[
-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + S
qrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 933

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{g\sqrt{f+gx}}{e\sqrt{a+cx^2}} \right) dx$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,d}{e^2}$$

$$= \frac{\left((ef-dg)^2\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{e^2\sqrt{a+cx^2}} + \frac{\left(2ag\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)}{\sqrt{-a}\,\sqrt{}}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}\,g(ef-dg)}{}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}\,g(ef-dg)}{}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}\,g(ef-dg)}{}$$

Mathematica [C] time = 1.08, size = 927, normalized size = 1.98

$$\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\left(-\frac{\sqrt{a}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2\sqrt{a}e}{i\sqrt{c}d+\sqrt{a}e};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)f^2}{i\sqrt{c}d+\sqrt{a}e} + \frac{2i\sqrt{a}g\sqrt{\frac{cx^2}{a}+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)f}{\sqrt{c}e} + \frac{2\sqrt{a}\,c}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]*(((2*I)*Sqrt[a]*f*g*
Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2
]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e) - (I*Sqrt[a]*d*g^
2*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt
[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e^2) + (g*Sqrt[(g*
(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*(I*Sqrt[a] + Sqrt[c]
*x)*((Sqrt[c]*f + I*Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(S
qrt[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
```

```
*g)] - I*Sqrt[a]*g*EllipticF[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I
*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*e*
Sqrt[(g*(Sqrt[a] - I*Sqrt[c]*x))/(I*Sqrt[c]*f + Sqrt[a]*g)]) - (Sqrt[a]*f^2
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), Ar
cSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f +
 Sqrt[a]*g)])/(I*Sqrt[c]*d + Sqrt[a]*e) + (2*Sqrt[a]*d*f*g*Sqrt[1 + (c*x^2)
/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*
Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(I*
Sqrt[c]*d*e + Sqrt[a]*e^2) - (Sqrt[a]*d^2*g^2*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sq
rt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(e^2*(I*Sqrt[c]*
d + Sqrt[a]*e))))/(Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{\frac{3}{2}}}{\sqrt{cx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

maple [B] time = 0.05, size = 959, normalized size = 2.04

$$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\left(-ae\,g^2\,\mathrm{EllipticE}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}},\sqrt{-\frac{-cf+\sqrt{-ac}}{cf+\sqrt{-ac}\,g}}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)/c*(EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*d*g^2
-EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*e*f*g+EllipticF((-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2
))*a*e*g^2-EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c
)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c*d*f*g+2*EllipticF((-(g*x+f)/(-c*f
+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/
2))*c*e*f^2-EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*
c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*e*g^2-EllipticE((-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2
))*c*e*f^2-EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)
^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2)
)*(-a*c)^(1/2)*d*g^2+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-
```

```
c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)
*g))^(1/2))*(-a*c)^(1/2)*e*f*g+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c
)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-
a*c)^(1/2)*g))^(1/2))*c*d*f*g-EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a
*c)^(1/2)*g))^(1/2))*c*e*f^2)/e^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}}{\sqrt{c\,x^2+a}\,\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^{\frac{3}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.645 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=457

$$\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,\left(9ae^2g^2-c\left(45d^2g^2-30defg+8e^2f^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{2\sqrt{-a}\sqrt{\cdots}}{\cdots}$$

```
[Out] -8/15*e^2*(-3*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2+2/5*e^2*(e*x+d)*
(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/15*e*(9*a*e^2*g^2-c*(45*d^2*g^2-30*d*e*
f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*
g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^
(1/2)/c^(3/2)/g^3/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)-2/15*(a*e^2*g^2*(-15*d*g+7*e*f)-c*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*
e^2*f^2*g+8*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {931, 1654, 844, 719, 424, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\left(ae^2g^2(7ef-15dg)-c\left(45d^2efg^2-15d^3g^3-30de^2f^2g+8e^3f^3\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\cdots\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-8*e^2*(e*f - 3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g^2) + (2*e^2*(d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + (2*Sqrt[-a]*e*(9*a*e^2*g^2
 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(15*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*(a*e^2*g^2*(7*e*f - 15*d*g)
- c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*Sqrt[(Sqrt[
c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/(15*c^(3/2)*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\int \frac{-5cd^3g+ae^2(2ef+dg)+e\left(3ae^2g+cd(2ef-15dg)\right)x+4ce^2(ef-3dg)x^2}{\sqrt{f+gx}\sqrt{a+cx^2}}}{5cg}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(15cd^3g\right.}{}}{}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\left(e\left(9ae^2g^2-c\right.\right.}{}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\left(2ae\left(9ae^2g^2-\right.\right.}{}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} + \frac{2\sqrt{-a}\,e\left(9ae^2g^2\right.}{}$$

Mathematica [C] time = 4.40, size = 625, normalized size = 1.37

$$\frac{2\sqrt{f+gx}\left(\frac{\sqrt{c}\,g\sqrt{f+gx}\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(9a^{3/2}e^3g^2+\sqrt{a}ce\left(-45d^2g^2+30defg-8e^2f^2\right)-ia\sqrt{c}e^2g(15dg+2ef)+15ic^{3/2}d^3g^2\right)F\left(i\sinh^{-1}\left(\sqrt{\right.\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e^2*g^2*(-4*e*f + 15*d*g + 3*e*g*x)*(a + c*x^2) + (e*g^
2*(-9*a^2*e^2*g^2 + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2)*x^2 + a*c*(-3
0*d*e*f*g + 45*d^2*g^2 + e^2*(8*f^2 - 9*g^2*x^2))))/(f + g*x) + I*c*e*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g + 45*
d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] + (Sqrt[c]*g*((15*I)*c^(3/2)*d^3*g^2 + 9*a^(3/2)*e^3*g^2 -
I*a*Sqrt[c]*e^2*g*(2*e*f + 15*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 30*d*e*f*g -
 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt
[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(15*c^2*g^4*Sqrt[a
+ c*x^2])
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+a}\,\sqrt{gx+f}}{cgx^3+cfx^2+agx+af},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)*sqrt(g*x
 + f)/(c*g*x^3 + c*f*x^2 + a*g*x + a*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

maple [B] time = 0.06, size = 2949, normalized size = 6.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/15*(4*a*c*e^3*f^2*g^2-45*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x
+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*
c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(
-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d^2*e*f*g^
3+30*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a
*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*El
lipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c
*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d*e^2*f^2*g^2+45*(-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c
*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(
1/2))*a*c*d*e^2*f*g^3-30*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-
a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^
(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*
f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*e^2*f*g^3-3*x^4*c^2*e^
3*g^4-15*x^2*c^2*d*e^2*f*g^3+15*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((
-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)
,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d^3*g^
4+45*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a
*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*El
lipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c
*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d^2*e*f^2*g^2-30*(-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*
c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d
*e^2*f^3*g-15*x*a*c*d*e^2*g^4+x*a*c*e^3*f*g^3-15*(-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a
*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^
(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*
c)^(1/2)*a*d*e^2*g^4+7*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*
c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1
/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+
(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e^3*f*g^3-8*(-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2
```

```
)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF(
(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c
)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*e^3*f^3*g-45*(-(g*x+f)/(-c*f+(-a*c)^(1/2)
*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c
)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d^
2*e*g^4-6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*
f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/
2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*
g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^3*f^2*g^2+45*(-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(
-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c
)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*
c*d^2*e*g^4-(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(
c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(
1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2
)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^3*f^2*g^2-15*(-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x
+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a
*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*
c^2*d^3*f*g^3-15*x^3*c^2*d*e^2*g^4+x^3*c^2*e^3*f*g^3-3*x^2*a*c*e^3*g^4+4*x^
2*c^2*e^3*f^2*g^2+9*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^
(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)
*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a
*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a^2*e^3*g^4-9*(-(g*x+f)/(-c*f+(-a
*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((
c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+
(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2
))*a^2*e^3*g^4+8*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/
2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)
^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e^3*f^4-15*a*c*d*e^2*f*g^3)*(g*x
+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/g^4/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.646 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=356

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(g^2\left(3cd^2-ae^2\right)+2cef(ef-3dg)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+4\sqrt{-a}e\cdots$$

```
[Out] 2/3*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+4/3*e*(-3*d*g+e*f)*EllipticE(1/2*
(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))
^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1
/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*((-a*e^2+3*c*d^2)*
g^2+2*c*e*f*(-3*d*g+e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1
/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+f)^(1/2
)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {931, 24, 844, 719, 424, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(g^2\left(3cd^2-ae^2\right)+2cef(ef-3dg)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+4\sqrt{-a}e\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) + (4*Sqrt[-a]*e*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*((3*c*d^2 - a*e^2)*g^2 + 2*c*e*f*(e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3
/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-d\left(3cd^2-ae^2\right)g+e\left(ae^2g+cd(2ef-9dg)\right)x+2ce^2(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-e^2\left(3cd^2-ae^2\right)g+2ce^3(ef-3dg)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{(2e(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2} + \frac{1}{3}\left(3d^2 - \frac{ae^2}{c} + \frac{2ef(ef-3\ldots}{g^2}\right.$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\left(4ae(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx\ldots}{\sqrt{-a}\left(cf-\frac{a\ldots}{\ldots}\right)}}}{\sqrt{1-x^2}}\ldots\right.}{3\sqrt{-a}\sqrt{c}g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} + \frac{4\sqrt{-a}\,e(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\ldots\right.}{3\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.54, size = 473, normalized size = 1.33

$$\frac{2\sqrt{f+gx}\left(\frac{g\sqrt{f+gx}\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(2\sqrt{a}\sqrt{c}e(ef-3dg)-iae^2g+3icd^2g\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}-\frac{2eg^2\left(a+cx^2\right)(ef-3dg)}{f+gx}\right.}{3cg^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(e^2*g^2*(a + c*x^2) - (2*e*g^2*(e*f - 3*d*g)*(a + c*x^2))
/(f + g*x) - (2*I)*c*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - 3*d*g)*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (g*((3*I)*c*d^2*g - I*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 3*d*g))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
)/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*g^3*Sqrt[a + c*x^2])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+a}\,\sqrt{gx+f}}{cgx^3+cfx^2+agx+af},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(g*x + f)/(c*g*x^3 +
 c*f*x^2 + a*g*x + a*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^2}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

maple [B] time = 0.05, size = 1769, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/3*((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a
*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*El
lipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c
*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*x+f)/(-c*f+(-a*c)^
```

```
(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+
(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*
c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(
-a*c)^(1/2)*c*d^2*g^3+6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a
*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(
1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f
+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*d*e*f*g^2-2*(-
(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/
2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF
((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*
c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g+6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)
*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c
)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*
e*g^3-3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2+3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*
g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)
^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d^2
*f*g^2-6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f
+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2
)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g
)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*d*e*g^3+2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g
)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^
(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e^2*
f*g^2-6*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+
(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)
*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)
/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*d*e*f^2*g+2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*
g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)
^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e^2
*f^3+c^2*e^2*g^3*x^3+c^2*e^2*f*g^2*x^2+a*c*e^2*g^3*x+a*c*e^2*f*g^2)*(g*x+f)
^(1/2)*(c*x^2+a)^(1/2)/c^2/g^3/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.647 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=288

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{\cdots}}$$

```
[Out] -2*e*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g/c^(
1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*(-d
*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+
f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)
/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {844, 719, 424, 419}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[a +
 c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
```

```
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{g}$$

$$= \frac{\left(2ae\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2a(-ef+dg)\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{-a}\,e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}(ef-dg)\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 1.68, size = 439, normalized size = 1.52

$$-\frac{2\left(\sqrt{c}g(f+gx)^{3/2}\left(\sqrt{a}e-i\sqrt{c}d\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-eg^2\left(a+cx^2\right)\sqrt{-\ldots}\right.}{cg^2\sqrt{a+cx^2}\sqrt{f+gx}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*(-(e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a + c*x^2)) + I*Sqrt[c]*e*(S
qrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[
-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)*g*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)]))/(c*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f + g*x]*Sqrt[a + c*x^
2])
```

fricas [F] time = 1.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\,(ex+d)\sqrt{gx+f}}{cgx^3+cfx^2+agx+af}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)/(c*g*x^3 + c*f*x^2 + a*g*x
 + a*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

maple [B] time = 0.04, size = 520, normalized size = 1.81

$$\frac{2\left(-aeg^2\,\mathrm{EllipticE}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}},\sqrt{-\frac{-cf+\sqrt{-ac}\,g}{cf+\sqrt{-ac}\,g}}\right)+aeg^2\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{-cf+\sqrt{-ac}\,g}},\sqrt{-\frac{-cf+\sqrt{-ac}\,g}{cf+\sqrt{-ac}\,g}}\right)+cdfg}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*
g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*e*g^2+EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c*d*f*
g-EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g
)/(c*f+(-a*c)^(1/2)*g))^(1/2))*(-a*c)^(1/2)*d*g^2+EllipticF((-(g*x+f)/(-c*f
+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/
2))*(-a*c)^(1/2)*e*f*g-EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(
-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*e*g^2-EllipticE((-(g*
x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/
2)*g))^(1/2))*c*e*f^2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*(
(-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*(-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+
a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.648 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] -2*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {719, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}f-ag}\right)}{\sqrt{c}\,\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.21, size = 186, normalized size = 1.37

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\,\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{a+cx^2}\,\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + c*x^2])
```

fricas [F] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\,\sqrt{gx+f}}{cgx^3+cfx^2+agx+af}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(g*x + f)/(c*g*x^3 + c*f*x^2 + a*g*x + a*f), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

maple [A] time = 0.04, size = 200, normalized size = 1.47

$$\frac{2\left(cf-\sqrt{-ac}\,g\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{-\frac{\left(gx+f\right)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+a}\;\text{EllipticF}\left(\sqrt{-\frac{\left(gx+f\right)c}{-cf+\sqrt{-ac}\,g}}, \sqrt{-\cdots}\right.}{\left(cg\,x^3+cf\,x^2+agx+af\right)cg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(c*f-(-a*c)^(1/2)*g)*EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(
-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*((c*x+(-a*c)^(1/2))/(-c
*f+(-a*c)^(1/2)*g)*g)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1
/2)*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/
g/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

3.649 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$

```
[Out] -2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(
-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+
1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(e+d*c^(1/2)/(-a)
^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*
EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/S
qrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = \frac{\sqrt{1+\frac{cx^2}{a}}\int \frac{1}{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\,\sqrt{1+\frac{\sqrt{c}\,x}{\sqrt{-a}}}\,(d+ex)\sqrt{f+gx}}\,dx}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}\,g}{\sqrt{c}}-\frac{\sqrt{-a}\,gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\right.}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}\,gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}\,g}{\sqrt{c}}\right)}}}\,dx, x\right.}{\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.91, size = 311, normalized size = 1.86

$$-\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\,\sqrt{-\frac{-gx+\frac{i\sqrt{a}\,g}{\sqrt{c}}}{f+gx}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-\Pi\left(\frac{\sqrt{c}\,(ef-dg)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right.\right.\right.}{\sqrt{a+cx^2}\,\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] - EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g))
, I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(e*f - d*g)*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

maple [A] time = 0.05, size = 235, normalized size = 1.41

$$\frac{2\left(cf-\sqrt{-ac}\,g\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{-cf+\sqrt{-ac}\,g}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{cf+\sqrt{-ac}\,g}}\,\sqrt{-\frac{\left(gx+f\right)c}{-cf+\sqrt{-ac}\,g}}\,\sqrt{c\,x^2+a}\,\sqrt{gx+f}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{\left(gx+f\right)c}{-cf+\sqrt{-ac}\,g}},\frac{\left(-cf+\cdots\right.}{\left(dg\cdots\right.}\right.}{\left(dg-ef\right)\left(cg\,x^3+cf\,x^2+agx+af\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
[Out] 2*(c*f-(-a*c)^(1/2)*g)*EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),
(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/
2)*g))^(1/2))*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((-c*x+(-a
*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)
^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + a}\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,\left(d + ex\right)\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x)), x)
```

3.650 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=746

$$-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}+\frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)(ef-dg)}-\frac{\sqrt{-a}\sqrt{\cdots}}{\cdots}$$

```
[Out] -e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-e*Ellip
ticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*
c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+
c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)+e*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-
a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+
f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2
^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^
2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)
/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^2*g-c*d*(-3*d*g+2*e*f))*Elli
pticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/
2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 2.11, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {940, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}+\frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)(ef-dg)}-\frac{\sqrt{-a}\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x
))) - (Sqrt[-a]*Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sq
rt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*e*f*Sqrt[(Sqrt[c]*(f + g*x)
)/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^
2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d
*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a]
+ e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[
c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\int \left(-\frac{cdg}{\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{cegx}{\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{ae\ldots}{(d+e\ldots}\right.}{2\left(cd^2+ae^2\right)(ef-dg\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} + \frac{(ceg)\int \frac{\ldots}{\sqrt{f+g\ldots}}}{2\left(cd^2+ae^2\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} + \frac{(ce)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} - \frac{(cef)\int \frac{\ldots}{\sqrt{f+gx}\ldots}}{2\left(cd^2+ae^2\right)\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,dg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin\ldots\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{f\ldots}}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\ldots\right.\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+\ldots}{\sqrt{c}\,f+\ldots}}}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\ldots\right.\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+\ldots}{\sqrt{c}\,f+\ldots}}}$$

Mathematica [C] time = 6.74, size = 1349, normalized size = 1.81

$$\sqrt{f+gx}\left(\frac{2\left(-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-2cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f-2icdeg\sqrt{\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*((-2*e^2*(a + c*x^2))/(d + e*x) + (2*(-(c*e^2*f^3*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*
e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c
*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g
*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)
/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (Sqrt[c]*d - I*Sqrt[a]*e)*g*(Sqrt[a]*e*g + I*Sqrt[c]*(e*f - 2*d*g))*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[
c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(
Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)] + I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[
(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)]))/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-(e*f) + d*g)*(f + g*
x))))/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^2\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

maple [B] time = 0.06, size = 5738, normalized size = 7.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)^2\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + a}\,{(d + e\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)^2\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x)), x)
```

3.651 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=1257

$$\frac{3\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{cx^2+a}\,e^2}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}-\frac{\sqrt{f+gx}\sqrt{cx^2+a}\,e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3\sqrt{-a}\sqrt{c}\left(ae^2g-cd(2ef-3dg\right.}{4\left(cd^2+ae\right.}$$

```
[Out] -1/2*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^2+3
/4*e^2*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*
d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/4*e*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+c*d^2)^
2/(-d*g+e*f)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(
1/2)+1/2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a
*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f
)^(1/2)/(c*x^2+a)^(1/2)-3/4*e*f*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*
(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))
^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)
+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/
2)+3/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^
(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a
*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*Ell
ipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1
/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/
2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f
)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-3/4*(a*e^2*g-c*d*(
-3*d*g+2*e*f))^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*
e^2+c*d^2)^2/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^
(1/2)
```

Rubi [A] time = 4.33, antiderivative size = 1257, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {940, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$\frac{3\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{cx^2+a}\,e^2}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}-\frac{\sqrt{f+gx}\sqrt{cx^2+a}\,e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3\sqrt{-a}\sqrt{c}\left(ae^2g-cd(2ef-3dg\right.}{4\left(cd^2+ae\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*
x)^2) + (3*e^2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[-a]*Sqrt[c]*e*(a
*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(
```

```
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*Sqrt[-
a]*Sqrt[c]*e*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sq
rt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 +
 a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c
]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2
*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*
e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*(a*e^2*g - c
*d*(2*e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[
1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)
])/(4*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g
*x]*Sqrt[a + c*x^2])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
```

```
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3ae^2g-4cd(ef-dg)+2ce(ef-2dg)x+ce^2gx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{3\left(ae^2g-cd(2ef-3dg)\right)}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\right.}{4\left(cd^2+ae^2\right)(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)} - \frac{(c(ef-3dg}{2\left(c\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+e}$$

Mathematica [C] time = 11.76, size = 2491, normalized size = 1.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-15*c^2*d^2*e^2*f^3 - 3*a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 9*c^2*d^3*e*f^
2*g + 9*a*c*d*e^3*f^2*g - 15*a*c*d^2*e^2*f*g^2 - 3*a^2*e^4*f*g^2 + 9*a*c*d^
3*e*g^3 + 3*a^2*d*e^3*g^3 + 30*c^2*d^2*e^2*f^2*(f + g*x) + 6*a*c*e^4*f^2*(f
 + g*x) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 18*c^2*d^3*e*f*g*(f + g*x) - 6*a
*c*d*e^3*f*g*(f + g*x) - 15*c^2*d^2*e^2*f*(f + g*x)^2 - 3*a*c*e^4*f*(f + g*
x)^2 + (6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 9*c^2*d^3*e*g*(f + g*x)^2 + 3*a*c*
d*e^3*g*(f + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*
e^2)*(e*f - d*g) - 3*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d + e*x)))/(d + e*x)
^2 + (3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*(a*e^2*g + c*
d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((Sqrt[
c]*d - I*Sqrt[a]*e)*(3*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*g*(e*f - 2*d*g
) - Sqrt[a]*c*e*(2*e^2*f^2 - 6*d*e*f*g + d^2*g^2) - I*c^(3/2)*d*(4*e^2*f^2
- 9*d*e*f*g + 8*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*A
rcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt
[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I
)*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*
Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(
e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqr
t[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqr
t[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[
c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]] - ((20*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]
*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticP
i[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
 + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((4*I)*a*c*d*e^3*f*g*S
qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
- g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + ((15*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x)
)/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/
2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSin
h[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g
)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((6*I)*a*c
*d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((3*I)*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*
g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
- I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
)/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

maple [B] time = 0.13, size = 20366, normalized size = 16.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Timed out
```

3.652 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=387

$$\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}+\frac{2\sqrt{-a}\sqrt{c}\,g\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\left(ag^2+cf^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{2e\sqrt{\frac{cx^2}{a}+1}\ \sqrt{}}{}$$

```
[Out] 2*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+2*g*EllipticE(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(-d*g+e*f)/(
a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/
2)-2*e*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/
2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^
2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)/(e
+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {958, 745, 21, 719, 424, 933, 168, 538, 537}

$$\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}+\frac{2\sqrt{-a}\sqrt{c}\,g\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\left(ag^2+cf^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{2e\sqrt{\frac{cx^2}{a}+1}\ \sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) + (2*Sq
rt[-a]*Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e
*f - d*g)*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[a + c*x^2]) - (2*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[S
qrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a
]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int\left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{e}{(ef-dg)(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{(2cg)\int \frac{-\frac{f}{2}-\frac{gx}{2}}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{(ef-dg)\left(cf^2+ag^2\right)} + \frac{\left(e\sqrt{1+\frac{cx^2}{a}}\right)\int}{(}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{(ef-dg)\left(cf^2+ag^2\right)} - \frac{\left(2e\sqrt{1+\frac{cx^2}{a}}\right)\mathrm{Su}}{}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(2e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{2-}}\right.}{(ef-dg)\sqrt{}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{2\sqrt{-a}\,\sqrt{c}\,g\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\right.\right.}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+}{\sqrt{c}f+\sqrt{}}}}$$

Mathematica [C] time = 3.23, size = 468, normalized size = 1.21

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(\left(\sqrt{c}(dg-2ef)+i\sqrt{a}eg\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+\sqrt{c}(ef-dg)E\right.}{\sqrt{a+cx^2}\left(\sqrt{c}f-i\sqrt{a}g\right)\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(Sqrt[c]*(e*f - d*g)*EllipticE[I*ArcSi
nh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*
g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (I*Sqrt[a]*e*g + Sqrt[c]*(-2*e*f + d*g))*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f - I*Sqrt[a]*g)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)]))/((Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

maple [B] time = 0.08, size = 2011, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c
)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*Elli
pticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f
+(-a*c)^(1/2)*g))^(1/2))*a*c*d*g^3-EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g
)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e*f*g^2
*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^
(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)+(-(g*x
+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g
)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticF((-(
g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(
1/2)*g))^(1/2))*c^2*d*f^2*g-EllipticF((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1
/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e*f^3*(-(g*x+f
)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*
g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)-(-(g*x+f)/(-c*f
+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2
)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-
c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^
(1/2))*a*c*d*g^3+(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/
2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)
*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)
^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e*f*g^2-(-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(
-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)*EllipticE((-(g*x+f)/(-c*f+(-a*c
)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^
2*d*f^2*g+(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*
f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/
2)*EllipticE((-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)*c)^(1/2),(-(-c*f+(-a*c)^(1/2)*
g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e*f^3-EllipticPi((-(g*x+f)/(-c*f+(-a*c)
^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)
*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*a*c*e*f*g^2*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g
)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^
(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/
2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/
(c*f+(-a*c)^(1/2)*g))^(1/2))*a*e*g^3*(-a*c)^(1/2)*(-(g*x+f)/(-c*f+(-a*c)^(1
/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-
a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)-EllipticPi((-(g*x+f)/(-c*f+(-a*c
)^(1/2)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2
)*g)/(c*f+(-a*c)^(1/2)*g))^(1/2))*c^2*e*f^3*(-(g*x+f)/(-c*f+(-a*c)^(1/2)*g)
```

```
*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(-a*c)^(
1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)+EllipticPi((-(g*x+f)/(-c*f+(-a*c)^(1/2
)*g)*c)^(1/2),(-c*f+(-a*c)^(1/2)*g)/(d*g-e*f)/c*e,(-(-c*f+(-a*c)^(1/2)*g)/(
c*f+(-a*c)^(1/2)*g))^(1/2))*c*e*f^2*g*(-a*c)^(1/2)*(-(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)*c)^(1/2)*((-c*x+(-a*c)^(1/2))/(c*f+(-a*c)^(1/2)*g)*g)^(1/2)*((c*x+(
-a*c)^(1/2))/(-c*f+(-a*c)^(1/2)*g)*g)^(1/2)-x^2*c^2*d*g^3+c^2*e*f*g^2*x^2-a
*c*d*g^3+a*c*e*f*g^2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)^2/(a*g^2+c*
f^2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,\left(d+ex\right)\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)
```

3.653 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=818

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)e^2}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f\cdots}{\sqrt{c}f+\cdots}}}$$

```
[Out] 2/3*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(3/2)+8/3*c*f*g^2*
(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)^2/(g*x+f)^(1/2)+2*e*g^2*(c*x^2+a)^
(1/2)/(-d*g+e*f)^2/(a*g^2+c*f^2)/(g*x+f)^(1/2)+8/3*c^(3/2)*f*g*EllipticE(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^
2)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*e*g
*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^
(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(
-d*g+e*f)^2/(a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)-2/3*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^
(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)/(a*g^2+c*
f^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*e^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(
1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/
2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)e^2}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f\cdots}{\sqrt{c}f+\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) + (
8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])
+ (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]) +
 (8*Sqrt[-a]*c^(3/2)*f*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqrt[c]*e*g*Sqrt[f + g*x]*Sqrt
[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*
Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(S
qrt[-a]*Sqrt[c]*f - a*g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]*Sqr
t[a + c*x^2]) - (2*e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
```

```
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int\left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{}{(ef}\right.$$

$$= \frac{e^2\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int\frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int\frac{1}{(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{(2c}{(e}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{}{(e}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{}{(e}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{}{(e}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{}{(e}$$

Mathematica [C] time = 8.97, size = 1917, normalized size = 2.34

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(a + c*x^2)*(a*g^2*(4*
e*f - d*g + 3*e*g*x) + c*f*(-(d*g*(5*f + 4*g*x)) + e*f*(8*f + 7*g*x))) - (f
 + g*x)*(7*c^2*e^2*f^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 11*c^2*d*e*f^4*g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]] + 10*a*c*e^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*a*c*d
*e*f^2*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*a*c*d^2*f*g^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + 3*a^2*e^2*f*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*
a^2*d*e*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*c^2*e^2*f^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 22*c^2*d*e*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x) - 8*c^2*d^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x) - 6*a*c*e^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*a*c
```

```
*d*e*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 7*c^2*e^2*f^3*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 11*c^2*d*e*f^2*g*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + 4*c^2*d^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 + 3*a*c*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*
x)^2 - 3*a*c*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]
*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (Sqrt[c]*f + I*Sqrt[a]*g)*(3*a^(3/2)*e^2*g^3 + (3*I)*a*Sqrt[c]*e
*g^2*(2*e*f - d*g) + Sqrt[a]*c*g*(2*e^2*f^2 + 2*d*e*f*g - d^2*g^2) + (3*I)*
c^(3/2)*f*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
 x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3*I)*c^2*e^2*f^4*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt
[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (6*I)*a*c*e^2
*f^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3
*I)*a^2*e^2*g^4*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sq
rt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*
f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)])))/(3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*(
f + g*x)^(3/2)*Sqrt[a + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

maple [B] time = 0.12, size = 9409, normalized size = 11.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)\left(gx + f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f + g\,x\right)^{5/2}\,\sqrt{c\,x^2 + a}\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,\left(d + ex\right)\left(f + gx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(5/2)), x)
```

3.654 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\Pi\left(\frac{2e}{\sqrt{-c}d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

```
[Out] -2*EllipticPi(1/2*(1-x*(-c)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*(-c)^(1/2)),2^(1/
2)*(g/(g+f*(-c)^(1/2)))^(1/2))*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/2)/
(e+d*(-c)^(1/2))/(g*x+f)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {932, 168, 538, 537}

$$-\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\Pi\left(\frac{2e}{\sqrt{-c}d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[-c]*(f + g*x))/(Sqrt[-c]*f + g)]*EllipticPi[(2*e)/(Sqrt[-c]*
d + e), ArcSin[Sqrt[1 - Sqrt[-c]*x]/Sqrt[2]], (2*g)/(Sqrt[-c]*f + g)])/((Sq
rt[-c]*d + e)*Sqrt[f + g*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{1-\sqrt{-c}\,x}\sqrt{1+\sqrt{-c}\,x}\,(d+ex)\sqrt{f+gx}}\,dx$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\sqrt{-c}\,d+e-ex^2\right)\sqrt{f+\frac{g}{\sqrt{-c}}-\frac{gx^2}{\sqrt{-c}}}}\,dx, x, \sqrt{1-\sqrt{-c}\right.\right.$$

$$= -\frac{\left(2\sqrt{1+\frac{g\left(-1+\sqrt{-c}\,x\right)}{\sqrt{-c}\,f+g}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\sqrt{-c}\,d+e-ex^2\right)\sqrt{1-\frac{gx^2}{\sqrt{-c}\left(f+\frac{g}{\sqrt{-c}}\right)}}}\,dx, x, \sqrt{1-\right.}{\sqrt{f+gx}}$$

$$= -\frac{2\sqrt{1-\frac{g\left(1-\sqrt{-c}\,x\right)}{\sqrt{-c}\,f+g}}\,\Pi\left(\frac{2e}{\sqrt{-c}\,d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}\,x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}\,f+g}\right)}{\left(\sqrt{-c}\,d+e\right)\sqrt{f+gx}}$$

Mathematica [C] time = 0.95, size = 261, normalized size = 2.37

$$-\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{ig}{\sqrt{c}}}{f+gx}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-ig}{\sqrt{c}\,f+ig}\right)-\Pi\left(\frac{\sqrt{c}\,(ef-dg)}{e\left(\sqrt{c}\,f+ig\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-ig}{\sqrt{c}\,f+ig}\right)\right)}{\sqrt{cx^2+1}\sqrt{-f-\frac{ig}{\sqrt{c}}}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(I/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)] - EllipticPi[(Sqrt[c]*(e*f - d*
g))/(e*(Sqrt[c]*f + I*g)), I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f + g*x]
], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)]))/(Sqrt[-f - (I*g)/Sqrt[c]]*(e*f -
d*g)*Sqrt[1 + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+1}\,(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

maple [B] time = 0.12, size = 215, normalized size = 1.95

$$\frac{2\left(\sqrt{-c}\, f+g\right)\sqrt{-\frac{\left(\sqrt{-c}\, x-1\right)g}{\sqrt{-c}\, f+g}}\,\sqrt{-\frac{\left(\sqrt{-c}\, x+1\right)g}{\sqrt{-c}\, f-g}}\,\sqrt{\frac{\left(gx+f\right)\sqrt{-c}}{\sqrt{-c}\, f+g}}\,\sqrt{c\,x^{2}+1}\,\sqrt{gx+f}\ \mathrm{EllipticPi}\left(\sqrt{\frac{\left(gx+f\right)\sqrt{-c}}{\sqrt{-c}\, f+g}},-\frac{\left(\sqrt{-c}\, f\right.}{\sqrt{-c}\,\left(d\right.}\right.}{\sqrt{-c}\,\left(dg-ef\right)\left(cg\,x^{3}+cf\,x^{2}+gx+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x)
```

```
[Out] 2*(g+f*(-c)^(1/2))/(-c)^(1/2)*EllipticPi(((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2
)))^(1/2),-(g+f*(-c)^(1/2))*e/(-c)^(1/2)/(d*g-e*f),((g+f*(-c)^(1/2))/(f*(-c
)^(1/2)-g))^(1/2))*(-(x*(-c)^(1/2)-1)*g/(g+f*(-c)^(1/2)))^(1/2)*(-(x*(-c)^(
1/2)+1)*g/(f*(-c)^(1/2)-g))^(1/2)*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/
2)*(c*x^2+1)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+1}\,(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+1}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)
```

3.655 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=454

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cd^2+ae^2}\sqrt{f\cdots}}{\sqrt[4]{cf^2+ag^2}\sqrt{d\cdots}}\right.\right.}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

```
[Out] -(a*g^2+c*f^2)^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2
)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arctan((a*e^2+c*d^2)^(
1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))*EllipticF(sin(2*arct
an((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/
2*(2+2*(a*e*g+c*d*f)/(a*e^2+c*d^2)^(1/2)/(a*g^2+c*f^2)^(1/2))^(1/2))*(1+(g*
x+f)*(a*e^2+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))*((-d*g+e*f)^2*(c*x^2+
a)/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)*((1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)
/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)/(1+(g*x+f)*(a*e^2
+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))^2)^(1/2)/(a*e^2+c*d^2)^(1/4)/(-d
*g+e*f)/(c*x^2+a)^(1/2)/(1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a
*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {936, 1103}

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cd^2+ae^2}\sqrt{f\cdots}}{\sqrt[4]{cf^2+ag^2}\sqrt{d\cdots}}\right.\right.}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^2
 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 +
a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)
*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2))/
(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^2]*El
lipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^2)^(1/
4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c*f^2 +
a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqrt[1 - (2
*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2 + a*e^2)*
(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```

Rule 936

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/
((c*f^2 + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sqrt[a + c*x^2]), Subst[Int[1/
Sqrt[1 - ((2*c*d*f + 2*a*e*g)*x^2)/(c*f^2 + a*g^2) + ((c*d^2 + a*e^2)*x^4)/
(c*f^2 + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+cx^2\right)}{\left(cf^2+ag^2\right)(d+ex)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(2cdf+2aeg)x^2}{cf^2+ag^2}+\frac{\left(cd^2+ae^2\right)x^4}{cf^2+ag^2}}}\,dx, x, \frac{\sqrt{f}}{\sqrt{d}}\right.}{(ef-dg)\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt[4]{cf^2+ag^2}\,(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+cx^2\right)}{\left(cf^2+ag^2\right)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}\,(f+gx)}{\sqrt{cf^2+ag^2}\,(d+ex)}\right)\sqrt{\frac{1-\frac{2(cdf+aeg)(f+}{\left(cf^2+ag^2\right)(d+}}{\left(1+\frac{\sqrt{cd}}{\sqrt{cf}}\right.}}}{\sqrt[4]{cd^2+ae^2}\,(ef-dg)\sqrt{a+cx^2}\sqrt{1}}$$

Mathematica [C] time = 1.40, size = 344, normalized size = 0.76

$$\frac{\sqrt{2}\left(\sqrt{c}\,x+i\sqrt{a}\right)\sqrt{d+ex}\sqrt{\frac{\frac{i\sqrt{c}\,dx}{\sqrt{a}}-\frac{i\sqrt{a}\,e}{\sqrt{c}}+d+ex}{d+ex}}\sqrt{\frac{(f+gx)\left(\sqrt{a}\,e+i\sqrt{c}\,d\right)}{(d+ex)\left(\sqrt{a}\,g+i\sqrt{c}\,f\right)}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(ef-dg)\left(\sqrt{c}\,x+i\sqrt{a}\right)}{\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)(d+ex)}}\right)\middle|-\frac{\frac{i\sqrt{c}\,df}{\sqrt{a}}-ef+dg+\frac{i\sqrt{}}{}}{2ef-2dg}\right.}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\sqrt{\frac{\left(\sqrt{c}\,x+i\sqrt{a}\right)(ef-dg)}{(d+ex)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[2]*(I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[(d - (I*Sqrt[a]*e)/Sqrt
[c] + (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*Sqrt[((I*Sqrt[c]*d + Sqrt[a
]*e)*(f + g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[Sqr
t[((e*f - d*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x
))]], -(((I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c])/(2*
e*f - 2*d*g))])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[((e*f - d*g)*(I*Sqrt[a] + S
qrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]*Sqrt[a + c*
x^2])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}{cegx^4+\left(cef+cdg\right)x^3+adf+\left(cdf+aeg\right)x^2+\left(aef+adg\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*e*f +
c*d*g)*x^3 + a*d*f + (c*d*f + a*e*g)*x^2 + (a*e*f + a*d*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

maple [A] time = 0.20, size = 433, normalized size = 0.95

$$\frac{2\left(c\,e^2 f\,x^2+2cdefx-\sqrt{-ac}\,e^2 g\,x^2+c\,d^2 f-2\sqrt{-ac}\,degx-\sqrt{-ac}\,d^2 g\right)\sqrt{\frac{(dg-ef)(cx+\sqrt{-ac})}{(-cf+\sqrt{-ac}\,g)(ex+d)}}\,\sqrt{\frac{(dg-ef)(-cx+\sqrt{-ac})}{(cf+\sqrt{-ac}\,g)(ex+d)}}\,\sqrt{}}{\sqrt{-\frac{(gx+f)(ex+d)(-cx+\sqrt{-ac})(cx+\sqrt{-ac})}{c}}\,\left(dg-ef\right)\left(cd-\sqrt{-ac}\,e\right)\sqrt{ceg\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(c*e^2*f*x^2-(-a*c)^(1/2)*x^2*e^2*g+2*c*d*e*f*x-2*(-a*c)^(1/2)*x*d*e*g+c*
d^2*f-(-a*c)^(1/2)*d^2*g)*EllipticF((((-a*c)^(1/2)*e-c*d)*(g*x+f)/(-c*f+(-a
*c)^(1/2)*g)/(e*x+d))^(1/2),((c*d+(-a*c)^(1/2)*e)*(-c*f+(-a*c)^(1/2)*g)/(c*
f+(-a*c)^(1/2)*g)/((-a*c)^(1/2)*e-c*d))^(1/2))*((d*g-e*f)*(c*x+(-a*c)^(1/2)
)/(-c*f+(-a*c)^(1/2)*g)/(e*x+d))^(1/2)*((d*g-e*f)*(-c*x+(-a*c)^(1/2))/(c*f+
(-a*c)^(1/2)*g)/(e*x+d))^(1/2)*(((-a*c)^(1/2)*e-c*d)*(g*x+f)/(-c*f+(-a*c)^(
1/2)*g)/(e*x+d))^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-1/c*(g
*x+f)*(e*x+d)*(-c*x+(-a*c)^(1/2))*(c*x+(-a*c)^(1/2)))^(1/2)/(d*g-e*f)/(c*d-
(-a*c)^(1/2)*e)/(c*e*g*x^4+c*d*g*x^3+c*e*f*x^3+a*e*g*x^2+c*d*f*x^2+a*d*g*x+
a*e*f*x+a*d*f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.656 $\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

```
[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)*(-x^2+1)^(1/2)/(-1+x)^(1/2)/(1+x)^(1/
2)/(2*x^2-1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {519, 421, 419}

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt[
1 + x]*Sqrt[-1 + 2*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{-1+x^2}\int \frac{1}{\sqrt{-1+x^2}\sqrt{-1+2x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}}$$
$$= \frac{\left(\sqrt{1-2x^2}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$
$$= \frac{\left(\sqrt{1-2x^2}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$
$$= \frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$

Mathematica [B] time = 0.24, size = 107, normalized size = 2.06

$$-\frac{2(x-1)^{3/2}\sqrt{\frac{x+1}{1-x}}\sqrt{\frac{1-2x^2}{(x-1)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{2}+2+\frac{1}{x-1}}}{2^{3/4}}\right)\middle|4\left(-4+3\sqrt{2}\right)\right)}{\sqrt{3+2\sqrt{2}}\sqrt{x+1}\sqrt{2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
[Out] (-2*(-1 + x)^(3/2)*Sqrt[(1 + x)/(1 - x)]*Sqrt[(1 - 2*x^2)/(-1 + x)^2]*Ellip
ticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]/2^(3/4)], 4*(-4 + 3*Sqrt[2])]
)/(Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2])
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}{2\,x^4-3\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)/(2*x^4 - 3*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

maple [A] time = 0.03, size = 58, normalized size = 1.12

$$\frac{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{2x^2-1}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\operatorname{EllipticF}\left(x,\sqrt{2}\right)}{2x^4-3x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^(1/2)/(x+1)^(1/2)/(2*x^2-1)^(1/2),x)
```

[Out] (x-1)^(1/2)*(x+1)^(1/2)*(2*x^2-1)^(1/2)/(2*x^4-3*x^2+1)*(-x^2+1)^(1/2)*(-2*x^2+1)^(1/2)*EllipticF(x,2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^2-1}\,\sqrt{x+1}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^2-1}\,\sqrt{x-1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)

[Out] int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)

[Out] Integral(1/(sqrt(x - 1)*sqrt(x + 1)*sqrt(2*x**2 - 1)), x)

3.657 $\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)}{35c^4d^4e\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{35c^3d^3e}$$

```
[Out] -16/35*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/35*(-a*e*g+c*d*f)*(g*x+f)^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+2/7*(g*x+f)^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+16/35*g*(-a*e*g+c*
d*f)^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] time = 0.42, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {870, 794, 648}

$$\frac{12(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{35c^2d^2\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{35c^3d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*Sqrt[d + e*x]) + (16*g*(c*d*f - a*e*
g)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3
*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(35*c^2*d^2*Sqrt[d + e*x]) + (2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
```

```
rQ[n])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cd\sqrt{d+ex}} + \frac{\left(6\left(cde^2f+cd^2eg-e\left(cd^2+a\right.\right.\right.}{7}$$

$$= \frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)^3\sqrt{ade}}{7c}$$

$$= \frac{16g(cdf-aeg)^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^3d^3e} + \frac{12(cdf-aeg)(f}{}$$

$$= -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^4d^4e\sqrt{d+ex}} +$$

Mathematica [A] time = 0.13, size = 136, normalized size = 0.51

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg\left(35f^2+14fgx+3g^2x^2\right)+c^3d^3\left(35f^3+35f\right.\right.}{35c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
+ g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^3
+ 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

fricas [A] time = 0.91, size = 193, normalized size = 0.72

$$\frac{2\left(5\,c^3d^3g^3x^3+35\,c^3d^3f^3-70\,ac^2d^2ef^2g+56\,a^2cde^2fg^2-16\,a^3e^3g^3+3\left(7\,c^3d^3fg^2-2\,ac^2d^2eg^3\right)x^2+\left(35\,c^3d^3f\right.\right.}{35\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 70*a*c^2*d^2*e*f^2*g + 56*a^2*c*
d*e^2*f*g^2 - 16*a^3*e^3*g^3 + 3*(7*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2
+ (35*c^3*d^3*f^2*g - 28*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^3}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^3/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx+ae\right)\left(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a\right.}{35\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-21*c^3*d^3*f*g^
2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-35*c^3*d^3*f^2*g*x+16*a^3*
e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)*(e*x+d)^(
1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.66, size = 218, normalized size = 0.81

$$\frac{2\sqrt{cdx+ae}\,f^3}{cd}+\frac{2\left(c^2d^2x^2-acdex-2\,a^2e^2\right)f^2g}{\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3\right)fg^2}{5\sqrt{cdx+ae}\,c^3d^3}+\frac{2\left(5\,c^4d^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^2
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*a
^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^4*d
^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16*a^4
*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

mupad [B] time = 3.66, size = 218, normalized size = 0.81

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-112\,a^2\,c\,d\,e^2\,f\,g^2+140\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,c^4\,d^4\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{7\,c\,d\,e}+\frac{6\,g^2\,x}{}\right.}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 140*a*c^2*d^2*e*f^2*g - 112*a^2*c*d*e^2*f*g^2))/(
35*c^4*d^4*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(7*c*d*e) + (6*g^2*x^2*(2*a*e*g
 - 7*c*d*f)*(d + e*x)^(1/2))/(35*c^2*d^2*e) - (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 35*c^2*d^2*f^2 - 28*a*c*d*e*f*g))/(35*c^3*d^3*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(f+gx\right)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.658 $\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{15c^3d^3e\sqrt{d+ex}}+\frac{8g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(c}{15c^2d^2e}$$

```
[Out] -8/15*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^3/d^3/e/(e*x+d)^(1/2)+2/5*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+8/15*g*(-a*e*g+c*d*f)*(e*x+d)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {870, 794, 648}

$$\frac{8g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{15c^2d^2e}-\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3e\right.}{15c^3d^3e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) + (8*g*(c*d*f - a*e*g)*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) +
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d +
 e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cd\sqrt{d+ex}} + \frac{\left(4\left(cde^2f+cd^2eg-e\left(cd^2\right.\right.\right.}{}$$

$$= \frac{8g(cdf-aeg)\sqrt{d+ex}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{a\ldots}}{5\ldots}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} + \ldots$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.44

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2\left(15f^2+10fgx+3g^2x^2\right)\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x) +
 c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

fricas [A] time = 0.94, size = 123, normalized size = 0.62

$$\frac{2\left(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-20\,acdefg+8\,a^2e^2g^2+2\left(5\,c^2d^2fg-2\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{e\ldots}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g + 8*a^2*e^2*g^2 +
 2*(5*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^2}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^2/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\left(cdx+ae\right)\left(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15f^2c^2d^2\right)\sqrt{ex+d}}{15\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x+10*c^2*d^2*f*g*x+8*a^2*
e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)*(e*x+d)^(1/2)/c^3/d^3/(c*d*e*x^2+a*e
^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.61, size = 133, normalized size = 0.66

$$\frac{2\sqrt{cdx+ae}\,f^2}{cd}+\frac{4\left(c^2d^2x^2-acdex-2\,a^2e^2\right)fg}{3\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3\right)g^2}{15\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*
a^2*c*d*e^2*x + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

mupad [B] time = 3.40, size = 142, normalized size = 0.71

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-40\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,c^3\,d^3\,e}+\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{5\,c\,d\,e}-\frac{4\,g\,x\,\left(2\,a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
2*g^2 + 30*c^2*d^2*f^2 - 40*a*c*d*e*f*g))/(15*c^3*d^3*e) + (2*g^2*x^2*(d +
e*x)^(1/2))/(5*c*d*e) - (4*g*x*(2*a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*c^2
*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(f+gx\right)^2}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.659 $\int \frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(3ef-dg)\right)}{3c^2d^2e\sqrt{d+ex}}$$

```
[Out] -2/3*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^2/d^2/e/(e*x+d)^(1/2)+2/3*g*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c/d/e
```

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(3ef-dg)\right)}{3c^2d^2e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2*e*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2g\sqrt{d+ex}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cde}+\frac{1}{3}\left(3f-\frac{dg}{e}-\frac{2aeg}{cd}\right)\int$$

$$=-\frac{2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2e\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}}{}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.42

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\,(cd(3f+gx)-2aeg)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

fricas [A] time = 1.05, size = 71, normalized size = 0.57

$$\frac{2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(cdgx + 3\,cdf - 2\,aeg\right)\sqrt{ex + d}}{3\left(c^2d^2ex + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*a*e*
g)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex + d}\left(gx + f\right)}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x), x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.54

$$-\frac{2\left(cdx + ae\right)\left(-cdgx + 2aeg - 3cdf\right)\sqrt{ex + d}}{3\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\;c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*g*x+2*a*e*g-3*c*d*f)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.55, size = 65, normalized size = 0.52

$$\frac{2\sqrt{cdx + ae}\,f}{cd} + \frac{2\left(c^2d^2x^2 - acdex - 2\,a^2e^2\right)g}{3\sqrt{cdx + ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*g/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

mupad [B] time = 3.23, size = 88, normalized size = 0.70

$$-\frac{\left(\frac{\left(4aeg-6cdf\right)\sqrt{d+ex}}{3c^2d^2e}-\frac{2gx\sqrt{d+ex}}{3cde}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] -((((4*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*c^2*d^2*e) - (2*g*x*(d + e*x)^(
1/2))/(3*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\left(f+gx\right)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.660 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.76

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

fricas [A] time = 0.77, size = 49, normalized size = 1.07

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

maple [A] time = 0.00, size = 50, normalized size = 1.09

$$\frac{2\left(cdx+ae\right)\sqrt{ex+d}}{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(e*x+d)^(1/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.50, size = 18, normalized size = 0.39

$$\frac{2\sqrt{cdx+ae}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)/(c*d)
```

mupad [B] time = 3.20, size = 54, normalized size = 1.17

$$\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d\,e\,\left(x+\frac{d}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

```
[Out] (2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(x
 + d/e))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.661 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=80

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

```
[Out] 2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/
2)/(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {874, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
 - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{\ldots}}{\ldots}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 1.16

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

fricas [A] time = 1.39, size = 252, normalized size = 3.15

$$\left[-\frac{\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - \left(cdef - \left(cd^2 + 2\,ae^2\right)g\right)x - 2\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{-cdfg + aeg^2}\,\sqrt{ex + d}}{egx^2 + df + \left(ef + dg\right)x}\right)}{cdfg - aeg^2}, -\frac{2\,\mathrm{arctan}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [-sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e
*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x))/(
c*d*f*g - a*e*g^2), -2*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x))/sqrt(c*d*f*g - a*e*g^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)), x)
```

maple [A] time = 0.03, size = 87, normalized size = 1.09

$$-\frac{2\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\;\mathrm{arctanh}\left(\frac{\sqrt{cdx + ae}\,g}{\sqrt{\left(aeg - cdf\right)g}}\right)}{\sqrt{ex + d}\,\sqrt{cdx + ae}\,\sqrt{\left(aeg - cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{\left(f+g\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

3.662 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

```
[Out] c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(3/2)/g^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}}{2(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{\left(cde^2\right)\operatorname{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(}\right.}{c}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right.}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

Mathematica [**A**] time = 0.11, size = 136, normalized size = 0.97

$$\frac{\sqrt{d+ex}\left(\sqrt{g}\,(ae+cdx)\sqrt{cdf-aeg}+cd(f+gx)\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}\,(f+gx)\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

fricas [**B**] time = 1.27, size = 703, normalized size = 5.02

$$\left[\frac{\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x+2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{egx^2+df+\left(ef+dg\right)x}\right.}{2\left(c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+\left(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4\right)x^2+\left(c^2d^2ef^3g+a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*g
^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*
g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^
2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g -
 2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f
*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a
*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x), -((c*d*e*g*x^2 + c*d
^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g - 2*a*c*d^2*e*
f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^
3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^2
*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 168, normalized size = 1.20

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(cdgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+cdf\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{\left(aeg-\right.}\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(aeg-cdf\right)\left(gx+f\right)\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2)*g)*x*c*d*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+
a*e)^(1/2)/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\,\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

3.663 $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}$$

```
[Out] 3/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)/g^(1/2)+1/2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+3/4*c*d*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2
)
```

Rubi [A] time = 0.31, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {872, 874, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^2) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*
d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{(3cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)}}}{4(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f-}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f-}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f-}$$

Mathematica [C] time = 0.05, size = 77, normalized size = 0.36

$$\frac{2c^2d^2\sqrt{(d+ex)(ae+cdx)}\ {}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{d+ex}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*c^2*d^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 3, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^3*Sqrt[d + e*x])
```

fricas [B] time = 0.78, size = 1283, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(5*c^2*d^2*f^2*g - 7*a*c*d*e*f*g
^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3*e*f^4*
g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 + (c^3*d^3*e*f^3*g^3 - 3*
a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^3 + (2*c^3*d^3
*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2
*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^5)*x^2
+ (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*
g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)
*f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g +
 c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g
^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g
```

```
^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (5*c^2*d
^2*f^2*g - 7*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^
5*g - 3*a*c^2*d^3*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 +
 (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e
^4*g^6)*x^3 + (2*c^3*d^3*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2
*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2
- 2*a^3*e^4)*f*g^5)*x^2 + (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4
 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*
a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 285, normalized size = 1.34

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^2d^2g^2x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+6c^2d^2fgx\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^2d^2f^2\,\mathrm{a}\right.}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^2*g^2+6*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^2*d^2*f^2-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c
*d*g+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g-5*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f
)^2/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.664 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=280

$$\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}\,(cdf-aeg)^{7/2}}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+$$

```
[Out] 5/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)/g^(1/2)+1/3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/12*c*d*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(
1/2)+5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(
g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}\,(cdf-aeg)^{7/2}}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^3) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(8*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (
5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*Sqrt[g]*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
```

```
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{(5cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)}}}{6(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.28

$$\frac{2c^3d^3\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{d+ex}\,(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 4, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^4*Sqrt[d + e*x])
```

fricas [B] time = 0.90, size = 2027, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*(33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*
f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^
3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^
6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3
```

```
*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2
*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*
d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2
*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^
3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^
7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*
e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2
*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*
c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^
5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 -
 a^4*e^5)*f^3*g^5)*x), -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*
d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 +
(c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/
(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (33*c^3*d^3*f^3*g - 59*a*c
^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3
 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3
*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e
^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)
*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*
f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(
c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 -
 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a
^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e
*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2
*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*
e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 450, normalized size = 1.61

$$\frac{\sqrt{cde\,x^2 + a\,e^2 x + c\,d^2 x + ade}\,\left(15c^3 d^3 g^3 x^3 \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right) + 45c^3 d^3 f\,g^2 x^2 \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right) + 45c^3 d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+45*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*x^2*c^2*d^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g^
2-40*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g-8*((a*e*g-c*d*
f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x
```

```
+a*e)^(1/2)*a*c*d*e*f*g-33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*
d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.665 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{16g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{5c^4d^4e\sqrt{d+ex}}+\frac{16g^2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{5c^3d^3e}$$

```
[Out] -2*(g*x+f)^3*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-16/5
*g*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/5*g*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+16/5*g^2*(-a*e*g+c*d*f)*(e*x+d)^(1/
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] time = 0.33, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {866, 870, 794, 648}

$$\frac{16g^2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{5c^3d^3e}+\frac{12g(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}}-\frac{16g\sqrt{x\left(ae^2\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^2*(c*d*
f - a*e*g)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^
3*d^3*e) + (12*g*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
5*c^2*d^2*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
```

```
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(6g)\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{12g(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right.}}{5c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{16g^2(cdf-aeg)\sqrt{d+ex}\,\sqrt{ade+}}{5c^3d^3e}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{16g(cdf-aeg)\left(2ae^2g-cd(3ef\right.}{5c^4d^{}}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 0.52

$$\frac{2\sqrt{d+ex}\,\left(16a^3e^3g^3+8a^2cde^2g^2(gx-5f)-2ac^2d^2eg\left(-15f^2+10fgx+g^2x^2\right)+c^3d^3\left(-5f^3+15f^2gx+5fg^{}\right.\right.}{5c^4d^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^2
*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x + 5*
f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.01, size = 216, normalized size = 0.84

$$\frac{2\left(c^3d^3g^3x^3-5c^3d^3f^3+30ac^2d^2ef^2g-40a^2cde^2fg^2+16a^3e^3g^3+\left(5c^3d^3fg^2-2ac^2d^2eg^3\right)x^2+\left(15c^3d^3f^2g\right.\right.}{5\left(c^5d^5ex^2+ac^4d^5e+\left(c^5d^6+ac^4d^4e^2\right)x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/5*(c^3*d^3*g^3*x^3 - 5*c^3*d^3*f^3 + 30*a*c^2*d^2*e*f^2*g - 40*a^2*c*d*e^
2*f*g^2 + 16*a^3*e^3*g^3 + (5*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 + (15*
c^3*d^3*f^2*g - 20*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x^2 + a*c^4*d^5*e +
(c^5*d^6 + a*c^4*d^4*e^2)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.71Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 187, normalized size = 0.73

$$\frac{2\left(cdx+ae\right)\left(g^{3}x^{3}c^{3}d^{3}-2a\,c^{2}d^{2}e\,g^{3}x^{2}+5c^{3}d^{3}f\,g^{2}x^{2}+8a^{2}cd\,e^{2}g^{3}x-20a\,c^{2}d^{2}ef\,g^{2}x+15c^{3}d^{3}f^{2}gx+16a^{3}e^{3}g^{3}-\cdots\right.}{5\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2/5*(c*d*x+a*e)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2*e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+
8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+15*c^3*d^3*f^2*g*x+16*a^3*e^3*g^
3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*g-5*c^3*d^3*f^3)*(e*x+d)^(3/2)/c^
4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.70, size = 165, normalized size = 0.64

$$-\frac{2\,f^{3}}{\sqrt{cdx+ae}\,cd}+\frac{6\left(cdx+2\,ae\right)f^{2}g}{\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{2\left(c^{2}d^{2}x^{2}-4\,acdex-8\,a^{2}e^{2}\right)fg^{2}}{\sqrt{cdx+ae}\,c^{3}d^{3}}+\frac{2\left(c^{3}d^{3}x^{3}-2\,ac^{2}d^{2}ex^{2}+8\,a^{2}cde^{2}x+16\,a^{3}\cdots\right.}{5\sqrt{cdx+ae}\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e)
*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*e*x^2 + 8*a^2*c*d*e^2*x + 16*
a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

mupad [B] time = 3.61, size = 252, normalized size = 0.98

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-80\,a^2\,c\,d\,e^2\,f\,g^2+60\,a\,c^2\,d^2\,e\,f^2\,g-10\,c^3\,d^3\,f^3\right)}{5\,c^5\,d^5\,e}+\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{5\,c^2\,d^2\,e}-\frac{2\,g^2\,x^2\,\left(2\,a\,e\,g\cdots\right.}{5\,c\cdots}\right.}{\frac{a}{c}+x^2+\frac{x\,\left(5\,c^5\,d^6+5\,a\,c^4\,d^4\,e^2\right)}{5\,c^5\,d^5\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e^
3*g^3 - 10*c^3*d^3*f^3 + 60*a*c^2*d^2*e*f^2*g - 80*a^2*c*d*e^2*f*g^2))/(5*c
```

```
^5*d^5*e) + (2*g^3*x^3*(d + e*x)^(1/2))/(5*c^2*d^2*e) - (2*g^2*x^2*(2*a*e*g
 - 5*c*d*f)*(d + e*x)^(1/2))/(5*c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2*
e^2*g^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(5*c^4*d^4*e)))/(a/c + x^2 + (x
*(5*c^5*d^6 + 5*a*c^4*d^4*e^2))/(5*c^5*d^5*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.666 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e}-\frac{2\sqrt{\cdots}}{cd\sqrt{x\left(ae\cdots\right.}}$$

```
[Out] -2*(g*x+f)^2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
g*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/e/(e*x+d)^(1/2)+8/3*g^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/c^2/d^2/e
```

Rubi [A] time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 794, 648}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e}-\frac{2\sqrt{\cdots}}{cd\sqrt{x\left(ae\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c^3*d^3*e*Sqrt[d + e*x]) + (8*g^2*Sqrt[d + e*x]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}dx = -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(4g)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{8g^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)}}{3c^2d^2e}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade}}{3c^3d^3e\sqrt{d+}}$$

Mathematica [A] time = 0.07, size = 88, normalized size = 0.49

$$\frac{2\sqrt{d+ex}\left(-8a^2e^2g^2-4acdeg(gx-3f)+c^2d^2\left(-3f^2+6fgx+g^2x^2\right)\right)}{3c^3d^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*f
^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.17, size = 147, normalized size = 0.81

$$\frac{2\left(c^2d^2g^2x^2-3c^2d^2f^2+12acdefg-8a^2e^2g^2+2\left(3c^2d^2fg-2acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+}}{3\left(c^4d^4ex^2+ac^3d^4e+\left(c^4d^5+ac^3d^3e^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^2*d^2*g^2*x^2 - 3*c^2*d^2*f^2 + 12*a*c*d*e*f*g - 8*a^2*e^2*g^2 + 2*(
3*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.25Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 116, normalized size = 0.64

$$-\frac{2(cdx+ae)\left(-g^2x^2c^2d^2+4acdeg^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3f^2c^2d^2\right)(ex+d)^{\frac{3}{2}}}{3\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^2*d^2*g^2*x^2+4*a*c*d*e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^
2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)*(e*x+d)^(3/2)/c^3/d^3/(c*d*e*x^2+a*e^2*
x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.63, size = 98, normalized size = 0.54

$$-\frac{2\,f^2}{\sqrt{cdx+ae}\,cd}+\frac{4\,(cdx+2\,ae)fg}{\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(c^2d^2x^2-4\,acdex-8\,a^2e^2\right)g^2}{3\,\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*c
^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a*e
)*c^3*d^3)
```

mupad [B] time = 3.43, size = 178, normalized size = 0.98

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-24\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2\right)}{3\,c^4\,d^4\,e}-\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-3\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}\right)}{\frac{a}{c}+x^2+\frac{x\,\left(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e^2\right)}{3\,c^4\,d^4\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^2*g^2 + 6*c^2*d^2*f^2 - 24*a*c*d*e*f*g))/(3*c^4*d^4*e) - (2*g^2*x^2*(d + e
*x)^(1/2))/(3*c^2*d^2*e) + (4*g*x*(2*a*e*g - 3*c*d*f)*(d + e*x)^(1/2))/(3*c
^3*d^3*e)))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.667 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\,\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] -2*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-2*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c^2/d^2/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {788, 648}

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\,\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2\left(-\frac{1}{2}e\left(2cdef-(c\right.\right.\right.}{}$$

$$=-\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\left(2ae^2g-cd(ef+\right.}{c^2d^2\,(}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.34

$$\frac{2\sqrt{d+ex}\,(2aeg+cd(gx-f))}{c^2d^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

fricas [A] time = 1.03, size = 96, normalized size = 0.64

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx-cdf+2\,aeg\right)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+\left(c^3d^4+ac^2d^2e^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - c*d*f + 2*a*e*g)*s
qrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.89Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 66, normalized size = 0.44

$$\frac{2\,(cdx+ae)\left(cdgx+2aeg-cdf\right)(ex+d)^{\frac{3}{2}}}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(c*d*g*x+2*a*e*g-c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(c*d*e*x^2+a*e^
2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.57, size = 48, normalized size = 0.32

$$-\frac{2\,f}{\sqrt{cdx+ae}\,cd}+\frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] -2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*d
^2)
```

mupad [B] time = 3.37, size = 118, normalized size = 0.79

$$\frac{\left(\frac{\left(4\,a\,e\,g-2\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\frac{a}{c}+x^2+\frac{x\,\left(c^3\,d^4+a\,c^2\,d^2\,e^2\right)}{c^3\,d^3\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2),x)
```

```
[Out] ((((4*a*e*g - 2*c*d*f)*(d + e*x)^(1/2))/(c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2
))/(c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2 +
 (x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

3.668 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] -2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.76

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 1.20, size = 74, normalized size = 1.61

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+\left(c^2d^3+acde^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^2
 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.66Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.00, size = 50, normalized size = 1.09

$$-\frac{2\left(cdx+ae\right)\left(ex+d\right)^{\frac{3}{2}}}{\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(e*x+d)^(3/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.51, size = 18, normalized size = 0.39

$$-\frac{2}{\sqrt{cdx+ae}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] -2/(sqrt(c*d*x + a*e)*c*d)
```

mupad [B] time = 3.27, size = 82, normalized size = 1.78

$$-\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^2
*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d+ex\right)^{\frac{3}{2}}}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

3.669 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

```
[Out] -2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(3/2)-2*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {868, 874, 205}

$$-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{g\int \frac{}{(f+gx)\sqrt{ade}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(2e^2g\right)\text{Subst}}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\sqrt{g}\tan^{-1}}{(c}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.53

$$-\frac{2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 1, 1/2, (g*(a*e + c*d*x))/(-(c*d*
f) + a*e*g)])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 0.99, size = 553, normalized size = 4.16

$$\left[-\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2-cd^2f+2adeg+2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdf-aeg\right)\sqrt{ex+d}\sqrt{-\frac{g}{cdf-aeg}}-}{egx^2+df+\left(ef+dg\right)x}\right.}{acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right)x^2+\left(\left(c^2d^3+acde^2\right)f-\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] [-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c
*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (
c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*d^2*e*f - a^2*d*e^2*g + (c
^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2
*e^3)*g)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d*f - a*
e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*s
qrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*
d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d
*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.51Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 128, normalized size = 0.96

$$-\frac{2\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(\sqrt{cdx+ae}\,g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{\left(aeg-cdf\right)g}\right)}{\sqrt{ex+d}\,\left(cdx+ae\right)\left(aeg-cdf\right)\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^
(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{3}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^{3/2}}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Timed out
```

3.670 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(a\cdots}}{\sqrt{d\cdots}}\right.}{(cdf-ae\cdots}$$

```
[Out] -3*c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(5/2)-2*(e*x+d)^(1/2)/(-a*e*g
+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3*g*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(a\cdots}}{\sqrt{d\cdots}}\right.}{(cdf-ae\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) - (3*c*d*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/(c*d*f - a*e*g)^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(3g)\int \cdots}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{ad\cdots}}{(cdf-\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{ad\cdots}}{(cdf-\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{ad\cdots}}{(cdf-\cdots}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.36

$$-\frac{2cd\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] (-2*c*d*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 2, 1/2, (g*(a*e + c*d*x))/(-(
c*d*f) + a*e*g)])/((c*d*f - a*e*g)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 0.81, size = 1067, normalized size = 5.28

$$\left[\frac{3\left(c^2d^2egx^3+acd^2ef+\left(c^2d^2ef+\left(c^2d^3+acde^2\right)g\right)x^2+\left(acd^2eg+\left(c^2d^3+acde^2\right)f\right)x\right)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2-\cdots}{\cdots}\right.}{2\left(ac^2d^3ef^3-2a^2cd^2e^2f^2g+a^3de^3fg^2+\left(c^3d^3ef^2g-2ac^2d^2e^2fg^2+a^2cde^3g^3\right)x^3+\left(c^3d^3ef^3+\left(c^3d^4-ac^2d^2e^2\cdots\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 + a*c*d*e^
2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(-g/(c*d*f - a*e
*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) -
(c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + 2*c*d*f + a*e*g)*sqrt
(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*d*e^3*f*g^2 + (c^
3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x^3 + (c^3*d^3*e*f
^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^2*c*d*e^3)*f*g^2
+ (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c^3*d^4 + a*c^2*d^
2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*d^2*e^2 - a^3*e^4
)*f*g^2)*x), -(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 +
 a*c*d*e^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(g/(c*d
*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x +
2*c*d*f + a*e*g)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g +
a^3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^
3)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e
+ a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3
+ (c^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^
2*c*d^2*e^2 - a^3*e^4)*f*g^2)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.58Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 225, normalized size = 1.11

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3\sqrt{cdx+ae}\;cd\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+3\sqrt{cdx+ae}\;cdfg\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+}}{\sqrt{(aeg-c}}\right.\right.}{\sqrt{ex+d}\,\left(cdx+ae\right)\left(aeg-cdf\right)^2\left(gx+f\right)\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c*d*g^2+3*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*c*d*f*g-3*((a*e*g-c*d*f)*g)^
(1/2)*x*c*d*g-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/2)*c*d*f
)/(e*x+d)^(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{{\left(f+g\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.671 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+\ldots}}{2\sqrt{d+ex}(f+gx)^2(cdf-\ldots}$$

```
[Out] -15/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(7/2)-2*(e*x+d)^(1/2)/
(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/2*g*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2
)-15/4*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+
f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$-\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+\ldots}}{2\sqrt{d+ex}(f+gx)^2(cdf-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) - (5*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) - (15*c*d*g*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (1
5*c^2*d^2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
```

```
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{(5g)\int}{}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ac}}{2(cdf}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ac}}{2(cdf}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ac}}{2(cdf}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ac}}{2(cdf}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.28

$$-\frac{2c^2d^2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 3, 1/2, (g*(a*e + c*d*x))
/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 1.11, size = 1863, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(15*(c^3*d^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^
4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4
 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)
*f^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g +
 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d
)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 +
d*f + (e*f + d*g)*x)) + 2*(15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f
*g - 2*a^2*e^2*g^2 + 5*(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^
2*f^4*g + 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2
- 3*a*c^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4
+ (2*c^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e
 - a^2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^
3*c*d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d
^5 - a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (
3*a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*
g^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*
e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 -
(5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x), -1/4*(15*(c^3*d^3*e*g^2*x^4 + a*c^
2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*
d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c
^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*f^2)*x)*sqrt(g/(c*d*f - a*e*g))*ar
ctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x
+ d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x))
 + (15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 5*
(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3*a^3*c*d^2*e
^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d^3*e^2*f^2*g
^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^4*e*f^4*g +
(c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^2
*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3 + a^4*e^5)*
g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*d^3*e^2)*f^4
*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d^3*e^2 + 5*a
^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 - (2*a^4*d*e^4
*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*
f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^2*e^3 - a^4*
e^5)*f^2*g^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.85Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.04, size = 379, normalized size = 1.38

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(15\sqrt{cdx+ae}\;c^2d^2g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\;g}{\sqrt{\left(aeg-cdf\right)g}}\right)+30\sqrt{cdx+ae}\;c^2d^2f\,g^2x\,\mathrm{arcta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*g^3+30*arctanh((c*
d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g^2
+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*
c^2*d^2*f^2*g-15*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^2*d^2*g^2-5*((a*e*g-c*d*f)*g
)^(1/2)*x*a*c*d*e*g^2-25*((a*e*g-c*d*f)*g)^(1/2)*x*c^2*d^2*f*g+2*((a*e*g-c*
d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*((a*e*g-c
*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+
f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{{\left(f+g\,x\right)}^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.672 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^4d^4e\sqrt{d+ex}}+\frac{16g^3\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^3d^3e}-\frac{}{c^2d}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-4*
g*(g*x+f)^2*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1
6/3*g^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c^4/d^4/e/(e*x+d)^(1/2)+16/3*g^3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] time = 0.28, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 794, 648}

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^4d^4e\sqrt{d+ex}}-\frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16g^3\sqrt{d+e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (4*g*Sqrt[d + e*x]*(f + g*x)^2)/(c^2*d^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^3*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(2g)\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$=-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$=-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$=-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

Mathematica [A] time = 0.11, size = 131, normalized size = 0.55

$$\frac{2(d+ex)^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-6ac^2d^2eg\left(f^2-6fgx+g^2x^2\right)+c^3d^3\left(-f^3-9f^2gx+9fg^2x^2+g\right.\right.}{3c^4d^4((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c^
2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2*x
^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [A] time = 0.75, size = 251, normalized size = 1.05

$$\frac{2\left(c^3d^3g^3x^3-c^3d^3f^3-6\,ac^2d^2ef^2g+24\,a^2cde^2fg^2-16\,a^3e^3g^3+3\left(3\,c^3d^3fg^2-2\,ac^2d^2eg^3\right)x^2-3\left(3\,c^3d^3f^2g-1\right.\right.}{3\left(c^6d^6ex^3+a^2c^4d^5e^2+\left(c^6d^7+2\,ac^5d^5e^2\right)x^2+\left(2\,ac^5d^6e+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^3*d^3*g^3*x^3 - c^3*d^3*f^3 - 6*a*c^2*d^2*e*f^2*g + 24*a^2*c*d*e^2*f
*g^2 - 16*a^3*e^3*g^3 + 3*(3*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 - 3*(3*
c^3*d^3*f^2*g - 12*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^6*d^6*e*x^3 + a^2*c^4*d^5*e^
2 + (c^6*d^7 + 2*a*c^5*d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.27Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 187, normalized size = 0.78

$$-\frac{2\left(cdx+ae\right)\left(-g^{3}x^{3}c^{3}d^{3}+6a\,c^{2}d^{2}e\,g^{3}x^{2}-9c^{3}d^{3}f\,g^{2}x^{2}+24a^{2}cd\,e^{2}g^{3}x-36a\,c^{2}d^{2}ef\,g^{2}x+9c^{3}d^{3}f^{2}gx+16a^{3}e\right.}{3\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^
2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x+9*c^3*d^3*f^2*g*x+16*a^3*e^3*
g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2*g+c^3*d^3*f^3)*(e*x+d)^(5/2)/c^4
/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [A] time = 0.74, size = 219, normalized size = 0.92

$$-\frac{2\left(3\,cdx+2\,ae\right)f^{2}g}{\left(c^{3}d^{3}x+ac^{2}d^{2}e\right)\sqrt{cdx+ae}}+\frac{2\left(3\,c^{2}d^{2}x^{2}+12\,acdex+8\,a^{2}e^{2}\right)fg^{2}}{\left(c^{4}d^{4}x+ac^{3}d^{3}e\right)\sqrt{cdx+ae}}+\frac{2\left(c^{3}d^{3}x^{3}-6\,ac^{2}d^{2}ex^{2}-24\,a^{2}cde^{2}x-16\right.}{3\left(c^{5}d^{5}x+ac^{4}d^{4}e\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*e*x^2 - 24*a^2*c*d*e
^2*x - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) - 2/3*
f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [B] time = 3.77, size = 278, normalized size = 1.16

$$-\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(\frac{32\,a^{3}\,e^{3}\,g^{3}}{3}-16\,a^{2}\,c\,d\,e^{2}\,f\,g^{2}+4\,a\,c^{2}\,d^{2}\,e\,f^{2}\,g+\frac{2\,c^{3}\,d^{3}\,f^{3}}{3}\right)}{c^{6}\,d^{6}\,e}-\frac{2\,g^{3}\,x^{3}\,\sqrt{d+e\,x}}{3\,c^{3}\,d^{3}\,e}+\frac{g^{2}\,x^{2}\,\left(4\,a\,e\right.}{}\right.}{x^{3}+\frac{a^{2}\,e}{c^{2}\,d}+\frac{a\,x\,\left(2\,c\,d^{2}+a\,e^{2}\right)}{c^{2}\,d^{2}}+\frac{x^{2}\,\left(c^{6}\,d^{7}+2\,a\,c^{5}\,d^{5}\,e^{2}\right)}{c^{6}\,d^{6}\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*((32*a^3*
e^3*g^3)/3 + (2*c^3*d^3*f^3)/3 + 4*a*c^2*d^2*e*f^2*g - 16*a^2*c*d*e^2*f*g^2
))/(c^6*d^6*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(3*c^3*d^3*e) + (g^2*x^2*(4*a*
e*g - 6*c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 3*c^2*d^2*f^2 - 12*a*c*d*e*f*g))/(c^5*d^5*e)))/(x^3 + (a^2*e)/(c
^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^6*d^7 + 2*a*c^5*d^5*e^2
))/(c^6*d^6*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.673 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\cdots}{3c\cdots}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/
3*g*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-8/3*g*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^3/d^3/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 788, 648}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\cdots}{3c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (8*g*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(3*c^2*d^2*(c*d^2 - a*e
^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*(c*d^2 -
 a*e^2)*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(4g)\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-aeg)(d+e}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2\right.}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-aeg)(d+e}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2\right.}}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.41

$$\frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2\left(f^2+6fgx-3g^2x^2\right)\right)}{3c^3d^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
+ 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [A] time = 1.14, size = 180, normalized size = 0.85

$$\frac{2\left(3\,c^2d^2g^2x^2-c^2d^2f^2-4\,acdefg+8\,a^2e^2g^2-6\left(c^2d^2fg-2\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3\left(c^5d^5ex^3+a^2c^3d^4e^2+\left(c^5d^6+2\,ac^4d^4e^2\right)x^2+\left(2\,ac^4d^5e+a^2c^3d^3e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(3*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 - 4*a*c*d*e*f*g + 8*a^2*e^2*g^2 - 6*(c
^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2
)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 4.4Unable to transpose Erro
r: Bad Argument Value
```

maple [A] time = 0.01, size = 116, normalized size = 0.55

$$\frac{2\,(cdx+ae)\left(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-f^2c^2d^2\right)(ex+d)^{\frac{5}{2}}}{3\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*c^2*d^2*g^2*x^2+12*a*c*d*e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e
^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)*(e*x+d)^(5/2)/c^3/d^3/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(5/2)
```

maxima [A] time = 0.67, size = 138, normalized size = 0.65

$$-\frac{4\,(3\,cdx+2\,ae)fg}{3\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}+\frac{2\left(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2\right)g^2}{3\left(c^4d^4x+ac^3d^3e\right)\sqrt{cdx+ae}}-\frac{2\,f^2}{3\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2/3*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [B] time = 3.61, size = 206, normalized size = 0.98

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3\,e}-\frac{\sqrt{d+e\,x}\,\left(-16\,a^2\,e^2\,g^2+8\,a\,c\,d\,e\,f\,g+2\,c^2\,d^2\,f^2\right)}{3\,c^5\,d^5\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-c\,d\,f\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}\right)}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(3\,c^5\,d^6+6\,a\,c^4\,d^4\,e^2\right)}{3\,c^5\,d^5\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^2*(d + e*x)^(1/2))
/(c^3*d^3*e) - ((d + e*x)^(1/2)*(2*c^2*d^2*f^2 - 16*a^2*e^2*g^2 + 8*a*c*d*e
*f*g))/(3*c^5*d^5*e) + (4*g*x*(2*a*e*g - c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e
)))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(3*c^
5*d^6 + 6*a*c^4*d^4*e^2))/(3*c^5*d^5*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.674 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] -2/3*(-a*e*g+c*d*f)*(e*x+d)^(5/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)+2/3*(2*a*e^2*g+c*d*(-3*d*g+e*f))*(e*x+d)^(1/2)/c^2/d^2/(-
a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {788, 648}

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt[d
 + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{\left(2ae^2g+cd(ef-3a\right.}{3c}$$

$$=-\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(2ae^2g+cc\right.}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{}}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.34

$$-\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)*
(d + e*x))^(3/2))
```

fricas [A] time = 1.20, size = 129, normalized size = 0.84

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(3\,cdgx+cdf+2\,aeg\right)\sqrt{ex+d}}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+\left(c^4d^5+2\,ac^3d^3e^2\right)x^2+\left(2\,ac^3d^4e+a^2c^2d^2e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*e
*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3
*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.14Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 66, normalized size = 0.43

$$-\frac{2\left(cdx+ae\right)\left(3cdgx+2aeg+cdf\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(3*c*d*g*x+2*a*e*g+c*d*f)*(e*x+d)^(5/2)/c^2/d^2/(c*d*e*x^2
+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [A] time = 0.60, size = 73, normalized size = 0.47

$$-\frac{2\left(3\,cdx+2\,ae\right)g}{3\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}-\frac{2\,f}{3\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] -2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2/
3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [B] time = 3.50, size = 149, normalized size = 0.97

$$-\frac{\left(\frac{\left(\frac{4\,a\,e\,g}{3}+\frac{2\,c\,d\,f}{3}\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,d^3\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^4\,d^5+2\,a\,c^3\,d^3\,e^2\right)}{c^4\,d^4\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2),x)
```

```
[Out] -(((((4*a*e*g)/3 + (2*c*d*f)/3)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d +
e*x)^(1/2))/(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^
3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5 + 2
*a*c^3*d^3*e^2))/(c^4*d^4*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

3.675 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {648}

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.77

$$-\frac{2(d+ex)^{3/2}}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 1.35, size = 107, normalized size = 2.23

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3\left(c^3d^3ex^3+a^2cd^2e^2+\left(c^3d^4+2\,ac^2d^2e^2\right)x^2+\left(2\,ac^2d^3e+a^2cde^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x
^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a^2
*c*d*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.06Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.00, size = 50, normalized size = 1.04

$$-\frac{2\left(cdx+ae\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(e*x+d)^(5/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [A] time = 0.52, size = 28, normalized size = 0.58

$$-\frac{2}{3\left(c^{2}d^{2}x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] -2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [B] time = 3.32, size = 110, normalized size = 2.29

$$-\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3*
d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x +
 2*a*c^2*d^2*e^2*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.676 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3}}{3\left(x\left(ae^2+cd^2\right)+ade+cd\right.}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+2
*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)+2*g*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {868, 874, 205}

$$\frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3}}{3\left(x\left(ae^2+cd^2\right)+ade+cd\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (2*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)
^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{g\int \frac{}{(f+gx)(ade+}}{c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf-aeg)^2}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf-aeg)^2}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf-aeg)^2}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.39

$$\frac{2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 1, -1/2, (g*(a*e + c*d*x))/(-(c*
d*f) + a*e*g)])/(3*(-(c*d*f) + a*e*g)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 1.24, size = 1015, normalized size = 5.40

$$\left[\frac{3\left(c^2d^2egx^3+a^2de^2g+\left(c^2d^3+2\,acde^2\right)gx^2+\left(2\,acd^2e+a^2e^3\right)gx\right)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg+2}{}\right.}{3\left(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+\left(c^4d^4ef^2-2\,ac^3d^3e^2fg+a^2c^2d^2e^3g^2\right)x^3+\left(\left(c^4d^5+2\,ac^3d^3e^2\right)f^2-2\left(a\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2
*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d
^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*
f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^
2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*
c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)
*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*
```

```
c*d*e^4)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x), 2/3*(3*(c^2*d^2*e*g*x^3
 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*
c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*
c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e +
2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a
*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g
 + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.14Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 219, normalized size = 1.16

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3\sqrt{cdx+ae}\;cd\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\;g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3\sqrt{cdx+ae}\;ae\,g^2\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{\sqrt{(ae}}\right.\right.}{3\sqrt{ex+d}\;\left(cdx+ae\right)^2\left(aeg-cdf\right)^2\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c*d*g^2+3*arctanh((c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f
)*g)^(1/2)*c*d*g*x-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*
c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^{5/2}}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

3.677 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+5*c*d*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)+10/3*g*(e*x+d)^(1
/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1
/2)
```

Rubi [A] time = 0.34, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (5*c
*d*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
```

```
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(5g)}{}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

Mathematica [C] time = 0.05, size = 75, normalized size = 0.28

$$-\frac{2cd(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (-2*c*d*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, (g*(a*e + c*d*x))/
(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 1.47, size = 1907, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^
4 + 2*a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2
*d^3*e + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*
c*d*e^3)*f*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt
(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*
g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 + 14*
a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^4 - 3*
a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^
5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2
*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4
*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)
*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d
^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2
- a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a
^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^
3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2
*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x), 1/3*
(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^4 + 2*
a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2*d^3*e
 + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*c*d*e^
3)*f*g)*x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2
 + 14*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^
4 - 3*a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (
c^5*d^5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c
^2*d^2*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*
(a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^
2*e^4)*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a
*c^4*d^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^
4*e^2 - a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3
- (2*a^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e +
a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a
^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 10.12Unable to transpose Er
ror: Bad Argument Value
```

maple [A] time = 0.04, size = 424, normalized size = 1.58

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15\sqrt{cdx+ae}\;c^2d^2g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15\sqrt{cdx+ae}\;acde\,g^3x\,\mathrm{arctanh}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*g^3+15*arctanh((c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c*d*e*g^3*(c*d*x+a*e)^(1/2)+15
*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c
^2*d^2*f*g^2+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*c*d*
e*f*g^2*(c*d*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-20*((a
*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-10*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x
-3*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f
*g+2*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*
g-c*d*f)^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{5/2}}{\left(f+g\,x\right)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.678 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=342

$$\frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade}}{6\sqrt{d+ex}\,(f+gx)^2(cdf-}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+35/4*c^2*d^2*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(9/2)+14/3*g*(
e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+35/6*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x
+f)^2/(e*x+d)^(1/2)+35/4*c*d*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-
a*e*g+c*d*f)^4/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$\frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade}}{6\sqrt{d+ex}\,(f+gx)^2(cdf-}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) + (14*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*g^2*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(6*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^
2) + (35*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a
*e*g)^4*Sqrt[d + e*x]*(f + g*x)) + (35*c^2*d^2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/(4*(c*d*f - a*e*g)^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(7g}{\cdots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{3(\cdots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{3(\cdots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{3(\cdots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{3(\cdots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\cdots}{3(\cdots}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 0.23

$$-\frac{2c^2d^2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (-2*c^2*d^2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 3, -1/2, (g*(a*e + c*d*
x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

fricas [B] time = 1.34, size = 2935, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [1/24*(105*(c^4*d^4*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2
+ (c^4*d^5 + 2*a*c^3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*
a*c^3*d^3*e^2)*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^
2*d^3*e^2*g^3 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*
c^2*d^2*e^3)*f*g^2)*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c
^2*d^2*e^3)*f^2*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f
+ 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)
*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3
 + 80*a*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*
d^3*f*g^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*
g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*
e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*
g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e
^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c
^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*
c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2
*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f
^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c
^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*
a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*
g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^
6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*
g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^
4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2
*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*
a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d
*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x), 1/12*(105*(c^4*d^4
*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2 + (c^4*d^5 + 2*a*c^
3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*a*c^3*d^3*e^2)*f*g^
2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^2*d^3*e^2*g^3 + (c^
4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f*g^2)
*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2*g)*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + (105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3 + 80*a
*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*d^3*f*g
^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*g^2 + 3
*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4
*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4
*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^
5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*
e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5
*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^
5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*
c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*
```

```
e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3
*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^
3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 +
 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4
*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a
^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^
4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3
*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f
^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 13.97Unable to transpose Er
ror: Bad Argument Value
```

maple [B] time = 0.04, size = 670, normalized size = 1.96

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(105\sqrt{cdx+ae}\;c^3d^3g^4x^3\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\;g}{\sqrt{(aeg-cdf)g}}\right)+105\sqrt{cdx+ae}\;a\,c^2d^2e\,g^4x^2\operatorname{arct}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -1/12*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(105*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^4*(c*d*x+a*e)^(1/2)+105*arctanh(
(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c^2*d^2*e*g^4*(c*d*x+a*e
)^(1/2)+210*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^
3*f*g^3*(c*d*x+a*e)^(1/2)+210*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*x*a*c^2*d^2*e*f*g^3*(c*d*x+a*e)^(1/2)+105*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g^2*(c*d*x+a*e)^(1/2)-105*((a*e*g
-c*d*f)*g)^(1/2)*x^3*c^3*d^3*g^3+105*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^
(1/2)*x^2*a*c^2*d^2*e*g^3-175*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^3*d^3*f*g^2-21*
((a*e*g-c*d*f)*g)^(1/2)*x*a^2*c*d*e^2*g^3-238*((a*e*g-c*d*f)*g)^(1/2)*x*a*c
^2*d^2*e*f*g^2-56*((a*e*g-c*d*f)*g)^(1/2)*x*c^3*d^3*f^2*g+6*((a*e*g-c*d*f)*
g)^(1/2)*a^3*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e*
g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/
(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.679 $\int \frac{(f+gx)^4 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=336

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^3\left(2ae^2g-cd(5ef-3dg)\right)}{3465c^5d^5e(d+ex)^{3/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/}}{1155c^4d^4e\sqrt{d+ex}}$$

```
[Out] -128/3465*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/e/(e*x+d)^(3/2)+32/231*(-a*e*g+c*d*f)^2*(g*x
+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)^(3/2)+16/99*(
-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*
x+d)^(3/2)+2/11*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+
d)^(3/2)+128/1155*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^4/d^4/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{99c^2d^2(d+ex)^{3/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg}{231c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(3465*c^5*d^5*e*(d + e*x)^(3/2)) + (128*g*(c*
d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4
*e*Sqrt[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (16*(c*d*f - a*e*
g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(99*c^2*d^2*(
d + e*x)^(3/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(11*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} + \frac{(8(cdf-aeg))}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}} + \frac{2(}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}} + \frac{1}{}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}} + \frac{32(cdf-}{}$$

$$= \frac{128(cdf-aeg)^3\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3465c^4d^4(d+ex)^{3/2}}$$

Mathematica [A] time = 0.18, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2\left(33f^2+22fgx+5g^2x^2\right)-8ac^3d\right.}{3465c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
1*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8*a
*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*d^4
*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4*x^4
)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

fricas [A] time = 1.09, size = 375, normalized size = 1.12

$$\frac{2\left(315\,c^5d^5g^4x^5+1155\,ac^4d^4ef^4-1848\,a^2c^3d^3e^2f^3g+1584\,a^3c^2d^2e^3f^2g^2-704\,a^4cde^4fg^3+128\,a^5e^5g^4+35\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*g^4*x^5 + 1155*a*c^4*d^4*e*f^4 - 1848*a^2*c^3*d^3*e^2*f
^3*g + 1584*a^3*c^2*d^2*e^3*f^2*g^2 - 704*a^4*c*d*e^4*f*g^3 + 128*a^5*e^5*g
^4 + 35*(44*c^5*d^5*f*g^3 + a*c^4*d^4*e*g^4)*x^4 + 10*(297*c^5*d^5*f^2*g^2
+ 22*a*c^4*d^4*e*f*g^3 - 4*a^2*c^3*d^3*e^2*g^4)*x^3 + 6*(462*c^5*d^5*f^3*g
+ 99*a*c^4*d^4*e*f^2*g^2 - 44*a^2*c^3*d^3*e^2*f*g^3 + 8*a^3*c^2*d^2*e^3*g^4
```

```
)*x^2 + (1155*c^5*d^5*f^4 + 924*a*c^4*d^4*e*f^3*g - 792*a^2*c^3*d^3*e^2*f^2
*g^2 + 352*a^3*c^2*d^2*e^3*f*g^3 - 64*a^4*c*d*e^4*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,(gx + f)^4}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^4/sqrt(e*x
+ d), x)
```

maple [A] time = 0.01, size = 283, normalized size = 0.84

$$\frac{2\,(cdx + ae)\left(315g^4x^4c^4d^4 - 280a\,c^3d^3e\,g^4x^3 + 1540c^4d^4f\,g^3x^3 + 240a^2c^2d^2e^2g^4x^2 - 1320a\,c^3d^3ef\,g^3x^2 + 2970c^4d\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d^
4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c^4
*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376*a*
c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^3*f*
g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f^4)*(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

maxima [A] time = 0.71, size = 320, normalized size = 0.95

$$\frac{2\,(cdx + ae)^{\frac{3}{2}}f^4}{3\,cd} + \frac{8\left(3\,c^2d^2x^2 + acdex - 2\,a^2e^2\right)\sqrt{cdx + ae}\,f^3g}{15\,c^2d^2} + \frac{4\left(15\,c^3d^3x^3 + 3\,ac^2d^2ex^2 - 4\,a^2cde^2x + 8\,a^3e^3\right)\sqrt{c\cdots}}{35\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^
2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^3)
+ 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3
*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(315*c^
5*d^5*x^5 + 35*a*c^4*d^4*e*x^4 - 40*a^2*c^3*d^3*e^2*x^3 + 48*a^3*c^2*d^2*e^
3*x^2 - 64*a^4*c*d*e^4*x + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

mupad [B] time = 3.60, size = 347, normalized size = 1.03

$$\frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}\,\left(\frac{2\,g^4\,x^5}{11} + \frac{256\,a^5\,e^5\,g^4 - 1408\,a^4\,c\,d\,e^4\,f\,g^3 + 3168\,a^3\,c^2\,d^2\,e^3\,f^2\,g^2 - 3696\,a^2\,c^3\,d^3\,e^2\,f^3\,g + 2310\,a\,c^4\,d^4\,e\,f^4\cdots}{3465\,c^5\,d^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5)/11 + (256*a^5*e
^5*g^4 + 2310*a*c^4*d^4*e*f^4 - 3696*a^2*c^3*d^3*e^2*f^3*g - 1408*a^4*c*d*e
^4*f*g^3 + 3168*a^3*c^2*d^2*e^3*f^2*g^2)/(3465*c^5*d^5) + (x*(2310*c^5*d^5*
f^4 - 128*a^4*c*d*e^4*g^4 + 704*a^3*c^2*d^2*e^3*f*g^3 + 1848*a*c^4*d^4*e*f^
3*g - 1584*a^2*c^3*d^3*e^2*f^2*g^2))/(3465*c^5*d^5) + (4*g*x^2*(8*a^3*e^3*g
^3 + 462*c^3*d^3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*
c^3*d^3) + (4*g^2*x^3*(297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(
693*c^2*d^2) + (2*g^3*x^4*(a*e*g + 44*c*d*f))/(99*c*d)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

3.680 $\int \frac{(f+gx)^3 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}} dx$

Optimal. Leaf size=269

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2\left(2ae^2g-cd(5ef-3dg)\right)}{315c^4d^4e(d+ex)^{3/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(}{105c^3d^3e\sqrt{d+ex}}$$

```
[Out] -16/315*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)/c^4/d^4/e/(e*x+d)^(3/2)+4/21*(-a*e*g+c*d*f)*(g*x+f)^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)+2/9*(g*x+f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+16/105*g*(-a*e*g
+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {870, 794, 648}

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}}-\frac{16\left(x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c^4*d^4*e*(d + e*x)^(3/2)) + (16*g*(c*d*f
 - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(21*c^2*d^2*(d + e*x)^(3/2)) + (2*(f + g*x)^3*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
```

```
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}+\frac{\left(2\left(cde^2f+cd\right.\right.}{}$$

$$=\frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}}+\frac{2(f}{}$$

$$=\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}}+\frac{4(cdf-a}{}$$

$$=\frac{16(cdf-aeg)^2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{315c^3d^3(d+ex)^{3/2}}$$

Mathematica [A] time = 0.12, size = 136, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg\left(21f^2+18fgx+5g^2x^2\right)+c^3d^3\left(105f^3\right.\right.}{315c^4d^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(3
*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(105*
f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3
/2))
```

fricas [A] time = 1.00, size = 264, normalized size = 0.98

$$\frac{2\left(35\,c^4d^4g^3x^4+105\,ac^3d^3ef^3-126\,a^2c^2d^2e^2f^2g+72\,a^3cde^3fg^2-16\,a^4e^4g^3+5\left(27\,c^4d^4fg^2+ac^3d^3eg^3\right)x^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 126*a^2*c^2*d^2*e^2*f^2*g
 + 72*a^3*c*d*e^3*f*g^2 - 16*a^4*e^4*g^3 + 5*(27*c^4*d^4*f*g^2 + a*c^3*d^3*
e*g^3)*x^3 + 3*(63*c^4*d^4*f^2*g + 9*a*c^3*d^3*e*f*g^2 - 2*a^2*c^2*d^2*e^2*
g^3)*x^2 + (105*c^4*d^4*f^3 + 63*a*c^3*d^3*e*f^2*g - 36*a^2*c^2*d^2*e^2*f*g
^2 + 8*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^3}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^3/sqrt(e*x
+ d), x)
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\,(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+\right.}{315\sqrt{ex+d}\;c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3*
f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*x+
16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

maxima [A] time = 0.65, size = 218, normalized size = 0.81

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^3}{3\,cd}+\frac{2\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,f^2g}{5\,c^2d^2}+\frac{2\left(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3\right)\sqrt{c}}{35\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3) + 2
/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3*c*
d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

mupad [B] time = 3.37, size = 242, normalized size = 0.90

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^3\,x^4}{9}-\frac{32\,a^4\,e^4\,g^3-144\,a^3\,c\,d\,e^3\,f\,g^2+252\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,c^4\,d^4}+\frac{x\left(16\,a^3\,c\,d\,e^3\,g^3-72\,a^2\,c\right.}{}\right.}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^3*x^4)/9 - (32*a^4*e^4
*g^3 - 210*a*c^3*d^3*e*f^3 + 252*a^2*c^2*d^2*e^2*f^2*g - 144*a^3*c*d*e^3*f*
g^2)/(315*c^4*d^4) + (x*(210*c^4*d^4*f^3 + 16*a^3*c*d*e^3*g^3 - 72*a^2*c^2*
d^2*e^2*f*g^2 + 126*a*c^3*d^3*e*f^2*g))/(315*c^4*d^4) + (2*g*x^2*(63*c^2*d^
2*f^2 - 2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(105*c^2*d^2) + (2*g^2*x^3*(a*e*g +
 27*c*d*f))/(63*c*d)))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)^{3}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/sqrt(d + e*x), x)
```

3.681 $\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)}{105c^3d^3e(d+ex)^{3/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(\ldots}{35c^2d^2e\sqrt{d+ex}}$$

```
[Out] -8/105*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(3/2)+2/7*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+8/35*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef\ldots\right.}{105c^3d^3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*(d + e*x)^(3/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*c^2*d^2*e*Sqrt[d +
 e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c
*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}} + \frac{\left(4\left(cde^2f+cd^2eg\right.\right.}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}} + \frac{2(f+gx)^2\left(ad\right.}{}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{105c^3d^3e(d+ex)^{3/2}}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2\left(35f^2+42fgx+15g^2x^2\right)\right)}{105c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g*
x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2
))
```

fricas [A] time = 1.14, size = 173, normalized size = 0.86

$$\frac{2\left(15\,c^3d^3g^2x^3+35\,ac^2d^2ef^2-28\,a^2cde^2fg+8\,a^3e^3g^2+3\left(14\,c^3d^3fg+ac^2d^2eg^2\right)x^2+\left(35\,c^3d^3f^2+14\,ac^2d^2efg\right.\right.}{105\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 28*a^2*c*d*e^2*f*g + 8*a^3
*e^3*g^2 + 3*(14*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 + 14*
a*c^2*d^2*e*f*g - 4*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^2}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^2/sqrt(e*x
+ d), x)
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\left(cdx+ae\right)\left(15g^2x^2c^2d^2-12acde\,g^2x+42c^2d^2fgx+8a^2e^2g^2-28acdefg+35f^2c^2d^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+}}{105\sqrt{ex+d}\;c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

maxima [A] time = 0.59, size = 133, normalized size = 0.66

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^2}{3\,cd}+\frac{4\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,fg}{15\,c^2d^2}+\frac{2\left(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3\right)\sqrt{cdx+ae}\,g^2}{105\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

mupad [B] time = 3.25, size = 157, normalized size = 0.78

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3}{7}+\frac{16\,a^3\,e^3\,g^2-56\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,c^3\,d^3}+\frac{x\,\left(-8\,a^2\,c\,d\,e^2\,g^2+28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,c^3\,d^3}+\dots\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3)/7 + (16*a^3*e^3
*g^2 + 70*a*c^2*d^2*e*f^2 - 56*a^2*c*d*e^2*f*g)/(105*c^3*d^3) + (x*(70*c^3*
d^3*f^2 - 8*a^2*c*d*e^2*g^2 + 28*a*c^2*d^2*e*f*g))/(105*c^3*d^3) + (2*g*x^2
*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)^2}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

3.682 $\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(2ae^2g-cd(5ef-3dg)\right)}{15c^2d^2e(d+ex)^{3/2}}$$

```
[Out] -2/15*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^2/d^2/e/(e*x+d)^(3/2)+2/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d
/e/(e*x+d)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(2ae^2g-cd(5ef-3dg)\right)}{15c^2d^2e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*c^2*d^2*e*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}+\frac{1}{5}\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\int \sqrt{}$$
$$=\frac{2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15cd(d+ex)^{3/2}}+\frac{2g\left(ade+\right.}{}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+3gx)-2aeg)}{15c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2*
d^2*(d + e*x)^(3/2))
```

fricas [A] time = 1.28, size = 102, normalized size = 0.82

$$\frac{2\left(3\,c^2d^2gx^2+5\,acdef-2\,a^2e^2g+\left(5\,c^2d^2f+acdeg\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{15\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 2*a^2*e^2*g + (5*c^2*d^2*f + a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*
x + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\left(gx+f\right)}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)/sqrt(e*x +
d), x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.54

$$-\frac{2\left(cdx+ae\right)\left(-3cdgx+2aeg-5cdf\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15\sqrt{ex+d}\;c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

maxima [A] time = 0.54, size = 65, normalized size = 0.52

$$\frac{2\left(cdx+ae\right)^{\frac{3}{2}}f}{3\,cd}+\frac{2\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,g}{15\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*e
^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

mupad [B] time = 3.13, size = 93, normalized size = 0.74

$$\frac{\left(\frac{2\,g\,x^2}{5}-\frac{4\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,c^2\,d^2}+\frac{x\,\left(10\,f\,c^2\,d^2+2\,a\,e\,g\,c\,d\right)}{15\,c^2\,d^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/
2),x)
```

```
[Out] (((2*g*x^2)/5 - (4*a^2*e^2*g - 10*a*c*d*e*f)/(15*c^2*d^2) + (x*(10*c^2*d^2*
f + 2*a*c*d*e*g))/(15*c^2*d^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

3.683 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

fricas [A] time = 0.78, size = 57, normalized size = 1.19

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdx+ae)\sqrt{ex+d}}{3\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
/(c*d*e*x + c*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d), x)
```

maple [A] time = 0.00, size = 50, normalized size = 1.04

$$\frac{2\left(cdx + ae\right)\sqrt{cde\,x^2 + a\,e^2 x + c\,d^2 x + ade}}{3\sqrt{ex + d}\;cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c/d/(e*x+d)^(1/2)
```

maxima [A] time = 0.49, size = 18, normalized size = 0.38

$$\frac{2\left(cdx + ae\right)^{\frac{3}{2}}}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

mupad [B] time = 3.05, size = 49, normalized size = 1.02

$$\frac{\left(\frac{2\,x}{3} + \frac{2\,a\,e}{3\,c\,d}\right)\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\sqrt{d + e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

```
[Out] (((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))
/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(d + ex\right)\left(ae + cdx\right)}}{\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

3.684 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx$

Optimal. Leaf size=124

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

```
[Out] -2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(3/2)+2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),x
]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt
[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)}dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cde^2f+cd^2eg-e(cd^2+ae^2)g)\int \frac{}{(f+g}}{e^2g}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(2e^2(cdf-aeg))\,\mathrm{Subst}\left(\int \frac{}{-e(cd^2+ae^2)g+c}\right.}{}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right.}{g^{3/2}}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 0.81

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}-\frac{\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g] - (Sqrt[c*d*f - a*e*g]*ArcTan[(Sq
rt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/Sqrt[a*e + c*d*x]))/(g^(3/2)
*Sqrt[d + e*x])
```

fricas [A] time = 1.33, size = 318, normalized size = 2.56

$$\left[\frac{(ex+d)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-(cdef-(cd^2+2\,ae^2)g)x}{egx^2+df+(ef+dg)x}\right)+2\sqrt{cdex^2+}}{egx+dg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*
d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
e*g*x + d*g), 2*((e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqr
t((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g*x + d*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 153, normalized size = 1.23

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(aeg\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-cdf\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{\left(aeg\right.}\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*a*e*g-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2)*g)*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x
+a*e)^(1/2)/g/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(f+g\,x\right)\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

3.685 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx$

Optimal. Leaf size=132

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)}$$

```
[Out] c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)-(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2)
,x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
+ (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c
*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg}\right.}{g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Mathematica [A] time = 0.19, size = 110, normalized size = 0.83

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}}-\frac{\sqrt{g}}{f+gx}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^2),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g]
*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

fricas [B] time = 0.94, size = 562, normalized size = 4.26

$$\left[-\frac{\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-cdfg+aeg^2}\,\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x-2\,\sqrt{cdex^2+ade+\left(cd^2\right.}}{egx^2+df+\left(ef+dg\right)x}\right.}{2\left(cd^2f^2g^2-adefg^3+\left(cdefg^3-ae^2g^4\right)x^2+\left(cdef^2g^2-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*
g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g
^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2
- a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4
+ (c*d^2 - a*e^2)*f*g^3)*x), -((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)
*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
 a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*
g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4
)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 161, normalized size = 1.22

$$\frac{\left(-cdgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-cdf\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(gx+f\right)\sqrt{\left(aeg-cdf\right)g}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x)
```

```
[Out] (-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c*d*g-arctanh((c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2))*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)/(c*d*x
+a*e)^(1/2)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^{2}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}{\sqrt{d+ex}\left(f+gx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**2), x)
```

3.686 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx$

Optimal. Leaf size=207

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}$$

```
[Out] 1/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-1/2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^2/(e*x+d)^(1/2)+1/4*c*d*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3)
,x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*g*Sqrt[d + e*x]*(f + g*x)^2
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)*
Sqrt[d + e*x]*(f + g*x)) + (c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(3/2)*(c*d
*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx)}+\frac{(c^2 \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx)}+\frac{(c^2 \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx)}+\frac{c^2d \cdots}{\cdots}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.38

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^3),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^3*(d + e*x)^(3/2))
```

fricas [B] time = 0.98, size = 1056, normalized size = 5.10

$$\left[\frac{\left(c^2d^2eg^2x^3+c^2d^3f^2+\left(2\,c^2d^2efg+c^2d^3g^2\right)x^2+\left(c^2d^2ef^2+2\,c^2d^3fg\right)x\right)\sqrt{-cdfg+aeg^2}\,\log\left(-\frac{cdegx^2-cd^2f+2\,adeg- \cdots}{\cdots}\right)\cdots}{8\left(c^2d^3f^4g^2-2\,acd^2ef^3g^3+a^2de^2f^2g^4+\left(c^2d^2ef^2g^4-2\,acde^2fg^5+a^2e^3g^6\right)x^3+\left(2\,c^2d^2ef^3g^3 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c^2*d^2*f^2*g - 3*a*c*d*e*f*g^2 +
```

```
2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e*f^3*g^3 + a
^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2*e^3*g^6)*x^
3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^4
- 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a^2*d*e^2*f*g
^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*f^2*g^4)*x),
 -1/4*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x
^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*f^2*g - 3*a*c*
d*e*f*g^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e
*f^3*g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2
*e^3*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e
^2)*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a
^2*d*e^2*f*g^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*
f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 285, normalized size = 1.38

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(c^2d^2g^2x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+2c^2d^2fgx\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+c^2d^2f^2\operatorname{ar}\right.}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^2*g^2+2*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2)*g)*c^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x-2*
((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+
f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^3\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,\left(f+gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**3), x)
```

3.687 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx$

Optimal. Leaf size=277

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-$$

```
[Out] 1/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(5/2)-1/3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^3/(e*x+d)^(1/2)+1/12*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*
d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x
+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4)
,x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*g*Sqrt[d + e*x]*(f + g*x)^3
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(8*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (c^3*d^3*ArcTan[
(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*
Sqrt[d + e*x])])/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{6g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\ldots$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\ldots$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\ldots$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\ldots$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.29

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^4),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^4*(d + e*x)^(3/2))
```

fricas [B] time = 1.08, size = 1732, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*(3*c^3*d^3*f^3*g - 17*a*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d
^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5
*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3
*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^
3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^
2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 +
 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g
^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^
2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*
d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e
^2 - a^3*e^4)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c
^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2
 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^3*d^3*f^3*g - 17*a
*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^
3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d
*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3
*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4
 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(
a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 + 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^
7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3
*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^
3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e -
a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 453, normalized size = 1.64

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^3d^3g^3x^3\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+9c^3d^3f\,g^2x^2\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+9c^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+9*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+9*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
```

```
d*f)*g)^(1/2)*g)*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^
2*d^2*g^2*x^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x-8*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+8*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/
(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*
e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}\left(gx + f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{{\left(f + g\,x\right)}^4\,\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,\left(f + gx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**4), x)
```

3.688 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx$

Optimal. Leaf size=347

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{96g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg}$$

```
[Out] 5/64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(7/2)-1/4*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^4/(e*x+d)^(1/2)+1/24*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/96*c^
2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/
(e*x+d)^(1/2)+5/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*
g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{96g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5)
,x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*g*Sqrt[d + e*x]*(f + g*x)^4
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^3*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*Sqrt[d
 + e*x]*(f + g*x)) + (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(3/2)*(c*d*f
- a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{8g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{(5\cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5c\cdots}{9\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5c\cdots}{9\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5c\cdots}{9\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5c\cdots}{9\cdots}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.23

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^5*(d + e*x)^(3/2))
```

fricas [B] time = 1.02, size = 2610, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2*g^3 - 184*
a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x
^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*
x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2*d^2*e^2*f*
g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2*d^3*e^2*f^
6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e*f^4*g^6 -
4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 +
 a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 16*
a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^7 + 2*(3
*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g
^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 6*a*c^3
*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^6 + 12*(a^2
*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^8
)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 8*a*c^3
*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c
^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7
)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - a*c^3*d^3*e
^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*(6*a^2*c^2*
d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^6)*x),
-1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c
*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c
*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*
x)) + (15*c^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2
*g^3 - 184*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d
^3*e*g^5)*x^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^
2*e^2*g^5)*x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2
*d^2*e^2*f*g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2
*d^3*e^2*f^6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e
*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*
e^4*f*g^9 + a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^
4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3
*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a
^4*e^5)*f*g^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^
5 - 6*a*c^3*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^
6 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e
^5)*f^2*g^8)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^
5 - 8*a*c^3*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 +
 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e
^5)*f^3*g^7)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 -
a*c^3*d^3*e^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*
(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^
4*g^6)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 696, normalized size = 2.01

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^4d^4g^4x^4\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+60c^4d^4f\,g^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+90c^4d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^4*d^4*g^4+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^4*d^4*f*g^3+90*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^4*d^4*f^2*g^2+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x*c^4*d^4*f^3*g-15*x^3*c^3*d^3*g^3*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*c^4*d^4*f^4+10*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)
*g)^(1/2)-55*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*
x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*x*a*c^2*d^2*
e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*x*c^3*d^3*f^2*g*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a^3*e^3*g^3+136*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2
*f*g^2-118*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d
^2*f^2)/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\,\left(gx+f\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^5\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

3.689 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)}{15015c^5d^5e(d+ex)^{5/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/}}{3003c^4d^4e(d+ex)^{3/2}}$$

```
[Out] -128/15015*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)/c^5/d^5/e/(e*x+d)^(5/2)+128/3003*g*(-a*e*g+c*d*f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(3/2)+32/429*(-a
*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e*
x+d)^(5/2)+16/143*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)/c^2/d^2/(e*x+d)^(5/2)+2/13*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] time = 0.61, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{143c^2d^2(d+ex)^{5/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg}{429c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2))/(15015*c^5*d^5*e*(d + e*x)^(5/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^
4*e*(d + e*x)^(3/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(143*c^2*d
^2*(d + e*x)^(5/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(13*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} + \frac{(8(cdf-aeg$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} +$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} +$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cd$$

$$= \frac{128(cdf-aeg)^3\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+c$$

Mathematica [A] time = 0.23, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(128a^4e^4g^4-64a^3cde^3g^3(13f+5gx)+16a^2c^2d^2e^2g^2\left(143f^2+130fgx+35g^2x^2\right)-8a}{150$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
3*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2) -
8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) + c^4
*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 1155*g
^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

fricas [A] time = 0.96, size = 472, normalized size = 1.40

$$\frac{2\left(1155\,c^6d^6g^4x^6+3003\,a^2c^4d^4e^2f^4-3432\,a^3c^3d^3e^3f^3g+2288\,a^4c^2d^2e^4f^2g^2-832\,a^5cde^5fg^3+128\,a^6e^6g^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^6*d^6*g^4*x^6 + 3003*a^2*c^4*d^4*e^2*f^4 - 3432*a^3*c^3*d^3
*e^3*f^3*g + 2288*a^4*c^2*d^2*e^4*f^2*g^2 - 832*a^5*c*d*e^5*f*g^3 + 128*a^6
*e^6*g^4 + 210*(26*c^6*d^6*f*g^3 + 7*a*c^5*d^5*e*g^4)*x^5 + 35*(286*c^6*d^6
*f^2*g^2 + 208*a*c^5*d^5*e*f*g^3 + a^2*c^4*d^4*e^2*g^4)*x^4 + 20*(429*c^6*d
^6*f^3*g + 715*a*c^5*d^5*e*f^2*g^2 + 13*a^2*c^4*d^4*e^2*f*g^3 - 2*a^3*c^3*d
^3*e^3*g^4)*x^3 + 3*(1001*c^6*d^6*f^4 + 4576*a*c^5*d^5*e*f^3*g + 286*a^2*c^
```

```
4*d^4*e^2*f^2*g^2 - 104*a^3*c^3*d^3*e^3*f*g^3 + 16*a^4*c^2*d^2*e^4*g^4)*x^2
 + 2*(3003*a*c^5*d^5*e*f^4 + 858*a^2*c^4*d^4*e^2*f^3*g - 572*a^3*c^3*d^3*e^
3*f^2*g^2 + 208*a^4*c^2*d^2*e^4*f*g^3 - 32*a^5*c*d*e^5*g^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}\left(gx + f\right)^4}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^4/(e*x +
d)^(3/2), x)
```

maple [A] time = 0.01, size = 283, normalized size = 0.84

$$\frac{2\,(cdx + ae)\left(1155g^4x^4c^4d^4 - 840a\,c^3d^3e\,g^4x^3 + 5460c^4d^4f\,g^3x^3 + 560a^2c^2d^2e^2g^4x^2 - 3640a\,c^3d^3ef\,g^3x^2 + 10010\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/15015*(c*d*x+a*e)*(1155*c^4*d^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*
d^4*f*g^3*x^3+560*a^2*c^2*d^2*e^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*
c^4*d^4*f^2*g^2*x^2-320*a^3*c*d*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720
*a*c^3*d^3*e*f^2*g^2*x+8580*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3
*f*g^3+2288*a^2*c^2*d^2*e^2*f^2*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^5/d^5/(e*x+d)^(3/2)
```

maxima [A] time = 0.74, size = 413, normalized size = 1.23

$$\frac{2\left(c^2d^2x^2 + 2\,acdex + a^2e^2\right)\sqrt{cdx + ae}\,f^4}{5\,cd} + \frac{8\left(5\,c^3d^3x^3 + 8\,ac^2d^2ex^2 + a^2cde^2x - 2\,a^3e^3\right)\sqrt{cdx + ae}\,f^3g}{35\,c^2d^2} + \frac{4\left(35\,c^4d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g^2
/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*
e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x
+ a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*e*x^5 +
 35*a^2*c^4*d^4*e^2*x^4 - 40*a^3*c^3*d^3*e^3*x^3 + 48*a^4*c^2*d^2*e^4*x^2 -
 64*a^5*c*d*e^5*x + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

mupad [B] time = 3.80, size = 445, normalized size = 1.32

$$\frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}\;\left(\frac{4\,g^3\,x^5\,\left(7\,a\,e\,g+26\,c\,d\,f\right)}{143} + \frac{256\,a^6\,e^6\,g^4-1664\,a^5\,c\,d\,e^5\,f\,g^3+4576\,a^4\,c^2\,d^2\,e^4\,f^2\,g^2-6864\,a^3\,c^3\,d^3\,e^3\,f^3\,g+}{15015\,c^5\,d^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^3*x^5*(7*a*e*g + 26*c*
d*f))/143 + (256*a^6*e^6*g^4 + 6006*a^2*c^4*d^4*e^2*f^4 - 6864*a^3*c^3*d^3*
e^3*f^3*g - 1664*a^5*c*d*e^5*f*g^3 + 4576*a^4*c^2*d^2*e^4*f^2*g^2)/(15015*c
^5*d^5) + (x^2*(6006*c^6*d^6*f^4 + 96*a^4*c^2*d^2*e^4*g^4 - 624*a^3*c^3*d^3
*e^3*f*g^3 + 27456*a*c^5*d^5*e*f^3*g + 1716*a^2*c^4*d^4*e^2*f^2*g^2))/(1501
5*c^5*d^5) + (x*(12012*a*c^5*d^5*e*f^4 - 128*a^5*c*d*e^5*g^4 + 3432*a^2*c^4
*d^4*e^2*f^3*g + 832*a^4*c^2*d^2*e^4*f*g^3 - 2288*a^3*c^3*d^3*e^3*f^2*g^2))
/(15015*c^5*d^5) + (2*c*d*g^4*x^6)/13 + (8*g*x^3*(429*c^3*d^3*f^3 - 2*a^3*e
^3*g^3 + 715*a*c^2*d^2*e*f^2*g + 13*a^2*c*d*e^2*f*g^2))/(3003*c^2*d^2) + (2
*g^2*x^4*(a^2*e^2*g^2 + 286*c^2*d^2*f^2 + 208*a*c*d*e*f*g))/(429*c*d)))/(d
+ e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.690 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)}{1155c^4d^4e(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(}{231c^3d^3e(d+ex)^{3/2}}$$

```
[Out] -16/1155*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(5/2)+16/231*g*(-a*e*g+c*d*f)^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(3/2)+4/33*(-a*e*g+c
*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/
2)+2/11*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] time = 0.41, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {870, 794, 648}

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}}-\frac{16\left(x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*c^4*d^4*e*(d + e*x)^(5/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*c^3*d^3*e*
(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*(d + e*x)^(5/2)) + (2*(f + g*x)^3*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}+\frac{(6(cdf-aeg}{}$$

$$=\frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}}+\frac{2}{}$$

$$=\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}}+\frac{4(cdf-}{}$$

$$=\frac{16(cdf-aeg)^2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{1155c^3d^3(d+ex)^{5/2}}$$

Mathematica [A] time = 0.17, size = 137, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(-16a^3e^3g^3+8a^2cde^2g^2(11f+5gx)-2ac^2d^2eg\left(99f^2+110fgx+35g^2x^2\right)+c^3d^3\left(23\right.\right.}{1155c^4d^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(11
*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3*(
231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d + e
*x)^(5/2))
```

fricas [A] time = 0.94, size = 340, normalized size = 1.26

$$\frac{2\left(105\,c^5d^5g^3x^5+231\,a^2c^3d^3e^2f^3-198\,a^3c^2d^2e^3f^2g+88\,a^4cde^4fg^2-16\,a^5e^5g^3+35\left(11\,c^5d^5fg^2+4\,ac^4d^4eg^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 198*a^3*c^2*d^2*e^3
*f^2*g + 88*a^4*c*d*e^4*f*g^2 - 16*a^5*e^5*g^3 + 35*(11*c^5*d^5*f*g^2 + 4*a
*c^4*d^4*e*g^3)*x^4 + 5*(99*c^5*d^5*f^2*g + 110*a*c^4*d^4*e*f*g^2 + a^2*c^3
*d^3*e^2*g^3)*x^3 + 3*(77*c^5*d^5*f^3 + 264*a*c^4*d^4*e*f^2*g + 11*a^2*c^3*
d^3*e^2*f*g^2 - 2*a^3*c^2*d^2*e^3*g^3)*x^2 + (462*a*c^4*d^4*e*f^3 + 99*a^2*
c^3*d^3*e^2*f^2*g - 44*a^3*c^2*d^2*e^3*f*g^2 + 8*a^4*c*d*e^4*g^3)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^3/(e*x +
d)^(3/2), x)
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx+ae\right)\left(-105g^{3}x^{3}c^{3}d^{3}+70a\,c^{2}d^{2}e\,g^{3}x^{2}-385c^{3}d^{3}f\,g^{2}x^{2}-40a^{2}cd\,e^{2}g^{3}x+220a\,c^{2}d^{2}ef\,g^{2}x-495c^{3}d^{3}f^{2}gx}{1155\left(ex+d\right)^{\frac{3}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-105*c^3*d^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^
3*f*g^2*x^2-40*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*
x+16*a^3*e^3*g^3-88*a^2*c*d*e^2*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^4/d^4/(e*x+d)^(3/2)
```

maxima [A] time = 0.69, size = 294, normalized size = 1.09

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdx+ae}\,f^{3}}{5\,cd}+\frac{6\left(5\,c^{3}d^{3}x^{3}+8\,ac^{2}d^{2}ex^{2}+a^{2}cde^{2}x-2\,a^{3}e^{3}\right)\sqrt{cdx+ae}\,f^{2}g}{35\,c^{2}d^{2}}+\frac{2\left(35\,c^{4}d^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^2/(
c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*e^
2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x +
a*e)*g^3/(c^4*d^4)
```

mupad [B] time = 3.65, size = 310, normalized size = 1.15

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^4\,\left(4\,a\,e\,g+11\,c\,d\,f\right)}{33}-\frac{32\,a^5\,e^5\,g^3-176\,a^4\,c\,d\,e^4\,f\,g^2+396\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,c^4\,d^4}+\frac{x^2\left(\cdots\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^4*(4*a*e*g + 11*c*
d*f))/33 - (32*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 396*a^3*c^2*d^2*e^3*
f^2*g - 176*a^4*c*d*e^4*f*g^2)/(1155*c^4*d^4) + (x^2*(462*c^5*d^5*f^3 - 12*
a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 + 1584*a*c^4*d^4*e*f^2*g))/(
1155*c^4*d^4) + (2*c*d*g^3*x^5)/11 + (2*g*x^3*(a^2*e^2*g^2 + 99*c^2*d^2*f^2
 + 110*a*c*d*e*f*g))/(231*c*d) + (2*a*e*x*(8*a^3*e^3*g^3 + 462*c^3*d^3*f^3
+ 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*c^3*d^3)))/(d + e*x)^
(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.691 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)}{315c^3d^3e(d+ex)^{5/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf}{63c^2d^2e(d+ex)^{3/2}}$$

```
[Out] -8/315*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(5/2)+8/63*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/e/(e*x+d)^(3/2)+2/9*(g*x+f)^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{63c^2d^2e(d+ex)^{3/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-\right.}{315c^3d^3e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2))/(315*c^3*d^3*e*(d + e*x)^(5/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*c^2*d^2*e*(d + e*x
)^(3/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9
*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}} + \frac{\left(4\left(cde^2f+\right.\right.}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)}{}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x\right.}{315c^3d^3e(d+ex)^{5/2}}$$

Mathematica [A] time = 0.12, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)+c^2d^2\left(63f^2+90fgx+35g^2x^2\right)\right)}{315c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g*
x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2
))
```

fricas [A] time = 0.87, size = 230, normalized size = 1.15

$$\frac{2\left(35\,c^4d^4g^2x^4+63\,a^2c^2d^2e^2f^2-36\,a^3cde^3fg+8\,a^4e^4g^2+10\left(9\,c^4d^4fg+5\,ac^3d^3eg^2\right)x^3+3\left(21\,c^4d^4f^2+48\,a\right.\right.}{315\left(c^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 36*a^3*c*d*e^3*f*g + 8
*a^4*e^4*g^2 + 10*(9*c^4*d^4*f*g + 5*a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f
^2 + 48*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2
+ 9*a^2*c^2*d^2*e^2*f*g - 2*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^2/(e*x +
d)^(3/2), x)
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\,(cdx+ae)\left(35g^2x^2c^2d^2-20acde\,g^2x+90c^2d^2fgx+8a^2e^2g^2-36acdefg+63f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x\right.}{315\,(ex+d)^{\frac{3}{2}}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-36*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(3/2)/c^3/d^3/(e*x+d)^(3/2)
```

maxima [A] time = 0.62, size = 192, normalized size = 0.96

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}\,f^2}{5\,cd}+\frac{4\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}\,fg}{35\,c^2d^2}+\frac{2\left(35\,c^4d^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a^2
*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(c^3*
d^3)
```

mupad [B] time = 3.43, size = 206, normalized size = 1.03

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g\,x^3\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{63}+\frac{16\,a^4\,e^4\,g^2-72\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,c^3\,d^3}+\frac{x^2\,\left(6\,a^2\,c^2\,d^2\,e^2\,g^2+288\,a\,c^3\,d\right.}{315\,c^3\,d^3}\right.}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g*x^3*(5*a*e*g + 9*c*d*f
))/63 + (16*a^4*e^4*g^2 + 126*a^2*c^2*d^2*e^2*f^2 - 72*a^3*c*d*e^3*f*g)/(31
5*c^3*d^3) + (x^2*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 + 288*a*c^3*d^3*
e*f*g))/(315*c^3*d^3) + (2*c*d*g^2*x^4)/9 + (4*a*e*x*(63*c^2*d^2*f^2 - 2*a^
2*e^2*g^2 + 9*a*c*d*e*f*g))/(315*c^2*d^2)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.692 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

```
[Out] -2/35*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)/c^2/d^2/e/(e*x+d)^(5/2)+2/7*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d
/e/(e*x+d)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*c^2*d^2*e*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}+\frac{1}{7}\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)$$

$$=\frac{2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35cd(d+ex)^{5/2}}+\frac{2g\,(a}{}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+5gx)-2aeg)}{35c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2*
d^2*(d + e*x)^(5/2))
```

fricas [A] time = 0.82, size = 137, normalized size = 1.10

$$\frac{2\left(5c^3d^3gx^3+7a^2cde^2f-2a^3e^3g+\left(7c^3d^3f+8ac^2d^2eg\right)x^2+\left(14ac^2d^2ef+a^2cde^2g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+\right.}}{35\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 2*a^3*e^3*g + (7*c^3*d^3*f + 8*a*
c^2*d^2*e*g)*x^2 + (14*a*c^2*d^2*e*f + a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 67, normalized size = 0.54

$$-\frac{2\left(cdx+ae\right)\left(-5cdgx+2aeg-7cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{35\left(ex+d\right)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c*d*g*x+2*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

maxima [A] time = 0.57, size = 107, normalized size = 0.86

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}\,f}{5\,cd}+\frac{2\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}\,g}{35\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35*
(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*x
+ a*e)*g/(c^2*d^2)
```

mupad [B] time = 3.25, size = 109, normalized size = 0.87

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^2\,\left(\frac{16\,a\,e\,g}{35}+\frac{2\,c\,d\,f}{5}\right)-\frac{4\,a^3\,e^3\,g-14\,a^2\,c\,d\,e^2\,f}{35\,c^2\,d^2}+\frac{2\,c\,d\,g\,x^3}{7}+\frac{2\,a\,e\,x\,\left(a\,e\,g+14\,c\,d\,f\right)}{35\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^2*((16*a*e*g)/35 + (2*c*d
*f)/5) - (4*a^3*e^3*g - 14*a^2*c*d*e^2*f)/(35*c^2*d^2) + (2*c*d*g*x^3)/7 +
(2*a*e*x*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\left(f+gx\right)}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2), x)
```

3.693 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

fricas [A] time = 0.96, size = 74, normalized size = 1.54

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{5\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
```

[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(3/2), x)

maple [A] time = 0.00, size = 50, normalized size = 1.04

$$\frac{2\,(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{5\,(ex+d)^{\frac{3}{2}}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)

[Out] 2/5*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c/d/(e*x+d)^(3/2)

maxima [A] time = 0.51, size = 43, normalized size = 0.90

$$\frac{2\left(c^2d^2x^2 + 2\,acdex + a^2e^2\right)\sqrt{cdx+ae}}{5\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")

[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)

mupad [B] time = 3.08, size = 62, normalized size = 1.29

$$\frac{\left(\frac{4\,a\,e\,x}{5}+\frac{2\,c\,d\,x^2}{5}+\frac{2\,a^2\,e^2}{5\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)

[Out] (((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)

[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)

3.694 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=179

$$\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2\left(x\left(ae^2+cd^2\right)+a\right.}{3g(d+ex}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+2*(-a*e*g+c*d*f
)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)-2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{864, 874, 205\}$

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+\right.}{3g(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[
d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x
)^(3/2)) + (2*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} - \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right.}{e^2g}$$

$$= -\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{3g(d+ex)^3}$$

$$= -\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{3g(d+ex)^3}$$

$$= -\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{3g(d+ex)^3}$$

Mathematica [A] time = 0.26, size = 132, normalized size = 0.74

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\sqrt{g}\sqrt{ae+cdx}\,(4aeg+cd(gx-3f))+3(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3g^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c*
d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [A] time = 0.97, size = 408, normalized size = 2.28

$$\left[-\frac{3\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-(c\right.}{egx^2+df+\left(ef+dg\right)x}}{3\left(eg^2x+dg^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-(c*d*f - a*e*g)/
g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^
2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^
2*x + d*g^2), -2/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt((c*d
*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^2*x + d*g^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 4.51Unable to transpose Error: Bad Argum
ent Value
```

maple [A] time = 0.02, size = 263, normalized size = 1.47

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3a^2e^2g^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-6acdefg\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+3c^2d^2f^2\operatorname{arc}\right.}{3\sqrt{ex+d}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*a^2*e^2*g^2-6*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2)*g)*a*c*d*e*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2)*g)*c^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x-4*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)), x)
```

3.695 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

Optimal. Leaf size=178

$$-\frac{3cd\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{g^2\sqrt{d+ex}}$$

```
[Out] -(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)-3*c*d*arct
an(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*
x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(5/2)+3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{3cd\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{g^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^2),x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*x))
- (3*c*d*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx}{2g}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

Mathematica [C] time = 0.05, size = 75, normalized size = 0.42

$$\frac{2cd((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (g*(a
*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^2*(d + e*x)^(5/2))
```

fricas [A] time = 1.01, size = 444, normalized size = 2.49

$$\left[\frac{3\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-(\cdots}{egx^2+df+\left(ef+dg\right)x}\right.}{2\left(eg^3x^2+dfg^2+\left(efg^2+dg^3\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-(c*d*f - a*e*
g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c
```

```
*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(e
*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x), (3*(c*d*e*g*x^2 + c*d^2*f + (c*d
*e*f + c*d^2*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f
 - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(
e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 306, normalized size = 1.72

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(-3acde\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+3c^2d^2fgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-3acdefg\,\mathrm{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)*(-3*arctanh((c*d*x+a*
e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c*d*e*g^2+3*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g-3*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*a*c*d*e*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^2*d^2*f^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d
*g*x-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1
/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

3.696 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

Optimal. Leaf size=195

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

```
[Out] -1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^2+3/4*
c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(1/2)-3/4*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^3),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(
f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*g*(d + e*x)^(3
/2)*(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(5/2)*Sqrt[c*d*f
 - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx}{4g}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Mathematica [A] time = 0.32, size = 135, normalized size = 0.69

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}} - \frac{\sqrt{g}\,(2aeg+cd(3f+5gx))}{(f+gx)^2}\right)}{4g^{5/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^3),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/(
f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]])/Sqrt[c*d*f - a*
e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x])
```

fricas [B] time = 1.15, size = 840, normalized size = 4.31

$$\left[-\frac{3\left(c^2d^2eg^2x^3 + c^2d^3f^2 + \left(2\,c^2d^2efg + c^2d^3g^2\right)x^2 + \left(c^2d^2ef^2 + 2\,c^2d^3fg\right)x\right)\sqrt{-cdfg+aeg^2}\,\log\left(-\frac{cdegx^2-cd^2f}{}\right.}{8\left(cd^2f^3g^3 - adef^2g^4 + \left(cdefg^5 - ae^2g\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*f^2*g - a*c*d*e*f*g^2
 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*f^2*g^4 + (c*d
*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c*d^2 - 2*a*e^2
)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*e^2)*f^2*g^4)*
x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g
^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan
(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e
```

```
*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^2*d^2*f^2*g -
 a*c*d*e*f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*
f^2*g^4 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c
*d^2 - 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*
e^2)*f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 276, normalized size = 1.42

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^2d^2g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+6c^2d^2fgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^2d^2f^2\,\mathrm{a}\right.}{4\sqrt{ex+d}\,\sqrt{cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^2*g^2+6*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^2*d^2*f^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d
*g*x+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^
2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

3.697 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

Optimal. Leaf size=265

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{(x(a$$

```
[Out] -1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^3+1/8*
c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(3/2)-1/4*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{(x(a$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^4),x]
```

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*(c
*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(3*g*(d + e*x)^(3/2)*(f + g*x)^3) + (c^3*d^3*ArcTan[(Sqrt[g]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*
x])])/(8*g^(5/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
```

```
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{(cd)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx}{2g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.30

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^4),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^4*(d + e*x)^(5/2))
```

fricas [B] time = 0.95, size = 1434, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^
3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d
^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
```

```
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) - 2*(3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3
 + 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f
^2*g^2 - 11*a*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^
4 + a^2*d*e^2*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^
8)*x^4 + (3*c^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2
*g^6 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*
e^2*f*g^7 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g
^6)*x^2 + (c^2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e
^2)*f^4*g^4 - (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*
x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^
2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*
g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*
g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) +
 (3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3 + 8*a^3*e^3*
g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f^2*g^2 - 11*a
*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^4 + a^2*d*e^2
*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^8)*x^4 + (3*c
^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^6 - (2*a*c
*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*e^2*f*g^7 + (
c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^6)*x^2 + (c^
2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g^4 -
 (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.03, size = 453, normalized size = 1.71

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^3d^3g^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+9c^3d^3f\,g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+9c^3d^3f^2g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+9*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+9*arctanh((c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2
*d^2*g^2*x^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+8*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-8*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(
e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^4\,\left(d + e\,x\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

3.698 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

Optimal. Leaf size=335

$$\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}+\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-$$

```
[Out] -1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^4+3/64
*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(5/2)-1/8*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^3/(e*x+d)^(1/2)+1/32*c^2*d^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)
+3/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^2/
(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}+\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^5),x]
```

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*Sqrt[d + e*x]*(f
+ g*x)^3) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2*(
c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x))
 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*g*(d + e*x)^(3/2)*(f +
g*x)^4) + (3*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(5/2)*(c*d*f - a*e*g)^(5
/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
```

```
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx}{8g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+g}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.24

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^5*(d + e*x)^(5/2))
```

fricas [B] time = 1.10, size = 2238, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [-1/128*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^
4*d^4*f^4*g - a*c^3*d^3*e*f^3*g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d
*e^3*f*g^4 - 16*a^4*e^4*g^5 - 3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11
*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*
c^4*d^4*f^3*g^2 - 55*a*c^3*d^3*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^
3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
)/(c^3*d^4*f^7*g^3 - 3*a*c^2*d^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*
d*e^3*f^4*g^6 + (c^3*d^3*e*f^3*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^
3*f*g^9 - a^3*e^4*g^10)*x^5 + (4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*
d^4 - 12*a*c^2*d^2*e^2)*f^3*g^7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 +
 (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*
d*e^3*f*g^9 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^
3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g
^5 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e
^4)*f^3*g^7)*x^2 + (c^3*d^3*e*f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 -
3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^
2*c*d^2*e^2 - a^3*e^4)*f^4*g^6)*x), -1/64*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f
^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4
*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*
e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^4*d^4*f^4*g - a*c^3*d^3*e*f^3*
g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d*e^3*f*g^4 - 16*a^4*e^4*g^5 -
3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^
3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*c^4*d^4*f^3*g^2 - 55*a*c^3*d^3
*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^7*g^3 - 3*a*c^2*d
^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*d*e^3*f^4*g^6 + (c^3*d^3*e*f^3
*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^3*f*g^9 - a^3*e^4*g^10)*x^5 +
(4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^
7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)
*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*d*e^3*f*g^9 + (2*c^3*d^4 - 9*a
*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^
2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*
f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^5 - 3*(3*a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^7)*x^2 + (c^3*d^3*e*
f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(
4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^6
)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 665, normalized size = 1.99

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^4d^4g^4x^4\,\text{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+12c^4d^4f\,g^3x^3\,\text{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+18c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x)
```

```
[Out] -1/64*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^4*d^4*g^4+12*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^4*d^4*f*g^3+18*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^4*d^4*f^2*g^2+12*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x*c^4*d^4*f^3*g-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)*c^3*d^3*g^3*x^3+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c^4*d^4*f^4+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*g^
3*x^2-11*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2+24*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*g^3*x-44*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+11*(c*d*x+a*e)^(1/2)*((a*e*g
-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x+16*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*a^3*e^3*g^3-24*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2*f*g
^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+3*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2)/(g*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^5\,(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

3.699 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

Optimal. Leaf size=405

$$\frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}+\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}$$

```
[Out] -1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^5+3/12
8*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(7/2)-3/40*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^4/(e*x+d)^(1/2)+1/80*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/
2)+1/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^
2/(g*x+f)^2/(e*x+d)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/g^2/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{80g^2\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^6),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(40*g^2*Sqrt[d + e*x]*
(f + g*x)^4) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(80*g^
2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)
^2) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^2*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(5*g*(d + e*x)^(3/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(128*g^(5/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{(3cd)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx}{10g}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.20

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^6),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^6*(d + e*x)^(5/2))
```

fricas [B] time = 1.03, size = 3204, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6
*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3
*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 +
 (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*
g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d)
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^
4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*a^4
*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 -
 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g^5
- 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f^3*
g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c*d*
e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4
*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3*c*
d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e^2*
f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^12)
*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e^2)
*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2*d^
3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)*x^
5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e^2)
*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^3*e
^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^4 +
 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f
^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^3*e
^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 + 5*
(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)*f^
7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3*e^
2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c^4*
d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*g^4
 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^2 -
 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x), -1/640*
(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^
5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c
^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^
5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e
*f^4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*
a^4*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*
x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^
3 - 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g
^5 - 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f
^3*g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^4*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3
*c*d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e
^2*f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^
12)*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e
```

```
^2)*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2
*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)
*x^5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e
^2)*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^
3*e^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^
4 + 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2
)*f^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^
3*e^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 +
 5*(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)
*f^7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3
*e^2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c
^4*d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*
g^4 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 955, normalized size = 2.36

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^5d^5g^5x^5\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+75c^5d^5f\,g^4x^4\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+150\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x)
```

```
[Out] 1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^5*c^5*d^5*g^5+75*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^5*d^5*f*g^4+150*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^5*d^5*f^2*g^3+150*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^5*d^5*f^3*g^2-15*x^4*c^4*d^4*g^4*((a*e*g-c
*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+75*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2)*g)*x*c^5*d^5*f^4*g+10*x^3*a*c^3*d^3*e*g^4*((a*e*g-c*d*f)*g)^(1/2
)*(c*d*x+a*e)^(1/2)-70*x^3*c^4*d^4*f*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e
)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^5*d^5*f^5
-8*x^2*a^2*c^2*d^2*e^2*g^4*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+46*x^2
*a*c^3*d^3*e*f*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-128*x^2*c^4*d^
4*f^2*g^2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-176*x*a^3*c*d*e^3*g^4*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+512*x*a^2*c^2*d^2*e^2*f*g^3*((a*e*
g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-466*x*a*c^3*d^3*e*f^2*g^2*((a*e*g-c*d*f
)*g)^(1/2)*(c*d*x+a*e)^(1/2)+70*x*c^4*d^4*f^3*g*((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)-128*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*e^4*g^4+33
6*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*f*g^3-248*((a*e*g-c
*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*((a*e*g-c*d*f)*
g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f^3*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5
/g^2/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^6\,\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

3.700 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)}{45045c^5d^5e(d+ex)^{7/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{6435c^4d^4e(d+ex)}$$

```
[Out] -128/45045*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/e/(e*x+d)^(7/2)+128/6435*g*(-a*e*g+c*d*f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(5/2)+32/715*(-a
*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*
x+d)^(7/2)+16/195*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(7/2)/c^2/d^2/(e*x+d)^(7/2)+2/15*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A] time = 0.62, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{195c^2d^2(d+ex)^{7/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-}{715c^3d^3(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(7/2))/(45045*c^5*d^5*e*(d + e*x)^(7/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^
4*e*(d + e*x)^(5/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(7/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(195*c^2*d
^2*(d + e*x)^(7/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(15*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} + \frac{(8(cdf-aeg))\int}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} + \frac{2(f}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} + \frac{16}{}$$

$$= \frac{128g(cdf-aeg)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} + \frac{32(cdf-}{}$$

$$= \frac{128(cdf-aeg)^3 \left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{45045c^4d^4(d+ex)^{7/2}}$$

Mathematica [A] time = 0.21, size = 205, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(128a^4e^4g^4-64a^3cde^3g^3(15f+7gx)+48a^2c^2d^2e^2g^2\left(65f^2+70fgx+21g^2x^2\right)\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3*
c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x + 21
*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 231*g
^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860*f*g
^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

fricas [A] time = 0.76, size = 567, normalized size = 1.69

$$\frac{2\left(3003\,c^7d^7g^4x^7+6435\,a^3c^4d^4e^3f^4-5720\,a^4c^3d^3e^4f^3g+3120\,a^5c^2d^2e^5f^2g^2-960\,a^6cde^6fg^3+128\,a^7e^7g^4+23\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^7*d^7*g^4*x^7 + 6435*a^3*c^4*d^4*e^3*f^4 - 5720*a^4*c^3*d^3
*e^4*f^3*g + 3120*a^5*c^2*d^2*e^5*f^2*g^2 - 960*a^6*c*d*e^6*f*g^3 + 128*a^7
*e^7*g^4 + 231*(60*c^7*d^7*f*g^3 + 31*a*c^6*d^6*e*g^4)*x^6 + 63*(390*c^7*d^
7*f^2*g^2 + 540*a*c^6*d^6*e*f*g^3 + 71*a^2*c^5*d^5*e^2*g^4)*x^5 + 35*(572*c
^7*d^7*f^3*g + 1794*a*c^6*d^6*e*f^2*g^2 + 636*a^2*c^5*d^5*e^2*f*g^3 + a^3*c
^4*d^4*e^3*g^4)*x^4 + 5*(1287*c^7*d^7*f^4 + 10868*a*c^6*d^6*e*f^3*g + 8814*
```

```
a^2*c^5*d^5*e^2*f^2*g^2 + 60*a^3*c^4*d^4*e^3*f*g^3 - 8*a^4*c^3*d^3*e^4*g^4)
*x^3 + 3*(6435*a*c^6*d^6*e*f^4 + 14300*a^2*c^5*d^5*e^2*f^3*g + 390*a^3*c^4*
d^4*e^3*f^2*g^2 - 120*a^4*c^3*d^3*e^4*f*g^3 + 16*a^5*c^2*d^2*e^5*g^4)*x^2 +
 (19305*a^2*c^5*d^5*e^2*f^4 + 2860*a^3*c^4*d^4*e^3*f^3*g - 1560*a^4*c^3*d^3
*e^4*f^2*g^2 + 480*a^5*c^2*d^2*e^5*f*g^3 - 64*a^6*c*d*e^6*g^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 17.12Done
```

maple [A] time = 0.01, size = 283, normalized size = 0.84

$$\frac{2\,(cdx+ae)\left(3003g^4x^4c^4d^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/45045*(c*d*x+a*e)*(3003*c^4*d^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^
4*d^4*f*g^3*x^3+1008*a^2*c^2*d^2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+245
70*c^4*d^4*f^2*g^2*x^2-448*a^3*c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-1
0920*a*c^3*d^3*e*f^2*g^2*x+20020*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*
d*e^3*f*g^3+3120*a^2*c^2*d^2*e^2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^
4*f^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^5/d^5/(e*x+d)^(5/2)
```

maxima [A] time = 0.76, size = 498, normalized size = 1.48

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f^4}{7\,cd}+\frac{8\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x\right.}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2)
+ 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f^2*g
^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*
d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5
*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*c^7*d^7*
x^7 + 7161*a*c^6*d^6*e*x^6 + 4473*a^2*c^5*d^5*e^2*x^5 + 35*a^3*c^4*d^4*e^3*
x^4 - 40*a^4*c^3*d^3*e^4*x^3 + 48*a^5*c^2*d^2*e^5*x^2 - 64*a^6*c*d*e^6*x +
128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

mupad [B] time = 4.09, size = 523, normalized size = 1.56

$$\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\;\left(\frac{2\,g^2\,x^5\,\left(71\,a^2\,e^2\,g^2+540\,a\,c\,d\,e\,f\,g+390\,c^2\,d^2\,f^2\right)}{715}+\frac{256\,a^7\,e^7\,g^4-1920\,a^6\,c\,d\,e^6\,f\,g^3+6240\,a^5\,c^2\,d^2\,e^5\,f^2\,g}{45045\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^5*(71*a^2*e^2*g^2
+ 390*c^2*d^2*f^2 + 540*a*c*d*e*f*g))/715 + (256*a^7*e^7*g^4 + 12870*a^3*c^
4*d^4*e^3*f^4 - 11440*a^4*c^3*d^3*e^4*f^3*g - 1920*a^6*c*d*e^6*f*g^3 + 6240
*a^5*c^2*d^2*e^5*f^2*g^2)/(45045*c^5*d^5) + (x^3*(12870*c^7*d^7*f^4 - 80*a^
4*c^3*d^3*e^4*g^4 + 600*a^3*c^4*d^4*e^3*f*g^3 + 108680*a*c^6*d^6*e*f^3*g +
88140*a^2*c^5*d^5*e^2*f^2*g^2))/(45045*c^5*d^5) + (2*c^2*d^2*g^4*x^7)/15 +
(2*c*d*g^3*x^6*(31*a*e*g + 60*c*d*f))/195 + (2*g*x^4*(a^3*e^3*g^3 + 572*c^3
*d^3*f^3 + 1794*a*c^2*d^2*e*f^2*g + 636*a^2*c*d*e^2*f*g^2))/(1287*c*d) + (2
*a^2*e^2*x*(19305*c^4*d^4*f^4 - 64*a^4*e^4*g^4 + 2860*a*c^3*d^3*e*f^3*g + 4
80*a^3*c*d*e^3*f*g^3 - 1560*a^2*c^2*d^2*e^2*f^2*g^2))/(45045*c^4*d^4) + (2*
a*e*x^2*(16*a^4*e^4*g^4 + 6435*c^4*d^4*f^4 + 14300*a*c^3*d^3*e*f^3*g - 120*
a^3*c*d*e^3*f*g^3 + 390*a^2*c^2*d^2*e^2*f^2*g^2))/(15015*c^3*d^3)))/(d + e*
x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.701 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)}{3003c^4d^4e(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7}}{429c^3d^3e(d+ex)^{5/2}}$$

```
[Out] -16/3003*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(7/2)+16/429*g*(-a*e*g+c*d*f)^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(5/2)+12/143*(-a*e*g
+c*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)^(
7/2)+2/13*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/
2)
```

Rubi [A] time = 0.40, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{12(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}}-\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*c^4*d^4*e*(d + e*x)^(7/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*c^3*d^3*e*
(d + e*x)^(5/2)) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(7/2)) + (2*(f + g*x)^3*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
```

```
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}} + \frac{(6(cdf-aeg))\int}{}$$

$$= \frac{12(cdf-aeg)(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} + \frac{2(f}{}$$

$$= \frac{16g(cdf-aeg)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} + \frac{12(cdf-a}{}$$

$$= \frac{16(cdf-aeg)^2 \left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{3003c^3d^3(d+ex)^{7/2}}$$

Mathematica [A] time = 0.16, size = 147, normalized size = 0.55

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(-16a^3e^3g^3+8a^2cde^2g^2(13f+7gx)-2ac^2d^2eg\left(143f^2+182fgx+63g^2x^2\right)+c\right.}{3003c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c
*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2*x
^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(300
3*c^4*d^4*Sqrt[d + e*x])
```

fricas [A] time = 0.86, size = 416, normalized size = 1.55

$$\frac{2\left(231\,c^6d^6g^3x^6+429\,a^3c^3d^3e^3f^3-286\,a^4c^2d^2e^4f^2g+104\,a^5cde^5fg^2-16\,a^6e^6g^3+63\left(13\,c^6d^6fg^2+9\,ac^5d^5eg^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3003*(231*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 286*a^4*c^2*d^2*e^4
*f^2*g + 104*a^5*c*d*e^5*f*g^2 - 16*a^6*e^6*g^3 + 63*(13*c^6*d^6*f*g^2 + 9*
a*c^5*d^5*e*g^3)*x^5 + 7*(143*c^6*d^6*f^2*g + 299*a*c^5*d^5*e*f*g^2 + 53*a^
2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 + 2717*a*c^5*d^5*e*f^2*g + 1469*a
^2*c^4*d^4*e^2*f*g^2 + 5*a^3*c^3*d^3*e^3*g^3)*x^3 + 3*(429*a*c^5*d^5*e*f^3
+ 715*a^2*c^4*d^4*e^2*f^2*g + 13*a^3*c^3*d^3*e^3*f*g^2 - 2*a^4*c^2*d^2*e^4*
g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 + 143*a^3*c^3*d^3*e^3*f^2*g - 52*a^4*c
^2*d^2*e^4*f*g^2 + 8*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 12.66Done
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx+ae\right)\left(-231g^3x^3c^3d^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3\right.}{3003\left(ex+d\right)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-231*c^3*d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d
^3*f*g^2*x^2-56*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*
g*x+16*a^3*e^3*g^3-104*a^2*c*d*e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^4/d^4/(e*x+d)^(5/2)
```

maxima [A] time = 0.70, size = 362, normalized size = 1.35

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f^3}{7\,cd}+\frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x\right.}{21\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2)
+ 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f*g^2
/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*d^
4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5*x
 - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

mupad [B] time = 3.81, size = 379, normalized size = 1.41

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g\,x^4\,\left(53\,a^2\,e^2\,g^2+299\,a\,c\,d\,e\,f\,g+143\,c^2\,d^2\,f^2\right)}{429}-\frac{32\,a^6\,e^6\,g^3-208\,a^5\,c\,d\,e^5\,f\,g^2+572\,a^4\,c^2\,d^2\,e^4\,f^2\,g-}{3003\,c^4\,d^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g*x^4*(53*a^2*e^2*g^2 +
143*c^2*d^2*f^2 + 299*a*c*d*e*f*g))/429 - (32*a^6*e^6*g^3 - 858*a^3*c^3*d^3
*e^3*f^3 + 572*a^4*c^2*d^2*e^4*f^2*g - 208*a^5*c*d*e^5*f*g^2)/(3003*c^4*d^4
) + (x^3*(858*c^6*d^6*f^3 + 10*a^3*c^3*d^3*e^3*g^3 + 2938*a^2*c^4*d^4*e^2*f
*g^2 + 5434*a*c^5*d^5*e*f^2*g))/(3003*c^4*d^4) + (2*c^2*d^2*g^3*x^6)/13 + (
6*c*d*g^2*x^5*(9*a*e*g + 13*c*d*f))/143 + (2*a^2*e^2*x*(8*a^3*e^3*g^3 + 128
7*c^3*d^3*f^3 + 143*a*c^2*d^2*e*f^2*g - 52*a^2*c*d*e^2*f*g^2))/(3003*c^3*d^
```

```
3) + (2*a*e*x^2*(429*c^3*d^3*f^3 - 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*f^2*g +
13*a^2*c*d*e^2*f*g^2))/(1001*c^2*d^2)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.702 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)}{693c^3d^3e(d+ex)^{7/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(\ldots}{99c^2d^2e(d+ex)^{5/2}}$$

```
[Out] -8/693*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(7/2)+8/99*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/e/(e*x+d)^(5/2)+2/11*(g*x+f)^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{99c^2d^2e(d+ex)^{5/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef\ldots\right.}{693c^3d^3e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(7/2))/(693*c^3*d^3*e*(d + e*x)^(7/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*c^2*d^2*e*(d + e*x
)^(5/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(1
1*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}} + \frac{(4(cdf-aeg))\int}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)^2\left(\right.}{}$$

$$= \frac{8(cdf-aeg)\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^2d^2(d+ex)^{7/2}}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 0.50

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^2g^2-4acdeg(11f+7gx)+c^2d^2\left(99f^2+154fgx+63g^2x^2\right)\right)}{693c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e
*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*d^
3*Sqrt[d + e*x])
```

fricas [A] time = 1.26, size = 284, normalized size = 1.42

$$\frac{2\left(63\,c^5d^5g^2x^5+99\,a^3c^2d^2e^3f^2-44\,a^4cde^4fg+8\,a^5e^5g^2+7\left(22\,c^5d^5fg+23\,ac^4d^4eg^2\right)x^4+\left(99\,c^5d^5f^2+418\,ac^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/693*(63*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 44*a^4*c*d*e^4*f*g + 8
*a^5*e^5*g^2 + 7*(22*c^5*d^5*f*g + 23*a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^
2 + 418*a*c^4*d^4*e*f*g + 113*a^2*c^3*d^3*e^2*g^2)*x^3 + 3*(99*a*c^4*d^4*e*
f^2 + 110*a^2*c^3*d^3*e^2*f*g + a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3
*e^2*f^2 + 22*a^3*c^2*d^2*e^3*f*g - 4*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 9.37Done
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\left(cdx+ae\right)\left(63g^{2}x^{2}c^{2}d^{2}-28acde\,g^{2}x+154c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-44acdefg+99f^{2}c^{2}d^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}\right.}{693\left(ex+d\right)^{\frac{5}{2}}c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/693*(c*d*x+a*e)*(63*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*
a^2*e^2*g^2-44*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(5/2)/c^3/d^3/(e*x+d)^(5/2)
```

maxima [A] time = 0.64, size = 243, normalized size = 1.22

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{cdx+ae}\,f^{2}}{7\,cd}+\frac{4\left(7\,c^{4}d^{4}x^{4}+19\,ac^{3}d^{3}ex^{3}+15\,a^{2}c^{2}d^{2}e^{2}x^{2}+a^{3}cde^{3}x\right.}{63\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) +
2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3*a
^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^2/(c^
3*d^3)
```

mupad [B] time = 3.56, size = 259, normalized size = 1.30

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^5\,g^2-88\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,c^3\,d^3}+\frac{x^3\,\left(226\,a^2\,c^3\,d^3\,e^2\,g^2+836\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,d^5\,f^2\right.}{693\,c^3\,d^3}\right.}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^5*g^2 + 198*a^3*c
^2*d^2*e^3*f^2 - 88*a^4*c*d*e^4*f*g)/(693*c^3*d^3) + (x^3*(198*c^5*d^5*f^2
+ 226*a^2*c^3*d^3*e^2*g^2 + 836*a*c^4*d^4*e*f*g))/(693*c^3*d^3) + (2*c^2*d^
2*g^2*x^5)/11 + (2*c*d*g*x^4*(23*a*e*g + 22*c*d*f))/99 + (2*a^2*e^2*x*(297*
c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(693*c^2*d^2) + (2*a*e*x^2*(
a^2*e^2*g^2 + 99*c^2*d^2*f^2 + 110*a*c*d*e*f*g))/(231*c*d)))/(d + e*x)^(1/2
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.703 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(2ae^2g-cd(9ef-7dg)\right)}{63c^2d^2e(d+ex)^{7/2}}$$

```
[Out] -2/63*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2
)/c^2/d^2/e/(e*x+d)^(7/2)+2/9*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
/e/(e*x+d)^(5/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(2ae^2g-cd(9ef-7dg)\right)}{63c^2d^2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*c^2*d^2*e*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}+\frac{1}{9}\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\int$$

$$=\frac{2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63cd(d+ex)^{7/2}}+\frac{2g\left(ade\right.}{}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.51

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\,(cd(9f+7gx)-2aeg)}{63c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*g
*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

fricas [A] time = 1.11, size = 173, normalized size = 1.38

$$\frac{2\left(7\,c^4d^4gx^4+9\,a^3cde^3f-2\,a^4e^4g+\left(9\,c^4d^4f+19\,ac^3d^3eg\right)x^3+3\left(9\,ac^3d^3ef+5\,a^2c^2d^2e^2g\right)x^2+\left(27\,a^2c^2d^2e^2\right.}{63\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

```
[Out] 2/63*(7*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 2*a^4*e^4*g + (9*c^4*d^4*f + 19*a
*c^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f + 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2
*c^2*d^2*e^2*f + a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 5.43Done
```

maple [A] time = 0.00, size = 67, normalized size = 0.54

$$-\frac{2\,(cdx+ae)\left(-7cdgx+2aeg-9cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{63\,(ex+d)^{\frac{5}{2}}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-7*c*d*g*x+2*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/c^2/d^2/(e*x+d)^(5/2)
```

maxima [A] time = 0.57, size = 141, normalized size = 1.13

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f}{7\,cd}+\frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x\right.}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*d^
2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

mupad [B] time = 3.37, size = 134, normalized size = 1.07

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,c^2\,d^2\,g\,x^4}{9}+\frac{2\,a\,e\,x^2\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{21}+\frac{2\,c\,d\,x^3\,\left(19\,a\,e\,g+9\,c\,d\,f\right)}{63}-\frac{2\,a^3\,e^3\,\left(2\,a\,e\,g-9\,c\,d\,f\right)}{63\,c^2\,d^2}+\frac{2\,a^2\,e^2}{}\right.}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*c^2*d^2*g*x^4)/9 + (2*a*
e*x^2*(5*a*e*g + 9*c*d*f))/21 + (2*c*d*x^3*(19*a*e*g + 9*c*d*f))/63 - (2*a^
3*e^3*(2*a*e*g - 9*c*d*f))/(63*c^2*d^2) + (2*a^2*e^2*x*(a*e*g + 27*c*d*f))/
(63*c*d)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2
),x)
```

```
[Out] Timed out
```

3.704 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

fricas [B] time = 0.87, size = 91, normalized size = 1.90

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{7\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 3.17Done
```

maple [A] time = 0.00, size = 50, normalized size = 1.04

$$\frac{2\,(cdx+ae)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{7\,(ex+d)^{\frac{5}{2}}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/7*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c/d/(e*x+d)^(5/2)
```

maxima [A] time = 0.51, size = 60, normalized size = 1.25

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{cdx+ae}}{7\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)/(c*d)
```

mupad [B] time = 3.16, size = 79, normalized size = 1.65

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{6\,a^2\,e^2\,x}{7}+\frac{2\,c^2\,d^2\,x^3}{7}+\frac{2\,a^3\,e^3}{7\,c\,d}+\frac{6\,a\,c\,d\,e\,x^2}{7}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d^
2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

3.705 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)} \, dx$

Optimal. Leaf size=236

$$-\frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right.\right.}{}$$

```
[Out] -2/3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/
2)+2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)-2*(-a*e*g+c*
d*f)^(5/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)+2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {864, 874, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{3g^2(d+ex)^{3/2}}-\frac{2(cdf-aeg)^{5/2}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)),x]
```

```
[Out] (2*(c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt
[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(3*g^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2))/(5*g*(d + e*x)^(5/2)) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/g^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int}{e^2g}$$

$$=-\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{5g(d+ex)^{5/}}$$

$$=\frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{2(cdf-aeg)\left(ade+\left(cd^2\right.\right.}{3g^2(d+e}$$

$$=\frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{2(cdf-aeg)\left(ade+\left(cd^2\right.\right.}{3g^2(d+e}$$

$$=\frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{2(cdf-aeg)\left(ade+\left(cd^2\right.\right.}{3g^2(d+e}$$

Mathematica [A] time = 0.36, size = 145, normalized size = 0.61

$$\frac{((d+ex)(ae+cdx))^{5/2}\left(-\frac{10(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{5/2}(ae+cdx)^{5/2}}+\frac{10(aeg-cdf)(4aeg+cd(gx-3f))}{3g^2(ae+cdx)^2}+2\right)}{5g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*(2 + (10*(-(c*d*f) + a*e*g)*(4*a*e*g + c*d
*(-3*f + g*x)))/(3*g^2*(a*e + c*d*x)^2) - (10*(c*d*f - a*e*g)^(5/2)*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(g^(5/2)*(a*e + c*d*x)^(5
/2))))/(5*g*(d + e*x)^(5/2))
```

fricas [A] time = 1.02, size = 587, normalized size = 2.49

$$\left[\frac{15\left(c^2d^3f^2-2\,acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2\,acde^2fg+a^2e^3g^2\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\,\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [1/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 -
 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x
^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)
```

```
/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 -
 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*g^3), 2
/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2
*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x +
d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (
3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c
^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d))/(e*g^3*x + d*g^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 431, normalized size = 1.83

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^3e^3g^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-45a^2cd\,e^2f\,g^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x)
```

```
[Out] -2/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*a^3*e^3*g^3-45*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*a^2*c*d*e^2*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g-15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*
c^2*d^2*g^2*x^2-11*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+
5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-23*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+35*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a
*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*
f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{\left(f+g\,x\right)\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

3.706 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2} dx$

Optimal. Leaf size=235

$$\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^3\sqrt{d+ex}}-\frac{\left(x\left(ae^2+cd\right.\right.}{g(d+}$$

```
[Out] 5/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)-(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)+5*c*d*(-a*e*g+c*d*f)
^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^(1/2)/(e*x+d)^(1/2))/g^(7/2)-5*c*d*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {862, 864, 874, 205}

$$-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^3\sqrt{d+ex}}+\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{\left(x\left(ae^2+c\right.\right.}{g(d-}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^2),x]
```

```
[Out] (-5*c*d*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*S
qrt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g^
2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d +
e*x)^(5/2)*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx}{2g}$$

$$= \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)\right.}{3g^2(d+ex)}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)\right.}{3g^2(d+ex)}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)\right.}{3g^2(d+ex)}$$

Mathematica [C] time = 0.07, size = 75, normalized size = 0.32

$$\frac{2cd((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (g*(a
*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^2*(d + e*x)^(7/2))
```

fricas [A] time = 1.27, size = 672, normalized size = 2.86

$$\left[-\frac{15\left(c^2d^3f^2-acd^2efg+\left(c^2d^2efg-acde^2g^2\right)x^2+\left(c^2d^2ef^2-acd^2eg^2+\left(c^2d^3-acde^2\right)fg\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\,\log\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^
2 + (c^2*d^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt(-(c
*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c
*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(2*c^
2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(5*c^2*
d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), -1/3*(15*(c^2*d^3*
f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^2 + (c^2*d^2*e*f^2
- a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arc
tan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)) - (2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*
e^2*g^2 - 2*(5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 523, normalized size = 2.23

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^2cd\,e^2g^3x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-30a\,c^2d^2ef\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x)
```

```
[Out] -1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x*a^2*c*d*e^2*g^3-30*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c^2*d^2*e*f*g^2+15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*a^2*c*d*e^2*f*g^2-30*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*c^2*d^2*g^2*x^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2
*x+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+3*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-20*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

3.707 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)\right.}{4g^2(d+ex}$$

```
[Out] -5/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)-
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^2-15/4*
c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(7/2)+15/4*c^2*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)\right.}{4g^2(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^3),x]
```

```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x
)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d
+ e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(4*g^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx}{4g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{4g^2(d+ex)^{3/2}(f+gx)}$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{4g^2(d+ex)^{3/2}(f+gx)}$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{4g^2(d+ex)^{3/2}(f+gx)}$$

Mathematica [C] time = 0.07, size = 79, normalized size = 0.32

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^3),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^3*(d + e*x)^(7/2))
```

fricas [A] time = 0.99, size = 683, normalized size = 2.78

$$\left[\frac{15\left(c^2d^2eg^2x^3+c^2d^3f^2+\left(2c^2d^2efg+c^2d^3g^2\right)x^2+\left(c^2d^2ef^2+2c^2d^3fg\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2adeg-2\ldots}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2
)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^
2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^5*x^3 + d*
f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/4*(15*(c^
2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*
d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*
sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (8*c
^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d
^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 +
2*d*f*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 526, normalized size = 2.14

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a\,c^2d^2e\,g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-15c^3d^3f\,g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c^2*d^2*e*g^3-15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+30*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c^2*d^2*e*f*g^2-30*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g-15*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*c^2*d^2*g^2*x^2+9*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^
2*x-25*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+2*((a*e*g-c*
d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

3.708 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

Optimal. Leaf size=253

$$\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

```
[Out] -5/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^3+5/
8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(1/2)-5/8*c^2*d^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^4),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*
x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g
^2*(d + e*x)^(3/2)*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)/(3*g*(d + e*x)^(5/2)*(f + g*x)^3) + (5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/(8*g^(7/2)*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx}{6g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

Mathematica [A] time = 0.37, size = 171, normalized size = 0.68

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{15c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}}-\frac{\sqrt{g}\left(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2\left(15f^2+40fgx+33g^2x^2\right)\right)}{(f+gx)^3}\right)}{24g^{7/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5*
f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (15
*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d
*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

fricas [B] time = 0.68, size = 1140, normalized size = 4.51

$$\left[-\frac{15\left(c^3d^3eg^3x^4+c^3d^4f^3+\left(3c^3d^3efg^2+c^3d^4g^3\right)x^3+3\left(c^3d^3ef^2g+c^3d^4fg^2\right)x^2+\left(c^3d^3ef^3+3c^3d^4f^2g\right)x\right)\sqrt{-c}}{48\left(cd^2f^4g^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d
*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
 + d*g)*x)) + 2*(15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^2*c*d*e^2*f
```

```
*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(20*c^3
*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d*e*f^3*g^5
+ (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 + (c*d^2 - 3
*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a*e^2)*f^2*g
^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f^3*g^5)*x),
 -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g
 + (c*d^2 + a*e^2)*g*x)) + (15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^
2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2
+ 2*(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d
*e*f^3*g^5 + (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 +
 (c*d^2 - 3*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a
*e^2)*f^2*g^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f
^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 441, normalized size = 1.74

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^3d^3g^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+45c^3d^3f\,g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+45*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3+33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*c^2*d^2*g^2*x^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2
*x+40*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+8*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

3.709 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

Optimal. Leaf size=323

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}$$

```
[Out] -5/24*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^3-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^4+5/
64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(3/2)-5/32*c^2*d^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^2/(e*x+d)^(1/2)+5/64*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^
(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^5),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e
*x]*(f + g*x)^2) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(64*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(4*g*(d + e*x)^(5/2)*(f + g*x)^4) +
 (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(7/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
```

```
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{(5cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx}{8g}$$

$$=-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{24g^2(d+ex)^{3/2}(f+gx)^3}$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.24

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^5*(d + e*x)^(7/2))
```

fricas [B] time = 0.93, size = 1862, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*f^2*g^3 - 56*a^3*c
*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 +
(73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2*d^2*e^2*g^5)*x^2
+ (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*c^2*d^2*e^2*f*g^4
+ 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*e^2*f^4*g^6 + (c^2
*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 + (4*c^2*d^2*e*f^3*g
^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2*(a*c*d^2*e - 2*a^
2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*f*g^9 + 2*(c^2*d^3
 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^8)*x^3 + 2*(2*c^2
*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^6 -
2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g^4 + 4*a^2*d*e^2*f
^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e - a^2*e^3)*f^4*g^
6)*x), -1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c
^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*
d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)
*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*
e^2)*g*x)) + (15*c^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*
f^2*g^3 - 56*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3
*d^3*e*g^5)*x^3 + (73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2
*d^2*e^2*g^5)*x^2 + (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*
c^2*d^2*e^2*f*g^4 + 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*
e^2*f^4*g^6 + (c^2*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 +
(4*c^2*d^2*e*f^3*g^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2
*(a*c*d^2*e - 2*a^2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*
f*g^9 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g
^8)*x^3 + 2*(2*c^2*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c
*d*e^2)*f^4*g^6 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g
^4 + 4*a^2*d*e^2*f^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e
 - a^2*e^3)*f^4*g^6)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 665, normalized size = 2.06

$$\frac{\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\,\left(15c^4d^4g^4x^4 \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right) + 60c^4d^4f\,g^3x^3 \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right) + 90c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^4*d^4*g^4+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^4*d^4*f*g^3+90*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^4*d^4*f^2*g^2+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x*c^4*d^4*f^3*g-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)*c^3*d^3*g^3*x^3+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*c^4*d^4*f^4-118*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2
*e*g^3*x^2+73*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2-1
36*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*g^3*x+36*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+55*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*
e)^(1/2)*a^3*e^3*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^
2*f*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*x+
a*e)^(1/2)/g^3/(a*e*g-c*d*f)/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^5\,(d + e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

3.710 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

Optimal. Leaf size=393

$$\frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}+\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg}$$

```
[Out] -1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^5+3/1
28*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(5/2)-1/16*c^2*d^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^3/(e*x+d)^(1/2)+1/64*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d
)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c
*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^6),x]
```

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*Sqrt[d + e*x
]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*
g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f +
 g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*
x)^(3/2)*(f + g*x)^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*g*
(d + e*x)^(5/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(128*g^(7
/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx}{2g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{8g^2(d+ex)^{3/2}(f+gx)^4}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.20

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^6),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^6*(d + e*x)^(7/2))
```

fricas [B] time = 1.07, size = 2750, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^
6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^
3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2
+ (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f
^4*g^2 - 2*a^2*c^3*d^3*e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*
c*d*e^4*f*g^5 - 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
- 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 +
2*(64*c^5*d^5*f^3*g^3 - 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5
 - 124*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^
3*g^3 - 279*a^2*c^3*d^3*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^3*d^4*f^8*g^4 - 3*a*c^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e
^3*f^5*g^7 + (c^3*d^3*e*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*
f*g^11 - a^3*e^4*g^12)*x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d
^4 - 15*a*c^2*d^2*e^2)*f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 +
 (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d
*e^3*f*g^11 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^3*g^9 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^
3*e*f^6*g^6 - a^3*d*e^3*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*
(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*
x^3 + 5*(c^3*d^3*e*f^7*g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2
*e^2)*f^6*g^6 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2
- a^3*e^4)*f^4*g^8)*x^2 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3
*d^4 - 3*a*c^2*d^2*e^2)*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 +
 (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^7)*x), -1/640*(15*(c^5*d^5*e*g^5*x^6 +
c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^
3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(
c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g
)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
+ a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^4*g^2 - 2*a^2*c^3*d^3*
e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*c*d*e^4*f*g^5 - 128*a^5
*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4
 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 + 2*(64*c^5*d^5*f^3*g^3 -
 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5 - 124*a^3*c^2*d^2*e^3*
g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^3*g^3 - 279*a^2*c^3*d^3
*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c*d*e^4*g^6)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^8*g^4 - 3*a*c
^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e^3*f^5*g^7 + (c^3*d^3*e
*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*f*g^11 - a^3*e^4*g^12)*
x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*
f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 + (3*a^2*c*d^2*e^2 - 5*a
^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d*e^3*f*g^11 + (c^3*d^4
- 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^9 + (3*a
^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^3*e*f^6*g^6 - a^3*d*e^3
*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*(a*c^2*d^3*e - a^2*c*d*
```

```
e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*x^3 + 5*(c^3*d^3*e*f^7*
g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^6 - 3*(2*a*
c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^8)*x^2
 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 + (15*a^2*c*d^2*e^2 - a^
3*e^4)*f^5*g^7)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 924, normalized size = 2.35

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^5d^5g^5x^5\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+75c^5d^5f\,g^4x^4\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+150c^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x)
```

```
[Out] -1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^5*c^5*d^5*g^5+75*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^5*d^5*f*g^4+150*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^5*d^5*f^2*g^3+150*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^5*d^5*f^3*g^2-15*((a*e*g-c*d*f)*g)^(1/2)*
(c*d*x+a*e)^(1/2)*c^4*d^4*g^4*x^4+75*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*g)*x*c^5*d^5*f^4*g+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
*a*c^3*d^3*e*g^4*x^3-70*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^4*d^4*f
*g^3*x^3+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^5*d^5*f^
5+248*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*g^4*x^2-466
*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f*g^3*x^2+128*((a*e*
g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^4*d^4*f^2*g^2*x^2+336*((a*e*g-c*d*f)*
g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*g^4*x-512*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f*g^3*x+46*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a
*e)^(1/2)*a*c^3*d^3*e*f^2*g^2*x+70*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*c^4*d^4*f^3*g*x+128*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*e^4*g^4
-176*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*f*g^3+8*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f^3*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^
5/g^3/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^6\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

3.711 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

Optimal. Leaf size=463

$$\frac{5c^6d^6\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}+\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}$$

```
[Out] -1/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^5-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^6+5/
512*c^6*d^6*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(7/2)-1/32*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^4/(e*x+d)^(1/2)+1/192*c^3*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x
+d)^(1/2)+5/768*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g
+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)+5/512*c^5*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192g^3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^7),x]
```

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x
]*(f + g*x)^4) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192
*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^4*d^4*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(768*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (5*c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*g^
3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)*(f + g*x)^5) - (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(6*g*(d + e*x)^(5/2)*(f + g*x)^6) + (5*c^
6*d^6*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*
d*f - a*e*g]*Sqrt[d + e*x])])/(512*g^(7/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx}{12g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4} - \frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^5}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.17

$$\frac{2c^6d^6((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},7;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^7),x]
```

```
[Out] (2*c^6*d^6*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 7, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^7*(d + e*x)^(7/2))
```

fricas [B] time = 1.42, size = 3872, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="fricas")
```

```
[Out] [1/3072*(15*(c^6*d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7
*g^6)*x^6 + 3*(5*c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*
f^3*g^3 + 3*c^6*d^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g
^3)*x^3 + 3*(2*c^6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 +
6*c^6*d^7*f^5*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f +
2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
 (e*f + d*g)*x)) - 2*(15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*
d^4*e^2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^
5 - 896*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5
*e*g^7)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*
e^2*g^7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4
*d^4*e^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a
*c^5*d^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^
6 + 72*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*
g^3 - 1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^
4*c^2*d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6
*a^2*c^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c
^4*d^4*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 -
 4*a^3*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^
4*g^14 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2
*d^2*e^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^
3*c*d^2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4
*f*g^13 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^
2*c^2*d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 -
(8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^
4*d*e^4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e
 - 2*a^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g
^10 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6
+ 4*a^4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3
*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^5*g^9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9
*g^5 + 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*
a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*
d*e^4)*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f
^10*g^4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6
*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d
*e^4)*f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x), -1/1536*(15*(c^6*
d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7*g^6)*x^6 + 3*(5*
c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*f^3*g^3 + 3*c^6*d
^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g^3)*x^3 + 3*(2*c^
6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 + 6*c^6*d^7*f^5*g)*
x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
```

```
x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
a*e^2)*g*x)) + (15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*d^4*e^
2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^5 - 89
6*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5*e*g^7
)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*e^2*g^
7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4*d^4*e
^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a*c^5*d
^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^6 + 72
*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*g^3 -
1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^4*c^2*
d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6*a^2*c
^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c^4*d^4
*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 - 4*a^3
*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^4*g^14
 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e
^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^3*c*d^
2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4*f*g^1
3 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*
d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 - (8*a^3
*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^4*d*e^
4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e - 2*a
^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^10 -
4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6 + 4*a^
4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3*d^4*e
 - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^
9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9*g^5 +
 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*a*c^3*
d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)
*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f^10*g^
4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6*(4*a*
c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 1261, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x)
```

```
[Out] 1/1536*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^6*d^6*f^6-15*x^5*c^5*d^5*g^5*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-85*x^4*c^5*d^5*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)-198*x^3*c^5*d^5*f^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)+198*x^2*c^5*d^5*f^3*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+85*
x*c^5*d^5*f^4*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+10*x^4*a*c^4*d^4*
e*g^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^6*c^6*d^6*g^6+90*arctanh((c*d*x+a*e)^(1/2)/(
```

```
(a*e*g-c*d*f)*g)^(1/2)*g)*x^5*c^6*d^6*f*g^5+225*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^6*d^6*f^2*g^4+300*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^6*d^6*f^3*g^3+225*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^6*d^6*f^4*g^2+90*arctanh((c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^6*d^6*f^5*g-8*x^3*a^2*c^3*d^3*e^2*g^5*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-432*x^2*a^3*c^2*d^2*e^3*g^5*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*x*a^4*c*d*e^4*g^5*(c*d*x+a*e)^(1/2)*
((a*e*g-c*d*f)*g)^(1/2)+640*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*c
*d*e^4*f*g^4-432*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c^2*d^2*e^3*
f^2*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^3*d^3*e^2*f^3*g^2
+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^4*d^4*e*f^4*g+56*x^3*a*c^
4*d^4*e*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1272*x^2*a^2*c^3*d^
3*e^2*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1188*x^2*a*c^4*d^4*e*
f^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1696*x*a^3*c^2*d^2*e^3*f*
g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1272*x*a^2*c^3*d^3*e^2*f^2*g^
3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*x*a*c^4*d^4*e*f^3*g^2*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-256*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)
^(1/2)*a^5*e^5*g^5+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^5*d^5*f^5
)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^6/g^3/(a*e*g-c*d*f)/(a^2*e^
2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^7\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**7,x)
```

```
[Out] Timed out
```

3.712 $\int \frac{\sqrt{d+ex}\,(f+gx)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=313

$$\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}$$

```
[Out] 5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/12*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+5/8*(-a*e*g+c*d*f)^2*
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {870, 891, 63, 217, 206}

$$\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{12c^2d^2\sqrt{d+ex}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*c^3*d^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*Sqrt[d + e*x]) + ((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[
g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{\left(5\left(cde^2f+cd^2eg-e\left(cd^2+a\right.\right.\right.}{6}$$

$$= \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade}}{3c}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

Mathematica [A] time = 0.61, size = 269, normalized size = 0.86

$$\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{ae+cdx}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\left(15a^2e^2g^2-10acdeg(4f+gx)+c^2d^2\left(33f^2+26fg\right.\right.\right.}{24c^{7/2}d^{7/2}\sqrt{g}\,\sqrt{(d+ex)(ae+cdx)}\,\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*(15*a^2*e^2*g^2 - 10*a
*c*d*e*g*(4*f + g*x) + c^2*d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2)) + 15*Sqrt[c
*d]*(c*d*f - a*e*g)^(5/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*c^(7/2)*d^(7/2)*Sqrt[g]*Sqrt[(a*e
 + c*d*x)*(d + e*x)]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
```

fricas [A] time = 2.70, size = 841, normalized size = 2.69

$$\left[\frac{4\left(8\,c^3d^3g^3x^2+33\,c^3d^3f^2g-40\,ac^2d^2efg^2+15\,a^2cde^2g^3+2\left(13\,c^3d^3fg^2-5\,ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*
a^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g*x + c^4*d^5*g),
1/48*(2*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*a
^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^3
 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e
*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-
c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sq
rt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g*x + c^4*d^5*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 511, normalized size = 1.63

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^3e^3g^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-45a^2cd\,e^2f\,g^2\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/
2))*a^3*e^3*g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2
*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)
*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^3-16*x^2*c^2*d^2*g^2*((g*x+f)*(c*d
*x+a*e))^(1/2)*(d*g*c)^(1/2)+20*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*x
*a*c*d*e*g^2-52*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*x*c^2*d^2*f*g-30*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2+80*((g*x+f)*(c*d*x+a*
e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*f*g-66*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c^3/d^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{5/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.713 $\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=244

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+$$

```
[Out] 3/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {870, 891, 63, 217, 206}

$$\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(4*c^2*d^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])])/(4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
```

```
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{\left(3\left(cde^2f+cd^2eg-e\left(cd^2+a\right.\right.\right.}{4}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

Mathematica [A] time = 0.47, size = 234, normalized size = 0.96

$$\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{ae+cdx}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\,(cd(5f+2gx)-3aeg)+3\sqrt{cd}\,(cdf-aeg)^{3/2}\sinh^{-1}\right.}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{(d+ex)(ae+cdx)}\,\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*(-3*a*e*g + c*d*(5*f +
 2*g*x)) + 3*Sqrt[c*d]*(c*d*f - a*e*g)^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(4*c^(5/2)*d^(5/2)
*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)
])
```

fricas [A] time = 2.47, size = 655, normalized size = 2.68

$$\left[\frac{4\left(2\,c^2d^2g^2x+5\,c^2d^2fg-3\,acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}\,+3\left(c^2d^3f^2-2\,acd^2efg+\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g*x
 + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c
^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2
*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g*x + c^3*
d^4*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 328, normalized size = 1.34

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-6acdefg\ln\left(\frac{2cdgx+ae\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/8*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
```

```
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*(c
*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*x*c*d*g-6*(c*d*g)^(1/2)*((g*x+f)*(c
*d*x+a*e))^(1/2)*a*e*g+10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/
(e*x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(f+gx\right)^{\frac{3}{2}}}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.714 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] (-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1
/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c/d/(e*x+d)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {870, 891, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e
*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
```

```
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right.}{2c}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\right.\right.}{2cde^2\sqrt{ade}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\right.\right.}{c^2}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\right.\right.}{c^2d^2e^2}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{ade+\left(cd^2+\right.}}$$

Mathematica [A] time = 0.19, size = 213, normalized size = 1.26

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}+\sqrt{cd}\sqrt{cdf-aeg}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)] + Sqrt[c*d]*Sqrt[c*d*f
 - a*e*g]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sq
rt[c*d*f - a*e*g])]))/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)
]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
```

fricas [A] time = 2.51, size = 521, normalized size = 3.08

$$\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\,cdg-\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{cdg}\,\log\left(-\frac{8c^2d^2eg^2x^3\cdots}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g*x + c^2*d^3*g), 1/2*(2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g - (
c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^2*d^2*e*g*x + c^2*d^3*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 201, normalized size = 1.19

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(aeg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-cdf\ln\left(\frac{2cdgx+aeg+cdf+2\cdots}{2\cdots}\right)\cdots\right)}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/2*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*
a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*c*d*f-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(e*
x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c/d/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{gx+f}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.715 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a
*e)^(1/2)*(e*x+d)^(1/2)/c^(1/2)/d^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {891, 63, 217, 206}

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\ &= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\ &= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\ &= \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 160, normalized size = 1.52

$$\frac{2\sqrt{cd}\sqrt{d+ex}\sqrt{ae+cdx}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (2*Sqrt[c*d]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[(c*d*
(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 2.02, size = 343, normalized size = 3.27

$$\left[\frac{\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2cdgx+cdf+aeg)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+(c^2d^3+a\right.}{ex+d}\right.}{2cdg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
+ a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d
*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d))/(c*d*g), -sqrt(-c*d*
g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e
*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*
d^2 + a*e^2)*g)*x))/(c*d*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 120, normalized size = 1.14

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)}{\sqrt{ex+d}\,\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))/(c*d*g)^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

3.716 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d + e
*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,\sqrt{f+gx}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])
```

fricas [B] time = 0.89, size = 114, normalized size = 1.87

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}}{cd^2f^2-adefg+(cdefg-ae^2g^2)x^2+(cdef^2-adeg^2+(cd^2-ae^2)fg)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(
c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^
2 + (c*d^2 - a*e^2)*f*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.03

$$-\frac{2\,(cdx+ae)\sqrt{ex+d}}{\sqrt{gx+f}\;\left(aeg-cdf\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/(g*x+f)^(1/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(e*x+d)^(1/2)/(c*d*e*x^2+a*e^2*x
+c*d^2*x+a*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(3/2)), x)
```

mupad [B] time = 4.64, size = 100, normalized size = 1.64

$$-\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(x\,\sqrt{f+g\,x}-\frac{\sqrt{f+g\,x}\,\left(c\,d^2\,f-a\,d\,e\,g\right)}{a\,e^2\,g-c\,d\,e\,f}\right)\,\left(a\,e^2\,g-c\,d\,e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((x*(f +
 g*x)^(1/2) - ((f + g*x)^(1/2)*(c*d^2*f - a*d*e*g))/(a*e^2*g - c*d*e*f))*(a
*e^2*g - c*d*e*f))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x)
```

3.717 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(e
*x+d)^(1/2)+4/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
2/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{(2cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2\right.}}}{3(cdf-aeg}$$

$$=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{4cd\sqrt{ade+\left(cd^2+ae^2\right)x}}{3(cdf-aeg)^2\sqrt{d+ex}\,\sqrt{}}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\,(cd(3f+2gx)-aeg)}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
- a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

fricas [B] time = 0.71, size = 288, normalized size = 2.23

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cd\right.}{3\left(c^2d^3f^4-2\,acd^2ef^3g+a^2de^2f^2g^2+\left(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4\right)x^3+\left(2\,c^2d^2ef^3g+a^2de^2g^4+\left(c^2d^3-4\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^4 - 2*a*c*d^2*e*f^3*g + a^2*d*e^2
*f^2*g^2 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^3*g^4)*x^3 + (2*c
^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^2 - 2*(a*c*d
^2*e - a^2*e^3)*f*g^3)*x^2 + (c^2*d^2*e*f^4 + 2*a^2*d*e^2*f*g^3 + 2*(c^2*d^
3 - a*c*d*e^2)*f^3*g - (4*a*c*d^2*e - a^2*e^3)*f^2*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 98, normalized size = 0.76

$$-\frac{2\,(cdx+ae)\left(-2cdgx+aeg-3cdf\right)\sqrt{ex+d}}{3\left(gx+f\right)^{\frac{3}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-2*c*d*g*x+a*e*g-3*c*d*f)*(e*x+d)^(1/2)/(g*x+f)^(3/2)/(a^
2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(5/2)), x)
```

mupad [B] time = 4.90, size = 147, normalized size = 1.14

$$-\frac{\left(\frac{\left(2\,a\,e\,g-6\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{d\,f\,\sqrt{f+g\,x}}{e\,g}+\frac{x\,\sqrt{f+g\,x}\,\left(d\,g+e\,f\right)}{e\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((2*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*e*g*(a*e*g - c*d*f)^2) - (4*c*d
*x*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (d*f*(f + g*x)^(1/2))/(e*g) + (x*(f
 + g*x)^(1/2)*(d*g + e*f))/(e*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.718 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(e
*x+d)^(1/2)+8/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)
^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)+16/15*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(5/2)) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(15*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (16*c^2*d^2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*
Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{(4cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2 \cdots}}}{5(cdf-aeg \cdots}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+(cd^2+ae^2)\cdots}}{15(cdf-aeg)^2\sqrt{d+ex}(\cdots}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+(cd^2+ae^2)\cdots}}{15(cdf-aeg)^2\sqrt{d+ex}(\cdots}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2e^2g^2-2acdeg(5f+2gx)+c^2d^2\left(15f^2+20fgx+8g^2x^2\right)\right)}{15\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x)
 + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d +
 e*x]*(f + g*x)^(5/2))
```

fricas [B] time = 1.01, size = 572, normalized size = 2.89

$$\overline{15\left(c^3d^4f^6-3\,ac^2d^3ef^5g+3\,a^2cd^2e^2f^4g^2-a^3de^3f^3g^3+\left(c^3d^3ef^3g^3-3\,ac^2d^2e^2f^2g^4+3\,a^2cde^3fg^5-a^3e^4g^6\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 3*a^2*e^2*g^2 +
 4*(5*c^2*d^2*f*g - a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^6 - 3*a*c^2*d^3*e*f^5*g + 3*a^2
*c*d^2*e^2*f^4*g^2 - a^3*d*e^3*f^3*g^3 + (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e
^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^4 + (3*c^3*d^3*e*f^4*g^2
- a^3*d*e^3*g^6 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^2*g^4 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^5)*x^3 + 3*(c^3*d^3*
e*f^5*g - a^3*d*e^3*f*g^5 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(a*c^2*
d^3*e - a^2*c*d*e^3)*f^3*g^3 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x^2 + (
c^3*d^3*e*f^6 - 3*a^3*d*e^3*f^2*g^4 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g - 3
*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^2 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^
3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2 \left(c d x+a e\right) \left(8 g^{2} x^{2} c^{2} d^{2}-4 a c d e g^{2} x+20 c^{2} d^{2} f g x+3 a^{2} e^{2} g^{2}-10 a c d e f g+15 f^{2} c^{2} d^{2}\right) \sqrt{e x+d}}{15 \left(g x+f\right)^{\frac{5}{2}} \left(a^{3} e^{3} g^{3}-3 a^{2} c d e^{2} f g^{2}+3 a c^{2} d^{2} e f^{2} g-f^{3} c^{3} d^{3}\right) \sqrt{c d e x^{2}+a e^{2} x+c d^{2} x+a d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2
*e^2*g^2-10*a*c*d*e*f*g+15*c^2*d^2*f^2)*(e*x+d)^(1/2)/(g*x+f)^(5/2)/(a^3*e^
3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2
*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(7/2)), x)
```

mupad [B] time = 5.17, size = 242, normalized size = 1.22

$$-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^2\,d^2\,x^2\,\sqrt{d+e\,x}}{15\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c\,d\,x\,\left(a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3\,\sqrt{f+g\,x}+\frac{d\,f^2\,\sqrt{f+g\,x}}{e\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(d\,g+2\,e\,f\right)}{e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(2\,d\,g+e\,f\right)}{e\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((d + e*x)^(1/2)*(6*a^2*e^2*g^2 + 30*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(15*
e*g^2*(a*e*g - c*d*f)^3) + (16*c^2*d^2*x^2*(d + e*x)^(1/2))/(15*e*(a*e*g -
c*d*f)^3) - (8*c*d*x*(a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*e*g*(a*e*g - c*
d*f)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2
) + (d*f^2*(f + g*x)^(1/2))/(e*g^2) + (x^2*(f + g*x)^(1/2)*(d*g + 2*e*f))/(
e*g) + (f*x*(f + g*x)^(1/2)*(2*d*g + e*f))/(e*g^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.719 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{35\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(7/2)/(e
*x+d)^(1/2)+12/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^2/(g*x+f)^(5/2)/(e*x+d)^(1/2)+16/35*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/35*c^3*d^3*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(
1/2)
```

Rubi [A] time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{872, 860\}$

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{35\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(7/2)) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(35*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (16*c^2*d^2*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d*f - a*e*g)^3*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(35*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{(6cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)}}}{7(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

Mathematica [A] time = 0.12, size = 152, normalized size = 0.57

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-5a^3e^3g^3+3a^2cde^2g^2(7f+2gx)-ac^2d^2eg\left(35f^2+28fgx+8g^2x^2\right)+c^3d^3\left(35f^3+70f^2\right.\right.}{35\sqrt{d+ex}\,(f+gx)^{7/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f +
 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3 +
 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d + e
*x]*(f + g*x)^(7/2))
```

fricas [B] time = 1.04, size = 953, normalized size = 3.57

$$\overline{35\left(c^4d^5f^8-4\,ac^3d^4ef^7g+6\,a^2c^2d^3e^2f^6g^2-4\,a^3cd^2e^3f^5g^3+a^4de^4f^4g^4+\left(c^4d^4ef^4g^4-4\,ac^3d^3e^2f^3g^5+6\,a^2c^2d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g + 21*a^2*c
*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c^2*d^2*e*g^3)*x^2 +
2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^
8 - 4*a*c^3*d^4*e*f^7*g + 6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*f^5*g
^3 + a^4*d*e^4*f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a
^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4*c^4*d^
4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^4 - 4*(a*c
^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^
4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g^7)*x^4 + 2*(3*c^4*d^4*e*f^6*g^
2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 - 2*(4*a*c^3*
d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3
*g^5 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d^4*e*f^7*g +
3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 - 12*(a*c^3*d^4
*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g
```

```
^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x^2 + (c^4*d^4*e*f^8 + 4*a^4*d*
e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^3*e^2)*f^7*g - 2*(8*a*c^3*d^4*e - 3*a^2*
c^2*d^2*e^3)*f^6*g^2 + 4*(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^3 - (16*a^
3*c*d^2*e^3 - a^4*e^5)*f^4*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2\left(cdx+ae\right)\left(-16g^{3}x^{3}c^{3}d^{3}+8a\,c^{2}d^{2}e\,g^{3}x^{2}-56c^{3}d^{3}f\,g^{2}x^{2}-6a^{2}cd\,e^{2}g^{3}x+28a\,c^{2}d^{2}ef\,g^{2}x-70c^{3}d^{3}f^{2}gx+5a\right.}{35\left(gx+f\right)^{\frac{7}{2}}\left(g^{4}e^{4}a^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g^{2}-4a\,c^{3}d^{3}e\,f^{3}g+f^{4}c^{4}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g
^2*x^2-6*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*
e^3*g^3-21*a^2*c*d*e^2*f*g^2+35*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)*(e*x+d)^(
1/2)/(g*x+f)^(7/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g
^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(9/2)), x)
```

mupad [B] time = 5.51, size = 357, normalized size = 1.34

$$-\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(10\,a^{3}\,e^{3}\,g^{3}-42\,a^{2}\,c\,d\,e^{2}\,f\,g^{2}+70\,a\,c^{2}\,d^{2}\,e\,f^{2}\,g-70\,c^{3}\,d^{3}\,f^{3}\right)}{35\,e\,g^{3}\,{\left(a\,e\,g-c\,d\,f\right)}^{4}}-\frac{32\,c^{3}\,d^{3}\,x^{3}\,\sqrt{d+e\,x}}{35\,e\,{\left(a\,e\,g-c\,d\,f\right)}^{4}}-\frac{4\,c\,d}{}\right.}{x^{4}\,\sqrt{f+g\,x}+\frac{d\,f^{3}\,\sqrt{f+g\,x}}{e\,g^{3}}+\frac{x^{3}\,\sqrt{f+g\,x}\,\left(d\,g+3\,e\,f\right)}{e\,g}+\frac{3\,f\,x^{2}\,\sqrt{f+g\,x}\,\left(d\,g+e\,f\right)}{e\,g^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(9/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(10*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 70*a*c^2*d^2*e*f^2*g - 42*a^2*c*d*e^2*f*g^2))/(35
*e*g^3*(a*e*g - c*d*f)^4) - (32*c^3*d^3*x^3*(d + e*x)^(1/2))/(35*e*(a*e*g -
 c*d*f)^4) - (4*c*d*x*(d + e*x)^(1/2)*(3*a^2*e^2*g^2 + 35*c^2*d^2*f^2 - 14*
```

```
a*c*d*e*f*g))/(35*e*g^2*(a*e*g - c*d*f)^4) + (16*c^2*d^2*x^2*(a*e*g - 7*c*d
*f)*(d + e*x)^(1/2))/(35*e*g*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (d
*f^3*(f + g*x)^(1/2))/(e*g^3) + (x^3*(f + g*x)^(1/2)*(d*g + 3*e*f))/(e*g) +
 (3*f*x^2*(f + g*x)^(1/2)*(d*g + e*f))/(e*g^2) + (f^2*x*(f + g*x)^(1/2)*(3*
d*g + e*f))/(e*g^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.720 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{15\sqrt{g}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{15g\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-}{4c^3d^3\sqrt{d+ex}}$$

```
[Out] -2*(g*x+f)^(5/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/2*g*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+15/4*g*(-a*e*g+c*d*f)*(g*x+f)^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{5g(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}}+\frac{15g\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4c^3d^3\sqrt{d+ex}}+\frac{15\sqrt{g}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*g*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c^3*d^3*Sqrt[d + e*x]) + (5*g*(f + g*x)^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*Sqrt[d + e*x]) + (15*Sqrt[g
]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a
*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}dx = -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(5g)\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2 \cdots\right.}}{2c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\cdots}}{4c^3d^3\sqrt{d+\cdots}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\cdots}}{4c^3d^3\sqrt{d+\cdots}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\cdots}}{4c^3d^3\sqrt{d+\cdots}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\cdots}}{4c^3d^3\sqrt{d+\cdots}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\cdots}}{4c^3d^3\sqrt{d+\cdots}}$$

Mathematica [C] time = 0.12, size = 100, normalized size = 0.33

$$-\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2)*Hypergeometric2F1[-5/2, -1/2, 1/2, (g*(a*
e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(
f + g*x))/(c*d*f - a*e*g))^(5/2))
```

fricas [A] time = 2.36, size = 971, normalized size = 3.23

$$\left[\frac{4\left(2\,c^2d^2g^2x^2-8\,c^2d^2f^2+25\,acdefg-15\,a^2e^2g^2+\left(9\,c^2d^2fg-5\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x), 1/8*(2*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.47Unable to transpose Error: Bad Argument Value

maple [B] time = 0.04, size = 648, normalized size = 2.15

$$\left(15a^2cd\,e^2g^3x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30a\,c^2d^2ef\,g^2x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)

[Out] 1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a^2*c*d*e^2*g^3-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a*c^2*d^2*e*f*g^2+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^3*d^3*f^2*g+15*a^3*e^3*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+15*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2-10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+18*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+

```
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-16*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)/c^
3/d^3/(e*x+d)^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{5/2}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.721 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=227

$$\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{c}}{cd\sqrt{x\left(ae^2\right.}}$$

```
[Out] -2*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
3*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^
(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*g*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{c}}{cd\sqrt{x\left(ae^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3*g*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(c^2*d^2*Sqrt[d + e*x]) + (3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*
x])])/(c^(5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
```

```
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(3g)\int\frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

Mathematica [C] time = 0.08, size = 100, normalized size = 0.44

$$-\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (g*(a*
e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(
f + g*x))/(c*d*f - a*e*g))^(3/2))
```

fricas [A] time = 2.33, size = 725, normalized size = 3.19

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx-2\,cdf+3\,aeg\right)\sqrt{ex+d}\,\sqrt{gx+f}\,-3\left(acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^
2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)
*g)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*
f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4
*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4
*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(
c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*a*e*g)*sqrt(e*x
+ d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^
2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c
^2*d^2*e^2)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.26Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.03, size = 396, normalized size = 1.74

$$-\frac{\left(3acde\,g^2x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^2d^2fgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3a^2e^2g^2\,\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a*c*d*e*g^2-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^2*d^2*f*g+3*a^2*e
^2*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))-3*a*c*d*e*f*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-2*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e
*g+4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/(c
*d*g)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.722 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*g^(1/2)*
(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {866, 891, 63, 217, 206}

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
```

```
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,d\ldots}{cd}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(g\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{\ldots}{\sqrt{ae+cd\ldots}}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+\ldots}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\mathrm{Subst}\left(\ldots\right)}{c^2d^2\sqrt{ade+\left(cd^2\ldots\right.}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\mathrm{Subst}\left(\ldots\right)}{c^2d^2\sqrt{ade+\left(cd^2+ae\ldots\right.}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\ldots}{c^{3/2}d^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)\ldots}}$$

Mathematica [A] time = 0.37, size = 176, normalized size = 1.09

$$\frac{2\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)-(cd)^{3/2}(f+gx)\right)}{(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-((c*d)^(3/2)*(f + g*x)) + Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[c
*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSi
nh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*
g])]))/((c*d)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 2.37, size = 569, normalized size = 3.53

$$\left[\frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)\sqrt{\frac{g}{cd}}\log\left(-\frac{8c^2d^2eg^2x^3 + c^2d^3f^2 + 6acd^2efg + a^2de^2g^2 + 8\left(c^2d^2efg + \left(c^2d^3 + acde^2\right)g^2\right)x^2 + 4\left(2c^2d^2gx + c^2d^2f + \cdots\right.}{\cdots}\right.}{2\left(c^2d^2e\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2
*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f
*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*
e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c
*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^
3 + a*c*d*e^2)*x), -((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d))
*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*
x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2
)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.34Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 210, normalized size = 1.30

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\,\left(cdgx\ln\left(\frac{2cdgx + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right) + aeg\ln\left(\frac{2cdgx + aeg + cdf + 2\sqrt{(g\cdots}}{2\sqrt{cd\cdots}}\right.\right.}{\sqrt{cdg}\,\left(cdx + ae\right)\sqrt{(gx+f)\,(cdx+ae)}\,\sqrt{ex+d}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] (g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c*d
*g+a*e*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2)/(c*d*x+a*e)/((g*x+f)*(c*d*x+a*e))^(1/2)/d/c/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^{\frac{3}{2}}\sqrt{f+gx}}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x
)
```

3.723 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

```
[Out] -2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {860}

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.82

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

fricas [B] time = 0.98, size = 125, normalized size = 2.05

$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{acd^2ef-a^2de^2g+(c^2d^2ef-acde^2g)x^2+((c^2d^3+acde^2)f-(acd^2e+a^2e^3)g)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 +
a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.03

$$\frac{2\sqrt{gx+f}\,(cdx+ae)(ex+d)^{\frac{3}{2}}}{(aeg-cdf)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(e*x+d)^(3/2)/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sq
rt(g*x + f)), x)
```

mupad [B] time = 4.68, size = 147, normalized size = 2.41

$$\frac{\left(\frac{2f\sqrt{d+ex}}{cde(aeg-cdf)}+\frac{2gx\sqrt{d+ex}}{cde(aeg-cdf)}\right)\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}{x^2\sqrt{f+gx}+\frac{a\sqrt{f+gx}}{c}+\frac{x\sqrt{f+gx}\,(cd^2+ae^2)}{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((2*f*(d + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)) + (2*g*x*(d + e*x)^(1/2))/(
c*d*e*(a*e*g - c*d*f)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2
*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1/2)*(a*e^2 + c*d^
2))/(c*d*e))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)(ae+cdx))^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)),
 x)
```

3.724 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-4*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(e*
x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(2g)}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{4g\sqrt{\cdots}}{(cdf\cdots}$$

Mathematica [**A**] time = 0.05, size = 64, normalized size = 0.52

$$-\frac{2\sqrt{d+ex}\,(aeg+cd(f+2gx))}{\sqrt{f+gx}\,\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [**B**] time = 0.89, size = 325, normalized size = 2.62

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2\cdots\right.}}{ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+\left(c^3d^3ef^2g-2\,ac^2d^2e^2fg^2+a^2cde^3g^3\right)x^3+\left(c^3d^3ef^3+\left(c^3d^4-ac^2d^2e^2\right)f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(e*x + d)*sqrt(g*x + f)/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*
d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x
^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^
2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c
^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*
d^2*e^2 - a^3*e^4)*f*g^2)*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.01, size = 97, normalized size = 0.78

$$-\frac{2\,(cdx+ae)\left(2cdgx+aeg+cdf\right)(ex+d)^{\frac{3}{2}}}{\sqrt{gx+f}\,\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(2*c*d*g*x+a*e*g+c*d*f)*(e*x+d)^(3/2)/(g*x+f)^(1/2)/(a^2*e^2
*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(3/2)), x)
```

mupad [B] time = 4.98, size = 151, normalized size = 1.22

$$-\frac{\left(\frac{4gx\sqrt{d+ex}}{e\left(aeg-cdf\right)^2}+\frac{\left(2aeg+2cdf\right)\sqrt{d+ex}}{cde\left(aeg-cdf\right)^2}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2\sqrt{f+gx}+\frac{a\sqrt{f+gx}}{c}+\frac{x\sqrt{f+gx}\left(cd^2+ae^2\right)}{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -(((4*g*x*(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^2) + ((2*a*e*g + 2*c*d*f)*(d
+ e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1
/2)*(a*e^2 + c*d^2))/(c*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.725 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-8/3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(
g*x+f)^(3/2)/(e*x+d)^(1/2)-16/3*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {868, 872, 860}

$$-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (8*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) - (16*c*d*g*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
```

```
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.55

$$-\frac{2\sqrt{d+ex}\left(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2\left(3f^2+12fgx+8g^2x^2\right)\right)}{3(f+gx)^{3/2}\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3*
f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d +
e*x)]*(f + g*x)^(3/2))
```

fricas [B] time = 1.02, size = 649, normalized size = 3.38

$$-\frac{\cdots}{3\left(ac^3d^4ef^5-3\,a^2c^2d^3e^2f^4g+3\,a^3cd^2e^3f^3g^2-a^4de^4f^2g^3+\left(c^4d^4ef^3g^2-3\,ac^3d^3e^2f^2g^3+3\,a^2c^2d^2e^3fg^4-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*c^2*d^2*g^2*x^2 + 3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - a^2*e^2*g^2 + 4*(
3*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3
*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d
^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^
4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*
d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3
 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*
d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d
^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 -
(2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c
^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^
2*e^3 - a^4*e^5)*f^2*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 168, normalized size = 0.88

$$-\frac{2 \left(c d x+a e\right) \left(-8 g^{2} x^{2} c^{2} d^{2}-4 a c d e \,g^{2} x -12 c^{2} d^{2} f g x +a^{2} e^{2} g^{2}-6 a c d e f g -3 f^{2} c^{2} d^{2}\right) \left(e x +d \right)^{\frac{3}{2}}}{3 \left(g x +f \right)^{\frac{3}{2}} \left(a^{3} e^{3} g^{3}-3 a^{2} c d \,e^{2} f \,g^{2}+3 a \,c^{2} d^{2} e \,f^{2} g -f^{3} c^{3} d^{3}\right) \left(c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e
^2*g^2-6*a*c*d*e*f*g-3*c^2*d^2*f^2)*(e*x+d)^(3/2)/(g*x+f)^(3/2)/(a^3*e^3*g^
3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2*x+c
*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(5/2)), x)
```

mupad [B] time = 5.33, size = 268, normalized size = 1.40

$$\frac{\left(\frac{8\,x\,\left(a\,e\,g+3\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{\sqrt{d+e\,x}\,\left(-2\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2\right)}{3\,c\,d\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c\,d\,g\,x^2\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3\,\sqrt{f+g\,x}+\frac{a\,f\,\sqrt{f+g\,x}}{c\,g}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,f\,d^2+a\,g\,d\,e+a\,f\,e^2\right)}{c\,d\,e\,g}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+c\,f\,d\,e+a\,g\,e^2\right)}{c\,d\,e\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((8*x*(a*e*g + 3*c*d*f)*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3) + ((d + e
*x)^(1/2)*(6*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 12*a*c*d*e*f*g))/(3*c*d*e*g*(a*e
*g - c*d*f)^3) + (16*c*d*g*x^2*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3))*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2) + (a*f*(f
 + g*x)^(1/2))/(c*g) + (x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + a*d*e*g))/(c
*d*e*g) + (x^2*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + c*d*e*f))/(c*d*e*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.726 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)^2}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-12/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/
(g*x+f)^(5/2)/(e*x+d)^(1/2)-16/5*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)-32/5*c^2*d^2*g*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {868, 872, 860}

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (12*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(5*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) - (16*c*d*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g
*x)^(3/2)) - (32*c^2*d^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

Mathematica [A] time = 0.09, size = 150, normalized size = 0.57

$$-\frac{2\sqrt{d+ex}\left(a^3e^3g^3-a^2cde^2g^2(5f+2gx)+ac^2d^2eg\left(15f^2+20fgx+8g^2x^2\right)+c^3d^3\left(5f^3+30f^2gx+40fg^2x^2\cdots\right.\right.}{5(f+gx)^{5/2}\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(5*f + 2*g*x) + a*c^2*d^2*
e*g*(15*f^2 + 20*f*g*x + 8*g^2*x^2) + c^3*d^3*(5*f^3 + 30*f^2*g*x + 40*f*g^
2*x^2 + 16*g^3*x^3)))/(5*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f
 + g*x)^(5/2))
```

fricas [B] time = 0.99, size = 1062, normalized size = 4.05

$$-\frac{\cdots}{5\left(ac^4d^5ef^7-4a^2c^3d^4e^2f^6g+6a^3c^2d^3e^3f^5g^2-4a^4cd^2e^4f^4g^3+a^5de^5f^3g^4+\left(c^5d^5ef^4g^3-4ac^4d^4e^2f^3g^4+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 5*a^2*c*d
*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*(1
5*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*f^7
- 4*a^2*c^3*d^4*e^2*f^6*g + 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^4*f^4
```

```
*g^3 + a^5*d*e^5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g^4 + 6
*a^2*c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7)*x^5 +
 (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3 - 2*(2*a*c^4*d
^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f
^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^6 + (a^4*c*d^2*e^4 + a^5*e^6
)*g^7)*x^4 + (3*c^5*d^5*e*f^6*g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3*a*c^4*d^4*
e^2)*f^5*g^2 - (11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2*(7*a^2*c^3*
d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 + 3*a^4*c*d*e^5
)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5*e*f^7 + 3*a^5
*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3*a*c^4*d^5*e + 2*a^2
*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4)*f^4*g^3 +
 (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^5)*f^3*g^4 - 3*(3*a^4*c*d^2*e^4 - a^5*e^
6)*f^2*g^5)*x^2 + (3*a^5*d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4*d^4*e^2)*f^7 - (a
*c^4*d^5*e + 4*a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^
4)*f^5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f^4*g^3 - (11*a^4*c*d^2*
e^4 - a^5*e^6)*f^3*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 259, normalized size = 0.99

$$-\frac{2\left(cdx+ae\right)\left(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3\right.}{5\left(gx+f\right)^{\frac{5}{2}}\left(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4\right)\left(cde\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/5*(c*d*x+a*e)*(16*c^3*d^3*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2
*x^2-2*a^2*c*d*e^2*g^3*x+20*a*c^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*
g^3-5*a^2*c*d*e^2*f*g^2+15*a*c^2*d^2*e*f^2*g+5*c^3*d^3*f^3)*(e*x+d)^(3/2)/(
g*x+f)^(5/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a
*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(7/2)), x)
```

mupad [B] time = 5.70, size = 414, normalized size = 1.58

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\sqrt{d+e\,x}\,\left(-a^2\,e^2\,g^2+10\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{\sqrt{d+e\,x}\,\left(\frac{2\,a^3\,e^3\,g^3}{5}-2\,a^2\,c\,d\,e^2\,f\,g^2+6\,a\,c^2\,d^2\,e\ldots}{c\,d\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right.}{x^4\,\sqrt{f+g\,x}+\frac{a\,f^2\,\sqrt{f+g\,x}}{c\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(2\,c\,d^2\,f\,g+c\,d\,e\,f^2+a\,d\,e\,g^2+2\,a\,e^2\,f\,g\right)}{c\,d\,e\,g^2}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+2\,c\,f\,d\ldots}{c\,d\,e\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(d + e*x)^(1/2)*(15*c
^2*d^2*f^2 - a^2*e^2*g^2 + 10*a*c*d*e*f*g))/(5*e*g*(a*e*g - c*d*f)^4) + ((d
 + e*x)^(1/2)*((2*a^3*e^3*g^3)/5 + 2*c^3*d^3*f^3 + 6*a*c^2*d^2*e*f^2*g - 2*
a^2*c*d*e^2*f*g^2))/(c*d*e*g^2*(a*e*g - c*d*f)^4) + (32*c^2*d^2*g*x^3*(d +
e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4) + (16*c*d*x^2*(a*e*g + 5*c*d*f)*(d + e*
x)^(1/2))/(5*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (a*f^2*(f + g*x)
^(1/2))/(c*g^2) + (x^2*(f + g*x)^(1/2)*(a*d*e*g^2 + c*d*e*f^2 + 2*a*e^2*f*g
 + 2*c*d^2*f*g))/(c*d*e*g^2) + (x^3*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + 2*
c*d*e*f))/(c*d*e*g) + (f*x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + 2*a*d*e*g))
/(c*d*e*g^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.727 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=289

$$\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5g^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}-\frac{10}{3c^2d^2\sqrt{x}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(5/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)-10/3*g*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+5*g^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2
)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{5g^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}+\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10}{3c^2d^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (10*g*Sqrt[d + e*x]*(f + g*x)^(3/2))/(3*c^2*d^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*Sqrt[d + e*x]) + (5*g^(3/2)*(c*d*f -
a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/
(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(7/2)*d^(7/2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(5g)\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}$$

Mathematica [C] time = 0.14, size = 102, normalized size = 0.35

$$-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*
((c*d*(f + g*x))/(c*d*f - a*e*g))^(5/2))
```

fricas [A] time = 2.39, size = 1055, normalized size = 3.65

$$\left[\frac{4\left(3c^2d^2g^2x^2-2c^2d^2f^2-10acdefg+15a^2e^2g^2-2\left(7c^2d^2fg-10acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g
^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3
*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^
2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d
*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d
^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e
*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*
d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a
*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 +
(c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x), 1/6
*(2*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g^2 -
2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3*g^2
+ (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*
g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d*e^3)
*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g
/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g
)*x)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 +
 (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.72Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.04, size = 652, normalized size = 2.26

$$-\frac{\left(15a\,c^2d^2e\,g^3x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-15c^3d^3f\,g^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*x^2*a*c^2*d^2*e*g^3-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^2*c^3*d^3*f
*g^2+30*a^2*c*d*e^2*g^3*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*a*c^2*d^2*e*f*g^2*x*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))+15*a^3*e^3*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-6*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2-40*(c*d*g)^(1/2)*((g
*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+28*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e
```

```
))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2
*g^2+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+4*((g*x+f)*(c
*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^2/(c*d*g)^(1
/2)/c^3/d^3/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{5/2}\,\left(d+e\,x\right)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.728 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx}{3cd\left(x\left(ae^2+cd^2\right)+ade\right.}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+2*g^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2)
)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-2*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = $\{866, 891, 63, 217, 206\}$

$$\frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx}{3cd\left(x\left(ae^2+cd^2\right)+ade\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (2*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(c^2*d^2*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(
5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
```

```
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

Mathematica [C] time = 0.11, size = 102, normalized size = 0.47

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*
((c*d*(f + g*x))/(c*d*f - a*e*g))^(3/2))
```

fricas [A] time = 2.22, size = 755, normalized size = 3.45

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(4\,cdgx+cdf+3\,aeg\right)\sqrt{ex+d}\,\sqrt{gx+f}-3\left(c^2d^2egx^3+a^2de^2g+\left(c^2d^3+2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*d*f + 3
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c
^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(g/(c*d))*lo
g(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8
*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d
^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)
*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^3 + a^2*c^2
*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e
^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*
d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2
*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 +
 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 5.17Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 343, normalized size = 1.57

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^2d^2g^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+6acde\,g^2x\ln\left(\frac{2cdg}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*x^2*c^2*d^2*g^2+6*a*c*d*e*g^2*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(
c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*a^2*e^2*g^2*ln(1/2*(2*c*d
*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)
)-8*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x
+f)*(c*d*x+a*e))^(1/2)*a*e*g-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*
```

```
d*f)/(c*d*g)^(1/2)/(c*d*x+a*e)^2/((g*x+f)*(c*d*x+a*e))^(1/2)/d^2/c^2/(e*x+d
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.729 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.83

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d +
 e*x))^(3/2))
```

fricas [B] time = 0.90, size = 193, normalized size = 3.06

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\left(gx+f\right)^{\frac{3}{2}}}{3\left(a^2cd^2e^2f-a^3de^3g+\left(c^3d^3ef-ac^2d^2e^2g\right)x^3+\left(\left(c^3d^4+2\,ac^2d^2e^2\right)f-\left(ac^2d^3e+2\,a^2cde^3\right)g\right)x^2+\left(\left(2\,ac^2d^3\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f)^(3
/2)/(a^2*c*d^2*e^2*f - a^3*d*e^3*g + (c^3*d^3*e*f - a*c^2*d^2*e^2*g)*x^3 +
((c^3*d^4 + 2*a*c^2*d^2*e^2)*f - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g)*x^2 + ((2
*a*c^2*d^3*e + a^2*c*d*e^3)*f - (2*a^2*c*d^2*e^2 + a^3*e^4)*g)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.22Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{2\left(gx+f\right)^{\frac{3}{2}}\left(cdx+ae\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(aeg-cdf\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 2/3*(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(e*x+d)^(5/2)/(c*d*e*x^2+a*e^2*
x+c*d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{5}{2}}\sqrt{gx+f}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(5/2), x)
```

mupad [B] time = 4.32, size = 169, normalized size = 2.68

$$\frac{\left(\frac{2\,f\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,\left(a\,e\,g-c\,d\,f\right)}+\frac{2\,g\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,\left(a\,e\,g-c\,d\,f\right)}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] (((2*f*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)) + (2*
g*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)))*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^
2 + 2*c*d^2))/(c^2*d^2) + (x^2*(2*a*e^2 + c*d^2))/(c*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.730 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+4/3*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f - a
*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{(2g)\int \frac{}{\sqrt{f+gx}}}{}$$

$$=-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{4}{3(cdf-aeg)}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\sqrt{f+gx}\,(3aeg-cd(f-2gx))}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(3*a*e*g - c*d*(f - 2*g*x)))/(3*(c*d*f - a
*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [B] time = 1.08, size = 318, normalized size = 2.48

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2\right.}}{3\left(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+\left(c^4d^4ef^2-2\,ac^3d^3e^2fg+a^2c^2d^2e^3g^2\right)x^3+\left(\left(c^4d^5+2\,ac^3d^3e^2\right)f^2-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g +
 a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2
)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e
^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^
2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g + (2*a^3*c*d^2
*e^3 + a^4*e^5)*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$\frac{2\sqrt{gx+f}\,\,(cdx+ae)\left(2cdgx+3aeg-cdf\right)(ex+d)^{\frac{5}{2}}}{3\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 2/3*(g*x+f)^(1/2)*(c*d*x+a*e)*(2*c*d*g*x+3*a*e*g-c*d*f)*(e*x+d)^(5/2)/(a^2*
e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sq
rt(g*x + f)), x)
```

mupad [B] time = 5.06, size = 246, normalized size = 1.92

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{\left(2\,c\,d\,f^2-6\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{x\,\left(6\,a\,e\,g^2+2\,c\,d\,f\,g\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^2*x^2*(d + e*x)^(1/2))
/(3*c*d*e*(a*e*g - c*d*f)^2) - ((2*c*d*f^2 - 6*a*e*f*g)*(d + e*x)^(1/2))/(3
*c^2*d^2*e*(a*e*g - c*d*f)^2) + (x*(6*a*e*g^2 + 2*c*d*f*g)*(d + e*x)^(1/2))
/(3*c^2*d^2*e*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2) + (a^2*e*(f + g*x)^
(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(c*d*e) + (a*x*(f
+ g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.731 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{}{3\sqrt{f+gx}\,\left(x\left(ae^2\right.\right.}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(
g*x+f)^(1/2)+8/3*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{}{3\sqrt{f+gx}\,\left(x\left(ae^2\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (8*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*Sqrt[f
 + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - $$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

Mathematica [A] time = 0.07, size = 103, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\left(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2\left(-f^2+4fgx+8g^2x^2\right)\right)}{3\sqrt{f+gx}\,((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(3*a^2*e^2*g^2 + 6*a*c*d*e*g*(f + 2*g*x) + c^2*d^2*(-f^2
 + 4*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3
/2)*Sqrt[f + g*x])
```

fricas [B] time = 0.99, size = 667, normalized size = 3.44

$$\frac{}{3\left(a^2c^3d^4e^2f^4-3a^3c^2d^3e^3f^3g+3a^4cd^2e^4f^2g^2-a^5de^5fg^3+\left(c^5d^5ef^3g-3ac^4d^4e^2f^2g^2+3a^2c^3d^3e^3fg^3-a^3c^2d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 + 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 4*(c
^2*d^2*f*g + 3*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^3*d^4*e^2*f^4 - 3*a^3*c^2*d^3*e^3*f^3*g
+ 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^5*e*f^3*g - 3*a*c^4*d^
4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2*e^4*g^4)*x^4 + (c^5*d
^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4*d^5*e + a^2*c^3*d^3*e
^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)*f*g^3 - (a^3*c^2*d^3*
e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d^4*e^2)*f^4 - (a*c^4*d
^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^2
*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a^4*c*d^2*e^4 + a^5*e^6
)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^
2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5
)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.87

$$-\frac{2 \left(c d x+a e\right) \left(8 g^{2} x^{2} c^{2} d^{2}+12 a c d e g^{2} x+4 c^{2} d^{2} f g x+3 a^{2} e^{2} g^{2}+6 a c d e f g-f^{2} c^{2} d^{2}\right) \left(e x+d\right)^{\frac{5}{2}}}{3 \sqrt{g x+f}\, \left(a^{3} e^{3} g^{3}-3 a^{2} c d\, e^{2} f\, g^{2}+3 a\, c^{2} d^{2} e\, f^{2} g-f^{3} c^{3} d^{3}\right) \left(c d e\, x^{2}+a\, e^{2} x+c\, d^{2} x+a d e\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*
e^2*g^2+6*a*c*d*e*f*g-c^2*d^2*f^2)*(e*x+d)^(5/2)/(g*x+f)^(1/2)/(a^3*e^3*g^3
-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e x+d\right)^{\frac{5}{2}}}{\left(c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x\right)^{\frac{5}{2}} \left(g x+f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(3/2)), x)
```

mupad [B] time = 5.28, size = 255, normalized size = 1.31

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g-2\,c^2\,d^2\,f^2\right)}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{8\,g\,x\,\left(3\,a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g^2*x^2*(d + e*x)^(1/2
))/(3*e*(a*e*g - c*d*f)^3) + ((d + e*x)^(1/2)*(6*a^2*e^2*g^2 - 2*c^2*d^2*f^
2 + 12*a*c*d*e*f*g))/(3*c^2*d^2*e*(a*e*g - c*d*f)^3) + (8*g*x*(3*a*e*g + c*
d*f)*(d + e*x)^(1/2))/(3*c*d*e*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2) +
(a^2*e*(f + g*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(
c*d*e) + (a*x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.732 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=260

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+4*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(3/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/3*c*d*g^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {868, 872, 860}

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*(f + g
*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f +
g*x)^(3/2)) + (32*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(
c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.10, size = 152, normalized size = 0.58

$$\frac{2(d+ex)^{3/2}\left(-a^3e^3g^3+3a^2cde^2g^2(3f+2gx)+3ac^2d^2eg\left(3f^2+12fgx+8g^2x^2\right)+c^3d^3\left(-f^3+6f^2gx+24fg\right.\right.}{3(f+gx)^{3/2}((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-(a^3*e^3*g^3) + 3*a^2*c*d*e^2*g^2*(3*f + 2*g*x) + 3*a*
c^2*d^2*e*g*(3*f^2 + 12*f*g*x + 8*g^2*x^2) + c^3*d^3*(-f^3 + 6*f^2*g*x + 24
*f*g^2*x^2 + 16*g^3*x^3)))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(
3/2)*(f + g*x)^(3/2))
```

fricas [B] time = 1.18, size = 1065, normalized size = 4.10

$$\frac{}{3\left(a^2c^4d^5e^2f^6-4\,a^3c^3d^4e^3f^5g+6\,a^4c^2d^3e^4f^4g^2-4\,a^5cd^2e^5f^3g^3+a^6de^6f^2g^4+\left(c^6d^6ef^4g^2-4\,ac^5d^5e^2f^3g^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^2
*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*d^
3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^4*d^5*e^2*f^6 - 4*a
^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3
```

```
+ a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*
c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 +
(2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e -
 a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g
^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2
*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e
^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e
^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f
^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g -
2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*
c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d
^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a
^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e
^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f
^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6
*e^7)*f^2*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 258, normalized size = 0.99

$$-\frac{2\left(cdx+ae\right)\left(-16g^{3}x^{3}c^{3}d^{3}-24a\,c^{2}d^{2}e\,g^{3}x^{2}-24c^{3}d^{3}f\,g^{2}x^{2}-6a^{2}cd\,e^{2}g^{3}x-36a\,c^{2}d^{2}ef\,g^{2}x-6c^{3}d^{3}f^{2}gx+a^{3}e^{3}g\right.}{3\left(gx+f\right)^{\frac{3}{2}}\left(g^{4}e^{4}a^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g^{2}-4a\,c^{3}d^{3}e\,f^{3}g+f^{4}c^{4}d^{4}\right)\left(cde\,x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g
^2*x^2-6*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3
*g^3-9*a^2*c*d*e^2*f*g^2-9*a*c^2*d^2*e*f^2*g+c^3*d^3*f^3)*(e*x+d)^(5/2)/(g*
x+f)^(3/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c
^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{5}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(5/2)), x)
```

mupad [B] time = 5.86, size = 416, normalized size = 1.60

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g\,x^2\,\left(a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\,x}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{\sqrt{d+e\,x}\,\left(2\,a^3\,e^3\,g^3-18\,a^2\,c\,d\,e^2\,f\,g^2-18\,a\,c^2\,d^2\,e\,f^2\,g+2\,c^3\,d^3\,f^3\right)}{3\,c^2\,d^2\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c}{3\,e}\right.}{x^4\,\sqrt{f+g\,x}+\frac{x^2\,\sqrt{f+g\,x}\,\left(g\,a^2\,e^3+2\,g\,a\,c\,d^2\,e+2\,f\,a\,c\,d\,e^2+f\,c^2\,d^3\right)}{c^2\,d^2\,e\,g}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,f\,d^2+a\,g\,d\,e+a\,f\,e^2\right)}{c^2\,d^2\,g}+\frac{a^2\,e\,f\,\sqrt{f+g}}{c^2\,d\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g*x^2*(a*e*g + c*d*f)*(
d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^4) - ((d + e*x)^(1/2)*(2*a^3*e^3*g^3 + 2
*c^3*d^3*f^3 - 18*a*c^2*d^2*e*f^2*g - 18*a^2*c*d*e^2*f*g^2))/(3*c^2*d^2*e*g
*(a*e*g - c*d*f)^4) + (32*c*d*g^2*x^3*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)
^4) + (4*x*(d + e*x)^(1/2)*(a^2*e^2*g^2 + c^2*d^2*f^2 + 6*a*c*d*e*f*g))/(c*
d*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (x^2*(f + g*x)^(1/2)*(a^2*e
^3*g + c^2*d^3*f + 2*a*c*d*e^2*f + 2*a*c*d^2*e*g))/(c^2*d^2*e*g) + (a*x*(f
+ g*x)^(1/2)*(a*e^2*f + 2*c*d^2*f + a*d*e*g))/(c^2*d^2*g) + (a^2*e*f*(f + g
*x)^(1/2))/(c^2*d*g) + (x^3*(f + g*x)^(1/2)*(2*a*e^2*g + c*d^2*g + c*d*e*f)
)/(c*d*e*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.733 $\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=385

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}}-$$

```
[Out] -5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(3/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/96*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+1/24*(a*e/c/d-f/
g)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/4*
(g*x+f)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-5/64*
(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/g/(e*x+d)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}}-$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c^3*d^3*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^2*d^2*g*Sqrt[d + e*x]) +
(((a*e)/(c*d) - f/g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(24*Sqrt[d + e*x]) + ((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{}}{\sqrt{ade}}}{8}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/}}{}$$

$$= -\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf}{}$$

Mathematica [A] time = 1.16, size = 300, normalized size = 0.78

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(15a^3e^3g^3-5a^2cde^2g^2(11f+2gx)+ac^2d^2eg\left(73f^2+36fgx+8\right.\right.\right.}{192c^{9/2}d^{9/2}g^{3/2}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)*(
f + g*x)*(15*a^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g
*(73*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^
2*x^2 + 48*g^3*x^3)) - 15*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d
*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*
d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(192*c^(9/2)*d^(9/2)*g^(3/2)*Sqrt[
(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 5.54, size = 1065, normalized size = 2.77

$$\left[\frac{4\left(48\,c^4d^4g^4x^3+15\,c^4d^4f^3g+73\,ac^3d^3ef^2g^2-55\,a^2c^2d^2e^2fg^3+15\,a^3cde^3g^4+8\left(17\,c^4d^4fg^3+ac^3d^3eg^4\right)x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d^3*e*f^2*g^2 -
 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g^3 - 5*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g^2*x +
 c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d
^3*e*f^2*g^2 - 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^
4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g
^3 - 5*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g^2*x + c^4*d
^5*g^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 870, normalized size = 2.26

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^4e^4g^4\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-60a^3cd\,e^3f\,g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*x^3*c^3*d
^3*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
```

```
/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3
+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln(1/2*(2*c*d*g*x
+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-16*x^
2*a*c^2*d^2*e*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)-272
*x^2*c^3*d^3*f*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+20
*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c*d*e^2*g^3-72
*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^2*d^2*e*f*g^2-
236*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^3*d^3*f^2*g-3
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3+110*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2-146*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-30*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1
/2)/g/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/c^3/d^3/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,(gx + f)^{\frac{5}{2}}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt(
e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^{5/2}\,\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.734 $\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=313

$$-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+$$

```
[Out] -1/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(3/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/12*(a*e/c/d-f/g)*(g*x+f)^(3/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] -((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(8*c^2*d^2*g*Sqrt[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^(3/2)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*Sqrt[d + e*x]) + ((f + g*x)^(5
/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)\int -}{\sqrt{}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+g}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

Mathematica [A] time = 0.84, size = 255, normalized size = 0.81

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(-\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(3a^2e^2g^2-2acdeg(4f+gx)-c^2d^2\left(3f^2+14fgx+8g^2x^2\right)\right)\right.}{24c^{7/2}d^{7/2}g^{3/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)
*(f + g*x)*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(4*f + g*x) - c^2*d^2*(3*f^2 + 14*f
*g*x + 8*g^2*x^2))) - 3*(c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(
f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[(a*
e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 3.27, size = 847, normalized size = 2.71

$$\left[\frac{4\left(8c^3d^3g^3x^2+3c^3d^3f^2g+8ac^2d^2efg^2-3a^2cde^2g^3+2\left(7c^3d^3fg^2+ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2
*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 +
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2*c*d
*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 602, normalized size = 1.92

$$\frac{\sqrt{gx+f}\ \sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\ \left(3a^3e^3g^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\ \sqrt{cdg}}{2\sqrt{cdg}}\right)-9a^2cd\,e^2f\,g^2\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*c*d
*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*e^3*g^3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*
g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*
ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c
^3*d^3*f^3+16*x^2*c^2*d^2*g^2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*
f)^(1/2)+4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c*d*e*
g^2+28*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^2*d^2*f*g-
6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*e^2*g^2+16*a*c*
d*e*f*g*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)+6*(c*d*g)^(1/
2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g/(c*
d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/d^2/c^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(
e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.735 $\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\,\sqrt{x}}{}$$

```
[Out] -1/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(3/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)+1/4*(a*e/c/d-f/g)*(g*x+f)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\,\sqrt{x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*
x],x]
```

```
[Out] (((a*e)/(c*d) - f/g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(4*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(4*c^(3/2)*d^(3/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
```

```
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{}{\sqrt{ad}}}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}}{}$$

Mathematica [A] time = 0.59, size = 215, normalized size = 0.89

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)(aeg+cd(f+2gx))-\sqrt{ae+cdx}(cdf-aeg)^{5/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh\right.}{4g^{3/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[
d + e*x],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^(5/2)*Sqrt[a*e +
 c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt
[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])))/(4*(c*d)^(5/2)*g^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 2.46, size = 657, normalized size = 2.73

$$\left[\frac{4\left(2c^2d^2g^2x+c^2d^2fg+acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+\left(c^2d^3f^2-2acd^2efg+a^2de^2g\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*c*
d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)
*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g +
 a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x +
 c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
+ (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*
e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^2*x + c
^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f
^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a
^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a
*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^2*x + c^2*d^3*g^
2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 385, normalized size = 1.60

$$-\frac{\sqrt{gx+f}\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\sqrt{cdg}}{2\sqrt{cdg}}\right)-2acdefg\ln\left(\frac{2cdg}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*a^2*e^2*g^2-2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*
x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*
c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*c^2*d^2*f^2-4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e
*f)^(1/2)*x*c*d*g-2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a
*e*g-2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c*d*f)/(e*x+d)
^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/d/g/c/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{gx + f}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*
x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f + g\,x}\,\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)(ae + cdx)}\,\sqrt{f + gx}}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

3.736 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\,g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] -(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(
1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/c^(1/2)/d^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/g/(e*x+d)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\,g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*x
]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x
]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+e\cdots}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2\cdots\right.}}}{2g}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+\cdots}\right.}{2g\sqrt{ade+\left(cd^2+\cdots\right.}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+\cdots}\right.}{cdg\sqrt{ade\cdots}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+\cdots}\right.}{cdg\sqrt{ade+\left(\cdots\right.}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+\cdots}}{\sqrt{c}\sqrt{d}\,g^{3/2}\sqrt{ade+\left(cd\cdots\right.}}$$

Mathematica [A] time = 0.81, size = 173, normalized size = 1.04

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}\,(f+gx)-\frac{\sqrt{c}\sqrt{d}\,(cdf-aeg)^{3/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f
 + g*x]),x]

[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x) - (Sqrt[c]*Sqrt[d]*(c*d*f
 - a*e*g)^(3/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt
[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/((c*d)^(3/
2)*Sqrt[a*e + c*d*x])))/(g^(3/2)*Sqrt[d + e*x]*Sqrt[f + g*x])
```

fricas [A] time = 2.29, size = 516, normalized size = 3.09

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}\,cdg-\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{cdg}\,\log\left(-\frac{8c^2d^2eg^2x^3+c^2d\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^2*x + c*d^2*g^2), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g + (c*
d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2
*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*
d*e*g^2*x + c*d^2*g^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 198, normalized size = 1.19

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\sqrt{gx+f}\,\left(aeg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-cdf\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+\cdots}}{2\sqrt{cdg}}\right.\right.}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)/(e*x+d)^(1/2)*(a*
e*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/
2))/(c*d*g)^(1/2))-c*d*f*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*
e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/((g*x+f)*(c*d*x+a*e))^(1/2)/g/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.737 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*c^(1/2)*
d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx = -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{(cd)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,d}{g}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\left(cd\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\,\sqrt{}}}{g\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\left(2\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\text{Subst}\left(\int -\right.}{g\sqrt{ade+\left(cd^2+ae\right.}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\left(2\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{}{1-}\right.}{g\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-}}{g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

Mathematica [A] time = 0.79, size = 169, normalized size = 1.07

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{cdf-aeg}\,\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\,\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\,\sqrt{ae+cdx}}-\sqrt{g}\right)}{g^{3/2}\sqrt{d+ex}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-Sqrt[g] + (Sqrt[c]*Sqrt[d]*Sqrt[c*d*f -
a*e*g]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/(Sqrt[c*d]*Sqrt[a*e
 + c*d*x])))/(g^(3/2)*Sqrt[d + e*x]*Sqrt[f + g*x])
```

fricas [A] time = 2.16, size = 521, normalized size = 3.30

$$\left[\frac{\left(egx^2+df+\left(ef+dg\right)x\right)\sqrt{\frac{cd}{g}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\left(2cdg^2x+cdfg+aeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{ex}\right)}{2\left(eg^2x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((e*g*x^2 + d*f + (e*f + d*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x), -((e*g*x^2 +
d*f + (e*f + d*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g +
 (e*f*g + d*g^2)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 197, normalized size = 1.25

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(cdgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+cdf\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}}{2\sqrt{cdg}}\right.\right.}{\sqrt{cdg}\,\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{ex+d}\,\sqrt{gx+f}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(c*d*g*x*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+c*d*f*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/((g*x+
f)*(c*d*x+a*e))^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\,\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\left(f+gx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

3.738 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(5
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(5/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f +
 g*x)^(3/2))
```

fricas [B] time = 0.95, size = 169, normalized size = 2.68

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdx+ae)\sqrt{ex+d}\,\sqrt{gx+f}}{3\left(cd^2f^3-adef^2g+\left(cdefg^2-ae^2g^3\right)x^3+\left(2\,cdef^2g-adeg^3+\left(cd^2-2\,ae^2\right)fg^2\right)x^2+\left(cdef^3-2\,adefg^2+\left(2\,cd^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
*sqrt(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 + (
2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 - 2*a
*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{2\,(cdx+ae)\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3\left(gx+f\right)^{\frac{3}{2}}\left(aeg-cdf\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/3/(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(1/2)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(5/2)), x)
```

mupad [B] time = 3.92, size = 136, normalized size = 2.16

$$-\frac{\left(\frac{2\,a\,e}{3\,a\,e\,g^2-3\,c\,d\,f\,g}+\frac{2\,c\,d\,x}{3\,a\,e\,g^2-3\,c\,d\,f\,g}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(3\,c\,d\,f^2-3\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,a\,e\,g^2-3\,c\,d\,f\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(5/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -(((2*a*e)/(3*a*e*g^2 - 3*c*d*f*g) + (2*c*d*x)/(3*a*e*g^2 - 3*c*d*f*g))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x*(f + g*x)^(1/2)*(d + e*x)^(1
```

```
/2) - ((f + g*x)^(1/2)*(3*c*d*f^2 - 3*a*e*f*g)*(d + e*x)^(1/2))/(3*a*e*g^2
- 3*c*d*f*g))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(5/2)), x)
```

3.739 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(5/2)+4/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(7
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx}{5(cdf-aeg)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+2gx)-3aeg)}{15(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*d
*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

fricas [B] time = 0.92, size = 402, normalized size = 3.12

$$\frac{2\left(2\,c^2d^2gx\right.}{15\left(c^2d^3f^5-2\,acd^2ef^4g+a^2de^2f^3g^2+\left(c^2d^2ef^2g^3-2\,acde^2fg^4+a^2e^3g^5\right)x^4+\left(3\,c^2d^2ef^3g^2+a^2de^2g^5+\left(c^2d^3-\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 3*a^2*e^2*g + (5*c^2*d^2*f - a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f)/(c^2*d^3*f^5 - 2*a*c*d^2*e*f^4*g + a^2*d*e^2*f^3*g^2 + (c^2*d^2*e*f^2*g^
3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^4 + (3*c^2*d^2*e*f^3*g^2 + a^2*d*e^2
*g^5 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^4)*x
^3 + 3*(c^2*d^2*e*f^4*g + a^2*d*e^2*f*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^2
 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x^2 + (c^2*d^2*e*f^5 + 3*a^2*d*e^2*f^2*
g^3 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g - (6*a*c*d^2*e - a^2*e^3)*f^3*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$-\frac{2\,(cdx+ae)\left(-2cdgx+3aeg-5cdf\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15\left(gx+f\right)^{\frac{5}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-2*c*d*g*x+3*a*e*g-5*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(g*x+f)^(5/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(7/2)), x)
```

mupad [B] time = 4.08, size = 187, normalized size = 1.45

$$\frac{\left(\frac{x\left(10c^2d^2f-2acdeg\right)}{15g^2\left(aeg-cdf\right)^2}-\frac{6a^2e^2g-10acdef}{15g^2\left(aeg-cdf\right)^2}+\frac{4c^2d^2x^2}{15g\left(aeg-cdf\right)^2}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2\sqrt{f+gx}\sqrt{d+ex}+\frac{f^2\sqrt{f+gx}\sqrt{d+ex}}{g^2}+\frac{2fx\sqrt{f+gx}\sqrt{d+ex}}{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(7/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] (((x*(10*c^2*d^2*f - 2*a*c*d*e*g))/(15*g^2*(a*e*g - c*d*f)^2) - (6*a^2*e^2*
g - 10*a*c*d*e*f)/(15*g^2*(a*e*g - c*d*f)^2) + (4*c^2*d^2*x^2)/(15*g*(a*e*g
 - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)
^(1/2)*(d + e*x)^(1/2) + (f^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (2*f*x
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(7/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.740 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(7/2)+8/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(5/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(9
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(7/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{(4cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx}{7(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2\left(35f^2+28fgx+8g^2x^2\right)\right)}{105(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*g
*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d +
 e*x)^(3/2)*(f + g*x)^(7/2))
```

fricas [B] time = 0.99, size = 748, normalized size = 3.78

$$\overline{105\left(c^3d^4f^7-3\,ac^2d^3ef^6g+3\,a^2cd^2e^2f^5g^2-a^3de^3f^4g^3+\left(c^3d^3ef^3g^4-3\,ac^2d^2e^2f^2g^5+3\,a^2cde^3fg^6-a^3e^4g^7\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 42*a^2*c*d*e^2*f*g + 15*a^3
*e^3*g^2 + 4*(7*c^3*d^3*f*g - a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 - 14*a
*c^2*d^2*e*f*g + 3*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^7 - 3*a*c^2*d^3*e*f^6*g + 3*
a^2*c*d^2*e^2*f^5*g^2 - a^3*d*e^3*f^4*g^3 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^
2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e*f^4*g
^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e
- 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4 + 2*(3*
c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g
^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e^2 - a^3*e
^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 + 3*(c^3*d^4
- 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^3 + (9
*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*a^3*d*e^3*f^3
*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3
)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2\,(cdx+ae)\left(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35f^2c^2d^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x}}{105\left(gx+f\right)^{\frac{7}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x)

[Out] -2/105*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*a^2*e^2*g^2-42*a*c*d*e*f*g+35*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(g*x+f)^(7/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(e*x+d)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x + f)^(9/2)), x)

mupad [B] time = 4.29, size = 289, normalized size = 1.46

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{30\,a^3\,e^3\,g^2-84\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x\,\left(6\,a^2\,c\,d\,e^2\,g^2-28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^3 \dots}{105\,g\,(a \dots}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(9/2)*(d + e*x)^(1/2)),x)

[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((30*a^3*e^3*g^2 + 70*a*c^2*d^2*e*f^2 - 84*a^2*c*d*e^2*f*g)/(105*g^3*(a*e*g - c*d*f)^3) + (x*(70*c^3*d^3*f^2 + 6*a^2*c*d*e^2*g^2 - 28*a*c^2*d^2*e*f*g))/(105*g^3*(a*e*g - c*d*f)^3) + (16*c^3*d^3*x^3)/(105*g*(a*e*g - c*d*f)^3) - (8*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(105*g^2*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x+d)**(1/2),x)

[Out] Timed out

3.741 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+c\right.}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(9/2)+4/21*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(7/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(5/2)+32/315*c^3*d^3*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(3/2)/(g*x+f)^
(3/2)
```

Rubi [A] time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+c\right.}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(1
1/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(21*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(315*(c*d*f - a*e*g)^4*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx}{3(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}}$$

Mathematica [A] time = 0.14, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(-35a^3e^3g^3+15a^2cde^2g^2(9f+2gx)-3ac^2d^2eg\left(63f^2+36fgx+8g^2x^2\right)+c^3d^3\left(105f^3+\right.\right.}{315(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(9
*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(10
5*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^4*(
d + e*x)^(3/2)*(f + g*x)^(9/2))
```

fricas [B] time = 0.91, size = 1179, normalized size = 4.42

$$\overline{315\left(c^4d^5f^9-4\,ac^3d^4ef^8g+6\,a^2c^2d^3e^2f^7g^2-4\,a^3cd^2e^3f^6g^3+a^4de^4f^5g^4+\left(c^4d^4ef^4g^5-4\,ac^3d^3e^2f^3g^6+6\,a^2c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2*e^2*f^2*g
 + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4*f*g^2 - a*c^3*d^3*
e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3*e*f*g^2 + a^2*c^2*d^2*e^2*g^
3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f*g^2
 - 5*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d)*sqrt(g*x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*d^3*e^
2*f^7*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e*f^4*g^
5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*d*e^4*f*g
^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^4*d^5 - 2
0*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^6 +
 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 5*a^4*
e^5)*f*g^8)*x^5 + 5*(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 - 8*a
*c^3*d^3*e^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 + 2*(3*
a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^
2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7 + (c^4*d^5 - 4*a*c^3
*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(3*a^
2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^
```

```
6)*x^3 + 5*(c^4*d^4*e*f^8*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^4*d^5 - 2*a*c^3*d^
3*e^2)*f^7*g^2 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^3 + 4*(3*a^2*c
^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^5)*x^
2 + (c^4*d^4*e*f^9 + 5*a^4*d*e^4*f^4*g^5 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^
8*g - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^2 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^3 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2\left(cdx+ae\right)\left(-16g^{3}x^{3}c^{3}d^{3}+24a\,c^{2}d^{2}e\,g^{3}x^{2}-72c^{3}d^{3}f\,g^{2}x^{2}-30a^{2}cd\,e^{2}g^{3}x+108a\,c^{2}d^{2}ef\,g^{2}x-126c^{3}d^{3}f^{2}gx}{315\left(gx+f\right)^{\frac{9}{2}}\left(g^{4}e^{4}a^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f
*g^2*x^2-30*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+3
5*a^3*e^3*g^3-135*a^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(g*x+f)^(9/2)/(a^4*e^4*g^4-4*a^3*c*
d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(e*x
+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(11/2)), x)
```

mupad [B] time = 4.50, size = 409, normalized size = 1.53

$$\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{x\left(-10\,a^{3}\,c\,d\,e^{3}\,g^{3}+54\,a^{2}\,c^{2}\,d^{2}\,e^{2}\,f\,g^{2}-126\,a\,c^{3}\,d^{3}\,e\,f^{2}\,g+210\,c^{4}\,d^{4}\,f^{3}\right)}{315\,g^{4}\left(a\,e\,g-c\,d\,f\right)^{4}}-\frac{70\,a^{4}\,e^{4}\,g^{3}-270\,a^{3}\,c\,d\,e^{3}\,f\,g}{315}}{x^{4}\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^{4}\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^{4}}+\frac{4\,f\,x^{3}\sqrt{f+g\,x}}{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(11/2)*(d + e*
x)^(1/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x*(210*c^4*d^4*f^3 - 10*a^
3*c*d*e^3*g^3 + 54*a^2*c^2*d^2*e^2*f*g^2 - 126*a*c^3*d^3*e*f^2*g))/(315*g^4
*(a*e*g - c*d*f)^4) - (70*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 378*a^2*c^2*d
^2*e^2*f^2*g - 270*a^3*c*d*e^3*f*g^2)/(315*g^4*(a*e*g - c*d*f)^4) + (32*c^4
*d^4*x^4)/(315*g*(a*e*g - c*d*f)^4) + (4*c^2*d^2*x^2*(a^2*e^2*g^2 + 21*c^2*
d^2*f^2 - 6*a*c*d*e*f*g))/(105*g^3*(a*e*g - c*d*f)^4) - (16*c^3*d^3*x^3*(a*
e*g - 9*c*d*f))/(315*g^2*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (6
*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x+
d)**(1/2),x)
```

```
[Out] Timed out
```

3.742 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=382

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}$$

```
[Out] 1/4*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+3
/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(5/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/32*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)*(
g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+3/64
*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2
/d^2/g^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (3*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*c^2*d^2*g^2*Sqrt[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*c*d*g^2*Sqrt[d + e*x]) - ((c
*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(8*g^2*Sqrt[d + e*x]) + ((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(5/2)*d^(5/2)*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} - \frac{(3(cdf-ae}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(}{}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(c}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(c}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(c}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(c}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(c}{}$$

Mathematica [A] time = 1.17, size = 302, normalized size = 0.79

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(3\sqrt{ae+cdx}\,(cdf-aeg)^{9/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\,\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)-\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}\,(f+gx)(ae+c}{64c^{7/2}d^{7/2}g^{5/2}\sqrt{f+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
d + e*x)^(3/2),x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)
*(f + g*x)*(3*a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(11*f + 2*g*x) - a*c^2*d^2*e*g*
(11*f^2 + 44*f*g*x + 24*g^2*x^2) + c^3*d^3*(3*f^3 - 2*f^2*g*x - 24*f*g^2*x^
2 - 16*g^3*x^3))) + 3*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f
+ g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
)/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(64*c^(7/2)*d^(7/2)*g^(5/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 5.26, size = 1059, normalized size = 2.77

$$\left[\frac{4\left(16\,c^4d^4g^4x^3-3\,c^4d^4f^3g+11\,ac^3d^3ef^2g^2+11\,a^2c^2d^2e^2fg^3-3\,a^3cde^3g^4+24\left(c^4d^4fg^3+ac^3d^3eg^4\right)x^2+2\left(c^4d\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/256*(4*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^2 +
11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^3*d^
3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*e^2*
g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f) + 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4
*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3
*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt
(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e
^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
 a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
 + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^3*x + c^3*d^4*g
^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^
2 + 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*
e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) - 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2
 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2
*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*
sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e
*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g^3*x + c^3*d^4*g^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 870, normalized size = 2.28

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3a^4e^4g^4\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-12a^3cd\,e^3f\,g^3\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 1/128*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(32*(c*d*g*x^2+
a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*g^3*x^3+3*a^4*e^4*g^4*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))-12*a^3*c*d*e^3*f*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
```

```
+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+18
*a^2*c^2*d^2*e^2*f^2*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x
+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-12*a*c^3*d^3*e*f^3*g*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))+3*c^4*d^4*f^4*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d
*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+48*(c*d*g
*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*g^3*x^2+48*(c*d
*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f*g^2*x^2+4*(c*d*
g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*g^3*x+88*(c*d*
g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f*g^2*x+4*(c*d
*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^2*g*x-6*(c*d*g)
^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3+22*(c*d*g)^(1/2)
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2+22*(c*d*g)^(1/2)
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-6*(c*d*g)^(1/2)*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c^2/d^2/
g^2/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} \left(gx + f\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*
x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f + g\,x\right)^{3/2}\,\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(d + e\,x\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Timed out
```

3.743 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}-\frac{(f-$$

```
[Out] 1/3*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+1
/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(5/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/4*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+1/8*(-a*e*g+c*d*f)^2*(g*x+f)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}-\frac{(f-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)
^(3/2),x]
```

```
[Out] ((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(8*c*d*g^2*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) + ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} - \frac{(cdf-aeg)\int \frac{\sqrt{.}}{}}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-}{}$$

Mathematica [A] time = 0.83, size = 254, normalized size = 0.82

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}\,(f+gx)(ae+cdx)\left(3a^2e^2g^2+2acdeg(4f+7gx)+c^2d^2\left(-3f^2+2fgx+8g^2x^2\right)\right)\right.}{24g^{5/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
+ e*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2
 + 2*f*g*x + 8*g^2*x^2)) + 3*(c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[(
c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e +
 c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g)])]))/(24*(c*d)^(5/2)*g^(5/2)*Sqrt[(a
*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 3.00, size = 847, normalized size = 2.73

$$\left[\frac{4\left(8c^3d^3g^3x^2-3c^3d^3f^2g+8ac^2d^2efg^2+3a^2cde^2g^3+2\left(c^3d^3fg^2+7ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2
*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 -
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d
*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 602, normalized size = 1.94

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3a^3e^3g^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-9a^2cd\,e^2f\,g^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*a^3*e^3*g^3*
ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))-9*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+9
*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f
*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-3*c^3*d^3*f^3*ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))-16*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^
2*g^2*x^2-28*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c*d*e*
g^2*x-4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f*g*x
-6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*e^2*g^2-16*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c*d*e*f*g+6*(c*d*g)^(1
/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c/d/
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x
+ d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + e x\right) \left(a e + c d x\right)\right)^{\frac{3}{2}} \sqrt{f + g x}}{\left(d + e x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)/(d + e*x)**(3/2), x
)
```

3.744 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=238

$$\frac{3\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}\,g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+$$

```
[Out] 1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g/(e*x+d)^(3/2)+3
/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/c^(1/2)/d^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}\,g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (-3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*g^2*Sqrt[d + e*x]) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(4*Sqrt[c]*Sqrt[d]*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
```

```
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}} - \frac{(3(cdf-aeg))\int \frac{\sqrt{ade+\left(cd^2+\right.}}{\sqrt{d+ex}}}{4g}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade\right.}{2}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade\right.}{2}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade\right.}{2}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade\right.}{2}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade\right.}{2}$$

Mathematica [A] time = 0.77, size = 193, normalized size = 0.81

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}(f+gx)(5aeg+cd(2gx-3f))+\frac{3\sqrt{c}\sqrt{d}(cdf-aeg)^{5/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{4g^{5/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x)*(5*a*e*g + c*d*(-3*f + 2*
g*x)) + (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^(5/2)*Sqrt[(c*d*(f + g*x))/(c*d*
f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*
Sqrt[c*d*f - a*e*g])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d
+ e*x]*Sqrt[f + g*x])
```

fricas [A] time = 2.35, size = 651, normalized size = 2.74

$$\left[\frac{4\left(2\,c^2d^2g^2x - 3\,c^2d^2fg + 5\,acdeg^2\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f} + 3\left(c^2d^3f^2 - 2\,acd^2efg + \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^3*x +
 c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2
*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f
*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2
*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^3*x + c*d^2*g^
3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 325, normalized size = 1.37

$$\frac{\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\,\sqrt{gx+f}\,\left(3a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right) - 6acdefg\ln\left(\frac{2cdgx+ae\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(3*a^2*e^2*g^2*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c
*d*g)^(1/2))-6*a*c*d*e*f*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*c^2*d^2*f^2*ln(1/2*(2*c*d*g*x+a
*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+4*(c
```

```
*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*g*x+10*(c*d*g)^(1/2)*((g*x+f)*(
c*d*x+a*e))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/
(e*x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sq
rt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\sqrt{f + g\,x}\,\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Timed out
```

3.745 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\cdots$$

```
[Out] -2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(1/2)-3*
(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1
/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+3*c*d*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{3cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (3*c*d*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt
[d + e*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)
^(3/2)*Sqrt[f + g*x]) - (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
```

```
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{g}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{g(d+ex)^{3/2}\sqrt{f+g}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{g(d+ex)^{3/2}\sqrt{f+g}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{g(d+ex)^{3/2}\sqrt{f+g}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{g(d+ex)^{3/2}\sqrt{f+g}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{g(d+ex)^{3/2}\sqrt{f+g}}$$

Mathematica [C] time = 0.16, size = 102, normalized size = 0.46

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd(d+ex)^{5/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^(3/2)*
Hypergeometric2F1[3/2, 5/2, 7/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*
c*d*(d + e*x)^(5/2)*(f + g*x)^(3/2))
```

fricas [A] time = 2.07, size = 663, normalized size = 2.99

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx+3\,cdf-2\,aeg\right)\sqrt{ex+d}\,\sqrt{gx+f}\,-3\left(cd^2f^2-adefg+\left(cdefg-ae^2g^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g
- a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g)*x)*sqrt(c*
d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*
g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^
2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*
f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^3*x^2 + d*f*g^2 + (e
*f*g^2 + d*g^3)*x), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
*g*x + 3*c*d*f - 2*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c*d^2*f^2 - a*d*
e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e
^2)*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*
d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^
2 + d*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 383, normalized size = 1.73

$$\frac{\left(3acde\,g^2x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^2d^2fgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3acdef\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x)
```

```
[Out] 1/2*(3*a*c*d*e*g^2*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^
(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-3*c^2*d^2*f*g*x*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*a*c*d*
e*f*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))-3*c^2*d^2*f^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+2*(c*d*g)^(1/2)*((g*x+f)*
(c*d*x+a*e))^(1/2)*c*d*g*x-4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e*
g+6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d
^2*x+a*d*e)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^2/(g*x+f)^(1/
2)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^{3/2}\,\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

3.746 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade\right.}{3g(d+ex)^{3/2}(f+}$$

```
[Out] -2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(3/2)+
2*c^(3/2)*d^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)
^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2
)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade\right.}{3g(d+ex)^{3/2}(f+}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*Sqr
t[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e
*x)^(3/2)*(f + g*x)^(3/2)) + (2*c^(3/2)*d^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/
(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
```

```
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx &= -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{g}\\
&= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\\
&= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\\
&= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\\
&= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\\
&= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.06, size = 188, normalized size = 0.88

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{c}\,\sqrt{d}\,(cdf-aeg)^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\,\sqrt{ae+cdx}}-\sqrt{g}\,(aeg+cd(3f+4gx))\right)}{3g^{5/2}\sqrt{d+ex}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]*(a*e*g + c*d*(3*f + 4*g*x))) +
(3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^(3/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^
```

```
(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c
*d*f - a*e*g])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/(3*g^(5/2)*Sqrt[d + e*x]*(
f + g*x)^(3/2))
```

fricas [A] time = 2.04, size = 685, normalized size = 3.20

$$\left[-\frac{4\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(4\,cdgx + 3\,cdf + aeg\right)\sqrt{ex+d}\,\sqrt{gx+f} - 3\left(cdeg^2x^3 + cd^2f^2 + \left(2\,cdefg + \right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f +
 a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d
*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(c*d/g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*
d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d
*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*
d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^4*x^3 + d*f^2*g^2 + (2*e*f*g^3 +
d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f + a*e*g)*sqrt(e*x + d)*sqrt(g*x + f)
 + 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^
2 + 2*c*d^2*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^
2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^3 + d*f^2*g^2
 + (2*e*f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 331, normalized size = 1.55

$$\frac{\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\,\left(3c^2d^2g^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right) + 6c^2d^2fgx\ln\left(\frac{2cdgx+aeg+cdf+}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*c^2*d^2*g^2*x^2*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))+6*c^2*d^2*f*g*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*c^2*d^2*f^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d
*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-8*(c*d*g)^(1
```

```
/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*g*x-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e
))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(c*d*g)^(
1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^{5/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

3.747 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f +
 g*x)^(5/2))
```

fricas [B] time = 0.87, size = 232, normalized size = 3.68

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex}}{5\left(cd^2f^4-adef^3g+\left(cdefg^3-ae^2g^4\right)x^4+\left(3\,cdef^2g^2-adeg^4+\left(cd^2-3\,ae^2\right)fg^3\right)x^3+3\left(cdef^3g-adefg^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*f*
g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*f*g
^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 + (c*
d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{2\left(cdx+ae\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{5\left(gx+f\right)^{\frac{5}{2}}\left(aeg-cdf\right)\left(ex+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x)
```

```
[Out] -2/5/(g*x+f)^(5/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/(e*x+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(7/2)), x)
```

mupad [B] time = 4.07, size = 232, normalized size = 3.68

$$-\frac{\left(\frac{2\,a^2\,e^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{2\,c^2\,d^2\,x^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{4\,a\,c\,d\,e\,x}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(5\,c\,d\,f^3-5\,a\,e\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{x\,\sqrt{f+g\,x}\,\left(10\,a\,e\,f\,g^2-10\,c\,d\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(7/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -(((2*a^2*e^2)/(5*a*e*g^3 - 5*c*d*f*g^2) + (2*c^2*d^2*x^2)/(5*a*e*g^3 - 5*c
*d*f*g^2) + (4*a*c*d*e*x)/(5*a*e*g^3 - 5*c*d*f*g^2))*(x*(a*e^2 + c*d^2) + a
```

```
*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^
(1/2)*(5*c*d*f^3 - 5*a*e*f^2*g)*(d + e*x)^(1/2))/(5*a*e*g^3 - 5*c*d*f*g^2)
+ (x*(f + g*x)^(1/2)*(10*a*e*f*g^2 - 10*c*d*f^2*g)*(d + e*x)^(1/2))/(5*a*e*
g^3 - 5*c*d*f*g^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

3.748 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(7/2)+4/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx}{7(cdf-aeg)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+2gx)-5aeg)}{35(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*d
*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

fricas [B] time = 0.74, size = 526, normalized size = 4.08

$$\overline{35\left(c^2d^3f^6 - 2\,acd^2ef^5g + a^2de^2f^4g^2 + \left(c^2d^2ef^2g^4 - 2\,acde^2fg^5 + a^2e^3g^6\right)x^5 + \left(4\,c^2d^2ef^3g^3 + a^2de^2g^6 + \left(c^2d\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c^
2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^6 - 2*a*c
*d^2*e*f^5*g + a^2*d*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 +
 a^2*e^3*g^6)*x^5 + (4*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 8*a*c
*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f*g^5)*x^4 + 2*(3*c^2*d^2*e*f^4
*g^2 + 2*a^2*d*e^2*f*g^5 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e
 - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d*e^2*f^2*g^4 + (
3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^3)*x^2 +
 (c^2*d^2*e*f^6 + 4*a^2*d*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g - (
8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$-\frac{2\,(cdx+ae)\left(-2cdgx+5aeg-7cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{35\left(gx+f\right)^{\frac{7}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)(ex+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/(g*x+f)^(7/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(9/2)), x)
```

mupad [B] time = 4.31, size = 247, normalized size = 1.91

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^2\,e^2\,\left(5\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^3\,d^3\,x^3}{35\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{4\,a\,c\,d\,e\,x\,\left(4\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(9/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^2*e^2*(5*a*e*g - 7*c*
d*f))/(35*g^3*(a*e*g - c*d*f)^2) - (4*c^3*d^3*x^3)/(35*g^2*(a*e*g - c*d*f)^
2) + (2*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2) + (4*a*c*
d*e*x*(4*a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

3.749 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-a}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(9/2)+8/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(5/2)/(g*x+f)^(7/2)+16/315*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Rubi [A] time = 0.23, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(9/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7}}$$

Mathematica [A] time = 0.13, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(35a^2e^2g^2-10acdeg(9f+2gx)+c^2d^2\left(63f^2+36fgx+8g^2x^2\right)\right)}{315(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^2*g^2 - 10*a*c*d*e*g*(9*f + 2*
g*x) + c^2*d^2*(63*f^2 + 36*f*g*x + 8*g^2*x^2)))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(9/2))
```

fricas [B] time = 1.03, size = 918, normalized size = 4.64

$$\overline{315\left(c^3d^4f^8-3\,ac^2d^3ef^7g+3\,a^2cd^2e^2f^6g^2-a^3de^3f^5g^3+\left(c^3d^3ef^3g^5-3\,ac^2d^2e^2f^2g^6+3\,a^2cde^3fg^7-a^3e^4g^8\right)x^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 90*a^3*c*d*e^3*f*g + 35
*a^4*e^4*g^2 + 4*(9*c^4*d^4*f*g - a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f^2
- 6*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2 - 72
*a^2*c^2*d^2*e^2*f*g + 25*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^8 - 3*a*c^2*d^3*e*f^7
*g + 3*a^2*c*d^2*e^2*f^6*g^2 - a^3*d*e^3*f^5*g^3 + (c^3*d^3*e*f^3*g^5 - 3*a
*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^6 + (5*c^3*d^3*
e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2
*d^3*e - 5*a^2*c*d*e^3)*f^2*g^6 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^7)*x^5
+ 5*(2*c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^
4*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*
e^4)*f^2*g^6)*x^4 + 10*(c^3*d^3*e*f^6*g^2 - a^3*d*e^3*f^2*g^6 + (c^3*d^4 -
3*a*c^2*d^2*e^2)*f^5*g^3 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (3*a^2*c
*d^2*e^2 - a^3*e^4)*f^3*g^5)*x^3 + 5*(c^3*d^3*e*f^7*g - 2*a^3*d*e^3*f^3*g^5
 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*
f^5*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^4)*x^2 + (c^3*d^3*e*f^8 - 5*a^3
*d*e^3*f^4*g^4 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^7*g - 3*(5*a*c^2*d^3*e - a
^2*c*d*e^3)*f^6*g^2 + (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2\left(cdx+ae\right)\left(8g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+36c^{2}d^{2}fgx+35a^{2}e^{2}g^{2}-90acdefg+63f^{2}c^{2}d^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}\right.}{315\left(gx+f\right)^{\frac{9}{2}}\left(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}c^{3}d^{3}\right)\left(ex+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+36*c^2*d^2*f*g*x+35*
a^2*e^2*g^2-90*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(3/2)/(g*x+f)^(9/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-
c^3*d^3*f^3)/(e*x+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(11/2)), x)
```

mupad [B] time = 4.48, size = 377, normalized size = 1.90

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{70\,a^4\,e^4\,g^2-180\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^2\,\left(6\,a^2\,c^2\,d^2\,e^2\,g^2-36\,a\,c^3\,d^3\,e\,f\,g+126\,c^4\,d^4\,f^2\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right.}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(11/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((70*a^4*e^4*g^2 + 126*a^2*
c^2*d^2*e^2*f^2 - 180*a^3*c*d*e^3*f*g)/(315*g^4*(a*e*g - c*d*f)^3) + (x^2*(
126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 - 36*a*c^3*d^3*e*f*g))/(315*g^4*(a*
e*g - c*d*f)^3) + (16*c^4*d^4*x^4)/(315*g^2*(a*e*g - c*d*f)^3) - (8*c^3*d^3
*x^3*(a*e*g - 9*c*d*f))/(315*g^3*(a*e*g - c*d*f)^3) + (4*a*c*d*e*x*(25*a^2*
e^2*g^2 + 63*c^2*d^2*f^2 - 72*a*c*d*e*f*g))/(315*g^4*(a*e*g - c*d*f)^3)))/(
x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))
/g^4 + (4*f*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

3.750 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+c\right.}{33(d+ex)^{5/2}(f+gx)^{9/2}(cd}$$

```
[Out] 2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(11/2)+4/33*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(9/2)+16/231*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(7/2)+32/1155*c^3*d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(5/2)/(g*x+
f)^(5/2)
```

Rubi [A] time = 0.32, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+c\right.}{33(d+ex)^{5/2}(f+gx)^{9/2}(cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(11/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(33*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2))/(1155*(c*d*f - a*e*g)^4*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx}{11(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx}$$

Mathematica [A] time = 0.18, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(-105a^3e^3g^3+35a^2cde^2g^2(11f+2gx)-5ac^2d^2eg\left(99f^2+44fgx+8g^2x^2\right)+c^3d^3\left(231f^3\right.\right.}{1155(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^3*g^3 + 35*a^2*c*d*e^2*g^2*(
11*f + 2*g*x) - 5*a*c^2*d^2*e*g*(99*f^2 + 44*f*g*x + 8*g^2*x^2) + c^3*d^3*(
231*f^3 + 198*f^2*g*x + 88*f*g^2*x^2 + 16*g^3*x^3)))/(1155*(c*d*f - a*e*g)^
4*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

fricas [B] time = 1.07, size = 1420, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 495*a^3*c^2*d^2*e^3*
f^2*g + 385*a^4*c*d*e^4*f*g^2 - 105*a^5*e^5*g^3 + 8*(11*c^5*d^5*f*g^2 - a*c
^4*d^4*e*g^3)*x^4 + 2*(99*c^5*d^5*f^2*g - 22*a*c^4*d^4*e*f*g^2 + 3*a^2*c^3*
d^3*e^2*g^3)*x^3 + (231*c^5*d^5*f^3 - 99*a*c^4*d^4*e*f^2*g + 33*a^2*c^3*d^3
*e^2*f*g^2 - 5*a^3*c^2*d^2*e^3*g^3)*x^2 + 2*(231*a*c^4*d^4*e*f^3 - 396*a^2*
c^3*d^3*e^2*f^2*g + 275*a^3*c^2*d^2*e^3*f*g^2 - 70*a^4*c*d*e^4*g^3)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^
5*f^10 - 4*a*c^3*d^4*e*f^9*g + 6*a^2*c^2*d^3*e^2*f^8*g^2 - 4*a^3*c*d^2*e^3*
f^7*g^3 + a^4*d*e^4*f^6*g^4 + (c^4*d^4*e*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7
+ 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 + a^4*e^5*g^10)*x^7 + (6*
c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^6 -
 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^7 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c
*d*e^4)*f^2*g^8 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^9)*x^6 + 3*(5*c^4*d^4
*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^5 - 2
*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^6 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^
3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^8)*x^5 + 5*(4*c^4*
d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^
4 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c^2*d^3*e^2 - 8
*a^3*c*d*e^4)*f^4*g^6 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7)*x^4 + 5*(3*c
^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*
```

```
g^3 - 2*(8*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^4 + 12*(2*a^2*c^2*d^3*e^2
 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^6)*x^3 + 3*(
2*c^4*d^4*e*f^9*g + 5*a^4*d*e^4*f^4*g^6 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8
*g^2 - 4*(5*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^3 + 2*(15*a^2*c^2*d^3*e^
2 - 4*a^3*c*d*e^4)*f^6*g^4 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^5)*x^2 +
(c^4*d^4*e*f^10 + 6*a^4*d*e^4*f^5*g^5 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9
*g - 6*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^2 + 4*(9*a^2*c^2*d^3*e^2 - a
^3*c*d*e^4)*f^7*g^3 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2\left(cdx+ae\right)\left(-16g^{3}x^{3}c^{3}d^{3}+40a\,c^{2}d^{2}e\,g^{3}x^{2}-88c^{3}d^{3}f\,g^{2}x^{2}-70a^{2}cd\,e^{2}g^{3}x+220a\,c^{2}d^{2}ef\,g^{2}x-198c^{3}d^{3}f^{2}gx\right.}{1155\left(gx+f\right)^{\frac{11}{2}}\left(g^{4}e^{4}a^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+40*a*c^2*d^2*e*g^3*x^2-88*c^3*d^3*
f*g^2*x^2-70*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-198*c^3*d^3*f^2*g*x+
105*a^3*e^3*g^3-385*a^2*c*d*e^2*f*g^2+495*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(g*x+f)^(11/2)/(a^4*e^4*g^4-4*a^3
*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(
e*x+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(13/2)), x)
```

mupad [B] time = 4.83, size = 519, normalized size = 1.94

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{210\,a^5\,e^5\,g^3-770\,a^4\,c\,d\,e^4\,f\,g^2+990\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,g^5\,\left(a\,e\,g-c\,d\,f\right)^4}-\frac{x^2\,\left(-10\,a^3\,c^2\,d^2\,e^3\,g^3+66\right.}{}\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(13/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((210*a^5*e^5*g^3 - 462*a^2
*c^3*d^3*e^2*f^3 + 990*a^3*c^2*d^2*e^3*f^2*g - 770*a^4*c*d*e^4*f*g^2)/(1155
*g^5*(a*e*g - c*d*f)^4) - (x^2*(462*c^5*d^5*f^3 - 10*a^3*c^2*d^2*e^3*g^3 +
66*a^2*c^3*d^3*e^2*f*g^2 - 198*a*c^4*d^4*e*f^2*g))/(1155*g^5*(a*e*g - c*d*f
)^4) - (32*c^5*d^5*x^5)/(1155*g^2*(a*e*g - c*d*f)^4) - (4*c^3*d^3*x^3*(3*a^
2*e^2*g^2 + 99*c^2*d^2*f^2 - 22*a*c*d*e*f*g))/(1155*g^4*(a*e*g - c*d*f)^4)
+ (16*c^4*d^4*x^4*(a*e*g - 11*c*d*f))/(1155*g^3*(a*e*g - c*d*f)^4) + (4*a*c
*d*e*x*(70*a^3*e^3*g^3 - 231*c^3*d^3*f^3 + 396*a*c^2*d^2*e*f^2*g - 275*a^2*
c*d*e^2*f*g^2))/(1155*g^5*(a*e*g - c*d*f)^4)))/(x^5*(f + g*x)^(1/2)*(d + e*
x)^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(
1/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (
10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

3.751 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=448

$$-\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{128c^2d^2g^3\sqrt{d+ex}}$$

```
[Out] -1/8*(-a*e*g+c*d*f)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g
^2/(e*x+d)^(3/2)+1/5*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
g/(e*x+d)^(5/2)-3/128*(-a*e*g+c*d*f)^5*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2
)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/64*(-a*e*g+c*d*f)^3*(g*
x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^3/(e*x+d)^(1/2)+1/
16*(-a*e*g+c*d*f)^2*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g
^3/(e*x+d)^(1/2)-3/128*(-a*e*g+c*d*f)^4*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/c^2/d^2/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}}-\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (-3*(c*d*f - a*e*g)^4*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(128*c^2*d^2*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^(3/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (
(c*d*f - a*e*g)^2*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(16*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(5/2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(5/2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) - (3
*(c*d*f - a*e*g)^5*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(128*c^(5/2)*d^(5/2)*g^(7/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(cdf-aeg)}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} -$$

$$= -\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} +$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

Mathematica [A] time = 6.01, size = 285, normalized size = 0.64

$$\frac{\sqrt{f+gx}\,((d+ex)(ae+cdx))^{7/2}\left(-\frac{15\sqrt{c}\sqrt{d}\sqrt{cd}(cdf-aeg)^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{g^{7/2}(ae+cdx)^{7/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}+\frac{15cd(cdf-aeg)^4}{g^3(ae+cdx)^3}-\frac{10cd(cdf-aeg)^3}{g^2(ae+cdx)^2}+8c\right.}{640c^3d^3(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(
d + e*x)^(5/2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(7/2)*Sqrt[f + g*x]*(48*c*d*(c*d*f - a*e*g) + (1
5*c*d*(c*d*f - a*e*g)^4)/(g^3*(a*e + c*d*x)^3) - (10*c*d*(c*d*f - a*e*g)^3)
/(g^2*(a*e + c*d*x)^2) + (8*c*d*(c*d*f - a*e*g)^2)/(g*(a*e + c*d*x)) + 128*
c^2*d^2*(f + g*x) - (15*Sqrt[c]*Sqrt[d]*Sqrt[c*d]*(c*d*f - a*e*g)^(9/2)*Arc
Sinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*
e*g])])/(g^(7/2)*(a*e + c*d*x)^(7/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
))/(640*c^3*d^3*(d + e*x)^(7/2))
```

fricas [A] time = 11.97, size = 1331, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/2560*(4*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2
 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*
g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 +
 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2
- 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^
5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f) - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 1
0*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^
5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d
^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(c*d*g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^4*x + c^3*d^4*g^4), 1/1280*(2*(
128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2 + 128*a^2*c
^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*g^5 + 16*(11
*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 + 64*a*c^4*d^
4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2 - 23*a*c^4*d
^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^5
*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 10*a^3*c^2*d^
3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^5*e*f^5 - 5*
a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d^2*e^4*f^2*g
^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^3*d^3*e*g^4*x + c^3*d^4*g^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 16.59Done
```

maple [B] time = 0.03, size = 1191, normalized size = 2.66

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^5e^5g^5\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-75a^4cd\,e^4f\,g^4\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 1/1280*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(256*x^4*c^4*d
^4*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+672*x^3*a*c^3*
d^3*e*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+352*x^3*c^4
*d^4*f*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*
(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x
^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*c*d*e^4*f
*g^4+150*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^
(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
/(c*d*g)^(1/2))*a^2*c^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^4
*d^4*e*f^4*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+
a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^5*d^5*f^5+496*x^2*a^2*c^2*d^2*
e^2*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+1024*x^2*a*c^
3*d^3*e*f*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+16*x^2*
c^4*d^4*f^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+20*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^3*c*d*e^3*g^4+932*(
c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c^2*d^2*e^2*f*g^
3+92*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^3*d^3*e*f^
2*g^2-20*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^4*d^4*f^
3*g-30*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^4*e^4*g^4+14
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*c*d*e^3*f*g^3+2
56*a^2*c^2*d^2*e^2*f^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^
(1/2)-140*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^3*d^3*e
*f^3*g+30*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^4*d^4*f^4
)/(e*x+d)^(1/2)/g^3/c^2/d^2/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/(c*d*g)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\left(gx + f\right)^{\frac{3}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e*
x + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

3.752 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{64cdg^3\sqrt{d+ex}}$$

```
[Out] -5/24*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
g^2/(e*x+d)^(3/2)+1/4*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
/g/(e*x+d)^(5/2)-5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2
)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/32*(-a*e*g+c*d*f)^2*(g*
x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)-5/64*(
-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g
^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{64cdg^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)
^(5/2),x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x]) - (5*(c*d*
f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
24*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(4*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(64*c^(3/2)*d^(3/2)*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{(5(cdf-aeg}{}$$

$$= -\frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} +$$

$$= \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5}{}$$

Mathematica [A] time = 1.13, size = 299, normalized size = 0.80

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}\,(f+gx)(ae+cdx)\left(15a^3e^3g^3+a^2cde^2g^2(73f+118gx)+ac^2d^2eg\left(-55f^2+36\right.\right.\right.}{192g^{7/2}(cd)^{5/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d
+ e*x)^(5/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d
^2*e*g*(-55*f^2 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x +
8*f*g^2*x^2 + 48*g^3*x^3)) - 15*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqr
t[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(192*(c*d)^(5/2)*g^(7/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

fricas [A] time = 5.35, size = 1065, normalized size = 2.83

$$\left[\frac{4\left(48\,c^4d^4g^4x^3+15\,c^4d^4f^3g-55\,ac^3d^3ef^2g^2+73\,a^2c^2d^2e^2fg^3+15\,a^3cde^3g^4+8\left(c^4d^4fg^3+17\,ac^3d^3eg^4\right)x^2-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d^3*e*f^2*g^2 +
 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f*g^3 + 17*a*c^
3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^3 - 59*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^4*x +
 c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d
^3*e*f^2*g^2 + 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f
*g^3 + 17*a*c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^
3 - 59*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^4*x + c^2*d
^3*g^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 7.91Done
```

maple [B] time = 0.02, size = 870, normalized size = 2.31

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^4e^4g^4\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-60a^3cd\,e^3f\,g^3\,\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*(c*d*g*x^
2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*g^3*x^3+15*a^4*e^4*g^4
*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c
*d*g)^(1/2))/(c*d*g)^(1/2))-60*a^3*c*d*e^3*f*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
+90*a^2*c^2*d^2*e^2*f^2*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*
g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-60*a*c^3*d^3*e*f^3*g
*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c
*d*g)^(1/2))/(c*d*g)^(1/2))+15*c^4*d^4*f^4*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-272*(
c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*g^3*x^2-16
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f*g^2*x^2-23
6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*g^3*x-7
2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f*g^2*x
+20*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^2*g*x-3
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3-146*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2+110*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-30*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1
/2)/c/d/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g^3/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx + f}}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

3.753 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=304

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8\sqrt{c}\,\sqrt{d}\,g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-$$

```
[Out] -5/12*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/
g^2/(e*x+d)^(3/2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*(g*x+f)^(1/2)
/g/(e*x+d)^(5/2)-5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(
1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/c^(1/2)
/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/8*(-a*e*g+c*d*f)^2*(g*x+
f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {864, 891, 63, 217, 206}

$$\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-\frac{5\sqrt{f+gx}\,\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}}-\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*g^3*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)) + (Sqrt[f + g*x]
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)) - (5*
(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
 + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*Sqrt[c]*Sqrt[d]*g^(7/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}} - \frac{(5(cdf-aeg))\int \frac{\left(ade+\left(cd\right.\right.}{(d+}}{6g}$$

$$= -\frac{5(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}\,(}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)}{}$$

Mathematica [A] time = 1.00, size = 229, normalized size = 0.75

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}\,(f+gx)\left(33a^2e^2g^2+2acdeg(13gx-20f)+c^2d^2\left(15f^2-10fgx+8g^2x^2\right)\right)-\frac{15\sqrt{c}\sqrt{d}}{}\right.}{24g^{7/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x)*(33*a^2*e^2*g^2 + 2*a*c*d
*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)) - (15*Sqrt
[c]*Sqrt[d]*(c*d*f - a*e*g)^(7/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*Arc
Sinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*
e*g])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

fricas [A] time = 3.12, size = 837, normalized size = 2.75

$$\left[\frac{4\left(8\,c^3d^3g^3x^2+15\,c^3d^3f^2g-40\,ac^2d^2efg^2+33\,a^2cde^2g^3-2\left(5\,c^3d^3fg^2-13\,ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*
a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^4*x + c*d^2*g^4), 1/
48*(2*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*a^2
*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^3*d^4*f^3 -
 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*
d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt
(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*
c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^4*x + c*d^2*g^4)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 508, normalized size = 1.67

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\sqrt{gx+f}\,\left(15a^3e^3g^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-45a^2cd\,e^2f\,g^2\ln\left(\frac{2cdg}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/48*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(15*a^3*e^3*g^3*
ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/
(c*d*g)^(1/2))-45*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+45*a*c^2*d^2*e*f^2*g*ln(
1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))-15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c
*d*g)^(1/2)*c^2*d^2*g^2*x^2+52*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*
c*d*e*g^2*x-20*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x+66*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-80*((g*x+f)*(c*d*x+a*e
))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*sq
rt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Timed out
```

3.754 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{15\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{15cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf}{4g^3\sqrt{d+ex}}$$

```
[Out] -2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(1/2)+5/
2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g^2/(e*x+d)^(3/
2)+15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/
(g*x+f)^(1/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/4*c*d*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{5cd\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{15cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^3\sqrt{d+ex}}+\frac{15\sqrt{c}\,\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (-15*c*d*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(4*g^3*Sqrt[d + e*x]) + (5*c*d*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(2*g^2*(d + e*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt[f + g*x]) + (15*Sqrt[c]*
Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*g^(7/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{g}$$

$$= \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+\right.}{g(d+ex)^{5/2}\sqrt{f+}}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}}{}$$

Mathematica [C] time = 0.10, size = 112, normalized size = 0.38

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2} {}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*x))/(c*d*f -
a*e*g))^(3/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (g*(a*e + c*d*x))/(-(c*d*f)
+ a*e*g)])/(7*c*d*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

fricas [A] time = 2.50, size = 915, normalized size = 3.11

$$\left[\frac{4\left(2\,c^2d^2g^2x^2-15\,c^2d^2f^2+25\,acdefg-8\,a^2e^2g^2-\left(5\,c^2d^2fg-9\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g
^2 - (5*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g
+ a^2*d*e^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2
 + (c^2*d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*
d^2*e - a^2*e^3)*f*g^2)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*
f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sq
rt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*
f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*
x + d)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), 1/8*(2*(2*c^2*d^2*g^2
*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 9
*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) - 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g + a^2*d*e^2*f*g^2 + (c^
2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2 + (c^2*d^2*e*f^3 + a^2
*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*d^2*e - a^2*e^3)*f*g^2)
*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^
4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 635, normalized size = 2.16

$$\frac{\left(15 a^{2} c d e^{2} g^{3} x \ln \left(\frac{2 c d g x+a e g+c d f+2 \sqrt{\left(g x+f\right)\left(c d x+a e\right)}\, \sqrt{c d g}}{2 \sqrt{c d g}}\right)-30 a c^{2} d^{2} e f g^{2} x \ln \left(\frac{2 c d g x+a e g+c d f+2 \sqrt{\left(g x+f\right)\left(c d x+a e\right)}\, \sqrt{c d g}}{2 \sqrt{c d g}}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x)
```

```
[Out] 1/8*(15*a^2*c*d*e^2*g^3*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*a*c^2*d^2*e*f*g^2*x*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))+15*c^3*d^3*f^2*g*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+15*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*
x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-3
0*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+4*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2+18*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)*c^2*d^2*f*g*x-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

3.755 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5c^2d^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}$$

```
[Out] -2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(3/2)-
10/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
(1/2)-5*c^(3/2)*d^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*c^2*d^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{5c^2d^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (5*c^2*d^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*
Sqrt[d + e*x]) - (10*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*Sqrt[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f + g*x)^(3/2)) - (5*c^(3/2)*d^(3/2)*(c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,d}{3g}$$

$$= -\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{3g(d+ex)^{5/2}(f+gx)^{3}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+a\right.\right.}{3g^2(d+ex)^{3/2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+a\right.\right.}{3g^2(d+ex)^{3/2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+a\right.\right.}{3g^2(d+ex)^{3/2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+a\right.\right.}{3g^2(d+ex)^{3/2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+a\right.\right.}{3g^2(d+ex)^{3/2}}$$

Mathematica [C] time = 0.12, size = 112, normalized size = 0.39

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}\,(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*x))/(c*d*f -
a*e*g))^(5/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (g*(a*e + c*d*x))/(-(c*d*f)
+ a*e*g)])/(7*c*d*Sqrt[d + e*x]*(f + g*x)^(5/2))
```

fricas [A] time = 2.31, size = 973, normalized size = 3.43

$$\left[\frac{4\left(3c^2d^2g^2x^2+15c^2d^2f^2-10acdefg-2a^2e^2g^2+2\left(10c^2d^2fg-7acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*g
^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - a*c*d^2*e*f^2*g
 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2*e*g
^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e*f*g^
2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f
^2*g^3 + 2*d*f*g^4)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c
*d*e*f*g - 2*a^2*e^2*g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3
*f^3 - a*c*d^2*e*f^2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2
*e*f^2*g - a*c*d^2*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*
f^3 - 2*a*c*d^2*e*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(-c*d/g)*ar
ctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2
*g^3 + 2*d*f*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 638, normalized size = 2.25

$$\left(15a\,c^2d^2e\,g^3x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-15c^3d^3f\,g^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x)
```

```
[Out] 1/6*(15*a*c^2*d^2*e*g^3*x^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*c^3*d^3*f*g^2*x^2*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))+30*a*c^2*d^2*e*f*g^2*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*c^3*d^3*f^2*g*x*ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
+15*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))
^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+6*((g*x+
f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2-28*(c*d*g)^(1/2)*((g*x+
f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+40*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^
(1/2)*c^2*d^2*f*g*x-4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2
-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
```

```
(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex + d\right)^{\frac{5}{2}}\left(gx + f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^{5/2}\,\left(d + e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

3.756 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=274

$$\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade\ldots\right.}{3g^2(d+ex)^{3/2}(f+\ldots}$$

```
[Out] -2/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
(3/2)-2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(
5/2)+2*c^(5/2)*d^(5/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*
x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+a\ldots\right.}{3g^2(d+ex)^{3/2}(f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e*x]
*Sqrt[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*(f + g*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f + g*x)^(5/2)) + (2*c^(5/2)*d^(5/2)*Sq
rt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx=-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{(cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx}{g}$$

$$=-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$=-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 1.37, size = 224, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{15\sqrt{c}\,\sqrt{d}\,(cdf-aeg)^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\,\sqrt{ae+cdx}}-\sqrt{g}\,\left(3a^2e^2g^2+acdeg(5f+11gx)+c\right.\right.}{15g^{7/2}\sqrt{d+ex}\,(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]*(3*a^2*e^2*g^2 + a*c*d*e*g*(5*f
 + 11*g*x) + c^2*d^2*(15*f^2 + 35*f*g*x + 23*g^2*x^2))) + (15*Sqrt[c]*Sqrt[
d]*(c*d*f - a*e*g)^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^(5/2)*ArcSinh[(S
qrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])
/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/(15*g^(7/2)*Sqrt[d + e*x]*(f + g*x)^(5/2))
```

fricas [A] time = 2.29, size = 933, normalized size = 3.41

$$\left[-\frac{4\left(23\,c^2d^2g^2x^2+15\,c^2d^2f^2+5\,acdefg+3\,a^2e^2g^2+\left(35\,c^2d^2fg+11\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{e}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] [-1/30*(4*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*
g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f
^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f
*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^
2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*
x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)
*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x), -1/15*(2
*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*g^2 + (35
*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f^3 + (3*c
^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f*g^2)*x^2
 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(
2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e
*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^
2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 511, normalized size = 1.86

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^3d^3g^3x^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+45c^3d^3f\,g^2x^2\ln\left(\frac{2cdgx+aeg+cd\cdots}{\cdots}\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x)
```

```
[Out] 1/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^3*c^3*d^3
*g^3+45*c^3*d^3*f*g^2*x^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+45*c^3*d^3*f^2*g*x*ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+
15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))-46*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*
c^2*d^2*g^2*x^2-22*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-
70*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-10*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*
f^2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^{7/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

3.757 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(g
*x+f)^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A] time = 0.08, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f +
 g*x)^(7/2))
```

fricas [B] time = 0.95, size = 299, normalized size = 4.75

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right.}{7\left(cd^2f^5-adef^4g+\left(cdefg^4-ae^2g^5\right)x^5+\left(4\,cdef^2g^3-adeg^5+\left(cd^2-4\,ae^2\right)fg^4\right)x^4+2\left(3\,cdef^3g^2-2\,adefg^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^5 -
 a*d*e*f^4*g + (c*d*e*f*g^4 - a*e^2*g^5)*x^5 + (4*c*d*e*f^2*g^3 - a*d*e*g^5
 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d*e*f^3*g^2 - 2*a*d*e*f*g^4 + (2*c
*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c*d*e*f^4*g - 3*a*d*e*f^2*g^3 + (3*c*d^
2 - 2*a*e^2)*f^3*g^2)*x^2 + (c*d*e*f^5 - 4*a*d*e*f^3*g^2 + (4*c*d^2 - a*e^2
)*f^4*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{2\,(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{7\left(gx+f\right)^{\frac{7}{2}}\left(aeg-cdf\right)(ex+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x)
```

```
[Out] -2/7/(g*x+f)^(7/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(5/2)/(e*x+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(9/2)), x)
```

mupad [B] time = 4.34, size = 325, normalized size = 5.16

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{2\,c^3\,d^3\,x^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a^2\,c\,d\,e^2\,x}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a\,c^2\,d^2\,e\,x^2}{7\,a\,e\,g^4-7\,c\,d\,f\,g}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(7\,c\,d\,f^4-7\,a\,e\,f^3\,g\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{x^2\,\sqrt{f+g\,x}\,\left(21\,a\,e\,f\,g^3-21\,c\,d\,f^2\,g^2\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}-\frac{x\,\sqrt{f+g\,x}\,\left(21\,c\,d\,f^3\,g-\right.}{7\,a\,e\,g^4-7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(9/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3)/(7*a*e*g^4 - 7
*c*d*f*g^3) + (2*c^3*d^3*x^3)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a^2*c*d*e^2*x)
/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a*c^2*d^2*e*x^2)/(7*a*e*g^4 - 7*c*d*f*g^3))
)/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^(1/2)*(7*c*d*f^4 - 7*a*
e*f^3*g)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) + (x^2*(f + g*x)^(1/2)*
(21*a*e*f*g^3 - 21*c*d*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3)
- (x*(f + g*x)^(1/2)*(21*c*d*f^3*g - 21*a*e*f^2*g^2)*(d + e*x)^(1/2))/(7*a*
e*g^4 - 7*c*d*f*g^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

3.758 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(g
*x+f)^(9/2)+4/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx}{9(cdf-aeg)}$$

$$=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\,(cd(9f+2gx)-7aeg)}{63\sqrt{d+ex}\,(f+gx)^{9/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-7*a*e*g + c*d*(9*f + 2*g
*x)))/(63*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(9/2))
```

fricas [B] time = 0.91, size = 639, normalized size = 4.95

$$\frac{}{63\left(c^2d^3f^7-2\,acd^2ef^6g+a^2de^2f^5g^2+\left(c^2d^2ef^2g^5-2\,acde^2fg^6+a^2e^3g^7\right)x^6+\left(5\,c^2d^2ef^3g^4+a^2de^2g^7+\left(c^2d^3-\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c^
3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*c^
2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*d*e^
2*f^5*g^2 + (c^2*d^2*e*f^2*g^5 - 2*a*c*d*e^2*f*g^6 + a^2*e^3*g^7)*x^6 + (5*
c^2*d^2*e*f^3*g^4 + a^2*d*e^2*g^7 + (c^2*d^3 - 10*a*c*d*e^2)*f^2*g^5 - (2*a
*c*d^2*e - 5*a^2*e^3)*f*g^6)*x^5 + 5*(2*c^2*d^2*e*f^4*g^3 + a^2*d*e^2*f*g^6
 + (c^2*d^3 - 4*a*c*d*e^2)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^5)*x^4 +
 10*(c^2*d^2*e*f^5*g^2 + a^2*d*e^2*f^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^4*g^
3 - (2*a*c*d^2*e - a^2*e^3)*f^3*g^4)*x^3 + 5*(c^2*d^2*e*f^6*g + 2*a^2*d*e^2
*f^3*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^5*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^4*g^
3)*x^2 + (c^2*d^2*e*f^7 + 5*a^2*d*e^2*f^4*g^3 + (5*c^2*d^3 - 2*a*c*d*e^2)*f
^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$-\frac{2\left(cdx+ae\right)\left(-2cdgx+7aeg-9cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{63\left(gx+f\right)^{\frac{9}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\left(ex+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-2*c*d*g*x+7*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/(g*x+f)^(9/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(11/2)), x)
```

mupad [B] time = 4.54, size = 315, normalized size = 2.44

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3\,\left(7\,a\,e\,g-9\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^4\,d^4\,x^4}{63\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(19\,a\,e\,g-\dots\right.}{63\,g^4\,\left(a\,e\,g-c\,d\dots\right.}\right.}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\dots}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(11/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3*(7*a*e*g - 9*c*
d*f))/(63*g^4*(a*e*g - c*d*f)^2) - (4*c^4*d^4*x^4)/(63*g^3*(a*e*g - c*d*f)^
2) + (2*c^3*d^3*x^3*(a*e*g - 9*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a^2*
c*d*e^2*x*(19*a*e*g - 27*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a*c^2*d^2*
e*x^2*(5*a*e*g - 9*c*d*f))/(21*g^4*(a*e*g - c*d*f)^2)))/(x^4*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

3.759 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-ae}$$

```
[Out] 2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(11/2)+8/99*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(7/2)/(g*x+f)^(9/2)+16/693*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Rubi [A] time = 0.23, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-ae}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(11/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(99*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*(c*d*f - a*e*g)^3*(
d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}}{11(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{99(cdf-aeg)^2(d+ex)^{7/2}(f+}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{99(cdf-aeg)^2(d+ex)^{7/2}(f+}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 0.58

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(63a^2e^2g^2-14acdeg(11f+2gx)+c^2d^2\left(99f^2+44fgx+8g^2x^2\right)\right)}{693\sqrt{d+ex}\,(f+gx)^{11/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^2*g^2 - 14*a*c*d
*e*g*(11*f + 2*g*x) + c^2*d^2*(99*f^2 + 44*f*g*x + 8*g^2*x^2)))/(693*(c*d*f
 - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^(11/2))
```

fricas [B] time = 0.94, size = 1101, normalized size = 5.56

$$\frac{}{693\left(c^3d^4f^9-3\,ac^2d^3ef^8g+3\,a^2cd^2e^2f^7g^2-a^3de^3f^6g^3+\left(c^3d^3ef^3g^6-3\,ac^2d^2e^2f^2g^7+3\,a^2cde^3fg^8-a^3e^4g^9\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/693*(8*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 154*a^4*c*d*e^4*f*g + 6
3*a^5*e^5*g^2 + 4*(11*c^5*d^5*f*g - a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^2
- 22*a*c^4*d^4*e*f*g + 3*a^2*c^3*d^3*e^2*g^2)*x^3 + (297*a*c^4*d^4*e*f^2 -
330*a^2*c^3*d^3*e^2*f*g + 113*a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3*e
^2*f^2 - 418*a^3*c^2*d^2*e^3*f*g + 161*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^9 - 3*a*
c^2*d^3*e*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3*d^3*e*
f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^3*e^4*g^9)*x^7
+ (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 - 18*a*c^2*d^2*e^2)*f^3*g
^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2*g^7 + 3*(a^2*c*d^2*e^2 - 2*a^3*e^4
)*f*g^8)*x^6 + 3*(5*c^3*d^3*e*f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c^3*d^4 - 15
*a*c^2*d^2*e^2)*f^4*g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3*g^6 + (6*a^
2*c*d^2*e^2 - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3 - 3*a^3*d*e^
3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^2*d^3*e - 4*a^
2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3*g^6)*x^4 + 5*(3*c^3*
d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^6*g^3
 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^5*g^4 + 3*(4*a^2*c*d^2*e^2 - a^3*e^4
)*f^4*g^5)*x^3 + 3*(2*c^3*d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g^5 + (5*c^3*d^4 -
6*a*c^2*d^2*e^2)*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^6*g^3 + (15*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e*f^9 - 6*a^3*d*e^3*f^5*
g^4 + 3*(2*c^3*d^4 - a*c^2*d^2*e^2)*f^8*g - 3*(6*a*c^2*d^3*e - a^2*c*d*e^3)
*f^7*g^2 + (18*a^2*c*d^2*e^2 - a^3*e^4)*f^6*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2\left(cdx+ae\right)\left(8g^{2}x^{2}c^{2}d^{2}-28acde\,g^{2}x+44c^{2}d^{2}fgx+63a^{2}e^{2}g^{2}-154acdefg+99f^{2}c^{2}d^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x\right.}{693\left(gx+f\right)^{\frac{11}{2}}\left(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}c^{3}d^{3}\right)\left(ex+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x)
```

```
[Out] -2/693*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+44*c^2*d^2*f*g*x+63*
a^2*e^2*g^2-154*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(5/2)/(g*x+f)^(11/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*
g-c^3*d^3*f^3)/(e*x+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(13/2)), x)
```

mupad [B] time = 4.82, size = 465, normalized size = 2.35

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{126\,a^5\,e^5\,g^2-308\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^3\,\left(6\,a^2\,c^3\,d^3\,e^2\,g^2-44\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,d^5\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4\,\sqrt{f+g\,x}\,\sqrt{}}{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(13/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((126*a^5*e^5*g^2 + 198*a^3
*c^2*d^2*e^3*f^2 - 308*a^4*c*d*e^4*f*g)/(693*g^5*(a*e*g - c*d*f)^3) + (x^3*
(198*c^5*d^5*f^2 + 6*a^2*c^3*d^3*e^2*g^2 - 44*a*c^4*d^4*e*f*g))/(693*g^5*(a
*e*g - c*d*f)^3) + (16*c^5*d^5*x^5)/(693*g^3*(a*e*g - c*d*f)^3) - (8*c^4*d^
4*x^4*(a*e*g - 11*c*d*f))/(693*g^4*(a*e*g - c*d*f)^3) + (2*a^2*c*d*e^2*x*(1
61*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 418*a*c*d*e*f*g))/(693*g^5*(a*e*g - c*d*
f)^3) + (2*a*c^2*d^2*e*x^2*(113*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 330*a*c*d*e
*f*g))/(693*g^5*(a*e*g - c*d*f)^3)))/(x^5*(f + g*x)^(1/2)*(d + e*x)^(1/2) +
 (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1/2)*(d +
```

```
e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (10*f^2*x^3
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/2)*(d + e*
x)^(1/2))/g^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

3.760 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cd\right.}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf}$$

```
[Out] 2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(13/2)+12/143*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(11/2)+16/429*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(9/2)+32/3003*c^3*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(7/2)/(g
*x+f)^(7/2)
```

Rubi [A] time = 0.32, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cd\right.}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(15/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(13/2)) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(7/2))/(143*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (16*c
^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*(c*d*f - a*e*g)^
3*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(3003*(c*d*f - a*e*g)^4*(d + e*x)^(7/2)*(f + g*x)^(7/2
))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}}{13(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{143(cdf-aeg)^2(d+ex)^{7/2}(f}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{143(cdf-aeg)^2(d+ex)^{7/2}(f}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{143(cdf-aeg)^2(d+ex)^{7/2}(f}$$

Mathematica [**A**] time = 0.14, size = 162, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(-231a^3e^3g^3+63a^2cde^2g^2(13f+2gx)-7ac^2d^2eg\left(143f^2+52fgx+8g^2x^2\right)\right.}{3003\sqrt{d+ex}(f+gx)^{13/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(15/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^3*g^3 + 63*a^2
*c*d*e^2*g^2*(13*f + 2*g*x) - 7*a*c^2*d^2*e*g*(143*f^2 + 52*f*g*x + 8*g^2*x
^2) + c^3*d^3*(429*f^3 + 286*f^2*g*x + 104*f*g^2*x^2 + 16*g^3*x^3)))/(3003*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*(f + g*x)^(13/2))
```

fricas [**B**] time = 1.05, size = 1648, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^4
*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 - a*
c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2*c^
4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*c^4*
d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2145*a
^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e^4*g^3
)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 2093*a^4*c
^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^4*e*f^10*
g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e^4*f^7*g^4
 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*e^3*f^2*g^9
 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*g^6 + a^4*d*
e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d^4*e - 21*a^2
*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4)*f^2*g^9 - (4
*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6*g^5 + a^4*d*e^
4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^3*d^4*e - 9*a^2*
c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^8 - (4*a^3
*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7*g^4 + 3*a^4*d*e^4
*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2*a*c^3*d^4*e - 5*a^
```

```
2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^4*g^7 - (
12*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^3*g^8)*x^5 + 35*(c^4*d^4*e*f^8*g^3 + a^4*d*
e^4*f^3*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^7*g^4 - 2*(2*a*c^3*d^4*e - 3*a^
2*c^2*d^2*e^3)*f^6*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^5*g^6 - (4
*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^7)*x^4 + 7*(3*c^4*d^4*e*f^9*g^2 + 5*a^4*d*e
^4*f^4*g^7 + (5*c^4*d^5 - 12*a*c^3*d^3*e^2)*f^8*g^3 - 2*(10*a*c^3*d^4*e - 9
*a^2*c^2*d^2*e^3)*f^7*g^4 + 6*(5*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^5 -
 (20*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^5*g^6)*x^3 + 7*(c^4*d^4*e*f^10*g + 3*a^4*
d*e^4*f^5*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^9*g^2 - 6*(2*a*c^3*d^4*e -
a^2*c^2*d^2*e^3)*f^8*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^7*g^4 -
(12*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^5)*x^2 + (c^4*d^4*e*f^11 + 7*a^4*d*e^4*f
^6*g^5 + (7*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^10*g - 2*(14*a*c^3*d^4*e - 3*a^2*c
^2*d^2*e^3)*f^9*g^2 + 2*(21*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^8*g^3 - (28*
a^3*c*d^2*e^3 - a^4*e^5)*f^7*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2\left(cdx+ae\right)\left(-16g^{3}x^{3}c^{3}d^{3}+56a\,c^{2}d^{2}e\,g^{3}x^{2}-104c^{3}d^{3}f\,g^{2}x^{2}-126a^{2}cd\,e^{2}g^{3}x+364a\,c^{2}d^{2}ef\,g^{2}x-286c^{3}d^{3}f^{2}gx}{3003\left(gx+f\right)^{\frac{13}{2}}\left(g^{4}e^{4}a^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+56*a*c^2*d^2*e*g^3*x^2-104*c^3*d^3
*f*g^2*x^2-126*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-286*c^3*d^3*f^2*g*
x+231*a^3*e^3*g^3-819*a^2*c*d*e^2*f*g^2+1001*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(g*x+f)^(13/2)/(a^4*e^4*g^4-4*
a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4
)/(e*x+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(15/2)), x)
```

mupad [B] time = 5.12, size = 627, normalized size = 2.35

$$-\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{462\,a^{6}\,e^{6}\,g^{3}-1638\,a^{5}\,c\,d\,e^{5}\,f\,g^{2}+2002\,a^{4}\,c^{2}\,d^{2}\,e^{4}\,f^{2}\,g-858\,a^{3}\,c^{3}\,d^{3}\,e^{3}\,f^{3}}{3003\,g^{6}\,\left(a\,e\,g-c\,d\,f\right)^{4}}-\frac{x^{3}\,\left(-10\,a^{3}\,c^{3}\,d^{3}\,e^{3}\,g^{3}+78\,a^{2}}{3}\right.}{x^{6}\,\sqrt{f+g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(15/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((462*a^6*e^6*g^3 - 858*a^3
*c^3*d^3*e^3*f^3 + 2002*a^4*c^2*d^2*e^4*f^2*g - 1638*a^5*c*d*e^5*f*g^2)/(30
03*g^6*(a*e*g - c*d*f)^4) - (x^3*(858*c^6*d^6*f^3 - 10*a^3*c^3*d^3*e^3*g^3
+ 78*a^2*c^4*d^4*e^2*f*g^2 - 286*a*c^5*d^5*e*f^2*g))/(3003*g^6*(a*e*g - c*d
*f)^4) - (32*c^6*d^6*x^6)/(3003*g^3*(a*e*g - c*d*f)^4) - (4*c^4*d^4*x^4*(3*
a^2*e^2*g^2 + 143*c^2*d^2*f^2 - 26*a*c*d*e*f*g))/(3003*g^5*(a*e*g - c*d*f)^
4) + (16*c^5*d^5*x^5*(a*e*g - 13*c*d*f))/(3003*g^4*(a*e*g - c*d*f)^4) + (2*
a^2*c*d*e^2*x*(567*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2717*a*c^2*d^2*e*f^2*g
- 2093*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4) + (2*a*c^2*d^2*e*x^
2*(371*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2145*a*c^2*d^2*e*f^2*g - 1469*a^2*c
*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4)))/(x^6*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^6*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^6 + (6*f*x^5*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (6*f^5*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (1
5*f^2*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (20*f^3*x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2))/g^3 + (15*f^4*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(15/2),x)
```

```
[Out] Timed out
```

3.761 $\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(d+ex)^{5/2}(f+gx)^{n+1}(ae+cdx)\,{}_2F_1\left(1,n-\frac{1}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^(5/2)*(g*x+f)^(1+n)*hypergeom([1, -1/2+n],[2+n],c*d*(g
*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((g*(a*e +
 c*d*x))/(c*d*f - a*e*g))])/(3*c*d*(a*e + c*d*x)*((c*d*(f + g*x))/(c*d*f -
a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 0.96

$$-\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, (g*(a*e +
 c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*((c*d*
(f + g*x))/(c*d*f - a*e*g))^n)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\left(gx+f\right)^n}{c^3d^3ex^4+a^3de^3+\left(c^3d^4+3\,ac^2d^2e^2\right)x^3+3\left(ac^2d^3e+a^2cde^3\right)x^2+\left(3\,a^2cd^2e^2+a^3e^4\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c^3*d^3*e*x^4 + a^3*d*e^3 + (c^3*d^4 + 3*a*c^2*d^2*e^2)*x^3 + 3*(a*c^2
*d^3*e + a^2*c*d*e^3)*x^2 + (3*a^2*c*d^2*e^2 + a^3*e^4)*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.73Unable to transpose Err
or: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^n}{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] int((e*x+d)^(5/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{n}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)^n\,{\left(d+e\,x\right)}^{5/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.762 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(d+ex)^{3/2}(f+gx)^{n+1}(ae+cdx)\,{}_2F_1\left(1,n+\frac{1}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^(3/2)*(g*x+f)^(1+n)*hypergeom([1, 1/2+n],[2+n],c*d*(g*
x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {891, 70, 69}

$$-\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((g*(a*e +
c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g))^n*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$= -\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 0.94

$$-\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (g*(a*e + c*
d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*
x))/(c*d*f - a*e*g))^n)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\left(gx+f\right)^n}{c^2d^2ex^3+a^2de^2+\left(c^2d^3+2\,acde^2\right)x^2+\left(2\,acd^2e+a^2e^3\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c^2*d^2*e*x^3 + a^2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2*e
 + a^2*e^3)*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.08Unable to transpose Err
or: Bad Argument Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^n}{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{n}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)^{n}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.763 $\int \frac{\sqrt{d+ex}\,(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{d+ex}\,(f+gx)^{n+1}(ae+cdx)\,{}_2F_1\left(1,n+\frac{3}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c
*d*f))*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{891, 70, 69\}$

$$\frac{2\sqrt{d+ex}\,(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (2*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g
))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{2(ae+cdx)\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 0.94

$$\frac{2(f+gx)^n\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2
, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[d + e*x]*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex+d}\left(gx+f\right)^n}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^n}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\left(gx+f\right)^n}{\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] int((e*x+d)^(1/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{n}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)^{n}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.764 $\int \frac{(f+gx)^n \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}} \, dx$

Optimal. Leaf size=104

$$-\frac{(f+gx)^{n+1}(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\ {}_2F_1\left(1,n+\frac{5}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\sqrt{d+ex}\,(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 5/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c
*d*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{891, 70, 69\}$

$$\frac{2(f+gx)^n(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\ {}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (2*(a*e + c*d*x)*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*Hy
pergeometric2F1[3/2, -n, 5/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(3*c*d
*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\int\sqrt{ae+cdx}\,(f+gx)^n\,dx}{\sqrt{ae+cdx}\,\sqrt{d+ex}}$$
$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right)\int\sqrt{ae+}$$
$$= \frac{2(ae+cdx)(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 0.96

$$\frac{2(f+gx)^n((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^n*Hypergeometric2F1[3/2, -n, 5
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*(d + e*x)^(3/2)*((c*d*(f
+ g*x))/(c*d*f - a*e*g))^n)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^n}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x +
 d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^n}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}\,\left(gx+f\right)^n}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((g*x+f)^n*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^n}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f + g\,x\right)^n\,\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\sqrt{d + e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(d + ex\right)\left(ae + cdx\right)}\left(f + gx\right)^n}{\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**n/sqrt(d + e*x), x)
```

3.765 $\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(f+gx)^{n+1}(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\,{}_2F_1\left(1,n+\frac{7}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(d+ex)^{3/2}(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*hypergeo
m([1, 7/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = $\{891, 70, 69\}$

$$\frac{2(f+gx)^n(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (2*(a*e + c*d*x)^2*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[5/2, -n, 7/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(5*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,\int (ae+cdx)^{3/2}(f+gx)^n\,dx}{\sqrt{ae+cdx}\,\sqrt{d+ex}}$$
$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right)\int (a}{\sqrt{ae+cdx}\,\sqrt{d+ex}}$$
$$= \frac{2(ae+cdx)^2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+}}{5cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.10, size = 100, normalized size = 0.96

$$\frac{2(f+gx)^n((d+ex)(ae+cdx))^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*x)^n*Hypergeometric2F1[5/2, -n, 7
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(d + e*x)^(5/2)*((c*d*(f
+ g*x))/(c*d*f - a*e*g))^n)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdx+ae)\left(gx+f\right)^n}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*(g*x + f
)^n/sqrt(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
d)^(3/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] int((g*x+f)^n*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}\left(gx + f\right)^n}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f + g\,x\right)^n\,\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.766 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(f+gx)^{n+1}(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\,{}_2F_1\left(1,n+\frac{9}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(d+ex)^{5/2}(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*hypergeo
m([1, 9/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[7/2, -n, 9/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(7*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\int (ae+cdx)^{5/2}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right)\int (ae+}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{2(ae+cdx)^3(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{7cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.07, size = 110, normalized size = 1.06

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric
2F1[7/2, -n, 9/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*c*d*Sqrt[d + e*
x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^n}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 6.4Done
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] int((g*x+f)^n*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}} \left(gx + f\right)^n}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f + g\,x\right)^n\,\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(d + e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.767 $\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=103

$$-\frac{(d+ex)^m(f+gx)^{n+1}(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(1,-m+n+2;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(cdf-aeg)}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^m*(g*x+f)^(1+n)*hypergeom([1, 2-m+n],[2+n],c*d*(g*x+f)
/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = $\{891, 70, 69\}$

$$\frac{(d+ex)^m(f+gx)^{n+1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(m,n+1;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{g(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m (f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \left((ae+cdx)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex\right.\right.$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+\right.\right.$$

$$= \frac{\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{1+n} \left(ade+\left(cd^2+\right.\right.}{g(1+}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 0.92

$$\frac{(d+ex)^m (f+gx)^{n+1} ((d+ex)(ae+cdx))^{-m} \left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(m,n+1;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{g(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] (((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*((a*e + c*d*x)*(d + e*x))^m)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m \left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m \left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m} (ex+d)^m \left(gx+f\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

[Out] int((e*x+d)^m*(g*x+f)^n/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.768 $\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=343

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m}{}$$

```
[Out] -6*(-a*e*g+c*d*f)^2*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^4/d^4/e/(m^2-7*m+12)/(m^2-3*m+2)+6*g*
(-a*e*g+c*d*f)^2*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/
e/(2-m)/(3-m)/(4-m)+3*(-a*e*g+c*d*f)*(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/(3-m)/(4-m)+(e*x+d)^(-1+m)*(g*x+f)^3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(4-m)
```

Rubi [A] time = 0.45, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {870, 794, 648}

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)
^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^4*d^4*e*(1 -
m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(2 - m)*(3 - m)*(4 - m))
+ (3*(c*d*f - a*e*g)*(d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^(-1 + m)*(
f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(4 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1}}{cd(4-m)}$$

$$= \frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2 \left(ade+\left(cd^2+ae^2\right.\right.}{c^2d^2(3-m)(4-m)}$$

$$= \frac{6g(cdf-aeg)^2(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{c^3d^3e(2-m)(3-m)(4-m)}$$

$$= -\frac{6(cdf-aeg)^2 \left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(}{c^4d^4e(1-m)(2-m)(}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.39

$$\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\left(\frac{3g^2(ae+cdx)^2(aeg-cdf)}{m-3}-\frac{3g(ae+cdx)(cdf-aeg)^2}{m-2}-\frac{(cdf-aeg)^3}{m-1}-\frac{g^3(ae+cdx)^3}{m-4}\right)}{c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3/
(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*d*
f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))/(c
^4*d^4)
```

fricas [B] time = 0.81, size = 705, normalized size = 2.06

$$-\frac{\left(ac^3d^3ef^3m^3-24\,ac^3d^3ef^3+36\,a^2c^2d^2e^2f^2g-24\,a^3cde^3fg^2+6\,a^4e^4g^3+\left(c^4d^4g^3m^3-6\,c^4d^4g^3m^2+11\,c^4d^4g^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^3*d^3*e*f^3*m^3 - 24*a*c^3*d^3*e*f^3 + 36*a^2*c^2*d^2*e^2*f^2*g - 24*
a^3*c*d*e^3*f*g^2 + 6*a^4*e^4*g^3 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2 +
11*c^4*d^4*g^3*m - 6*c^4*d^4*g^3)*x^4 - (24*c^4*d^4*f*g^2 - (3*c^4*d^4*f*g^
2 + a*c^3*d^3*e*g^3)*m^3 + 3*(7*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m^2 - 2*(2
1*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m)*x^3 - 3*(3*a*c^3*d^3*e*f^3 - a^2*c^2*
d^2*e^2*f^2*g)*m^2 - 3*(12*c^4*d^4*f^2*g - (c^4*d^4*f^2*g + a*c^3*d^3*e*f*g
^2)*m^3 + (8*c^4*d^4*f^2*g + 5*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m^2
 - (19*c^4*d^4*f^2*g + 4*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m)*x^2 +
(26*a*c^3*d^3*e*f^3 - 21*a^2*c^2*d^2*e^2*f^2*g + 6*a^3*c*d*e^3*f*g^2)*m - (
24*c^4*d^4*f^3 - (c^4*d^4*f^3 + 3*a*c^3*d^3*e*f^2*g)*m^3 + 3*(3*c^4*d^4*f^3
 + 7*a*c^3*d^3*e*f^2*g - 2*a^2*c^2*d^2*e^2*f*g^2)*m^2 - 2*(13*c^4*d^4*f^3 +
 18*a*c^3*d^3*e*f^2*g - 12*a^2*c^2*d^2*e^2*f*g^2 + 3*a^3*c*d*e^3*g^3)*m)*x)
*(e*x + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m
 + 24*c^4*d^4)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

giac [B] time = 0.33, size = 2024, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 3*(x*e + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) - 6*(x*e + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + (x*e + d)^m*a*c^3*d^3*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) + 3*(x*e + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e) -
m*log(x*e + d)) - 21*(x*e + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*log(c*d*x + a*
e) - m*log(x*e + d)) + 11*(x*e + d)^m*c^4*d^4*g^3*m*x^4*e^(-m*log(c*d*x + a
*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f*g^2*m^3*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d) + 1) - 3*(x*e + d)^m*a*c^3*d^3*g^3*m^2*x^3*e^(-m
*log(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*c^4*d^4*f^3*m^3*x*e^(
-m*log(c*d*x + a*e) - m*log(x*e + d)) - 24*(x*e + d)^m*c^4*d^4*f^2*g*m^2*x^
2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 42*(x*e + d)^m*c^4*d^4*f*g^2*m
*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 6*(x*e + d)^m*c^4*d^4*g^3*x
^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f^2*g
*m^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 15*(x*e + d)^m*a*c^3*
d^3*f*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 2*(x*e + d
)^m*a*c^3*d^3*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 9*(x
*e + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 57*(
x*e + d)^m*c^4*d^4*f^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 2
4*(x*e + d)^m*c^4*d^4*f*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) +
3*(x*e + d)^m*a^2*c^2*d^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) + (x*e + d)^m*a*c^3*d^3*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - 21*(x*e + d)^m*a*c^3*d^3*f^2*g*m^2*x*e^(-m*log(c*d*x + a*e) - m
*log(x*e + d) + 1) + 12*(x*e + d)^m*a*c^3*d^3*f*g^2*m*x^2*e^(-m*log(c*d*x +
 a*e) - m*log(x*e + d) + 1) + 26*(x*e + d)^m*c^4*d^4*f^3*m*x*e^(-m*log(c*d*
x + a*e) - m*log(x*e + d)) - 36*(x*e + d)^m*c^4*d^4*f^2*g*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d)) + 6*(x*e + d)^m*a^2*c^2*d^2*f*g^2*m^2*x*e^(-m*l
og(c*d*x + a*e) - m*log(x*e + d) + 2) - 3*(x*e + d)^m*a^2*c^2*d^2*g^3*m*x^2
*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 9*(x*e + d)^m*a*c^3*d^3*f^3
*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 36*(x*e + d)^m*a*c^3*d^
3*f^2*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 24*(x*e + d)^m*c
^4*d^4*f^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a^2*c
^2*d^2*f^2*g*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 24*(x*e + d
)^m*a^2*c^2*d^2*f*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) + 26
*(x*e + d)^m*a*c^3*d^3*f^3*m*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) +
 6*(x*e + d)^m*a^3*c*d*g^3*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 3)
 - 21*(x*e + d)^m*a^2*c^2*d^2*f^2*g*m*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) - 24*(x*e + d)^m*a*c^3*d^3*f^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
 d) + 1) + 6*(x*e + d)^m*a^3*c*d*f*g^2*m*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 3) + 36*(x*e + d)^m*a^2*c^2*d^2*f^2*g*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) - 24*(x*e + d)^m*a^3*c*d*f*g^2*e^(-m*log(c*d*x + a*e) - m*l
og(x*e + d) + 3) + 6*(x*e + d)^m*a^4*g^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 4))/(c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m +
 24*c^4*d^4)
```

maple [A] time = 0.01, size = 527, normalized size = 1.54

$$-\frac{\left(c^{3} d^{3} g^{3} m^{3} x^{3}+3 c^{3} d^{3} f\, g^{2} m^{3} x^{2}-6 c^{3} d^{3} g^{3} m^{2} x^{3}+3 a\, c^{2} d^{2} e\, g^{3} m^{2} x^{2}+3 c^{3} d^{3} f^{2} g\, m^{3} x-21 c^{3} d^{3} f\, g^{2} m^{2} x^{2}+11 c^{3} d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^3/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*x
^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x^2
```

```
+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+c^3
*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^3*x^
3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m*x+6*
a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3*f*g^2
*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*m+24*a*
c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3-24*a^2*
c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c*d*e*x^2+
a*e^2*x+c*d^2*x+a*d*e)^m)/c^4/d^4/(m^4-10*m^3+35*m^2-50*m+24)
```

maxima [A] time = 0.61, size = 331, normalized size = 0.97

$$-\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)}-\frac{3\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)f^2g}{\left(m^2-3m+2\right)(cdx+ae)^m c^2d^2}-\frac{3\left(\left(m^2-3m+2\right)c^3d^3x^3+\left(m^2-m\right)ac^2d^2ex^2+\right.}{\left(m^3-6m^2+11m-6\right)(cdx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - 3
*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m
*x + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3) -
((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*e*x^3
 + 3*(m^2 - m)*a^2*c^2*d^2*e^2*x^2 + 6*a^3*c*d*e^3*m*x + 6*a^4*e^4)*g^3/((m
^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

mupad [B] time = 3.75, size = 615, normalized size = 1.79

$$-\frac{\frac{g^3x^4(d+ex)^m\left(m^3-6m^2+11m-6\right)}{m^4-10m^3+35m^2-50m+24}+\frac{x(d+ex)^m\left(6a^3cde^3g^3m+6a^2c^2d^2e^2fg^2m^2-24a^2c^2d^2e^2fg^2m+3ac^3d^3ef^2gm^3-21ac^3d^3ef^2gm^2+\right.}{c^4d^4\left(m^4-10m^3+35m^2-50m+24\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^3*x^4*(d + e*x)^m*(11*m - 6*m^2 + m^3 - 6))/(35*m^2 - 50*m - 10*m^3 +
m^4 + 24) + (x*(d + e*x)^m*(26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3 - 9*c^4*d^4*f
^3*m^2 + c^4*d^4*f^3*m^3 + 6*a^3*c*d*e^3*g^3*m - 24*a^2*c^2*d^2*e^2*f*g^2*m
 + 36*a*c^3*d^3*e*f^2*g*m + 6*a^2*c^2*d^2*e^2*f*g^2*m^2 - 21*a*c^3*d^3*e*f^
2*g*m^2 + 3*a*c^3*d^3*e*f^2*g*m^3))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4
+ 24)) + (a*e*(d + e*x)^m*(6*a^3*e^3*g^3 - 24*c^3*d^3*f^3 + 26*c^3*d^3*f^3*
m - 9*c^3*d^3*f^3*m^2 + c^3*d^3*f^3*m^3 + 36*a*c^2*d^2*e*f^2*g - 24*a^2*c*d
*e^2*f*g^2 - 21*a*c^2*d^2*e*f^2*g*m + 6*a^2*c*d*e^2*f*g^2*m + 3*a*c^2*d^2*e
*f^2*g*m^2))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (3*g*x^2*(m -
1)*(d + e*x)^m*(12*c^2*d^2*f^2 + a^2*e^2*g^2*m - 7*c^2*d^2*f^2*m + c^2*d^2*
f^2*m^2 - 4*a*c*d*e*f*g*m + a*c*d*e*f*g*m^2))/(c^2*d^2*(35*m^2 - 50*m - 10*
m^3 + m^4 + 24)) + (g^2*x^3*(d + e*x)^m*(a*e*g*m - 12*c*d*f + 3*c*d*f*m)*(m
^2 - 3*m + 2))/(c*d*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)))/(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^m
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

3.769 $\int (d+ex)^m (f+gx)^2 \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{-m} dx$

Optimal. Leaf size=246

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(}{}$$

```
[Out] -2*(-a*e*g+c*d*f)*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/e/(1-m)/(2-m)/(3-m)+2*g*(-a*e*g+c
*d*f)*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/e/(2-m)/(3-
m)+(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(3-
m)
```

Rubi [A] time = 0.20, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {870, 794, 648}

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(
-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(1 - m)
*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^(-1 + m
)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)
)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1}}{cd(3-m)}$$
$$= \frac{2g(cdf-aeg)(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{c^2d^2e(2-m)(3-m)}$$
$$= -\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d}{c^3d^3e(1-m)(2-}$$

Mathematica [A] time = 0.11, size = 131, normalized size = 0.53

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\left(2a^2e^2g^2+2acdeg(f(m-3)+g(m-1)x)+c^2d^2\left(f^2\left(m^2-5m+6\right)+2fg\left(\right.\right.\right.}{c^3d^3(m-3)(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2*
a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*f*
g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 + m)
*(-1 + m)))
```

fricas [A] time = 0.92, size = 350, normalized size = 1.42

$$-\frac{\left(ac^2d^2ef^2m^2+6\,ac^2d^2ef^2-6\,a^2cde^2fg+2\,a^3e^3g^2+\left(c^3d^3g^2m^2-3\,c^3d^3g^2m+2\,c^3d^3g^2\right)x^3+\left(6\,c^3d^3fg+\left(2\,c^3\right.\right.\right.}{\left(c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^2*d^2*e*f^2*m^2 + 6*a*c^2*d^2*e*f^2 - 6*a^2*c*d*e^2*f*g + 2*a^3*e^3*g
^2 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 + (6*c^3*d^3*f
*g + (2*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*m^2 - (8*c^3*d^3*f*g + a*c^2*d^2*e*g
^2)*m)*x^2 - (5*a*c^2*d^2*e*f^2 - 2*a^2*c*d*e^2*f*g)*m + (6*c^3*d^3*f^2 + (
c^3*d^3*f^2 + 2*a*c^2*d^2*e*f*g)*m^2 - (5*c^3*d^3*f^2 + 6*a*c^2*d^2*e*f*g -
 2*a^2*c*d*e^2*g^2)*m)*x)*(e*x + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^
3*d^3*m - 6*c^3*d^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

giac [B] time = 0.28, size = 981, normalized size = 3.99

$$-\frac{(xe+d)^m c^3d^3g^2m^2x^3e^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+2\,(xe+d)^m c^3d^3fgm^2x^2e^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}-3\,(xe+d)^m c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 2*(x*e + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) - 3*(x*e + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (x*e + d)^m*a*c^2*d^2*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 1) + (x*e + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
```

```
+ d)) - 8*(x*e + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + 2*(x*e + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) + 2*(x*e + d)^m*a*c^2*d^2*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - (x*e + d)^m*a*c^2*d^2*g^2*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 1) - 5*(x*e + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 6*(x*e + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d)) + (x*e + d)^m*a*c^2*d^2*f^2*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) - 6*(x*e + d)^m*a*c^2*d^2*f*g*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 6*(x*e + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 2*(x*e + d)^m*a^2*c*d*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) - 5*(x*e + d)^m*a*c^2*d^2*f^2*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 2*(x*e + d)^m*a^2*c*d*f*g*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) + 6*(x*e + d)^m*a*c^2*d^2*f^2*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) - 6*(x*e + d)^m*a^2*c*d*f*g*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) + 2*(x*e + d)^m*a^3*g^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 3))/(c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^3*d^3*m - 6*c^3*d^3)
```

maple [A] time = 0.01, size = 235, normalized size = 0.96

$$-\frac{(cdx+ae)\left(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2c^2d^2\right.}{\left(m^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+2
*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*c*
d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-6*a
*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c
^3/d^3/(m^3-6*m^2+11*m-6)
```

maxima [A] time = 0.56, size = 193, normalized size = 0.78

$$-\frac{(cdx+ae)f^2}{(cdx+ae)^m cd(m-1)}-\frac{2\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)fg}{\left(m^2-3\,m+2\right)(cdx+ae)^m c^2d^2}-\frac{\left(\left(m^2-3\,m+2\right)c^3d^3x^3+\left(m^2-m\right)ac^2d^2ex^2+\right.}{\left(m^3-6\,m^2+11\,m-6\right)(cdx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - ((m
^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m*x +
 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

mupad [B] time = 3.52, size = 327, normalized size = 1.33

$$-\frac{\frac{g^2\,x^3\,(d+e\,x)^m\,\left(m^2-3\,m+2\right)}{m^3-6\,m^2+11\,m-6}+\frac{x\,(d+e\,x)^m\,\left(2\,a^2\,c\,d\,e^2\,g^2\,m+2\,a\,c^2\,d^2\,e\,f\,g\,m^2-6\,a\,c^2\,d^2\,e\,f\,g\,m+c^3\,d^3\,f^2\,m^2-5\,c^3\,d^3\,f^2\,m+6\,c^3\,d^3\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}+\frac{a\,e\,(d+e\,x)}{}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^2*x^3*(d + e*x)^m*(m^2 - 3*m + 2))/(11*m - 6*m^2 + m^3 - 6) + (x*(d +
e*x)^m*(6*c^3*d^3*f^2 - 5*c^3*d^3*f^2*m + c^3*d^3*f^2*m^2 + 2*a^2*c*d*e^2*g
^2*m + 2*a*c^2*d^2*e*f*g*m^2 - 6*a*c^2*d^2*e*f*g*m))/(c^3*d^3*(11*m - 6*m^2
```

```
 + m^3 - 6)) + (a*e*(d + e*x)^m*(2*a^2*e^2*g^2 + 6*c^2*d^2*f^2 - 5*c^2*d^2*
f^2*m + c^2*d^2*f^2*m^2 - 6*a*c*d*e*f*g + 2*a*c*d*e*f*g*m))/(c^3*d^3*(11*m
- 6*m^2 + m^3 - 6)) + (g*x^2*(m - 1)*(d + e*x)^m*(a*e*g*m - 6*c*d*f + 2*c*d
*f*m))/(c*d*(11*m - 6*m^2 + m^3 - 6)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^m
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

3.770 $\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=150

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-\ldots\right.}{c^2d^2e(1-m)(2-m)}$$

```
[Out] -(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1-m)/c^2/d^2/e/(1-m)/(2-m)+g*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1-m)/c/d/e/(2-m)
```

Rubi [A] time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {794, 648}

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-\ldots\right.}{c^2d^2e(1-m)(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(
d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx &= \frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}-\ldots\\ &= -\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\ldots}{c^2d^2e(1-m)(2-\ldots}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.45

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}(aeg+cd(f(m-2)+g(m-1)x))}{c^2d^2(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-2
 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

fricas [A] time = 0.91, size = 145, normalized size = 0.97

$$-\frac{\left(acdefm-2\,acdef+a^2e^2g+\left(c^2d^2gm-c^2d^2g\right)x^2-\left(2\,c^2d^2f-\left(c^2d^2f+acdeg\right)m\right)x\right)(ex+d)^m}{\left(c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] -(a*c*d*e*f*m - 2*a*c*d*e*f + a^2*e^2*g + (c^2*d^2*g*m - c^2*d^2*g)*x^2 - (
2*c^2*d^2*f - (c^2*d^2*f + a*c*d*e*g)*m)*x)*(e*x + d)^m/((c^2*d^2*m^2 - 3*c
^2*d^2*m + 2*c^2*d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

giac [B] time = 0.25, size = 369, normalized size = 2.46

$$-\frac{(xe+d)^m c^2d^2gmx^2e^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+(xe+d)^m c^2d^2fmxe^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}-(xe+d)^m c^2d^2gx^2e^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] -((x*e + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x
*e + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - (x*e + d
)^m*c^2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*
c*d*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*c^2*
d^2*f*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*c*d*f*m*e^
(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*a*c*d*f*e^(-m*lo
g(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*a^2*g*e^(-m*log(c*d*x +
a*e) - m*log(x*e + d) + 2))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

maple [A] time = 0.00, size = 89, normalized size = 0.59

$$-\frac{\left(cdgmx+cdfm-cdgx+aeg-2cdf\right)(cdx+ae)(ex+d)^m\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{\left(m^2-3m+2\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

maxima [A] time = 0.52, size = 94, normalized size = 0.63

$$-\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)}-\frac{\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)g}{\left(m^2-3\,m+2\right)(cdx+ae)^m c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

[Out] -(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*c
*d*e*m*x + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)

mupad [B] time = 3.36, size = 139, normalized size = 0.93

$$-\frac{\frac{g\,x^2\,(m-1)\,(d+e\,x)^m}{m^2-3\,m+2}+\frac{x\,(d+e\,x)^m\,\left(a\,e\,g\,m-2\,c\,d\,f+c\,d\,f\,m\right)}{c\,d\,\left(m^2-3\,m+2\right)}+\frac{a\,e\,(d+e\,x)^m\,\left(a\,e\,g-2\,c\,d\,f+c\,d\,f\,m\right)}{c^2\,d^2\,\left(m^2-3\,m+2\right)}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)

[Out] -((g*x^2*(m - 1)*(d + e*x)^m)/(m^2 - 3*m + 2) + (x*(d + e*x)^m*(a*e*g*m - 2
*c*d*f + c*d*f*m))/(c*d*(m^2 - 3*m + 2)) + (a*e*(d + e*x)^m*(a*e*g - 2*c*d*
f + c*d*f*m))/(c^2*d^2*(m^2 - 3*m + 2)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)

[Out] Exception raised: TypeError

3.771 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=54

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

```
[Out] (e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {648}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.78

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}}{cd(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

fricas [A] time = 0.95, size = 57, normalized size = 1.06

$$-\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fri
cas")
```

```
[Out] -(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^m)
```

giac [A] time = 0.22, size = 87, normalized size = 1.61

$$-\frac{(xe+d)^m cdxe^{(-m\log(cdx+ae)-m\log(xe+d))} + (xe+d)^m ae^{(-m\log(cdx+ae)-m\log(xe+d)+1)}}{cdm-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gia
c")
```

```
[Out] -((x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*
a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1))/(c*d*m - c*d)
```

maple [A] time = 0.00, size = 57, normalized size = 1.06

$$-\frac{(cdx+ae)(ex+d)^m \left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{(m-1)\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

maxima [A] time = 0.47, size = 33, normalized size = 0.61

$$-\frac{cdx+ae}{(cdx+ae)^m cd(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="max
ima")
```

```
[Out] -(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

mupad [B] time = 3.25, size = 57, normalized size = 1.06

$$-\frac{(a\,e+c\,d\,x)\,(d+e\,x)^m}{c\,d\,(m-1)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^m)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Exception raised: TypeError
```

3.772 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx$

Optimal. Leaf size=99

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

```
[Out] (c*d*x+a*e)*(e*x+d)^m*hypergeom([1, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*f
))/(-a*e*g+c*d*f)/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {891, 68}

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e + c
*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^m)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae$$

$$=\frac{(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(1,1\right.}{(cdf-aeg)(1-m)}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.83

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\,{}_2F_1\left(1,1-m;2-m;\frac{g(ae+cdx)}{aeg-cdf}\right)}{(m-1)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2F1[1
, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)*(-1
 + m)))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{(gx+f)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] integral((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
[Out] int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
[Out] Timed out
```

3.773 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$

Optimal. Leaf size=101

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

```
[Out] c*d*(c*d*x+a*e)*(e*x+d)^m*hypergeom([2, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c
*d*f))/(-a*e*g+c*d*f)^2/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {891, 68}

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*e
 + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^m)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int$$

$$= \frac{cd(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}{}_2F_1}{(cdf-aeg)^2(1-m)}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.83

$$-\frac{cd(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}{}_2F_1\left(2,1-m;2-m;\frac{g(ae+cdx)}{aeg-cdf}\right)}{(m-1)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c*d*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2
F1[2, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)
^2*(-1 + m)))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{\left(g^2x^2+2\,fgx+f^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\left(gx+f\right)^2\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^2/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^2/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\left(gx+f\right)^2\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{\left(f+g\,x\right)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
[Out] int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
[Out] Timed out
```

3.774 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$

Optimal. Leaf size=105

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

```
[Out] c^2*d^2*(c*d*x+a*e)*(e*x+d)^m*hypergeom([3, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e
*g+c*d*f))/(-a*e*g+c*d*f)^3/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {891, 68}

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g*
(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae}{(}$$

$$= \frac{c^2d^2(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\Big(}{(cdf-aeg)^3(1-m)}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.84

$$-\frac{c^2d^2(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\,{}_2F_1\left(3,1-m;2-m;\frac{g(ae+cdx)}{aeg-cdf}\right)}{(m-1)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c^2*d^2*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeomet
ric2F1[3, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*
e*g)^3*(-1 + m)))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{\left(g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^m}{\left(gx+f\right)^3\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int\frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^3/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^3/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^m}{\left(gx+f\right)^3\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.775 $\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$ d

Optimal. Leaf size=105

$$\frac{2(f+gx)^{5/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

```
[Out] 2/5*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(5/2)*hypergeom([5/
2, m],[7/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^{5/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \left((ae+cdx)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.\right.$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+c\right.\right.$$

$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{5/2} \left(ade+\left(cd^2+a\right.\right.}{5g}$$

Mathematica [A] time = 0.05, size = 93, normalized size = 0.89

$$\frac{2(f+gx)^{5/2}(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*((a*e +
c*d*x)*(d + e*x))^m)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(gx+f\right)^{\frac{3}{2}}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{\frac{3}{2}}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{\frac{3}{2}} \left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m} (ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^(3/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(3/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{\frac{3}{2}}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)^{3/2}\,{\left(d+e\,x\right)}^m}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.776 $\int (d+ex)^m \sqrt{f+gx} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=105

$$\frac{2(f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

```
[Out] 2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(3/2)*hypergeom([3/
2, m],[5/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}dx=\left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.\right.$$
$$=\left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+\right.\right.$$
$$=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+\right.\right.}{3g}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.89

$$\frac{2(f+gx)^{3/2}(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e +
c*d*x)*(d + e*x))^m)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{gx+f}\,(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}\,(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{gx+f}\,\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^(1/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(1/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}\,(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,{(d+e\,x)}^m}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.777 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{f+gx}\,(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

```
[Out] 2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([1/2, m],[3/2],c*d*
(g*x+f)/(-a*e*g+c*d*f))*(g*x+f)^(1/2)/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
m)
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2\sqrt{f+gx}\,(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int$$

$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{g}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.88

$$\frac{2\sqrt{f+gx}\,(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m\, {}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m),x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*
x)*(d + e*x))^m)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(1/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(1/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.778 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

```
[Out] -2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-1/2, m],[1/2],c*
d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)/(g*x+f)^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right.$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{g\sqrt{f+gx}}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.88

$$-\frac{2(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*x)*(d + e*x)
)^m*Sqrt[f + g*x])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{gx+f}\,(ex+d)^m}{\left(g^2x^2+2\,fgx+f^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m}{(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(3/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(3/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{\left(f+g\,x\right)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.779 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

```
[Out] -2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-3/2, m],[-1/2]
,c*d*(g*x+f)/(-a*e*g+c*d*f))/g/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^m)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae}{(f}$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\ {}_2F}{3g(f+gx)^{3/2}}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.89

$$-\frac{2(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e + c*d*x)*(d + e
*x))^m*(f + g*x)^(3/2))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{gx+f}\,(ex+d)^m}{\left(g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^
3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\left(gx+f\right)^{\frac{5}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}(ex+d)^m}{\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(5/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(5/2)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{{\left(f+g\,x\right)}^{5/2}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(5/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.780 $\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=65

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}(ae+cdx)^n}{cd(-m+n+1)}$$

[Out] (c*d*x+a*e)^n*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m+n)

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {858}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}(ae+cdx)^n}{cd(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] ((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c*d*(1 - m + n))
```

Rule 858

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && EqQ[c*e*f + c*d*g - b
*e*g, 0] && NeQ[m - n - 1, 0]
```

Rubi steps

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(ae+cdx)^n(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.82

$$\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m}(ae+cdx)^{n+1}}{-cdm+cdn+cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] ((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/((c*d - c*d*m + c*d*n)*((a*e + c*d*x)*(
d + e*x))^m)
```

fricas [A] time = 0.74, size = 66, normalized size = 1.02

$$-\frac{(cdx+ae)(cdx+ae)^n(ex+d)^m e^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] -(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d))/(c*d*m - c*d*n - c*d)
```

giac [A] time = 0.25, size = 114, normalized size = 1.75

$$-\frac{(cdx+ae)^n(xe+d)^m cdxe^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+(cdx+ae)^n(xe+d)^m ae^{\left(-m\log(cdx+ae)-m\log(xe+d)+1\right)}}{cdm-cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] -((c*d*x + a*e)^n*(x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (c*d*x + a*e)^n*(x*e + d)^m*a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) +
 1))/(c*d*m - c*d*n - c*d)
```

maple [A] time = 0.00, size = 64, normalized size = 0.98

$$-\frac{(ex+d)^m\,(cdx+ae)^{n+1}\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{(m-n-1)\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+a*e)^n*(e*x+d)^m/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(c*d*x+a*e)^(n+1)/c/d/(-1+m-n)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^m)
```

maxima [A] time = 0.50, size = 49, normalized size = 0.75

$$-\frac{(cdx+ae)e^{\left(-m\log(cdx+ae)+n\log(cdx+ae)\right)}}{cd(m-n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] -(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n - 1
))
```

mupad [B] time = 3.54, size = 63, normalized size = 0.97

$$\frac{(a\,e+c\,d\,x)^{n+1}\,(d+e\,x)^m}{c\,d\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m\,(n-m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*e + c*d*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] ((a*e + c*d*x)^(n + 1)*(d + e*x)^m)/(c*d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m*(n - m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.781 $\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade\right.$

Optimal. Leaf size=78

$$-\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

```
[Out] -(e*x+d)^m*(-c*d*e^2*g*x-a*e^3*g)^m*ln(c*d*x+a*e)/c/d/e^2/g/((a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$, Rules used = $\{891, 23, 31\}$

$$-\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))/
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \left((ae+cdx)^m(d+ex\right.$$

$$= \left((d+ex)^m \left(cd^2eg-\right.\right.$$

$$= -\frac{(d+ex)^m \left(-ae^3g-\right.}{}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.82

$$-\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m}\log(ae+cdx)\left(-e^2g(ae+cdx)\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
+ m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a*
e + c*d*x)*(d + e*x))^m))
```

fricas [A] time = 0.92, size = 35, normalized size = 0.45

$$-\frac{\log(cdx+ae)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

```
[Out] -log(c*d*x + a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-cde^2gx+cd^2eg-\left(cd^2+ae^2\right)eg\right)^{m-1}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

```
[Out] integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m - 1)*(e*x + d
)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int\left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}\left(-cd\,e^2gx+c\,d^2eg-\left(a\,e^2+c\,d^2\right)eg\right)^{m-1}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(m-1)/((c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(m-1)/((c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [A] time = 0.50, size = 32, normalized size = 0.41

$$-\frac{e^{2\,m-2}\left(-g\right)^m\log(cdx+ae)}{cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

```
[Out] -e^(2*m - 2)*(-g)^m*log(c*d*x + a*e)/(c*d*g)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m\,\left(c\,d^2\,e\,g-e\,g\,\left(c\,d^2+a\,e^2\right)-c\,d\,e^2\,g\,x\right)^{m-1}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)/
((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

3.782 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{d+ex}\,(f+gx)^{n+1}(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\,{}_2F_1\left(1,n+\frac{3}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{cdg(n+1)(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}+\frac{2e(f+gx)^{n+1}\ldots}{cd\ldots}$$

```
[Out] (2*a*e^2*g*(1+n)+c*d*(e*f-d*g*(3+2*n)))*(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom
([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))*(e*x+d)^(1/2)/c/d/g/(-a*e*g+c
*d*f)/(1+n)/(3+2*n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e*(g*x+f)^(1+
n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(3+2*n)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {880, 891, 70, 69}

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,(f+gx)^n(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\ldots}{c^2d^2g(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*
(3 + 2*n)*Sqrt[d + e*x]) - (2*(2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*n)
))*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c^2*d^2*g*(3 + 2*n)*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
```

```
IntegerQ[2*p]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left(2ae^2g(1+n)+cd(ef-\right.}{}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left(\left(2ae^2g(1+n)+cd(ef\right.\right.}{cdg(3+}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left(\left(2ae^2g(1+n)+cd(ef\right.\right.}{}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{2\left(2ae^2g(1+n)+cd(ef\right.}{}$$

Mathematica [A] time = 0.13, size = 145, normalized size = 0.68

$$\frac{(f+gx)^n\sqrt{(d+ex)(ae+cdx)}\left(\left(cd(dg(2n+3)-ef)-2ae^2g(n+1)\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)+cde\right.}{c^2d^2g\left(n+\frac{3}{2}\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*(c*d*e*(f + g*x) + ((-2*a*e^2*g*
(1 + n) + c*d*(-(e*f) + d*g*(3 + 2*n)))*Hypergeometric2F1[1/2, -n, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*(f + g*x))/(c*d*f - a*e*g))^n))/(
c^2*d^2*g*(3/2 + n)*Sqrt[d + e*x])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\left(gx+f\right)^n}{cdx+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c*d*x + a*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{n}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{n}}{\sqrt{cde\,x^{2}+ade+\left(a\,e^{2}+c\,d^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{n}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{n}\,\left(d+e\,x\right)^{3/2}}{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.783 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=501

$$\frac{128\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{3465c^6d^6eg\sqrt{d+ex}}-\frac{128\sqrt{d+ex}\,\sqrt{x}}{}$$

```
[Out] 128/3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(2*a*e^2*g-c*d*(-d
*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^6/d^6/e/g/(e*x+d)^(1/2
)-32/1155*(-a*e*g+c*d*f)^2*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/g/(e*x+d)^(1/2)-16/693*(-a*e*g+c*d
*f)*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/99*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+
f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/11*e
*(g*x+f)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)-128/
3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(e*x+d)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e
```

Rubi [A] time = 0.89, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(10ae^2g+cd(ef-11dg)\right)}{99c^2d^2g\sqrt{d+ex}}-\frac{16(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(}{693c^3d^3g\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(2*a*e^2*g - c*d*(
3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^6*d^6*e*
g*Sqrt[d + e*x]) - (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))
*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^5*d^5*e
) - (32*(c*d*f - a*e*g)^2*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1155*c^4*d^4*g*Sqrt[d + e*x]) -
(16*(c*d*f - a*e*g)*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(693*c^3*d^3*g*Sqrt[d + e*x]) - (2*(10*
a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(99*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^5*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{11cdg\sqrt{d+ex}} - \frac{1}{11}\left(-11d+\frac{10ae^2}{cd}+\frac{ef}{g}\right)$$

$$= -\frac{2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} +$$

$$= -\frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)}}{693c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{32(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right.}}{1155c^4d^4g\sqrt{d+ex}}$$

$$= -\frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right.}}{3465c^5d^5e}$$

$$= \frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ad}}{3465c^6d^6eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.42, size = 246, normalized size = 0.49

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3465\left(cd^2-ae^2\right)(cdf-aeg)^4-385g^3(ae+cdx)^4\left(5ae^2g-cd(dg+4ef)\right)+990g^2(ae+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3465*(c*d^2 - a*e^2)*(c*d*f - a*e*g)^4 +
1155*(c*d*f - a*e*g)^3*(-5*a*e^2*g + c*d*(e*f + 4*d*g))*(a*e + c*d*x) + 138
6*g*(c*d*f - a*e*g)^2*(-5*a*e^2*g + c*d*(2*e*f + 3*d*g))*(a*e + c*d*x)^2 +
990*g^2*(c*d*f - a*e*g)*(-5*a*e^2*g + c*d*(3*e*f + 2*d*g))*(a*e + c*d*x)^3
- 385*g^3*(5*a*e^2*g - c*d*(4*e*f + d*g))*(a*e + c*d*x)^4 + 315*e*g^4*(a*e
+ c*d*x)^5))/(3465*c^6*d^6*Sqrt[d + e*x])
```

fricas [A] time = 0.67, size = 597, normalized size = 1.19

$$\frac{2\left(315\,c^5d^5eg^4x^5+1155\left(3\,c^5d^6-2\,ac^4d^4e^2\right)f^4-1848\left(5\,ac^4d^5e-4\,a^2c^3d^3e^3\right)f^3g+1584\left(7\,a^2c^3d^4e^2-6\,a^3c^2d^2e^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*e*g^4*x^5 + 1155*(3*c^5*d^6 - 2*a*c^4*d^4*e^2)*f^4 - 18
48*(5*a*c^4*d^5*e - 4*a^2*c^3*d^3*e^3)*f^3*g + 1584*(7*a^2*c^3*d^4*e^2 - 6*
a^3*c^2*d^2*e^4)*f^2*g^2 - 704*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*f*g^3 +
128*(11*a^4*c*d^2*e^4 - 10*a^5*e^6)*g^4 + 35*(44*c^5*d^5*e*f*g^3 + (11*c^5*
d^6 - 10*a*c^4*d^4*e^2)*g^4)*x^4 + 10*(297*c^5*d^5*e*f^2*g^2 + 22*(9*c^5*d^
6 - 8*a*c^4*d^4*e^2)*f*g^3 - 4*(11*a*c^4*d^5*e - 10*a^2*c^3*d^3*e^3)*g^4)*x
^3 + 6*(462*c^5*d^5*e*f^3*g + 99*(7*c^5*d^6 - 6*a*c^4*d^4*e^2)*f^2*g^2 - 44
*(9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*f*g^3 + 8*(11*a^2*c^3*d^4*e^2 - 10*a^3
*c^2*d^2*e^4)*g^4)*x^2 + (1155*c^5*d^5*e*f^4 + 924*(5*c^5*d^6 - 4*a*c^4*d^4
*e^2)*f^3*g - 792*(7*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^2*g^2 + 352*(9*a^2*
c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*f*g^3 - 64*(11*a^3*c^2*d^3*e^3 - 10*a^4*c*
d*e^5)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c
^6*d^6*e*x + c^6*d^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^4}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^4/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [A] time = 0.01, size = 641, normalized size = 1.28

$$-\frac{2\,(cdx+ae)\left(-315eg^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3465*(c*d*x+a*e)*(-315*c^5*d^5*e*g^4*x^5+350*a*c^4*d^4*e^2*g^4*x^4-385*c
^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400*a^2*c^3*d^3*e^3*g^4*x^3+440*a*c
^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*x^3-1980*c^5*d^6*f*g^3*x^3-2970*c
^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*g^4*x^2-528*a^2*c^3*d^4*e^2*g^4*x^
2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*c^4*d^5*e*f*g^3*x^2+3564*a*c^4*d^4*
e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^2-2772*c^5*d^5*e*f^3*g*x^2-640*a^4*c
```

*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x+2816*a^3*c^2*d^2*e^4*f*g^3*x-3168*a^2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^3*e^3*f^2*g^2*x+5544*a*c^4*d^5*e*f^2*g^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620*c^5*d^6*f^3*g*x-1155*c^5*d^5*e*f^4*x+1280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*g^4-5632*a^4*c*d*e^5*f*g^3+6336*a^3*c^2*d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4*f^2*g^2-11088*a^2*c^3*d^4*e^2*f^2*g^2-7392*a^2*c^3*d^3*e^3*f^3*g+9240*a*c^4*d^5*e*f^3*g+2310*a*c^4*d^4*e^2*f^4-3465*c^5*d^6*f^4)*(e*x+d)^(1/2)/c^6/d^6/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)

maxima [A] time = 0.74, size = 693, normalized size = 1.38

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^4}{3\sqrt{cdx+ae}\,c^2d^2}+\frac{8\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x\right.}{15\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*e*x^4 + 56*a^3*
c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
 - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f^2*g^
2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4
 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2
*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^
3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^5) + 2/346
5*(315*c^6*d^6*e*x^6 + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(11*c^6*d^7 -
 a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x^4 + 8*(11*a
^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*e^3 - 10*a^4*
c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x)*g^4/(sqrt(c*
d*x + a*e)*c^6*d^6)
```

mupad [B] time = 4.09, size = 653, normalized size = 1.30

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5\,\sqrt{d+e\,x}}{11\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(2560\,a^5\,e^6\,g^4-2816\,a^4\,c\,d^2\,e^4\,g^4-11264\,a^4\,c\,d\,e^5\,f\,g^3+12672\,a^3\,c^2\,d^3\,e^3\,f\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5*(d + e*x)^(1/2))
/(11*c*d) - ((d + e*x)^(1/2)*(2560*a^5*e^6*g^4 - 6930*c^5*d^6*f^4 + 4620*a*
c^4*d^4*e^2*f^4 - 2816*a^4*c*d^2*e^4*g^4 - 14784*a^2*c^3*d^3*e^3*f^3*g + 12
672*a^3*c^2*d^3*e^3*f*g^3 + 18480*a*c^4*d^5*e*f^3*g - 11264*a^4*c*d*e^5*f*g
^3 - 22176*a^2*c^3*d^4*e^2*f^2*g^2 + 19008*a^3*c^2*d^2*e^4*f^2*g^2))/(3465*
c^6*d^6*e) + (x*(d + e*x)^(1/2)*(2310*c^5*d^5*e*f^4 + 9240*c^5*d^6*f^3*g -
1408*a^3*c^2*d^3*e^3*g^4 + 1280*a^4*c*d*e^5*g^4 - 7392*a*c^4*d^4*e^2*f^3*g
- 11088*a*c^4*d^5*e*f^2*g^2 + 6336*a^2*c^3*d^4*e^2*f*g^3 - 5632*a^3*c^2*d^2
*e^4*f*g^3 + 9504*a^2*c^3*d^3*e^3*f^2*g^2))/(3465*c^6*d^6*e) + (x^2*(d + e*
x)^(1/2)*(8316*c^5*d^6*f^2*g^2 + 1056*a^2*c^3*d^4*e^2*g^4 - 960*a^3*c^2*d^2
*e^4*g^4 + 5544*c^5*d^5*e*f^3*g - 7128*a*c^4*d^4*e^2*f^2*g^2 + 4224*a^2*c^3
*d^3*e^3*f*g^3 - 4752*a*c^4*d^5*e*f*g^3))/(3465*c^6*d^6*e) + (4*g^2*x^3*(d
+ e*x)^(1/2)*(40*a^2*e^3*g^2 + 297*c^2*d^2*e*f^2 + 198*c^2*d^3*f*g - 44*a*c
*d^2*e*g^2 - 176*a*c*d*e^2*f*g))/(693*c^3*d^3*e) + (2*g^3*x^4*(d + e*x)^(1/
2)*(11*c*d^2*g - 10*a*e^2*g + 44*c*d*e*f))/(99*c^2*d^2*e)))/(x + d/e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.784 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=412

$$\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{315c^5d^5eg\sqrt{d+ex}}-\frac{16\sqrt{d+ex}\,\sqrt{x(}$$

```
[Out] 16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3
*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e/g/(e*x+d)^(1/2)-4/
105*(-a*e*g+c*d*f)*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/63*(8*a*e^2*g+c*d*(-9*d*g+
e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(
1/2)+2/9*e*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^
(1/2)-16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(e*x+d)^(1/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e
```

Rubi [A] time = 0.63, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(8ae^2g+cd(ef-9dg)\right)}{63c^2d^2g\sqrt{d+ex}}-\frac{4(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(c}{105c^3d^3g\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(2*a*e^2*g - c*d*(3*e
*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^5*d^5*e*g*Sq
rt[d + e*x]) - (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*Sqrt[d
 + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^4*d^4*e) - (4*(
c*d*f - a*e*g)*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*g*Sqrt[d + e*x]) - (2*(8*a*e^2*g
+ c*d*(e*f - 9*d*g))*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(63*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
```

```
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{9cdg\sqrt{d+ex}} - \frac{1}{9}\left(-9d+\frac{8ae^2}{cd}+\frac{ef}{g}\right)\int \frac{\ldots}{\sqrt{a\ldots}}$$

$$= -\frac{2\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} + \frac{2e(f\ldots}{\ldots}$$

$$= -\frac{4(cdf-aeg)\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+c\ldots}}{105c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+\ldots}}{315c^4d^4e}$$

$$= \frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(c\ldots\right.}}{315c^5d^5eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.27, size = 264, normalized size = 0.64

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(128a^4e^5g^3-16a^3cde^3g^2(9dg+27ef+4egx)+24a^2c^2d^2e^2g\left(3dg(7f+gx)+e\left(21f^2+9f\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27*
e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*f^
2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*g^2
*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^4*(9
*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 189*f^
2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

fricas [A] time = 0.96, size = 408, normalized size = 0.99

$$\frac{2\left(35\,c^4d^4eg^3x^4+105\left(3\,c^4d^5-2\,ac^3d^3e^2\right)f^3-126\left(5\,ac^3d^4e-4\,a^2c^2d^2e^3\right)f^2g+72\left(7\,a^2c^2d^3e^2-6\,a^3cde^4\right)fg^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*e*g^3*x^4 + 105*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^3 - 126*(
5*a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3)*f^2*g + 72*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*
d*e^4)*f*g^2 - 16*(9*a^3*c*d^2*e^3 - 8*a^4*e^5)*g^3 + 5*(27*c^4*d^4*e*f*g^2
 + (9*c^4*d^5 - 8*a*c^3*d^3*e^2)*g^3)*x^3 + 3*(63*c^4*d^4*e*f^2*g + 9*(7*c^
4*d^5 - 6*a*c^3*d^3*e^2)*f*g^2 - 2*(9*a*c^3*d^4*e - 8*a^2*c^2*d^2*e^3)*g^3)
*x^2 + (105*c^4*d^4*e*f^3 + 63*(5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^2*g - 36*(7*
a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f*g^2 + 8*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e
^4)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*
d^5*e*x + c^5*d^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^3}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^3/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [A] time = 0.01, size = 425, normalized size = 1.03

$$\frac{2\,(cdx+ae)\left(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^4*d^4*e*g^3*x^4-40*a*c^3*d^3*e^2*g^3*x^3+45*c^4*d^5
*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2*d^2*e^3*g^3*x^2-54*a*c^3*d^4*e*
g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^4*d^5*f*g^2*x^2+189*c^4*d^4*e*f^2
*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^3*e^2*g^3*x+216*a^2*c^2*d^2*e^3*f*
g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*d^3*e^2*f^2*g*x+315*c^4*d^5*f^2*g*x
+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-144*a^3*c*d^2*e^3*g^3-432*a^3*c*d*e^4*
f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^2*c^2*d^2*e^3*f^2*g-630*a*c^3*d^4*e*f
^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^5*f^3)*(e*x+d)^(1/2)/c^5/d^5/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.69, size = 484, normalized size = 1.17

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^3}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x\right.}{5\,\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^
3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f*g^2/(
sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4 +
128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2*c^
3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^3*c
^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

mupad [B] time = 3.86, size = 438, normalized size = 1.06

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,a^4\,e^5\,g^3-288\,a^3\,c\,d^2\,e^3\,g^3-864\,a^3\,c\,d\,e^4\,f\,g^2+1008\,a^2\,c^2\,d^3\,e^2\,f\,g^2+1008\,a^2\,c^2\,d^2\,e^3\,f^2\,g-126\ldots\right.}{315\,c^5\,d^5\,e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(256*a^4*e
^5*g^3 + 630*c^4*d^5*f^3 - 420*a*c^3*d^3*e^2*f^3 - 288*a^3*c*d^2*e^3*g^3 +
1008*a^2*c^2*d^2*e^3*f^2*g + 1008*a^2*c^2*d^3*e^2*f*g^2 - 1260*a*c^3*d^4*e*
f^2*g - 864*a^3*c*d*e^4*f*g^2))/(315*c^5*d^5*e) + (2*g^3*x^4*(d + e*x)^(1/2
))/(9*c*d) + (x*(d + e*x)^(1/2)*(210*c^4*d^4*e*f^3 + 630*c^4*d^5*f^2*g + 14
4*a^2*c^2*d^3*e^2*g^3 - 128*a^3*c*d*e^4*g^3 - 504*a*c^3*d^3*e^2*f^2*g + 432
*a^2*c^2*d^2*e^3*f*g^2 - 504*a*c^3*d^4*e*f*g^2))/(315*c^5*d^5*e) + (2*g*x^2
*(d + e*x)^(1/2)*(16*a^2*e^3*g^2 + 63*c^2*d^2*e*f^2 + 63*c^2*d^3*f*g - 18*a
*c*d^2*e*g^2 - 54*a*c*d*e^2*f*g))/(105*c^3*d^3*e) + (2*g^2*x^3*(d + e*x)^(1
/2)*(9*c*d^2*g - 8*a*e^2*g + 27*c*d*e*f))/(63*c^2*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.785 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=321

$$\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{105c^4d^4eg\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\,\sqrt{x\left(ae^2\right.}}{}$$

```
[Out] 8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3*e*
f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e/g/(e*x+d)^(1/2)-2/35*
(6*a*e^2*g+c*d*(-7*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/7*e*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c/d/g/(e*x+d)^(1/2)-8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*
f))*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A] time = 0.42, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(6ae^2g+cd(ef-7dg)\right)}{35c^2d^2g\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(c}{105c^3d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f
- d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*Sqrt[
d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*Sqrt[d + e*x
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^
2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(35*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
```

```
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cdg\sqrt{d+ex}} - \frac{1}{7}\left(-7d+\frac{6ae^2}{cd}+\frac{ef}{g}\right)\int \frac{}{\sqrt{a}}$$

$$= -\frac{2\left(6ae^2g+cd(ef-7dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}} + \frac{2e(f}{}$$

$$= -\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{105c^3d^3e}$$

$$= \frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2\right.}}{105c^4d^4eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.18, size = 169, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-48a^3e^4g^2+8a^2cde^2g(7dg+14ef+3egx)-2ac^2d^2e\left(14dg(5f+gx)+e\left(35f^2+28fgx+\right.\right.\right.}{105c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*f
 + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*f*
g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(35*
f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

fricas [A] time = 0.90, size = 256, normalized size = 0.80

$$\frac{2\left(15\,c^3d^3eg^2x^3+35\left(3\,c^3d^4-2\,ac^2d^2e^2\right)f^2-28\left(5\,ac^2d^3e-4\,a^2cde^3\right)fg+8\left(7\,a^2cd^2e^2-6\,a^3e^4\right)g^2+3\left(14\,c^3d^3e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*e*g^2*x^3 + 35*(3*c^3*d^4 - 2*a*c^2*d^2*e^2)*f^2 - 28*(5*
a*c^2*d^3*e - 4*a^2*c*d*e^3)*f*g + 8*(7*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^2 + 3*
(14*c^3*d^3*e*f*g + (7*c^3*d^4 - 6*a*c^2*d^2*e^2)*g^2)*x^2 + (35*c^3*d^3*e*
f^2 + 14*(5*c^3*d^4 - 4*a*c^2*d^2*e^2)*f*g - 4*(7*a*c^2*d^3*e - 6*a^2*c*d*e
^3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*
d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{2}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^2/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [A] time = 0.01, size = 255, normalized size = 0.79

$$-\frac{2\left(cdx+ae\right)\left(-15e\,g^{2}x^{3}c^{3}d^{3}+18a\,c^{2}d^{2}e^{2}g^{2}x^{2}-21c^{3}d^{4}g^{2}x^{2}-42c^{3}d^{3}efg\,x^{2}-24a^{2}cd\,e^{3}g^{2}x+28a\,c^{2}d^{3}e\,g^{2}x+\right.}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(-15*c^3*d^3*e*g^2*x^3+18*a*c^2*d^2*e^2*g^2*x^2-21*c^3*d
^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*e^3*g^2*x+28*a*c^2*d^3*e*g^2*x+5
6*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*c^3*d^3*e*f^2*x+48*a^3*e^4*g^2-56
*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+140*a*c^2*d^3*e*f*g+70*a*c^2*d^2*e^2
*f^2-105*c^3*d^4*f^2)*(e*x+d)^(1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)
```

maxima [A] time = 0.63, size = 309, normalized size = 0.96

$$\frac{2\left(c^{2}d^{2}ex^{2}+3\,acd^{2}e-2\,a^{2}e^{3}+\left(3\,c^{2}d^{3}-acde^{2}\right)x\right)f^{2}}{3\,\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{4\left(3\,c^{3}d^{3}ex^{3}-10\,a^{2}cd^{2}e^{2}+8\,a^{3}e^{4}+\left(5\,c^{3}d^{4}-ac^{2}d^{2}e^{2}\right)x\right.}{15\,\sqrt{cdx+ae}\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g^2/(sq
rt(c*d*x + a*e)*c^4*d^4)
```

mupad [B] time = 3.71, size = 279, normalized size = 0.87

$$\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{2\,g^{2}\,x^{3}\,\sqrt{d+e\,x}}{7\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(96\,a^{3}\,e^{4}\,g^{2}-112\,a^{2}\,c\,d^{2}\,e^{2}\,g^{2}-224\,a^{2}\,c\,d\,e^{3}\,f\,g+280\,a\,c^{2}\,d^{3}\,e\,f\,g+140\,a\,c^{2}\right.}{105\,c^{4}\,d^{4}\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3*(d + e*x)^(1/2))
/(7*c*d) - ((d + e*x)^(1/2)*(96*a^3*e^4*g^2 - 210*c^3*d^4*f^2 + 140*a*c^2*d
^2*e^2*f^2 - 112*a^2*c*d^2*e^2*g^2 + 280*a*c^2*d^3*e*f*g - 224*a^2*c*d*e^3*
f*g))/(105*c^4*d^4*e) + (x*(d + e*x)^(1/2)*(70*c^3*d^3*e*f^2 + 140*c^3*d^4*
f*g - 56*a*c^2*d^3*e*g^2 + 48*a^2*c*d*e^3*g^2 - 112*a*c^2*d^2*e^2*f*g))/(10
5*c^4*d^4*e) + (2*g*x^2*(d + e*x)^(1/2)*(7*c*d^2*g - 6*a*e^2*g + 14*c*d*e*f
))/(35*c^2*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)^{2}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.786 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=209

$$-\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(4ae^2g-cd(5ef-dg)\right)}{15c^3d^3e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^2d^2e}$$

```
[Out] 2/5*g*(e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e-4/15*(-a*
e^2+c*d^2)*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c^3/d^3/e/(e*x+d)^(1/2)-2/15*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(e*x+d)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

Rubi [A] time = 0.20, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {794, 656, 648}

$$-\frac{2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(4ae^2g-cd(5ef-dg)\right)}{15c^2d^2e}-\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^3d^3e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) - (2*(4*a*e^2*g - c*d*(5
*e*f - d*g))*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15
*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*c*d*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2g(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cde} + \frac{1}{5}\left(5f-\frac{dg}{e}-\frac{4aeg}{cd}\right)\int \frac{}{\sqrt{a}}$$

$$= -\frac{2\left(4ae^2g-cd(5ef-dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e} + \frac{2g(d}{}$$

$$= -\frac{4\left(cd^2-ae^2\right)\left(4ae^2g-cd(5ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} - \frac{2(}{}$$

Mathematica [A] time = 0.10, size = 96, normalized size = 0.46

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^3g-2acde(5dg+5ef+2egx)+c^2d^2(5d(3f+gx)+ex(5f+3gx))\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sqrt
[d + e*x])
```

fricas [A] time = 0.93, size = 141, normalized size = 0.67

$$\frac{2\left(3c^2d^2egx^2+5\left(3c^2d^3-2acde^2\right)f-2\left(5acd^2e-4a^2e^3\right)g+\left(5c^2d^2ef+\left(5c^2d^3-4acde^2\right)g\right)x\right)\sqrt{cdex^2+ade+}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*e*g*x^2 + 5*(3*c^2*d^3 - 2*a*c*d*e^2)*f - 2*(5*a*c*d^2*e -
4*a^2*e^3)*g + (5*c^2*d^2*e*f + (5*c^2*d^3 - 4*a*c*d*e^2)*g)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [A] time = 0.00, size = 131, normalized size = 0.63

$$\frac{2(cdx+ae)\left(3egx^2c^2d^2-4acde^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10acd^2eg-10acde^2f+15d^3fc^2\right)\sqrt{ex+}}{15\sqrt{cdex^2+ae^2x+cd^2x+ade}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*e*g*x^2-4*a*c*d*e^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2
*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*e^2*f+15*c^2*d^3*f)*(e*x+d)^(1/2
)/c^3/d^3/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.57, size = 168, normalized size = 0.80

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2}{15\,\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 + 8
*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^3
)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

mupad [B] time = 3.48, size = 152, normalized size = 0.73

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,a^2\,e^3-20\,g\,a\,c\,d^2\,e-20\,f\,a\,c\,d\,e^2+30\,f\,c^2\,d^3\right)}{15\,c^3\,d^3\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}}{5\,c\,d}+\frac{2\,x\,\sqrt{d+e\,x}\,\left(5\,c\,g\,d^2+\right.}{15\,c^2\,d}\right.}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
3*g + 30*c^2*d^3*f - 20*a*c*d*e^2*f - 20*a*c*d^2*e*g))/(15*c^3*d^3*e) + (2*
g*x^2*(d + e*x)^(1/2))/(5*c*d) + (2*x*(d + e*x)^(1/2)*(5*c*d^2*g - 4*a*e^2*
g + 5*c*d*e*f))/(15*c^2*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.787 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=109

$$\frac{4(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

```
[Out] 4/3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^
(1/2)+2/3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d
```

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {656, 648}

$$\frac{4(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= \frac{2\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x}}}{3d}\\ &= \frac{4(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{ade+(cd^2+ae}}{3cd}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.50

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(cd(3d+ex)-2ae^2\right)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

fricas [A] time = 0.88, size = 73, normalized size = 0.67

$$\frac{2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(cdex + 3\,cd^2 - 2\,ae^2\right)\sqrt{ex + d}}{3\left(c^2d^2ex + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^
2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

maple [A] time = 0.00, size = 69, normalized size = 0.63

$$-\frac{2\left(cdx + ae\right)\left(-cdex + 2a\,e^2 - 3c\,d^2\right)\sqrt{ex + d}}{3\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\;c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*e*x+2*a*e^2-3*c*d^2)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.50, size = 65, normalized size = 0.60

$$\frac{2\left(c^2d^2ex^2 + 3\,acd^2e - 2\,a^2e^3 + \left(3\,c^2d^3 - acde^2\right)x\right)}{3\sqrt{cdx + ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

mupad [B] time = 3.36, size = 85, normalized size = 0.78

$$\frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d} - \frac{\left(4\,a\,e^2-6\,c\,d^2\right)\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x + \frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

```
[Out] (((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3*
c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.788 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[Out] -2*(-d*g+e*f)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)+2*e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {880, 874, 205}

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] (2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2\left(\frac{1}{2}cde^2f-\frac{1}{2}cd^2eg\right)\right)\int \frac{}{(f+gx}}{cdeg}$$

$$= \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2e^2(ef-dg)\right)\text{Subst}\left(\int \frac{}{-e\left(cd^2\right.}\right.}{}$$

$$= \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2\right.}}{\sqrt{cdf-aeg}}\right.}{g^{3/2}\sqrt{cdf-aeg}}$$

<u>**Mathematica**</u> **[A]** time = 0.11, size = 140, normalized size = 1.01

$$\frac{2\sqrt{d+ex}\left(e\sqrt{g}\,(ae+cdx)\sqrt{cdf-aeg}+cd(dg-ef)\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{cdg^{3/2}\sqrt{(d+ex)(ae+cdx)}\,\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f)
 + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

<u>**fricas**</u> **[A]** time = 0.67, size = 511, normalized size = 3.68

$$\left[\frac{\left(cd^2ef-cd^3g+\left(cde^2f-cd^2eg\right)x\right)\sqrt{-cdfg+aeg^2}\,\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)}}{egx^2+df+\left(ef+dg\right)x}\right.}{c^2d^3fg^2-acd^2eg^3+\left(c^2d^2efg^2-acde^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((c*d^2*e*f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(-c*d*f*g + a*e*g^2
)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)
*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)
*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(c*d*e*f*g - a*e^2*g^2
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2
 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x), 2*((c*d^2*e*f - c*
d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c
*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*f*g - a*e^2*g^2)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2 - a*c
*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x)]
```

<u>**giac**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

maple [A] time = 0.02, size = 163, normalized size = 1.17

$$-\frac{2\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(c\,d^{2}g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-cdef\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{(a\cdots}\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\,cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*c*d^2*g-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*c*d*e*f-(c*d*x+a*e)^(1/2)*e*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/
(c*d*x+a*e)^(1/2)/c/d/g/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)\,\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

3.789 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=170

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}$$

```
[Out] -(2*a*e^2*g-c*d*(d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-(-d
*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e
*x+d)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {878, 874, 205}

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*e
*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[
d + e*x]])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{\left(e\left(\frac{1}{2}cde^2f+\frac{3}{2}cd\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} - \frac{\left(e^2\left(2ae^2g-cd(e\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} - \frac{\left(2ae^2g-cd(ef+\right.}{}$$

Mathematica [A] time = 0.15, size = 155, normalized size = 0.91

$$\frac{\sqrt{d+ex}\left(-\frac{\sqrt{ae+cdx}\left(cd(dg+ef)-2ae^2g\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}}-\frac{\sqrt{g}(dg-ef)(ae+cdx)}{f+gx}\right)}{g^{3/2}\sqrt{(d+ex)(ae+cdx)}\,(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(-((Sqrt[g]*(-(e*f) + d*g)*(a*e + c*d*x))/(f + g*x)) - ((-2*
a*e^2*g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d
*x])/Sqrt[c*d*f - a*e*g]])/Sqrt[c*d*f - a*e*g]))/(g^(3/2)*(-(c*d*f) + a*e*g
)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 1.06, size = 896, normalized size = 5.27

$$\left[-\frac{\left(cd^2ef^2+\left(cd^3-2\,ade^2\right)fg+\left(cde^2fg+\left(cd^2e-2\,ae^3\right)g^2\right)x^2+\left(cde^2f^2+2\left(cd^2e-ae^3\right)fg+\left(cd^3-2\,ade^2\right)g\right.\right.}{2\left(c^2d^3f^3g^2-2\,acd^2ef^2g^3+a^2de^2fg^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2
*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*
e^2)*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) + 2*(c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^
2*e*f^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^
2*e^3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^
2)*f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x), -((c*d^2*e*f^2 + (c*d^3 - 2
*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2
+ 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*
```

```
f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^2*e*f^2*g^3 + a^2*d*e
^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^2 + (c^2
*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g^3 - (2*a*c*d
^2*e - a^2*e^3)*f*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 347, normalized size = 2.04

$$\frac{\left(-2a e^2 g^2 x \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right) + c d^2 g^2 x \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right) + cdefgx \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right) - 2a e^2 f g \operatorname{arct}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] (-2*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*e^2*g^2+arctan
h((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c*d^2*g^2+arctanh((c*d*x+a
*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c*d*e*f*g-2*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*e^2*f*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2)*g)*c*d^2*f*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c*d*e*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*d*g+((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*e*f)/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.790 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{cd\left(4ae^2g-cd(3dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+\right.}}{}$$

```
[Out] -1/4*c*d*(4*a*e^2*g-c*d*(3*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)
^(5/2)-1/2*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d
*f)/(g*x+f)^2/(e*x+d)^(1/2)-1/4*(4*a*e^2*g-c*d*(3*d*g+e*f))*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {878, 872, 874, 205}

$$-\frac{cd\left(4ae^2g-cd(3dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(
f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4
*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{7}{2}cd\right.\right.}{}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+\right.}{4g(cdf}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+\right.}{4g(cdf}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+\right.}{4g(cdf}$$

Mathematica [A] time = 0.42, size = 189, normalized size = 0.72

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd\left(2ae^2g-\frac{1}{2}cd(3dg+ef)\right)\left(\frac{cdf-aeg}{cdf+cdgx}+\frac{\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{ae+cdx}}\right)}{(cdf-aeg)^2}+\frac{ef-dg}{(f+gx)^2}\right)}{2g\sqrt{d+ex}\,(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((e*f - d*g)/(f + g*x)^2 + (c*d*(2*a*e^2*g -
 (c*d*(e*f + 3*d*g))/2)*((c*d*f - a*e*g)/(c*d*f + c*d*g*x) + (Sqrt[c*d*f -
a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*Sq
rt[a*e + c*d*x])))/(c*d*f - a*e*g)^2))/(2*g*(-(c*d*f) + a*e*g)*Sqrt[d + e*x
])
```

fricas [B] time = 0.72, size = 1704, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g
^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d
^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*d^2
*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2*e^2
)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
d*g)*x)) - 2*(c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g^4 - (5*c^2*d^3 - a*c*d*e^2)*f
^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c^2*d^2*e*f^2*g^2 + (3*c^2*d^3
- 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e^3)*g^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f
^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*f^2*g^5 + (c^3*d^3*e*f^3*g^4 -
 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^3 + (2*c^3*
d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*
c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x
^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^4 + (6*a^2*c*d^2*e^2 - a^3
*e^4)*f^2*g^5)*x), -1/4*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g
 + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e
^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*
g^3)*x^2 + (c^2*d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*
d^4 - 4*a*c*d^2*e^2)*f*g^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e
*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g
^4 - (5*c^2*d^3 - a*c*d*e^2)*f^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c
^2*d^2*e*f^2*g^2 + (3*c^2*d^3 - 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e
^3)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3
*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*
f^2*g^5 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^
6 - a^3*e^4*g^7)*x^3 + (2*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*
a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c
*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 +
 (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^
3*g^4 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 673, normalized size = 2.58

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;\left(4acd\,e^2g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\;g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3c^2d^3g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\;g}{\sqrt{\left(aeg-cdf\right)g}}\right)-c^2d^2ef\,g^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(4*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c*d*e^2*g^3-3*arctanh((c*d*x+a*e)^(1/2)/((a*
```

```
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^3*g^3-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2)*g)*x^2*c^2*d^2*e*f*g^2+8*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2)*g)*x*a*c*d*e^2*f*g^2-6*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*g)*x*c^2*d^3*f*g^2-2*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2)*g)*x*c^2*d^2*e*f^2*g+4*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2)*g)*a*c*d*e^2*f^2*g-3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c^2*d^3*f^2*g-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^2
*d^2*e*f^3-4*x*a*e^2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*x*c*d^
2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+x*c*d*e*f*g*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-2*a*e^2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*g-c
*d*f)^2/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^3\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.791 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=351

$$-\frac{c^2d^2\left(6ae^2g-cd(5dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg\right.}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}$$

```
[Out] -1/8*c^2*d^2*(6*a*e^2*g-c*d*(5*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*
d*f)^(7/2)-1/3*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g
+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)-1/12*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)-1/
8*c*d*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g
/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {878, 872, 874, 205}

$$-\frac{c^2d^2\left(6ae^2g-cd(5dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg\right.}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*
(f + g*x)^2) - (c*d*(6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c^
2*d^2*(6*a*e^2*g - c*d*(e*f + 5*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*g^(3/2)*(c
*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{11}{2}c\right.\right.}{}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+\right.}{12g(cdf}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+\right.}{12g(cdf}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+\right.}{12g(cdf}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+\right.}{12g(cdf}$$

Mathematica [C] time = 0.10, size = 132, normalized size = 0.38

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{ef-dg}{(f+gx)^3}-\frac{c^2d^2\left(cd(5dg+ef)-6ae^2g\right){}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{(cdf-aeg)^3}\right)}{3g\sqrt{d+ex}\,(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((e*f - d*g)/(f + g*x)^3 - (c^2*d^2*(-6*a*e^
2*g + c*d*(e*f + 5*d*g))*Hypergeometric2F1[1/2, 3, 3/2, (g*(a*e + c*d*x))/(
-(c*d*f) + a*e*g)])/(c*d*f - a*e*g)^3))/(3*g*(-(c*d*f) + a*e*g)*Sqrt[d + e*
x])
```

fricas [B] time = 1.08, size = 2736, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (c^3*d^3*e
^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e^2*f^2*g^
2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)
*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f^2*g^2 +
(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4*c^3*d^4*
e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(
3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*a*c^2*d^2*e^2)*f^3*g
^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*a^2*c*d^2*e^2 - 2*a^
3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*c^2*d^2*e^2)*f*g^4 -
 (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^3*e*f^3*g^2 + (20*c^
3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 31*a^2*c*d*e^3)*f*g^4
 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4*e*f^6*g^3 + 6*a^2*c
^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^4*f^3*g^6 + (c^4*d^4
*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*
d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^
4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f
^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 3
*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 -
4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 +
2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*c*d^2*e^3 - a^4*e^5)
*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^7 + (3*c^4*d^5 - 4*a
*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(9*
a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3
*g^6)*x), -1/24*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (
c^3*d^3*e^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e
^2*f^2*g^2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2
*d^3*e^2)*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f
^2*g^2 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4
*c^3*d^4*e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^
2 + a*e^2)*g*x)) + (3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*
a*c^2*d^2*e^2)*f^3*g^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*
c^2*d^2*e^2)*f*g^4 - (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^
3*e*f^3*g^2 + (20*c^3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 3
1*a^2*c*d*e^3)*f*g^4 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4
*e*f^6*g^3 + 6*a^2*c^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^
4*f^3*g^6 + (c^4*d^4*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^
3*f^2*g^7 - 4*a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 +
 a^4*d*e^4*g^9 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e -
9*a^2*c^2*d^2*e^3)*f^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 -
(4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4
*f*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2
*d^2*e^3)*f^4*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*
c*d^2*e^3 - a^4*e^5)*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^
7 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*
e^3)*f^5*g^4 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^
2*e^3 - a^4*e^5)*f^3*g^6)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 1142, normalized size = 3.25

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(18a\,c^2d^2e^2g^4x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-15c^3d^4g^4x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-3*c^2*d^2*e*f^3*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2)*g)*x^3*c^3*d^4*g^4-15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2)*g)*c^3*d^4*f^3*g-3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*
g)*c^3*d^3*e*f^4-50*x*a*c*d*e^2*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)-45*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^4*f*
g^3-45*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^4*f^2*g
^2+15*x^2*c^2*d^3*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+12*x*a^2*e^
3*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+8*a^2*d*e^2*g^3*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+4*a^2*e^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2)+33*c^2*d^3*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-18
*x^2*a*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+40*x*c^2*d^3*f
*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+18*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*a*c^2*d^2*e^2*g^4-3*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*e*f*g^3-9*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*e*f^2*g^2-9*arctanh((c*d*x+a*e)^(
1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*e*f^3*g+18*arctanh((c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e^2*f^3*g+3*x^2*c^2*d^2*e*f*g^2*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-10*x*a*c*d^2*e*g^3*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)+8*x*c^2*d^2*e*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d
*f)*g)^(1/2)-26*a*c*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1
6*a*c*d*e^2*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+54*arctanh((c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c^2*d^2*e^2*f*g^3+54*arctanh
((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c^2*d^2*e^2*f^2*g^2)/(e*x
+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*
a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.792 $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=324

$$-\frac{b\sqrt{1-d^2x^2}\,(45a^2d^4+60acd^2+10b^2d^2+24c^2)}{15d^6}-\frac{x\sqrt{1-d^2x^2}\,(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c$$

```
[Out] 1/16*(16*a^3*d^6+24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c^3)
*arcsin(d*x)/d^7-1/15*b*(45*a^2*d^4+60*a*c*d^2+10*b^2*d^2+24*c^2)*(-d^2*x^2
+1)^(1/2)/d^6-1/16*(24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c
^3)*x*(-d^2*x^2+1)^(1/2)/d^6-1/15*b*(30*a*c*d^2+5*b^2*d^2+12*c^2)*x^2*(-d^2
*x^2+1)^(1/2)/d^4-1/24*c*(18*a*c*d^2+18*b^2*d^2+5*c^2)*x^3*(-d^2*x^2+1)^(1/
2)/d^4-3/5*b*c^2*x^4*(-d^2*x^2+1)^(1/2)/d^2-1/6*c^3*x^5*(-d^2*x^2+1)^(1/2)/
d^2
```

Rubi [A] time = 0.93, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {899, 1815, 641, 216}

$$-\frac{x\sqrt{1-d^2x^2}\,(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3)}{16d^6}-\frac{b\sqrt{1-d^2x^2}\,(45a^2d^4+60acd^2+10b^2d^2+24c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*Sqrt[1 - d^2*x^2])/(15*
d^6) - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)
*x*Sqrt[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*S
qrt[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*Sqrt[
1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*Sqrt[1 - d^2*x^2])/(5*d^2) - (c^3*x^5
*Sqrt[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*
b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(16*d^7)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\int \frac{\left(a+bx+cx^2\right)^3}{\sqrt{1-d^2x^2}}\,dx$$

$$=-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}-\frac{\int \frac{-6a^3d^2-18a^2bd^2x-18a\left(b^2+ac\right)d^2x^2-6b\left(b^2+6ac\right)d^2x^3-c\left(5c^2+18b^2d^2+18acd^2\right)x^4-18b}{\sqrt{1-d^2x^2}}}{6d^2}$$

$$=-\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}+\frac{\int \frac{30a^3d^4+90a^2bd^4x+90a\left(b^2+ac\right)d^4x^2+6bd^2\left(12c^2+5b^2d^2+30\right.}{\sqrt{1-d^2x^2}}}{30d^4}$$

$$=-\frac{c\left(5c^2+18b^2d^2+18acd^2\right)x^3\sqrt{1-d^2x^2}}{24d^4}-\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}-\int$$

$$=-\frac{b\left(12c^2+5b^2d^2+30acd^2\right)x^2\sqrt{1-d^2x^2}}{15d^4}-\frac{c\left(5c^2+18b^2d^2+18acd^2\right)x^3\sqrt{1-d^2x^2}}{24d^4}-$$

$$=-\frac{\left(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4\right)x\sqrt{1-d^2x^2}}{16d^6}-\frac{b\left(12c^2+5b^2d^2+30a\right.}{15d}$$

$$=-\frac{b\left(24c^2+10b^2d^2+60acd^2+45a^2d^4\right)\sqrt{1-d^2x^2}}{15d^6}-\frac{\left(5c^3+18b^2cd^2+18ac^2d^2+24ab\right.}{16d^6}$$

$$=-\frac{b\left(24c^2+10b^2d^2+60acd^2+45a^2d^4\right)\sqrt{1-d^2x^2}}{15d^6}-\frac{\left(5c^3+18b^2cd^2+18ac^2d^2+24ab\right.}{16d^6}$$

Mathematica [A] time = 0.26, size = 229, normalized size = 0.71

$$\frac{15\sin^{-1}(dx)\left(16a^3d^6+24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)-d\sqrt{1-d^2x^2}\left(48b\left(15a^2d^4+10acd^2\left(d^2x\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2 +
c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8 +
 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2) +
 c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4)))) + 15*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2
*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(240*d^7)
```

fricas [A] time = 0.83, size = 251, normalized size = 0.77

$$-\frac{\left(40\,c^3d^5x^5+144\,bc^2d^5x^4+720\,a^2bd^5+384\,bc^2d+160\left(b^3+6\,abc\right)d^3+10\left(5\,c^3d^3+18\left(b^2c+ac^2\right)d^5\right)x^3+16\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*((40*c^3*d^5*x^5 + 144*b*c^2*d^5*x^4 + 720*a^2*b*d^5 + 384*b*c^2*d +
 160*(b^3 + 6*a*b*c)*d^3 + 10*(5*c^3*d^3 + 18*(b^2*c + a*c^2)*d^5)*x^3 + 16
*(12*b*c^2*d^3 + 5*(b^3 + 6*a*b*c)*d^5)*x^2 + 15*(24*(a*b^2 + a^2*c)*d^5 +
5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(16*
```

```
a^3*d^6 + 24*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 18*(b^2*c + a*c^2)*d^2)*arctan((
sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^7
```

giac [A] time = 0.62, size = 412, normalized size = 1.27

$$-\frac{\left(\left(2\left((dx+1)\left(4\,(dx+1)\left(\frac{5\,(dx+1)c^3}{d^6}+\frac{18\,bc^2d^{37}-25\,c^3d^{36}}{d^{42}}\right)+\frac{9\left(10\,b^2cd^{38}+10\,ac^2d^{38}-32\,bc^2d^{37}+25\,c^3d^{36}\right)}{d^{42}}\right)+\frac{40\,b^3d^{39}+240\,abcd^{39}}{}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/240*(((2*((d*x + 1)*(4*(d*x + 1)*(5*(d*x + 1)*c^3/d^6 + (18*b*c^2*d^37 -
 25*c^3*d^36)/d^42) + 9*(10*b^2*c*d^38 + 10*a*c^2*d^38 - 32*b*c^2*d^37 + 25
*c^3*d^36)/d^42) + (40*b^3*d^39 + 240*a*b*c*d^39 - 270*b^2*c*d^38 - 270*a*c
^2*d^38 + 528*b*c^2*d^37 - 275*c^3*d^36)/d^42)*(d*x + 1) + 5*(72*a*b^2*d^40
 + 72*a^2*c*d^40 - 32*b^3*d^39 - 192*a*b*c*d^39 + 162*b^2*c*d^38 + 162*a*c^
2*d^38 - 192*b*c^2*d^37 + 85*c^3*d^36)/d^42)*(d*x + 1) + 15*(48*a^2*b*d^41
- 24*a*b^2*d^40 - 24*a^2*c*d^40 + 16*b^3*d^39 + 96*a*b*c*d^39 - 30*b^2*c*d^
38 - 30*a*c^2*d^38 + 48*b*c^2*d^37 - 11*c^3*d^36)/d^42)*sqrt(d*x + 1)*sqrt(
-d*x + 1) - 30*(16*a^3*d^6 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 18*b^2*c*d^2 + 1
8*a*c^2*d^2 + 5*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^6)/d
```

maple [C] time = 0.06, size = 602, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(40\sqrt{-d^2x^2+1}\,c^3d^5x^5\mathrm{csgn}(d)+144\sqrt{-d^2x^2+1}\,bc^2d^5x^4\mathrm{csgn}(d)+180\sqrt{-d^2x^2+1}\,ac^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(40*csgn(d)*x^5*c^3*d^5*(-d^2*x^2+1)^(1
/2)+144*csgn(d)*x^4*b*c^2*d^5*(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*a*c^2*d^5*
(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*b^2*c*d^5*(-d^2*x^2+1)^(1/2)+480*csgn(d)
*x^2*a*b*c*d^5*(-d^2*x^2+1)^(1/2)+80*csgn(d)*x^2*b^3*d^5*(-d^2*x^2+1)^(1/2)
+50*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^3*c^3+360*csgn(d)*d^5*(-d^2*x^2+1)^(1/
2)*x*a^2*c+360*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*x*a*b^2+192*csgn(d)*d^3*(-d^2
*x^2+1)^(1/2)*x^2*b*c^2+720*(-d^2*x^2+1)^(1/2)*csgn(d)*d^5*a^2*b-240*arctan
(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^3*d^6+270*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)
*x*a*c^2+270*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2*c+960*(-d^2*x^2+1)^(1/2)*
csgn(d)*d^3*a*b*c+160*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b^3-360*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^4-360*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*a*b^2*d^4+75*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c^3+384*(-d^2*x^2+1)^(1/2)*c
sgn(d)*d*b*c^2-270*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^2-270*arc
tan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*c*d^2-75*arctan(csgn(d)*d*x/(-d^2*x
^2+1)^(1/2))*c^3)*csgn(d)/d^7/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.98, size = 365, normalized size = 1.13

$$-\frac{\sqrt{-d^2x^2+1}\,c^3x^5}{6\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}\,bc^2x^4}{5\,d^2}+\frac{a^3\arcsin(dx)}{d}-\frac{5\,\sqrt{-d^2x^2+1}\,c^3x^3}{24\,d^4}-\frac{3\,\sqrt{-d^2x^2+1}\,\left(b^2c+ac^2\right)x^3}{4\,d^2}-\frac{3\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*sqrt(-d^2*x^2 + 1)*c^3*x^5/d^2 - 3/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^4/d^2
+ a^3*arcsin(d*x)/d - 5/24*sqrt(-d^2*x^2 + 1)*c^3*x^3/d^4 - 3/4*sqrt(-d^2*x
```

```
^2 + 1)*(b^2*c + a*c^2)*x^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*a^2*b/d^2 - 4/5*sqrt
(-d^2*x^2 + 1)*b*c^2*x^2/d^4 - 1/3*sqrt(-d^2*x^2 + 1)*(b^3 + 6*a*b*c)*x^2/d
^2 - 3/2*sqrt(-d^2*x^2 + 1)*(a*b^2 + a^2*c)*x/d^2 + 3/2*(a*b^2 + a^2*c)*arc
sin(d*x)/d^3 - 5/16*sqrt(-d^2*x^2 + 1)*c^3*x/d^6 - 9/8*sqrt(-d^2*x^2 + 1)*(
b^2*c + a*c^2)*x/d^4 - 8/5*sqrt(-d^2*x^2 + 1)*b*c^2/d^6 - 2/3*sqrt(-d^2*x^2
 + 1)*(b^3 + 6*a*b*c)/d^4 + 5/16*c^3*arcsin(d*x)/d^7 + 9/8*(b^2*c + a*c^2)*
arcsin(d*x)/d^5
```

mupad [B] time = 31.33, size = 1768, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^23*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6
*a^2*c*d^4 + (9*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2)
 - 1)*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (9*b^2*c*d
^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((175*c^3)/12 + 6*
a*b^2*d^4 + (105*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1
)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((175*c^3)/12 + 6*a*b^2*d^4 + (1
05*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^2
1 + (((1 - d*x)^(1/2) - 1)^5*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)
/2 + 126*a^2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^19*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)/2 + 126*a^
2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^19 + (((1 - d*x)^(1/2)
- 1)^7*((8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 +
 (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^17*(
(8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 + (1533*b
^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^11*((25295*
c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((
d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(1/2) - 1)^13*((25295*c^3)/2 + 420*a*b
^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((d*x + 1)^(1/2) -
 1)^13 - (((1 - d*x)^(1/2) - 1)^9*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^
2*d^2 - 804*a^2*c*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*
x)^(1/2) - 1)^15*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^2*d^2 - 804*a^2*c
*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^6*
((1024*b^3*d^3)/3 + 1080*a^2*b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x
+ 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^18*((1024*b^3*d^3)/3 + 1080*a^2*
b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^18 + (((1 - d
*x)^(1/2) - 1)^10*(1024*b^3*d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*
a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)^14*(1024*b^3*
d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*a*b*c*d^3))/((d*x + 1)^(1/2)
 - 1)^14 + (((1 - d*x)^(1/2) - 1)^12*((3200*b^3*d^3)/3 + 6048*a^2*b*d^5 + (
32768*b*c^2*d)/5 + 6400*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^
(1/2) - 1)^4*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c*d^3))/((d*x + 1)^(1/2)
 - 1)^4 + (((1 - d*x)^(1/2) - 1)^20*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c
*d^3))/((d*x + 1)^(1/2) - 1)^20 + (((1 - d*x)^(1/2) - 1)^8*(768*b^3*d^3 + 2
880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2
) - 1)^16*(768*b^3*d^3 + 2880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2)
 - 1)^16 + (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 +
 (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22)/(d^7 + (
12*d^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (66*d^7*((1 - d*x
)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (220*d^7*((1 - d*x)^(1/2) - 1)^6)
/((d*x + 1)^(1/2) - 1)^6 + (495*d^7*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/
2) - 1)^8 + (792*d^7*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (
924*d^7*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (792*d^7*((1 -
 d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (495*d^7*((1 - d*x)^(1/2) -
 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (220*d^7*((1 - d*x)^(1/2) - 1)^18)/((d*x
 + 1)^(1/2) - 1)^18 + (66*d^7*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) -
```

```
1)^20 + (12*d^7*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^7*(
(1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24) - (atan(((1 - d*x)^(1/2)
 - 1)/((d*x + 1)^(1/2) - 1))*(5*c^3 + 16*a^3*d^6 + 24*a*b^2*d^4 + 18*a*c^2*
d^2 + 24*a^2*c*d^4 + 18*b^2*c*d^2))/(4*d^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.793 $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=166

$$\frac{\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1}}{3d^2}$$

```
[Out] 1/8*(8*a^2*d^4+8*a*c*d^2+4*b^2*d^2+3*c^2)*arcsin(d*x)/d^5-2/3*b*(3*a*d^2+2*
c)*(-d^2*x^2+1)^(1/2)/d^4-1/8*(4*b^2+c*(8*a+3*c/d^2))*x*(-d^2*x^2+1)^(1/2)/
d^2-2/3*b*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/4*c^2*x^3*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A] time = 0.32, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 1815, 641, 216}

$$\frac{\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*b*(2*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)
/d^2))*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2)
 - (c^2*x^3*Sqrt[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
8*a^2*d^4)*ArcSin[d*x])/(8*d^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(a+bx+cx^2)^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} - \frac{\int \frac{-4a^2d^2-8abd^2x-(3c^2+4b^2d^2+8acd^2)x^2-8bcd^2x^3}{\sqrt{1-d^2x^2}}dx}{4d^2}$$
$$= -\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{\int \frac{12a^2d^4+8bd^2(2c+3ad^2)x+3d^2(3c^2+4b^2d^2+8acd^2)x^2}{\sqrt{1-d^2x^2}}dx}{12d^4}$$
$$= -\frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} - \frac{\int \frac{-3d^2(}{}}{}$$
$$= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}$$
$$= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}$$

Mathematica [A] time = 0.12, size = 114, normalized size = 0.69

$$\frac{3\sin^{-1}(dx)(8a^2d^4+8acd^2+4b^2d^2+3c^2) - d\sqrt{1-d^2x^2}\left(16b(3ad^2+cd^2x^2+2c)+3cx(8ad^2+2cd^2x^2+3c)\right.}{24d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*b^2*d^2*x + 16*b*(2*c + 3*a*d^2 + c*d^2*x^2) + 3
*c*x*(3*c + 8*a*d^2 + 2*c*d^2*x^2))) + 3*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8
*a^2*d^4)*ArcSin[d*x])/(24*d^5)
```

fricas [A] time = 0.68, size = 134, normalized size = 0.81

$$-\frac{(6c^2d^3x^3+16bcd^3x^2+48abd^3+32bcd+3(4(b^2+2ac)d^3+3c^2d)x)\sqrt{dx+1}\sqrt{-dx+1}+6(8a^2d^4+4(}{24d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*((6*c^2*d^3*x^3 + 16*b*c*d^3*x^2 + 48*a*b*d^3 + 32*b*c*d + 3*(4*(b^2
+ 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*a^2*d^4 + 4*
(b^2 + 2*a*c)*d^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

giac [A] time = 0.42, size = 196, normalized size = 1.18

$$-\frac{\left((dx+1)\left(2(dx+1)\left(\frac{3(dx+1)c^2}{d^4}+\frac{8bcd^{17}-9c^2d^{16}}{d^{20}}\right)+\frac{12b^2d^{18}+24acd^{18}-32bcd^{17}+27c^2d^{16}}{d^{20}}\right)+\frac{3(16abd^{19}-4b^2d^{18}-8acd^{18}+16bcd}{d^{20}}\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)*c^2/d^4 + (8*b*c*d^17 - 9*c^2*d
^16)/d^20) + (12*b^2*d^18 + 24*a*c*d^18 - 32*b*c*d^17 + 27*c^2*d^16)/d^20)
```

```
+ 3*(16*a*b*d^19 - 4*b^2*d^18 - 8*a*c*d^18 + 16*b*c*d^17 - 5*c^2*d^16)/d^20
)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2 + 3*c
^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)/d
```

maple [C] time = 0.03, size = 291, normalized size = 1.75

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6\sqrt{-d^2x^2+1}\,c^2d^3x^3\mathrm{csgn}(d)+16\sqrt{-d^2x^2+1}\,bc\,d^3x^2\mathrm{csgn}(d)-24a^2d^4\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*csgn(d)*x^3*c^2*d^3*(-d^2*x^2+1)^(1/2
)+16*csgn(d)*x^2*b*c*d^3*(-d^2*x^2+1)^(1/2)+24*csgn(d)*d^3*(-d^2*x^2+1)^(1/
2)*x*a*c+12*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2+48*(-d^2*x^2+1)^(1/2)*csgn
(d)*d^3*a*b-24*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a^2*d^4+9*csgn(d)*d
*(-d^2*x^2+1)^(1/2)*x*c^2+32*(-d^2*x^2+1)^(1/2)*csgn(d)*d*b*c-24*arctan(1/(
-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a*c*d^2-12*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*c
sgn(d))*b^2*d^2-9*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*c^2)*csgn(d)/d^5
/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 171, normalized size = 1.03

$$-\frac{\sqrt{-d^2x^2+1}\,c^2x^3}{4\,d^2}-\frac{2\,\sqrt{-d^2x^2+1}\,bcx^2}{3\,d^2}+\frac{a^2\arcsin(dx)}{d}-\frac{2\,\sqrt{-d^2x^2+1}\,ab}{d^2}-\frac{\sqrt{-d^2x^2+1}\left(b^2+2\,ac\right)x}{2\,d^2}-\frac{3\,\sqrt{-d^2x^2+}}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*sqrt(-d^2*x^2 + 1)*b*c*x^2/d^2 +
a^2*arcsin(d*x)/d - 2*sqrt(-d^2*x^2 + 1)*a*b/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(
b^2 + 2*a*c)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*c^2*x/d^4 + 1/2*(b^2 + 2*a*c)*a
rcsin(d*x)/d^3 - 4/3*sqrt(-d^2*x^2 + 1)*b*c/d^4 + 3/8*c^2*arcsin(d*x)/d^5
```

mupad [B] time = 13.85, size = 897, normalized size = 5.40

$$-\frac{\frac{\left(\sqrt{1-d\,x}-1\right)^{15}\left(2\,b^2\,d^2+\frac{3\,c^2}{2}+4\,a\,c\,d^2\right)}{\left(\sqrt{d\,x+1}-1\right)^{15}}+\frac{\left(\sqrt{1-d\,x}-1\right)^{3}\left(6\,b^2\,d^2-\frac{23\,c^2}{2}+12\,a\,c\,d^2\right)}{\left(\sqrt{d\,x+1}-1\right)^{3}}-\frac{\left(\sqrt{1-d\,x}-1\right)^{13}\left(6\,b^2\,d^2-\frac{23\,c^2}{2}+12\,a\,c\,d^2\right)}{\left(\sqrt{d\,x+1}-1\right)^{13}}+\frac{\left(\sqrt{1-d\,x}-1\right)^{5}\left(30\,b\right.}{\left(\sqrt{d\right.}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^15*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1
)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^3*(6*b^2*d^2 - (23*c^2)/2 + 12*a*c
*d^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*(6*b^2*d^2 - (23
*c^2)/2 + 12*a*c*d^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*
((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^11*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2)
 - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*(22*b^2*d^2 - (671*c^2)/2 + 44*a*c*d^2)
)/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*(22*b^2*d^2 - (671*c^2
)/2 + 44*a*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(128*
b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(1
28*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8
```

```
*((256*b*c*d)/3 + 320*a*b*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2)
 - 1)^6*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x
)^(1/2) - 1)^10*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^10 - (
((1 - d*x)^(1/2) - 1)*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1)^(1/2)
 - 1) + (16*a*b*d^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (16*
a*b*d^3*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8
+ (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1
- d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) -
1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)
^(1/2) - 1)^16) - (atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(3*c^2
 + 8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2))/(2*d^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.794 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.71

$$\frac{(2ad^2+c)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}\,(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

fricas [A] time = 1.00, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1} + 2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

giac [A] time = 0.27, size = 76, normalized size = 1.21

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left(\frac{(dx+1)c}{d^2} + \frac{2\,bd^5-cd^4}{d^6}\right) - \frac{2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
[Out] -1/2*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6
) - 2*(2*a*d^2 + c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.02, size = 117, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-2a\,d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right) + \sqrt{-d^2x^2+1}\;cdx\,\mathrm{csgn}(d) + 2\sqrt{-d^2x^2+1}\;bd\,\mathrm{csgn}(d) - c\,\mathrm{ar}\right.}{2\sqrt{-d^2x^2+1}\;d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

maxima [A] time = 0.97, size = 57, normalized size = 0.90

$$\frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}\,b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

mupad [B] time = 7.76, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\left(\sqrt{1-d\,x}\right.}{\left(\sqrt{d\,x+1}-\right.}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 49.71, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.795 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

```
[Out] -c*arctanh(1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2
)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2
)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+c*arctanh(1/2*
(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*
d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+
2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 985, 725, 206}

$$\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] -((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*
c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (
Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2
 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^
2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}} + \frac{(2c)\,\text{Sub}}{}$$

$$= -\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}} + \frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{}{\sqrt{2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2}}$$

Mathematica [A] time = 0.56, size = 260, normalized size = 0.92

$$\frac{2\sqrt{2}c\left(\frac{\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{1-d^2x^2}\sqrt{-2bd^2\left(\sqrt{b^2-4ac}+b\right)+4acd^2+4c^2}}\right)}{2\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}} - \frac{\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{2\sqrt{bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*Sqrt[2]*c*(-1/2*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^
2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]/Sqrt[
2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2] + ArcTanh[(2*c + (b + S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*
c])*d^2]*Sqrt[1 - d^2*x^2])]/(2*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2])))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 1.20, size = 4313, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b
^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^
2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*
```

a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*

```
b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((
a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*
d^2)))/x) + 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 +
b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2
*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sq
rt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4
 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqr
t(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d
^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(2)*(((a^3*b^3 - 4
*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^
5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 +
(b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^2))*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d
^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^
2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4
*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5
 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b
^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (
b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x)
```

giac [B] time = 1.50, size = 684, normalized size = 2.43

$$-\frac{\left(ad^2-bd+c\right)\left(\frac{\left(ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}\right)d}{ad^2-bd+c}-d\right)\sqrt{\frac{ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}{ad^2-bd+c}}\arctan\left(-\frac{\cdots}{2\sqrt{\cdots}}\right.}{\left(ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}\right)\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d^2 - b*d + c)*((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c)
+ (a*d^2 - c)^2))*d/(a*d^2 - b*d + c) - d)*sqrt((a*d^2 - c + sqrt(-(a*d^2 +
 b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c))*arctan(-1/
2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt
(-d*x + 1)))/sqrt((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) +
(a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(
a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c
) + (a*d^2 - c)^2)) + (a*d^2 - b*d + c)*((a*d^2 - c - sqrt(-(a*d^2 + b*d +
c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*d/(a*d^2 - b*d + c) - d)*sqrt((a*d^2
 - c - sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 -
 b*d + c))*arctan(-1/2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x
 + 1)/(sqrt(2) - sqrt(-d*x + 1)))/sqrt((a*d^2 - c - sqrt(-(a*d^2 + b*d + c)
*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c - sqrt
(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d
+ c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))
```

maple [C] time = 0.14, size = 1759, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -32*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*c^2*(ln(2*(x*b*d^2-(-4*a*c+b^2)^
(1/2)*x*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d
^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(2*c*x-(-4*a*c+b^2
)^(1/2)+b))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c
+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+x*b*d^
2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))
*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2
)-2*a*c+b^2)/a^2/c^2)^(1/2)+2*ln(2*(x*b*d^2-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*
x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^
(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(2*c*x-(-4*a*c+b^2)^(1/2)+b))*a*c*d^
2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-
b^2)/a^2/c^2)^(1/2)-ln(2*(x*b*d^2-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*x^2+1)^(1/
2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*
c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(2*c*x-(-4*a*c+b^2)^(1/2)+b))*b^2*d^2*(-(b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c
^2)^(1/2)-2*ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+x*b*d^2+(-d^2*x^2+1)^(1/2)*(-(b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^
2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*a*c*d^2*(-(b*(-4*a*c+b^2)
^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2
)+ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+x*b*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^
2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/
2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a
*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)+ln(2*(x*
b*d^2-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2
*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)
/(2*c*x-(-4*a*c+b^2)^(1/2)+b))*c^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^
2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/
2)*x*d^2+x*b*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a
^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a
*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2))/(-d^2*x^2+1)^(1/2)/(b*d-d*(-4*a*c+b
^2)^(1/2)+2*c)/(d*(-4*a*c+b^2)^(1/2)+b*d+2*c)/(b*d-d*(-4*a*c+b^2)^(1/2)-2*c
)/(-4*a*c+b^2)^(1/2)/(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a
*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)/(d*(-4*a*c+b^2)^(1/2)+b*d-2*c)/(-(b
*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a
^2/c^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)\sqrt{dx+1}\,\sqrt{-dx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

mupad [B] time = 82.37, size = 33018, normalized size = 117.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^
2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 3
```

2*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d ^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16 *a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^ (1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2 *c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32 *a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^ 2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16* a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8 *a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2* c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32* a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a ^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 1 6*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1 /2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8* d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628388371660 8*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392 *a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a ^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6* d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473 5460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792 *a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c ^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1 /2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340 029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5 *b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816* a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c ^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992 8776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b ^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^ 5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054 7964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4 *d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4831838208 0*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a ^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^1 0 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 10812 58016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184 *a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c ^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116 460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c ^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 2961 8094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b *c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^1 0 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 1556 4961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1 /2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 574494825 51296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^1 3 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 12884901888 0*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4* c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 12197707 12064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^ 8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 123695058124 8*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072* a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 3994319 58528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5* b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(26800595927 04*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^ 9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 644245094 40*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4 *d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 4 29496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^ 8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266 704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6 *d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 6871947 6736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c ^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 257698 03776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c ^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 3049426 78016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d ^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 11338713 66144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3 *b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^ 14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 348751344435 2*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d ^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 1047972020 2240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728* a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 16 32087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^ 2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b* c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 3006 4771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3 *c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 687194 76736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233 382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^ 3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 1 0960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^ 2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 31525 05995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450 944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2 *d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960* a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 14 6028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^1 0 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i + (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6* a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 +

10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889

785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i)/((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^

(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2 *c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32 *a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^ 2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16* a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8 *a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2* c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32* a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a ^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 1 6*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1 /2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8* a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c ^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a ^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741 824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283 883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^ 6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 22677 42732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c ^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934 592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4 *c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 417 3634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 14023927214 8992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6 *b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 10 73741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 115448720916 48*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d ^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 68719476 7360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b ^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^1 1 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 374 5211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 34359738368 0*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 21 47483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224* a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 11 5964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8 *c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 54975581 3888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2* c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5 738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208 320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b ^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704* a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^1 0 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 4724464025 6*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5 *d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 37795712 2048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(3 0236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5 *c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778 116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2* b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d

^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672
924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272
*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 773
09411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^
2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 254262
0639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^
3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 305
8016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768
*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12
+ (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*
b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 1724
8588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b
*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11
- 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 1889785
61024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147
483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 +
44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a
^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2
045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*
b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^
10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 158
57019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1
/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244
096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 -
137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b
^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 103
0792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048
*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*
c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(139500
53777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*
c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360
777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a
^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1511828
4881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^
11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388
352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2
*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 22333829939
2*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10
+ 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*
a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d
^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) -
1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616
*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7
- 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*
d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b
^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312
512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^1
0 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416
*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 +
149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*
b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(214
7483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4
*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^
5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^
(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 14602
8888064*a*b*c^4*d^8) - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b
^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^
3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/

2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396
983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4
*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(
16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2
 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2))
)^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b
^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*
d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*
a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^
2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b
^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2
*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^
4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d
^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 -
 d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*
d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 472446
40256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*
c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 177
21035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026
624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b
^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 +
b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*
c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a
^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)
*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^1
4 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a
^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 +
29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*
b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 6
6571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*
b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8
 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263
779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473546
0352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^
5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*
d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2)
 - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029
796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*
c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42
949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2
*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*
d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57
999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992877
6704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*
c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b
^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054796
4928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^
14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a
*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8
589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*
b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 -
 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 10812580
16768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^
5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*
d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*(302365697638
40*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 -
 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a

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*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7
 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 149464
8619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a
^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^
3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*
b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 32
98534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5
*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 61
8475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*
a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^
3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*
x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 -
 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*
a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 +
 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 141733920
76800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7
*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*
b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401
664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12
 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 204547817472
0*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14
- 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 742084
44940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832
*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2
- 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6
*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 1374389534
72*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10
 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*
a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^
4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 -
25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^
2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 +
14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a
^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*
d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2
*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 377957
1220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2
*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 +
 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4
*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 123695058
1248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d
^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 6871
9476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 22
33382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*
b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x
 + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 +
(((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 +
 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*
b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 315
2505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2
) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 64424
50944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c
^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a
^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 214748364
8*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) +
 283467841536*a*c^4*d^8 + (2*((1 - d*x)^(1/2) - 1)^2*(519691042816*a*c^4*d^
8 + 1073741824*a*b^2*c^2*d^10))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a*b^2*
c^2*d^10 + (34359738368*a*b*c^3*d^9*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
 - 1)))*(-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
```

8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*2i - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c

^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(18897856

10240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/(
(d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^
10 + 146028888064*a*b*c^4*d^8)*1i + (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4
+ b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c
^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(((
(1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d
^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*
b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^
2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 3
2*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*
a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*
d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(1
6*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2
+ 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))
^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^
2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 3
2*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d
^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16
*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^
(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 284498633687
04*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*
d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 4724464
0256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*
c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10
 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 37795
7122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2
 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2
)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c
^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c
d^2)))^(1/2)((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 214748364
8*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 362
83883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^
6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 76
0209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b
^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 181419
41858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^
3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c
^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12
 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 247
8196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184
*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1
 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*
d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758
715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*
c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720
238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488
*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*
c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13
 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503
724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 107
3741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^
8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 116135915683
84*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 +
48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b
^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 6184
75290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b

^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^
10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 2149
2016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712
*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*
(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a
^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 17
78116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^
2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5
*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 46
72924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 35734127902
72*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 7
7309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*
c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542
620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*
c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3
058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 17523466567
68*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^1
2 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^
2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17
248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5
*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^1
1 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 18897
8561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(21
47483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6
+ 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928
*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 -
2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^
4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*
d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 1
5857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^
(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 7559142
44096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12
+ 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a
*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1
030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 35991825940
48*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^
2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(1395
0053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^
5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 3
60777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240
*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118
284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*
d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 99299643
88352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b
^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299
392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^1
0 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 25769803776
0*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2
*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3
*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 1245
54051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c
*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^
3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 226774273228
8*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 254
6915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4
*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((
d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2
*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676
416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 1

88978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1
) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a
*b*c^4*d^8)*1i)/((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2
 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*
c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^
2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^
2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*
c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*
c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2
- 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c
*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2
 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2
)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c
^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c
d^2)))^(1/2)((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 -
 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*
d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2
+ b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)
/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^
3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*
d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648
*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628
3883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6
*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760
209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^
8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 1814194
1858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3
*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^
4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12
+ 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478
196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*
a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1
- d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d
^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 421937587
15904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c
^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 107202
38370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*
a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c
^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13
- 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 1675037
24544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073
741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8
 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 1161359156838
4*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4
8318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^
6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 61847
5290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^
6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^1
0 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492
016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*
a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2
*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 176737904230
4*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^
12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244
096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^
5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^

12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4

*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d

^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 824
6337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 94918777241
6*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c
^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551
296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13
+ 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*
a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^
4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4
260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712
064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*
c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*
a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 -
30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^
4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 +
910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958
528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^
3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704
*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9
+ 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440
*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d
^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429
496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) -
1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*
d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 8579626670
4896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d
^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 687194767
36*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5
*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 +
2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803
776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 -
549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5
*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678
016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^1
2 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366
144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b
^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14
) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*
a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^1
0 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 104797202022
40*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)
^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^
3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632
087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*
b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))
/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^
5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 300647
71072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c
^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d
x)^(1/2) - 1)(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 -
197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352
*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767
360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(10
73741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8
+ 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a
*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10
+ 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^
2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 91912
3001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2)
- 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*

```
b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 343597383
68*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) + 283467841536*a*c^4*d^8 + (2
*((1 - d*x)^(1/2) - 1)^2*(519691042816*a*c^4*d^8 + 1073741824*a*b^2*c^2*d^1
0))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a*b^2*c^2*d^10 + (34359738368*a*b*
c^3*d^9*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))*(-(8*a*c^3 - 2*b^2*c
^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d
^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3
*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4
*c*d^2)))^(1/2)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)), x)
```

3.796 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=571

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-\right.}}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\right.}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

```
[Out] -(b*(b^2*d^2-c*(3*a*d^2+c))-c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)*(-d^2*x^2+1)^(1/
2)/(-4*a*c+b^2)/(b^2*d^2-(a*d^2+c)^2)/(c*x^2+b*x+a)-1/2*c*arctanh(1/2*(2*c+
d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b
*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3+12*a*c^2*d^2-a*b*d^4*(b+(-4*a*c+
b^2)^(1/2))-c*d^2*(5*b^2-8*a^2*d^2-b*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2
)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2
)))^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^
2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3
+12*a*c^2*d^2-2*a*b^2*d^4-4*c*d^2*(-2*a^2*d^2+b^2)-b*d^2*(-a*d^2+c)*(b+(-4*
a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 5.23, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 975, 1034, 725, 206}

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-\right.}}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\right.}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)))
 - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^
2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a
*c^2*d^2 - 2*a*b^2*d^4 - b*(b + Sqrt[b^2 - 4*a*c])*d^2*(c - a*d^2) - 4*c*d^
2*(b^2 - 2*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]
*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\,\sqrt{1+dx}\,\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} + \cdots$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} - \cdots$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} - \cdots$$

Mathematica [A] time = 1.28, size = 508, normalized size = 0.89

$$\frac{\frac{c\left(cd^2\left(8a^2d^2+b\sqrt{b^2-4ac}-5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2}}+\frac{c\left(cd^2\left(-8a^2d^2+b\sqrt{b^2-4ac}+\right.\right.}{}}{\left(b^2-4ac\right)\left(\left(a\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] (((b^3*d^2 - b*c*(c + 3*a*d^2) + b^2*c*d^2*x - 2*c^2*(c + a*d^2)*x)*Sqrt[1
- d^2*x^2])/(a + x*(b + c*x)) + (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^
2 - 4*a*c])*d^4 + c*d^2*(-5*b^2 + b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2))*ArcTanh
[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b +
Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2]) + (c*(-4*c^3 - 12*a*
c^2*d^2 + a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(5*b^2 + b*Sqrt[b^2 - 4*a
*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2
 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[
2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2
]))/((b^2 - 4*a*c)*(-(b^2*d^2) + (c + a*d^2)^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [C] time = 0.81, size = 41837, normalized size = 73.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^2\sqrt{dx+1}\sqrt{-dx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.797 $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-\frac{3\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2\right.}{8d^7}$$

```
[Out] -3/8*(8*a^2*c*d^4+8*a*b^2*d^4+12*a*c^2*d^2+12*b^2*c*d^2+5*c^3)*arcsin(d*x)/
d^7+(b*(3*a^2+3*c^2/d^4+b^2/d^2+6*a*c/d^2)*d^4+(a*d^2+c)*(a^2*d^4+2*a*c*d^2
+3*b^2*d^2+c^2)*x)/d^6/(-d^2*x^2+1)^(1/2)+b*(6*a*c*d^2+b^2*d^2+5*c^2)*(-d^2
*x^2+1)^(1/2)/d^6+1/8*c*(12*a*c*d^2+12*b^2*d^2+7*c^2)*x*(-d^2*x^2+1)^(1/2)/
d^6+b*c^2*x^2*(-d^2*x^2+1)^(1/2)/d^4+1/4*c^3*x^3*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] time = 0.60, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 1814, 1815, 641, 216}

$$\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-\frac{3\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2\right.}{8d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 + 3
*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (b*(5*c^2 + b^
2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2])/d^6 + (c*(7*c^2 + 12*b^2*d^2 + 12*a*c
*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^6) + (b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^4 + (c
^3*x^3*Sqrt[1 - d^2*x^2])/(4*d^4) - (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2
 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/(8*d^7)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{\left(a+bx+cx^2\right)^3}{\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} - \int \frac{\frac{c^3+3ac^2d^2}{}}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{c^3x^3\sqrt{1-}}{4d^4}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{bc^2x^2\sqrt{1-}}{d^4}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{c\left(7c^2+12\right.}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{b\left(5c^2+b^2\right.}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{b\left(5c^2+b^2\right.}{}$$

Mathematica [A] time = 0.24, size = 239, normalized size = 0.87

$$\frac{-3\sqrt{1-d^2x^2}\,\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3\right)-8b\left(-3a^2d^5+6acd^3\left(d^2x^2-2\right)+c^2d\left(d^4x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (-8*b^3*d^3*(-2 + d^2*x^2) - 12*b^2*d^3*x*(-2*a*d^2 + c*(-3 + d^2*x^2)) + d
*x*(24*a^2*c*d^4 + 8*a^3*d^6 - 12*a*c^2*d^2*(-3 + d^2*x^2) + c^3*(15 - 5*d^
2*x^2 - 2*d^4*x^4)) - 8*b*(-3*a^2*d^5 + 6*a*c*d^3*(-2 + d^2*x^2) + c^2*d*(-
8 + 4*d^2*x^2 + d^4*x^4)) - 3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 8*a*b^
2*d^4 + 8*a^2*c*d^4)*Sqrt[1 - d^2*x^2]*ArcSin[d*x])/(8*d^7*Sqrt[1 - d^2*x^2
])
```

fricas [A] time = 0.70, size = 376, normalized size = 1.36

$$-\frac{24\,a^2bd^5+64\,bc^2d+16\left(b^3+6\,abc\right)d^3-8\left(3\,a^2bd^7+8\,bc^2d^3+2\left(b^3+6\,abc\right)d^5\right)x^2-\left(2\,c^3d^5x^5+8\,bc^2d^5x^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/8*(24*a^2*b*d^5 + 64*b*c^2*d + 16*(b^3 + 6*a*b*c)*d^3 - 8*(3*a^2*b*d^7 +
 8*b*c^2*d^3 + 2*(b^3 + 6*a*b*c)*d^5)*x^2 - (2*c^3*d^5*x^5 + 8*b*c^2*d^5*x^
4 - 24*a^2*b*d^5 - 64*b*c^2*d - 16*(b^3 + 6*a*b*c)*d^3 + (5*c^3*d^3 + 12*(b
^2*c + a*c^2)*d^5)*x^3 + 8*(4*b*c^2*d^3 + (b^3 + 6*a*b*c)*d^5)*x^2 - (8*a^3
*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*sqrt(
d*x + 1)*sqrt(-d*x + 1) + 6*(8*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 12*(b^2*c + a*
c^2)*d^2 - (8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^9*x^2 - d^7)
```

giac [B] time = 0.61, size = 732, normalized size = 2.65

$$\left(\left((dx+1)\left(2\,(dx+1)\left(\frac{(dx+1)c^3}{d^7}+\frac{4\,bc^2d^{36}-5\,c^3d^{35}}{d^{42}}\right)+\frac{12\,b^2cd^{37}+12\,ac^2d^{37}-32\,bc^2d^{36}+25\,c^3d^{35}}{d^{42}}\right)+\frac{8\,b^3d^{38}+48\,abcd^{38}-36\,b^2cd^{37}-36\,}{d^{42}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)*c^3/d^7 + (4*b*c^2*d^36 - 5*c^3*d^3
5)/d^42) + (12*b^2*c*d^37 + 12*a*c^2*d^37 - 32*b*c^2*d^36 + 25*c^3*d^35)/d^
42) + (8*b^3*d^38 + 48*a*b*c*d^38 - 36*b^2*c*d^37 - 36*a*c^2*d^37 + 80*b*c^
2*d^36 - 35*c^3*d^35)/d^42)*(d*x + 1) - 2*(2*a^3*d^41 + 6*a^2*b*d^40 + 6*a*
b^2*d^39 + 6*a^2*c*d^39 + 10*b^3*d^38 + 60*a*b*c*d^38 - 6*b^2*c*d^37 - 6*a*
c^2*d^37 + 54*b*c^2*d^36 - 7*c^3*d^35)/d^42)*sqrt(d*x + 1)*sqrt(-d*x + 1)/(
d*x - 1) - 3/4*(8*a*b^2*d^4 + 8*a^2*c*d^4 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 5
*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^7 + 1/4*(a^3*d^6*(sqrt(2) - sqrt(
-d*x + 1))/sqrt(d*x + 1) - 3*a^2*b*d^5*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) + 3*a*b^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a^2*c*d^4*(
sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - b^3*d^3*(sqrt(2) - sqrt(-d*x + 1)
)/sqrt(d*x + 1) - 6*a*b*c*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*
b^2*c*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a*c^2*d^2*(sqrt(2) -
 sqrt(-d*x + 1))/sqrt(d*x + 1) - 3*b*c^2*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(
d*x + 1) + c^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^7 - 1/4*(a^3*d^6
 - 3*a^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d^3 + 3*b^2*
c*d^2 + 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*sqrt(d*x + 1)/(d^7*(sqrt(2) - sqrt(-
d*x + 1)))
```

maple [C] time = 0.04, size = 755, normalized size = 2.74

$$\sqrt{-dx+1}\,\left(2\sqrt{-d^2x^2+1}\,c^3d^5x^5\mathrm{csgn}(d)+8\sqrt{-d^2x^2+1}\,b\,c^2d^5x^4\mathrm{csgn}(d)-8\sqrt{-d^2x^2+1}\,a^3d^7x\,\mathrm{csgn}(d)-24a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] 1/8*(-d*x+1)^(1/2)*(-16*(-d^2*x^2+1)^(1/2)*b^3*d^3*csgn(d)+24*a^2*c*d^4*arc
tan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+24*a*b^2*d^4*arctan(1/(-d^2*x^2+1)^(1
/2)*d*x*csgn(d))+36*a*c^2*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+36*b
^2*c*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+15*c^3*arctan(1/(-d^2*x^2
+1)^(1/2)*d*x*csgn(d))-15*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*c^3*
d^2+2*(-d^2*x^2+1)^(1/2)*c^3*d^5*x^5*csgn(d)-36*arctan(1/(-d^2*x^2+1)^(1/2)
*d*x*csgn(d))*x^2*a*c^2*d^4-36*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2
*b^2*c*d^4-24*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*a^2*c*d^6-24*arc
tan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*a*b^2*d^6-8*csgn(d)*d^7*(-d^2*x^2
+1)^(1/2)*x*a^3-15*(-d^2*x^2+1)^(1/2)*c^3*d*x*csgn(d)-64*(-d^2*x^2+1)^(1/2)
```

```
*b*c^2*d*csgn(d)+8*(-d^2*x^2+1)^(1/2)*b^3*d^5*x^2*csgn(d)+5*(-d^2*x^2+1)^(1
/2)*c^3*d^3*x^3*csgn(d)-24*(-d^2*x^2+1)^(1/2)*a^2*b*d^5*csgn(d)+8*(-d^2*x^2
+1)^(1/2)*b*c^2*d^5*x^4*csgn(d)+12*(-d^2*x^2+1)^(1/2)*a*c^2*d^5*x^3*csgn(d)
+12*(-d^2*x^2+1)^(1/2)*b^2*c*d^5*x^3*csgn(d)-24*(-d^2*x^2+1)^(1/2)*a^2*c*d^
5*x*csgn(d)-24*(-d^2*x^2+1)^(1/2)*a*b^2*d^5*x*csgn(d)+32*(-d^2*x^2+1)^(1/2)
*b*c^2*d^3*x^2*csgn(d)-36*(-d^2*x^2+1)^(1/2)*a*c^2*d^3*x*csgn(d)-36*(-d^2*x
^2+1)^(1/2)*b^2*c*d^3*x*csgn(d)-96*(-d^2*x^2+1)^(1/2)*a*b*c*d^3*csgn(d)+48*
(-d^2*x^2+1)^(1/2)*a*b*c*d^5*x^2*csgn(d))*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2
)/d^7/(d*x+1)^(1/2)
```

maxima [A] time = 0.98, size = 371, normalized size = 1.34

$$-\frac{c^3x^5}{4\sqrt{-d^2x^2+1}\,d^2}-\frac{bc^2x^4}{\sqrt{-d^2x^2+1}\,d^2}+\frac{a^3x}{\sqrt{-d^2x^2+1}}-\frac{5\,c^3x^3}{8\sqrt{-d^2x^2+1}\,d^4}-\frac{3\left(b^2c+ac^2\right)x^3}{2\sqrt{-d^2x^2+1}\,d^2}+\frac{3\,a^2b}{\sqrt{-d^2x^2+1}\,d^2}-\frac{4\,bc^2x}{\sqrt{-d^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2)
+ a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(b^
2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*d^2
) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d^2*x
^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2 + a^
2*c)*arcsin(d*x)/d^3 + 15/8*c^3*x/(sqrt(-d^2*x^2 + 1)*d^6) + 9/2*(b^2*c + a
*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(d*x)/d^7 - 9/2*(b^2*c +
a*c^2)*arcsin(d*x)/d^5 + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3 + 6*a*b*
c)/(sqrt(-d^2*x^2 + 1)*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^3}{(1-d\,x)^{3/2}\,(d\,x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

3.798 $\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}} \, dx$

Optimal. Leaf size=135

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\sin^{-1}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

```
[Out] -1/2*(2*b^2+c*(4*a+3*c/d^2))*arcsin(d*x)/d^3+(2*b*(a+c/d^2)*d^2+(a^2*d^4+2*
a*c*d^2+b^2*d^2+c^2)*x)/d^4/(-d^2*x^2+1)^(1/2)+2*b*c*(-d^2*x^2+1)^(1/2)/d^4
+1/2*c^2*x*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {899, 1814, 1815, 641, 216}

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\sin^{-1}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[1
 - d^2*x^2]) + (2*b*c*Sqrt[1 - d^2*x^2])/d^4 + (c^2*x*Sqrt[1 - d^2*x^2])/(2
*d^4) - ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
```

```
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{\left(a+bx+cx^2\right)^2}{\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} - \int \frac{\frac{c^2+b^2d^2+2acd^2}{d^4}+\frac{2bcx}{d^2}+\frac{c^2x^2}{d^2}}{\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \frac{\int \frac{-2b^2-c\left(4a+\frac{3c}{d^2}\right)-4b}{\sqrt{1-d^2x^2}}}{2d^2}$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} -$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} -$$

Mathematica [A] time = 0.12, size = 127, normalized size = 0.94

$$\frac{dx\left(2a^2d^4+4acd^2+c^2\left(3-d^2x^2\right)\right)-\sqrt{1-d^2x^2}\,\sin^{-1}(dx)\left(4acd^2+2b^2d^2+3c^2\right)+4bd\left(ad^2+c\left(2-d^2x^2\right)\right)+2b}{2d^5\sqrt{1-d^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b^2*d^3*x + 4*b*d*(a*d^2 + c*(2 - d^2*x^2)) + d*x*(4*a*c*d^2 + 2*a^2*d^4
 + c^2*(3 - d^2*x^2)) - (3*c^2 + 2*b^2*d^2 + 4*a*c*d^2)*Sqrt[1 - d^2*x^2]*A
rcSin[d*x])/(2*d^5*Sqrt[1 - d^2*x^2])
```

fricas [A] time = 0.89, size = 204, normalized size = 1.51

$$-\frac{4\,abd^3+8\,bcd-4\left(abd^5+2\,bcd^3\right)x^2-\left(c^2d^3x^3+4\,bcd^3x^2-4\,abd^3-8\,bcd-\left(2\,a^2d^5+2\left(b^2+2\,ac\right)d^3+3\,c^2d\right)x}{2\left(d^7x^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a*b*d^3 + 8*b*c*d - 4*(a*b*d^5 + 2*b*c*d^3)*x^2 - (c^2*d^3*x^3 + 4*
b*c*d^3*x^2 - 4*a*b*d^3 - 8*b*c*d - (2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^
2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*(b^2 + 2*a*c)*d^2 - (2*(b^2 + 2
*a*c)*d^4 + 3*c^2*d^2)*x^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/(d^7*x^2 - d^5)
```

giac [B] time = 0.39, size = 387, normalized size = 2.87

$$\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{(dx+1)c^2}{d^5}+\frac{4\,bcd^{16}-3\,c^2d^{15}}{d^{20}}\right)-\frac{a^2d^{19}+2\,abd^{18}+b^2d^{17}+2\,acd^{17}+10\,bcd^{16}-c^2d^{15}}{d^{20}}\right)}{2\,(dx-1)}-\frac{\left(2\,b^2d^2+4\,acd^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*((d*x + 1)*c^2/d^5 + (4*b*c*d^1
6 - 3*c^2*d^15)/d^20) - (a^2*d^19 + 2*a*b*d^18 + b^2*d^17 + 2*a*c*d^17 + 10
*b*c*d^16 - c^2*d^15)/d^20)/(d*x - 1) - (2*b^2*d^2 + 4*a*c*d^2 + 3*c^2)*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1))/d^5 + 1/4*(a^2*d^4*(sqrt(2) - sqrt(-d*x + 1)
)/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + b^2*
d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) - sqrt(-d
*x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) +
 c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^5 - 1/4*(a^2*d^4 - 2*a*b*d
^3 + b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^5*(sqrt(2) - sqr
t(-d*x + 1)))
```

maple [C] time = 0.03, size = 380, normalized size = 2.81

$$\frac{\sqrt{-dx+1}\,\left(-2\sqrt{-d^2x^2+1}\;a^2d^5x\,\mathrm{csgn}(d)-4ac\,d^4x^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-2b^2d^4x^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+\sqrt{-d^2\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*((-d^2*x^2+1)^(1/2)*c^2*d^3*x^3*csgn(d)-2*csgn(d)*d^5*(-
d^2*x^2+1)^(1/2)*x*a^2+4*(-d^2*x^2+1)^(1/2)*b*c*d^3*x^2*csgn(d)-4*arctan(1/
(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*a*c*d^4-2*arctan(1/(-d^2*x^2+1)^(1/2)*d
*x*csgn(d))*x^2*b^2*d^4-4*(-d^2*x^2+1)^(1/2)*a*c*d^3*x*csgn(d)-2*(-d^2*x^2+
1)^(1/2)*b^2*d^3*x*csgn(d)-4*(-d^2*x^2+1)^(1/2)*a*b*d^3*csgn(d)-3*arctan(1/
(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*c^2*d^2-3*(-d^2*x^2+1)^(1/2)*c^2*d*x*cs
gn(d)-8*(-d^2*x^2+1)^(1/2)*b*c*d*csgn(d)+4*a*c*d^2*arctan(1/(-d^2*x^2+1)^(1
/2)*d*x*csgn(d))+2*b^2*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+3*c^2*a
rctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)
/d^5/(d*x+1)^(1/2)
```

maxima [A] time = 0.97, size = 176, normalized size = 1.30

$$\frac{a^2x}{\sqrt{-d^2x^2+1}}-\frac{c^2x^3}{2\sqrt{-d^2x^2+1}\,d^2}-\frac{2\,bcx^2}{\sqrt{-d^2x^2+1}\,d^2}+\frac{2\,ab}{\sqrt{-d^2x^2+1}\,d^2}+\frac{\left(b^2+2\,ac\right)x}{\sqrt{-d^2x^2+1}\,d^2}-\frac{\left(b^2+2\,ac\right)\arcsin\left(dx\right)}{d^3}+\frac{\cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] a^2*x/sqrt(-d^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-d^2*x^2 + 1)*d^2) - 2*b*c*x^2
/(sqrt(-d^2*x^2 + 1)*d^2) + 2*a*b/(sqrt(-d^2*x^2 + 1)*d^2) + (b^2 + 2*a*c)*
x/(sqrt(-d^2*x^2 + 1)*d^2) - (b^2 + 2*a*c)*arcsin(d*x)/d^3 + 3/2*c^2*x/(sqr
t(-d^2*x^2 + 1)*d^4) - 3/2*c^2*arcsin(d*x)/d^5 + 4*b*c/(sqrt(-d^2*x^2 + 1)*
d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^2}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
[Out] Timed out
```

3.799 $\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{x\left(ad^2+c\right)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

```
[Out] -c*arcsin(d*x)/d^3+(b+(a*d^2+c)*x)/d^2/(-d^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1814, 12, 216}

$$\frac{x\left(ad^2+c\right)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{a+bx+cx^2}{\left(1-d^2x^2\right)^{3/2}}\,dx$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \int \frac{c}{d^2\sqrt{1-d^2x^2}}\,dx$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \frac{c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{d^2}$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.98

$$\frac{\frac{d\left(x\left(ad^2+c\right)+b\right)}{\sqrt{1-d^2x^2}} - c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] ((d*(b + (c + a*d^2)*x))/Sqrt[1 - d^2*x^2] - c*ArcSin[d*x])/d^3
```

fricas [B] time = 0.97, size = 101, normalized size = 2.52

$$\frac{bd^3x^2 - \left(bd + \left(ad^3 + cd\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1} - bd + 2\left(cd^2x^2 - c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{d^5x^2 - d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^3*x^2 - (b*d + (a*d^3 + c*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - b*d + 2
*(c*d^2*x^2 - c)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*x^2
 - d^3)
```

giac [B] time = 0.30, size = 182, normalized size = 4.55

$$-\frac{2\,c\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^3} + \frac{\frac{ad^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} - \frac{bd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} + \frac{c\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{4\,d^3} - \frac{\left(ad^2 - bd + c\right)\sqrt{dx+1}}{4\,d^3\left(\sqrt{2}-\sqrt{-dx+1}\right)} - \frac{\left(ad^5 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*c*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3 + 1/4*(a*d^2*(sqrt(2) - sqrt(-d*
x + 1))/sqrt(d*x + 1) - b*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c*(s
qrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^3 - 1/4*(a*d^2 - b*d + c)*sqrt(d*
x + 1)/(d^3*(sqrt(2) - sqrt(-d*x + 1))) - 1/2*(a*d^5 + b*d^4 + c*d^3)*sqrt(
d*x + 1)*sqrt(-d*x + 1)/((d*x - 1)*d^6)
```

maple [C] time = 0.03, size = 151, normalized size = 3.78

$$\frac{\left(-\sqrt{-d^2x^2+1}\;a\,d^3x\,\mathrm{csgn}(d) - c\,d^2x^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-(dx-1)(dx+1)}}\right) - \sqrt{-d^2x^2+1}\;cdx\,\mathrm{csgn}(d) - \sqrt{-d^2x^2+1}\;bd\,\mathrm{csgn}(d\right.}{(dx-1)\sqrt{-d^2x^2+1}\;\sqrt{dx+1}\;d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] (-(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*x*a-arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^
(1/2))*x^2*c*d^2-(-d^2*x^2+1)^(1/2)*c*d*x*csgn(d)-(-d^2*x^2+1)^(1/2)*b*d*cs
gn(d)+arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^(1/2))*c)*(-d*x+1)^(1/2)*csgn(d
)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^3/(d*x+1)^(1/2)
```

maxima [A] time = 0.97, size = 61, normalized size = 1.52

$$\frac{ax}{\sqrt{-d^2x^2+1}} + \frac{cx}{\sqrt{-d^2x^2+1}\,d^2} - \frac{c\arcsin(dx)}{d^3} + \frac{b}{\sqrt{-d^2x^2+1}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*x/sqrt(-d^2*x^2 + 1) + c*x/(sqrt(-d^2*x^2 + 1)*d^2) - c*arcsin(d*x)/d^3 +
 b/(sqrt(-d^2*x^2 + 1)*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c\,x^2+b\,x+a}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

3.800 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=443

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}$$

```
[Out] d^2*(b-(a*d^2+c)*x)/(b^2*d^2-(a*d^2+c)^2)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(
1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*
a*c+b^2)^(1/2)))/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*c*arctanh(1/2*(2*c+d^2*x*(b
+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))/
(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+
(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 976, 1034, 725, 206}

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) + (
c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b - S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2
- 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) - (c
*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sq
rt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\int \frac{2d^2\left(c^2-b^2d^2+acd^2\right)-2bcd^4x}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,dx}{2d^2\left(b^2d^2-\left(c+ad^2\right)^2\right)}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-\right.\right.\right.}{\sqrt{b^2-4ac}\ \left(\right.}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-\right.\right.\right.}{}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2a}}$$

Mathematica [A] time = 2.39, size = 335, normalized size = 0.76

$$\frac{d^2\left(x\left(ad^2+c\right)-b\right)}{\sqrt{1-d^2x^2}\left(a^2d^4+2acd^2-b^2d^2+c^2\right)}-\frac{2\sqrt{2}\,c^3\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{\sqrt{b^2-4ac}\left(bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2\right)^{3/2}}+\frac{2\sqrt{2}\,c^3\tanh^{-1}\left(\cdots\right.}{\sqrt{b^2-4ac}\left(-bd^2\left(\sqrt{\cdots}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(-b + (c + a*d^2)*x))/((c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*Sqrt[1 -
d^2*x^2]) - (2*Sqrt[2]*c^3*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(S
qrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2]
)])/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2)
^(3/2)) + (2*Sqrt[2]*c^3*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqr
t[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])
/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)^(3/
2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.12, size = 11141, normalized size = 25.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-d\,x)^{3/2}\,(d\,x+1)^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)),x)
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.801 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=939

$$-\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(2c^4-d^2\left(b^2+6a^2d^2\right)c^2-\left(4a^3d^6+6ab^2d^4\right)c+b^2d^4\left(2b^2+a\right.\right.}{\left(b^2-4ac\right)\left(ad^2-bd+c\right)^2\left(ad^2+bd+c\right)^2\sqrt{1-d^2x^2}}$$

```
[Out] -d^2*(b*(-11*a^2*c*d^4+3*a*b^2*d^4-10*a*c^2*d^2+2*b^2*c*d^2+c^3)-(2*c^4+b^2
*d^4*(a^2*d^2+2*b^2)-c^2*d^2*(6*a^2*d^2+b^2)-c*(4*a^3*d^6+6*a*b^2*d^4))*x)/
(-4*a*c+b^2)/(a*d^2-b*d+c)^2/(a*d^2+b*d+c)^2/(-d^2*x^2+1)^(1/2)+(-b*(b^2*d^
2-c*(3*a*d^2+c))+c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)/(-4*a*c+b^2)/(b^2*d^2-(a*d^
2+c)^2)/(c*x^2+b*x+a)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b-(-
4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4
*a*c+b^2)^(1/2)))^(1/2))*(4*c^5+24*a*c^4*d^2+3*a*b^3*d^6*(b+(-4*a*c+b^2)^(1
/2))-c^3*d^2*(9*b^2-36*a^2*d^2-b*(-4*a*c+b^2)^(1/2))-2*a*c^2*d^4*(7*b^2-8*a
^2*d^2+5*b*(-4*a*c+b^2)^(1/2))+b*c*d^4*(2*b^3-17*a^2*b*d^2+2*b^2*(-4*a*c+b^
2)^(1/2)-11*a^2*d^2*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(a^2*d^4+2*a*c*
d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2
+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*c^5*d^2
-24*a*c^4*d^4-6*a*b^4*d^8-4*b^2*c*d^6*(-7*a^2*d^2+b^2)+2*c^3*(-18*a^2*d^6+4
*b^2*d^4)+8*c^2*(-2*a^3*d^8+3*a*b^2*d^6)+b*d^4*(-11*a^2*c*d^4+3*a*b^2*d^4-1
0*a*c^2*d^2+2*b^2*c*d^2+c^3)*(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/d^2
/(a^2*d^4+2*a*c*d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*
c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 11.84, antiderivative size = 938, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {899, 975, 1062, 1034, 725, 206}

$$-\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(2c^4-d^2\left(b^2+6a^2d^2\right)c^2-\left(4a^3d^6+6ab^2d^4\right)c+b^2d^4\left(2b^2+a\right.\right.}{\left(b^2-4ac\right)\left(ad^2-bd+c\right)^2\left(ad^2+bd+c\right)^2\sqrt{1-d^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) -
 (2*c^4 + b^2*d^4*(2*b^2 + a^2*d^2) - c^2*d^2*(b^2 + 6*a^2*d^2) - c*(6*a*b^
2*d^4 + 4*a^3*d^6))*x))/((b^2 - 4*a*c)*(c - b*d + a*d^2)^2*(c + b*d + a*d^2
)^2*Sqrt[1 - d^2*x^2])) - (b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d
^2 + 2*a*c*d^2)*x)/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^
2)*Sqrt[1 - d^2*x^2]) + (c*(4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 -
4*a*c])*d^6 - c^3*d^2*(9*b^2 - b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*
d^4*(7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*ArcTanh[(2*
c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b -
 Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*
Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*
a*c*d^2 + a^2*d^4)^2) - (c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*
c*d^6*(b^2 - 7*a^2*d^2) - b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2
```

```
- 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6
) - 8*c^2*(3*a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])
*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sq
rt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*d^2*Sqrt[2*c^2 + 2*a*c*d^2
- b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1062

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*(2*a*
f)) + c*(A*(2*c^2*d - c*(2*a*f)) - B*(-2*a*c*e) + C*(-2*a*(c*d - a*f)))*x))
/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((-4*a*c)*(a*c*e
^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*S
imp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1) + (2*(A*c*(c*
d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*
c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) -
(2*f*((A*c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*((Plus[2])*a*f)))*(p +
q + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(-(c*e*(2*p + q
```

```
+ 4))))*x - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2
- 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !Intege
rQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}} - \frac{\int \frac{-2c^3}{\cdots}}{\cdots}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(\cdots\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ac\cdots\right.\right.}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(\cdots\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ac\cdots\right.\right.}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(\cdots\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ac\cdots\right.\right.}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(\cdots\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ac\cdots\right.\right.}$$

Mathematica [A] time = 9.22, size = 890, normalized size = 0.95

$$\frac{\frac{\sqrt{2}\left(-3ab\left(b+\sqrt{b^2-4ac}\right)d^4+20ac^2d^2-c\left(7b^2-3\sqrt{b^2-4ac}\,b-16a^2d^2\right)d^2+4c^3\right)\tanh^{-1}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{2}\sqrt{2c^2+2ad^2c-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)c^3}{\sqrt{b^2-4ac}\left(2c^2+2ad^2c-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)^{3/2}} - \frac{\sqrt{2}\left(-3ab\left(b-\sqrt{b^2-4\cdots}\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((-(b^3*d^2) + b*c*(c + 3*a*d^2) + c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/((b^
2 - 4*a*c)*(-(b^2*d^2) + (c + a*d^2)^2)*(a + b*x + c*x^2)*Sqrt[1 - d^2*x^2]
)) + ((2*d^2*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/Sqrt[1 - d^2*x^2] - (c*(3*b*d
^2*(c - a*d^2) + (4*c^3 - 7*b^2*c*d^2 + 20*a*c^2*d^2 - 3*a*b^2*d^4 + 16*a^2
*c*d^4)/Sqrt[b^2 - 4*a*c])*(2*c - (b - Sqrt[b^2 - 4*a*c])*d^2*x))/((4*c^2 -
 (b - Sqrt[b^2 - 4*a*c])^2*d^2)*Sqrt[1 - d^2*x^2]) - (c*(3*b*d^2*(c - a*d^2
) - (4*c^3 - 7*b^2*c*d^2 + 20*a*c^2*d^2 - 3*a*b^2*d^4 + 16*a^2*c*d^4)/Sqrt[
b^2 - 4*a*c])*(2*c - (b + Sqrt[b^2 - 4*a*c])*d^2*x))/((4*c^2 - (b + Sqrt[b^
```

```
2 - 4*a*c])^2*d^2)*Sqrt[1 - d^2*x^2]) + (Sqrt[2]*c^3*(4*c^3 + 20*a*c^2*d^2
- 3*a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(7*b^2 - 3*b*Sqrt[b^2 - 4*a*c]
- 16*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[
2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sq
rt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)^(3/2))
- (Sqrt[2]*c^3*(4*c^3 + 20*a*c^2*d^2 - 3*a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 -
c*d^2*(7*b^2 + 3*b*Sqrt[b^2 - 4*a*c] - 16*a^2*d^2))*ArcTanh[(2*c + (b + Sqr
t[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b
*(b + Sqrt[b^2 - 4*a*c])*d^2)^(3/2)))/((b^2 - 4*a*c)*(-(b^2*d^2) + (c + a*d
^2)^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [C] time = 4.14, size = 108974, normalized size = 116.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)^2 (dx+1)^{\frac{3}{2}} (-dx+1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}\,{\left(c\,x^2+b\,x+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.802 $\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=54

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

```
[Out] x*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2,-c*x^2/a)/((c*x^2/a+1)^p)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {517, 430, 429}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c
*x^2)/a)^p
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rubi steps

$$\begin{aligned}
\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx &= \int \left(a+cx^2\right)^p\left(1-e^2x^2\right)^m\,dx\\
&= \left(\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p\left(1-e^2x^2\right)^m\,dx\\
&= x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)
\end{aligned}$$

Mathematica [B] time = 0.20, size = 167, normalized size = 3.09

$$\frac{3ax\left(1-e^2x^2\right)^m\left(a+cx^2\right)^pF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)}{2x^2\left(cpF_1\left(\frac{3}{2};1-p,-m;\frac{5}{2};-\frac{cx^2}{a},e^2x^2\right)-ae^2mF_1\left(\frac{3}{2};-p,1-m;\frac{5}{2};-\frac{cx^2}{a},e^2x^2\right)\right)+3aF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a
), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x^2
*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*Appell
F1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2+a\right)^p(ex+1)^m(-ex+1)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+a\right)^p(ex+1)^m(-ex+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(-ex+1\right)^m\left(ex+1\right)^m\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

```
[Out] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+a\right)^p(ex+1)^m(-ex+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2+a\right)}^p\,{\left(1-e\,x\right)}^m\,{\left(e\,x+1\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m,x)
```

```
[Out] int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.803 $\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=89

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d-ex)^m(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*(-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-c*x^
2/a)/((c*x^2/a+1)^p)/((1-e^2*x^2/d^2)^m)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {519, 430, 429}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d-ex)^m(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^
2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx &= \left((d-ex)^m(d+ex)^m\left(d^2-e^2x^2\right)^{-m}\right)\int \left(a+cx^2\right)^p\left(d^2-e^2x^2\right)^m\,dx\\ &= \left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2-e^2x^2\right)^{-m}\right)\int \left(1+\frac{cx^2}{a}\right)^p (d\\ &= \left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int \left(1+\frac{cx^2}{a}\right)^p (1\\ &= x(d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};\right.\end{aligned}$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
[Out] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p, x]
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+a\right)^p(-ex+d)^m(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
[Out] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^p\,(d+e\,x)^m\,(d-e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m,x)
[Out] int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)
[Out] Timed out
```

3.804 $\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=92

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}(df-efx)^mF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*(e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-
c*x^2/a)/((c*x^2/a+1)^p)/((1-e^2*x^2/d^2)^m)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {519, 430, 429}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}(df-efx)^mF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((
c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx = \left((d+ex)^m(df-efx)^m\left(d^2f-e^2fx^2\right)^{-m}\right)\int\left(a+cx^2\right)^p\left(d^2f-e^2fx^2\right)$$

$$=\left((d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2f-e^2fx^2\right)^{-m}\right)\int\left(1\right.$$

$$=\left((d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int\left(1+\right.$$

$$=x(d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-\right.$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p, x]
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-efx+df\right)^m\left(cx^2+a\right)^p(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-efx+df\right)^m\left(cx^2+a\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+a\right)^p(ex+d)^m\left(-efx+df\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
[Out] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-efx+df\right)^m\left(cx^2+a\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,f-e\,f\,x\right)^m\,\left(c\,x^2+a\right)^p\,\left(d+e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m,x)
[Out] int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.805 $\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)dx$

Optimal. Leaf size=275

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg\right.\right.}{g^6(n+3)}$$

```
[Out] -(-d*g+e*f)^3*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^6/(1+n)+(-d*g+e*f)^2
*(3*a*e*g^2+c*(2*d^2*g^2-10*d*e*f*g+5*e^2*f^2))*(g*x+f)^(2+n)/g^6/(2+n)-e*(
-d*g+e*f)*(3*a*e*g^2+c*(7*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3+n)/g^6
/(3+n)+e^2*(a*e*g^2+c*(9*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(4+n)/g^6/
(4+n)-5*c*e^3*(-d*g+e*f)*(g*x+f)^(5+n)/g^6/(5+n)+c*e^4*(g*x+f)^(6+n)/g^6/(6
+n)
```

Rubi [A] time = 0.26, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {947}

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg\right.\right.}{g^6(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 + n
))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(
f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2
 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*e*g^
2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(4 + n
)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(f + g*
x)^(6 + n))/(g^6*(6 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)dx=\int\left(\frac{(ef-dg)^3\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^5}+\frac{(ef-dg)^2\left(3aeg^2\right.}{}\right.$$

$$=-\frac{(ef-dg)^3\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^6(1+n)}+\frac{(ef-dg)^2\left(3aeg^2+\right.}{}$$

Mathematica [A] time = 0.31, size = 249, normalized size = 0.91

$$\frac{(f+gx)^{n+1}\left(\frac{e^2(f+gx)^3\left(aeg^2+c\left(9d^2g^2-20defg+10e^2f^2\right)\right)}{n+4}-\frac{e(f+gx)^2(ef-dg)\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2f^2\right)\right)}{n+3}+\frac{(f+gx)(ef-dg)^2\left(3aeg^2+c\left(2\right.\right.}{n+2}\right.}{g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(-(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n))
+ ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(f +
g*x))/(2 + n) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g + 7*
d^2*g^2))*(f + g*x)^2)/(3 + n) + (e^2*(a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g
 + 9*d^2*g^2))*(f + g*x)^3)/(4 + n) - (5*c*e^3*(e*f - d*g)*(f + g*x)^4)/(5
+ n) + (c*e^4*(f + g*x)^5)/(6 + n)))/g^6
```

fricas [B] time = 0.85, size = 2032, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^3*f*g^5*n^5 - 120*c*e^4*f^6 + 720*c*d*e^3*f^5*g + 720*a*d^3*f*g^5 - 18
0*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 240*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 360
*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4 + (c*e^4*g^6*n^5 + 15*c*e^4*g^6*n^4 + 85*c*e
^4*g^6*n^3 + 225*c*e^4*g^6*n^2 + 274*c*e^4*g^6*n + 120*c*e^4*g^6)*x^6 + (72
0*c*d*e^3*g^6 + (c*e^4*f*g^5 + 5*c*d*e^3*g^6)*n^5 + 10*(c*e^4*f*g^5 + 8*c*d
*e^3*g^6)*n^4 + 5*(7*c*e^4*f*g^5 + 95*c*d*e^3*g^6)*n^3 + 50*(c*e^4*f*g^5 +
26*c*d*e^3*g^6)*n^2 + 12*(2*c*e^4*f*g^5 + 135*c*d*e^3*g^6)*n)*x^5 + (20*a*d
^3*f*g^5 - (2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n^4 + (180*(9*c*d^2*e^2 + a*e^3)*
g^6 + (5*c*d*e^3*f*g^5 + (9*c*d^2*e^2 + a*e^3)*g^6)*n^5 - (5*c*e^4*f^2*g^4
- 60*c*d*e^3*f*g^5 - 17*(9*c*d^2*e^2 + a*e^3)*g^6)*n^4 - (30*c*e^4*f^2*g^4
- 235*c*d*e^3*f*g^5 - 107*(9*c*d^2*e^2 + a*e^3)*g^6)*n^3 - (55*c*e^4*f^2*g^
4 - 360*c*d*e^3*f*g^5 - 307*(9*c*d^2*e^2 + a*e^3)*g^6)*n^2 - 6*(5*c*e^4*f^2
*g^4 - 30*c*d*e^3*f*g^5 - 66*(9*c*d^2*e^2 + a*e^3)*g^6)*n)*x^4 + (155*a*d^3
*f*g^5 + 2*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 18*(2*c*d^4 + 3*a*d^2*e)*f^2*g
^4)*n^3 + (240*(7*c*d^3*e + 3*a*d*e^2)*g^6 + ((9*c*d^2*e^2 + a*e^3)*f*g^5 +
 (7*c*d^3*e + 3*a*d*e^2)*g^6)*n^5 - 2*(10*c*d*e^3*f^2*g^4 - 7*(9*c*d^2*e^2
+ a*e^3)*f*g^5 - 9*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^4 + (20*c*e^4*f^3*g^3 - 1
80*c*d*e^3*f^2*g^4 + 65*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 121*(7*c*d^3*e + 3*a*
d*e^2)*g^6)*n^3 + 4*(15*c*e^4*f^3*g^3 - 100*c*d*e^3*f^2*g^4 + 28*(9*c*d^2*e
^2 + a*e^3)*f*g^5 + 93*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^2 + 4*(10*c*e^4*f^3*g
^3 - 60*c*d*e^3*f^2*g^4 + 15*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 127*(7*c*d^3*e +
 3*a*d*e^2)*g^6)*n)*x^3 + (580*a*d^3*f*g^5 - 6*(9*c*d^2*e^2 + a*e^3)*f^4*g^
2 + 30*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 119*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)
*n^2 + (360*(2*c*d^4 + 3*a*d^2*e)*g^6 + ((7*c*d^3*e + 3*a*d*e^2)*f*g^5 + (2
*c*d^4 + 3*a*d^2*e)*g^6)*n^5 - (3*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 16*(7*c*d
^3*e + 3*a*d*e^2)*f*g^5 - 19*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^4 + (60*c*d*e^3*f
^3*g^3 - 36*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 + 89*(7*c*d^3*e + 3*a*d*e^2)*f*g^
5 + 137*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^3 - (60*c*e^4*f^4*g^2 - 420*c*d*e^3*f^
3*g^3 + 123*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 194*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 461*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^2 - 6*(10*c*e^4*f^4*g^2 - 60*c*d*e^3*
f^3*g^3 + 15*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 20*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 117*(2*c*d^4 + 3*a*d^2*e)*g^6)*n)*x^2 + 2*(60*c*d*e^3*f^5*g + 522*a*d^
3*f*g^5 - 33*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^3
*g^3 - 171*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n + (720*a*d^3*g^6 + (a*d^3*g^6 +
 (2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^5 + 2*(10*a*d^3*g^6 - (7*c*d^3*e + 3*a*d*e^
2)*f^2*g^4 + 9*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^4 + (155*a*d^3*g^6 + 6*(9*c*d
^2*e^2 + a*e^3)*f^3*g^3 - 30*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 + 119*(2*c*d^4
 + 3*a*d^2*e)*f*g^5)*n^3 - 2*(60*c*d*e^3*f^4*g^2 - 290*a*d^3*g^6 - 33*(9*c*
d^2*e^2 + a*e^3)*f^3*g^3 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 - 171*(2*c*d^
4 + 3*a*d^2*e)*f*g^5)*n^2 + 12*(10*c*e^4*f^5*g - 60*c*d*e^3*f^4*g^2 + 87*a*
d^3*g^6 + 15*(9*c*d^2*e^2 + a*e^3)*f^3*g^3 - 20*(7*c*d^3*e + 3*a*d*e^2)*f^2
*g^4 + 30*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n)*x)*(g*x + f)^n/(g^6*n^6 + 21*g^6*
n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^6)
```

giac [B] time = 0.59, size = 3760, normalized size = 13.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^6*n^5*x^6*e^4 + 5*(g*x + f)^n*c*d*g^6*n^5*x^5*e^3 + 9*(g*x
 + f)^n*c*d^2*g^6*n^5*x^4*e^2 + 7*(g*x + f)^n*c*d^3*g^6*n^5*x^3*e + 2*(g*x
+ f)^n*c*d^4*g^6*n^5*x^2 + (g*x + f)^n*c*f*g^5*n^5*x^5*e^4 + 15*(g*x + f)^n
*c*g^6*n^4*x^6*e^4 + 5*(g*x + f)^n*c*d*f*g^5*n^5*x^4*e^3 + 80*(g*x + f)^n*c
*d*g^6*n^4*x^5*e^3 + 9*(g*x + f)^n*c*d^2*f*g^5*n^5*x^3*e^2 + 153*(g*x + f)^
n*c*d^2*g^6*n^4*x^4*e^2 + 7*(g*x + f)^n*c*d^3*f*g^5*n^5*x^2*e + 126*(g*x +
f)^n*c*d^3*g^6*n^4*x^3*e + 2*(g*x + f)^n*c*d^4*f*g^5*n^5*x + 38*(g*x + f)^n
*c*d^4*g^6*n^4*x^2 + 10*(g*x + f)^n*c*f*g^5*n^4*x^5*e^4 + 85*(g*x + f)^n*c*
g^6*n^3*x^6*e^4 + 60*(g*x + f)^n*c*d*f*g^5*n^4*x^4*e^3 + (g*x + f)^n*a*g^6*
n^5*x^4*e^3 + 475*(g*x + f)^n*c*d*g^6*n^3*x^5*e^3 + 126*(g*x + f)^n*c*d^2*f
*g^5*n^4*x^3*e^2 + 3*(g*x + f)^n*a*d*g^6*n^5*x^3*e^2 + 963*(g*x + f)^n*c*d^
2*g^6*n^3*x^4*e^2 + 112*(g*x + f)^n*c*d^3*f*g^5*n^4*x^2*e + 3*(g*x + f)^n*a
*d^2*g^6*n^5*x^2*e + 847*(g*x + f)^n*c*d^3*g^6*n^3*x^3*e + 36*(g*x + f)^n*c
*d^4*f*g^5*n^4*x + (g*x + f)^n*a*d^3*g^6*n^5*x + 274*(g*x + f)^n*c*d^4*g^6*
n^3*x^2 - 5*(g*x + f)^n*c*f^2*g^4*n^4*x^4*e^4 + 35*(g*x + f)^n*c*f*g^5*n^3*
x^5*e^4 + 225*(g*x + f)^n*c*g^6*n^2*x^6*e^4 - 20*(g*x + f)^n*c*d*f^2*g^4*n^
4*x^3*e^3 + (g*x + f)^n*a*f*g^5*n^5*x^3*e^3 + 235*(g*x + f)^n*c*d*f*g^5*n^3
*x^4*e^3 + 17*(g*x + f)^n*a*g^6*n^4*x^4*e^3 + 1300*(g*x + f)^n*c*d*g^6*n^2*
x^5*e^3 - 27*(g*x + f)^n*c*d^2*f^2*g^4*n^4*x^2*e^2 + 3*(g*x + f)^n*a*d*f*g^
5*n^5*x^2*e^2 + 585*(g*x + f)^n*c*d^2*f*g^5*n^3*x^3*e^2 + 54*(g*x + f)^n*a*
d*g^6*n^4*x^3*e^2 + 2763*(g*x + f)^n*c*d^2*g^6*n^2*x^4*e^2 - 14*(g*x + f)^n
*c*d^3*f^2*g^4*n^4*x*e + 3*(g*x + f)^n*a*d^2*f*g^5*n^5*x*e + 623*(g*x + f)^
n*c*d^3*f*g^5*n^3*x^2*e + 57*(g*x + f)^n*a*d^2*g^6*n^4*x^2*e + 2604*(g*x +
f)^n*c*d^3*g^6*n^2*x^3*e - 2*(g*x + f)^n*c*d^4*f^2*g^4*n^4 + (g*x + f)^n*a*
d^3*f*g^5*n^5 + 238*(g*x + f)^n*c*d^4*f*g^5*n^3*x + 20*(g*x + f)^n*a*d^3*g^
6*n^4*x + 922*(g*x + f)^n*c*d^4*g^6*n^2*x^2 - 30*(g*x + f)^n*c*f^2*g^4*n^3*
x^4*e^4 + 50*(g*x + f)^n*c*f*g^5*n^2*x^5*e^4 + 274*(g*x + f)^n*c*g^6*n*x^6*
e^4 - 180*(g*x + f)^n*c*d*f^2*g^4*n^3*x^3*e^3 + 14*(g*x + f)^n*a*f*g^5*n^4*
x^3*e^3 + 360*(g*x + f)^n*c*d*f*g^5*n^2*x^4*e^3 + 107*(g*x + f)^n*a*g^6*n^3
*x^4*e^3 + 1620*(g*x + f)^n*c*d*g^6*n*x^5*e^3 - 324*(g*x + f)^n*c*d^2*f^2*g
^4*n^3*x^2*e^2 + 48*(g*x + f)^n*a*d*f*g^5*n^4*x^2*e^2 + 1008*(g*x + f)^n*c*
d^2*f*g^5*n^2*x^3*e^2 + 363*(g*x + f)^n*a*d*g^6*n^3*x^3*e^2 + 3564*(g*x + f
)^n*c*d^2*g^6*n*x^4*e^2 - 210*(g*x + f)^n*c*d^3*f^2*g^4*n^3*x*e + 54*(g*x +
 f)^n*a*d^2*f*g^5*n^4*x*e + 1358*(g*x + f)^n*c*d^3*f*g^5*n^2*x^2*e + 411*(g
*x + f)^n*a*d^2*g^6*n^3*x^2*e + 3556*(g*x + f)^n*c*d^3*g^6*n*x^3*e - 36*(g*
x + f)^n*c*d^4*f^2*g^4*n^3 + 20*(g*x + f)^n*a*d^3*f*g^5*n^4 + 684*(g*x + f)
^n*c*d^4*f*g^5*n^2*x + 155*(g*x + f)^n*a*d^3*g^6*n^3*x + 1404*(g*x + f)^n*c
*d^4*g^6*n*x^2 + 20*(g*x + f)^n*c*f^3*g^3*n^3*x^3*e^4 - 55*(g*x + f)^n*c*f^
2*g^4*n^2*x^4*e^4 + 24*(g*x + f)^n*c*f*g^5*n*x^5*e^4 + 120*(g*x + f)^n*c*g^
6*x^6*e^4 + 60*(g*x + f)^n*c*d*f^3*g^3*n^3*x^2*e^3 - 3*(g*x + f)^n*a*f^2*g^
4*n^4*x^2*e^3 - 400*(g*x + f)^n*c*d*f^2*g^4*n^2*x^3*e^3 + 65*(g*x + f)^n*a*
f*g^5*n^3*x^3*e^3 + 180*(g*x + f)^n*c*d*f*g^5*n*x^4*e^3 + 307*(g*x + f)^n*a
*g^6*n^2*x^4*e^3 + 720*(g*x + f)^n*c*d*g^6*x^5*e^3 + 54*(g*x + f)^n*c*d^2*f
^3*g^3*n^3*x*e^2 - 6*(g*x + f)^n*a*d*f^2*g^4*n^4*x*e^2 - 1107*(g*x + f)^n*c
*d^2*f^2*g^4*n^2*x^2*e^2 + 267*(g*x + f)^n*a*d*f*g^5*n^3*x^2*e^2 + 540*(g*x
 + f)^n*c*d^2*f*g^5*n*x^3*e^2 + 1116*(g*x + f)^n*a*d*g^6*n^2*x^3*e^2 + 1620
*(g*x + f)^n*c*d^2*g^6*x^4*e^2 + 14*(g*x + f)^n*c*d^3*f^3*g^3*n^3*e - 3*(g*
x + f)^n*a*d^2*f^2*g^4*n^4*e - 1036*(g*x + f)^n*c*d^3*f^2*g^4*n^2*x*e + 357
*(g*x + f)^n*a*d^2*f*g^5*n^3*x*e + 840*(g*x + f)^n*c*d^3*f*g^5*n*x^2*e + 13
83*(g*x + f)^n*a*d^2*g^6*n^2*x^2*e + 1680*(g*x + f)^n*c*d^3*g^6*x^3*e - 238
*(g*x + f)^n*c*d^4*f^2*g^4*n^2 + 155*(g*x + f)^n*a*d^3*f*g^5*n^3 + 720*(g*x
 + f)^n*c*d^4*f*g^5*n*x + 580*(g*x + f)^n*a*d^3*g^6*n^2*x + 720*(g*x + f)^n
```

```
*c*d^4*g^6*x^2 + 60*(g*x + f)^n*c*f^3*g^3*n^2*x^3*e^4 - 30*(g*x + f)^n*c*f^
2*g^4*n*x^4*e^4 + 420*(g*x + f)^n*c*d*f^3*g^3*n^2*x^2*e^3 - 36*(g*x + f)^n*
a*f^2*g^4*n^3*x^2*e^3 - 240*(g*x + f)^n*c*d*f^2*g^4*n*x^3*e^3 + 112*(g*x +
f)^n*a*f*g^5*n^2*x^3*e^3 + 396*(g*x + f)^n*a*g^6*n*x^4*e^3 + 594*(g*x + f)^
n*c*d^2*f^3*g^3*n^2*x*e^2 - 90*(g*x + f)^n*a*d*f^2*g^4*n^3*x*e^2 - 810*(g*x
 + f)^n*c*d^2*f^2*g^4*n*x^2*e^2 + 582*(g*x + f)^n*a*d*f*g^5*n^2*x^2*e^2 + 1
524*(g*x + f)^n*a*d*g^6*n*x^3*e^2 + 210*(g*x + f)^n*c*d^3*f^3*g^3*n^2*e - 5
4*(g*x + f)^n*a*d^2*f^2*g^4*n^3*e - 1680*(g*x + f)^n*c*d^3*f^2*g^4*n*x*e +
1026*(g*x + f)^n*a*d^2*f*g^5*n^2*x*e + 2106*(g*x + f)^n*a*d^2*g^6*n*x^2*e -
 684*(g*x + f)^n*c*d^4*f^2*g^4*n + 580*(g*x + f)^n*a*d^3*f*g^5*n^2 + 1044*(
g*x + f)^n*a*d^3*g^6*n*x - 60*(g*x + f)^n*c*f^4*g^2*n^2*x^2*e^4 + 40*(g*x +
 f)^n*c*f^3*g^3*n*x^3*e^4 - 120*(g*x + f)^n*c*d*f^4*g^2*n^2*x*e^3 + 6*(g*x
+ f)^n*a*f^3*g^3*n^3*x*e^3 + 360*(g*x + f)^n*c*d*f^3*g^3*n*x^2*e^3 - 123*(g
*x + f)^n*a*f^2*g^4*n^2*x^2*e^3 + 60*(g*x + f)^n*a*f*g^5*n*x^3*e^3 + 180*(g
*x + f)^n*a*g^6*x^4*e^3 - 54*(g*x + f)^n*c*d^2*f^4*g^2*n^2*e^2 + 6*(g*x + f
)^n*a*d*f^3*g^3*n^3*e^2 + 1620*(g*x + f)^n*c*d^2*f^3*g^3*n*x*e^2 - 444*(g*x
 + f)^n*a*d*f^2*g^4*n^2*x*e^2 + 360*(g*x + f)^n*a*d*f*g^5*n*x^2*e^2 + 720*(
g*x + f)^n*a*d*g^6*x^3*e^2 + 1036*(g*x + f)^n*c*d^3*f^3*g^3*n*e - 357*(g*x
+ f)^n*a*d^2*f^2*g^4*n^2*e + 1080*(g*x + f)^n*a*d^2*f*g^5*n*x*e + 1080*(g*x
 + f)^n*a*d^2*g^6*x^2*e - 720*(g*x + f)^n*c*d^4*f^2*g^4 + 1044*(g*x + f)^n*
a*d^3*f*g^5*n + 720*(g*x + f)^n*a*d^3*g^6*x - 60*(g*x + f)^n*c*f^4*g^2*n*x^
2*e^4 - 720*(g*x + f)^n*c*d*f^4*g^2*n*x*e^3 + 66*(g*x + f)^n*a*f^3*g^3*n^2*
x*e^3 - 90*(g*x + f)^n*a*f^2*g^4*n*x^2*e^3 - 594*(g*x + f)^n*c*d^2*f^4*g^2*
n*e^2 + 90*(g*x + f)^n*a*d*f^3*g^3*n^2*e^2 - 720*(g*x + f)^n*a*d*f^2*g^4*n*
x*e^2 + 1680*(g*x + f)^n*c*d^3*f^3*g^3*e - 1026*(g*x + f)^n*a*d^2*f^2*g^4*n
*e + 720*(g*x + f)^n*a*d^3*f*g^5 + 120*(g*x + f)^n*c*f^5*g*n*x*e^4 + 120*(g
*x + f)^n*c*d*f^5*g*n*e^3 - 6*(g*x + f)^n*a*f^4*g^2*n^2*e^3 + 180*(g*x + f)
^n*a*f^3*g^3*n*x*e^3 - 1620*(g*x + f)^n*c*d^2*f^4*g^2*e^2 + 444*(g*x + f)^n
*a*d*f^3*g^3*n*e^2 - 1080*(g*x + f)^n*a*d^2*f^2*g^4*e + 720*(g*x + f)^n*c*d
*f^5*g*e^3 - 66*(g*x + f)^n*a*f^4*g^2*n*e^3 + 720*(g*x + f)^n*a*d*f^3*g^3*e
^2 - 120*(g*x + f)^n*c*f^6*e^4 - 180*(g*x + f)^n*a*f^4*g^2*e^3)/(g^6*n^6 +
21*g^6*n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^
6)
```

maple [B] time = 0.02, size = 2017, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e^4*g^5*n^5*x^5+5*c*d*e^3*g^5*n^5*x^4+15*c*e^4*g^5*n^4*x^5
+9*c*d^2*e^2*g^5*n^5*x^3+80*c*d*e^3*g^5*n^4*x^4-5*c*e^4*f*g^4*n^4*x^4+85*c*
e^4*g^5*n^3*x^5+a*e^3*g^5*n^5*x^3+7*c*d^3*e*g^5*n^5*x^2+153*c*d^2*e^2*g^5*n
^4*x^3-20*c*d*e^3*f*g^4*n^4*x^3+475*c*d*e^3*g^5*n^3*x^4-50*c*e^4*f*g^4*n^3*
x^4+225*c*e^4*g^5*n^2*x^5+3*a*d*e^2*g^5*n^5*x^2+17*a*e^3*g^5*n^4*x^3+2*c*d^
4*g^5*n^5*x+126*c*d^3*e*g^5*n^4*x^2-27*c*d^2*e^2*f*g^4*n^4*x^2+963*c*d^2*e^
2*g^5*n^3*x^3-240*c*d*e^3*f*g^4*n^3*x^3+1300*c*d*e^3*g^5*n^2*x^4+20*c*e^4*f
^2*g^3*n^3*x^3-175*c*e^4*f*g^4*n^2*x^4+274*c*e^4*g^5*n*x^5+3*a*d^2*e*g^5*n^
5*x+54*a*d*e^2*g^5*n^4*x^2-3*a*e^3*f*g^4*n^4*x^2+107*a*e^3*g^5*n^3*x^3+38*c
*d^4*g^5*n^4*x-14*c*d^3*e*f*g^4*n^4*x+847*c*d^3*e*g^5*n^3*x^2-378*c*d^2*e^2
*f*g^4*n^3*x^2+2763*c*d^2*e^2*g^5*n^2*x^3+60*c*d*e^3*f^2*g^3*n^3*x^2-940*c*
d*e^3*f*g^4*n^2*x^3+1620*c*d*e^3*g^5*n*x^4+120*c*e^4*f^2*g^3*n^2*x^3-250*c*
e^4*f*g^4*n*x^4+120*c*e^4*g^5*x^5+a*d^3*g^5*n^5+57*a*d^2*e*g^5*n^4*x-6*a*d*
e^2*f*g^4*n^4*x+363*a*d*e^2*g^5*n^3*x^2-42*a*e^3*f*g^4*n^3*x^2+307*a*e^3*g^
5*n^2*x^3-2*c*d^4*f*g^4*n^4+274*c*d^4*g^5*n^3*x-224*c*d^3*e*f*g^4*n^3*x+260
4*c*d^3*e*g^5*n^2*x^2+54*c*d^2*e^2*f^2*g^3*n^3*x-1755*c*d^2*e^2*f*g^4*n^2*x
^2+3564*c*d^2*e^2*g^5*n*x^3+540*c*d*e^3*f^2*g^3*n^2*x^2-1440*c*d*e^3*f*g^4*
n*x^3+720*c*d*e^3*g^5*x^4-60*c*e^4*f^3*g^2*n^2*x^2+220*c*e^4*f^2*g^3*n*x^3-
120*c*e^4*f*g^4*x^4+20*a*d^3*g^5*n^4-3*a*d^2*e*f*g^4*n^4+411*a*d^2*e*g^5*n^
```

```
3*x-96*a*d*e^2*f*g^4*n^3*x+1116*a*d*e^2*g^5*n^2*x^2+6*a*e^3*f^2*g^3*n^3*x-1
95*a*e^3*f*g^4*n^2*x^2+396*a*e^3*g^5*n*x^3-36*c*d^4*f*g^4*n^3+922*c*d^4*g^5
*n^2*x+14*c*d^3*e*f^2*g^3*n^3-1246*c*d^3*e*f*g^4*n^2*x+3556*c*d^3*e*g^5*n*x
^2+648*c*d^2*e^2*f^2*g^3*n^2*x-3024*c*d^2*e^2*f*g^4*n*x^2+1620*c*d^2*e^2*g^
5*x^3-120*c*d*e^3*f^3*g^2*n^2*x+1200*c*d*e^3*f^2*g^3*n*x^2-720*c*d*e^3*f*g^
4*x^3-180*c*e^4*f^3*g^2*n*x^2+120*c*e^4*f^2*g^3*x^3+155*a*d^3*g^5*n^3-54*a*
d^2*e*f*g^4*n^3+1383*a*d^2*e*g^5*n^2*x+6*a*d*e^2*f^2*g^3*n^3-534*a*d*e^2*f*
g^4*n^2*x+1524*a*d*e^2*g^5*n*x^2+72*a*e^3*f^2*g^3*n^2*x-336*a*e^3*f*g^4*n*x
^2+180*a*e^3*g^5*x^3-238*c*d^4*f*g^4*n^2+1404*c*d^4*g^5*n*x+210*c*d^3*e*f^2
*g^3*n^2-2716*c*d^3*e*f*g^4*n*x+1680*c*d^3*e*g^5*x^2-54*c*d^2*e^2*f^3*g^2*n
^2+2214*c*d^2*e^2*f^2*g^3*n*x-1620*c*d^2*e^2*f*g^4*x^2-840*c*d*e^3*f^3*g^2*
n*x+720*c*d*e^3*f^2*g^3*x^2+120*c*e^4*f^4*g*n*x-120*c*e^4*f^3*g^2*x^2+580*a
*d^3*g^5*n^2-357*a*d^2*e*f*g^4*n^2+2106*a*d^2*e*g^5*n*x+90*a*d*e^2*f^2*g^3*
n^2-1164*a*d*e^2*f*g^4*n*x+720*a*d*e^2*g^5*x^2-6*a*e^3*f^3*g^2*n^2+246*a*e^
3*f^2*g^3*n*x-180*a*e^3*f*g^4*x^2-684*c*d^4*f*g^4*n+720*c*d^4*g^5*x+1036*c*
d^3*e*f^2*g^3*n-1680*c*d^3*e*f*g^4*x-594*c*d^2*e^2*f^3*g^2*n+1620*c*d^2*e^2
*f^2*g^3*x+120*c*d*e^3*f^4*g*n-720*c*d*e^3*f^3*g^2*x+120*c*e^4*f^4*g*x+1044
*a*d^3*g^5*n-1026*a*d^2*e*f*g^4*n+1080*a*d^2*e*g^5*x+444*a*d*e^2*f^2*g^3*n-
720*a*d*e^2*f*g^4*x-66*a*e^3*f^3*g^2*n+180*a*e^3*f^2*g^3*x-720*c*d^4*f*g^4+
1680*c*d^3*e*f^2*g^3-1620*c*d^2*e^2*f^3*g^2+720*c*d*e^3*f^4*g-120*c*e^4*f^5
+720*a*d^3*g^5-1080*a*d^2*e*f*g^4+720*a*d*e^2*f^2*g^3-180*a*e^3*f^3*g^2)/g^
6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

maxima [B] time = 0.60, size = 811, normalized size = 2.95

$$\frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)\left(gx+f\right)^n cd^4}{\left(n^2+3n+2\right)g^2}+\frac{7\left(\left(n^2+3n+2\right)g^3x^3+\left(n^2+n\right)fg^2x^2-2f^2gnx+2f^3\right)\left(gx+f\right)^n cd}{\left(n^3+6n^2+11n+6\right)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^4/((n^2 + 3*n + 2)*g^2)
 + 7*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^3*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 3*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d^2*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)
*a*d^3/(g*(n + 1)) + 9*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2
*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*
c*d^2*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + 3*((n^2 + 3*n + 2)*g^
3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*d*e^2/((n^
3 + 6*n^2 + 11*n + 6)*g^3) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5
 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x
^3 + 12*(n^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*d*e^3/
((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5) + ((n^3 + 6*n^2 + 11*
n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 +
6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*a*e^3/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)
*g^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^6*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*f*g^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*
f^2*g^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*f^3*g^3*x^3 - 60*(n^2 + n)*f^4*g^2*x^2
 + 120*f^5*g*n*x - 120*f^6)*(g*x + f)^n*c*e^4/((n^6 + 21*n^5 + 175*n^4 + 73
5*n^3 + 1624*n^2 + 1764*n + 720)*g^6)
```

mupad [B] time = 3.90, size = 1943, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^3*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(720*a*d^3*g^6 + 580*a*d^3*g^6*n^2 + 155*a*d^3*g^6*n^3 + 20*
a*d^3*g^6*n^4 + a*d^3*g^6*n^5 + 1044*a*d^3*g^6*n + 720*c*d^4*f*g^5*n + 120*
c*e^4*f^5*g*n + 180*a*e^3*f^3*g^3*n + 684*c*d^4*f*g^5*n^2 + 238*c*d^4*f*g^5
*n^3 + 36*c*d^4*f*g^5*n^4 + 2*c*d^4*f*g^5*n^5 + 66*a*e^3*f^3*g^3*n^2 + 6*a*
e^3*f^3*g^3*n^3 - 444*a*d*e^2*f^2*g^4*n^2 - 90*a*d*e^2*f^2*g^4*n^3 - 6*a*d*
e^2*f^2*g^4*n^4 + 1620*c*d^2*e^2*f^3*g^3*n - 120*c*d*e^3*f^4*g^2*n^2 - 1036
*c*d^3*e*f^2*g^4*n^2 - 210*c*d^3*e*f^2*g^4*n^3 - 14*c*d^3*e*f^2*g^4*n^4 + 1
080*a*d^2*e*f*g^5*n + 594*c*d^2*e^2*f^3*g^3*n^2 + 54*c*d^2*e^2*f^3*g^3*n^3
- 720*a*d*e^2*f^2*g^4*n + 1026*a*d^2*e*f*g^5*n^2 + 357*a*d^2*e*f*g^5*n^3 +
54*a*d^2*e*f*g^5*n^4 + 3*a*d^2*e*f*g^5*n^5 - 720*c*d*e^3*f^4*g^2*n - 1680*c
*d^3*e*f^2*g^4*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n
^6 + 720)) - ((f + g*x)^n*(120*c*e^4*f^6 + 180*a*e^3*f^4*g^2 + 720*c*d^4*f^
2*g^4 - 720*a*d^3*f*g^5 - 720*c*d*e^3*f^5*g - 1044*a*d^3*f*g^5*n - 720*a*d*
e^2*f^3*g^3 + 1080*a*d^2*e*f^2*g^4 - 1680*c*d^3*e*f^3*g^3 - 580*a*d^3*f*g^5
*n^2 - 155*a*d^3*f*g^5*n^3 - 20*a*d^3*f*g^5*n^4 - a*d^3*f*g^5*n^5 + 66*a*e^
3*f^4*g^2*n + 684*c*d^4*f^2*g^4*n + 1620*c*d^2*e^2*f^4*g^2 + 6*a*e^3*f^4*g^
2*n^2 + 238*c*d^4*f^2*g^4*n^2 + 36*c*d^4*f^2*g^4*n^3 + 2*c*d^4*f^2*g^4*n^4
- 90*a*d*e^2*f^3*g^3*n^2 + 357*a*d^2*e*f^2*g^4*n^2 - 6*a*d*e^2*f^3*g^3*n^3
+ 54*a*d^2*e*f^2*g^4*n^3 + 3*a*d^2*e*f^2*g^4*n^4 + 594*c*d^2*e^2*f^4*g^2*n
- 210*c*d^3*e*f^3*g^3*n^2 - 14*c*d^3*e*f^3*g^3*n^3 - 120*c*d*e^3*f^5*g*n +
54*c*d^2*e^2*f^4*g^2*n^2 - 444*a*d*e^2*f^3*g^3*n + 1026*a*d^2*e*f^2*g^4*n -
 1036*c*d^3*e*f^3*g^3*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) + (c*e^4*x^6*(f + g*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n
^4 + n^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 72
0) + (x^2*(f + g*x)^n*(n + 1)*(720*c*d^4*g^4 + 238*c*d^4*g^4*n^2 + 36*c*d^4
*g^4*n^3 + 2*c*d^4*g^4*n^4 + 1080*a*d^2*e*g^4 + 684*c*d^4*g^4*n - 60*c*e^4*
f^4*n + 1026*a*d^2*e*g^4*n + 357*a*d^2*e*g^4*n^2 + 54*a*d^2*e*g^4*n^3 + 3*a
*d^2*e*g^4*n^4 - 90*a*e^3*f^2*g^2*n - 33*a*e^3*f^2*g^2*n^2 - 3*a*e^3*f^2*g^
2*n^3 - 810*c*d^2*e^2*f^2*g^2*n + 360*a*d*e^2*f*g^3*n + 360*c*d*e^3*f^3*g*n
 + 840*c*d^3*e*f*g^3*n - 297*c*d^2*e^2*f^2*g^2*n^2 - 27*c*d^2*e^2*f^2*g^2*n
^3 + 222*a*d*e^2*f*g^3*n^2 + 45*a*d*e^2*f*g^3*n^3 + 3*a*d*e^2*f*g^3*n^4 + 6
0*c*d*e^3*f^3*g*n^2 + 518*c*d^3*e*f*g^3*n^2 + 105*c*d^3*e*f*g^3*n^3 + 7*c*d
^3*e*f*g^3*n^4))/(g^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6
 + 720)) + (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(840*c*d^3*g^3 + 105*c*d^3*g^
3*n^2 + 7*c*d^3*g^3*n^3 + 360*a*d*e*g^3 + 518*c*d^3*g^3*n + 20*c*e^3*f^3*n
+ 45*a*d*e*g^3*n^2 + 3*a*d*e*g^3*n^3 + 30*a*e^2*f*g^2*n + 11*a*e^2*f*g^2*n^
2 + a*e^2*f*g^2*n^3 + 222*a*d*e*g^3*n - 120*c*d*e^2*f^2*g*n + 270*c*d^2*e*f
*g^2*n - 20*c*d*e^2*f^2*g*n^2 + 99*c*d^2*e*f*g^2*n^2 + 9*c*d^2*e*f*g^2*n^3)
)/(g^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (e^2
*x^4*(f + g*x)^n*(11*n + 6*n^2 + n^3 + 6)*(270*c*d^2*g^2 + 30*a*e*g^2 + 9*c
*d^2*g^2*n^2 + 11*a*e*g^2*n + a*e*g^2*n^2 + 99*c*d^2*g^2*n - 5*c*e^2*f^2*n
+ 5*c*d*e*f*g*n^2 + 30*c*d*e*f*g*n))/(g^2*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (c*e^3*x^5*(f + g*x)^n*(30*d*g + 5*d*g*n + e
*f*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(g*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Timed out
```

3.806 $\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=208

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+$$

```
[Out] (-d*g+e*f)^2*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^5/(1+n)-2*(-d*g+e*f)*
(a*e*g^2+c*(d^2*g^2-4*d*e*f*g+2*e^2*f^2))*(g*x+f)^(2+n)/g^5/(2+n)+e*(a*e*g^
2+c*(5*d^2*g^2-12*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3+n)/g^5/(3+n)-4*c*e^2*(-d*g
+e*f)*(g*x+f)^(4+n)/g^5/(4+n)+c*e^3*(g*x+f)^(5+n)/g^5/(5+n)
```

Rubi [A] time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{947\}$

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n))
 - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)
^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g
^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 +
 n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right) dx = \int \left(\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^4}+\frac{2(ef-dg)\left(-aeg^2\right.}{}\right.$$

$$= \frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^5(1+n)}-\frac{2(ef-dg)\left(aeg^2+c\right.}{}$$

Mathematica [A] time = 0.20, size = 187, normalized size = 0.90

$$\frac{(f+gx)^{n+1}\left(\frac{e(f+gx)^2\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{n+3}-\frac{2(f+gx)(ef-dg)\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{n+2}+\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)}{n+1}-\frac{4ce^2}{}\right.}{g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n) - (
2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x))/(2
 + n) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^2)/
(3 + n) - (4*c*e^2*(e*f - d*g)*(f + g*x)^3)/(4 + n) + (c*e^3*(f + g*x)^4)/(
5 + n)))/g^5
```

fricas [B] time = 1.32, size = 1122, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^2*f*g^4*n^4 + 24*c*e^3*f^5 - 120*c*d*e^2*f^4*g + 120*a*d^2*f*g^4 + 40*
(5*c*d^2*e + a*e^2)*f^3*g^2 - 120*(c*d^3 + a*d*e)*f^2*g^3 + (c*e^3*g^5*n^4
+ 10*c*e^3*g^5*n^3 + 35*c*e^3*g^5*n^2 + 50*c*e^3*g^5*n + 24*c*e^3*g^5)*x^5
+ (120*c*d*e^2*g^5 + (c*e^3*f*g^4 + 4*c*d*e^2*g^5)*n^4 + 2*(3*c*e^3*f*g^4 +
 22*c*d*e^2*g^5)*n^3 + (11*c*e^3*f*g^4 + 164*c*d*e^2*g^5)*n^2 + 2*(3*c*e^3*
f*g^4 + 122*c*d*e^2*g^5)*n)*x^4 + 2*(7*a*d^2*f*g^4 - (c*d^3 + a*d*e)*f^2*g^
3)*n^3 + (40*(5*c*d^2*e + a*e^2)*g^5 + (4*c*d*e^2*f*g^4 + (5*c*d^2*e + a*e^
2)*g^5)*n^4 - 4*(c*e^3*f^2*g^3 - 8*c*d*e^2*f*g^4 - 3*(5*c*d^2*e + a*e^2)*g^
5)*n^3 - (12*c*e^3*f^2*g^3 - 68*c*d*e^2*f*g^4 - 49*(5*c*d^2*e + a*e^2)*g^5)
*n^2 - 2*(4*c*e^3*f^2*g^3 - 20*c*d*e^2*f*g^4 - 39*(5*c*d^2*e + a*e^2)*g^5)*
n)*x^3 + (71*a*d^2*f*g^4 + 2*(5*c*d^2*e + a*e^2)*f^3*g^2 - 24*(c*d^3 + a*d*
e)*f^2*g^3)*n^2 + (120*(c*d^3 + a*d*e)*g^5 + ((5*c*d^2*e + a*e^2)*f*g^4 + 2
*(c*d^3 + a*d*e)*g^5)*n^4 - 2*(6*c*d*e^2*f^2*g^3 - 5*(5*c*d^2*e + a*e^2)*f*
g^4 - 13*(c*d^3 + a*d*e)*g^5)*n^3 + (12*c*e^3*f^3*g^2 - 72*c*d*e^2*f^2*g^3
+ 29*(5*c*d^2*e + a*e^2)*f*g^4 + 118*(c*d^3 + a*d*e)*g^5)*n^2 + 2*(6*c*e^3*
f^3*g^2 - 30*c*d*e^2*f^2*g^3 + 10*(5*c*d^2*e + a*e^2)*f*g^4 + 107*(c*d^3 +
a*d*e)*g^5)*n)*x^2 - 2*(12*c*d*e^2*f^4*g - 77*a*d^2*f*g^4 - 9*(5*c*d^2*e +
a*e^2)*f^3*g^2 + 47*(c*d^3 + a*d*e)*f^2*g^3)*n + (120*a*d^2*g^5 + (a*d^2*g^
5 + 2*(c*d^3 + a*d*e)*f*g^4)*n^4 + 2*(7*a*d^2*g^5 - (5*c*d^2*e + a*e^2)*f^2
*g^3 + 12*(c*d^3 + a*d*e)*f*g^4)*n^3 + (24*c*d*e^2*f^3*g^2 + 71*a*d^2*g^5 -
 18*(5*c*d^2*e + a*e^2)*f^2*g^3 + 94*(c*d^3 + a*d*e)*f*g^4)*n^2 - 2*(12*c*e
^3*f^4*g - 60*c*d*e^2*f^3*g^2 - 77*a*d^2*g^5 + 20*(5*c*d^2*e + a*e^2)*f^2*g
^3 - 60*(c*d^3 + a*d*e)*f*g^4)*n)*x)*(g*x + f)^n/(g^5*n^5 + 15*g^5*n^4 + 85
*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

giac [B] time = 0.23, size = 2114, normalized size = 10.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^5*n^4*x^5*e^3 + 4*(g*x + f)^n*c*d*g^5*n^4*x^4*e^2 + 5*(g*x
 + f)^n*c*d^2*g^5*n^4*x^3*e + 2*(g*x + f)^n*c*d^3*g^5*n^4*x^2 + (g*x + f)^n
*c*f*g^4*n^4*x^4*e^3 + 10*(g*x + f)^n*c*g^5*n^3*x^5*e^3 + 4*(g*x + f)^n*c*d
*f*g^4*n^4*x^3*e^2 + 44*(g*x + f)^n*c*d*g^5*n^3*x^4*e^2 + 5*(g*x + f)^n*c*d
^2*f*g^4*n^4*x^2*e + 60*(g*x + f)^n*c*d^2*g^5*n^3*x^3*e + 2*(g*x + f)^n*c*d
^3*f*g^4*n^4*x + 26*(g*x + f)^n*c*d^3*g^5*n^3*x^2 + 6*(g*x + f)^n*c*f*g^4*n
^3*x^4*e^3 + 35*(g*x + f)^n*c*g^5*n^2*x^5*e^3 + 32*(g*x + f)^n*c*d*f*g^4*n^
3*x^3*e^2 + (g*x + f)^n*a*g^5*n^4*x^3*e^2 + 164*(g*x + f)^n*c*d*g^5*n^2*x^4
*e^2 + 50*(g*x + f)^n*c*d^2*f*g^4*n^3*x^2*e + 2*(g*x + f)^n*a*d*g^5*n^4*x^2
*e + 245*(g*x + f)^n*c*d^2*g^5*n^2*x^3*e + 24*(g*x + f)^n*c*d^3*f*g^4*n^3*x
 + (g*x + f)^n*a*d^2*g^5*n^4*x + 118*(g*x + f)^n*c*d^3*g^5*n^2*x^2 - 4*(g*x
 + f)^n*c*f^2*g^3*n^3*x^3*e^3 + 11*(g*x + f)^n*c*f*g^4*n^2*x^4*e^3 + 50*(g*
x + f)^n*c*g^5*n*x^5*e^3 - 12*(g*x + f)^n*c*d*f^2*g^3*n^3*x^2*e^2 + (g*x +
f)^n*a*f*g^4*n^4*x^2*e^2 + 68*(g*x + f)^n*c*d*f*g^4*n^2*x^3*e^2 + 12*(g*x +
 f)^n*a*g^5*n^3*x^3*e^2 + 244*(g*x + f)^n*c*d*g^5*n*x^4*e^2 - 10*(g*x + f)^
```

```
n*c*d^2*f^2*g^3*n^3*x*e + 2*(g*x + f)^n*a*d*f*g^4*n^4*x*e + 145*(g*x + f)^n
*c*d^2*f*g^4*n^2*x^2*e + 26*(g*x + f)^n*a*d*g^5*n^3*x^2*e + 390*(g*x + f)^n
*c*d^2*g^5*n*x^3*e - 2*(g*x + f)^n*c*d^3*f^2*g^3*n^3 + (g*x + f)^n*a*d^2*f*
g^4*n^4 + 94*(g*x + f)^n*c*d^3*f*g^4*n^2*x + 14*(g*x + f)^n*a*d^2*g^5*n^3*x
 + 214*(g*x + f)^n*c*d^3*g^5*n*x^2 - 12*(g*x + f)^n*c*f^2*g^3*n^2*x^3*e^3 +
 6*(g*x + f)^n*c*f*g^4*n*x^4*e^3 + 24*(g*x + f)^n*c*g^5*x^5*e^3 - 72*(g*x +
 f)^n*c*d*f^2*g^3*n^2*x^2*e^2 + 10*(g*x + f)^n*a*f*g^4*n^3*x^2*e^2 + 40*(g*
x + f)^n*c*d*f*g^4*n*x^3*e^2 + 49*(g*x + f)^n*a*g^5*n^2*x^3*e^2 + 120*(g*x
+ f)^n*c*d*g^5*x^4*e^2 - 90*(g*x + f)^n*c*d^2*f^2*g^3*n^2*x*e + 24*(g*x + f
)^n*a*d*f*g^4*n^3*x*e + 100*(g*x + f)^n*c*d^2*f*g^4*n*x^2*e + 118*(g*x + f)
^n*a*d*g^5*n^2*x^2*e + 200*(g*x + f)^n*c*d^2*g^5*x^3*e - 24*(g*x + f)^n*c*d
^3*f^2*g^3*n^2 + 14*(g*x + f)^n*a*d^2*f*g^4*n^3 + 120*(g*x + f)^n*c*d^3*f*g
^4*n*x + 71*(g*x + f)^n*a*d^2*g^5*n^2*x + 120*(g*x + f)^n*c*d^3*g^5*x^2 + 1
2*(g*x + f)^n*c*f^3*g^2*n^2*x^2*e^3 - 8*(g*x + f)^n*c*f^2*g^3*n*x^3*e^3 + 2
4*(g*x + f)^n*c*d*f^3*g^2*n^2*x*e^2 - 2*(g*x + f)^n*a*f^2*g^3*n^3*x*e^2 - 6
0*(g*x + f)^n*c*d*f^2*g^3*n*x^2*e^2 + 29*(g*x + f)^n*a*f*g^4*n^2*x^2*e^2 +
78*(g*x + f)^n*a*g^5*n*x^3*e^2 + 10*(g*x + f)^n*c*d^2*f^3*g^2*n^2*e - 2*(g*
x + f)^n*a*d*f^2*g^3*n^3*e - 200*(g*x + f)^n*c*d^2*f^2*g^3*n*x*e + 94*(g*x
+ f)^n*a*d*f*g^4*n^2*x*e + 214*(g*x + f)^n*a*d*g^5*n*x^2*e - 94*(g*x + f)^n
*c*d^3*f^2*g^3*n + 71*(g*x + f)^n*a*d^2*f*g^4*n^2 + 154*(g*x + f)^n*a*d^2*g
^5*n*x + 12*(g*x + f)^n*c*f^3*g^2*n*x^2*e^3 + 120*(g*x + f)^n*c*d*f^3*g^2*n
*x*e^2 - 18*(g*x + f)^n*a*f^2*g^3*n^2*x*e^2 + 20*(g*x + f)^n*a*f*g^4*n*x^2*
e^2 + 40*(g*x + f)^n*a*g^5*x^3*e^2 + 90*(g*x + f)^n*c*d^2*f^3*g^2*n*e - 24*
(g*x + f)^n*a*d*f^2*g^3*n^2*e + 120*(g*x + f)^n*a*d*f*g^4*n*x*e + 120*(g*x
+ f)^n*a*d*g^5*x^2*e - 120*(g*x + f)^n*c*d^3*f^2*g^3 + 154*(g*x + f)^n*a*d^
2*f*g^4*n + 120*(g*x + f)^n*a*d^2*g^5*x - 24*(g*x + f)^n*c*f^4*g*n*x*e^3 -
24*(g*x + f)^n*c*d*f^4*g*n*e^2 + 2*(g*x + f)^n*a*f^3*g^2*n^2*e^2 - 40*(g*x
+ f)^n*a*f^2*g^3*n*x*e^2 + 200*(g*x + f)^n*c*d^2*f^3*g^2*e - 94*(g*x + f)^n
*a*d*f^2*g^3*n*e + 120*(g*x + f)^n*a*d^2*f*g^4 - 120*(g*x + f)^n*c*d*f^4*g*
e^2 + 18*(g*x + f)^n*a*f^3*g^2*n*e^2 - 120*(g*x + f)^n*a*d*f^2*g^3*e + 24*(
g*x + f)^n*c*f^5*e^3 + 40*(g*x + f)^n*a*f^3*g^2*e^2)/(g^5*n^5 + 15*g^5*n^4
+ 85*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

maple [B] time = 0.01, size = 1048, normalized size = 5.04

$$\frac{\left(c e^3 g^4 n^4 x^4 + 4cd e^2 g^4 n^4 x^3 + 10c e^3 g^4 n^3 x^4 + 5c d^2 e g^4 n^4 x^2 + 44cd e^2 g^4 n^3 x^3 - 4c e^3 f g^3 n^3 x^3 + 35c e^3 g^4 n^2 x^4 + a e^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e^3*g^4*n^4*x^4+4*c*d*e^2*g^4*n^4*x^3+10*c*e^3*g^4*n^3*x^4
+5*c*d^2*e*g^4*n^4*x^2+44*c*d*e^2*g^4*n^3*x^3-4*c*e^3*f*g^3*n^3*x^3+35*c*e^
3*g^4*n^2*x^4+a*e^2*g^4*n^4*x^2+2*c*d^3*g^4*n^4*x+60*c*d^2*e*g^4*n^3*x^2-12
*c*d*e^2*f*g^3*n^3*x^2+164*c*d*e^2*g^4*n^2*x^3-24*c*e^3*f*g^3*n^2*x^3+50*c*
e^3*g^4*n*x^4+2*a*d*e*g^4*n^4*x+12*a*e^2*g^4*n^3*x^2+26*c*d^3*g^4*n^3*x-10*
c*d^2*e*f*g^3*n^3*x+245*c*d^2*e*g^4*n^2*x^2-96*c*d*e^2*f*g^3*n^2*x^2+244*c*
d*e^2*g^4*n*x^3+12*c*e^3*f^2*g^2*n^2*x^2-44*c*e^3*f*g^3*n*x^3+24*c*e^3*g^4*
x^4+a*d^2*g^4*n^4+26*a*d*e*g^4*n^3*x-2*a*e^2*f*g^3*n^3*x+49*a*e^2*g^4*n^2*x
^2-2*c*d^3*f*g^3*n^3+118*c*d^3*g^4*n^2*x-100*c*d^2*e*f*g^3*n^2*x+390*c*d^2*
e*g^4*n*x^2+24*c*d*e^2*f^2*g^2*n^2*x-204*c*d*e^2*f*g^3*n*x^2+120*c*d*e^2*g^
4*x^3+36*c*e^3*f^2*g^2*n*x^2-24*c*e^3*f*g^3*x^3+14*a*d^2*g^4*n^3-2*a*d*e*f*
g^3*n^3+118*a*d*e*g^4*n^2*x-20*a*e^2*f*g^3*n^2*x+78*a*e^2*g^4*n*x^2-24*c*d^
3*f*g^3*n^2+214*c*d^3*g^4*n*x+10*c*d^2*e*f^2*g^2*n^2-290*c*d^2*e*f*g^3*n*x+
200*c*d^2*e*g^4*x^2+144*c*d*e^2*f^2*g^2*n*x-120*c*d*e^2*f*g^3*x^2-24*c*e^3*
f^3*g*n*x+24*c*e^3*f^2*g^2*x^2+71*a*d^2*g^4*n^2-24*a*d*e*f*g^3*n^2+214*a*d*
e*g^4*n*x+2*a*e^2*f^2*g^2*n^2-58*a*e^2*f*g^3*n*x+40*a*e^2*g^4*x^2-94*c*d^3*
f*g^3*n+120*c*d^3*g^4*x+90*c*d^2*e*f^2*g^2*n-200*c*d^2*e*f*g^3*x-24*c*d*e^2
*f^3*g*n+120*c*d*e^2*f^2*g^2*x-24*c*e^3*f^3*g*x+154*a*d^2*g^4*n-94*a*d*e*f*
```

```
g^3*n+120*a*d*e*g^4*x+18*a*e^2*f^2*g^2*n-40*a*e^2*f*g^3*x-120*c*d^3*f*g^3+2
00*c*d^2*e*f^2*g^2-120*c*d*e^2*f^3*g+24*c*e^3*f^4+120*a*d^2*g^4-120*a*d*e*f
*g^3+40*a*e^2*f^2*g^2)/g^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

maxima [B] time = 0.54, size = 512, normalized size = 2.46

$$\frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)\left(gx+f\right)^n cd^3}{\left(n^2+3n+2\right)g^2}+\frac{5\left(\left(n^2+3n+2\right)g^3x^3+\left(n^2+n\right)fg^2x^2-2f^2gnx+2f^3\right)\left(gx+f\right)^n}{\left(n^3+6n^2+11n+6\right)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^3/((n^2 + 3*n + 2)*g^2)
 + 5*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^2*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 2*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a
*d^2/(g*(n + 1)) + 4*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n
)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*
d*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + ((n^2 + 3*n + 2)*g^3*x^3
+ (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*e^2/((n^3 + 6*n^
2 + 11*n + 6)*g^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5 + (n^4 +
6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x^3 + 12*(n
^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*e^3/((n^5 + 15*n
^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5)
```

mupad [B] time = 3.52, size = 1133, normalized size = 5.45

$$\frac{\left(f+g\,x\right)^n\,\left(-2\,c\,d^3\,f^2\,g^3\,n^3-24\,c\,d^3\,f^2\,g^3\,n^2-94\,c\,d^3\,f^2\,g^3\,n-120\,c\,d^3\,f^2\,g^3+10\,c\,d^2\,e\,f^3\,g^2\,n^2+90\,c\,d^2\,e\,f^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^2*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] ((f + g*x)^n*(24*c*e^3*f^5 + 40*a*e^2*f^3*g^2 - 120*c*d^3*f^2*g^3 + 120*a*d
^2*f*g^4 - 120*a*d*e*f^2*g^3 - 120*c*d*e^2*f^4*g + 154*a*d^2*f*g^4*n + 200*
c*d^2*e*f^3*g^2 + 71*a*d^2*f*g^4*n^2 + 14*a*d^2*f*g^4*n^3 + a*d^2*f*g^4*n^4
 + 18*a*e^2*f^3*g^2*n - 94*c*d^3*f^2*g^3*n + 2*a*e^2*f^3*g^2*n^2 - 24*c*d^3
*f^2*g^3*n^2 - 2*c*d^3*f^2*g^3*n^3 + 10*c*d^2*e*f^3*g^2*n^2 - 94*a*d*e*f^2*
g^3*n - 24*c*d*e^2*f^4*g*n - 24*a*d*e*f^2*g^3*n^2 - 2*a*d*e*f^2*g^3*n^3 + 9
0*c*d^2*e*f^3*g^2*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (x*(f + g*x)^n*(120*a*d^2*g^5 + 71*a*d^2*g^5*n^2 + 14*a*d^2*g^5*n^3 + a*
d^2*g^5*n^4 + 154*a*d^2*g^5*n + 120*c*d^3*f*g^4*n - 24*c*e^3*f^4*g*n - 40*a
*e^2*f^2*g^3*n + 94*c*d^3*f*g^4*n^2 + 24*c*d^3*f*g^4*n^3 + 2*c*d^3*f*g^4*n^
4 - 18*a*e^2*f^2*g^3*n^2 - 2*a*e^2*f^2*g^3*n^3 + 120*a*d*e*f*g^4*n + 24*c*d
*e^2*f^3*g^2*n^2 - 90*c*d^2*e*f^2*g^3*n^2 - 10*c*d^2*e*f^2*g^3*n^3 + 94*a*d
*e*f*g^4*n^2 + 24*a*d*e*f*g^4*n^3 + 2*a*d*e*f*g^4*n^4 + 120*c*d*e^2*f^3*g^2
*n - 200*c*d^2*e*f^2*g^3*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5
+ 120)) + (c*e^3*x^5*(f + g*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*
n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^2*(f + g*x)^n*(n + 1)*(120*
c*d^3*g^3 + 24*c*d^3*g^3*n^2 + 2*c*d^3*g^3*n^3 + 120*a*d*e*g^3 + 94*c*d^3*g
^3*n + 12*c*e^3*f^3*n + 24*a*d*e*g^3*n^2 + 2*a*d*e*g^3*n^3 + 20*a*e^2*f*g^2
*n + 9*a*e^2*f*g^2*n^2 + a*e^2*f*g^2*n^3 + 94*a*d*e*g^3*n - 60*c*d*e^2*f^2*
g*n + 100*c*d^2*e*f*g^2*n - 12*c*d*e^2*f^2*g*n^2 + 45*c*d^2*e*f*g^2*n^2 + 5
*c*d^2*e*f*g^2*n^3))/(g^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
+ (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(100*c*d^2*g^2 + 20*a*e*g^2 + 5*c*d^2*
g^2*n^2 + 9*a*e*g^2*n + a*e*g^2*n^2 + 45*c*d^2*g^2*n - 4*c*e^2*f^2*n + 4*c*
d*e*f*g*n^2 + 20*c*d*e*f*g*n))/(g^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^
```

```
5 + 120)) + (c*e^2*x^4*(f + g*x)^n*(20*d*g + 4*d*g*n + e*f*n)*(11*n + 6*n^2
 + n^3 + 6))/(g*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
```

sympy [A] time = 11.82, size = 11946, normalized size = 57.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x**2 + 5*c*
d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0)), (-3*a*d**2*g**4/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 2*a*d*e*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**
2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 8*a*d*e*g**4*x/(12*f**4*g**5 + 48*f**3
*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - a*e**2*f**2*
g**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
12*g**9*x**4) - 4*a*e**2*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*
g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 6*a*e**2*g**4*x**2/(12*f**4*g*
*5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
2*c*d**3*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g
**8*x**3 + 12*g**9*x**4) - 8*c*d**3*g**4*x/(12*f**4*g**5 + 48*f**3*g**6*x +
 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 5*c*d**2*e*f**2*g**2/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 20*c*d**2*e*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g*
*7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 30*c*d**2*e*g**4*x**2/(12*f**4*g
**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
 12*c*d*e**2*f**3*g/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48
*f*g**8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*f**2*g**2*x/(12*f**4*g**5 + 48*f
**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 72*c*d*e*
*2*f*g**3*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g*
*8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*g**4*x**3/(12*f**4*g**5 + 48*f**3*g**
6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*f**4*l
og(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*
x**3 + 12*g**9*x**4) + 25*c*e**3*f**4/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f
**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f**3*g*x*log(f/g
 + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
 12*g**9*x**4) + 88*c*e**3*f**3*g*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**
2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 72*c*e**3*f**2*g**2*x**2*log
(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x*
*3 + 12*g**9*x**4) + 108*c*e**3*f**2*g**2*x**2/(12*f**4*g**5 + 48*f**3*g**6
*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*
x**3*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f
*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*x**3/(12*f**4*g**5 + 48*f**3*
g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*g**
4*x**4*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48
*f*g**8*x**3 + 12*g**9*x**4), Eq(n, -5)), (-a*d**2*g**4/(3*f**3*g**5 + 9*f*
*2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*d*e*f*g**3/(3*f**3*g**5 + 9*f*
*2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*d*e*g**4*x/(3*f**3*g**5 + 9*
f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*e**2*f**2*g**2/(3*f**3*g**5
+ 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*f*g**3*x/(3*f**3*
g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*g**4*x**2/(3
*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - c*d**3*f*g**3/(
3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*c*d**3*g**4*
x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 5*c*d**2*e*
f**2*g**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 15*
c*d**2*e*f*g**3*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**
3) - 15*c*d**2*e*g**4*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3
*g**8*x**3) + 12*c*d*e**2*f**3*g*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x
+ 9*f*g**7*x**2 + 3*g**8*x**3) + 22*c*d*e**2*f**3*g/(3*f**3*g**5 + 9*f**2*g
```

```
**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f**2*g**2*x*log(f/g + x)
/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 54*c*d*e**2*
f**2*g**2*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 3
6*c*d*e**2*f*g**3*x**2*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7
*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f*g**3*x**2/(3*f**3*g**5 + 9*f**2*g**6*x
 + 9*f*g**7*x**2 + 3*g**8*x**3) + 12*c*d*e**2*g**4*x**3*log(f/g + x)/(3*f**
3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f**4*log(
f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 22*c
*e**3*f**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36
*c*e**3*f**3*g*x*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2
 + 3*g**8*x**3) - 54*c*e**3*f**3*g*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7
*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2*log(f/g + x)/(3*f**3*g**5 +
 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2/(3
*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f*g**
3*x**3*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x
**3) + 3*c*e**3*g**4*x**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*
g**8*x**3), Eq(n, -4)), (-a*d**2*g**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) - 2*a*d*e*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4*a*d*e*g**
4*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 2*a*e**2*f**2*g**2*log(f/g +
 x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 3*a*e**2*f**2*g**2/(2*f**2*g
**5 + 4*f*g**6*x + 2*g**7*x**2) + 4*a*e**2*f*g**3*x*log(f/g + x)/(2*f**2*g*
*5 + 4*f*g**6*x + 2*g**7*x**2) + 4*a*e**2*f*g**3*x/(2*f**2*g**5 + 4*f*g**6*
x + 2*g**7*x**2) + 2*a*e**2*g**4*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*
x + 2*g**7*x**2) - 2*c*d**3*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2)
 - 4*c*d**3*g**4*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*f
**2*g**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 15*c*d**2*
e*f**2*g**2/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3*x
*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3
*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*g**4*x**2*log(f/g
 + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f**3*g*log(f/g
 + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 36*c*d*e**2*f**3*g/(2*f**2
*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x*log(f/g + x)/(2
*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x/(2*f**2*g*
*5 + 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f*g**3*x**2*log(f/g + x)/(2*f*
*2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 8*c*d*e**2*g**4*x**3/(2*f**2*g**5 + 4
*f*g**6*x + 2*g**7*x**2) + 12*c*e**3*f**4*log(f/g + x)/(2*f**2*g**5 + 4*f*g
**6*x + 2*g**7*x**2) + 18*c*e**3*f**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) + 24*c*e**3*f**3*g*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) + 24*c*e**3*f**3*g*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 12*c*e*
*3*f**2*g**2*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4
*c*e**3*f*g**3*x**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + c*e**3*g**4*
x**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2), Eq(n, -3)), (-3*a*d**2*g**4/
(3*f*g**5 + 3*g**6*x) + 6*a*d*e*f*g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) +
 6*a*d*e*f*g**3/(3*f*g**5 + 3*g**6*x) + 6*a*d*e*g**4*x*log(f/g + x)/(3*f*g*
*5 + 3*g**6*x) - 6*a*e**2*f**2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 6*
a*e**2*f**2*g**2/(3*f*g**5 + 3*g**6*x) - 6*a*e**2*f*g**3*x*log(f/g + x)/(3*
f*g**5 + 3*g**6*x) + 3*a*e**2*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*
g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*g**3/(3*f*g**5 + 3*g**
6*x) + 6*c*d**3*g**4*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**
2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**2*g**2/(3*f*g**5
 + 3*g**6*x) - 30*c*d**2*e*f*g**3*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 15
*c*d**2*e*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g*log(f/g + x)
/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g/(3*f*g**5 + 3*g**6*x) + 36*c*d*
e**2*f**2*g**2*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 18*c*d*e**2*f*g**3*x*
*2/(3*f*g**5 + 3*g**6*x) + 6*c*d*e**2*g**4*x**3/(3*f*g**5 + 3*g**6*x) - 12*
c*e**3*f**4*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 12*c*e**3*f**4/(3*f*g**5 +
 3*g**6*x) - 12*c*e**3*f**3*g*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*e*
*3*f**2*g**2*x**2/(3*f*g**5 + 3*g**6*x) - 2*c*e**3*f*g**3*x**3/(3*f*g**5 +
3*g**6*x) + c*e**3*g**4*x**4/(3*f*g**5 + 3*g**6*x), Eq(n, -2)), (a*d**2*log
```

```
(f/g + x)/g - 2*a*d*e*f*log(f/g + x)/g**2 + 2*a*d*e*x/g + a*e**2*f**2*log(f
/g + x)/g**3 - a*e**2*f*x/g**2 + a*e**2*x**2/(2*g) - 2*c*d**3*f*log(f/g + x
)/g**2 + 2*c*d**3*x/g + 5*c*d**2*e*f**2*log(f/g + x)/g**3 - 5*c*d**2*e*f*x/
g**2 + 5*c*d**2*e*x**2/(2*g) - 4*c*d*e**2*f**3*log(f/g + x)/g**4 + 4*c*d*e*
*2*f**2*x/g**3 - 2*c*d*e**2*f*x**2/g**2 + 4*c*d*e**2*x**3/(3*g) + c*e**3*f*
*4*log(f/g + x)/g**5 - c*e**3*f**3*x/g**4 + c*e**3*f**2*x**2/(2*g**3) - c*e
**3*f*x**3/(3*g**2) + c*e**3*x**4/(4*g), Eq(n, -1)), (a*d**2*f*g**4*n**4*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g
**5*n + 120*g**5) + 14*a*d**2*f*g**4*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*f
*g**4*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 154*a*d**2*f*g**4*n*(f + g*x)**n/(g**5*n**
5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
120*a*d**2*f*g**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 2
25*g**5*n**2 + 274*g**5*n + 120*g**5) + a*d**2*g**5*n**4*x*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 14*a*d**2*g**5*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*g**5*n**2*x*(f
+ g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g*
*5*n + 120*g**5) + 154*a*d**2*g**5*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d**2*g**
5*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 +
 274*g**5*n + 120*g**5) - 2*a*d*e*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 +
15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*a
*d*e*f**2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 +
 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 94*a*d*e*f**2*g**3*n*(f + g*x)**n
/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 12
0*g**5) - 120*a*d*e*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g
**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*a*d*e*f*g**4*n**4*x*(
f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*
g**5*n + 120*g**5) + 24*a*d*e*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g*
*5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*a*d*e*
f*g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g
**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d*e*f*g**4*n*x*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 2*a*d*e*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 26*a*d*e*g**5*n**3*x**2*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g
**5*n + 120*g**5) + 118*a*d*e*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g
**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*a*d*
e*g**5*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g
**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d*e*g**5*x**2*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 2*a*e**2*f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 18*a*e**2*f**3*g**2*n*(f +
 g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**
5*n + 120*g**5) + 40*a*e**2*f**3*g**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**
4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 2*a*e**2*f**2*g
**3*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) - 18*a*e**2*f**2*g**3*n**2*x*(f + g*x)**n/(g
**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g
**5) - 40*a*e**2*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + a*e**2*f*g**4*n**4*x**
2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) + 10*a*e**2*f*g**4*n**3*x**2*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 29
*a*e**2*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 20*a*e**2*f*g**4*n*x**2*(f +
 g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**
```

5*n + 120*g**5) + a*e**2*g**5*n**4*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n
4 + 85*g5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 12*a*e**2*g**
5*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + 49*a*e**2*g**5*n**2*x**3*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 78*a*e**2*g**5*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 40*a*e**2*g**5*x**3*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) - 2*c*d**3*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d**3*f*
*2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) - 94*c*d**3*f**2*g**3*n*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) - 120*c*d**3*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
3 + 225*g5*n**2 + 274*g**5*n + 120*g**5) + 2*c*d**3*f*g**4*n**4*x*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + 24*c*d**3*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*c*d**3*f*
g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d**3*f*g**4*n*x*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 2*c*d**3*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 26*c*d**3*g**5*n**3*x**2*(
f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*
g**5*n + 120*g**5) + 118*c*d**3*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15
*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*c*
d**3*g**5*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 22
5*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d**3*g**5*x**2*(f + g*x)**n/(g
5*n5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g
5) + 10*c*d2*e*f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 90*c*d**2*e*f**3*g*
*2*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + 200*c*d**2*e*f**3*g**2*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 10*
c*d**2*e*f**2*g**3*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 90*c*d**2*e*f**2*g**3*n**2*
x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) - 200*c*d**2*e*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*
c*d**2*e*f*g**4*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 50*c*d**2*e*f*g**4*n**3*x**
2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) + 145*c*d**2*e*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n*
*5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
 100*c*d**2*e*f*g**4*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*c*d**2*e*g**5*n**4*x**3
*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 27
4*g**5*n + 120*g**5) + 60*c*d**2*e*g**5*n**3*x**3*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 245
*c*d**2*e*g**5*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
3 + 225*g5*n**2 + 274*g**5*n + 120*g**5) + 390*c*d**2*e*g**5*n*x**3*(f
+ g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g*
*5*n + 120*g**5) + 200*c*d**2*e*g**5*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d*e**2
*f**4*g*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*
n**2 + 274*g**5*n + 120*g**5) - 120*c*d*e**2*f**4*g*(f + g*x)**n/(g**5*n**5
 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2
4*c*d*e**2*f**3*g**2*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*f**3*g**2*n*

```
x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) - 12*c*d*e**2*f**2*g**3*n**3*x**2*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 - 72*c*d*e**2*f**2*g**3*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 60*c*d*e**2*f**2*g
**3*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 4*c*d*e**2*f*g**4*n**4*x**3*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 32*c*d*e**2*f*g**4*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 68*c*d*e**2*f*g*
*4*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) + 40*c*d*e**2*f*g**4*n*x**3*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 4*c*d*e**2*g**5*n**4*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 44*c*d*e**2*g**5*n*
*3*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*
*2 + 274*g**5*n + 120*g**5) + 164*c*d*e**2*g**5*n**2*x**4*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 244*c*d*e**2*g**5*n*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g
**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*g**5*x**4*
(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274
*g**5*n + 120*g**5) + 24*c*e**3*f**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*e**3*f**4*g
*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + 12*c*e**3*f**3*g**2*n**2*x**2*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 12*c*e**3*f**3*g**2*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 4*c*e**3*f**2*g**3*n**
3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**
2 + 274*g**5*n + 120*g**5) - 12*c*e**3*f**2*g**3*n**2*x**3*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) - 8*c*e**3*f**2*g**3*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85
*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + c*e**3*f*g**4*n**4*x*
*4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 +
274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n**3*x**4*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 11
*c*e**3*f*g**4*n**2*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n*x**4*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + c*e**3*g**5*n**4*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 10*c*e**3*g**5
*n**3*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 35*c*e**3*g**5*n**2*x**5*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 50*c*e**3*g**5*n*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24*c*e**3*g**5*x**5*(f + g
*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*
n + 120*g**5), True))
```

3.807 $\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=146

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f\cdot}{g^4(n+3}$$

```
[Out] -(-d*g+e*f)*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^4/(1+n)+(a*e*g^2+c*(2*
d^2*g^2-6*d*e*f*g+3*e^2*f^2))*(g*x+f)^(2+n)/g^4/(2+n)-3*c*e*(-d*g+e*f)*(g*x
+f)^(3+n)/g^4/(3+n)+c*e^2*(g*x+f)^(4+n)/g^4/(4+n)
```

Rubi [A] time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {771}

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f\cdot}{g^4(n+3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n))
) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))/(
g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*e^2
*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)dx &= \int \left(\frac{(ef-dg)\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^3}+\frac{\left(aeg^2+c\left(3e^2f^2\right.\right.}{}\right.\\ &= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)}+\frac{\left(aeg^2+c\left(3e^2f^2-\right.\right.}{}\end{aligned}$$

Mathematica [A] time = 0.28, size = 141, normalized size = 0.97

$$\frac{(f+gx)^{n+1}\left(\frac{2(f+gx)\left(aeg^2(n+3)+c\left(-d^2g^2n-6defg+3e^2f^2\right)\right)}{g^2(n+2)}+\frac{6(dg-ef)\left(ag^2+cf(ef-2dg)\right)}{g^2(n+1)}+(a+cx(2d+ex))(dg(n+6)-3ef\right.}{g^2(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*((6*(-(e*f) + d*g)*(a*g^2 + c*f*(e*f - 2*d*g)))/(g^2*(1
+ n)) + (2*(a*e*g^2*(3 + n) + c*(3*e^2*f^2 - 6*d*e*f*g - d^2*g^2*n))*(f + g
*x))/(g^2*(2 + n)) + (-3*e*f + d*g*(6 + n) + e*g*(3 + n)*x)*(a + c*x*(2*d +
 e*x))))/(g^2*(3 + n)*(4 + n))
```

fricas [B] time = 1.18, size = 549, normalized size = 3.76

$$\frac{\left(adfg^3n^3 - 6\,ce^2f^4 + 24\,cdef^3g + 24\,adfg^3 - 12\left(2\,cd^2 + ae\right)f^2g^2 + \left(ce^2g^4n^3 + 6\,ce^2g^4n^2 + 11\,ce^2g^4n + 6\,ce^2g^4\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*f*g^3*n^3 - 6*c*e^2*f^4 + 24*c*d*e*f^3*g + 24*a*d*f*g^3 - 12*(2*c*d^2
+ a*e)*f^2*g^2 + (c*e^2*g^4*n^3 + 6*c*e^2*g^4*n^2 + 11*c*e^2*g^4*n + 6*c*e^
2*g^4)*x^4 + (24*c*d*e*g^4 + (c*e^2*f*g^3 + 3*c*d*e*g^4)*n^3 + 3*(c*e^2*f*g
^3 + 7*c*d*e*g^4)*n^2 + 2*(c*e^2*f*g^3 + 21*c*d*e*g^4)*n)*x^3 + (9*a*d*f*g^
3 - (2*c*d^2 + a*e)*f^2*g^2)*n^2 + (12*(2*c*d^2 + a*e)*g^4 + (3*c*d*e*f*g^3
 + (2*c*d^2 + a*e)*g^4)*n^3 - (3*c*e^2*f^2*g^2 - 15*c*d*e*f*g^3 - 8*(2*c*d^
2 + a*e)*g^4)*n^2 - (3*c*e^2*f^2*g^2 - 12*c*d*e*f*g^3 - 19*(2*c*d^2 + a*e)*
g^4)*n)*x^2 + (6*c*d*e*f^3*g + 26*a*d*f*g^3 - 7*(2*c*d^2 + a*e)*f^2*g^2)*n
+ (24*a*d*g^4 + (a*d*g^4 + (2*c*d^2 + a*e)*f*g^3)*n^3 - (6*c*d*e*f^2*g^2 -
9*a*d*g^4 - 7*(2*c*d^2 + a*e)*f*g^3)*n^2 + 2*(3*c*e^2*f^3*g - 12*c*d*e*f^2*
g^2 + 13*a*d*g^4 + 6*(2*c*d^2 + a*e)*f*g^3)*n)*x)*(g*x + f)^n/(g^4*n^4 + 10
*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 24*g^4)
```

giac [B] time = 0.39, size = 1018, normalized size = 6.97

$$\frac{\left(gx+f\right)^n cg^4n^3x^4e^2 + 3\left(gx+f\right)^n cdg^4n^3x^3e + 2\left(gx+f\right)^n cd^2g^4n^3x^2 + \left(gx+f\right)^n cfg^3n^3x^3e^2 + 6\left(gx+f\right)^n cg^4n^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^4*n^3*x^4*e^2 + 3*(g*x + f)^n*c*d*g^4*n^3*x^3*e + 2*(g*x +
 f)^n*c*d^2*g^4*n^3*x^2 + (g*x + f)^n*c*f*g^3*n^3*x^3*e^2 + 6*(g*x + f)^n*c
*g^4*n^2*x^4*e^2 + 3*(g*x + f)^n*c*d*f*g^3*n^3*x^2*e + 21*(g*x + f)^n*c*d*g
^4*n^2*x^3*e + 2*(g*x + f)^n*c*d^2*f*g^3*n^3*x + 16*(g*x + f)^n*c*d^2*g^4*n
^2*x^2 + 3*(g*x + f)^n*c*f*g^3*n^2*x^3*e^2 + 11*(g*x + f)^n*c*g^4*n*x^4*e^2
 + 15*(g*x + f)^n*c*d*f*g^3*n^2*x^2*e + (g*x + f)^n*a*g^4*n^3*x^2*e + 42*(g
*x + f)^n*c*d*g^4*n*x^3*e + 14*(g*x + f)^n*c*d^2*f*g^3*n^2*x + (g*x + f)^n*
a*d*g^4*n^3*x + 38*(g*x + f)^n*c*d^2*g^4*n*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n^
2*x^2*e^2 + 2*(g*x + f)^n*c*f*g^3*n*x^3*e^2 + 6*(g*x + f)^n*c*g^4*x^4*e^2 -
 6*(g*x + f)^n*c*d*f^2*g^2*n^2*x*e + (g*x + f)^n*a*f*g^3*n^3*x*e + 12*(g*x
+ f)^n*c*d*f*g^3*n*x^2*e + 8*(g*x + f)^n*a*g^4*n^2*x^2*e + 24*(g*x + f)^n*c
*d*g^4*x^3*e - 2*(g*x + f)^n*c*d^2*f^2*g^2*n^2 + (g*x + f)^n*a*d*f*g^3*n^3
+ 24*(g*x + f)^n*c*d^2*f*g^3*n*x + 9*(g*x + f)^n*a*d*g^4*n^2*x + 24*(g*x +
f)^n*c*d^2*g^4*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n*x^2*e^2 - 24*(g*x + f)^n*c*d
*f^2*g^2*n*x*e + 7*(g*x + f)^n*a*f*g^3*n^2*x*e + 19*(g*x + f)^n*a*g^4*n*x^2
*e - 14*(g*x + f)^n*c*d^2*f^2*g^2*n + 9*(g*x + f)^n*a*d*f*g^3*n^2 + 26*(g*x
 + f)^n*a*d*g^4*n*x + 6*(g*x + f)^n*c*f^3*g*n*x*e^2 + 6*(g*x + f)^n*c*d*f^3
*g*n*e - (g*x + f)^n*a*f^2*g^2*n^2*e + 12*(g*x + f)^n*a*f*g^3*n*x*e + 12*(g
*x + f)^n*a*g^4*x^2*e - 24*(g*x + f)^n*c*d^2*f^2*g^2 + 26*(g*x + f)^n*a*d*f
*g^3*n + 24*(g*x + f)^n*a*d*g^4*x + 24*(g*x + f)^n*c*d*f^3*g*e - 7*(g*x + f
)^n*a*f^2*g^2*n*e + 24*(g*x + f)^n*a*d*f*g^3 - 6*(g*x + f)^n*c*f^4*e^2 - 12
*(g*x + f)^n*a*f^2*g^2*e)/(g^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 2
4*g^4)
```

maple [B] time = 0.01, size = 449, normalized size = 3.08

$$\frac{\left(c\,e^2g^3n^3x^3 + 3cde\,g^3n^3x^2 + 6c\,e^2g^3n^2x^3 + 2c\,d^2g^3n^3x + 21cde\,g^3n^2x^2 - 3c\,e^2f\,g^2n^2x^2 + 11c\,e^2g^3n\,x^3 + ae\,g^3n^3x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e^2*g^3*n^3*x^3+3*c*d*e*g^3*n^3*x^2+6*c*e^2*g^3*n^2*x^3+2*
c*d^2*g^3*n^3*x+21*c*d*e*g^3*n^2*x^2-3*c*e^2*f*g^2*n^2*x^2+11*c*e^2*g^3*n*x
^3+a*e*g^3*n^3*x+16*c*d^2*g^3*n^2*x-6*c*d*e*f*g^2*n^2*x+42*c*d*e*g^3*n*x^2-
9*c*e^2*f*g^2*n*x^2+6*c*e^2*g^3*x^3+a*d*g^3*n^3+8*a*e*g^3*n^2*x-2*c*d^2*f*g
^2*n^2+38*c*d^2*g^3*n*x-30*c*d*e*f*g^2*n*x+24*c*d*e*g^3*x^2+6*c*e^2*f^2*g*n
*x-6*c*e^2*f*g^2*x^2+9*a*d*g^3*n^2-a*e*f*g^2*n^2+19*a*e*g^3*n*x-14*c*d^2*f*
g^2*n+24*c*d^2*g^3*x+6*c*d*e*f^2*g*n-24*c*d*e*f*g^2*x+6*c*e^2*f^2*g*x+26*a*
d*g^3*n-7*a*e*f*g^2*n+12*a*e*g^3*x-24*c*d^2*f*g^2+24*c*d*e*f^2*g-6*c*e^2*f^
3+24*a*d*g^3-12*a*e*f*g^2)/g^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 0.51, size = 289, normalized size = 1.98

$$\frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)\left(gx+f\right)^n cd^2}{\left(n^2+3n+2\right)g^2}+\frac{3\left(\left(n^2+3n+2\right)g^3x^3+\left(n^2+n\right)fg^2x^2-2f^2gnx+2f^3\right)\left(gx+f\right)^{n}}{\left(n^3+6n^2+11n+6\right)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^2/((n^2 + 3*n + 2)*g^2)
 + 3*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g^2*(n + 1)*x^2 + f*g*n
*x - f^2)*(g*x + f)^n*a*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a*d/(g*
(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^
3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*e^2/((n^4
+ 10*n^3 + 35*n^2 + 50*n + 24)*g^4)
```

mupad [B] time = 3.29, size = 572, normalized size = 3.92

$$\frac{x\left(f+g\,x\right)^n\,\left(2\,c\,d^2\,f\,g^3\,n^3+14\,c\,d^2\,f\,g^3\,n^2+24\,c\,d^2\,f\,g^3\,n-6\,c\,d\,e\,f^2\,g^2\,n^2-24\,c\,d\,e\,f^2\,g^2\,n+a\,d\,g^4\,n^3+9\,a\,\cdots\right.}{g^4\,\left(n^4+10\,n^3+35\,n^2+50\,n\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(24*a*d*g^4 + 26*a*d*g^4*n + 9*a*d*g^4*n^2 + a*d*g^4*n^3 + 7
*a*e*f*g^3*n^2 + a*e*f*g^3*n^3 + 24*c*d^2*f*g^3*n + 6*c*e^2*f^3*g*n + 14*c*
d^2*f*g^3*n^2 + 2*c*d^2*f*g^3*n^3 + 12*a*e*f*g^3*n - 24*c*d*e*f^2*g^2*n - 6
*c*d*e*f^2*g^2*n^2))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - ((f + g*x)
^n*(6*c*e^2*f^4 + 24*c*d^2*f^2*g^2 - 24*a*d*f*g^3 + 12*a*e*f^2*g^2 - 9*a*d*
f*g^3*n^2 - a*d*f*g^3*n^3 + 7*a*e*f^2*g^2*n + a*e*f^2*g^2*n^2 + 14*c*d^2*f^
2*g^2*n - 24*c*d*e*f^3*g - 26*a*d*f*g^3*n + 2*c*d^2*f^2*g^2*n^2 - 6*c*d*e*f
^3*g*n))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e^2*x^4*(f + g*x)^n
*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24) + (x^2*(f +
g*x)^n*(n + 1)*(24*c*d^2*g^2 + 12*a*e*g^2 + 2*c*d^2*g^2*n^2 + 7*a*e*g^2*n +
 a*e*g^2*n^2 + 14*c*d^2*g^2*n - 3*c*e^2*f^2*n + 3*c*d*e*f*g*n^2 + 12*c*d*e*
f*g*n))/(g^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e*x^3*(f + g*x)^n*(1
2*d*g + 3*d*g*n + e*f*n)*(3*n + n^2 + 2))/(g*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24))
```

sympy [A] time = 5.27, size = 4952, normalized size = 33.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d*x + a*e*x**2/2 + c*d**2*x**2 + c*d*e*x**3 + c*e**2*x**
4/4), Eq(g, 0)), (-2*a*d*g**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - a*e*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - 3*a*e*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x
**2 + 6*g**7*x**3) - 2*c*d**2*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g
**6*x**2 + 6*g**7*x**3) - 6*c*d**2*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 1
8*f*g**6*x**2 + 6*g**7*x**3) - 6*c*d*e*f**2*g/(6*f**3*g**4 + 18*f**2*g**5*x
 + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*f*g**2*x/(6*f**3*g**4 + 18*f**2
*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*g**3*x**2/(6*f**3*g**4 +
 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*f**3*log(f/g + x
)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 11*c*e**2
*f**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*
e**2*f**2*g*x*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 +
 6*g**7*x**3) + 27*c*e**2*f**2*g*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**
6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2*log(f/g + x)/(6*f**3*g**4 + 1
8*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2/(6*f*
*3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*g**3*x*
*3*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**
3), Eq(n, -4)), (-a*d*g**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - a*e*f
*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 2*a*e*g**3*x/(2*f**2*g**4
+ 4*f*g**5*x + 2*g**6*x**2) - 2*c*d**2*f*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2
*g**6*x**2) - 4*c*d**2*g**3*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*
c*d*e*f**2*g*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 9*c*d*
e*f**2*g/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x*log(f
/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x/(2*f**
2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*c*d*e*g**3*x**2*log(f/g + x)/(2*f**2
*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 6*c*e**2*f**3*log(f/g + x)/(2*f**2*g**4
 + 4*f*g**5*x + 2*g**6*x**2) - 9*c*e**2*f**3/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) -
6*c*e**2*f*g**2*x**2*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2)
+ 2*c*e**2*g**3*x**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2), Eq(n, -3)),
(-2*a*d*g**3/(2*f*g**4 + 2*g**5*x) + 2*a*e*f*g**2*log(f/g + x)/(2*f*g**4 +
2*g**5*x) + 2*a*e*f*g**2/(2*f*g**4 + 2*g**5*x) + 2*a*e*g**3*x*log(f/g + x)/
(2*f*g**4 + 2*g**5*x) + 4*c*d**2*f*g**2*log(f/g + x)/(2*f*g**4 + 2*g**5*x)
+ 4*c*d**2*f*g**2/(2*f*g**4 + 2*g**5*x) + 4*c*d**2*g**3*x*log(f/g + x)/(2*f
*g**4 + 2*g**5*x) - 12*c*d*e*f**2*g*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 12
*c*d*e*f**2*g/(2*f*g**4 + 2*g**5*x) - 12*c*d*e*f*g**2*x*log(f/g + x)/(2*f*g
**4 + 2*g**5*x) + 6*c*d*e*g**3*x**2/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3*l
og(f/g + x)/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3/(2*f*g**4 + 2*g**5*x) + 6
*c*e**2*f**2*g*x*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 3*c*e**2*f*g**2*x**2/
(2*f*g**4 + 2*g**5*x) + c*e**2*g**3*x**3/(2*f*g**4 + 2*g**5*x), Eq(n, -2)),
 (a*d*log(f/g + x)/g - a*e*f*log(f/g + x)/g**2 + a*e*x/g - 2*c*d**2*f*log(f
/g + x)/g**2 + 2*c*d**2*x/g + 3*c*d*e*f**2*log(f/g + x)/g**3 - 3*c*d*e*f*x/
g**2 + 3*c*d*e*x**2/(2*g) - c*e**2*f**3*log(f/g + x)/g**4 + c*e**2*f**2*x/g
**3 - c*e**2*f*x**2/(2*g**2) + c*e**2*x**3/(3*g), Eq(n, -1)), (a*d*f*g**3*n
**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 9*a*d*f*g**3*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 26*a*d*f*g**3*n*(f + g*x)**n/(g**4*n**4 + 10*
g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*f*g**3*(f + g*x)**
n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*d*g**
4*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n
+ 24*g**4) + 9*a*d*g**4*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 26*a*d*g**4*n*x*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*g**4*x*(f + g
*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - a*
e*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50
*g**4*n + 24*g**4) - 7*a*e*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
```

```
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 12*a*e*f**2*g**2*(f + g*x)**n/(g
**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*f*g**3*
n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n +
24*g**4) + 7*a*e*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*f*g**3*n*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*g**4*n**3*x**2
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) + 8*a*e*g**4*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 19*a*e*g**4*n*x**2*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*g**4*x**2*(f +
g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 2
*c*d**2*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**
2 + 50*g**4*n + 24*g**4) - 14*c*d**2*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d**2*f**2*g**2*(f
 + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4)
+ 2*c*d**2*f*g**3*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n
**2 + 50*g**4*n + 24*g**4) + 14*c*d**2*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d**2*f*g**3*n
*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g
**4) + 2*c*d**2*g**4*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 16*c*d**2*g**4*n**2*x**2*(f + g*x)**n/(g
**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 38*c*d**2*g
**4*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*
n + 24*g**4) + 24*c*d**2*g**4*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 +
 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*d*e*f**3*g*n*(f + g*x)**n/(g**4*
n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d*e*f**3*g
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) - 6*c*d*e*f**2*g**2*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g
**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d*e*f**2*g**2*n*x*(f + g*x)**n/(g**4
*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c*d*e*f*g**3
*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*
n + 24*g**4) + 15*c*d*e*f*g**3*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*
n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*c*d*e*f*g**3*n*x**2*(f + g*
x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c
*d*e*g**4*n**3*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 +
 50*g**4*n + 24*g**4) + 21*c*d*e*g**4*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 1
0*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 42*c*d*e*g**4*n*x**3*(f
 + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4)
+ 24*c*d*e*g**4*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2
+ 50*g**4*n + 24*g**4) - 6*c*e**2*f**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e**2*f**3*g*n*x*(f + g*x)**n
/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 3*c*e**2
*f**2*g**2*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2
+ 50*g**4*n + 24*g**4) - 3*c*e**2*f**2*g**2*n*x**2*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*f*g**3*n**3*x
**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 3*c*e**2*f*g**3*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 +
35*g**4*n**2 + 50*g**4*n + 24*g**4) + 2*c*e**2*f*g**3*n*x**3*(f + g*x)**n/(
g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*g**
4*n**3*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4
*n + 24*g**4) + 6*c*e**2*g**4*n**2*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n
**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 11*c*e**2*g**4*n*x**4*(f + g*x)
**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e
**2*g**4*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g*
*4*n + 24*g**4), True))
```

3.808 $\int (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=84

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

```
[Out] (a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^3/(1+n)-2*c*(-d*g+e*f)*(g*x+f)^(2+
n)/g^3/(2+n)+c*e*(g*x+f)^(3+n)/g^3/(3+n)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f -
 d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3 +
n))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx = \int \left(\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^2}+\frac{2c(-ef+dg)(f+gx)^{1+n}}{g^2}+\frac{ce(f+gx)^{2+n}}{g^2}\right) dx$$
$$=\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^3(1+n)}-\frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)}+\frac{ce(f+gx)^{3+n}}{g^3(3+n)}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 0.87

$$\frac{(f+gx)^{n+1}\left(\frac{ag^2+cf(ef-2dg)}{n+1}-\frac{2c(f+gx)(ef-dg)}{n+2}+\frac{ce(f+gx)^2}{n+3}\right)}{g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*((a*g^2 + c*f*(e*f - 2*d*g))/(1 + n) - (2*c*(e*f - d*g)*
(f + g*x))/(2 + n) + (c*e*(f + g*x)^2)/(3 + n)))/g^3
```

fricas [B] time = 1.08, size = 218, normalized size = 2.60

$$\frac{\left(afg^2n^2+2cef^3-6cdf^2g+6afg^2+\left(ceg^3n^2+3ceg^3n+2ceg^3\right)x^3+\left(6cdg^3+\left(cefg^2+2cdg^3\right)n^2+\left(cefg^2+\right.\right.\right.}{g^3n^3+6g^3n^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*f*g^2*n^2 + 2*c*e*f^3 - 6*c*d*f^2*g + 6*a*f*g^2 + (c*e*g^3*n^2 + 3*c*e*g
^3*n + 2*c*e*g^3)*x^3 + (6*c*d*g^3 + (c*e*f*g^2 + 2*c*d*g^3)*n^2 + (c*e*f*g
^2 + 8*c*d*g^3)*n)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d*f*
g^2 + a*g^3)*n^2 - (2*c*e*f^2*g - 6*c*d*f*g^2 - 5*a*g^3)*n)*x)*(g*x + f)^n/
(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)
```

giac [B] time = 0.19, size = 373, normalized size = 4.44

$$\frac{(gx+f)^n cg^3n^2x^3e + 2(gx+f)^n cdg^3n^2x^2 + (gx+f)^n cfg^2n^2x^2e + 3(gx+f)^n cg^3nx^3e + 2(gx+f)^n cdfg^2n^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^3*n^2*x^3*e + 2*(g*x + f)^n*c*d*g^3*n^2*x^2 + (g*x + f)^n*
c*f*g^2*n^2*x^2*e + 3*(g*x + f)^n*c*g^3*n*x^3*e + 2*(g*x + f)^n*c*d*f*g^2*n
^2*x + 8*(g*x + f)^n*c*d*g^3*n*x^2 + (g*x + f)^n*c*f*g^2*n*x^2*e + 2*(g*x +
 f)^n*c*g^3*x^3*e + 6*(g*x + f)^n*c*d*f*g^2*n*x + (g*x + f)^n*a*g^3*n^2*x +
 6*(g*x + f)^n*c*d*g^3*x^2 - 2*(g*x + f)^n*c*f^2*g*n*x*e - 2*(g*x + f)^n*c*
d*f^2*g*n + (g*x + f)^n*a*f*g^2*n^2 + 5*(g*x + f)^n*a*g^3*n*x - 6*(g*x + f)
^n*c*d*f^2*g + 5*(g*x + f)^n*a*f*g^2*n + 6*(g*x + f)^n*a*g^3*x + 2*(g*x + f
)^n*c*f^3*e + 6*(g*x + f)^n*a*f*g^2)/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^
3)
```

maple [A] time = 0.00, size = 147, normalized size = 1.75

$$\frac{\left(ce\,g^2n^2x^2 + 2cd\,g^2n^2x + 3ce\,g^2n\,x^2 + 8cd\,g^2nx - 2cefgnx + 2ce\,x^2g^2 + a\,g^2n^2 - 2cdfgn + 6cd\,g^2x - 2cefgx \cdots}{\left(n^3+6n^2+11n+6\right)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*c*e*g^2*n*x^2+8*c*d*g^2*n*
x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*g*n+6*c*d*g^2*x-2*c*e*f*g*x
+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)/g^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 0.47, size = 135, normalized size = 1.61

$$\frac{2\left(g^2(n+1)x^2 + fgnx - f^2\right)\left(gx+f\right)^n cd}{\left(n^2+3n+2\right)g^2} + \frac{\left(\left(n^2+3n+2\right)g^3x^3 + \left(n^2+n\right)fg^2x^2 - 2f^2gnx + 2f^3\right)\left(gx+f\right)^n ce \cdots}{\left(n^3+6n^2+11n+6\right)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d/((n^2 + 3*n + 2)*g^2) +
 ((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x
 + f)^n*c*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g*x + f)^(n + 1)*a/(g*(n + 1)
)
```

mupad [B] time = 3.07, size = 211, normalized size = 2.51

$$\left(f+g\,x\right)^n\left(\frac{f\,\left(2\,c\,e\,f^2-2\,c\,d\,f\,g\,n-6\,c\,d\,f\,g+a\,g^2\,n^2+5\,a\,g^2\,n+6\,a\,g^2\right)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{x\,\left(-2\,c\,e\,f^2\,g\,n+2\,c\,d\,f\,g^2\,n^2+\cdots\right)}{g^3\,\left(n^3+\cdots\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (f + g*x)^n*((f*(6*a*g^2 + a*g^2*n^2 + 2*c*e*f^2 + 5*a*g^2*n - 6*c*d*f*g -
2*c*d*f*g*n))/(g^3*(11*n + 6*n^2 + n^3 + 6)) + (x*(6*a*g^3 + a*g^3*n^2 + 5*
a*g^3*n + 2*c*d*f*g^2*n^2 + 6*c*d*f*g^2*n - 2*c*e*f^2*g*n))/(g^3*(11*n + 6*
n^2 + n^3 + 6)) + (c*e*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (c*x
^2*(n + 1)*(6*d*g + 2*d*g*n + e*f*n))/(g*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [A] time = 2.21, size = 1489, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*g**2/(2*f**2*
g**3 + 4*f*g**4*x + 2*g**5*x**2) - 2*c*d*f*g/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) - 4*c*d*g**2*x/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 2*c*e*
f**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 3*c*e*f**2/(2*
f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x*log(f/g + x)/(2*f**2*g*
*3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) + 2*c*e*g**2*x**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**
5*x**2), Eq(n, -3)), (-a*g**2/(f*g**3 + g**4*x) + 2*c*d*f*g*log(f/g + x)/(f
*g**3 + g**4*x) + 2*c*d*f*g/(f*g**3 + g**4*x) + 2*c*d*g**2*x*log(f/g + x)/(
f*g**3 + g**4*x) - 2*c*e*f**2*log(f/g + x)/(f*g**3 + g**4*x) - 2*c*e*f**2/(
f*g**3 + g**4*x) - 2*c*e*f*g*x*log(f/g + x)/(f*g**3 + g**4*x) + c*e*g**2*x*
*2/(f*g**3 + g**4*x), Eq(n, -2)), (a*log(f/g + x)/g - 2*c*d*f*log(f/g + x)/
g**2 + 2*c*d*x/g + c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 + c*e*x**2/(2*
g), Eq(n, -1)), (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**
3*n + 6*g**3) + a*g**3*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 5*a*g**3*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 6*a*g**3*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3
*n + 6*g**3) - 2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) - 6*c*d*f**2*g*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 2*c*d*f*g**2*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2
+ 11*g**3*n + 6*g**3) + 6*c*d*f*g**2*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n
**2 + 11*g**3*n + 6*g**3) + 2*c*d*g**3*n**2*x**2*(f + g*x)**n/(g**3*n**3 +
6*g**3*n**2 + 11*g**3*n + 6*g**3) + 8*c*d*g**3*n*x**2*(f + g*x)**n/(g**3*n*
*3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*c*d*g**3*x**2*(f + g*x)**n/(g**3
*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*f**3*(f + g*x)**n/(g**3*n
**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) - 2*c*e*f**2*g*n*x*(f + g*x)**n/(g*
*3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n**2*x**2*(f + g*x
)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n*x**2*(f
+ g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*g**3*n**2*x*
*3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 3*c*e*g**3
*n*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e
*g**3*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3), Tru
e))
```

3.809 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx$

Optimal. Leaf size=114

$$\frac{(cd^2-ae)(f+gx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)}-\frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

```
[Out] -c*(-d*g+e*f)*(g*x+f)^(1+n)/e/g^2/(1+n)+c*(g*x+f)^(2+n)/g^2/(2+n)+(c*d^2-a*
e)*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f
)/(1+n)
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {951, 80, 68}

$$\frac{(cd^2-ae)(f+gx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)}-\frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] -((c*(e*f - d*g)*(f + g*x)^(1 + n))/(e*g^2*(1 + n))) + (c*(f + g*x)^(2 + n)
)/(g^2*(2 + n)) + ((c*d^2 - a*e)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(f+gx)^n \left(a+2cdx+cex^2\right)}{d+ex}\,dx = \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\int \frac{(f+gx)^n(-eg(cdf-ag)(2+n)-ceg(ef-dg)(2+n)x)}{d+ex}\,dx}{eg^2(2+n)}$$

$$= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} - \frac{\left(cd^2-ae\right)\int \frac{(f+gx)^n}{d+ex}\,dx}{e}$$

$$= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\left(cd^2-ae\right)(f+gx)^{1+n}\,{}_2F_1\left(1,1+\right.}{e(ef-dg)(1+n)}$$

Mathematica [A] time = 0.15, size = 93, normalized size = 0.82

$$\frac{(f+gx)^{n+1}\left(\frac{\left(cd^2-ae\right){}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{ef-dg}+\frac{c(dg(n+2)-ef+eg(n+1)x)}{g^2(n+2)}\right)}{e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] ((f + g*x)^(1 + n)*((c*(-(e*f) + d*g*(2 + n) + e*g*(1 + n)*x))/(g^2*(2 + n)
) + ((c*d^2 - a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f -
d*g)])/(e*f - d*g)))/(e*(1 + n))
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(ce\,x^2+2cdx+a\right)\left(gx+f\right)^n}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2 + 2\,cdx + a\right)\left(gx + f\right)^n}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f + g\,x\right)^n\,\left(c\,e\,x^2 + 2\,c\,d\,x + a\right)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^n\left(a + 2cdx + cex^2\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d),x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x), x)
```

3.810 $\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=88

$$\frac{c(f+gx)^{n+1}}{eg(n+1)}-\frac{g\left(cd^2-ae\right)(f+gx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

```
[Out] c*(g*x+f)^(1+n)/e/g/(1+n)-(c*d^2-a*e)*g*(g*x+f)^(1+n)*hypergeom([2, 1+n],[2
+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^2/(1+n)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {947, 68}

$$\frac{c(f+gx)^{n+1}}{eg(n+1)}-\frac{g\left(cd^2-ae\right)(f+gx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] (c*(f + g*x)^(1 + n))/(e*g*(1 + n)) - ((c*d^2 - a*e)*g*(f + g*x)^(1 + n)*Hy
pergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)
^2*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^2}\,dx &= \int \left(\frac{c(f+gx)^n}{e}+\frac{\left(-cd^2+ae\right)(f+gx)^n}{e(d+ex)^2}\right)dx\\
&= \frac{c(f+gx)^{1+n}}{eg(1+n)}+\frac{\left(-cd^2+ae\right)\int \frac{(f+gx)^n}{(d+ex)^2}\,dx}{e}\\
&= \frac{c(f+gx)^{1+n}}{eg(1+n)}-\frac{\left(cd^2-ae\right)g(f+gx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.94

$$\frac{(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right)\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\right)}{eg(n+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] ((f + g*x)^(1 + n)*(c*(e*f - d*g)^2 + (-(c*d^2) + a*e)*g^2*Hypergeometric2F
1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*g*(e*f - d*g)^2*(1 + n))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cex^2 + 2\,cdx + a\right)\left(gx + f\right)^n}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2 + 2\,cdx + a\right)\left(gx + f\right)^n}{\left(ex + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(ce\,x^2 + 2cdx + a\right)\left(gx + f\right)^n}{\left(ex + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2 + 2\,cdx + a\right)\left(gx + f\right)^n}{\left(ex + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f + g\,x\right)^n\,\left(c\,e\,x^2 + 2\,c\,d\,x + a\right)}{\left(d + e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x)
```

```
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.811 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=193

$$\frac{(f+gx)^{n+1}\left(aeg^2(1-n)n-c\left(d^2g^2\left(-n^2+n+2\right)-4defg+2e^2f^2\right)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3}-\frac{g(1-n)\left(c\right.}{2e(d}$$

```
[Out] -1/2*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^2-1/2*(c*d^2-a*e)*g*(1-n)
*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)+1/2*(a*e*g^2*(1-n)*n-c*(2*e^2*f^2-4*d
*e*f*g+d^2*g^2*(-n^2+n+2)))*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f
)/(-d*g+e*f))/e/(-d*g+e*f)^3/(1+n)
```

Rubi [A] time = 0.22, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {949, 78, 68}

$$\frac{(f+gx)^{n+1}\left(aeg^2(1-n)n-c\left(d^2g^2\left(-n^2+n+2\right)-4defg+2e^2f^2\right)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3}-\frac{g(1-n)\left(c\right.}{2e(d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

```
[Out] -((a - (c*d^2)/e)*(f + g*x)^(1 + n))/(2*(e*f - d*g)*(d + e*x)^2) - ((c*d^2
- a*e)*g*(1 - n)*(f + g*x)^(1 + n))/(2*e*(e*f - d*g)^2*(d + e*x)) + ((a*e*g
^2*(1 - n)*n - c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2*(2 + n - n^2)))*(f + g*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(2*
e*(e*f - d*g)^3*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx &= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{\int \frac{(f+gx)^n\left(ag(1-n)-\frac{cd(2ef-dg(1+n))}{e}-2c(ef-dg)x\right)}{(d+ex)^2}\,dx}{2(ef-dg)} \\
&= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{\left(cd^2-ae\right)g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)}-\frac{\left(aeg^2(1-n)n-c\right.}{} \\
&= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{\left(cd^2-ae\right)g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)}+\frac{\left(aeg^2(1-n)n-c\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.55

$$
-\frac{(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right)\,{}_2F_1\left(3,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)\right)}{e(n+1)(ef-dg)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

```
[Out] -((((f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + n, 2 + n, (e
*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[3, 1 + n,
 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^3*(1 + n)))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2
*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$
\int \frac{\left(ce\,x^2+2cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f+g\,x\right)}^n\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^n\left(a+2cdx+cex^2\right)}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**3,x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**3, x)
```

3.812 $\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx$

Optimal. Leaf size=197

$$\frac{g(f+gx)^{n+1}\left(aeg^2\left(n^2-3n+2\right)+c\left(d^2g^2\left(-n^2+3n+4\right)-12defg+6e^2f^2\right)\right){}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4}-\frac{g(2$$

```
[Out] -1/3*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^3-1/6*(c*d^2-a*e)*g*(2-n)
*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)^2+1/6*g*(a*e*g^2*(n^2-3*n+2)+c*(6*e^2
*f^2-12*d*e*f*g+d^2*g^2*(-n^2+3*n+4)))*(g*x+f)^(1+n)*hypergeom([2, 1+n],[2+
n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^4/(1+n)
```

Rubi [A] time = 0.23, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {949, 78, 68}

$$\frac{g(f+gx)^{n+1}\left(aeg^2\left(n^2-3n+2\right)+c\left(d^2g^2\left(-n^2+3n+4\right)-12defg+6e^2f^2\right)\right){}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4}-\frac{g(2$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

```
[Out] -((a - (c*d^2)/e)*(f + g*x)^(1 + n))/(3*(e*f - d*g)*(d + e*x)^3) - ((c*d^2
 - a*e)*g*(2 - n)*(f + g*x)^(1 + n))/(6*e*(e*f - d*g)^2*(d + e*x)^2) + (g*(a
*e*g^2*(2 - 3*n + n^2) + c*(6*e^2*f^2 - 12*d*e*f*g + d^2*g^2*(4 + 3*n - n^2
)))*(f + g*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f
 - d*g)])/(6*e*(e*f - d*g)^4*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{\int \frac{(f+gx)^n\left(ag(2-n)-\frac{cd(3ef-dg(1+n))}{e}-3c(ef-dg)x\right)}{(d+ex)^3}dx}{3(ef-dg)}$$
$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{\left(cd^2-ae\right)g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{\left(aeg^2\left(2-3n\right.\right.}{}$$
$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{\left(cd^2-ae\right)g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{g\left(aeg^2\left(2-3\right.\right.}{}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.54

$$\frac{g(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right){}_2F_1\left(4,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)\right)}{e(n+1)(ef-dg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

```
[Out] (g*(f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[2, 1 + n, 2 + n, (e
*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[4, 1 + n,
 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^4*(1 + n))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2
*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(ce\,x^2+2cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cex^2 + 2\,cdx + a\right)\left(gx + f\right)^n}{\left(ex + d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f+g\,x\right)}^n\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{n} \left(a + 2cdx + cex^{2}\right)}{\left(d + ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**4,x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**4, x)
```

3.813 $\int (d+ex)^m (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=231

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}(g(m+n+2)(aeg(m+n+3)-cd(dg(n+1)+ef(m+2)))+c(m+2)(ef-\ldots}{e^2g^2(m+1)(m+n+2)(m+n+3)}$$

```
[Out] -c*(-d*g+e*f)*(2+m)*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e/g^2/(2+m+n)/(3+m+n)+c*(e*
x+d)^(2+m)*(g*x+f)^(1+n)/e/g/(3+m+n)+(c*(-d*g+e*f)*(2+m)*(e*f*(1+m)+d*g*(1+
n))+g*(2+m+n)*(a*e*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n))))*(e*x+d)^(1+m)*(g*x
+f)^n*hypergeom([-n, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^2/g^2/(1+m)/(2+m+n
)/(3+m+n)/((e*(g*x+f)/(-d*g+e*f))^n)
```

Rubi [A] time = 0.26, antiderivative size = 227, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {951, 80, 70, 69}

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(aeg(m+n+3)+\frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right)\ldots}{e^2g(m+1)(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -((c*(e*f - d*g)*(2 + m)*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(e*g^2*(2 + m
 + n)*(3 + m + n))) + (c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e*g*(3 + m +
 n)) + ((a*e*g*(3 + m + n) + (c*(e*f - d*g)*(2 + m)*(e*f*(1 + m) + d*g*(1 +
 n)))/(g*(2 + m + n)) - c*d*(e*f*(2 + m) + d*g*(1 + n)))*(d + e*x)^(1 + m)*
(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g)
)])/(e^2*g*(1 + m)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
```

```
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int (d+ex)^m (f+gx)^n \left(a+2cdx+cex^2\right) dx = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{\int (d+ex)^m (f+gx)^n \left(e(aeg(3+m+n)\right.}{}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)}{eg(3+m+n)}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)}{eg(3+m+n)}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+m}(f+gx)}{eg(3+m+n)}$$

Mathematica [A] time = 0.18, size = 179, normalized size = 0.77

$$\frac{(d+ex)^{m+1}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} \left(e\left(ag^2+cf(ef-2dg)\right) {}_2F_1\left(m+1,-n;m+2;\frac{g(d+ex)}{dg-ef}\right)+c(ef-dg)^2 {}_2F_1\left(m+1,\right.\right.}{e^2g^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -2
 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] - 2*c*(e*f - d*g)^2*Hypergeometr
ic2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(a*g^2 + c*f*
(e*f - 2*d*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x))/(-(e*f) +
d*g)]))/(e^2*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cex^2+2\,cdx+a\right)(ex+d)^m\left(gx+f\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cex^2+2\,cdx+a\right)(ex+d)^m\left(gx+f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(ce\,x^{2}+2cdx+a\right)\left(ex+d\right)^{m}\left(gx+f\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
[Out] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cex^{2}+2\,cdx+a\right)(ex+d)^{m}\left(gx+f\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^n\,{\left(d+e\,x\right)}^m\,\left(c\,e\,x^2+2\,c\,d\,x+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2),x)
[Out] int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.814 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=83

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

```
[Out] c*x/e/g+(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^2/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)*ln
(g*x+f)/g^2/(-d*g+e*f)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {893}

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - ((
c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx &= \int \left(\frac{c}{eg}+\frac{cd^2-bde+ae^2}{e(ef-dg)(d+ex)}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)(f+gx)}\right)dx\\ &= \frac{cx}{eg}+\frac{\left(cd^2-bde+ae^2\right)\log(d+ex)}{e^2(ef-dg)}-\frac{\left(cf^2-bfg+ag^2\right)\log(f+gx)}{g^2(ef-dg)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.02

$$-\frac{\log(d+ex)\left(-ae^2+bde-cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) - ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) -
 ((c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

fricas [A] time = 1.21, size = 99, normalized size = 1.19

$$\frac{\left(cd^2-bde+ae^2\right)g^2\log\left(ex+d\right)+\left(ce^2fg-cdeg^2\right)x-\left(ce^2f^2-be^2fg+ae^2g^2\right)\log\left(gx+f\right)}{e^3fg^2-de^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*g^2*log(e*x + d) + (c*e^2*f*g - c*d*e*g^2)*x - (c*
e^2*f^2 - b*e^2*f*g + a*e^2*g^2)*log(g*x + f))/(e^3*f*g^2 - d*e^2*g^3)
```

giac [A] time = 0.18, size = 88, normalized size = 1.06

$$\frac{cxe^{(-1)}}{g} + \frac{\left(cf^2 - bfg + ag^2\right)\log\left(\left|gx + f\right|\right)}{dg^3 - fg^2e} - \frac{\left(cd^2 - bde + ae^2\right)\log\left(\left|xe + d\right|\right)}{dge^2 - fe^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] c*x*e^(-1)/g + (c*f^2 - b*f*g + a*g^2)*log(abs(g*x + f))/(d*g^3 - f*g^2*e)
- (c*d^2 - b*d*e + a*e^2)*log(abs(x*e + d))/(d*g*e^2 - f*e^3)
```

maple [A] time = 0.01, size = 142, normalized size = 1.71

$$-\frac{a\ln\left(ex+d\right)}{dg-ef}+\frac{a\ln\left(gx+f\right)}{dg-ef}+\frac{bd\ln\left(ex+d\right)}{\left(dg-ef\right)e}-\frac{bf\ln\left(gx+f\right)}{\left(dg-ef\right)g}-\frac{c\,d^2\ln\left(ex+d\right)}{\left(dg-ef\right)e^2}+\frac{c\,f^2\ln\left(gx+f\right)}{\left(dg-ef\right)g^2}+\frac{cx}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/e/g+1/(d*g-e*f)*ln(g*x+f)*a-1/g/(d*g-e*f)*ln(g*x+f)*b*f+1/g^2/(d*g-e*f)
*ln(g*x+f)*c*f^2-1/(d*g-e*f)*ln(e*x+d)*a+1/(d*g-e*f)/e*ln(e*x+d)*b*d-1/(d*g
-e*f)/e^2*ln(e*x+d)*c*d^2
```

maxima [A] time = 0.44, size = 87, normalized size = 1.05

$$\frac{\left(cd^2 - bde + ae^2\right)\log\left(ex + d\right)}{e^3 f - de^2 g} - \frac{\left(cf^2 - bfg + ag^2\right)\log\left(gx + f\right)}{efg^2 - dg^3} + \frac{cx}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(e^3*f - d*e^2*g) - (c*f^2 - b*f*g + a
*g^2)*log(g*x + f)/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

mupad [B] time = 3.42, size = 84, normalized size = 1.01

$$\frac{\ln\left(d+e\,x\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^3\,f-d\,e^2\,g}+\frac{\ln\left(f+g\,x\right)\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^2\,\left(d\,g-e\,f\right)}+\frac{c\,x}{e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(a*e^2 + c*d^2 - b*d*e))/(e^3*f - d*e^2*g) + (log(f + g*x)*(a
*g^2 + c*f^2 - b*f*g))/(g^2*(d*g - e*f)) + (c*x)/(e*g)
```

sympy [B] time = 9.52, size = 420, normalized size = 5.06

$$\frac{cx}{eg}+\frac{\left(ag^2-bfg+cf^2\right)\log\left(x+\frac{adeg^2+ae^2fg-2bdefg+cd^2fg+cdef^2-\frac{d^2eg\left(ag^2-bfg+cf^2\right)}{dg-ef}+\frac{2de^2f\left(ag^2-bfg+cf^2\right)}{dg-ef}-\frac{e^3f^2\left(ag^2-bfg+cf^2\right)}{g\left(dg-ef\right)}}{2ae^2g^2-bdeg^2-be^2fg+cd^2g^2+ce^2f^2}\right)}{g^2\left(dg-ef\right)}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*
b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2)/(d
*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2*(a
*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e*
*2*f*g + c*d**2*g**2 + c*e**2*f**2))/(g**2*(d*g - e*f)) - (a*e**2 - b*d*e +
 c*d**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g + c*d*
e*f**2 + d**2*g**3*(a*e**2 - b*d*e + c*d**2)/(e*(d*g - e*f)) - 2*d*f*g**2*(
a*e**2 - b*d*e + c*d**2)/(d*g - e*f) + e*f**2*g*(a*e**2 - b*d*e + c*d**2)/(
d*g - e*f))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2 + c*e**2
*f**2))/(e**2*(d*g - e*f))
```

3.815 $\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=184

$$\frac{x\left(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^4(ef-dg)}-\frac{\log(f+gx}{g}$$

```
[Out] (b^2*e^2*g^2-2*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x/
e^3/g^3-1/2*c*(-2*b*e*g+c*d*g+c*e*f)*x^2/e^2/g^2+1/3*c^2*x^3/e/g+(a*e^2-b*d
*e+c*d^2)^2*ln(e*x+d)/e^4/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^2*ln(g*x+f)/g^4/(-
d*g+e*f)
```

Rubi [A] time = 0.31, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {893}

$$\frac{x\left(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^4(ef-dg)}-\frac{\log(f+gx}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) + (c
^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d*g)
) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)(f+gx)}\,dx=\int \left(\frac{b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2\left(e^2f^2+defg+d^2g^2\right)}{e^3g^3}-\frac{c(cef+cdg-2beg}{e^2g^2}\right.$$

$$=\frac{\left(b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2\left(e^2f^2+defg+d^2g^2\right)\right)x}{e^3g^3}-\frac{c(cef+cdg-2beg)}{2e^2g^2}$$

Mathematica [A] time = 0.15, size = 177, normalized size = 0.96

$$-\frac{egx(dg-ef)\left(6ceg(2aeg+b(-2dg-2ef+egx))+6b^2e^2g^2+c^2\left(6d^2g^2-3deg(gx-2f)+e^2\left(6f^2-3fgx+\right.\right.\right.}{6e^4g^4(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/6*(e*g*(-(e*f) + d*g)*x*(6*b^2*e^2*g^2 + 6*c*e*g*(2*a*e*g + b*(-2*e*f -
2*d*g + e*g*x)) + c^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*
g*x + 2*g^2*x^2))) - 6*(c*d^2 + e*(-(b*d) + a*e))^2*g^4*Log[d + e*x] + 6*e^
4*(c*f^2 + g*(-(b*f) + a*g))^2*Log[f + g*x])/(e^4*g^4*(e*f - d*g))
```

fricas [A] time = 1.68, size = 313, normalized size = 1.70

$$\frac{6\left(c^2d^4 - 2\,bcd^3e - 2\,abde^3 + a^2e^4 + \left(b^2 + 2\,ac\right)d^2e^2\right)g^4\log\left(ex + d\right) + 2\left(c^2e^4fg^3 - c^2de^3g^4\right)x^3 - 3\left(c^2e^4f^2g^2 - 2\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e
^2)*g^4*log(e*x + d) + 2*(c^2*e^4*f*g^3 - c^2*d*e^3*g^4)*x^3 - 3*(c^2*e^4*f
^2*g^2 - 2*b*c*e^4*f*g^3 - (c^2*d^2*e^2 - 2*b*c*d*e^3)*g^4)*x^2 + 6*(c^2*e^
4*f^3*g - 2*b*c*e^4*f^2*g^2 + (b^2 + 2*a*c)*e^4*f*g^3 - (c^2*d^3*e - 2*b*c*
d^2*e^2 + (b^2 + 2*a*c)*d*e^3)*g^4)*x - 6*(c^2*e^4*f^4 - 2*b*c*e^4*f^3*g -
2*a*b*e^4*f*g^3 + a^2*e^4*g^4 + (b^2 + 2*a*c)*e^4*f^2*g^2)*log(g*x + f))/(e
^5*f*g^4 - d*e^4*g^5)
```

giac [A] time = 0.17, size = 281, normalized size = 1.53

$$\frac{\left(c^2f^4 - 2\,bcf^3g + b^2f^2g^2 + 2\,acf^2g^2 - 2\,abfg^3 + a^2g^4\right)\log\left(\left|gx + f\right|\right)}{dg^5 - fg^4e} - \frac{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,ab\right.}{dge^4 - fe^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^2*f^4 - 2*b*c*f^3*g + b^2*f^2*g^2 + 2*a*c*f^2*g^2 - 2*a*b*f*g^3 + a^2*g^
4)*log(abs(g*x + f))/(d*g^5 - f*g^4*e) - (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e
^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*log(abs(x*e + d))/(d*g*e^4 - f*
e^5) + 1/6*(2*c^2*g^2*x^3*e^2 - 3*c^2*d*g^2*x^2*e + 6*c^2*d^2*g^2*x - 3*c^2
*f*g*x^2*e^2 + 6*b*c*g^2*x^2*e^2 + 6*c^2*d*f*g*x*e - 12*b*c*d*g^2*x*e + 6*c
^2*f^2*x*e^2 - 12*b*c*f*g*x*e^2 + 6*b^2*g^2*x*e^2 + 12*a*c*g^2*x*e^2)*e^(-3
)/g^3
```

maple [B] time = 0.01, size = 444, normalized size = 2.41

$$-\frac{a^2\ln\left(ex + d\right)}{dg - ef} + \frac{a^2\ln\left(gx + f\right)}{dg - ef} + \frac{2abd\ln\left(ex + d\right)}{\left(dg - ef\right)e} - \frac{2abf\ln\left(gx + f\right)}{\left(dg - ef\right)g} - \frac{2acd^2\ln\left(ex + d\right)}{\left(dg - ef\right)e^2} + \frac{2acf^2\ln\left(gx + f\right)}{\left(dg - ef\right)g^2} - \frac{b^2d^2}{\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/3*c^2*x^3/e/g+1/e/g*x^2*b*c-1/2/e^2/g*x^2*c^2*d-1/2/e/g^2*x^2*c^2*f+2/e/g
*a*c*x+1/e/g*b^2*x-2/e^2/g*b*c*d*x-2/e/g^2*b*c*f*x+1/e^3/g*c^2*d^2*x+1/e^2/
g^2*c^2*d*f*x+1/e/g^3*c^2*f^2*x+1/(d*g-e*f)*ln(g*x+f)*a^2-2/g/(d*g-e*f)*ln(
g*x+f)*a*b*f+2/g^2/(d*g-e*f)*ln(g*x+f)*a*c*f^2+1/g^2/(d*g-e*f)*ln(g*x+f)*b^
2*f^2-2/g^3/(d*g-e*f)*ln(g*x+f)*b*c*f^3+1/g^4/(d*g-e*f)*ln(g*x+f)*c^2*f^4-1
/(d*g-e*f)*ln(e*x+d)*a^2+2/e/(d*g-e*f)*ln(e*x+d)*a*b*d-2/e^2/(d*g-e*f)*ln(e
*x+d)*a*c*d^2-1/e^2/(d*g-e*f)*ln(e*x+d)*b^2*d^2+2/e^3/(d*g-e*f)*ln(e*x+d)*b
*c*d^3-1/e^4/(d*g-e*f)*ln(e*x+d)*c^2*d^4
```

maxima [A] time = 0.45, size = 255, normalized size = 1.39

$$\frac{\left(c^2d^4 - 2\,bcd^3e - 2\,abde^3 + a^2e^4 + \left(b^2 + 2\,ac\right)d^2e^2\right)\log\left(ex + d\right)}{e^5f - de^4g} - \frac{\left(c^2f^4 - 2\,bcf^3g - 2\,abfg^3 + a^2g^4 + \left(b^2 + 2\,ac\right)f\right.}{efg^4 - dg^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log
(e*x + d)/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*g^
4 + (b^2 + 2*a*c)*f^2*g^2)*log(g*x + f)/(e*f*g^4 - d*g^5) + 1/6*(2*c^2*e^2*
g^2*x^3 - 3*(c^2*e^2*f*g + (c^2*d*e - 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*e^2*f^2
+ (c^2*d*e - 2*b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e + (b^2 + 2*a*c)*e^2)*g^2
)*x)/(e^3*g^3)
```

mupad [B] time = 3.51, size = 266, normalized size = 1.45

$$x\left(\frac{b^2+2\,a\,c}{e\,g}+\frac{\left(\frac{c^2\,\left(d\,g+e\,f\right)}{e^2\,g^2}-\frac{2\,b\,c}{e\,g}\right)\,\left(d\,g+e\,f\right)}{e\,g}-\frac{c^2\,d\,f}{e^2\,g^2}\right)-x^2\,\left(\frac{c^2\,\left(d\,g+e\,f\right)}{2\,e^2\,g^2}-\frac{b\,c}{e\,g}\right)+\frac{\ln\left(d+e\,x\right)\,\left(e^2\,\left(b^2\,d^2+2\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((f + g*x)*(d + e*x)),x)
```

```
[Out] x*((2*a*c + b^2)/(e*g) + (((c^2*(d*g + e*f))/(e^2*g^2) - (2*b*c)/(e*g))*(d*
g + e*f))/(e*g) - (c^2*d*f)/(e^2*g^2)) - x^2*((c^2*(d*g + e*f))/(2*e^2*g^2)
 - (b*c)/(e*g)) + (log(d + e*x)*(e^2*(b^2*d^2 + 2*a*c*d^2) + a^2*e^4 + c^2*
d^4 - 2*a*b*d*e^3 - 2*b*c*d^3*e))/(e^5*f - d*e^4*g) + (log(f + g*x)*(g^2*(b
^2*f^2 + 2*a*c*f^2) + a^2*g^4 + c^2*f^4 - 2*a*b*f*g^3 - 2*b*c*f^3*g))/(d*g^
5 - e*f*g^4) + (c^2*x^3)/(3*e*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

3.816 $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=531

$$-\frac{x\left(-3ce^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg\left(aeg\left(d^2g\right.\right.\right.}{e^5g}$$

```
[Out] -(b^2*e^3*g^3*(-3*a*e*g+b*d*g+b*e*f)-c^3*(d^4*g^4+d^3*e*f*g^3+d^2*e^2*f^2*g
^2+d*e^3*f^3*g+e^4*f^4)-3*c*e^2*g^2*(a^2*e^2*g^2-2*a*b*e*g*(d*g+e*f)+b^2*(d
^2*g^2+d*e*f*g+e^2*f^2))-3*c^2*e*g*(a*e*g*(d^2*g^2+d*e*f*g+e^2*f^2)-b*(d^3*
g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)))*x/e^5/g^5+1/2*(b^3*e^3*g^3-3*b*c*e^2
*g^2*(-2*a*e*g+b*d*g+b*e*f)-c^3*(d^3*g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)-3
*c^2*e*g*(a*e*g*(d*g+e*f)-b*(d^2*g^2+d*e*f*g+e^2*f^2)))*x^2/e^4/g^4+1/3*c*(
3*b^2*e^2*g^2-3*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x
^3/e^3/g^3-1/4*c^2*(-3*b*e*g+c*d*g+c*e*f)*x^4/e^2/g^2+1/5*c^3*x^5/e/g+(a*e^
2-b*d*e+c*d^2)^3*ln(e*x+d)/e^6/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^3*ln(g*x+f)/g
^6/(-d*g+e*f)
```

Rubi [A] time = 0.99, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {893}

$$-\frac{x\left(-3ce^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg\left(aeg\left(d^2g\right.\right.\right.}{e^5g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -(((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d^
2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b*e
*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(e^2
*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^
3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g
) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(a*e*g*
(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (c*(3*b^
2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g + d^2*
g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^2*g^2) +
(c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6*(e*f - d*
g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)(f+gx)}\,dx = \int \left(\frac{-b^2e^3g^3(bef+bdg-3aeg)+c^3\left(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4\right)+\cdots}{\cdots} \right.$$

$$= -\frac{\left(b^2e^3g^3(bef+bdg-3aeg)-c^3\left(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4\right)-3ce^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.42, size = 476, normalized size = 0.90

$$-\frac{egx\left(-30ce^2g^2(ef-dg)\left(6a^2e^2g^2+6abeg(-2dg-2ef+egx)+b^2\left(6d^2g^2-3deg(gx-2f)+e^2\left(6f^2-3fgx\cdots\right.\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/60*(e*g*x*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e*
g*x)) + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^3*
g^5*x^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^2 +
 15*f*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2 + 6*
a*b*e*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) +
e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g)*(6*d
^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) + b*(-12
*d^4*g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^4*f*(12*
f^3 - 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)))) - 60*(c*d^2 + e*(-(b*d) + a*e
))^3*g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f + g*x])/(
e^6*g^6*(e*f - d*g))
```

fricas [A] time = 9.83, size = 736, normalized size = 1.39

$$\frac{60\left(c^3d^6-3\,bc^2d^5e-3\,a^2bde^5+a^3e^6+3\left(b^2c+ac^2\right)d^4e^2-\left(b^3+6\,abc\right)d^3e^3+3\left(ab^2+a^2c\right)d^2e^4\right)g^6\log\left(ex+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/60*(60*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*
c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*g^6*log
(e*x + d) + 12*(c^3*e^6*f*g^5 - c^3*d*e^5*g^6)*x^5 - 15*(c^3*e^6*f^2*g^4 -
3*b*c^2*e^6*f*g^5 - (c^3*d^2*e^4 - 3*b*c^2*d*e^5)*g^6)*x^4 + 20*(c^3*e^6*f^
3*g^3 - 3*b*c^2*e^6*f^2*g^4 + 3*(b^2*c + a*c^2)*e^6*f*g^5 - (c^3*d^3*e^3 -
3*b*c^2*d^2*e^4 + 3*(b^2*c + a*c^2)*d*e^5)*g^6)*x^3 - 30*(c^3*e^6*f^4*g^2 -
 3*b*c^2*e^6*f^3*g^3 + 3*(b^2*c + a*c^2)*e^6*f^2*g^4 - (b^3 + 6*a*b*c)*e^6*
f*g^5 - (c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 - (b^3 +
 6*a*b*c)*d*e^5)*g^6)*x^2 + 60*(c^3*e^6*f^5*g - 3*b*c^2*e^6*f^4*g^2 + 3*(b^
2*c + a*c^2)*e^6*f^3*g^3 - (b^3 + 6*a*b*c)*e^6*f^2*g^4 + 3*(a*b^2 + a^2*c)*
e^6*f*g^5 - (c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c + a*c^2)*d^3*e^3 - (b^3
 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e^5)*g^6)*x - 60*(c^3*e^6*f^6 - 3
*b*c^2*e^6*f^5*g - 3*a^2*b*e^6*f*g^5 + a^3*e^6*g^6 + 3*(b^2*c + a*c^2)*e^6*
f^4*g^2 - (b^3 + 6*a*b*c)*e^6*f^3*g^3 + 3*(a*b^2 + a^2*c)*e^6*f^2*g^4)*log(
g*x + f))/(e^7*f*g^6 - d*e^6*g^7)
```

giac [A] time = 0.17, size = 907, normalized size = 1.71

$$\frac{\left(c^3f^6-3\,bc^2f^5g+3\,b^2cf^4g^2+3\,ac^2f^4g^2-b^3f^3g^3-6\,abcf^3g^3+3\,ab^2f^2g^4+3\,a^2cf^2g^4-3\,a^2bfg^5+a^3g^6\right)\log\cdots}{dg^7-fg^6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^3*f^6 - 3*b*c^2*f^5*g + 3*b^2*c*f^4*g^2 + 3*a*c^2*f^4*g^2 - b^3*f^3*g^3
- 6*a*b*c*f^3*g^3 + 3*a*b^2*f^2*g^4 + 3*a^2*c*f^2*g^4 - 3*a^2*b*f*g^5 + a^3
*g^6)*log(abs(g*x + f))/(d*g^7 - f*g^6*e) - (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^
2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2
*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*log(abs(x*e + d))/(d*g*e^
6 - f*e^7) + 1/60*(12*c^3*g^4*x^5*e^4 - 15*c^3*d*g^4*x^4*e^3 + 20*c^3*d^2*g
^4*x^3*e^2 - 30*c^3*d^3*g^4*x^2*e + 60*c^3*d^4*g^4*x - 15*c^3*f*g^3*x^4*e^4
 + 45*b*c^2*g^4*x^4*e^4 + 20*c^3*d*f*g^3*x^3*e^3 - 60*b*c^2*d*g^4*x^3*e^3 -
 30*c^3*d^2*f*g^3*x^2*e^2 + 90*b*c^2*d^2*g^4*x^2*e^2 + 60*c^3*d^3*f*g^3*x*e
 - 180*b*c^2*d^3*g^4*x*e + 20*c^3*f^2*g^2*x^3*e^4 - 60*b*c^2*f*g^3*x^3*e^4
+ 60*b^2*c*g^4*x^3*e^4 + 60*a*c^2*g^4*x^3*e^4 - 30*c^3*d*f^2*g^2*x^2*e^3 +
90*b*c^2*d*f*g^3*x^2*e^3 - 90*b^2*c*d*g^4*x^2*e^3 - 90*a*c^2*d*g^4*x^2*e^3
+ 60*c^3*d^2*f^2*g^2*x*e^2 - 180*b*c^2*d^2*f*g^3*x*e^2 + 180*b^2*c*d^2*g^4*
x*e^2 + 180*a*c^2*d^2*g^4*x*e^2 - 30*c^3*f^3*g*x^2*e^4 + 90*b*c^2*f^2*g^2*x
^2*e^4 - 90*b^2*c*f*g^3*x^2*e^4 - 90*a*c^2*f*g^3*x^2*e^4 + 30*b^3*g^4*x^2*e
^4 + 180*a*b*c*g^4*x^2*e^4 + 60*c^3*d*f^3*g*x*e^3 - 180*b*c^2*d*f^2*g^2*x*e
^3 + 180*b^2*c*d*f*g^3*x*e^3 + 180*a*c^2*d*f*g^3*x*e^3 - 60*b^3*d*g^4*x*e^3
 - 360*a*b*c*d*g^4*x*e^3 + 60*c^3*f^4*x*e^4 - 180*b*c^2*f^3*g*x*e^4 + 180*b
^2*c*f^2*g^2*x*e^4 + 180*a*c^2*f^2*g^2*x*e^4 - 60*b^3*f*g^3*x*e^4 - 360*a*b
*c*f*g^3*x*e^4 + 180*a*b^2*g^4*x*e^4 + 180*a^2*c*g^4*x*e^4)*e^(-5)/g^5
```

maple [B] time = 0.02, size = 1232, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/5*c^3*x^5/e/g+1/2/g/e*x^2*b^3-6/g/e^2*a*b*c*d*x-6/g^2/e*a*b*c*f*x+3/g^2/e
^2*a*c^2*d*f*x+3/g^2/e^2*b^2*c*d*f*x-3/g^2/e^3*b*c^2*d^2*f*x-6/g^3/(d*g-e*f
)*ln(g*x+f)*a*b*c*f^3+6/e^3/(d*g-e*f)*ln(e*x+d)*a*b*c*d^3+1/(d*g-e*f)*ln(g*
x+f)*a^3-1/(d*g-e*f)*ln(e*x+d)*a^3-3/g/(d*g-e*f)*ln(g*x+f)*a^2*b*f+3/e/(d*g
-e*f)*ln(e*x+d)*a^2*b*d-3/e^2/(d*g-e*f)*ln(e*x+d)*a^2*c*d^2-3/e^2/(d*g-e*f)
*ln(e*x+d)*a*b^2*d^2-3/e^4/(d*g-e*f)*ln(e*x+d)*a*c^2*d^4-3/e^4/(d*g-e*f)*ln
(e*x+d)*b^2*c*d^4+3/e^5/(d*g-e*f)*ln(e*x+d)*b*c^2*d^5+3/2/g^3/e*x^2*b*c^2*f
^2-1/2/g^2/e^3*x^2*c^3*d^2*f-1/2/g^3/e^2*x^2*c^3*d*f^2+3/g/e^3*a*c^2*d^2*x+
3/g^3/e*a*c^2*f^2*x+3/g/e^3*b^2*c*d^2*x+3/g^3/e*b^2*c*f^2*x+3/g^2/(d*g-e*f)
*ln(g*x+f)*a^2*c*f^2+3/g^2/(d*g-e*f)*ln(g*x+f)*a*b^2*f^2+3/g^4/(d*g-e*f)*ln
(g*x+f)*a*c^2*f^4+3/g^4/(d*g-e*f)*ln(g*x+f)*b^2*c*f^4-3/g^5/(d*g-e*f)*ln(g*
x+f)*b*c^2*f^5-1/g/e^2*x^3*b*c^2*d-1/g^2/e*x^3*b*c^2*f+1/3/g^2/e^2*x^3*c^3*
d*f+1/g/e*x^3*a*c^2+1/g^5/e*c^3*f^4*x-3/g/e^4*b*c^2*d^3*x-3/g^4/e*b*c^2*f^3
*x+1/g^2/e^4*c^3*d^3*f*x+1/g^3/e^3*c^3*d^2*f^2*x+1/g^4/e^2*c^3*d*f^3*x+1/g/
e^5*c^3*d^4*x+3/4/g/e*x^4*b*c^2-1/4/g/e^2*x^4*c^3*d-1/4/g^2/e*x^4*c^3*f+1/3
/g/e^3*x^3*c^3*d^2+1/3/g^3/e*x^3*c^3*f^2-1/2/g/e^4*x^2*c^3*d^3-1/2/g^4/e*x^
2*c^3*f^3+3/g/e*a^2*c*x+3/g/e*a*b^2*x-1/g/e^2*b^3*d*x-1/g^2/e*b^3*f*x+1/g/e
*x^3*b^2*c-1/g^3/(d*g-e*f)*ln(g*x+f)*b^3*f^3+1/g^6/(d*g-e*f)*ln(g*x+f)*c^3*
f^6+1/e^3/(d*g-e*f)*ln(e*x+d)*b^3*d^3-1/e^6/(d*g-e*f)*ln(e*x+d)*c^3*d^6+3/g
/e*x^2*a*b*c-3/2/g/e^2*x^2*a*c^2*d-3/2/g^2/e*x^2*a*c^2*f-3/2/g/e^2*x^2*b^2*
c*d-3/2/g^2/e*x^2*b^2*c*f+3/2/g/e^3*x^2*b*c^2*d^2-3/g^3/e^2*b*c^2*d*f^2*x+3
/2/g^2/e^2*x^2*b*c^2*d*f
```

maxima [A] time = 0.49, size = 721, normalized size = 1.36

$$\frac{\left(c^3d^6 - 3\,bc^2d^5e - 3\,a^2bde^5 + a^3e^6 + 3\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 + 3\left(ab^2 + a^2c\right)d^2e^4\right)\log\left(ex + d\right)}{e^7f - de^6g} - \frac{\left(c^3f^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*
e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e*x + d)/(e^
7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^3*g^6 + 3*(b^
2*c + a*c^2)*f^4*g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^2*c)*f^2*g^4)
*log(g*x + f)/(e*f*g^6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*(c^3*e^4*f*
g^3 + (c^3*d*e^3 - 3*b*c^2*e^4)*g^4)*x^4 + 20*(c^3*e^4*f^2*g^2 + (c^3*d*e^3
 - 3*b*c^2*e^4)*f*g^3 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^
4)*g^4)*x^3 - 30*(c^3*e^4*f^3*g + (c^3*d*e^3 - 3*b*c^2*e^4)*f^2*g^2 + (c^3*
d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^4)*f*g^3 + (c^3*d^3*e - 3*b*c
^2*d^2*e^2 + 3*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*g^4)*x^2 + 60*(
c^3*e^4*f^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*f^3*g + (c^3*d^2*e^2 - 3*b*c^2*d*e^
3 + 3*(b^2*c + a*c^2)*e^4)*f^2*g^2 + (c^3*d^3*e - 3*b*c^2*d^2*e^2 + 3*(b^2*
c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*f*g^3 + (c^3*d^4 - 3*b*c^2*d^3*e +
3*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + 3*(a*b^2 + a^2*c)*e^4)*
g^4)*x)/(e^5*g^5)
```

mupad [B]   time = 4.20, size = 794, normalized size = 1.50

$$x^4\left(\frac{3\,b\,c^2}{4\,e\,g}-\frac{c^3\,\left(d\,g+e\,f\right)}{4\,e^2\,g^2}\right)-x^3\left(\frac{\left(d\,g+e\,f\right)\,\left(\frac{3\,b\,c^2}{e\,g}-\frac{c^3\,\left(d\,g+e\,f\right)}{e^2\,g^2}\right)}{3\,e\,g}-\frac{c\,\left(b^2+a\,c\right)}{e\,g}+\frac{c^3\,d\,f}{3\,e^2\,g^2}\right)+x^2\left(\frac{b^3+6\,a\,c\,b}{2\,e\,g}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((f + g*x)*(d + e*x)),x)
```

```
[Out] x^4*((3*b*c^2)/(4*e*g) - (c^3*(d*g + e*f))/(4*e^2*g^2)) - x^3*(((d*g + e*f)
*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(3*e*g) - (c*(a*c + b^2))
/(e*g) + (c^3*d*f)/(3*e^2*g^2)) + x^2*((b^3 + 6*a*b*c)/(2*e*g) + ((d*g + e*
f)*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (
3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(2*e*g) - (d*f*((3*b*c^2)/(e
*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(2*e*g)) + x*((3*a*(a*c + b^2))/(e*g) -
 ((d*g + e*f)*((b^3 + 6*a*b*c)/(e*g) + ((d*g + e*f)*(((d*g + e*f)*((3*b*c^2
)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (
c^3*d*f)/(e^2*g^2)))/(e*g) - (d*f*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2
*g^2)))/(e*g)))/(e*g) + (d*f*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e
*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(e
*g)) + (log(d + e*x)*(e^4*(3*a*b^2*d^2 + 3*a^2*c*d^2) + e^2*(3*a*c^2*d^4 +
3*b^2*c*d^4) - e^3*(b^3*d^3 + 6*a*b*c*d^3) + a^3*e^6 + c^3*d^6 - 3*a^2*b*d*
e^5 - 3*b*c^2*d^5*e))/(e^7*f - d*e^6*g) + (log(f + g*x)*(g^4*(3*a*b^2*f^2 +
 3*a^2*c*f^2) + g^2*(3*a*c^2*f^4 + 3*b^2*c*f^4) - g^3*(b^3*f^3 + 6*a*b*c*f^
3) + a^3*g^6 + c^3*f^6 - 3*a^2*b*f*g^5 - 3*b*c^2*f^5*g))/(d*g^7 - e*f*g^6)
+ (c^3*x^5)/(5*e*g)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

3.817 $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx$

Optimal. Leaf size=246

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2}{(ef-dg)}$$

```
[Out] e^2*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)-g^2*ln(g*x+f)/(-d*g+e*f)/(a*g^
2-b*f*g+c*f^2)-1/2*(-b*e*g+c*d*g+c*e*f)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)
/(c*f^2-g*(-a*g+b*f))-(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*arctanh((
2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(c*f^2-g*(-a*g+b*f))/(-4*a
*c+b^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {893, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2}{(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)) -
(g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*d*g
- b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f -
 a*g)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \int\left(-\frac{e^3}{\left(cd^2-bde+ae^2\right)(-ef+dg)(d+ex)}-\frac{g^3}{(ef-dg)\left(cf^2-bfg+ag^2\right)\right.$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)}-\frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)}+\frac{\int\frac{c^2df+b^2eg-}{}}{\left(cd^2-bd\right.}$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)}-\frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)}+\frac{(-cef-}{2\left(cd^2-bd\right.}$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)}-\frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)}-\frac{(cef+cdg}{2\left(cd^2-bd\right.}$$

$$= -\frac{\left(2c^2df+b^2eg-c(bef+bdg+2aeg)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2\log}{\left(cd^2-bde+\right.}$$

Mathematica [A] time = 0.32, size = 246, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)\left(g(ag-bf)+cf^2\right)}+\frac{e^2\log(d+ex)}{(ef-dg)\left(e(ae-bd)+cd^2\right)}-\frac{\log(a+x(b+cx))(-beg}{2\left(e(ae-bd)+cd^2\right)\left(g(ag\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] ((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(c*f^2 + g
*(-(b*f) + a*g))) + (e^2*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))*(e*f - d
*g)) - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) - ((c*e*
f + c*d*g - b*e*g)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))*(c*f
^2 + g*(-(b*f) + a*g)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 392, normalized size = 1.59

$$\frac{g^3\log\left(\left|gx+f\right|\right)}{cdf^2g^2-bdfg^3+adg^4-cf^3ge+bf^2g^2e-afg^3e}-\frac{\left(cdg+cfe-bge\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2f^2-bcd^2fg+acd^2g^2-bcdf^2e+b^2dfge-abdg^2e+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] g^3*log(abs(g*x + f))/(c*d*f^2*g^2 - b*d*f*g^3 + a*d*g^4 - c*f^3*g*e + b*f^
2*g^2*e - a*f*g^3*e) - 1/2*(c*d*g + c*f*e - b*g*e)*log(c*x^2 + b*x + a)/(c^
2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b^2*d*f*g*e - a*b*d*g
^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2) - e^3*log(abs(x*e + d))/(c*
d^3*g*e - c*d^2*f*e^2 - b*d^2*g*e^2 + b*d*f*e^3 + a*d*g*e^3 - a*f*e^4) + (2
*c^2*d*f - b*c*d*g - b*c*f*e + b^2*g*e - 2*a*c*g*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b
^2*d*f*g*e - a*b*d*g^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2)*sqrt(-b
^2 + 4*a*c))
```

maple [B] time = 0.01, size = 606, normalized size = 2.46

$$-\frac{2aceg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)\left(a\,g^2-bfg+c\,f^2\right)\sqrt{4ac-b^2}}+\frac{b^2eg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)\left(a\,g^2-bfg+c\,f^2\right)\sqrt{4ac-b^2}}-\frac{}{\left(a\,e^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*ln(c*x^2+b*x+a)*g*e*b-1/2/(a*e^
2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*c*ln(c*x^2+b*x+a)*g*d-1/2/(a*e^2-b*d*e+c
*d^2)/(a*g^2-b*f*g+c*f^2)*c*ln(c*x^2+b*x+a)*f*e-2/(a*e^2-b*d*e+c*d^2)/(a*g^
2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*e*
g+1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*b^2*e*g-1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*g-1/(a*e^2-b*d*e+
c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b*c*e*f+2/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d*f+g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2
)*ln(g*x+f)-e^2/(a*e^2-b*d*e+c*d^2)/(d*g-e*f)*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 19.25, size = 12173, normalized size = 49.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 - a^3*b^2*e^5*
g^5*(b^2 - 4*a*c)^(1/2) - c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) - c^5*d^5*f^3
*g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 1
8*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 + 4*a^4*c*e
^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*
```

a*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^2*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 + a^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g^5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*b^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f^3*g^2*x - 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b*c^3*e^5*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d^5*f*g^4*(b^2 - 4*a*c)^(1/2) + 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c^2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c*d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e^5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*f^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5*g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f^5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*x + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x - 6*b^5*c*e^5*f^3*g^2*x + 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) + a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) - a^2*b^3*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^3*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21*a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^3 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4*g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3 - b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3*x + 3*a*b*c^4*d*e^4*f^5 + 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^4*d^5*f*g^4 + 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4*f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x + a*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5*f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e^2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c^4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^4*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g^5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d^3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x + 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f^3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x + 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) - 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 24*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) + b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 9*a^2*c^3*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3*d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2*g^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*b^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f^2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17*b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*e^3*f^2*g^3*x - 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) - 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*c*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b^5*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2

*c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10
*a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4
 + 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^3*e^2*f^3*g^2
*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) - b^3*c^
2*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^3*d^4*e*g^5*x - 14*a*b^4
*c*d^2*e^3*g^5*x - 5*a^2*b^3*c*d*e^4*g^5*x + 4*a^3*b*c^2*d*e^4*g^5*x - 26*a
*b^2*c^3*e^5*f^4*g*x - 14*a*b^4*c*e^5*f^2*g^3*x - 5*a^2*b^3*c*e^5*f*g^4*x +
 4*a^3*b*c^2*e^5*f*g^4*x - 6*a*c^5*d^2*e^3*f^4*g*x - 6*a*c^5*d^4*e*f^2*g^3*
x + 12*a^3*c^3*d*e^4*f*g^4*x + 3*b*c^5*d^3*e^2*f^4*g*x + 3*b*c^5*d^4*e*f^3*
g^2*x - 12*b^3*c^3*d*e^4*f^4*g*x - 12*b^3*c^3*d^4*e*f*g^4*x + 8*b^5*c*d*e^4
*f^2*g^3*x + 8*b^5*c*d^2*e^3*f*g^4*x + 6*a*b^2*c^2*d^4*e*g^5*(b^2 - 4*a*c)^
(1/2) - 6*a*b^3*c*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*e^5*f^4*g*(
b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^
2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2)
 - 6*a^3*c^2*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + b*c^4*d^3*e^2*f^4*g*(b^2 - 4
*a*c)^(1/2) + b*c^4*d^4*e*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e^4*g^5
x(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) - 6*b^4*
c*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e^5*f*g^4*x*(b^2 - 4*a*c)^(
1/2) + 6*b^3*c^2*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 6*b^4*c*e^5*f^3*g^2*x*(b
^2 - 4*a*c)^(1/2) + 5*c^5*d^3*e^2*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^4*e
*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 16*a*b*c^4*d^3*e^2*f^3*g^2 + 2*a*b^3*c^2*d
*e^4*f^3*g^2 + 2*a*b^3*c^2*d^3*e^2*f*g^4 - 5*a^2*b*c^3*d*e^4*f^3*g^2 - 5*a^
2*b*c^3*d^3*e^2*f*g^4 + 15*b^2*c^3*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) +
15*b^2*c^3*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 25*b^3*c^2*d^2*e^3*f^2*g
^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*
a*b^3*c*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c*d*e^4*f*g^4*(b^2 -
 4*a*c)^(1/2) - 10*a*b^3*c*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*
d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1
/2) - 3*a^2*b^2*c*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^2*e^3*f^4*g*x
*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^4*e*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 12*b^2
*c^3*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 12*b^2*c^3*d^4*e*f*g^4*x*(b^2 - 4*
a*c)^(1/2) + 8*b^4*c*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 8*b^4*c*d^2*e^3*
f*g^4*x*(b^2 - 4*a*c)^(1/2) - 60*a*b*c^4*d^2*e^3*f^3*g^2*x - 60*a*b*c^4*d^3
*e^2*f^2*g^3*x - 18*a*b^2*c^3*d*e^4*f^3*g^2*x - 18*a*b^2*c^3*d^3*e^2*f*g^4*
x - 38*a*b^3*c^2*d*e^4*f^2*g^3*x - 38*a*b^3*c^2*d^2*e^3*f*g^4*x + 27*a^2*b*
c^3*d*e^4*f^2*g^3*x + 27*a^2*b*c^3*d^2*e^3*f*g^4*x - 36*a^2*b^2*c^2*d*e^4*f
*g^4*x + 20*a*b*c^3*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^3*d^3*e^
2*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/
2) + 6*a*b^2*c^2*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d*e^4*f^2
g^3(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) +
 20*a*b^2*c^2*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d^2*e^3*g^5*
x*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 13
*a^2*b*c^2*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 41*a*b*c^4*d*e^4*f^4*g*x + 4
1*a*b*c^4*d^4*e*f*g^4*x + 28*a*b^4*c*d*e^4*f*g^4*x - 20*a*c^4*d^2*e^3*f^3*g
^2*x*(b^2 - 4*a*c)^(1/2) - 20*a*c^4*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) +
 a^2*c^3*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + a^2*c^3*d^2*e^3*f*g^4*x*(b^2
 - 4*a*c)^(1/2) - 20*b*c^4*d^3*e^2*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 4*b^3*c^
2*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 4*b^3*c^2*d^3*e^2*f*g^4*x*(b^2 - 4*
a*c)^(1/2) + 114*a*b^2*c^3*d^2*e^3*f^2*g^3*x - 14*a*b*c^3*d*e^4*f^4*g*(b^2
 - 4*a*c)^(1/2) - 14*a*b*c^3*d^4*e*f*g^4*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*d^
4*e*g^5*x*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2)
 + 13*a*c^4*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 13*a*c^4*d^4*e*f*g^4*x*(b^2
 - 4*a*c)^(1/2) - 27*a*b^2*c^2*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 60*a*b*
c^3*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^2*d*e^4*f^2*g^3*x*(b
^2 - 4*a*c)^(1/2) - 26*a*b^2*c^2*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 18*a
*b^3*c*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 6*a*b*c^3*d*e^4*f^3*g^2*x*(b^2 -
 4*a*c)^(1/2) - 6*a*b*c^3*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2
*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2))*(b^2*c*d*g - 4*a*c^2*d*g - 4*a*c^2*e*f
 - b^3*e*g + b^2*c*e*f - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*e*g*(b^2 - 4*a*

c)^(1/2) + 4*a*b*c*e*g + 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a
*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2
*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^
2*f^2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b
^2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*
c*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g)) - (
log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 + a^3*b^2*e^5*g
^5*(b^2 - 4*a*c)^(1/2) + c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) + c^5*d^5*f^3*
g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 18
*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 - 4*a^4*c*e^
5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*a
*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^2
*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 + a
^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g^
5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*b
^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f^
3*g^2*x + 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b*c^3*e^5*f^5*(b^2 -
4*a*c)^(1/2) - 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) - 7*a*c^4*d^5*f*g^4*(b
^2 - 4*a*c)^(1/2) - 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^5*g^5
x(b^2 - 4*a*c)^(1/2) - 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c^
2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c*
d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e^
5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*f
^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5*
g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f^
5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*x
 + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x -
 6*b^5*c*e^5*f^3*g^2*x - 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) - a^2*b^3*
d*e^4*g^5*(b^2 - 4*a*c)^(1/2) + b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) + 7*a
^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(1
/2) - a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) + b*c^4*d^5*f^2*g^3*(b^2 - 4*a*
c)^(1/2) + 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) + a^2*b^3*e^5*g^5*x*(b^2
 - 4*a*c)^(1/2) + 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c^3*e^5*f
^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*
d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2)
 + 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21*
a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^3
 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4*
g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3 -
 b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3*
x + 3*a*b*c^4*d*e^4*f^5 - 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*b
*c^4*d^5*f*g^4 - 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4*
f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x + a
*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5*
f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e^
2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c^
4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^4
*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g^
5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d^
3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x +
 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f^
3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x - 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*c
)^(1/2) + 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*e^5*f^
3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 2
4*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*c^3*d*e^4*f^3*g^2*(b^2
 - 4*a*c)^(1/2) + 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^2*
e^3*f^4*g*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) -
 b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 9*a^2*c^3*d^3*e^2*g^5*x*(b^2 -
 4*a*c)^(1/2) + 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3*

d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2*g
^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*b
^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f^
2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17*
b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*e
^3*f^2*g^3*x + 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*c*e^5*f*g^
4*(b^2 - 4*a*c)^(1/2) + 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c
*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) - a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*
b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) - a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/
2) - 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b^5*d*e^4*f*g^4*x*(b^2 - 4
*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2*
c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10*
a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4
- 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^3*e^2*f^3*g^2*
(b^2 - 4*a*c)^(1/2) + b^3*c^2*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2
*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^3*d^4*e*g^5*x - 14*a*b^4*
c*d^2*e^3*g^5*x - 5*a^2*b^3*c*d*e^4*g^5*x + 4*a^3*b*c^2*d*e^4*g^5*x - 26*a*
b^2*c^3*e^5*f^4*g*x - 14*a*b^4*c*e^5*f^2*g^3*x - 5*a^2*b^3*c*e^5*f*g^4*x +
4*a^3*b*c^2*e^5*f*g^4*x - 6*a*c^5*d^2*e^3*f^4*g*x - 6*a*c^5*d^4*e*f^2*g^3*x
 + 12*a^3*c^3*d*e^4*f*g^4*x + 3*b*c^5*d^3*e^2*f^4*g*x + 3*b*c^5*d^4*e*f^3*g
^2*x - 12*b^3*c^3*d*e^4*f^4*g*x - 12*b^3*c^3*d^4*e*f*g^4*x + 8*b^5*c*d*e^4*
f^2*g^3*x + 8*b^5*c*d^2*e^3*f*g^4*x - 6*a*b^2*c^2*d^4*e*g^5*(b^2 - 4*a*c)^(
1/2) + 6*a*b^3*c*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*e^5*f^4*g*(b
^2 - 4*a*c)^(1/2) + 6*a*b^3*c*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^2
*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2)
+ 6*a^3*c^2*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^3*e^2*f^4*g*(b^2 - 4*
a*c)^(1/2) - b*c^4*d^4*e*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e^4*g^5*
x*(b^2 - 4*a*c)^(1/2) - 6*b^3*c^2*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) + 6*b^4*c
*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*e^5*f*g^4*x*(b^2 - 4*a*c)^(1
/2) - 6*b^3*c^2*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 6*b^4*c*e^5*f^3*g^2*x*(b^
2 - 4*a*c)^(1/2) - 5*c^5*d^3*e^2*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^4*e*
f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 16*a*b*c^4*d^3*e^2*f^3*g^2 + 2*a*b^3*c^2*d*
e^4*f^3*g^2 + 2*a*b^3*c^2*d^3*e^2*f*g^4 - 5*a^2*b*c^3*d*e^4*f^3*g^2 - 5*a^2
*b*c^3*d^3*e^2*f*g^4 - 15*b^2*c^3*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 1
5*b^2*c^3*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 25*b^3*c^2*d^2*e^3*f^2*g^
3*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a
*b^3*c*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) - 17*a^2*b^2*c*d*e^4*f*g^4*(b^2 -
4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*b^2*c*d
*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/
2) + 3*a^2*b^2*c*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^2*e^3*f^4*g*x*
(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^4*e*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 12*b^2*
c^3*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 12*b^2*c^3*d^4*e*f*g^4*x*(b^2 - 4*a
*c)^(1/2) - 8*b^4*c*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 8*b^4*c*d^2*e^3*f
*g^4*x*(b^2 - 4*a*c)^(1/2) - 60*a*b*c^4*d^2*e^3*f^3*g^2*x - 60*a*b*c^4*d^3*
e^2*f^2*g^3*x - 18*a*b^2*c^3*d*e^4*f^3*g^2*x - 18*a*b^2*c^3*d^3*e^2*f*g^4*x
 - 38*a*b^3*c^2*d*e^4*f^2*g^3*x - 38*a*b^3*c^2*d^2*e^3*f*g^4*x + 27*a^2*b*c
^3*d*e^4*f^2*g^3*x + 27*a^2*b*c^3*d^2*e^3*f*g^4*x - 36*a^2*b^2*c^2*d*e^4*f*
g^4*x - 20*a*b*c^3*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^3*d^3*e^2
*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2
) - 6*a*b^2*c^2*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d*e^4*f^2*
g^3*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) -
20*a*b^2*c^2*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d^2*e^3*g^5*x
*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 13*
a^2*b*c^2*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 41*a*b*c^4*d*e^4*f^4*g*x + 41
*a*b*c^4*d^4*e*f*g^4*x + 28*a*b^4*c*d*e^4*f*g^4*x + 20*a*c^4*d^2*e^3*f^3*g^
2*x*(b^2 - 4*a*c)^(1/2) + 20*a*c^4*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) -
a^2*c^3*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - a^2*c^3*d^2*e^3*f*g^4*x*(b^2
- 4*a*c)^(1/2) + 20*b*c^4*d^3*e^2*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 4*b^3*c^2
*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 4*b^3*c^2*d^3*e^2*f*g^4*x*(b^2 - 4*a

```
*c)^(1/2) + 114*a*b^2*c^3*d^2*e^3*f^2*g^3*x + 14*a*b*c^3*d*e^4*f^4*g*(b^2 -
 4*a*c)^(1/2) + 14*a*b*c^3*d^4*e*f*g^4*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*d^4
*e*g^5*x*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) -
 13*a*c^4*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 13*a*c^4*d^4*e*f*g^4*x*(b^2 -
 4*a*c)^(1/2) + 27*a*b^2*c^2*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 60*a*b*c
^3*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 26*a*b^2*c^2*d*e^4*f^2*g^3*x*(b^
2 - 4*a*c)^(1/2) + 26*a*b^2*c^2*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 18*a*
b^3*c*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^3*d*e^4*f^3*g^2*x*(b^2 -
4*a*c)^(1/2) + 6*a*b*c^3*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*b*c^2*
d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2))*(b^3*e*g + 4*a*c^2*d*g + 4*a*c^2*e*f - b
^2*c*d*g - b^2*c*e*f - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*e*g*(b^2 - 4*a*c
)^(1/2) - 4*a*b*c*e*g + 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a*
c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*
g^2 - a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2
*f^2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^
2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c
*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g)) + (e
^2*log(d + e*x))/(a*e^3*f - c*d^3*g - a*d*e^2*g - b*d*e^2*f + b*d^2*e*g + c
*d^2*e*f) + (g^2*log(f + g*x))/(a*d*g^3 - c*e*f^3 - a*e*f*g^2 - b*d*f*g^2 +
 b*e*f^2*g + c*d*f^2*g)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.818 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=644

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-\right.}$$

```
[Out] (-b^3*e*g+b^2*c*(d*g+e*f)-2*a*c^2*(d*g+e*f)-b*c*(-3*a*e*g+c*d*f)-c*(2*c^2*d
*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
f^2-g*(-a*g+b*f))/(c*x^2+b*x+a)+2*c*(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e
*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+
c*d^2)/(c*f^2-g*(-a*g+b*f))+e^4*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)-
g^4*ln(g*x+f)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2-1/2*(-b*e*g+c*d*g+c*e*f)*(c*
(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c
*d^2)^2/(c*f^2-g*(-a*g+b*f))^2+(b^2*e^2*g^2*(-2*a*e*g+b*d*g+b*e*f)-2*c^3*d*
f*(d^2*g^2+d*e*f*g+e^2*f^2)+2*c*e*g*(a^2*e^2*g^2+a*b*e*g*(d*g+e*f)-b^2*(d*g
+e*f)^2)-c^2*(4*a*d*e^2*f*g^2-b*(d^3*g^3+5*d^2*e*f*g^2+5*d*e^2*f^2*g+e^3*f^
3)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^2/(c*f^2-g*(
-a*g+b*f))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 2.05, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {893, 638, 618, 206, 634, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*g
) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (2*c
*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e^2*f^2
 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) - b^2*(e
*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g + 5*d^2*e*
f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Log[d + e*x]
)/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)
*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^
2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e
 + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \int\left(-\frac{e^5}{\left(cd^2-bde+ae^2\right)^2(-ef+dg)(d+ex)} - \frac{g^5}{(ef-dg)\left(cf^2-bfg+ag\right.}\right.$$

$$= \frac{e^4\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2(ef-dg)} - \frac{g^4\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)^2} + \frac{\int \frac{-b^2e^2g^2}{}}{}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+\right.}$$

Mathematica [A] time = 2.63, size = 710, normalized size = 1.10

$$\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2c^3\left(2a^2eg\left(d^2g^2-5defg+e^2f^2\right)+ab\left(3d^3g^3+11d^2efg^2+11de^2f^2g+3e^3f^3\right)-4b^2d^2ef^2g\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(f -
 g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*(a + x*(b + c*x))) + ((4*c^5
*d^3*f^3 + b^4*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*b^2*c*e*g*(-6*a^2*e^2*
g^2 + 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) + 2*c^4*d*
f*(-3*b*d*f*(e*f + d*g) + 2*a*(3*e^2*f^2 + d*e*f*g + 3*d^2*g^2)) + c^2*(-12
*a^3*e^3*g^3 - 6*a^2*b*e^2*g^2*(e*f + d*g) + 12*a*b^2*e*g*(e^2*f^2 + d*e*f*
g + d^2*g^2) + b^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3)) - 2*c^3
*(-4*b^2*d^2*e*f^2*g + 2*a^2*e*g*(e^2*f^2 - 5*d*e*f*g + d^2*g^2) + a*b*(3*e
^3*f^3 + 11*d*e^2*f^2*g + 11*d^2*e*f*g^2 + 3*d^3*g^3)))*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2*(c*
f^2 + g*(-(b*f) + a*g))^2) + (e^4*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))
^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g)
)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(
e*f + d*g)))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(c*f^2 +
 g*(-(b*f) + a*g))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.28, size = 3315, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] g^5*log(abs(g*x + f))/(c^2*d*f^4*g^2 - 2*b*c*d*f^3*g^3 + b^2*d*f^2*g^4 + 2*
a*c*d*f^2*g^4 - 2*a*b*d*f*g^5 + a^2*d*g^6 - c^2*f^5*g*e + 2*b*c*f^4*g^2*e -
 b^2*f^3*g^3*e - 2*a*c*f^3*g^3*e + 2*a*b*f^2*g^4*e - a^2*f*g^5*e) - 1/2*(c^
2*d^3*g^3 + c^2*d^2*f*g^2*e - 2*b*c*d^2*g^3*e + c^2*d*f^2*g*e^2 - 2*b*c*d*f
*g^2*e^2 + b^2*d*g^3*e^2 + 2*a*c*d*g^3*e^2 + c^2*f^3*e^3 - 2*b*c*f^2*g*e^3
+ b^2*f*g^2*e^3 + 2*a*c*f*g^2*e^3 - 2*a*b*g^3*e^3)*log(c*x^2 + b*x + a)/(c^
4*d^4*f^4 - 2*b*c^3*d^4*f^3*g + b^2*c^2*d^4*f^2*g^2 + 2*a*c^3*d^4*f^2*g^2 -
 2*a*b*c^2*d^4*f*g^3 + a^2*c^2*d^4*g^4 - 2*b*c^3*d^3*f^4*e + 4*b^2*c^2*d^3*
f^3*g*e - 2*b^3*c*d^3*f^2*g^2*e - 4*a*b*c^2*d^3*f^2*g^2*e + 4*a*b^2*c*d^3*f
*g^3*e - 2*a^2*b*c*d^3*g^4*e + b^2*c^2*d^2*f^4*e^2 + 2*a*c^3*d^2*f^4*e^2 -
2*b^3*c*d^2*f^3*g*e^2 - 4*a*b*c^2*d^2*f^3*g*e^2 + b^4*d^2*f^2*g^2*e^2 + 4*a
*b^2*c*d^2*f^2*g^2*e^2 + 4*a^2*c^2*d^2*f^2*g^2*e^2 - 2*a*b^3*d^2*f*g^3*e^2
- 4*a^2*b*c*d^2*f*g^3*e^2 + a^2*b^2*d^2*g^4*e^2 + 2*a^3*c*d^2*g^4*e^2 - 2*a
*b*c^2*d*f^4*e^3 + 4*a*b^2*c*d*f^3*g*e^3 - 2*a*b^3*d*f^2*g^2*e^3 - 4*a^2*b*
c*d*f^2*g^2*e^3 + 4*a^2*b^2*d*f*g^3*e^3 - 2*a^3*b*d*g^4*e^3 + a^2*c^2*f^4*e
^4 - 2*a^2*b*c*f^3*g*e^4 + a^2*b^2*f^2*g^2*e^4 + 2*a^3*c*f^2*g^2*e^4 - 2*a^
3*b*f*g^3*e^4 + a^4*g^4*e^4) - e^5*log(abs(x*e + d))/(c^2*d^5*g*e - c^2*d^4
*f*e^2 - 2*b*c*d^4*g*e^2 + 2*b*c*d^3*f*e^3 + b^2*d^3*g*e^3 + 2*a*c*d^3*g*e^
3 - b^2*d^2*f*e^4 - 2*a*c*d^2*f*e^4 - 2*a*b*d^2*g*e^4 + 2*a*b*d*f*e^5 + a^2
*d*g*e^5 - a^2*f*e^6) - (4*c^5*d^3*f^3 - 6*b*c^4*d^3*f^2*g + 12*a*c^4*d^3*f
*g^2 + b^3*c^2*d^3*g^3 - 6*a*b*c^3*d^3*g^3 - 6*b*c^4*d^2*f^3*e + 8*b^2*c^3*
d^2*f^2*g*e + 4*a*c^4*d^2*f^2*g*e + b^3*c^2*d^2*f*g^2*e - 22*a*b*c^3*d^2*f*
g^2*e - 2*b^4*c*d^2*g^3*e + 12*a*b^2*c^2*d^2*g^3*e - 4*a^2*c^3*d^2*g^3*e +
```

```
12*a*c^4*d*f^3*e^2 + b^3*c^2*d*f^2*g*e^2 - 22*a*b*c^3*d*f^2*g*e^2 - 2*b^4*c
*d*f*g^2*e^2 + 12*a*b^2*c^2*d*f*g^2*e^2 + 20*a^2*c^3*d*f*g^2*e^2 + b^5*d*g^
3*e^2 - 4*a*b^3*c*d*g^3*e^2 - 6*a^2*b*c^2*d*g^3*e^2 + b^3*c^2*f^3*e^3 - 6*a
*b*c^3*f^3*e^3 - 2*b^4*c*f^2*g*e^3 + 12*a*b^2*c^2*f^2*g*e^3 - 4*a^2*c^3*f^2
*g*e^3 + b^5*f*g^2*e^3 - 4*a*b^3*c*f*g^2*e^3 - 6*a^2*b*c^2*f*g^2*e^3 - 2*a*
b^4*g^3*e^3 + 12*a^2*b^2*c*g^3*e^3 - 12*a^3*c^2*g^3*e^3)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/((b^2*c^4*d^4*f^4 - 4*a*c^5*d^4*f^4 - 2*b^3*c^3*d^4*f^
3*g + 8*a*b*c^4*d^4*f^3*g + b^4*c^2*d^4*f^2*g^2 - 2*a*b^2*c^3*d^4*f^2*g^2 -
 8*a^2*c^4*d^4*f^2*g^2 - 2*a*b^3*c^2*d^4*f*g^3 + 8*a^2*b*c^3*d^4*f*g^3 + a^
2*b^2*c^2*d^4*g^4 - 4*a^3*c^3*d^4*g^4 - 2*b^3*c^3*d^3*f^4*e + 8*a*b*c^4*d^3
*f^4*e + 4*b^4*c^2*d^3*f^3*g*e - 16*a*b^2*c^3*d^3*f^3*g*e - 2*b^5*c*d^3*f^2
*g^2*e + 4*a*b^3*c^2*d^3*f^2*g^2*e + 16*a^2*b*c^3*d^3*f^2*g^2*e + 4*a*b^4*c
*d^3*f*g^3*e - 16*a^2*b^2*c^2*d^3*f*g^3*e - 2*a^2*b^3*c*d^3*g^4*e + 8*a^3*b
*c^2*d^3*g^4*e + b^4*c^2*d^2*f^4*e^2 - 2*a*b^2*c^3*d^2*f^4*e^2 - 8*a^2*c^4*
d^2*f^4*e^2 - 2*b^5*c*d^2*f^3*g*e^2 + 4*a*b^3*c^2*d^2*f^3*g*e^2 + 16*a^2*b*
c^3*d^2*f^3*g*e^2 + b^6*d^2*f^2*g^2*e^2 - 12*a^2*b^2*c^2*d^2*f^2*g^2*e^2 -
16*a^3*c^3*d^2*f^2*g^2*e^2 - 2*a*b^5*d^2*f*g^3*e^2 + 4*a^2*b^3*c*d^2*f*g^3*
e^2 + 16*a^3*b*c^2*d^2*f*g^3*e^2 + a^2*b^4*d^2*g^4*e^2 - 2*a^3*b^2*c*d^2*g^
4*e^2 - 8*a^4*c^2*d^2*g^4*e^2 - 2*a*b^3*c^2*d*f^4*e^3 + 8*a^2*b*c^3*d*f^4*e
^3 + 4*a*b^4*c*d*f^3*g*e^3 - 16*a^2*b^2*c^2*d*f^3*g*e^3 - 2*a*b^5*d*f^2*g^2
*e^3 + 4*a^2*b^3*c*d*f^2*g^2*e^3 + 16*a^3*b*c^2*d*f^2*g^2*e^3 + 4*a^2*b^4*d
*f*g^3*e^3 - 16*a^3*b^2*c*d*f*g^3*e^3 - 2*a^3*b^3*d*g^4*e^3 + 8*a^4*b*c*d*g
^4*e^3 + a^2*b^2*c^2*f^4*e^4 - 4*a^3*c^3*f^4*e^4 - 2*a^2*b^3*c*f^3*g*e^4 +
8*a^3*b*c^2*f^3*g*e^4 + a^2*b^4*f^2*g^2*e^4 - 2*a^3*b^2*c*f^2*g^2*e^4 - 8*a
^4*c^2*f^2*g^2*e^4 - 2*a^3*b^3*f*g^3*e^4 + 8*a^4*b*c*f*g^3*e^4 + a^4*b^2*g^
4*e^4 - 4*a^5*c*g^4*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^4*d^3*f^3 - 2*b^2*c^3*d
^3*f^2*g + 2*a*c^4*d^3*f^2*g + b^3*c^2*d^3*f*g^2 - a*b*c^3*d^3*f*g^2 - a*b^
2*c^2*d^3*g^3 + 2*a^2*c^3*d^3*g^3 - 2*b^2*c^3*d^2*f^3*e + 2*a*c^4*d^2*f^3*e
 + 4*b^3*c^2*d^2*f^2*g*e - 7*a*b*c^3*d^2*f^2*g*e - 2*b^4*c*d^2*f*g^2*e + 3*
a*b^2*c^2*d^2*f*g^2*e + 2*a^2*c^3*d^2*f*g^2*e + 2*a*b^3*c*d^2*g^3*e - 5*a^2
*b*c^2*d^2*g^3*e + b^3*c^2*d*f^3*e^2 - a*b*c^3*d*f^3*e^2 - 2*b^4*c*d*f^2*g*
e^2 + 3*a*b^2*c^2*d*f^2*g*e^2 + 2*a^2*c^3*d*f^2*g*e^2 + b^5*d*f*g^2*e^2 - a
*b^3*c*d*f*g^2*e^2 - 3*a^2*b*c^2*d*f*g^2*e^2 - a*b^4*d*g^3*e^2 + 2*a^2*b^2*
c*d*g^3*e^2 + 2*a^3*c^2*d*g^3*e^2 - a*b^2*c^2*f^3*e^3 + 2*a^2*c^3*f^3*e^3 +
 2*a*b^3*c*f^2*g*e^3 - 5*a^2*b*c^2*f^2*g*e^3 - a*b^4*f*g^2*e^3 + 2*a^2*b^2*
c*f*g^2*e^3 + 2*a^3*c^2*f*g^2*e^3 + a^2*b^3*g^3*e^3 - 3*a^3*b*c*g^3*e^3 + (
2*c^5*d^3*f^3 - 3*b*c^4*d^3*f^2*g + b^2*c^3*d^3*f*g^2 + 2*a*c^4*d^3*f*g^2 -
 a*b*c^3*d^3*g^3 - 3*b*c^4*d^2*f^3*e + 5*b^2*c^3*d^2*f^2*g*e - 2*a*c^4*d^2*
f^2*g*e - 2*b^3*c^2*d^2*f*g^2*e - a*b*c^3*d^2*f*g^2*e + 2*a*b^2*c^2*d^2*g^3
*e - 2*a^2*c^3*d^2*g^3*e + b^2*c^3*d*f^3*e^2 + 2*a*c^4*d*f^3*e^2 - 2*b^3*c^
2*d*f^2*g*e^2 - a*b*c^3*d*f^2*g*e^2 + b^4*c*d*f*g^2*e^2 + 2*a^2*c^3*d*f*g^2
*e^2 - a*b^3*c*d*g^3*e^2 + a^2*b*c^2*d*g^3*e^2 - a*b*c^3*f^3*e^3 + 2*a*b^2*
c^2*f^2*g*e^3 - 2*a^2*c^3*f^2*g*e^3 - a*b^3*c*f*g^2*e^3 + a^2*b*c^2*f*g^2*e
^3 + a^2*b^2*c*g^3*e^3 - 2*a^3*c^2*g^3*e^3)*x)/((c*d^2 - b*d*e + a*e^2)^2*(
c*f^2 - b*f*g + a*g^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [B] time = 0.05, size = 9103, normalized size = 14.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 32.63, size = 130035, normalized size = 201.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((b^3*e*g + 2*a*c^2*d*g + 2*a*c^2*e*f + b*c^2*d*f - b^2*c*d*g - b^2*c*e*f -
 3*a*b*c*e*g)/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*
c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^3*d*e*g^2 + b^3*c*d
*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2 - a*b^2*c*e^2*f^2
- b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 - 4*a*b*c^2*d^2*f*g -
 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g) - (x*(2*a*c^2*e*g - 2*c^3*d*f + b*c
^2*d*g + b*c^2*e*f - b^2*c*e*g))/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b
^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^
3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2
 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 -
4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g))/(a + b*x + c*x^
2) + symsum(log((12*a^2*c^5*e^6*g^6 - 3*b^2*c^5*d^2*e^4*g^6 - 3*b^2*c^5*e^6
*f^2*g^4 + 4*c^7*d^2*e^4*f^2*g^4 - 2*a*b^2*c^4*e^6*g^6 + 16*a*c^6*d^2*e^4*g
^6 + 3*b^3*c^4*d*e^5*g^6 + 16*a*c^6*e^6*f^2*g^4 + 3*b^3*c^4*e^6*f*g^5 - 4*b
*c^6*d*e^5*f^2*g^4 - 4*b*c^6*d^2*e^4*f*g^5 - 16*a*b*c^5*d*e^5*g^6 - 16*a*b*
c^5*e^6*f*g^5 + 16*a*c^6*d*e^5*f*g^5)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4
 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e
^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4
- 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32
*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c
^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2
*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4
- 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^
3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b
*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^
3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c
^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*
c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^
2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e
^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^
2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2
*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3
*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^
4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*
a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f
^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a
*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f
^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5
*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g
- 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*
d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*
a^4*b^2*c^2*d*e^3*f*g^3) - root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6
*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^
5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f
^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*
z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 5
```

12*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10
*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^1
0*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*
e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g
^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4
+ 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a
^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^
5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9
*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*
z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*
a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8
*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f
^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*
z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16
*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*
c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3
*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*
z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*
b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5
*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*
a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 56
32*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 +
5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^
4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7
*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*
g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f
^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^
4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5
*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*
d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c
^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^
2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6
*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*
a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 33
60*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 -
3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^
4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7
*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*
g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f
^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^
2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3
*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*
d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c
^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^
6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5
*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*
a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 17
28*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 -
1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^
4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2
*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*
g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f
^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^
8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7
*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*
d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c
^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^
3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4
*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*

a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 13
60*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 -
 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^
4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8
*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*
g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f
^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7
*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^
2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e
^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*
e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8
*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^
5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d
^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*
d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2
*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^
4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c
^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*
c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7
*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^
7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b
^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*
b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3
*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^
3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a
^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*
a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504
*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 50
4*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 4
16*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 -
352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 -
 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4
- 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4
 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^
4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z
^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*
z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^
7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*
g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5
*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^
2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f
^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4
*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e
^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*
e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*
e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e
^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^
4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3
*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2
*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^
5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4
 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4
+ 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 -
 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 -
2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2
112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 21
12*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 166
4*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664
*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*

a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a
^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^
6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5
*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*
b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b
^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*
c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c
^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*
c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c
^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^
4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5
*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d
^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*
e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3
*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*
g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8
*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^
4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 +
640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 64
0*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a
^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*
b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c
^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^
3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d
^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^
9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5
*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*
g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g
*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^
4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 -
288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 28
8*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a
^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*
b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c
^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*
d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2
*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^
7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f
^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*
g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^
6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*
z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4
- 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 -
196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192
*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^
2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b
^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*
c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2
*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*
f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5
*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^
3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z
^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4
+ 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 2
8*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a
^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^
11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c
^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8
*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^

5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f
^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^
3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 -
 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*
c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*
e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*
g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 -
 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c
^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^
6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5
*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*
g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z
^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 -
2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*
a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c
^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^
4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8
*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*
g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z
^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 +
40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b
^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e
^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*
g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4
 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*
b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*
e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6
*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*
z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 -
 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*
a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^
6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*
e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^
7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^
4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 3
20*a^4*b^3*c^7*d^10*f^3*g^7*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^
4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2
*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*
d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^
2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z
^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6
*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*
b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c
^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^
8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6
*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*
z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 +
48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*
b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8
*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*
e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f
^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z
^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 +
90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*
b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^1
0*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^
8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 -
14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^
7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4

*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11
*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^
9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a
^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*
e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*
z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*
a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3
*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*
g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^
4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^
8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4
*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^
9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4
 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*
c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10
*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z
^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6
*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*
f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4
 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a
^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*
d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*
z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^
9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7
*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^1
0*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*
a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d
^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z
^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*
a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7
*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^
10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*
a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e
*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 1
28*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9
*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 -
256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^
5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^
4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6
*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4
 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^
10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6
*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^
5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^
10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*
c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^1
0*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a
^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g
^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^
6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^1
0*g^10*z^4 + 64*a^7*c^7*e^10*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6
*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b
^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*
z^4 - b^10*c^4*d^4*e^6*f^10*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^
8*g^10*z^4 - a^4*b^10*d^6*e^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^
6*c^4*d^10*g^10*z^4 + 272*a^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e
^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^
6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 +
 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4

```
*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^
6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g
^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 -
60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^
8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7
*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*
z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b
^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^
7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e
^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*
e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3
*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^
3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d
^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*
d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d
^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^
2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2
*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*
e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^
6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2
 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d
*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^
2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 +
 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 20
8*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a
^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*
b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c
^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^
5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5
*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5
*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*
g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z
^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 -
54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*
b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5
*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*
d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e
^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*
g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z
^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 -
32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*
b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c
^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^
2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f
^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*
z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 5
0*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7
*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*
f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z
^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6
*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^
4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z
^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*
c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2
*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*
z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 3
76*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*
c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5
*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^
```

6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z
^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 31
0*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b
^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*
e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*
g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 -
471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^
3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c
^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*
e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*
z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*
b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e
*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2
+ 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^
3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2
- 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*
c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^
3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 6
24*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*
c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2
*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b
^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e
^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2
- 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*
c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8
*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9
*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^
2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*
e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a
*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*
g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*
c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^
2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^
4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 -
36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f
^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*
c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4
*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2
- 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8
*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8
*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7
*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 18
8*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*
g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^
8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 -
232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2
*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^
3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*
z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*
c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^
6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8
*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^
8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^
2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*
g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 4
4*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c
^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*
g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*
b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e

^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4
*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^
6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*
c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*
b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4
*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*
c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4
*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*
d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g
^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a
^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g
^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g
*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7
*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5
*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5
*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*
z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*
e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 1
6*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2
*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b
^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^
6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3
*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z
- 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*
d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a
*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4
*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*
a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z
 - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^
7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3
*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g
^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g
^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7
*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z +
3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*
z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c
^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5
*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g
^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c
^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2
*c^4*e^6*g^6, z, k)*((13*a^2*b^5*c^2*e^7*g^7 - 56*a^3*b^3*c^3*e^7*g^7 + 24*
a^2*c^7*d^5*e^2*g^7 - 2*b^4*c^5*d^5*e^2*g^7 + b^5*c^4*d^4*e^3*g^7 + b^6*c^3
*d^3*e^4*g^7 - 2*b^7*c^2*d^2*e^5*g^7 + 24*a^2*c^7*e^7*f^5*g^2 - 2*b^4*c^5*e
^7*f^5*g^2 + b^5*c^4*e^7*f^4*g^3 + b^6*c^3*e^7*f^3*g^4 - 2*b^7*c^2*e^7*f^2*
g^5 - a*b^7*c*e^7*g^7 + b^8*c*d*e^6*g^7 + b^8*c*e^7*f*g^6 + 80*a^4*b*c^4*e^
7*g^7 - 28*a^4*c^5*d*e^6*g^7 + b^3*c^6*d^6*e*g^7 - 28*a^4*c^5*e^7*f*g^6 + b
^3*c^6*e^7*f^6*g + 4*c^9*d^3*e^4*f^6*g + 4*c^9*d^6*e*f^3*g^4 - 12*a*b^6*c^2
*d*e^6*g^7 - 12*a*b^6*c^2*e^7*f*g^6 - 4*b*c^8*d^2*e^5*f^6*g - 4*b*c^8*d^6*e
*f^2*g^5 - b^2*c^7*d*e^6*f^6*g - b^2*c^7*d^6*e*f*g^6 - 2*b^7*c^2*d*e^6*f*g^
6 + 2*a*b^2*c^6*d^5*e^2*g^7 + 10*a*b^3*c^5*d^4*e^3*g^7 - 20*a*b^4*c^4*d^3*e
^4*g^7 + 25*a*b^5*c^3*d^2*e^5*g^7 - 56*a^2*b*c^6*d^4*e^3*g^7 + 44*a^2*b^4*c
^3*d*e^6*g^7 + 76*a^3*b*c^5*d^2*e^5*g^7 - 40*a^3*b^2*c^4*d*e^6*g^7 + 2*a*b^
2*c^6*e^7*f^5*g^2 + 10*a*b^3*c^5*e^7*f^4*g^3 - 20*a*b^4*c^4*e^7*f^3*g^4 + 2
5*a*b^5*c^3*e^7*f^2*g^5 - 56*a^2*b*c^6*e^7*f^4*g^3 + 44*a^2*b^4*c^3*e^7*f*g
^6 + 76*a^3*b*c^5*e^7*f^2*g^5 - 40*a^3*b^2*c^4*e^7*f*g^6 + 16*a*c^8*d^2*e^5
*f^5*g^2 + 24*a*c^8*d^3*e^4*f^4*g^3 + 24*a*c^8*d^4*e^3*f^3*g^4 + 16*a*c^8*d
^5*e^2*f^2*g^5 + 28*a^2*c^7*d*e^6*f^4*g^3 + 28*a^2*c^7*d^4*e^3*f*g^6 - 80*a
^3*c^6*d*e^6*f^2*g^5 - 80*a^3*c^6*d^2*e^5*f*g^6 - 12*b*c^8*d^3*e^4*f^5*g^2
- 12*b*c^8*d^5*e^2*f^3*g^4 + 6*b^3*c^6*d*e^6*f^5*g^2 + 6*b^3*c^6*d^5*e^2*f*

g^6 - 9*b^4*c^5*d*e^6*f^4*g^3 - 9*b^4*c^5*d^4*e^3*f*g^6 + 4*b^5*c^4*d*e^6*f ^3*g^4 + 4*b^5*c^4*d^3*e^4*f*g^6 + b^6*c^3*d*e^6*f^2*g^5 + b^6*c^3*d^2*e^5* f*g^6 - 4*a*b*c^7*d^6*e*g^7 - 4*a*b*c^7*e^7*f^6*g + 8*a*c^8*d*e^6*f^6*g + 8 *a*c^8*d^6*e*f*g^6 + 65*a^2*b^2*c^5*d^3*e^4*g^7 - 88*a^2*b^3*c^4*d^2*e^5*g^ 7 + 65*a^2*b^2*c^5*e^7*f^3*g^4 - 88*a^2*b^3*c^4*e^7*f^2*g^5 + 68*a^2*c^7*d^ 2*e^5*f^3*g^4 + 68*a^2*c^7*d^3*e^4*f^2*g^5 + 8*b^2*c^7*d^2*e^5*f^5*g^2 + 9* b^2*c^7*d^3*e^4*f^4*g^3 + 9*b^2*c^7*d^4*e^3*f^3*g^4 + 8*b^2*c^7*d^5*e^2*f^2 *g^5 - b^3*c^6*d^2*e^5*f^4*g^3 + 4*b^3*c^6*d^3*e^4*f^3*g^4 - b^3*c^6*d^4*e^ 3*f^2*g^5 - 9*b^4*c^5*d^2*e^5*f^3*g^4 - 9*b^4*c^5*d^3*e^4*f^2*g^5 + 7*b^5*c ^4*d^2*e^5*f^2*g^5 + 74*a*b^2*c^6*d^2*e^5*f^3*g^4 + 74*a*b^2*c^6*d^3*e^4*f^ 2*g^5 - 28*a*b^3*c^5*d^2*e^5*f^2*g^5 - 120*a^2*b*c^6*d^2*e^5*f^2*g^5 + 159* a^2*b^2*c^5*d*e^6*f^2*g^5 + 159*a^2*b^2*c^5*d^2*e^5*f*g^6 - 36*a*b*c^7*d*e^ 6*f^5*g^2 - 36*a*b*c^7*d^5*e^2*f*g^6 + 28*a*b^5*c^3*d*e^6*f*g^6 + 104*a^3*b *c^5*d*e^6*f*g^6 - 56*a*b*c^7*d^2*e^5*f^4*g^3 - 96*a*b*c^7*d^3*e^4*f^3*g^4 - 56*a*b*c^7*d^4*e^3*f^2*g^5 + 44*a*b^2*c^6*d*e^6*f^4*g^3 + 44*a*b^2*c^6*d^ 4*e^3*f*g^6 - 32*a*b^4*c^4*d*e^6*f^2*g^5 - 32*a*b^4*c^4*d^2*e^5*f*g^6 - 116 *a^2*b*c^6*d*e^6*f^3*g^4 - 116*a^2*b*c^6*d^3*e^4*f*g^6 - 112*a^2*b^3*c^4*d* e^6*f*g^6)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16* a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^ 4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a ^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6* c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3 *e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4* f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a ^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c ^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5* c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^ 2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5 *d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3* e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3 *f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e* f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2 *f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^ 2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3 *c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a *b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32* a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2 *e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4* a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3 *f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b ^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^ 2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3* d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) - root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9 *b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6* d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f ^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4* z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a ^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7 *c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6* e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9 *g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^ 10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c *d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^ 7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b

^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^
4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f
^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^
4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*
a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*
c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^
5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 -
9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^
4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4
*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*
g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f
^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^
5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3
*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*
d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c
^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^
3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7
*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*
a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 37
76*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 +
3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^
4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2
*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*
g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f
^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^
3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3
*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*
d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c
^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^
4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2
*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*
a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 18
28*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 -
1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^
4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8
*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*
g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f
^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^
8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7
*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*
d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c
^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^
2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7
*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*
a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 14
08*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 +
1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^
4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5
*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*
g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f
^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^
5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2
*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*
d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c
^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*
c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5
*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^
6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b
^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*

b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5
*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^
2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a
^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*
a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694
*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 69
4*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 6
92*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 +
692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 +
672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4
+ 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4
- 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^
4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z
^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*
z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4
*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^
6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g
^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*
g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4
*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^
7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f
^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*
f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8
*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^
6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e
^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^
3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d
^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*
d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4
*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^
5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10
*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^
7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b
^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^
5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5
*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*
c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c
^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*
c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*
c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3
*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^
3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f
*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*
g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g
^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^
5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9
*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*
z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z
^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^
4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4
+ 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4
+ 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 -
1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 -
1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1
280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 12
80*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 115
2*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152
*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*
a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a

^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*
b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5
*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^
4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d
^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^
8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f
^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*
g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g
*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^
4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 -
480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 40
0*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a
^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*
b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6
*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^
3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d
^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*
e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4
*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*
g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3
*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^
4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 -
280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 25
6*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a
^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^
6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7
*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*
d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3
*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*
e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4
*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^
5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g
^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*
z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4
+ 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 +
156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*
a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8
*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c
*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^
7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e
^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*
f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2
*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^
6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^
4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 +
24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a
^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^1
1*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11
*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*
d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4
*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^
3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 2
0*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c
^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^
6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2
*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 46
08*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^
7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6
*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*

e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f
^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^
7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4
 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 17
92*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^
9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9
*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*
e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f
^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7
*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20
*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^1
1*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e
^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g
^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 -
5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^1
2*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*
e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*
f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g
^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^
4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68
*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b
^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*
e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^
4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7
*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4
 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192
*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b
^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*
d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f
^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z
^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 +
320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a
^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^
3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5
*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^
2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10
*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 -
6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3
*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*
c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d
^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^
4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10
*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 +
48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2
*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9
*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z
^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*
c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7
*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*
b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f
^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 -
768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9
*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^
3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 -
 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^
10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*
f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 -
14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e
^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4

+ 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^
6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5
*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*
a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*
c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*
g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 1
2*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3
*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g
^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 +
 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c
^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8
*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 76
8*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*
c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^
6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4
- 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9
*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*
f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4
 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*
b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e
^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4
+ 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d
*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4
- 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*
e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^
4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5
*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^
2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c
^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6
*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^
3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g
^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^
10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^
8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^
7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8
*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 -
6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5
*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 1
2*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^1
0*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4
*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a
^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10
*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e
^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a
^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^
4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7
*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^
5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 -
76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*
c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e
^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*
z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*
a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c
*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2
*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 +
2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2
*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^
2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b
^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*

b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 -

26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6* z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*(root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^

5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4
*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*
d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c
^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^
2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4
*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*
a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 34
40*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 -
3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^
4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2
*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*
g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f
^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^
5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2
*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*
d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c
^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^
4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5
*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*
a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 17
40*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 -
1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^
4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6
*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*
g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f
^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^
2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8
*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*
d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c
^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^
3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5
*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*
a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 13
60*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 -
1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^
4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5
*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*
g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f
^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^
3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8
*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^
2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d
^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*
d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4
*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^
7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c
^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*
c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8
*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^
8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b
^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*
b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4
*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^
3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a
^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*
a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544
*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 54
4*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 5
36*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 -

536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d

^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*

b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*
e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f
^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^
3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4
 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 19
2*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b
^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2
*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f
^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*
z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 +
 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*
a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b
^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6
*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f
^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2
*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 4
80*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^
8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4
*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*
d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e
^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10
*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 +
 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*
b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c
^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^
5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4
*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10
*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 +
 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2
*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c
^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6
*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 1
4*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*
e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z
^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^
6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2
*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^
9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e
^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*
z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a
^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9
*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*
z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4
*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2
*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^
3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*
f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 +
 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*
c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^
10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^
4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*
b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*
f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4
 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*
b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^1
0*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z
^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*
b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^
3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10

*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a
^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7
*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z
^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192
*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^
5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^1
0*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*
b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f
*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*
a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f
^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 25
6*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*
f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4
+ 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^1
0*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 +
256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11
*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4
+ 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4
*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10
*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*
d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*
z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*
b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*
z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*
b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10
*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f
^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6
*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 -
b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^1
0*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2
*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*
e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g
^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2
+ 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4
*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*
e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7
*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 +
44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*
c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e
^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z
^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^
7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d
^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*
d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4
*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^
5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c
^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*
c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*
c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c
^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^
6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3
*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*
d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^
7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 -
4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*
e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7
*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^
2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 -
208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 16

4*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a
^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^
2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3
*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*
d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e
^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^
2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4
*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2
- 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*
a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b
^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^
2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e
^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^
6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2
*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2
- 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*
a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5
*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3
*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^
3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6
*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 3
2*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7
*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*
f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z
^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*
c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4
*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^
2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c
^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*
f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5
*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 +
320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3
*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^
6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f
^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*
z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 2
32*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*
b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5
*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*
g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2
+ 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a
^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*
c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6
*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*
z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^
3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3
*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 -
4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5
*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2
- 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c
^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^
7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 54
8*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^
2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z
^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2
*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^
5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 -
68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*
d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z

^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d
*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 -
24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f
*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*
b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*
z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^
7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 +
128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*
e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a
*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*
g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*
f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^
3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*
c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 -
2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z
^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*
g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^
6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z
^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6
*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 18
8*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*
f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^
8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2
+ 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*
g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*
z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 -
48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^1
0*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d
*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*
g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 1
20*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d
^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^
6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^
4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f
^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a
*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g
^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*
e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6
*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^
5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5
*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^
4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^
3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^
5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2
*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*
a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*
a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z +
14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4
*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^
2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z +
52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*
f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3
*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^
5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2
*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z
- 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^
2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32
*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^
5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*

d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*((64*a^6*c^7*d^7*e^2*g^9 + 64*a^7*c^6*d^5*e^4*g^9 - 64*a^8*c^5*d^3*e^6*g^9 + 64*a^6*c^7*e^9*f^7*g^2 + 64*a^7*c^6*e^9*f^5*g^4 - 64*a^8*c^5*e^9*f^3*g^6 - 64*a^9*c^4*d*e^8*g^9 - 64*a^9*c^4*e^9*f*g^8 + 16*a^5*b*c^7*d^8*e*g^9 + a^6*b^6*c*d*e^8*g^9 + 16*a^5*b*c^7*e^9*f^8*g + a^6*b^6*c*e^9*f*g^8 - 128*a^5*c^8*d*e^8*f^8*g - 128*a^5*c^8*d^8*e*f*g^8 + a^3*b^5*c^5*d^8*e*g^9 + a^3*b^9*c*d^4*e^5*g^9 - 8*a^4*b^3*c^6*d^8*e*g^9 - a^4*b^8*c*d^3*e^6*g^9 - a^5*b^7*c*d^2*e^7*g^9 - 144*a^6*b*c^6*d^6*e^3*g^9 - 80*a^7*b*c^5*d^4*e^5*g^9 - 12*a^7*b^4*c^2*d*e^8*g^9 + 80*a^8*b*c^4*d^2*e^7*g^9 + 48*a^8*b^2*c^3*d*e^8*g^9 + a^3*b^5*c^5*e^9*f^8*g + a^3*b^9*c*e^9*f^4*g^5 - 8*a^4*b^3*c^6*e^9*f^8*g - a^4*b^8*c*e^9*f^3*g^6 - a^5*b^7*c*e^9*f^2*g^7 - 144*a^6*b*c^6*e^9*f^6*g^3 - 80*a^7*b*c^5*e^9*f^4*g^5 - 12*a^7*b^4*c^2*e^9*f*g^8 + 80*a^8*b*c^4*e^9*f^2*g^7 + 48*a^8*b^2*c^3*e^9*f*g^8 - 128*a^3*c^10*d^5*e^4*f^8*g - 128*a^3*c^10*d^8*e*f^5*g^4 - 256*a^4*c^9*d^3*e^6*f^8*g - 256*a^4*c^9*d^8*e*f^3*g^6 - 448*a^6*c^7*d*e^8*f^6*g^3 - 448*a^6*c^7*d^6*e^3*f*g^8 - 576*a^7*c^6*d*e^8*f^4*g^5 - 576*a^7*c^6*d^4*e^5*f*g^8 - 320*a^8*c^5*d*e^8*f^2*g^7 - 320*a^8*c^5*d^2*e^7*f*g^8 + b^5*c^8*d^6*e^3*f^8*g + b^5*c^8*d^8*e*f^6*g^3 - b^6*c^7*d^5*e^4*f^8*g - b^6*c^7*d^8*e*f^5*g^4 - b^7*c^6*d^4*e^5*f^8*g - b^7*c^6*d^8*e*f^4*g^5 + b^8*c^5*d^3*e^6*f^8*g + b^8*c^5*d^8*e*f^3*g^6 + b^12*c*d^3*e^6*f^4*g^5 + b^12*c*d^4*e^5*f^3*g^6 - 4*a^3*b^6*c^4*d^7*e^2*g^9 + 6*a^3*b^7*c^3*d^6*e^3*g^9 - 4*a^3*b^8*c^2*d^5*e^4*g^9 + 36*a^4*b^4*c^5*d^7*e^2*g^9 - 57*a^4*b^5*c^4*d^6*e^3*g^9 + 37*a^4*b^6*c^3*d^5*e^4*g^9 - 7*a^4*b^7*c^2*d^4*e^5*g^9 - 96*a^5*b^2*c^6*d^7*e^2*g^9 + 168*a^5*b^3*c^5*d^6*e^3*g^9 - 100*a^5*b^4*c^4*d^5*e^4*g^9 + 3*a^5*b^5*c^3*d^4*e^5*g^9 + 10*a^5*b^6*c^2*d^3*e^6*g^9 + 48*a^6*b^2*c^5*d^5*e^4*g^9 + 56*a^6*b^3*c^4*d^4*e^5*g^9 - 36*a^6*b^4*c^3*d^3*e^6*g^9 + 13*a^6*b^5*c^2*d^2*e^7*g^9 + 64*a^7*b^2*c^4*d^3*e^6*g^9 - 56*a^7*b^3*c^3*d^2*e^7*g^9 - 4*a^3*b^6*c^4*e^9*f^7*g^2 + 6*a^3*b^7*c^3*e^9*f^6*g^3 - 4*a^3*b^8*c^2*e^9*f^5*g^4 + 36*a^4*b^4*c^5*e^9*f^7*g^2 - 57*a^4*b^5*c^4*e^9*f^6*g^3 + 37*a^4*b^6*c^3*e^9*f^5*g^4 - 7*a^4*b^7*c^2*e^9*f^4*g^5 - 96*a^5*b^2*c^6*e^9*f^7*g^2 + 168*a^5*b^3*c^5*e^9*f^6*g^3 - 100*a^5*b^4*c^4*e^9*f^5*g^4 + 3*a^5*b^5*c^3*e^9*f^4*g^5 + 10*a^5*b^6*c^2*e^9*f^3*g^6 + 48*a^6*b^2*c^5*e^9*f^5*g^4 + 56*a^6*b^3*c^4*e^9*f^4*g^5 - 36*a^6*b^4*c^3*e^9*f^3*g^6 + 13*a^6*b^5*c^2*e^9*f^2*g^7 + 64*a^7*b^2*c^4*e^9*f^3*g^6 - 56*a^7*b^3*c^3*e^9*f^2*g^7 + 64*a^3*c^10*d^6*e^3*f^7*g^2 + 64*a^3*c^10*d^7*e^2*f^6*g^3 + 192*a^4*c^9*d^4*e^5*f^7*g^2 - 320*a^4*c^9*d^5*e^4*f^6*g^3 - 320*a^4*c^9*d^6*e^3*f^5*g^4 + 192*a^4*c^9*d^7*e^2*f^4*g^5 + 192*a^5*c^8*d^2*e^7*f^7*g^2 - 832*a^5*c^8*d^3*e^6*f^6*g^3 - 192*a^5*c^8*d^4*e^5*f^5*g^4 - 192*a^5*c^8*d^5*e^4*f^4*g^5 - 832*a^5*c^8*d^6*e^3*f^3*g^6 + 192*a^5*c^8*d^7*e^2*f^2*g^7 + 64*a^6*c^7*d^2*e^7*f^5*g^4 - 960*a^6*c^7*d^3*e^6*f^4*g^5 - 960*a^6*c^7*d^4*e^5*f^3*g^6 + 64*a^6*c^7*d^5*e^4*f^2*g^7 - 448*a^7*c^6*d^2*e^7*f^3*g^6 - 448*a^7*c^6*d^3*e^6*f^2*g^7 - 2*b^5*c^8*d^7*e^2*f^7*g^2 + 2*b^6*c^7*d^6*e^3*f^7*g^2 + 2*b^6*c^7*d^7*e^2*f^6*g^3 - 2*b^7*c^6*d^5*e^4*f^7*g^2 - 6*b^7*c^6*d^6*e^3*f^6*g^3 - 2*b^7*c^6*d^7*e^2*f^5*g^4 + 6*b^8*c^5*d^4*e^5*f^7*g^2 + 8*b^8*c^5*d^5*e^4*f^6*g^3 + 8*b^8*c^5*d^6*e^3*f^5*g^4 + 6*b^8*c^5*d^7*e^2*f^4*g^5 - 4*b^9*c^4*d^3*e^6*f^7*g^2 - 11*b^9*c^4*d^4*e^5*f^6*g^3 - 10*b^9*c^4*d^5*e

^4*f^5*g^4 - 11*b^9*c^4*d^6*e^3*f^4*g^5 - 4*b^9*c^4*d^7*e^2*f^3*g^6 + 6*b^1
0*c^3*d^3*e^6*f^6*g^3 + 9*b^10*c^3*d^4*e^5*f^5*g^4 + 9*b^10*c^3*d^5*e^4*f^4
*g^5 + 6*b^10*c^3*d^6*e^3*f^3*g^6 - 4*b^11*c^2*d^3*e^6*f^5*g^4 - 4*b^11*c^2
*d^4*e^5*f^4*g^5 - 4*b^11*c^2*d^5*e^4*f^3*g^6 + 16*a*b^3*c^9*d^7*e^2*f^7*g^
2 - 12*a*b^4*c^8*d^6*e^3*f^7*g^2 - 12*a*b^4*c^8*d^7*e^2*f^6*g^3 + 30*a*b^5*
c^7*d^5*e^4*f^7*g^2 + 30*a*b^5*c^7*d^6*e^3*f^6*g^3 + 30*a*b^5*c^7*d^7*e^2*f
^5*g^4 - 100*a*b^6*c^6*d^4*e^5*f^7*g^2 - 56*a*b^6*c^6*d^5*e^4*f^6*g^3 - 56*
a*b^6*c^6*d^6*e^3*f^5*g^4 - 100*a*b^6*c^6*d^7*e^2*f^4*g^5 + 62*a*b^7*c^5*d^
3*e^6*f^7*g^2 + 128*a*b^7*c^5*d^4*e^5*f^6*g^3 + 42*a*b^7*c^5*d^5*e^4*f^5*g^
4 + 128*a*b^7*c^5*d^6*e^3*f^4*g^5 + 62*a*b^7*c^5*d^7*e^2*f^3*g^6 + 4*a*b^8*
c^4*d^2*e^7*f^7*g^2 - 76*a*b^8*c^4*d^3*e^6*f^6*g^3 - 48*a*b^8*c^4*d^4*e^5*f
^5*g^4 - 48*a*b^8*c^4*d^5*e^4*f^4*g^5 - 76*a*b^8*c^4*d^6*e^3*f^3*g^6 + 4*a*
b^8*c^4*d^7*e^2*f^2*g^7 - 6*a*b^9*c^3*d^2*e^7*f^6*g^3 + 28*a*b^9*c^3*d^3*e^
6*f^5*g^4 - 20*a*b^9*c^3*d^4*e^5*f^4*g^5 + 28*a*b^9*c^3*d^5*e^4*f^3*g^6 - 6
*a*b^9*c^3*d^6*e^3*f^2*g^7 + 4*a*b^10*c^2*d^2*e^7*f^5*g^4 + 14*a*b^10*c^2*d
^3*e^6*f^4*g^5 + 14*a*b^10*c^2*d^4*e^5*f^3*g^6 + 4*a*b^10*c^2*d^5*e^4*f^2*g
^7 - 32*a^2*b*c^10*d^7*e^2*f^7*g^2 + 48*a^2*b^2*c^9*d^5*e^4*f^8*g + 48*a^2*
b^2*c^9*d^8*e*f^5*g^4 - 168*a^2*b^3*c^8*d^4*e^5*f^8*g - 168*a^2*b^3*c^8*d^8
*e*f^4*g^5 + 80*a^2*b^4*c^7*d^3*e^6*f^8*g + 80*a^2*b^4*c^7*d^8*e*f^3*g^6 +
27*a^2*b^5*c^6*d^2*e^7*f^8*g + 27*a^2*b^5*c^6*d^8*e*f^2*g^7 + 4*a^2*b^7*c^4
*d*e^8*f^7*g^2 + 4*a^2*b^7*c^4*d^7*e^2*f*g^8 - 6*a^2*b^8*c^3*d*e^8*f^6*g^3
- 6*a^2*b^8*c^3*d^6*e^3*f*g^8 + 4*a^2*b^9*c^2*d*e^8*f^5*g^4 + 4*a^2*b^9*c^2
*d^5*e^4*f*g^8 + 16*a^2*b^10*c*d^2*e^7*f^3*g^6 + 16*a^2*b^10*c*d^3*e^6*f^2*
g^7 + 224*a^3*b*c^9*d^5*e^4*f^7*g^2 - 288*a^3*b*c^9*d^6*e^3*f^6*g^3 + 224*a
^3*b*c^9*d^7*e^2*f^5*g^4 - 32*a^3*b^2*c^8*d^3*e^6*f^8*g - 32*a^3*b^2*c^8*d^
8*e*f^3*g^6 - 168*a^3*b^3*c^7*d^2*e^7*f^8*g - 168*a^3*b^3*c^7*d^8*e*f^2*g^7
 - 14*a^3*b^5*c^5*d*e^8*f^7*g^2 - 14*a^3*b^5*c^5*d^7*e^2*f*g^8 + 40*a^3*b^6
*c^4*d*e^8*f^6*g^3 + 40*a^3*b^6*c^4*d^6*e^3*f*g^8 - 44*a^3*b^7*c^3*d*e^8*f^
5*g^4 - 44*a^3*b^7*c^3*d^5*e^4*f*g^8 + 24*a^3*b^8*c^2*d*e^8*f^4*g^5 + 24*a^
3*b^8*c^2*d^4*e^5*f*g^8 - 30*a^3*b^9*c*d^2*e^7*f^2*g^7 + 544*a^4*b*c^8*d^3*
e^6*f^7*g^2 + 256*a^4*b*c^8*d^4*e^5*f^6*g^3 + 1632*a^4*b*c^8*d^5*e^4*f^5*g^
4 + 256*a^4*b*c^8*d^6*e^3*f^4*g^5 + 544*a^4*b*c^8*d^7*e^2*f^3*g^6 - 80*a^4*
b^3*c^6*d*e^8*f^7*g^2 - 80*a^4*b^3*c^6*d^7*e^2*f*g^8 - 60*a^4*b^4*c^5*d*e^8
*f^6*g^3 - 60*a^4*b^4*c^5*d^6*e^3*f*g^8 + 234*a^4*b^5*c^4*d*e^8*f^5*g^4 + 2
34*a^4*b^5*c^4*d^5*e^4*f*g^8 - 208*a^4*b^6*c^3*d*e^8*f^4*g^5 - 208*a^4*b^6*
c^3*d^4*e^5*f*g^8 + 50*a^4*b^7*c^2*d*e^8*f^3*g^6 + 50*a^4*b^7*c^2*d^3*e^6*f
*g^8 + 416*a^5*b*c^7*d^2*e^7*f^6*g^3 + 2592*a^5*b*c^7*d^3*e^6*f^5*g^4 + 105
6*a^5*b*c^7*d^4*e^5*f^4*g^5 + 2592*a^5*b*c^7*d^5*e^4*f^3*g^6 + 416*a^5*b*c^
7*d^6*e^3*f^2*g^7 + 96*a^5*b^2*c^6*d*e^8*f^6*g^3 + 96*a^5*b^2*c^6*d^6*e^3*f
*g^8 - 784*a^5*b^3*c^5*d*e^8*f^5*g^4 - 784*a^5*b^3*c^5*d^5*e^4*f*g^8 + 732*
a^5*b^4*c^4*d*e^8*f^4*g^5 + 732*a^5*b^4*c^4*d^4*e^5*f*g^8 - 18*a^5*b^5*c^3*
d*e^8*f^3*g^6 - 18*a^5*b^5*c^3*d^3*e^6*f*g^8 - 184*a^5*b^6*c^2*d*e^8*f^2*g^
7 - 184*a^5*b^6*c^2*d^2*e^7*f*g^8 + 1024*a^6*b*c^6*d^2*e^7*f^4*g^5 + 3552*a
^6*b*c^6*d^3*e^6*f^3*g^6 + 1024*a^6*b*c^6*d^4*e^5*f^2*g^7 - 736*a^6*b^2*c^5
*d*e^8*f^4*g^5 - 736*a^6*b^2*c^5*d^4*e^5*f*g^8 - 720*a^6*b^3*c^4*d*e^8*f^3*
g^6 - 720*a^6*b^3*c^4*d^3*e^6*f*g^8 + 684*a^6*b^4*c^3*d*e^8*f^2*g^7 + 684*a
^6*b^4*c^3*d^2*e^7*f*g^8 + 992*a^7*b*c^5*d^2*e^7*f^2*g^7 - 736*a^7*b^2*c^4*
d*e^8*f^2*g^7 - 736*a^7*b^2*c^4*d^2*e^7*f*g^8 - 10*a^5*b^7*c*d*e^8*f*g^8 +
608*a^8*b*c^4*d*e^8*f*g^8 - 144*a^2*b^3*c^8*d^5*e^4*f^7*g^2 + 48*a^2*b^3*c^
8*d^6*e^3*f^6*g^3 - 144*a^2*b^3*c^8*d^7*e^2*f^5*g^4 + 524*a^2*b^4*c^7*d^4*e
^5*f^7*g^2 + 44*a^2*b^4*c^7*d^5*e^4*f^6*g^3 + 44*a^2*b^4*c^7*d^6*e^3*f^5*g^
4 + 524*a^2*b^4*c^7*d^7*e^2*f^4*g^5 - 270*a^2*b^5*c^6*d^3*e^6*f^7*g^2 - 480
*a^2*b^5*c^6*d^4*e^5*f^6*g^3 + 246*a^2*b^5*c^6*d^5*e^4*f^5*g^4 - 480*a^2*b^
5*c^6*d^6*e^3*f^4*g^5 - 270*a^2*b^5*c^6*d^7*e^2*f^3*g^6 - 90*a^2*b^6*c^5*d^
2*e^7*f^7*g^2 + 286*a^2*b^6*c^5*d^3*e^6*f^6*g^3 - 180*a^2*b^6*c^5*d^4*e^5*f
^5*g^4 - 180*a^2*b^6*c^5*d^5*e^4*f^4*g^5 + 286*a^2*b^6*c^5*d^6*e^3*f^3*g^6
- 90*a^2*b^6*c^5*d^7*e^2*f^2*g^7 + 104*a^2*b^7*c^4*d^2*e^7*f^6*g^3 + 4*a^2*
b^7*c^4*d^3*e^6*f^5*g^4 + 520*a^2*b^7*c^4*d^4*e^5*f^4*g^5 + 4*a^2*b^7*c^4*d
^5*e^4*f^3*g^6 + 104*a^2*b^7*c^4*d^6*e^3*f^2*g^7 - 30*a^2*b^8*c^3*d^2*e^7*f

^5*g^4 - 186*a^2*b^8*c^3*d^3*e^6*f^4*g^5 - 186*a^2*b^8*c^3*d^4*e^5*f^3*g^6 - 30*a^2*b^8*c^3*d^5*e^4*f^2*g^7 - 27*a^2*b^9*c^2*d^2*e^7*f^4*g^5 + 70*a^2*b^9*c^2*d^3*e^6*f^3*g^6 - 27*a^2*b^9*c^2*d^4*e^5*f^2*g^7 - 928*a^3*b^2*c^8*d^4*e^5*f^7*g^2 + 288*a^3*b^2*c^8*d^5*e^4*f^6*g^3 + 288*a^3*b^2*c^8*d^6*e^3*f^5*g^4 - 928*a^3*b^2*c^8*d^7*e^2*f^4*g^5 + 208*a^3*b^3*c^7*d^3*e^6*f^7*g^2 + 512*a^3*b^3*c^7*d^4*e^5*f^6*g^3 - 1424*a^3*b^3*c^7*d^5*e^4*f^5*g^4 + 512*a^3*b^3*c^7*d^6*e^3*f^4*g^5 + 208*a^3*b^3*c^7*d^7*e^2*f^3*g^6 + 540*a^3*b^4*c^6*d^2*e^7*f^7*g^2 - 228*a^3*b^4*c^6*d^3*e^6*f^6*g^3 + 1428*a^3*b^4*c^6*d^4*e^5*f^5*g^4 + 1428*a^3*b^4*c^6*d^5*e^4*f^4*g^5 - 228*a^3*b^4*c^6*d^6*e^3*f^3*g^6 + 540*a^3*b^4*c^6*d^7*e^2*f^2*g^7 - 518*a^3*b^5*c^5*d^2*e^7*f^6*g^3 - 190*a^3*b^5*c^5*d^3*e^6*f^5*g^4 - 2110*a^3*b^5*c^5*d^4*e^5*f^4*g^5 - 190*a^3*b^5*c^5*d^5*e^4*f^3*g^6 - 518*a^3*b^5*c^5*d^6*e^3*f^2*g^7 - 88*a^3*b^6*c^4*d^2*e^7*f^5*g^4 + 368*a^3*b^6*c^4*d^3*e^6*f^4*g^5 + 368*a^3*b^6*c^4*d^4*e^5*f^3*g^6 - 88*a^3*b^6*c^4*d^5*e^4*f^2*g^7 + 404*a^3*b^7*c^3*d^2*e^7*f^4*g^5 + 12*a^3*b^7*c^3*d^3*e^6*f^3*g^6 + 404*a^3*b^7*c^3*d^4*e^5*f^2*g^7 - 140*a^3*b^8*c^2*d^2*e^7*f^3*g^6 - 140*a^3*b^8*c^2*d^3*e^6*f^2*g^7 - 1024*a^4*b^2*c^7*d^2*e^7*f^7*g^2 - 128*a^4*b^2*c^7*d^3*e^6*f^6*g^3 - 2016*a^4*b^2*c^7*d^4*e^5*f^5*g^4 - 2016*a^4*b^2*c^7*d^5*e^4*f^4*g^5 - 128*a^4*b^2*c^7*d^6*e^3*f^3*g^6 - 1024*a^4*b^2*c^7*d^7*e^2*f^2*g^7 + 688*a^4*b^3*c^6*d^2*e^7*f^6*g^3 - 720*a^4*b^3*c^6*d^3*e^6*f^5*g^4 + 2160*a^4*b^3*c^6*d^4*e^5*f^4*g^5 - 720*a^4*b^3*c^6*d^5*e^4*f^3*g^6 + 688*a^4*b^3*c^6*d^6*e^3*f^2*g^7 + 1124*a^4*b^4*c^5*d^2*e^7*f^5*g^4 + 1060*a^4*b^4*c^5*d^3*e^6*f^4*g^5 + 1060*a^4*b^4*c^5*d^4*e^5*f^3*g^6 + 1124*a^4*b^4*c^5*d^5*e^4*f^2*g^7 - 1616*a^4*b^5*c^4*d^2*e^7*f^4*g^5 - 674*a^4*b^5*c^4*d^3*e^6*f^3*g^6 - 1616*a^4*b^5*c^4*d^4*e^5*f^2*g^7 + 186*a^4*b^6*c^3*d^2*e^7*f^3*g^6 + 186*a^4*b^6*c^3*d^3*e^6*f^2*g^7 + 334*a^4*b^7*c^2*d^2*e^7*f^2*g^7 - 2208*a^5*b^2*c^6*d^2*e^7*f^5*g^4 - 2592*a^5*b^2*c^6*d^3*e^6*f^4*g^5 - 2592*a^5*b^2*c^6*d^4*e^5*f^3*g^6 - 2208*a^5*b^2*c^6*d^5*e^4*f^2*g^7 + 1728*a^5*b^3*c^5*d^2*e^7*f^4*g^5 - 304*a^5*b^3*c^5*d^3*e^6*f^3*g^6 + 1728*a^5*b^3*c^5*d^4*e^5*f^2*g^7 + 1108*a^5*b^4*c^4*d^2*e^7*f^3*g^6 + 1108*a^5*b^4*c^4*d^3*e^6*f^2*g^7 - 1170*a^5*b^5*c^3*d^2*e^7*f^2*g^7 - 2432*a^6*b^2*c^5*d^2*e^7*f^3*g^6 - 2432*a^6*b^2*c^5*d^3*e^6*f^2*g^7 + 1008*a^6*b^3*c^4*d^2*e^7*f^2*g^7 - 8*a*b^3*c^9*d^6*e^3*f^8*g - 8*a*b^3*c^9*d^8*e*f^6*g^3 + 27*a*b^5*c^7*d^4*e^5*f^8*g + 27*a*b^5*c^7*d^8*e*f^4*g^5 - 18*a*b^6*c^6*d^3*e^6*f^8*g - 18*a*b^6*c^6*d^8*e*f^3*g^6 - a*b^7*c^5*d^2*e^7*f^8*g - a*b^7*c^5*d^8*e*f^2*g^7 - a*b^11*c*d^2*e^7*f^4*g^5 - 10*a*b^11*c*d^3*e^6*f^3*g^6 - a*b^11*c*d^4*e^5*f^2*g^7 + 16*a^2*b*c^10*d^6*e^3*f^8*g + 16*a^2*b*c^10*d^8*e*f^6*g^3 - a^2*b^6*c^5*d*e^8*f^8*g - a^2*b^6*c^5*d^8*e*f*g^8 - a^2*b^10*c*d*e^8*f^4*g^5 - a^2*b^10*c*d^4*e^5*f*g^8 + 304*a^3*b*c^9*d^4*e^5*f^8*g + 304*a^3*b*c^9*d^8*e*f^4*g^5 - 6*a^3*b^9*c*d*e^8*f^3*g^6 - 6*a^3*b^9*c*d^3*e^6*f*g^8 + 304*a^4*b*c^8*d^2*e^7*f^8*g + 304*a^4*b*c^8*d^8*e*f^2*g^7 + 48*a^4*b^2*c^7*d*e^8*f^8*g + 48*a^4*b^2*c^7*d^8*e*f*g^8 + 16*a^4*b^8*c*d*e^8*f^2*g^7 + 16*a^4*b^8*c*d^2*e^7*f*g^8 + 288*a^5*b*c^7*d*e^8*f^7*g^2 + 288*a^5*b*c^7*d^7*e^2*f*g^8 + 1184*a^6*b*c^6*d*e^8*f^5*g^4 + 1184*a^6*b*c^6*d^5*e^4*f*g^8 + 118*a^6*b^5*c^2*d*e^8*f*g^8 + 1504*a^7*b*c^5*d*e^8*f^3*g^6 + 1504*a^7*b*c^5*d^3*e^6*f*g^8 - 464*a^7*b^3*c^3*d*e^8*f*g^8)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^

3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) - (x*(128*a^9*c^4*e^9*g^9 + 24*a^7*b^4*c^2*e^9*g^9 - 96*a^8*b^2*c^3*e^9*g^9 + 288*a^6*c^7*d^6*e^3*g^9 + 416*a^7*c^6*d^4*e^5*g^9 + 352*a^8*c^5*d^2*e^7*g^9 + 288*a^6*c^7*e^9*f^6*g^3 + 416*a^7*c^6*e^9*f^4*g^5 + 352*a^8*c^5*e^9*f^2*g^7 - 2*a^6*b^6*c*e^9*g^9 + 96*a^5*c^8*d^8*e*g^9 + 96*a^5*c^8*e^9*f^8*g + 6*a^5*b^7*c*d*e^8*g^9 - 384*a^8*b*c^4*d*e^8*g^9 + 6*a^5*b^7*c*e^9*f*g^8 - 384*a^8*b*c^4*e^9*f*g^8 + 64*a^8*c^5*d*e^8*f*g^8 - 2*a^2*b^6*c^5*d^8*e*g^9 - 2*a^2*b^10*c*d^4*e^5*g^9 + 22*a^3*b^4*c^6*d^8*e*g^9 + 6*a^3*b^9*c*d^3*e^6*g^9 - 80*a^4*b^2*c^7*d^8*e*g^9 - 8*a^4*b^8*c*d^2*e^7*g^9 - 416*a^5*b*c^7*d^7*e^2*g^9 - 960*a^6*b*c^6*d^5*e^4*g^9 - 72*a^6*b^5*c^2*d*e^8*g^9 - 928*a^7*b*c^5*d^3*e^6*g^9 + 288*a^7*b^3*c^3*d*e^8*g^9 - 2*a^2*b^6*c^5*e^9*f^8*g - 2*a^2*b^10*c*e^9*f^4*g^5 + 22*a^3*b^4*c^6*e^9*f^8*g + 6*a^3*b^9*c*e^9*f^3*g^6 - 80*a^4*b^2*c^7*e^9*f^8*g - 8*a^4*b^8*c*e^9*f^2*g^7 - 416*a^5*b*c^7*e^9*f^7*g^2 - 960*a^6*b*c^6*e^9*f^5*g^4 - 72*a^6*b^5*c^2*e^9*f*g^8 - 928*a^7*b*c^5*e^9*f^3*g^6 + 288*a^7*b^3*c^3*e^9*f*g^8 - 32*a^2*c^11*d^6*e^3*f^8*g - 32*a^2*c^11*d^8*e*f^6*g^3 + 32*a^3*c^10*d^4*e^5*f^8*g + 32*a^3*c^10*d^8*e*f^4*g^5 + 160*a^4*c^9*d^2*e^7*f^8*g + 160*a^4*c^9*d^8*e*f^2*g^7 + 64*a^5*c^8*d*e^8*f^7*g^2 + 64*a^5*c^8*d^7*e^2*f*g^8 + 192*a^6*c^7*d*e^8*f^5*g^4 + 192*a^6*c^7*d^5*e^4*f*g^8 + 192*a^7*c^6*d*e^8*f^3*g^6 + 192*a^7*c^6*d^3*e^6*f*g^8 - 2*b^4*c^9*d^6*e^3*f^8*g - 2*b^4*c^9*d^8*e*f^6*g^3 + 6*b^5*c^8*d^5*e^4*f^8*g + 6*b^5*c^8*d^8*e*f^5*g^4 - 8*b^6*c^7*d^4*e^5*f^8*g - 8*b^6*c^7*d^8*e*f^4*g^5 + 6*b^7*c^6*d^3*e^6*f^8*g + 6*b^7*c^6*d^8*e*f^3*g^6 - 2*b^8*c^5*d^2*e^7*f^8*g - 2*b^8*c^5*d^8*e*f^2*g^7 - 2*b^12*c*d^2*e^7*f^4*g^5 + 2*b^12*c*d^3*e^6*f^3*g^6 - 2*b^12*c*d^4*e^5*f^2*g^7 + 8*a^2*b^7*c^4*d^7*e^2*g^9 - 12*a^2*b^8*c^3*d^6*e^3*g^9 + 8*a^2*b^9*c^2*d^5*e^4*g^9 - 90*a^3*b^5*c^5*d^7*e^2*g^9 + 132*a^3*b^6*c^4*d^6*e^3*g^9 - 76*a^3*b^7*c^3*d^5*e^4*g^9 + 6*a^3*b^8*c^2*d^4*e^5*g^9 + 336*a^4*b^3*c^6*d^7*e^2*g^9 - 462*a^4*b^4*c^5*d^6*e^3*g^9 + 164*a^4*b^5*c^4*d^5*e^4*g^9 + 106*a^4*b^6*c^3*d^4*e^5*g^9 - 56*a^4*b^7*c^2*d^3*e^6*g^9 + 432*a^5*b^2*c^6*d^6*e^3*g^9 + 288*a^5*b^3*c^5*d^5*e^4*g^9 - 598*a^5*b^4*c^4*d^4*e^5*g^9 + 102*a^5*b^5*c^3*d^3*e^6*g^9 + 90*a^5*b^6*c^2*d^2*e^7*g^9 + 720*a^6*b^2*c^5*d^4*e^5*g^9 + 336*a^6*b^3*c^4*d^3*e^6*g^9 - 314*a^6*b^4*c^3*d^2*e^7*g^9 + 240*a^7*b^2*c^4*d^2*e^7*g^9 + 8*a^2*b^7*c^4*e^9*f^7*g^2 - 12*a^2*b^8*c^3*e^9*f^6*g^3 + 8*a^2*b^9*c^2*e^9*f^5*g^4 - 90*a^3*b^5*c^5*e^9*f^7*g^2 + 132*a^3*b^6*c^4*e^9*f^6*g^3 - 76*a^3*b^7*c^3*e^9*f^5*g^4 + 6*a^3*b^8*c^2*e^9*f^4*g^5 + 336*a^4*b^3*c^6*e^9*f^7*g^2 - 462*a^4*b^4*c^5*e^9*f^6*g^3 + 164*a^4*b^5*c^4*e^9*f^5*g^4 + 106*a^4*b^6*c^3*e^9*f^4*g^5 - 56*a^4*b^7*c^2*e^9*f^3*g^6 + 432*a^5*b^2*c^6*e^9*f^6*g^3 + 288*a^5*b^3*c^5*e^9*f^5*g^4 - 598*a^5*b^4*c^4*e^9*f^4*g^5 + 102*a^5*b^5*c^3*e^9*f^3*g^6 + 90*a^5*b^6*c^2*e^9*f^2*g^7 + 720*a^6*b^2*c^5*e^9*f^4*g^5 + 336*a^6*b^3*c^4*e^9*f^3*g^6 - 314*a^6*b^4*c^3*e^9*f^2*g^7 + 240*a^7*b^2*c^4*e^9*f^2*g^7 + 64*a^2*c^11*d^7*e^2*f^7*g^2 + 192*a^3*c^10*d^5*e^4*f^7*g^2 - 320*a^3*c^10*d^6*e^3*f^6*g^3 + 192*a^3*c^10*d^7*e^2*f^5*g^4 + 192*a^4*c^9*d^3*e^6*f^7*g^2 - 256*a^4*c^9*d^4*e^5*f^6*g^3 + 576*a^4*c^9*d^5*e^4*f^5*g^4 - 256*a^4*c^9*d^6*e^3*f^4*g^5 + 192*a^4*c^9*d^7*e^2*f^3*g^6 + 320*a^5*c^8*d^2*e^7*f^6*g^3 + 576*a^5*c^8*d^3*e^6*f^5*g^4 - 192*a^5*c^8*d^4*e^5*f^4*g^5 + 576*a^5*c^8*d^5*e^4*f^3*g^6 + 320*a^5*c^8*d^6*e^3*f^2*g^7 + 512*a^6*c^7*d^2*e^7*f^4*g^5 + 576*a^6*c^7*d^3*e^6*f^3*g^6 + 512*a^6*c^7*d^4*e^5*f^2*g^7 + 704*a^7*c^6*d^2*e^7*f^2*g^7 + 4*b^4*c^9*d^7*e^2*f^7*g^2 - 6*b^5*c^8*d^6*e^3*f^7* g^2 - 6*b^5*c^8*d^7*e^2*f^6*g^3 - 6*b^6*c^7*d^5*e^4*f^7*g^2 + 26*b^6*c^7*d^6*e^3*f^6*g^3 - 6*b^6*c^7*d^7*e^2*f^5*g^4 + 22*b^7*c^6*d^4*e^5*f^7*g^2 - 22*b^7*c^6*d^5*e^4*f^6*g^3 - 22*b^7*c^6*d^6*e^3*f^5*g^4 + 22*b^7*c^6*d^7*e^2*f^4*g^5 - 22*b^8*c^5*d^3*e^6*f^7*g^2 - 12*b^8*c^5*d^4*e^5*f^6*g^3 + 42*b^8*c^5*d^5*e^4*f^5*g^4 - 12*b^8*c^5*d^6*e^3*f^4*g^5 - 22*b^8*c^5*d^7*e^2*f^3*g^6 + 8*b^9*c^4*d^2*e^7*f^7*g^2 + 28*b^9*c^4*d^3*e^6*f^6*g^3 - 16*b^9*c^4*d^4*e^5*f^5*g^4 - 16*b^9*c^4*d^5*e^4*f^4*g^5 + 28*b^9*c^4*d^6*e^3*f^3*g^6 + 8*b^9*c^4*d^7*e^2*f^2*g^7 - 12*b^10*c^3*d^2*e^7*f^6*g^3 - 12*b^10*c^3*d^3*e^6*f^5*g^4 + 18*b^10*c^3*d^4*e^5*f^4*g^5 - 12*b^10*c^3*d^5*e^4*f^3*g^6 - 12*b^10*c^3*d^6*e^3*f^2*g^7 + 8*b^11*c^2*d^2*e^7*f^5*g^4 - 2*b^11*c^2*d^3*e^6*f^4*g^5 - 2*b^11*c^2*d^4*e^5*f^3*g^6 + 8*b^11*c^2*d^5*e^4*f^2*g^7 - 32*a*b^2*c^10*d^7*e^2*f^7*g^2 + 48*a*b^3*c^9*d^6*e^3*f^7*g^2 + 48*a*b^3*c^9*d^7*e^2*f^6*g^3 + 60*a*b^4*c^8*d^5*e^4*f^7*g^2 - 228*a*b^4*c^8*d^6*e^3*f^6*g^3 + 60*a*b^4*c^8*d^7*e^2*f^5*g^4 - 214*a*b^5*c^7*d^4*e^5*f^7*g^2 + 194*a*b^5*c^7*d^5*e^4*f^6*g^3 + 194*a*b^5*c^7*d^6*e^3*f^5*g^4 - 214*a*b^5*c^7*d^7*e^2*f^4*g^5 + 216*a*b^6*c^6*d^3*e^6*f^7*g^2 + 148*a*b^6*c^6*d^4*e^5*f^6*g^3 - 408*a*b^6*c^6*d^5*e^4*f^5*g^4 + 148*a*b^6*c^6*d^6*e^3*f^4*g^5 + 216*a*b^6*c^6*d^7*e^2*f^3*g^6 - 62*a*b^7*c^5*d^2*e^7*f^7*g^2 - 302*a*b^7*c^5*d^3*e^6*f^6*g^3 + 150*a*b^7*c^5*d^4*e^5*f^5*g^4 + 150*a*b^7*c^5*d^5*e^4*f^4*g^5 - 302*a*b^7*c^5*d^6*e^3*f^3*g^6 - 62*a*b^7*c^5*d^7*e^2*f^2*g^7 + 100*a*b^8*c^4*d^2*e^7*f^6*g^3 + 136*a*b^8*c^4*d^3*e^6*f^5*g^4 - 200*a*b^8*c^4*d^4*e^5*f^4*g^5 + 136*a*b^8*c^4*d^5*e^4*f^3*g^6 + 100*a*b^8*c^4*d^6*e^3*f^2*g^7 - 68*a*b^9*c^3*d^2*e^7*f^5*g^4 + 32*a*b^9*c^3*d^3*e^6*f^4*g^5 + 32*a*b^9*c^3*d^4*e^5*f^3*g^6 - 68*a*b^9*c^3*d^5*e^4*f^2*g^7 + 14*a*b^10*c^2*d^2*e^7*f^4*g^5 - 32*a*b^10*c^2*d^3*e^6*f^3*g^6 + 14*a*b^10*c^2*d^4*e^5*f^2*g^7 - 96*a^2*b*c^10*d^6*e^3*f^7*g^2 - 96*a^2*b*c^10*d^7*e^2*f^6*g^3 - 144*a^2*b^2*c^9*d^4*e^5*f^8*g - 144*a^2*b^2*c^9*d^8*e*f^4*g^5 + 128*a^2*b^3*c^8*d^3*e^6*f^8*g + 128*a^2*b^3*c^8*d^8*e*f^3*g^6 - 6*a^2*b^4*c^7*d^2*e^7*f^8*g - 6*a^2*b^4*c^7*d^8*e*f^2*g^7 + 174*a^2*b^6*c^5*d*e^8*f^7*g^2 + 174*a^2*b^6*c^5*d^7*e^2*f*g^8 - 260*a^2*b^7*c^4*d*e^8*f^6*g^3 - 260*a^2*b^7*c^4*d^6*e^3*f*g^8 + 156*a^2*b^8*c^3*d*e^8*f^5*g^4 + 156*a^2*b^8*c^3*d^5*e^4*f*g^8 - 18*a^2*b^9*c^2*d*e^8*f^4*g^5 - 18*a^2*b^9*c^2*d^4*e^5*f*g^8 - 6*a^2*b^10*c*d^2*e^7*f^2*g^7 - 608*a^3*b*c^9*d^4*e^5*f^7*g^2 + 288*a^3*b*c^9*d^5*e^4*f^6*g^3 + 288*a^3*b*c^9*d^6*e^3*f^5*g^4 - 608*a^3*b*c^9*d^7*e^2*f^4*g^5 - 112*a^3*b^2*c^8*d^2*e^7*f^8*g - 112*a^3*b^2*c^8*d^8*e*f^2*g^7 - 620*a^3*b^4*c^6*d*e^8*f^7*g^2 - 620*a^3*b^4*c^6*d^7*e^2*f*g^8 + 894*a^3*b^5*c^5*d*e^8*f^6*g^3 + 894*a^3*b^5*c^5*d^6*e^3*f*g^8 - 384*a^3*b^6*c^4*d*e^8*f^5*g^4 - 384*a^3*b^6*c^4*d^5*e^4*f*g^8 - 140*a^3*b^7*c^3*d*e^8*f^4*g^5 - 140*a^3*b^7*c^3*d^4*e^5*f*g^8 + 92*a^3*b^8*c^2*d*e^8*f^3*g^6 + 92*a^3*b^8*c^2*d^3*e^6*f*g^8 - 928*a^4*b*c^8*d^2*e^7*f^7*g^2 - 160*a^4*b*c^8*d^3*e^6*f^6*g^3 - 672*a^4*b*c^8*d^4*e^5*f^5*g^4 - 672*a^4*b*c^8*d^5*e^4*f^4*g^5 - 160*a^4*b*c^8*d^6*e^3*f^3*g^6 - 928*a^4*b*c^8*d^7*e^2*f^2*g^7 + 704*a^4*b^2*c^7*d*e^8*f^7*g^2 + 704*a^4*b^2*c^7*d^7*e^2*f*g^8 - 816*a^4*b^3*c^6*d*e^8*f^6*g^3 - 816*a^4*b^3*c^6*d^6*e^3*f*g^8 - 308*a^4*b^4*c^5*d*e^8*f^5*g^4 - 308*a^4*b^4*c^5*d^5*e^4*f*g^8 + 898*a^4*b^5*c^4*d*e^8*f^4*g^5 + 898*a^4*b^5*c^4*d^4*e^5*f*g^8 - 150*a^4*b^6*c^3*d*e^8*f^3*g^6 - 150*a^4*b^6*c^3*d^3*e^6*f*g^8 - 154*a^4*b^7*c^2*d*e^8*f^2*g^7 - 154*a^4*b^7*c^2*d^2*e^7*f*g^8 - 1824*a^5*b*c^7*d^2*e^7*f^5*g^4 - 1056*a^5*b*c^7*d^3*e^6*f^4*g^5 - 1056*a^5*b*c^7*d^4*e^5*f^3*g^6 - 1824*a^5*b*c^7*d^5*e^4*f^2*g^7 + 1440*a^5*b^2*c^6*d*e^8*f^5*g^4 + 1440*a^5*b^2*c^6*d^5*e^4*f*g^8 - 976*a^5*b^3*c^5*d*e^8*f^4*g^5 - 976*a^5*b^3*c^5*d^4*e^5*f*g^8 - 644*a^5*b^4*c^4*d*e^8*f^3*g^6 - 644*a^5*b^4*c^4*d^3*e^6*f*g^8 + 498*a^5*b^5*c^3*d*e^8*f^2*g^7 + 498*a^5*b^5*c^3*d^2*e^7*f*g^8 - 1888*a^6*b*c^6*d^2*e^7*f^3*g^6 - 1888*a^6*b*c^6*d^3*e^6*f^2*g^7 + 1600*a^6*b^2*c^5*d*e^8*f^3*g^6 + 1600*a^6*b^2*c^5*d^3*e^6*f*g^8 - 176*a^6*b^3*c^4*d*e^8*f^2*g^7 - 176*a^6*b^3*c^4*d^2*e^7*f*g^8 + 4*a*b^7*c^5*d*e^8*f^8*g + 4*a*b^7*c^5*d^8*e*f*g^8 + 4*a*b^11*c*d*e^8*f^4*g^5 + 4*a*b^11*c*d^4*e^5*f*g^8 - 160*a^4*b*c^8*d*e^8*f^8*g - 160*a^4*b*c^8*d^8*e*f*g^8 - 14*a^4*b^8*c*d*e^8*f*g^8 - 192*a^2*b^2*c^9*d^5*e^4*f^7*g^2 + 576*a^2*b^2*c^9*d^6*e^3*f^6*g^3 - 192*a^2*b^2*c^9*d^7*e^2*f^5*g^4 + 656*a^2*b^3*c^8*d^4*e^5*f^7*g^2 - 496*a^2*b^3*c^8*d^5*e^4*f^6

*g^3 - 496*a^2*b^3*c^8*d^6*e^3*f^5*g^4 + 656*a^2*b^3*c^8*d^7*e^2*f^4*g^5 -
660*a^2*b^4*c^7*d^3*e^6*f^7*g^2 - 624*a^2*b^4*c^7*d^4*e^5*f^6*g^3 + 1284*a^
2*b^4*c^7*d^5*e^4*f^5*g^4 - 624*a^2*b^4*c^7*d^6*e^3*f^4*g^5 - 660*a^2*b^4*c
^7*d^7*e^2*f^3*g^6 + 54*a^2*b^5*c^6*d^2*e^7*f^7*g^2 + 1062*a^2*b^5*c^6*d^3*
e^6*f^6*g^3 - 474*a^2*b^5*c^6*d^4*e^5*f^5*g^4 - 474*a^2*b^5*c^6*d^5*e^4*f^4
*g^5 + 1062*a^2*b^5*c^6*d^6*e^3*f^3*g^6 + 54*a^2*b^5*c^6*d^7*e^2*f^2*g^7 -
130*a^2*b^6*c^5*d^2*e^7*f^6*g^3 - 482*a^2*b^6*c^5*d^3*e^6*f^5*g^4 + 850*a^2
*b^6*c^5*d^4*e^5*f^4*g^5 - 482*a^2*b^6*c^5*d^5*e^4*f^3*g^6 - 130*a^2*b^6*c^
5*d^6*e^3*f^2*g^7 + 108*a^2*b^7*c^4*d^2*e^7*f^5*g^4 - 228*a^2*b^7*c^4*d^3*e
^6*f^4*g^5 - 228*a^2*b^7*c^4*d^4*e^5*f^3*g^6 + 108*a^2*b^7*c^4*d^5*e^4*f^2*
g^7 - 18*a^2*b^8*c^3*d^2*e^7*f^4*g^5 + 192*a^2*b^8*c^3*d^3*e^6*f^3*g^6 - 18
*a^2*b^8*c^3*d^4*e^5*f^2*g^7 - 2*a^2*b^9*c^2*d^2*e^7*f^3*g^6 - 2*a^2*b^9*c^
2*d^3*e^6*f^2*g^7 + 544*a^3*b^2*c^8*d^3*e^6*f^7*g^2 + 960*a^3*b^2*c^8*d^4*e
^5*f^6*g^3 - 1440*a^3*b^2*c^8*d^5*e^4*f^5*g^4 + 960*a^3*b^2*c^8*d^6*e^3*f^4
*g^5 + 544*a^3*b^2*c^8*d^7*e^2*f^3*g^6 + 496*a^3*b^3*c^7*d^2*e^7*f^7*g^2 -
1168*a^3*b^3*c^7*d^3*e^6*f^6*g^3 + 688*a^3*b^3*c^7*d^4*e^5*f^5*g^4 + 688*a^
3*b^3*c^7*d^5*e^4*f^4*g^5 - 1168*a^3*b^3*c^7*d^6*e^3*f^3*g^6 + 496*a^3*b^3*
c^7*d^7*e^2*f^2*g^7 - 668*a^3*b^4*c^6*d^2*e^7*f^6*g^3 + 436*a^3*b^4*c^6*d^3
*e^6*f^5*g^4 - 1820*a^3*b^4*c^6*d^4*e^5*f^4*g^5 + 436*a^3*b^4*c^6*d^5*e^4*f
^3*g^6 - 668*a^3*b^4*c^6*d^6*e^3*f^2*g^7 + 238*a^3*b^5*c^5*d^2*e^7*f^5*g^4
+ 734*a^3*b^5*c^5*d^3*e^6*f^4*g^5 + 734*a^3*b^5*c^5*d^4*e^5*f^3*g^6 + 238*a
^3*b^5*c^5*d^5*e^4*f^2*g^7 + 144*a^3*b^6*c^4*d^2*e^7*f^4*g^5 - 416*a^3*b^6*
c^4*d^3*e^6*f^3*g^6 + 144*a^3*b^6*c^4*d^4*e^5*f^2*g^7 - 156*a^3*b^7*c^3*d^2
*e^7*f^3*g^6 - 156*a^3*b^7*c^3*d^3*e^6*f^2*g^7 + 44*a^3*b^8*c^2*d^2*e^7*f^2
*g^7 + 1344*a^4*b^2*c^7*d^2*e^7*f^6*g^3 + 192*a^4*b^2*c^7*d^3*e^6*f^5*g^4 +
1920*a^4*b^2*c^7*d^4*e^5*f^4*g^5 + 192*a^4*b^2*c^7*d^5*e^4*f^3*g^6 + 1344*
a^4*b^2*c^7*d^6*e^3*f^2*g^7 + 80*a^4*b^3*c^6*d^2*e^7*f^5*g^4 - 560*a^4*b^3*
c^6*d^3*e^6*f^4*g^5 - 560*a^4*b^3*c^6*d^4*e^5*f^3*g^6 + 80*a^4*b^3*c^6*d^5*
e^4*f^2*g^7 - 1280*a^4*b^4*c^5*d^2*e^7*f^4*g^5 - 220*a^4*b^4*c^5*d^3*e^6*f^
3*g^6 - 1280*a^4*b^4*c^5*d^4*e^5*f^2*g^7 + 714*a^4*b^5*c^4*d^2*e^7*f^3*g^6
+ 714*a^4*b^5*c^4*d^3*e^6*f^2*g^7 + 58*a^4*b^6*c^3*d^2*e^7*f^2*g^7 + 2304*a
^5*b^2*c^6*d^2*e^7*f^4*g^5 + 1248*a^5*b^2*c^6*d^3*e^6*f^3*g^6 + 2304*a^5*b^
2*c^6*d^4*e^5*f^2*g^7 - 272*a^5*b^3*c^5*d^2*e^7*f^3*g^6 - 272*a^5*b^3*c^5*d
^3*e^6*f^2*g^7 - 996*a^5*b^4*c^4*d^2*e^7*f^2*g^7 + 1600*a^6*b^2*c^5*d^2*e^7
*f^2*g^7 + 16*a*b^2*c^10*d^6*e^3*f^8*g + 16*a*b^2*c^10*d^8*e*f^6*g^3 - 48*a
*b^3*c^9*d^5*e^4*f^8*g - 48*a*b^3*c^9*d^8*e*f^5*g^4 + 66*a*b^4*c^8*d^4*e^5*
f^8*g + 66*a*b^4*c^8*d^8*e*f^4*g^5 - 52*a*b^5*c^7*d^3*e^6*f^8*g - 52*a*b^5*
c^7*d^8*e*f^3*g^6 + 14*a*b^6*c^6*d^2*e^7*f^8*g + 14*a*b^6*c^6*d^8*e*f^2*g^7
- 16*a*b^8*c^4*d*e^8*f^7*g^2 - 16*a*b^8*c^4*d^7*e^2*f*g^8 + 24*a*b^9*c^3*d
*e^8*f^6*g^3 + 24*a*b^9*c^3*d^6*e^3*f*g^8 - 16*a*b^10*c^2*d*e^8*f^5*g^4 - 1
6*a*b^10*c^2*d^5*e^4*f*g^8 + 2*a*b^11*c*d^2*e^7*f^3*g^6 + 2*a*b^11*c*d^3*e^
6*f^2*g^7 + 96*a^2*b*c^10*d^5*e^4*f^8*g + 96*a^2*b*c^10*d^8*e*f^5*g^4 - 42*
a^2*b^5*c^6*d*e^8*f^8*g - 42*a^2*b^5*c^6*d^8*e*f*g^8 - 10*a^2*b^10*c*d*e^8*
f^3*g^6 - 10*a^2*b^10*c*d^3*e^6*f*g^8 - 64*a^3*b*c^9*d^3*e^6*f^8*g - 64*a^3
*b*c^9*d^8*e*f^3*g^6 + 144*a^3*b^3*c^7*d*e^8*f^8*g + 144*a^3*b^3*c^7*d^8*e*
f*g^8 + 14*a^3*b^9*c*d*e^8*f^2*g^7 + 14*a^3*b^9*c*d^2*e^7*f*g^8 - 544*a^5*b
*c^7*d*e^8*f^6*g^3 - 544*a^5*b*c^7*d^6*e^3*f*g^8 + 168*a^5*b^6*c^2*d*e^8*f*
g^8 - 992*a^6*b*c^6*d*e^8*f^4*g^5 - 992*a^6*b*c^6*d^4*e^5*f*g^8 - 668*a^6*b
^4*c^3*d*e^8*f*g^8 - 992*a^7*b*c^5*d*e^8*f^2*g^7 - 992*a^7*b*c^5*d^2*e^7*f*
g^8 + 864*a^7*b^2*c^4*d*e^8*f*g^8))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 +
16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4
*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 -
8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a
^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5
*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f
^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 -
2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*
c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c
^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*

c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2
*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*
e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*
e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3
*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*
g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d
^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c
^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*
c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*
b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2
*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b
^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3
*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c
*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g -
64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^
3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^
4*b^2*c^2*d*e^3*f*g^3)) - (48*a^6*b^2*c^3*e^8*g^8 - 12*a^5*b^4*c^2*e^8*g^8
- 64*a^7*c^4*e^8*g^8 + 40*a^4*c^7*d^6*e^2*g^8 + 80*a^5*c^6*d^4*e^4*g^8 - 24
*a^6*c^5*d^2*e^6*g^8 + 40*a^4*c^7*e^8*f^6*g^2 + 80*a^5*c^6*e^8*f^4*g^4 - 24
*a^6*c^5*e^8*f^2*g^6 + a^4*b^6*c*e^8*g^8 + a*b^5*c^5*d^7*e*g^8 + a*b^9*c*d^
3*e^5*g^8 + 20*a^3*b*c^7*d^7*e*g^8 + a*b^5*c^5*e^8*f^7*g + a*b^9*c*e^8*f^3*
g^5 + 20*a^3*b*c^7*e^8*f^7*g + 8*a*c^10*d^5*e^3*f^7*g + 8*a*c^10*d^7*e*f^5*
g^3 + 8*a^3*c^8*d*e^7*f^7*g + 8*a^3*c^8*d^7*e*f*g^7 + 304*a^6*c^5*d*e^7*f*g
^7 - b^6*c^5*d*e^7*f^7*g - b^6*c^5*d^7*e*f*g^7 - b^10*c*d*e^7*f^3*g^5 - b^1
0*c*d^3*e^5*f*g^7 - 4*a*b^6*c^4*d^6*e^2*g^8 + 6*a*b^7*c^3*d^5*e^3*g^8 - 4*a
*b^8*c^2*d^4*e^4*g^8 - 9*a^2*b^3*c^6*d^7*e*g^8 - 2*a^2*b^8*c*d^2*e^6*g^8 -
88*a^4*b*c^6*d^5*e^3*g^8 - 172*a^5*b*c^5*d^3*e^5*g^8 - 4*a*b^6*c^4*e^8*f^6*
g^2 + 6*a*b^7*c^3*e^8*f^5*g^3 - 4*a*b^8*c^2*e^8*f^4*g^4 - 9*a^2*b^3*c^6*e^8
*f^7*g - 2*a^2*b^8*c*e^8*f^2*g^6 - 88*a^4*b*c^6*e^8*f^5*g^3 - 172*a^5*b*c^5
*e^8*f^3*g^5 - 16*a*c^10*d^6*e^2*f^6*g^2 + 16*a^2*c^9*d^3*e^5*f^7*g + 16*a^
2*c^9*d^7*e*f^3*g^5 + 192*a^4*c^7*d*e^7*f^5*g^3 + 192*a^4*c^7*d^5*e^3*f*g^7
 + 488*a^5*c^6*d*e^7*f^3*g^5 + 488*a^5*c^6*d^3*e^5*f*g^7 - 2*b^2*c^9*d^5*e^
3*f^7*g - 2*b^2*c^9*d^7*e*f^5*g^3 + 3*b^3*c^8*d^4*e^4*f^7*g + 3*b^3*c^8*d^7
*e*f^4*g^4 + 4*b^7*c^4*d*e^7*f^6*g^2 + 4*b^7*c^4*d^6*e^2*f*g^7 - 6*b^8*c^3*
d*e^7*f^5*g^3 - 6*b^8*c^3*d^5*e^3*f*g^7 + 4*b^9*c^2*d*e^7*f^4*g^4 + 4*b^9*c
^2*d^4*e^4*f*g^7 - b^10*c*d^2*e^6*f^2*g^6 + 38*a^2*b^4*c^5*d^6*e^2*g^8 - 58
*a^2*b^5*c^4*d^5*e^3*g^8 + 36*a^2*b^6*c^3*d^4*e^4*g^8 - 5*a^2*b^7*c^2*d^3*e
^5*g^8 - 98*a^3*b^2*c^6*d^6*e^2*g^8 + 158*a^3*b^3*c^5*d^5*e^3*g^8 - 80*a^3*
b^4*c^4*d^4*e^4*g^8 - 22*a^3*b^5*c^3*d^3*e^5*g^8 + 22*a^3*b^6*c^2*d^2*e^6*g
^8 - 20*a^4*b^2*c^5*d^4*e^4*g^8 + 147*a^4*b^3*c^4*d^3*e^5*g^8 - 80*a^4*b^4*
c^3*d^2*e^6*g^8 + 102*a^5*b^2*c^4*d^2*e^6*g^8 + 38*a^2*b^4*c^5*e^8*f^6*g^2
- 58*a^2*b^5*c^4*e^8*f^5*g^3 + 36*a^2*b^6*c^3*e^8*f^4*g^4 - 5*a^2*b^7*c^2*e
^8*f^3*g^5 - 98*a^3*b^2*c^6*e^8*f^6*g^2 + 158*a^3*b^3*c^5*e^8*f^5*g^3 - 80*
a^3*b^4*c^4*e^8*f^4*g^4 - 22*a^3*b^5*c^3*e^8*f^3*g^5 + 22*a^3*b^6*c^2*e^8*f
^2*g^6 - 20*a^4*b^2*c^5*e^8*f^4*g^4 + 147*a^4*b^3*c^4*e^8*f^3*g^5 - 80*a^4*
b^4*c^3*e^8*f^2*g^6 + 102*a^5*b^2*c^4*e^8*f^2*g^6 - 56*a^2*c^9*d^4*e^4*f^6*
g^2 + 80*a^2*c^9*d^5*e^3*f^5*g^3 - 56*a^2*c^9*d^6*e^2*f^4*g^4 + 264*a^3*c^8
*d^3*e^5*f^5*g^3 - 96*a^3*c^8*d^4*e^4*f^4*g^4 + 264*a^3*c^8*d^5*e^3*f^3*g^5
 + 40*a^4*c^7*d^2*e^6*f^4*g^4 + 736*a^4*c^7*d^3*e^5*f^3*g^5 + 40*a^4*c^7*d^
4*e^4*f^2*g^6 + 16*a^5*c^6*d^2*e^6*f^2*g^6 + 4*b^2*c^9*d^6*e^2*f^6*g^2 - 3*
b^3*c^8*d^5*e^3*f^6*g^2 - 3*b^3*c^8*d^6*e^2*f^5*g^3 - 4*b^4*c^7*d^4*e^4*f^6
*g^2 + 8*b^4*c^7*d^5*e^3*f^5*g^3 - 4*b^4*c^7*d^6*e^2*f^4*g^4 - b^6*c^5*d^2*
e^6*f^6*g^2 - b^6*c^5*d^3*e^5*f^5*g^3 - b^6*c^5*d^4*e^4*f^4*g^4 - b^6*c^5*d
^5*e^3*f^3*g^5 - b^6*c^5*d^6*e^2*f^2*g^6 + 4*b^7*c^4*d^2*e^6*f^5*g^3 + 4*b^
7*c^4*d^3*e^5*f^4*g^4 + 4*b^7*c^4*d^4*e^4*f^3*g^5 + 4*b^7*c^4*d^5*e^3*f^2*g
^6 - 6*b^8*c^3*d^2*e^6*f^4*g^4 - 6*b^8*c^3*d^3*e^5*f^3*g^5 - 6*b^8*c^3*d^4*
e^4*f^2*g^6 + 4*b^9*c^2*d^2*e^6*f^3*g^5 + 4*b^9*c^2*d^3*e^5*f^2*g^6 + 30*a*
b^2*c^8*d^4*e^4*f^6*g^2 - 52*a*b^2*c^8*d^5*e^3*f^5*g^3 + 30*a*b^2*c^8*d^6*e
^2*f^4*g^4 - 6*a*b^3*c^7*d^3*e^5*f^6*g^2 + 8*a*b^3*c^7*d^4*e^4*f^5*g^3 + 8*

a*b^3*c^7*d^5*e^3*f^4*g^4 - 6*a*b^3*c^7*d^6*e^2*f^3*g^5 + 20*a*b^4*c^6*d^2*e^6*f^6*g^2 + 26*a*b^4*c^6*d^3*e^5*f^5*g^3 - 28*a*b^4*c^6*d^4*e^4*f^4*g^4 + 26*a*b^4*c^6*d^5*e^3*f^3*g^5 + 20*a*b^4*c^6*d^6*e^2*f^2*g^6 - 61*a*b^5*c^5*d^2*e^6*f^5*g^3 - 43*a*b^5*c^5*d^3*e^5*f^4*g^4 - 43*a*b^5*c^5*d^4*e^4*f^3*g^5 - 61*a*b^5*c^5*d^5*e^3*f^2*g^6 + 80*a*b^6*c^4*d^2*e^6*f^4*g^4 + 68*a*b^6*c^4*d^3*e^5*f^3*g^5 + 80*a*b^6*c^4*d^4*e^4*f^2*g^6 - 44*a*b^7*c^3*d^2*e^6*f^3*g^5 - 44*a*b^7*c^3*d^3*e^5*f^2*g^6 + 4*a*b^8*c^2*d^2*e^6*f^2*g^6 + 24*a^2*b*c^8*d^3*e^5*f^6*g^2 - 32*a^2*b*c^8*d^4*e^4*f^5*g^3 - 32*a^2*b*c^8*d^5*e^3*f^4*g^4 + 24*a^2*b*c^8*d^6*e^2*f^3*g^5 + 113*a^2*b^3*c^6*d*e^7*f^6*g^2 + 113*a^2*b^3*c^6*d^6*e^2*f*g^7 - 152*a^2*b^4*c^5*d*e^7*f^5*g^3 - 152*a^2*b^4*c^5*d^5*e^3*f*g^7 + 34*a^2*b^5*c^4*d*e^7*f^4*g^4 + 34*a^2*b^5*c^4*d^4*e^4*f*g^7 + 64*a^2*b^6*c^3*d*e^7*f^3*g^5 + 64*a^2*b^6*c^3*d^3*e^5*f*g^7 - 31*a^2*b^7*c^2*d*e^7*f^2*g^6 - 31*a^2*b^7*c^2*d^2*e^6*f*g^7 - 260*a^3*b*c^7*d^2*e^6*f^5*g^3 - 476*a^3*b*c^7*d^3*e^5*f^4*g^4 - 476*a^3*b*c^7*d^4*e^4*f^3*g^5 - 260*a^3*b*c^7*d^5*e^3*f^2*g^6 - 16*a^3*b^2*c^6*d*e^7*f^5*g^3 - 16*a^3*b^2*c^6*d^5*e^3*f*g^7 + 282*a^3*b^3*c^5*d*e^7*f^4*g^4 + 282*a^3*b^3*c^5*d^4*e^4*f*g^7 - 316*a^3*b^4*c^4*d*e^7*f^3*g^5 - 316*a^3*b^4*c^4*d^3*e^5*f*g^7 + 70*a^3*b^5*c^3*d*e^7*f^2*g^6 + 70*a^3*b^5*c^3*d^2*e^6*f*g^7 - 928*a^4*b*c^6*d^2*e^6*f^3*g^5 - 928*a^4*b*c^6*d^3*e^5*f^2*g^6 + 246*a^4*b^2*c^5*d*e^7*f^3*g^5 + 246*a^4*b^2*c^5*d^3*e^5*f*g^7 + 173*a^4*b^3*c^4*d*e^7*f^2*g^6 + 173*a^4*b^3*c^4*d^2*e^6*f*g^7 - 12*a*b*c^9*d^4*e^4*f^7*g - 12*a*b*c^9*d^7*e*f^4*g^4 + 10*a*b^4*c^6*d*e^7*f^7*g + 10*a*b^4*c^6*d^7*e*f*g^7 + 3*a*b^9*c*d*e^7*f^2*g^6 + 3*a*b^9*c*d^2*e^6*f*g^7 - 2*a^2*b^8*c*d*e^7*f*g^7 - 64*a^2*b^2*c^7*d^2*e^6*f^6*g^2 - 154*a^2*b^2*c^7*d^3*e^5*f^5*g^3 + 152*a^2*b^2*c^7*d^4*e^4*f^4*g^4 - 154*a^2*b^2*c^7*d^5*e^3*f^3*g^5 - 64*a^2*b^2*c^7*d^6*e^2*f^2*g^6 + 245*a^2*b^3*c^6*d^2*e^6*f^5*g^3 + 227*a^2*b^3*c^6*d^3*e^5*f^4*g^4 + 227*a^2*b^3*c^6*d^4*e^4*f^3*g^5 + 245*a^2*b^3*c^6*d^5*e^3*f^2*g^6 - 346*a^2*b^4*c^5*d^2*e^6*f^4*g^4 - 280*a^2*b^4*c^5*d^3*e^5*f^3*g^5 - 346*a^2*b^4*c^5*d^4*e^4*f^2*g^6 + 120*a^2*b^5*c^4*d^2*e^6*f^3*g^5 + 120*a^2*b^5*c^4*d^3*e^5*f^2*g^6 + 70*a^2*b^6*c^3*d^2*e^6*f^2*g^6 + 478*a^3*b^2*c^6*d^2*e^6*f^4*g^4 + 232*a^3*b^2*c^6*d^3*e^5*f^3*g^5 + 478*a^3*b^2*c^6*d^4*e^4*f^2*g^6 + 200*a^3*b^3*c^5*d^2*e^6*f^3*g^5 + 200*a^3*b^3*c^5*d^3*e^5*f^2*g^6 - 528*a^3*b^4*c^4*d^2*e^6*f^2*g^6 + 988*a^4*b^2*c^5*d^2*e^6*f^2*g^6 + 12*a*b*c^9*d^5*e^3*f^6*g^2 + 12*a*b*c^9*d^6*e^2*f^5*g^3 - 4*a*b^2*c^8*d^3*e^5*f^7*g - 4*a*b^2*c^8*d^7*e*f^3*g^5 - 2*a*b^3*c^7*d^2*e^6*f^7*g - 2*a*b^3*c^7*d^7*e*f^2*g^6 - 41*a*b^5*c^5*d*e^7*f^6*g^2 - 41*a*b^5*c^5*d^6*e^2*f*g^7 + 60*a*b^6*c^4*d*e^7*f^5*g^3 + 60*a*b^6*c^4*d^5*e^3*f*g^7 - 34*a*b^7*c^3*d*e^7*f^4*g^4 - 34*a*b^7*c^3*d^4*e^4*f*g^7 + 2*a*b^8*c^2*d*e^7*f^3*g^5 + 2*a*b^8*c^2*d^3*e^5*f*g^7 + 8*a^2*b*c^8*d^2*e^6*f^7*g + 8*a^2*b*c^8*d^7*e*f^2*g^6 - 26*a^2*b^2*c^7*d*e^7*f^7*g - 26*a^2*b^2*c^7*d^7*e*f*g^7 - 52*a^3*b*c^7*d*e^7*f^6*g^2 - 52*a^3*b*c^7*d^6*e^2*f*g^7 + 24*a^3*b^6*c^2*d*e^7*f*g^7 - 520*a^4*b*c^6*d*e^7*f^4*g^4 - 520*a^4*b*c^6*d^4*e^4*f*g^7 - 80*a^4*b^4*c^3*d*e^7*f*g^7 - 596*a^5*b*c^5*d*e^7*f^2*g^6 - 596*a^5*b*c^5*d^2*e^6*f*g^7 - 12*a^5*b^2*c^4*d*e^7*f*g^7)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4 *c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) + (x*(48*a^4*b^5*c^2*e^8*g^8 - 192*a^5*b^3*c^3*e^8*g^8 - 256*a^4*c^7*d^5*e^3*g^8 - 464*a^5*c^6*d^3*e^5*g^8 - 256*a^4*c^7*e^8*f^5*g^3 - 464*a^5*c^6*e^8*f^3*g^5 - 4*a^3*b^7*c*e^8*g^8 + 256*a^6*b*c^4*e^8*g^8 - 48*a^3*c^8*d^7*e*g^8 - 256*a^6*c^5*d*e^7*g^8 - 48*a^3*c^8*e^8*f^7*g - 256*a^6*c^5*e^8*f*g^7 - 2*a*b^4*c^6*d^7*e*g^8 - 2*a*b^9*c*d^2*e^6*g^8 + 6*a^2*b^8*c*d*e^7*g^8 - 2*a*b^4*c^6*e^8*f^7*g - 2*a*b^9*c*e^8*f^2*g^6 + 6*a^2*b^8*c*e^8*f*g^7 - 16*a*c^10*d^4*e^4*f^7*g - 16*a*c^10*d^7*e*f^4*g^4 + 2*b^5*c^6*d*e^7*f^7*g + 2*b^5*c^6*d^7*e*f*g^7 + 2*b^10*c*d*e^7*f^2*g^6 + 2*b^10*c*d^2*e^6*f*g^7 + 6*a*b^5*c^5*d^6*e^2*g^8 - 4*a*b^6*c^4*d^5*e^3*g^8 - 4*a*b^7*c^3*d^4*e^4*g^8 + 6*a*b^8*c^2*d^3*e^5*g^8 + 20*a^2*b^2*c^7*d^7*e*g^8 + 144*a^3*b*c^7*d^6*e^2*g^8 - 68*a^3*b^6*c^2*d*e^7*g^8 + 640*a^4*b*c^6*d^4*e^4*g^8 + 240*a^4*b^4*c^3*d*e^7*g^8 + 848*a^5*b*c^5*d^2*e^6*g^8 - 192*a^5*b^2*c^4*d*e^7*g^8 + 6*a*b^5*c^5*e^8*f^6*g^2 - 4*a*b^6*c^4*e^8*f^5*g^3 - 4*a*b^7*c^3*e^8*f^4*g^4 + 6*a*b^8*c^2*e^8*f^3*g^5 + 20*a^2*b^2*c^7*e^8*f^7*g + 144*a^3*b*c^7*e^8*f^6*g^2 - 68*a^3*b^6*c^2*e^8*f*g^7 + 640*a^4*b*c^6*e^8*f^4*g^4 + 240*a^4*b^4*c^3*e^8*f*g^7 + 848*a^5*b*c^5*e^8*f^2*g^6 - 192*a^5*b^2*c^4*e^8*f*g^7 + 16*a*c^10*d^5*e^3*f^6*g^2 + 16*a*c^10*d^6*e^2*f^5*g^3 - 64*a^2*c^9*d^2*e^6*f^7*g - 64*a^2*c^9*d^7*e*f^2*g^6 + 48*a^3*c^8*d*e^7*f^6*g^2 + 48*a^3*c^8*d^6*e^2*f*g^7 - 304*a^5*c^6*d*e^7*f^2*g^6 - 304*a^5*c^6*d^2*e^6*f*g^7 + 4*b^2*c^9*d^4*e^4*f^7*g + 4*b^2*c^9*d^7*e*f^4*g^4 - 8*b^3*c^8*d^3*e^5*f^7*g - 8*b^3*c^8*d^7*e*f^3*g^5 + 2*b^4*c^7*d^2*e^6*f^7*g + 2*b^4*c^7*d^7*e*f^2*g^6 - 6*b^6*c^5*d*e^7*f^6*g^2 - 6*b^6*c^5*d^6*e^2*f*g^7 + 4*b^7*c^4*d*e^7*f^5*g^3 + 4*b^7*c^4*d^5*e^3*f*g^7 + 4*b^8*c^3*d*e^7*f^4*g^4 + 4*b^8*c^3*d^4*e^4*f*g^7 - 6*b^9*c^2*d*e^7*f^3*g^5 - 6*b^9*c^2*d^3*e^5*f*g^7 - 60*a^2*b^3*c^6*d^6*e^2*g^8 + 30*a^2*b^4*c^5*d^5*e^3*g^8 + 64*a^2*b^5*c^4*d^4*e^4*g^8 - 72*a^2*b^6*c^3*d^3*e^5*g^8 + 12*a^2*b^7*c^2*d^2*e^6*g^8 + 8*a^3*b^2*c^6*d^5*e^3*g^8 - 352*a^3*b^3*c^5*d^4*e^4*g^8 + 268*a^3*b^4*c^4*d^3*e^5*g^8 + 52*a^3*b^5*c^3*d^2*e^6*g^8 - 188*a^4*b^2*c^5*d^3*e^5*g^8 - 484*a^4*b^3*c^4*d^2*e^6*g^8 - 60*a^2*b^3*c^6*e^8*f^6*g^2 + 30*a^2*b^4*c^5*e^8*f^5*g^3 + 64*a^2*b^5*c^4*e^8*f^4*g^4 - 72*a^2*b^6*c^3*e^8*f^3*g^5 + 12*a^2*b^7*c^2*e^8*f^2*g^6 + 8*a^3*b^2*c^6*e^8*f^5*g^3 - 352*a^3*b^3*c^5*e^8*f^4*g^4 + 268*a^3*b^4*c^4*e^8*f^3*g^5 + 52*a^3*b^5*c^3*e^8*f^2*g^6 - 188*a^4*b^2*c^5*e^8*f^3*g^5 - 484*a^4*b^3*c^4*e^8*f^2*g^6 + 64*a^2*c^9*d^3*e^5*f^6*g^2 + 64*a^2*c^9*d^6*e^2*f^3*g^5 - 272*a^3*c^8*d^2*e^6*f^5*g^3 + 16*a^3*c^8*d^3*e^5*f^4*g^4 + 16*a^3*c^8*d^4*e^4*f^3*g^5 - 272*a^3*c^8*d^5*e^3*f^2*g^6 - 512*a^4*c^7*d^2*e^6*f^3*g^5 - 512*a^4*c^7*d^3*e^5*f^2*g^6 - 4*b^2*c^9*d^5*e^3*f^6*g^2 - 4*b^2*c^9*d^6*e^2*f^5*g^3 - 4*b^3*c^8*d^4*e^4*f^6*g^2 + 24*b^3*c^8*d^5*e^3*f^5*g^3 - 4*b^3*c^8*d^6*e^2*f^4*g^4 + 22*b^4*c^7*d^3*e^5*f^6*g^2 - 24*b^4*c^7*d^4*e^4*f^5*g^3 - 24*b^4*c^7*d^5*e^3*f^4*g^4 + 22*b^4*c^7*d^6*e^2*f^3*g^5 - 8*b^5*c^6*d^2*e^6*f^6*g^2 - 14*b^5*c^6*d^3*e^5*f^5*g^3 + 40*b^5*c^6*d^4*e^4*f^4*g^4 - 14*b^5*c^6*d^5*e^3*f^3*g^5 - 8*b^5*c^6*d^6*e^2*f^2*g^6 + 14*b^6*c^5*d^2*e^6*f^5*g^3 - 4*b^6*c^5*d^3*e^5*f^4*g^4 - 4*b^6*c^5*d^4*e^4*f^3*g^5 + 14*b^6*c^5*d^5*e^3*f^2*g^6 - 16*b^7*c^4*d^2*e^6*f^4*g^4 - 4*b^7*c^4*d^3*e^5*f^3*g^5 - 16*b^7*c^4*d^4*e^4*f^2*g^6 + 14*b^8*c^3*d^2*e^6*f^3*g^5 + 14*b^8*c^3*d^3*e^5*f^2*g^6 - 8*b^9*c^2*d^2*e^6*f^2*g^6 - 8*a*b^9*c*d*e^7*f*g^7 - 104*a*b^2*c^8*d^3*e^5*f^6*g^2 + 96*a*b^2*c^8*d^4*e^4*f^5*g^3 + 96*a*b^2*c^8*d^5*e^3*f^4*g^4 - 104*a*b^2*c^8*d^6*e^2*f^3*g^5 + 104*a*b^3*c^7*d^3*e^5*f^5*g^3 - 160*a*b^3*c^7*d^4*e^4*f^4*g^4 + 104*a*b^3*c^7*d^5*e^3*f^3*g^5 - 78*a*b^4*c^6*d^2*e^6*f^5*g^3 - 42*a*b^4*c^6*d^3*e^5*f^4*g^4 - 42*a*b^4*c^6*d^4*e^4*f^

3*g^5 - 78*a*b^4*c^6*d^5*e^3*f^2*g^6 + 166*a*b^5*c^5*d^2*e^6*f^4*g^4 + 88*a*b^5*c^5*d^3*e^5*f^3*g^5 + 166*a*b^5*c^5*d^4*e^4*f^2*g^6 - 148*a*b^6*c^4*d^2*e^6*f^3*g^5 - 148*a*b^6*c^4*d^3*e^5*f^2*g^6 + 60*a*b^7*c^3*d^2*e^6*f^2*g^6 + 128*a^2*b*c^8*d^2*e^6*f^6*g^2 - 192*a^2*b*c^8*d^3*e^5*f^5*g^3 - 192*a^2*b*c^8*d^5*e^3*f^3*g^5 + 128*a^2*b*c^8*d^6*e^2*f^2*g^6 - 212*a^2*b^2*c^7*d*e^7*f^6*g^2 - 212*a^2*b^2*c^7*d^6*e^2*f*g^7 + 96*a^2*b^3*c^6*d*e^7*f^5*g^3 + 96*a^2*b^3*c^6*d^5*e^3*f*g^7 + 266*a^2*b^4*c^5*d*e^7*f^4*g^4 + 266*a^2*b^4*c^5*d^4*e^4*f*g^7 - 196*a^2*b^5*c^4*d*e^7*f^3*g^5 - 196*a^2*b^5*c^4*d^3*e^5*f*g^7 - 108*a^2*b^6*c^3*d*e^7*f^2*g^6 - 108*a^2*b^6*c^3*d^2*e^6*f*g^7 + 656*a^3*b*c^7*d^2*e^6*f^4*g^4 - 64*a^3*b*c^7*d^3*e^5*f^3*g^5 + 656*a^3*b*c^7*d^4*e^4*f^2*g^6 - 488*a^3*b^2*c^6*d*e^7*f^4*g^4 - 488*a^3*b^2*c^6*d^4*e^4*f*g^7 + 16*a^3*b^3*c^5*d*e^7*f^3*g^5 + 16*a^3*b^3*c^5*d^3*e^5*f*g^7 + 612*a^3*b^4*c^4*d*e^7*f^2*g^6 + 612*a^3*b^4*c^4*d^2*e^6*f*g^7 + 1536*a^4*b*c^6*d^2*e^6*f^2*g^6 - 772*a^4*b^2*c^5*d*e^7*f^2*g^6 - 772*a^4*b^2*c^5*d^2*e^6*f*g^7 + 32*a*b*c^9*d^3*e^5*f^7*g + 32*a*b*c^9*d^7*e*f^3*g^5 - 24*a*b^3*c^7*d*e^7*f^7*g - 24*a*b^3*c^7*d^7*e*f*g^7 + 64*a^2*b*c^8*d*e^7*f^7*g + 64*a^2*b*c^8*d^7*e*f*g^7 + 608*a^5*b*c^5*d*e^7*f*g^7 + 156*a^2*b^2*c^7*d^2*e^6*f^5*g^3 + 228*a^2*b^2*c^7*d^3*e^5*f^4*g^4 + 228*a^2*b^2*c^7*d^4*e^4*f^3*g^5 + 156*a^2*b^2*c^7*d^5*e^3*f^2*g^6 - 572*a^2*b^3*c^6*d^2*e^6*f^4*g^4 - 272*a^2*b^3*c^6*d^3*e^5*f^3*g^5 - 572*a^2*b^3*c^6*d^4*e^4*f^2*g^6 + 424*a^2*b^4*c^5*d^2*e^6*f^3*g^5 + 424*a^2*b^4*c^5*d^3*e^5*f^2*g^6 + 24*a^2*b^5*c^4*d^2*e^6*f^2*g^6 - 96*a^3*b^2*c^6*d^2*e^6*f^3*g^5 - 96*a^3*b^2*c^6*d^3*e^5*f^2*g^6 - 928*a^3*b^3*c^5*d^2*e^6*f^2*g^6 + 16*a*b*c^9*d^4*e^4*f^6*g^2 - 96*a*b*c^9*d^5*e^3*f^5*g^3 + 16*a*b*c^9*d^6*e^2*f^4*g^4 + 8*a*b^2*c^8*d^2*e^6*f^7*g + 8*a*b^2*c^8*d^7*e*f^2*g^6 + 74*a*b^4*c^6*d*e^7*f^6*g^2 + 74*a*b^4*c^6*d^6*e^2*f*g^7 - 48*a*b^5*c^5*d*e^7*f^5*g^3 - 48*a*b^5*c^5*d^5*e^3*f*g^7 - 52*a*b^6*c^4*d*e^7*f^4*g^4 - 52*a*b^6*c^4*d^4*e^4*f*g^7 + 64*a*b^7*c^3*d*e^7*f^3*g^5 + 64*a*b^7*c^3*d^3*e^5*f*g^7 - 6*a*b^8*c^2*d*e^7*f^2*g^6 - 6*a*b^8*c^2*d^2*e^6*f*g^7 + 84*a^2*b^7*c^2*d*e^7*f*g^7 + 128*a^3*b*c^7*d*e^7*f^5*g^3 + 128*a^3*b*c^7*d^5*e^3*f*g^7 - 248*a^3*b^5*c^3*d*e^7*f*g^7 + 512*a^4*b*c^6*d*e^7*f^3*g^5 + 512*a^4*b*c^6*d^3*e^5*f*g^7 + 8*a^4*b^3*c^4*d*e^7*f*g^7))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) + (x*(b^8*c*e^7*g^7 + 104*a^4*c^5*e^7*g^7 + 50*a^2*b^4*c^3*e^7*g^7 - 96*a^3*b^2*c^4*e^7*g^7 + 36*a^2*c^7*d^4*e^3*g^7 + 72*a^3*c^6*d^2*e^5*g^7 - 2*b^3*c^6*d^5*e^2*g

^7 + b^4*c^5*d^4*e^3*g^7 + b^6*c^3*d^2*e^5*g^7 + 36*a^2*c^7*e^7*f^4*g^3 + 7
2*a^3*c^6*e^7*f^2*g^5 - 2*b^3*c^6*e^7*f^5*g^2 + b^4*c^5*e^7*f^4*g^3 + b^6*c
^3*e^7*f^2*g^5 - 12*a*b^6*c^2*e^7*g^7 + b^2*c^7*d^6*e*g^7 - 2*b^7*c^2*d*e^6
*g^7 + b^2*c^7*e^7*f^6*g - 2*b^7*c^2*e^7*f*g^6 + 4*c^9*d^2*e^5*f^6*g + 4*c^
9*d^6*e*f^2*g^5 - 4*a*b*c^7*d^5*e^2*g^7 + 22*a*b^5*c^3*d*e^6*g^7 - 16*a^3*b
*c^5*d*e^6*g^7 - 4*a*b*c^7*e^7*f^5*g^2 + 22*a*b^5*c^3*e^7*f*g^6 - 16*a^3*b*
c^5*e^7*f*g^6 + 8*a*c^8*d*e^6*f^5*g^2 + 8*a*c^8*d^5*e^2*f*g^6 - 112*a^3*c^6
*d*e^6*f*g^6 + 4*b^6*c^3*d*e^6*f*g^6 + 2*a*b^2*c^6*d^4*e^3*g^7 + 10*a*b^3*c
^5*d^3*e^4*g^7 - 18*a*b^4*c^4*d^2*e^5*g^7 - 80*a^2*b*c^6*d^3*e^4*g^7 - 56*a
^2*b^3*c^4*d*e^6*g^7 + 2*a*b^2*c^6*e^7*f^4*g^3 + 10*a*b^3*c^5*e^7*f^3*g^4 -
 18*a*b^4*c^4*e^7*f^2*g^5 - 80*a^2*b*c^6*e^7*f^3*g^4 - 56*a^2*b^3*c^4*e^7*f
*g^6 + 40*a*c^8*d^2*e^5*f^4*g^3 + 40*a*c^8*d^4*e^3*f^2*g^5 + 16*a^2*c^7*d*e
^6*f^3*g^4 + 16*a^2*c^7*d^3*e^4*f*g^6 - 12*b*c^8*d^2*e^5*f^5*g^2 - 12*b*c^8
*d^5*e^2*f^2*g^5 + 10*b^2*c^7*d*e^6*f^5*g^2 + 10*b^2*c^7*d^5*e^2*f*g^6 - 14
*b^4*c^5*d*e^6*f^3*g^4 - 14*b^4*c^5*d^3*e^4*f*g^6 + 6*b^5*c^4*d*e^6*f^2*g^5
 + 6*b^5*c^4*d^2*e^5*f*g^6 - 4*b*c^8*d*e^6*f^6*g - 4*b*c^8*d^6*e*f*g^6 + 54
*a^2*b^2*c^5*d^2*e^5*g^7 + 54*a^2*b^2*c^5*e^7*f^2*g^5 + 168*a^2*c^7*d^2*e^5
*f^2*g^5 + 5*b^2*c^7*d^2*e^5*f^4*g^3 + 5*b^2*c^7*d^4*e^3*f^2*g^5 + 10*b^3*c
^6*d^2*e^5*f^3*g^4 + 10*b^3*c^6*d^3*e^4*f^2*g^5 - 12*b^4*c^5*d^2*e^5*f^2*g^
5 + 36*a*b^2*c^6*d^2*e^5*f^2*g^5 - 60*a*b*c^7*d*e^6*f^4*g^3 - 60*a*b*c^7*d^
4*e^3*f*g^6 - 72*a*b^4*c^4*d*e^6*f*g^6 - 80*a*b*c^7*d^2*e^5*f^3*g^4 - 80*a*
b*c^7*d^3*e^4*f^2*g^5 + 92*a*b^2*c^6*d*e^6*f^3*g^4 + 92*a*b^2*c^6*d^3*e^4*f
*g^6 + 6*a*b^3*c^5*d*e^6*f^2*g^5 + 6*a*b^3*c^5*d^2*e^5*f*g^6 - 192*a^2*b*c^
6*d*e^6*f^2*g^5 - 192*a^2*b*c^6*d^2*e^5*f*g^6 + 276*a^2*b^2*c^5*d*e^6*f*g^6
))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*
e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*
b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d
^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*
e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2
*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*
a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e
^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3
*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*
g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*
g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3
*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*
g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2
 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g -
2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 1
6*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^
4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4
*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d
*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*
c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2
*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*
d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g -
 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2
*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a
^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^
2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) + (x*(4*b
^3*c^4*e^6*g^6 - 16*a*b*c^5*e^6*g^6 + 16*a*c^6*d*e^5*g^6 + 16*a*c^6*e^6*f*g
^5 - 4*b^2*c^5*d*e^5*g^6 - 4*b^2*c^5*e^6*f*g^5))/(16*a^2*c^6*d^4*f^4 + a^4*
b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 1
6*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c
^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e
^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2
 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 +
b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5
*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*

g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3))*root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*

f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e
^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^
7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3
*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*
c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b
^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^
5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828
*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1
828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4
+ 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z
^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^
4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8
*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*
f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e
^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^
2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6
*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*
c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b
^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^
7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408
*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1
408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4
+ 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z
^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^
5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4
*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*
f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e
^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^
8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2
*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c
^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*
c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6
*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^
3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b
^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*
b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2
*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^
3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a
^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*
a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694
*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 69
4*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 6
92*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 +
692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 +
672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4
+ 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4
+ 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^
4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z
^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*
z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7
*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^
4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g
^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*
g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6
*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^
2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f
^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*
f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2
*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^

2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7

*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^
7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6
*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^
4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*
e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^
3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f
^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*
g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9
*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^
4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 +
156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 15
6*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5
*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^
5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d
^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*
e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7
*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^
7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g
^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*
z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 +
24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24
*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^
11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^1
1*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2
*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*
e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6
*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4
- 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^1
1*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^
8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6
*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4
- 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*
a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c
^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^
3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4
*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*
g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z
^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 +
1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*
a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c
^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^
6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8
*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*
g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4
- 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*
b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^
4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f
^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^
4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2
*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^
4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^1
0*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4
*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*
z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 +
90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^1
0*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c
^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*
f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g
^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^

4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 1
92*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^
2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c
^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10
*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g
^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4
- 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320
*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*
b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c
^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6
*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g
^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4
 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a
^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^
3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6
*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*
e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f
^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4
 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a
^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*
c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^
9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4
*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e
^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^
4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*
d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z
^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^
5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*
e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*
z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a
^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^
7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z
^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*
b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g
^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3
*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f
^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 -
768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*
b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10
*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^
4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b
^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f
^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z
^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^
4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^1
0*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^
4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8
*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5
*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10
*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a
^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^
9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*
z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68
*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*
d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^1
0*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6
*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g
^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8
*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*

g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b
^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*
f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 -
 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f
^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 +
256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^
10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 +
384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*
d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^
4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d
^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*
z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7
*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z
^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b
^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4
 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g
^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^1
0*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*
b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^
4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b
^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*
e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f
^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^
2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^
3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d
^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*
g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 +
 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5
*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^
7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z
^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^
7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4
*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^
2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b
^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*
b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3
*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^
2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2
*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*
b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b
^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^
5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*
c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e
*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4
*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^
4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3
*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^
3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z
^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2
+ 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 1
22*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*
a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b
^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c
^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^
2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4
*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*
g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z
^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 +
48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^

2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3
*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^
6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2
*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*
g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z
^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 +
20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*
b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c
^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^
7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g
^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 +
24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^
6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2
*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z
^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*
c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5
*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^
2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c
^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^
6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g
^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2
- 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a
^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*
d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4
*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^
4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 +
229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^
4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^
2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5
*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^
2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229
*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^
3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^
4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8
*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2
*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^
2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*
b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f
^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2
- 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^
8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g
^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 18
2*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e
^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^
2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b
*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*
e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 5
8*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d
^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2
+ 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*
f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*
a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7
*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*
c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2
- 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^
8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 4
8*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5
*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2
*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^

2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g
^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^
3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d
^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5
*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 -
8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^
4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^
8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 +
36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2
*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a
^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*
z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^
2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 4
5*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 +
4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b
^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*
c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*
b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^
3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2
*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 1
20*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^
3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4
*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2
*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2
*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*
b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z
- 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*
g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d
^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*
z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d
^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z
+ b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2
*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4
*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c
*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6
*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c
*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12
*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f
*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^
6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*
a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2
*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5
*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*
z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^
2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4
*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*
e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^
2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g
^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*
z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^
4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5
*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*
d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8
*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c
*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c
*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2
*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^
7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^
7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5

```
+ 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b
*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5
*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6
*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^
6 - 36*a^2*c^4*e^6*g^6, z, k), k, 1, 4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.819 $\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=287

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg\right.}{5g}$$

```
[Out] 2/3*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(3
/2)/g^6+2/5*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+1
0*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*
g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*e^2*(-b*e*g-3*c*d*g+5*c*e
*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e*f)^3*(a*g^2-b
*f*g+c*f^2)*(g*x+f)^(1/2)/g^6
```

Rubi [A] time = 0.50, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1153}

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg\right.}{5g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*
g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2))
/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2
- 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b
*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/2))/(
7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{\sqrt{f+gx}}dx = \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^3\left(cf^2-bfg+ag^2\right)}{g^5}+\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))x^2}{g^5}+\frac{(ef-\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4\cdots}{3g^6\cdots}$$

Mathematica [A] time = 0.41, size = 249, normalized size = 0.87

$$\frac{2\sqrt{f+gx}\left(-495e(f+gx)^3\left(c\left(-3d^2g^2+12defg-10e^2f^2\right)-eg(aeg+3bdg-4bef)\right)+693(f+gx)^2(ef-dg)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*(e*f - d*g)^3*(c*f^2 + g*(-(b*f) + a*g)) + 1155*(e*
f - d*g)^2*(c*f*(5*e*f - 2*d*g) + g*(-4*b*e*f + b*d*g + 3*a*e*g))*(f + g*x)
 + 693*(e*f - d*g)*(-3*e*g*(-2*b*e*f + b*d*g + a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^2 - 495*e*(-(e*g*(-4*b*e*f + 3*b*d*g + a*e*g))
 + c*(-10*e^2*f^2 + 12*d*e*f*g - 3*d^2*g^2))*(f + g*x)^3 - 385*e^2*(5*c*e*f
 - 3*c*d*g - b*e*g)*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

fricas [A] time = 1.26, size = 429, normalized size = 1.49

$$\frac{2\left(315\,ce^3g^5x^5-1280\,ce^3f^5+3465\,ad^3g^5+1408\left(3\,cde^2+be^3\right)f^4g-1584\left(3\,cd^2e+3\,bde^2+ae^3\right)f^3g^2+1848\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 3465*a*d^3*g^5 + 1408*(3*c*d*e
^2 + b*e^3)*f^4*g - 1584*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 + 1848*(c*
d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - 2310*(b*d^3 + 3*a*d^2*e)*f*g^4 - 35*
(10*c*e^3*f*g^4 - 11*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3 - 8
8*(3*c*d*e^2 + b*e^3)*f*g^4 + 99*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 -
 3*(160*c*e^3*f^3*g^2 - 176*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 198*(3*c*d^2*e +
3*b*d*e^2 + a*e^3)*f*g^4 - 231*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (
640*c*e^3*f^4*g - 704*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 792*(3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 1155*(b*
d^3 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/g^6
```

giac [B] time = 0.20, size = 565, normalized size = 1.97

$$\frac{2\left(3465\,\sqrt{gx+f}\,ad^3+\frac{1155\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)bd^3}{g}+\frac{3465\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)ad^2e}{g}+\frac{231\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+\cdots}\right.}{g^2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*b*d^3/g + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e/g + 231*(3*
(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^3/g^2 +
693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d^2
*e/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x +
 f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 297*(5*(g*x + f)^(7/2)
- 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*d
*e^2/g^3 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqr
t(g*x + f)*f^4)*c*d*e^2/g^4 + 11*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*
f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f
^4)*b*e^3/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x +
 f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*s
qrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

maple [B] time = 0.01, size = 540, normalized size = 1.88

$$\frac{2\sqrt{gx+f}\ \left(315e^3c\,x^5g^5+385b\,e^3g^5x^4+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485bd\,e^2g^5x^3-440b\,e^3\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+385*b*e^3*g^5*x^4+1155*c*d*e^2*g^5*
x^4-350*c*e^3*f*g^4*x^4+495*a*e^3*g^5*x^3+1485*b*d*e^2*g^5*x^3-440*b*e^3*f*
g^4*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e^3*f^2*g^3*x^3+2
079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+2079*b*d^2*e*g^5*x^2-1782*b*d*e^2*f
*g^4*x^2+528*b*e^3*f^2*g^3*x^2+693*c*d^3*g^5*x^2-1782*c*d^2*e*f*g^4*x^2+158
4*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a*d^2*e*g^5*x-2772*a*d*e^2
*f*g^4*x+792*a*e^3*f^2*g^3*x+1155*b*d^3*g^5*x-2772*b*d^2*e*f*g^4*x+2376*b*d
*e^2*f^2*g^3*x-704*b*e^3*f^3*g^2*x-924*c*d^3*f*g^4*x+2376*c*d^2*e*f^2*g^3*x
-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5-6930*a*d^2*e*f*g^4
+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2-2310*b*d^3*f*g^4+5544*b*d^2*e*f^2*
g^3-4752*b*d*e^2*f^3*g^2+1408*b*e^3*f^4*g+1848*c*d^3*f^2*g^3-4752*c*d^2*e*f
^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.46, size = 429, normalized size = 1.49

$$\frac{2\left(315\left(gx+f\right)^{\frac{11}{2}}ce^3-385\left(5\,ce^3f-\left(3\,cde^2+be^3\right)g\right)\left(gx+f\right)^{\frac{9}{2}}+495\left(10\,ce^3f^2-4\left(3\,cde^2+be^3\right)fg+\left(3\,cd^2e+\ldots\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g
)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^
2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 6*(3*c*
d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3
*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 4*(3*c*d*e
^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3/2) - 3
465*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 +
3*a*d^2*e)*f*g^4)*sqrt(g*x + f))/g^6
```

mupad [B] time = 0.15, size = 283, normalized size = 0.99

$$\frac{(f+gx)^{9/2}\left(2be^3g-10ce^3f+6cde^2g\right)}{9g^6}+\frac{(f+gx)^{7/2}\left(6cd^2eg^2-24cde^2fg+6bde^2g^2+20ce^3f^2-8\right.}{7g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(9/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(9*g^6) + ((f + g*
x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g^2 + 6*c*d^
2*e*g^2 - 24*c*d*e^2*f*g))/(7*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*e^
2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8*c*d*e*f*g)
)/(5*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^3*(a*g^2 + c*f^2 - b*f*g))/g^6 +
 (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^2 + 5*c*e*f^2 - 4*b*e*
f*g - 2*c*d*f*g))/(3*g^6) + (2*c*e^3*(f + g*x)^(11/2))/(11*g^6)
```

sympy [A] time = 164.70, size = 1544, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**3*f/sqrt(f + g*x) - 2*a*d**3*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 6*a*d**2*e*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 6*a*d*e**2
*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 6*a
*d*e**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
(f + g*x)**(5/2)/5)/g**2 - 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f
+ g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*a*e**3*(f**4/sqr
t(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)
**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*b*d**3*f*(-f/sqrt(f + g*x) - sqrt(
f + g*x))/g - 2*b*d**3*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)*
*(3/2)/3)/g - 6*b*d**2*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g**2 - 6*b*d**2*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
 + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 6*b*d*e**2*f*(-f**3/sqrt
(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)
/g**3 - 6*b*d*e**2*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f +
 g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*b*e**3
*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4
*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*b*e**3*(-f**5/sqrt(f +
 g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x
)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 - 2*c*d**3*f*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**
3*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g
*x)**(5/2)/5)/g**2 - 6*c*d**2*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 6*c*d**2*e*(f**4/sqrt(
f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**
(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f
**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f +
 g*x)**(7/2)/7)/g**4 - 6*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*
x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**
(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 - 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f*
*4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5
*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**5 - 2*c*e**3*(f**6/sqrt(f +
g*x) + 6*f**5*sqrt(f + g*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**(
5/2) - 15*f**2*(f + g*x)**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11
/2)/11)/g**5)/g, Ne(g, 0)), ((a*d**3*x + c*e**3*x**6/6 + x**5*(b*e**3 + 3*c
*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 +
 3*b*d**2*e + c*d**3)/3 + x**2*(3*a*d**2*e + b*d**3)/2)/sqrt(f), True))
```

3.820 $\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=212

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\,(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(\ldots}{\ldots}$$

```
[Out] -2/3*(-d*g+e*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(3/
2)/g^5-2/5*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(
g*x+f)^(5/2)/g^5-2/7*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(7/2)/g^5+2/9*c*e^2
*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^5
```

Rubi [A] time = 0.34, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {897, 1153}

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\,(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*g
)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/(3
*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*
x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$=\frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4}+\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))x^2}{g^4}+\frac{(-eg(3b\ldots}{\ldots}\right.\right.}{g}$$

$$=\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-\ldots}{3g^5}$$

Mathematica [A] time = 0.34, size = 184, normalized size = 0.87

$$2\sqrt{f+gx}\left(-63(f+gx)^2\left(-eg(aeg+2bdg-3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)+315(ef-dg)^2\left(g(ag-bf)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g)) - 105*(e*f -
 d*g)*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*(f + g*x) - 63
*(-(e*g*(-3*b*e*f + 2*b*d*g + a*e*g)) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 45*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^3 + 35*c*e^2*(f
+ g*x)^4))/(315*g^5)
```

fricas [A] time = 0.74, size = 260, normalized size = 1.23

$$2\left(35\,ce^2g^4x^4+128\,ce^2f^4+315\,ad^2g^4-144\left(2\,cde+be^2\right)f^3g+168\left(cd^2+2\,bde+ae^2\right)f^2g^2-210\left(bd^2+2\,ad\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 + 315*a*d^2*g^4 - 144*(2*c*d*e + b*
e^2)*f^3*g + 168*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 - 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 5*(8*c*e^2*f*g^3 - 9*(2*c*d*e + b*e^2)*g^4)*x^3 + 3*(16*c*e^2*f^2*g
^2 - 18*(2*c*d*e + b*e^2)*f*g^3 + 21*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (
64*c*e^2*f^3*g - 72*(2*c*d*e + b*e^2)*f^2*g^2 + 84*(c*d^2 + 2*b*d*e + a*e^2
)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/g^5
```

giac [A] time = 0.23, size = 363, normalized size = 1.71

$$2\left(315\sqrt{gx+f}\,ad^2+\frac{105\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)bd^2}{g}+\frac{210\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ade}{g}+\frac{21\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)c}{g^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
b*d^2/g + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e/g + 21*(3*(g*x +
f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2/g^2 + 42*(3*(
g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d*e/g^2 + 2
1*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*e^2/g
^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
- 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*e^2/g^3 + (35*(g*x
+ f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f
)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

maple [A] time = 0.01, size = 315, normalized size = 1.49

$$2\sqrt{gx+f}\;\left(35e^2c\,x^4g^4+45b\,e^2g^4x^3+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+126bde\,g^4x^2-54b\,e^2f\,g^3x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+45*b*e^2*g^4*x^3+90*c*d*e*g^4*x^3-40*
c*e^2*f*g^3*x^3+63*a*e^2*g^4*x^2+126*b*d*e*g^4*x^2-54*b*e^2*f*g^3*x^2+63*c*
d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+210*a*d*e*g^4*x-84*a*e
^2*f*g^3*x+105*b*d^2*g^4*x-168*b*d*e*f*g^3*x+72*b*e^2*f^2*g^2*x-84*c*d^2*f*
g^3*x+144*c*d*e*f^2*g^2*x-64*c*e^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+16
8*a*e^2*f^2*g^2-210*b*d^2*f*g^3+336*b*d*e*f^2*g^2-144*b*e^2*f^3*g+168*c*d^2
*f^2*g^2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 261, normalized size = 1.23

$$2\left(35\left(gx+f\right)^{\frac{9}{2}}ce^{2}-45\left(4\,ce^{2}f-\left(2\,cde+be^{2}\right)g\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(6\,ce^{2}f^{2}-3\left(2\,cde+be^{2}\right)fg+\left(cd^{2}+2\,bde+ae^{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*x
 + f)^(7/2) + 63*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
+ a*e^2)*g^2)*(g*x + f)^(5/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2*
g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*(g*x + f)^(3
/2) + 315*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2*b*d
*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)*sqrt(g*x + f))/g^5
```

mupad [B] time = 3.17, size = 204, normalized size = 0.96

$$\frac{\left(f+g\,x\right)^{7/2}\,\left(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g\right)}{7\,g^5}+\frac{\left(f+g\,x\right)^{5/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g\right.}{5\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(7*g^5) + ((f + g*x)^
(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d*e*g^2 - 6*b*e^2*f*g
 - 12*c*d*e*f*g))/(5*g^5) + (2*(f + g*x)^(3/2)*(d*g - e*f)*(2*a*e*g^2 + b*d
*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*
g - e*f)^2*(a*g^2 + c*f^2 - b*f*g))/g^5 + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

sympy [A] time = 105.54, size = 1001, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**2*f/sqrt(f + g*x) - 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 - 2*b*d**2*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*b*d**2
*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 4*b*d*e*
f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 4*b*
d*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
 g*x)**(5/2)/5)/g**2 - 2*b*e**2*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*b*e**2*(f**4/sqrt(f
+ g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5
```

```
/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqr
t(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f
**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 4*c*d*e
*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
g*x)**(5/2)/5)/g**3 - 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
 - 2*c*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)
**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*c*e**2*(-f*
*5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f*
*2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g,
 Ne(g, 0)), ((a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(
a*e**2 + 2*b*d*e + c*d**2)/3 + x**2*(2*a*d*e + b*d**2)/2)/sqrt(f), True))
```

3.821 $\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{f+gx}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-be}{5g}$$

```
[Out] 2/3*(c*f*(-2*d*g+3*e*f)-g*(-a*e*g-b*d*g+2*b*e*f))*(g*x+f)^(3/2)/g^4-2/5*(-b
*e*g-c*d*g+3*c*e*f)*(g*x+f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f
)*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^4
```

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {771}

$$-\frac{2\sqrt{f+gx}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-be}{5g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*f
 - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3*c
*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7
*g^4)
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx=\int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+}}{g^3}\right.$$

$$=-\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg}{3g^4}$$

Mathematica [A] time = 0.19, size = 131, normalized size = 0.96

$$\frac{2\sqrt{f+gx}\,\left(7g\left(5ag(3dg-2ef+egx)+5bdg(gx-2f)+be\left(8f^2-4fgx+3g^2x^2\right)\right)+c\left(7dg\left(8f^2-4fgx+3g^2x^2\right.\right.\right.}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(7*g*(5*b*d*g*(-2*f + g*x) + 5*a*g*(-2*e*f + 3*d*g + e*g*x
) + b*e*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + c*(7*d*g*(8*f^2 - 4*f*g*x + 3*g^2*
x^2) - 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*g^4)
```

fricas [A] time = 0.90, size = 125, normalized size = 0.91

$$\frac{2\left(15\,ceg^3x^3 - 48\,cef^3 + 105\,adg^3 + 56\,(cd+be)f^2g - 70\,(bd+ae)fg^2 - 3\left(6\,cefg^2 - 7\,(cd+be)g^3\right)x^2 + \left(24\,ce\right.\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 105*a*d*g^3 + 56*(c*d + b*e)*f^2*g - 7
0*(b*d + a*e)*f*g^2 - 3*(6*c*e*f*g^2 - 7*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2
*g - 28*(c*d + b*e)*f*g^2 + 35*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/g^4
```

giac [A] time = 0.19, size = 199, normalized size = 1.45

$$\frac{2\left(105\sqrt{gx+f}\,ad + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}\,f\right)bd}{g} + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}\,f\right)ae}{g} + \frac{7\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}\,f^2\right)cd}{g^2} + \frac{7}{\cdots}\right.}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b*d
/g + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2)
- 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 7*(3*(g*x + f)^(5/
2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*e/g^2 + 3*(5*(g*x + f)^
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^
3)*c*e/g^3)/g
```

maple [A] time = 0.01, size = 144, normalized size = 1.05

$$\frac{2\sqrt{gx+f}\,\left(15ce\,x^3g^3 + 21be\,g^3x^2 + 21cd\,g^3x^2 - 18cef\,g^2x^2 + 35ae\,g^3x + 35bd\,g^3x - 28bef\,g^2x - 28cdf\,g^2x + \right.}{105g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(15*c*e*g^3*x^3+21*b*e*g^3*x^2+21*c*d*g^3*x^2-18*c*e*f*
g^2*x^2+35*a*e*g^3*x+35*b*d*g^3*x-28*b*e*f*g^2*x-28*c*d*f*g^2*x+24*c*e*f^2*
g*x+105*a*d*g^3-70*a*e*f*g^2-70*b*d*f*g^2+56*b*e*f^2*g+56*c*d*f^2*g-48*c*e*
f^3)/g^4
```

maxima [A] time = 0.46, size = 129, normalized size = 0.94

$$\frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce - 21\,\left(3\,cef - (cd+be)g\right)(gx+f)^{\frac{5}{2}} + 35\,\left(3\,cef^2 - 2\,(cd+be)fg + (bd+ae)g^2\right)(gx+f)^{\frac{3}{2}} - 1\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(5/2
) + 35*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*(g*x + f)^(3/2) -
105*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)*sqrt(g*x +
f))/g^4
```

mupad [B] time = 0.08, size = 125, normalized size = 0.91

$$\frac{\left(f+g\,x\right)^{5/2}\,\left(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f\right)}{5\,g^4}+\frac{\left(f+g\,x\right)^{3/2}\,\left(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g\right)}{3\,g^4}+\frac{2\,\sqrt{f+g}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(5*g^4) + ((f + g*x)^(3/2)*
(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g - 4*c*d*f*g))/(3*g^4) + (2*(
f + g*x)^(1/2)*(d*g - e*f)*(a*g^2 + c*f^2 - b*f*g))/g^4 + (2*c*e*(f + g*x)^
(7/2))/(7*g^4)
```

sympy [A] time = 55.83, size = 549, normalized size = 4.01

$$\begin{cases} \frac{-\frac{2adf}{\sqrt{f+gx}}-2ad\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)-\frac{2aef\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}-\frac{2ae\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}-\frac{2bdf\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}-\frac{2bd\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}}{} \\ \frac{adx+\frac{cex^4}{4}+\frac{x^3(be+cd)}{3}+\frac{x^2(ae+bd)}{2}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d*f/sqrt(f + g*x) - 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) - 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*b*d*f*(-f/sqrt(f + g*x)
- sqrt(f + g*x))/g - 2*b*d*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g - 2*b*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*
x)**(3/2)/3)/g**2 - 2*b*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(
f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 2*c*d*f*(f**2/sqrt(f + g*x) +
2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d*(-f**3/sqrt(f + g*x) -
 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 2*c
*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**3 - 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
)/g, Ne(g, 0)), ((a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x**2*(a*e + b*d
)/2)/sqrt(f), True))
```

3.822 $\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{f+gx}\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[Out] -2/3*(-b*g+2*c*f)*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2-b*f*g+
c*f^2)*(g*x+f)^(1/2)/g^3
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {698}

$$\frac{2\sqrt{f+gx}\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^
(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2\sqrt{f+gx}}+\frac{(-2cf+bg)\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.74

$$\frac{2\sqrt{f+gx}\left(5g(3ag-2bf+bgx)+c\left(8f^2-4fgx+3g^2x^2\right)\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^2
*x^2)))/(15*g^3)
```

fricas [A] time = 0.80, size = 54, normalized size = 0.74

$$\frac{2\left(3\,cg^2x^2+8\,cf^2-10\,bfg+15\,ag^2-\left(4\,cfg-5\,bg^2\right)x\right)\sqrt{gx+f}}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)*
sqrt(g*x + f)/g^3
```

giac [A] time = 0.17, size = 77, normalized size = 1.05

$$\frac{2\left(15\sqrt{gx+f}\,a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)b}{g}+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

maple [A] time = 0.00, size = 53, normalized size = 0.73

$$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2+5b\,g^2x-4cfgx+15a\,g^2-10bfg+8c\,f^2\right)}{15g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(3*c*g^2*x^2+5*b*g^2*x-4*c*f*g*x+15*a*g^2-10*b*f*g+8*c*f
^2)/g^3
```

maxima [A] time = 0.44, size = 77, normalized size = 1.05

$$\frac{2\left(15\sqrt{gx+f}\,a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)b}{g}+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

mupad [B] time = 3.12, size = 58, normalized size = 0.79

$$\frac{2\sqrt{f+g\,x}\,\left(3\,c\,{\left(f+g\,x\right)}^2+15\,a\,g^2+15\,c\,f^2+5\,b\,g\,\left(f+g\,x\right)-10\,c\,f\,\left(f+g\,x\right)-15\,b\,f\,g\right)}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(1/2),x)
```

```
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 + 5*b*g*(f + g*x)
 - 10*c*f*(f + g*x) - 15*b*f*g))/(15*g^3)
```

sympy [A] time = 10.87, size = 223, normalized size = 3.05

$$\begin{cases} \frac{-\frac{2af}{\sqrt{f+gx}} - 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2bf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2b\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+\cdots}\right.}{\cdots}}{g} \\ \frac{ax + \frac{bx^2}{2} + \frac{cx^3}{3}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*f/sqrt(f + g*x) - 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) -
 2*b*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*b*(f**2/sqrt(f + g*x) + 2*f
*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sq
rt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*
sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0))
, ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))
```

3.823 $\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/
2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)+2*(b*e*g-c*(d*g+e*f))*(g*x+f)
^(1/2)/e^2/g^2
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {897, 1153, 208}

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))
/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{beg-c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2-bde+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}\cdots}\right.}{e^2g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2\left(cd^2-bde+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+g\cdots}}{\sqrt{ef-dg}}\right.}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] time = 0.19, size = 118, normalized size = 1.02

$$\frac{2\left(-\frac{g^2\left(cd^2-e(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2}+\frac{c(f+gx)^{3/2}}{3e}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(((b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/e^2 + (c*(f + g*x)^(3/2))/(3*e)
 - ((c*d^2 - e*(b*d - a*e))*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
d*g]])/(e^(5/2)*Sqrt[e*f - d*g])))/g^2
```

fricas [A] time = 0.87, size = 341, normalized size = 2.94

$$\left[\frac{3\left(cd^2-bde+ae^2\right)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2ef-dg-2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^2+\left(cde^2-3be^3\right)fg-3\left(cd^2e-\cdots\right.\right.}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 - b*d*e + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f
- d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 +
(c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)*g^2 - (c*e^3*f*g - c*d*e^2*
g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 - b*d*e + a*e
^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g
*x + e*f)) - (2*c*e^3*f^2 + (c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)
*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3)]
```

giac [A] time = 0.17, size = 128, normalized size = 1.10

$$\frac{2\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\sqrt{gx+f}\,cdg^5e-(gx+f)^{\frac{3}{2}}cg^4e^2+3\sqrt{gx+f}\,cfg^4e^2-3\sqrt{gx}\cdots\right.}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2
)/sqrt(d*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*
g^4*e^2 + 3*sqrt(g*x + f)*c*f*g^4*e^2 - 3*sqrt(g*x + f)*b*g^5*e^2)*e^(-3)/g
^6
```

maple [A] time = 0.01, size = 189, normalized size = 1.63

$$\frac{2a\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)\,e}}-\frac{2bd\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)\,e}\,e}+\frac{2c\,d^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)\,e}\,e^2}+\frac{2\sqrt{gx+f}\,b}{eg}-\frac{2\sqrt{gx+f}\,cd}{e^2g}-\frac{2\sqrt{gx+f}}{e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3*(g*x+f)^(3/2)*c/e/g^2+2/g/e*b*(g*x+f)^(1/2)-2*(g*x+f)^(1/2)*c*d/e^2/g-2
*(g*x+f)^(1/2)*c/e*f/g^2+2/((d*g-e*f)*e)^(1/2)*a*arctan((g*x+f)^(1/2)/((d*g
-e*f)*e)^(1/2)*e)-2/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e
)^(1/2)*e)*b*d+2/((d*g-e*f)*e)^(1/2)*c*d^2/e^2*arctan((g*x+f)^(1/2)/((d*g-e
*f)*e)^(1/2)*e)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.14, size = 117, normalized size = 1.01

$$\sqrt{f+g\,x}\,\left(\frac{2\,b\,g-4\,c\,f}{e\,g^2}-\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}\right)+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}}+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (f + g*x)^(1/2)*((2*b*g - 4*c*f)/(e*g^2) - (2*c*(d*g^3 - e*f*g^2))/(e^2*g^4
)) + (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2 -
b*d*e))/(e^(5/2)*(d*g - e*f)^(1/2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

sympy [A] time = 37.63, size = 112, normalized size = 0.97

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3eg^2}-\frac{2\left(ae^2-bde+cd^2\right)\mathrm{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\,\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\left(dg-ef\right)}+\frac{2\sqrt{f+gx}\left(beg-cdg-cef\right)}{e^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*(a*e**2 - b*d*e + c*d**2)*atan(1/(sqrt(
e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sqrt(e/(d*g - e*f))*(d*g - e*f)) + 2*s
qrt(f + g*x)*(b*e*g - c*d*g - c*e*f)/(e**2*g**2)
```

3.824 $\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=140

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{2c\sqrt{f+gx}}{e^2g}$$

```
[Out] (c*d*(-3*d*g+4*e*f)-e*(-a*e*g-b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+d*(-
b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
```

Rubi [A] time = 0.29, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1157, 388, 208}

$$-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e
*f - d*g)*(d + e*x)) + ((c*d*(4*e*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g))
*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/
2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{2cf^2}{g^2}+\frac{2bf}{g}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} - \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\,\text{Sub}}{e^2g(ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\tan}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.56, size = 150, normalized size = 1.07

$$\frac{\sqrt{f+gx}\left(eg(bd-ae)+c\left(-3d^2g+2de(f-gx)+2e^2fx\right)\right)}{e^2g(d+ex)(ef-dg)} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-aeg-bdg+2bef)+cd(3d}{e^{5/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(e*(b*d - a*e)*g + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x)
)))/(e^2*g*(e*f - d*g)*(d + e*x)) - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f - b
*d*g - a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(
e*f - d*g)^(3/2))
```

fricas [B] time = 1.24, size = 637, normalized size = 4.55

$$\left[-\frac{\sqrt{e^2f-deg}\left(2\left(2\,cd^2e-bde^2\right)fg-\left(3\,cd^3-bd^2e-ade^2\right)g^2+\left(2\left(2\,cde^2-be^3\right)fg-\left(3\,cd^2e-bde^2-ae^3\right)g^2\right)x}{2\left(de^5f^2g-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(e^2*f - d*e*g)*(2*(2*c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*
e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^
3)*g^2)*x)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/
(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 + a*e^4)*f*g + (3*c*
d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2
*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6
*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -(sqrt(-e^2*f + d*e*g)*(2*(2*c*d^
2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*
e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^3)*g^2)*x)*arctan(sqrt(-e^2*f + d*e*g
)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 +
```

```
a*e^4)*f*g + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e
^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 +
d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

giac [A] time = 0.18, size = 175, normalized size = 1.25

$$\frac{2\sqrt{gx+f}\,ce^{(-2)}}{g}-\frac{\left(3\,cd^2g-4\,cdfe-bdge+2\,bfe^2-age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge^2-fe^3\right)\sqrt{dge-fe^2}}+\frac{\sqrt{gx+f}\,cd^2g-\sqrt{gx+f}\,bdge+\sqrt{}}{\left(dge^2-fe^3\right)\left(dg+\left(gx+f\right)e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - b*d*g*e + 2*b*f*e^2 -
 a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sq
rt(d*g*e - f*e^2)) + (sqrt(g*x + f)*c*d^2*g - sqrt(g*x + f)*b*d*g*e + sqrt(
g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g*x + f)*e - f*e))
```

maple [B] time = 0.02, size = 371, normalized size = 2.65

$$\frac{ag\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}+\frac{bdg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}-\frac{2bf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}-\frac{3c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e^2}+\frac{4cdf\,\mathrm{ar}}{\left(dg-e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*c/e^2/g+g/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*a-g/e/(d*g-e*
f)*(g*x+f)^(1/2)/(e*g*x+d*g)*b*d+g/e^2/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*
c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1
/2)*e)*a+g/e/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2)*e)*b*d-2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-
e*f)*e)^(1/2)*e)*b*f-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(
1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2+4/e/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*d*c*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.23, size = 146, normalized size = 1.04

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(a\,e^2\,g-2\,b\,e^2\,f-3\,c\,d^2\,g+b\,d\,e\,g+4\,c\,d\,e\,f\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2-b\,g\,d\,e+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)}+\frac{2\,c\,\sqrt{}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 2*b*e^2*f - 3
*c*d^2*g + b*d*e*g + 4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(
1/2)*(a*e^2*g + c*d^2*g - b*d*e*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d
*e^2*g)) + (2*c*(f + g*x)^(1/2))/(e^2*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.825 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=206

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(eg(-3aeg-bdg+4bef)-c\left(3d^2g^2-8defg+8e^2f^2\right)\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\,(cd}{}$$

```
[Out] 1/4*(e*g*(-3*a*e*g-b*d*g+4*b*e*f)-c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctan
h(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+d
*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(-5*d*g+8*e*f)
-e*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A] time = 0.39, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = $\{897, 1157, 385, 208\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(eg(-3aeg-bdg+4bef)-c\left(3d^2g^2-8defg+8e^2f^2\right)\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\,(cd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((
c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/(4*e^2*
(e*f - d*g)^2*(d + e*x)) + ((e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2
 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
```

```
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\mathrm{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{4cf^2}{g^2}+\frac{4bf}{g}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{(eg(\ldots}{\ldots}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{(eg(\ldots}{\ldots}$$

Mathematica [A] time = 0.65, size = 297, normalized size = 1.44

$$\frac{-\frac{2\sqrt{e}\sqrt{f+gx}\left(e(ae-bd)+cd^2\right)}{(d+ex)^2(ef-dg)} - \frac{3g\left(e(ae-bd)+cd^2\right)\left(g(d+ex)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)-\sqrt{e}\sqrt{f+gx}\sqrt{ef-dg}\right)}{(d+ex)(ef-dg)^{5/2}} - \frac{4\sqrt{e}\sqrt{f+gx}(be-2cd)}{(d+ex)(ef-dg)} - \frac{4g(2cd-be)\,\mathrm{ta}\ldots}{(ef\ldots}}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] ((-2*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f + g*x])/((e*f - d*g)*(d + e*
x)^2) - (4*Sqrt[e]*(-2*c*d + b*e)*Sqrt[f + g*x])/((e*f - d*g)*(d + e*x)) -
(4*(2*c*d - b*e)*g*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e*f -
 d*g)^(3/2) - (8*c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e
*f - d*g] - (3*(c*d^2 + e*(-(b*d) + a*e))*g*(-(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt
[f + g*x]) + g*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]))
/((e*f - d*g)^(5/2)*(d + e*x)))/(4*e^(5/2))
```

fricas [B] time = 0.98, size = 1096, normalized size = 5.32

$$\left[\frac{\left(8cd^2e^2f^2-4\left(2cd^3e+bd^2e^2\right)fg+\left(3cd^4+bd^3e+3ad^2e^2\right)g^2+\left(8ce^4f^2-4\left(2cde^3+be^4\right)fg+\left(3cd^2e^2+b\ldots\right.\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d^4 + b*d^3*e
 + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g + (3*c*d^2*e
^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^2*e^2 + b*d*
```

```
e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*
log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d))
+ 2*(2*(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a
*d*e^4)*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b
*e^5)*f^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2
*e^3 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d
^4 + b*d^3*e + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g
+ (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^
2*e^2 + b*d*e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(-e^
2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*
(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a*d*e^4)
*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b*e^5)*f
^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2*e^3 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

giac [B] time = 0.20, size = 373, normalized size = 1.81

$$\frac{\left(3\,cd^2g^2 - 8\,cdfge + bdg^2e + 8\,cf^2e^2 - 4\,bfge^2 + 3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^2g^2e^2 - 2\,dfge^3 + f^2e^4\right)\sqrt{dge-fe^2}} - \frac{3\,\sqrt{gx+f}\,cd^3g^3 + 5\left(gx+f\right)^{\frac{3}{2}}cd^2g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + b*d*g^2*e + 8*c*f^2*e^2 - 4*b*f*g*e^2 + 3*
a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*
f*g*e^3 + f^2*e^4)*sqrt(d*g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 +
5*(g*x + f)^(3/2)*c*d^2*g^2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e + sqrt(g*x +
 f)*b*d^2*g^3*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sqrt(g*x + f)*c*d*f^2*g
*e^2 - (g*x + f)^(3/2)*b*d*g^2*e^2 + 3*sqrt(g*x + f)*b*d*f*g^2*e^2 - 5*sqrt
(g*x + f)*a*d*g^3*e^2 + 4*(g*x + f)^(3/2)*b*f*g*e^3 - 4*sqrt(g*x + f)*b*f^2
*g*e^3 - 3*(g*x + f)^(3/2)*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g
^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

maple [B] time = 0.02, size = 538, normalized size = 2.61

$$\frac{3a\,g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(d^2g^2 - 2defg + e^2f^2\right)\sqrt{\left(dg-ef\right)e}} + \frac{bd\,g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(d^2g^2 - 2defg + e^2f^2\right)\sqrt{\left(dg-ef\right)e}\,e} - \frac{bfg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(d^2g^2 - 2defg + e^2f^2\right)\sqrt{\left(dg - \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g+b*d*e*g-4*b*e^2*f-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e
*f*g+e^2*f^2)*(g*x+f)^(3/2)+1/8*(5*a*e^2*g-b*d*e*g-4*b*e^2*f-3*c*d^2*g+8*c*
d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2))/(d*g-e*f+(g*x+f)*e)^2+3/4/(d^2*g^2-2*
d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*a*g^2*arctan((g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2)*e)+1/4/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan((g
*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*d*g^2-1/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((
d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*f*g+3/4/(d^
2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*c*d^2/e^2*g^2*arctan((g*x+f)^(
```

```
1/2)/((d*g-e*f)*e)^(1/2)*e)-2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/
2)*c*d/e*f*g*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)+2/(d^2*g^2-2*d*e*f
*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*c*f^2*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1
/2)*e)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [**B**] time = 0.28, size = 270, normalized size = 1.31

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+b\,d\,e\,g^2+8\,c\,e^2\,f^2-4\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,\left(d\,g-e\,f\right)^{5/2}}-\frac{\frac{\sqrt{f+g\,x}\,\left(3\,c\,d^2\,g^2+b\,d\,e\,g^2-8\,c\,f\,d}{4\,e^2\,\left(d\,g-e\,f\right.}}{e^2\,\left(f+g\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a*e^2*g^2 + 3*c*d^2*g
^2 + 8*c*e^2*f^2 + b*d*e*g^2 - 4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g
- e*f)^(5/2)) - (((f + g*x)^(1/2)*(3*c*d^2*g^2 - 5*a*e^2*g^2 + b*d*e*g^2 +
4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^2*(d*g - e*f)) - ((f + g*x)^(3/2)*(3*a*e^2
*g^2 - 5*c*d^2*g^2 + b*d*e*g^2 - 4*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*
f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 + e^2*f^2
 - 2*d*e*f*g)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.826 $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg\right.}{3g}$$

```
[Out] 2/3*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+10*e^2*f^
2))*(g*x+f)^(3/2)/g^6-2/5*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*g^2-12*d
*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e^2*(-b*e*g-3*c*d*g+5*c*e*f)*(g*x
+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2*(-d*g+e*f)^3*(a*g^2-b*f*g+c*f^2
)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*
b*e*f))*(g*x+f)^(1/2)/g^6
```

Rubi [A] time = 0.41, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {897, 1261}

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg\right.}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/
g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g
- a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(5*g^6
) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\mathrm{Subst}\left(\int\left(\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))}{g^5}+\frac{(-ef+dg)^3\left(cf^2-bfg+ag^2\right)}{g^5x^2}+\frac{(ef-dg}{}\right.\right.}{}$$

$$= \frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg}{g^6}$$

Mathematica [A] time = 0.73, size = 249, normalized size = 0.87

$$\frac{2\left(-63e(f+gx)^3\left(c\left(-3d^2g^2+12defg-10e^2f^2\right)-eg(aeg+3bdg-4bef)\right)+105(f+gx)^2(ef-dg)\left(-3eg(aeg\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*(e*f - d*g)^3*(c*f^2 + g*(-(b*f) + a*g)) + 315*(e*f - d*g)^2*(c*f*(
5*e*f - 2*d*g) + g*(-4*b*e*f + b*d*g + 3*a*e*g))*(f + g*x) + 105*(e*f - d*g
)*(-3*e*g*(-2*b*e*f + b*d*g + a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 63*e*(-(e*g*(-4*b*e*f + 3*b*d*g + a*e*g)) + c*(-10*e^2*f^2
+ 12*d*e*f*g - 3*d^2*g^2))*(f + g*x)^3 - 45*e^2*(5*c*e*f - 3*c*d*g - b*e*g)
*(f + g*x)^4 + 35*c*e^3*(f + g*x)^5))/(315*g^6*Sqrt[f + g*x])
```

fricas [A] time = 0.91, size = 438, normalized size = 1.54

$$\frac{2\left(35\,ce^3g^5x^5+1280\,ce^3f^5-315\,ad^3g^5-1152\left(3\,cde^2+be^3\right)f^4g+1008\left(3\,cd^2e+3\,bde^2+ae^3\right)f^3g^2-840\left(cd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 315*a*d^3*g^5 - 1152*(3*c*d*e^2
+ b*e^3)*f^4*g + 1008*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 - 840*(c*d^3
+ 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + 630*(b*d^3 + 3*a*d^2*e)*f*g^4 - 5*(10*c*
e^3*f*g^4 - 9*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + (80*c*e^3*f^2*g^3 - 72*(3*c*d*
e^2 + b*e^3)*f*g^4 + 63*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 - (160*c*e
^3*f^3*g^2 - 144*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 126*(3*c*d^2*e + 3*b*d*e^2 +
 a*e^3)*f*g^4 - 105*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f
^4*g - 576*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 504*(3*c*d^2*e + 3*b*d*e^2 + a*e^3
)*f^2*g^3 - 420*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 315*(b*d^3 + 3*a*d^
2*e)*g^5)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

giac [B] time = 0.24, size = 669, normalized size = 2.35

$$-\frac{2\left(cd^3f^2g^3-bd^3fg^4+ad^3g^5-3\,cd^2f^3g^2e+3\,bd^2f^2g^3e-3\,ad^2fg^4e+3\,cdf^4ge^2-3\,bdf^3g^2e^2+3\,adf^2g^3e^2\right.}{\sqrt{gx+f}\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 - b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f
^2*g^3*e - 3*a*d^2*f*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^
2*g^3*e^2 - c*f^5*e^3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*g^6) +
2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 + 31
5*sqrt(g*x + f)*b*d^3*g^52 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x +
f)^(3/2)*c*d^2*f*g^50*e + 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 315*(g*x +
f)^(3/2)*b*d^2*g^51*e - 1890*sqrt(g*x + f)*b*d^2*f*g^51*e + 945*sqrt(g*x +
f)*a*d^2*g^52*e + 135*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*
d*f*g^49*e^2 + 1890*(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c
*d*f^3*g^49*e^2 + 189*(g*x + f)^(5/2)*b*d*g^50*e^2 - 945*(g*x + f)^(3/2)*b*
d*f*g^50*e^2 + 2835*sqrt(g*x + f)*b*d*f^2*g^50*e^2 + 315*(g*x + f)^(3/2)*a*
d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*x + f)^(9/2)*c*g^48*
e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^(5/2)*c*f^2*g^48*e^3
 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x + f)*c*f^4*g^48*e^3
+ 45*(g*x + f)^(7/2)*b*g^49*e^3 - 252*(g*x + f)^(5/2)*b*f*g^49*e^3 + 630*(g
*x + f)^(3/2)*b*f^2*g^49*e^3 - 1260*sqrt(g*x + f)*b*f^3*g^49*e^3 + 63*(g*x
+ f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a*f*g^50*e^3 + 945*sqrt(g*x + f
)*a*f^2*g^50*e^3)/g^54
```

maple [B] time = 0.01, size = 540, normalized size = 1.89

$$-\frac{2\left(-35e^3c\,x^5g^5-45b\,e^3g^5x^4-135cd\,e^2g^5x^4+50c\,e^3f\,g^4x^4-63a\,e^3g^5x^3-189bd\,e^2g^5x^3+72b\,e^3f\,g^4x^3-189c\,d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/315/(g*x+f)^(1/2)*(-35*c*e^3*g^5*x^5-45*b*e^3*g^5*x^4-135*c*d*e^2*g^5*x^
4+50*c*e^3*f*g^4*x^4-63*a*e^3*g^5*x^3-189*b*d*e^2*g^5*x^3+72*b*e^3*f*g^4*x^
3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2*g^3*x^3-315*a*d*e^
2*g^5*x^2+126*a*e^3*f*g^4*x^2-315*b*d^2*e*g^5*x^2+378*b*d*e^2*f*g^4*x^2-144
*b*e^3*f^2*g^3*x^2-105*c*d^3*g^5*x^2+378*c*d^2*e*f*g^4*x^2-432*c*d*e^2*f^2*
g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5*x+1260*a*d*e^2*f*g^4*x-504*a*
e^3*f^2*g^3*x-315*b*d^3*g^5*x+1260*b*d^2*e*f*g^4*x-1512*b*d*e^2*f^2*g^3*x+5
76*b*e^3*f^3*g^2*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*f^2*g^3*x+1728*c*d*e^2*f^
3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2*e*f*g^4+2520*a*d*e^2*f^2
*g^3-1008*a*e^3*f^3*g^2-630*b*d^3*f*g^4+2520*b*d^2*e*f^2*g^3-3024*b*d*e^2*f
^3*g^2+1152*b*e^3*f^4*g+840*c*d^3*f^2*g^3-3024*c*d^2*e*f^3*g^2+3456*c*d*e^2
*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.46, size = 437, normalized size = 1.53

$$2\left(\frac{35\left(gx+f\right)^{\frac{9}{2}}ce^3-45\left(5\,ce^3f-\left(3\,cde^2+be^3\right)g\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(10\,ce^3f^2-4\left(3\,cde^2+be^3\right)fg+\left(3\,cd^2e+3\,bde^2+ae^3\right)g^2\right)\left(gx+f\right)^{\frac{5}{2}}-105\left(10\,ce^3f^3-6\left(3\,cde^2+be\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g)*(
g*x + f)^(7/2) + 63*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(5/2) - 105*(10*c*e^3*f^3 - 6*(3*c*d*e^
2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 4*(3*c*d*e^2 +
b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*sqrt(g*x + f))/g^5 + 315
*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d*e^
```

```
2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 + 3*a
*d^2*e)*f*g^4)/(sqrt(g*x + f)*g^5))/g
```

mupad [B] time = 0.12, size = 394, normalized size = 1.38

$$\frac{(f+g\,x)^{7/2}\,\left(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g\right)}{7\,g^6}-\frac{2\,c\,d^3\,f^2\,g^3-2\,b\,d^3\,f\,g^4+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2+6\,b\,d^2\,e\,f^2\,g^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(7*g^6) - (2*a*d^3
*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g^3 - 2*b*d^3*f*g^4 + 2*
b*e^3*f^4*g - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*b*d
*e^2*f^3*g^2 + 6*b*d^2*e*f^2*g^3 - 6*c*d^2*e*f^3*g^2)/(g^6*(f + g*x)^(1/2))
 + ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g
^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f*g))/(5*g^6) + (2*(f + g*x)^(3/2)*(d*g - e
*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8
*c*d*e*f*g))/(3*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^
2 + 5*c*e*f^2 - 4*b*e*f*g - 2*c*d*f*g))/g^6 + (2*c*e^3*(f + g*x)^(9/2))/(9*
g^6)
```

sympy [A] time = 158.56, size = 452, normalized size = 1.59

$$\frac{2ce^3\left(f+gx\right)^{\frac{9}{2}}}{9g^6}+\frac{\left(f+gx\right)^{\frac{7}{2}}\left(2be^3g+6cde^2g-10ce^3f\right)}{7g^6}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(2ae^3g^2+6bde^2g^2-8be^3fg+6cd^2eg^2-24\right.}{5g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(2*b*e**3*g + 6*c*d*e
**2*g - 10*c*e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*b*d*e**
2*g**2 - 8*b*e**3*f*g + 6*c*d**2*e*g**2 - 24*c*d*e**2*f*g + 20*c*e**3*f**2)
/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3 - 6*a*e**3*f*g**2 + 6*b*d**2*
e*g**3 - 18*b*d*e**2*f*g**2 + 12*b*e**3*f**2*g + 2*c*d**3*g**3 - 18*c*d**2*
e*f*g**2 + 36*c*d*e**2*f**2*g - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6
*a*d**2*e*g**4 - 12*a*d*e**2*f*g**3 + 6*a*e**3*f**2*g**2 + 2*b*d**3*g**4 -
12*b*d**2*e*f*g**3 + 18*b*d*e**2*f**2*g**2 - 8*b*e**3*f**3*g - 4*c*d**3*f*g
**3 + 18*c*d**2*e*f**2*g**2 - 24*c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2
*(d*g - e*f)**3*(a*g**2 - b*f*g + c*f**2)/(g**6*sqrt(f + g*x))
```

3.827 $\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(}{}$$

```
[Out] -2/3*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)
^(3/2)/g^5-2/5*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+
f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)/g^5/(g*x+f)^(1/2)-2*(-d*g+e
*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/g^5
```

Rubi [A] time = 0.29, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1261}

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])/g
^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g
^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^
(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))}{g^4}+\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4x^2}+\frac{(-eg(3\ldots}{}\right.\right.}{}$$

$$= -\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)}{g^5\sqrt{f+gx}} - \frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg\ldots}{g^5}$$

Mathematica [A] time = 0.36, size = 184, normalized size = 0.88

$$\frac{2\left(-35(f+gx)^2\left(-eg(aeg+2bdg-3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)-105(ef-dg)^2\left(g(ag-bf)+cf^2\right)-1\ldots\right.}{105g\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g)) - 105*(e*f - d*g)*(2*c*f*
(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*(f + g*x) - 35*(-(e*g*(-3*b
*e*f + 2*b*d*g + a*e*g)) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^2
 - 21*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(1
05*g^5*Sqrt[f + g*x])
```

fricas [A] time = 0.99, size = 269, normalized size = 1.28

$$\frac{2\left(15\,ce^2g^4x^4-384\,ce^2f^4-105\,ad^2g^4+336\left(2\,cde+be^2\right)f^3g-280\left(cd^2+2\,bde+ae^2\right)f^2g^2+210\left(bd^2+2\,ad\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 - 105*a*d^2*g^4 + 336*(2*c*d*e + b*
e^2)*f^3*g - 280*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 + 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 3*(8*c*e^2*f*g^3 - 7*(2*c*d*e + b*e^2)*g^4)*x^3 + (48*c*e^2*f^2*g^2
 - 42*(2*c*d*e + b*e^2)*f*g^3 + 35*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (19
2*c*e^2*f^3*g - 168*(2*c*d*e + b*e^2)*f^2*g^2 + 140*(c*d^2 + 2*b*d*e + a*e^
2)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

giac [B] time = 0.23, size = 404, normalized size = 1.92

$$-\frac{2\left(cd^2f^2g^2-bd^2fg^3+ad^2g^4-2\,cdf^3ge+2\,bdf^2g^2e-2\,adfg^3e+cf^4e^2-bf^3ge^2+af^2g^2e^2\right)}{\sqrt{gx+f}\,g^5}+\frac{2\left(35\left(gx+\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 - b*d^2*f*g^3 + a*d^2*g^4 - 2*c*d*f^3*g*e + 2*b*d*f^2*g^2
*e - 2*a*d*f*g^3*e + c*f^4*e^2 - b*f^3*g*e^2 + a*f^2*g^2*e^2)/(sqrt(g*x + f
)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32 - 210*sqrt(g*x + f)*c*d^2*f*g
^32 + 105*sqrt(g*x + f)*b*d^2*g^33 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g
```

```
*x + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 70*(g*x + f
)^(3/2)*b*d*g^32*e - 420*sqrt(g*x + f)*b*d*f*g^32*e + 210*sqrt(g*x + f)*a*d
*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^30*e^2 +
 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*e^2 + 21
*(g*x + f)^(5/2)*b*g^31*e^2 - 105*(g*x + f)^(3/2)*b*f*g^31*e^2 + 315*sqrt(g
*x + f)*b*f^2*g^31*e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*
a*f*g^32*e^2)/g^35
```

maple [A] time = 0.01, size = 315, normalized size = 1.50

$$-\frac{2\left(-15e^2c\,x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-21*b*e^2*g^4*x^3-42*c*d*e*g^4*x^3+2
4*c*e^2*f*g^3*x^3-35*a*e^2*g^4*x^2-70*b*d*e*g^4*x^2+42*b*e^2*f*g^3*x^2-35*c
*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*g^2*x^2-210*a*d*e*g^4*x+140*a*
e^2*f*g^3*x-105*b*d^2*g^4*x+280*b*d*e*f*g^3*x-168*b*e^2*f^2*g^2*x+140*c*d^2
*f*g^3*x-336*c*d*e*f^2*g^2*x+192*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^
3+280*a*e^2*f^2*g^2-210*b*d^2*f*g^3+560*b*d*e*f^2*g^2-336*b*e^2*f^3*g+280*c
*d^2*f^2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 269, normalized size = 1.28

$$\frac{2\left(\frac{15\left(gx+f\right)^{\frac{7}{2}}ce^2-21\left(4ce^2f-\left(2cde+be^2\right)g\right)\left(gx+f\right)^{\frac{5}{2}}+35\left(6ce^2f^2-3\left(2cde+be^2\right)fg+\left(cd^2+2bde+ae^2\right)g^2\right)\left(gx+f\right)^{\frac{3}{2}}-105\left(4ce^2f^3-3\left(2cde+be^2\right)f^2g+2\right.}{g^4}\right.}{105g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*
x + f)^(5/2) + 35*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
 + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2
*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*sqrt(g*x +
f))/g^4 - 105*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2
*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)/(sqrt(g*x + f)*g^4))/g
```

mupad [B] time = 3.13, size = 270, normalized size = 1.29

$$\frac{\left(f+g\,x\right)^{5/2}\,\left(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g\right)}{5\,g^5}-\frac{2\,c\,d^2\,f^2\,g^2-2\,b\,d^2\,f\,g^3+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g+4\,b\,d\,e\,f^2\,g^2-4\,a\,d\,e}{g^5\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(5*g^5) - (2*a*d^2*g^
4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 - 2*b*d^2*f*g^3 - 2*b*e
^2*f^3*g + 4*b*d*e*f^2*g^2 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^
(1/2)) + ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d
*e*g^2 - 6*b*e^2*f*g - 12*c*d*e*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*g - e
*f)*(2*a*e*g^2 + b*d*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/g^5 + (2*c*e
^2*(f + g*x)^(7/2))/(7*g^5)
```

sympy [A] time = 79.49, size = 272, normalized size = 1.30

$$\frac{2ce^2\left(f+gx\right)^{\frac{7}{2}}}{7g^5}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(2be^2g+4cdeg-8ce^2f\right)}{5g^5}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2ae^2g^2+4bdeg^2-6be^2fg+2cd^2g^2-12cde\right.}{3g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(2*b*e**2*g + 4*c*d*e
*g - 8*c*e**2*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 4*b*d*e*g**2
- 6*b*e**2*f*g + 2*c*d**2*g**2 - 12*c*d*e*f*g + 12*c*e**2*f**2)/(3*g**5) +
sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g**2 + 2*b*d**2*g**3 - 8*b*d*e*f*g
**2 + 6*b*e**2*f**2*g - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/
g**5 - 2*(d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f + g*x))
```

3.828 $\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3c}{3g^4}$$

```
[Out] -2/3*(-b*e*g-c*d*g+3*c*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(
-d*g+e*f)*(a*g^2-b*f*g+c*f^2)/g^4/(g*x+f)^(1/2)+2*(c*f*(-2*d*g+3*e*f)-g*(-a
*e*g-b*d*g+2*b*e*f))*(g*x+f)^(1/2)/g^4
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {771}

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3c}{3g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*f
- c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx=\int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{cf(3ef-2dg)-g(2bef-bdg-aeg)}{g^3\sqrt{f+gx}}+\frac{(-3c}{}\right.$$
$$=\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^4}$$

Mathematica [A] time = 0.18, size = 128, normalized size = 0.95

$$\frac{2\left(5g\left(3ag(-dg+2ef+egx)+3bdg(2f+gx)+be\left(-8f^2-4fgx+g^2x^2\right)\right)+c\left(5dg\left(-8f^2-4fgx+g^2x^2\right)+3e(\right.\right.}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 -
4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3 +
 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(15*g^4*Sqrt[f + g*x])
```

fricas [A] time = 0.95, size = 135, normalized size = 1.00

$$\frac{2\left(3\,ceg^3x^3+48\,cef^3-15\,adg^3-40\,(cd+be)f^2g+30\,(bd+ae)fg^2-\left(6\,cefg^2-5\,(cd+be)g^3\right)x^2+\left(24\,cef^2g\right.}{15\left(g^5x+fg^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 15*a*d*g^3 - 40*(c*d + b*e)*f^2*g + 30*(
b*d + a*e)*f*g^2 - (6*c*e*f*g^2 - 5*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2*g -
20*(c*d + b*e)*f*g^2 + 15*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

giac [A] time = 0.18, size = 204, normalized size = 1.51

$$-\frac{2\left(cdf^2g-bdfg^2+adg^3-cf^3e+bf^2ge-afg^2e\right)}{\sqrt{gx+f}\,g^4}+\frac{2\left(5\left(gx+f\right)^{\frac{3}{2}}cdg^{17}-30\sqrt{gx+f}\,cdfg^{17}+15\sqrt{gx+f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g - b*d*f*g^2 + a*d*g^3 - c*f^3*e + b*f^2*g*e - a*f*g^2*e)/(sqr
t(g*x + f)*g^4) + 2/15*(5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f
*g^17 + 15*sqrt(g*x + f)*b*d*g^18 + 3*(g*x + f)^(5/2)*c*g^16*e - 15*(g*x +
f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e + 5*(g*x + f)^(3/2)*b*g
^17*e - 30*sqrt(g*x + f)*b*f*g^17*e + 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

maple [A] time = 0.00, size = 144, normalized size = 1.07

$$-\frac{2\left(-3ce\,x^3g^3-5be\,g^3x^2-5cd\,g^3x^2+6cef\,g^2x^2-15ae\,g^3x-15bd\,g^3x+20bef\,g^2x+20cdf\,g^2x-24ce\,f^2gx\right.}{15\sqrt{gx+f}\;g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/15/(g*x+f)^(1/2)*(-3*c*e*g^3*x^3-5*b*e*g^3*x^2-5*c*d*g^3*x^2+6*c*e*f*g^2
*x^2-15*a*e*g^3*x-15*b*d*g^3*x+20*b*e*f*g^2*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x
+15*a*d*g^3-30*a*e*f*g^2-30*b*d*f*g^2+40*b*e*f^2*g+40*c*d*f^2*g-48*c*e*f^3)
/g^4
```

maxima [A] time = 0.44, size = 137, normalized size = 1.01

$$\frac{2\left(\frac{3\left(gx+f\right)^{\frac{5}{2}}ce-5\left(3\,cef-(cd+be)g\right)\left(gx+f\right)^{\frac{3}{2}}+15\left(3\,cef^2-2\,(cd+be)fg+(bd+ae)g^2\right)\sqrt{gx+f}}{g^3}+\frac{15\left(cef^3-adg^3-(cd+be)f^2g+(bd+ae)fg^2\right)}{\sqrt{gx+f}\,g^3}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(3/2)
+ 15*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*sqrt(g*x + f))/g^3 +
 15*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)/(sqrt(g*x +
 f)*g^3))/g
```

mupad [B] time = 3.13, size = 147, normalized size = 1.09

$$\frac{\left(f+g\,x\right)^{3/2}\,\left(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f\right)}{3\,g^4}-\frac{2\,a\,d\,g^3-2\,c\,e\,f^3-2\,a\,e\,f\,g^2-2\,b\,d\,f\,g^2+2\,b\,e\,f^2\,g+2\,c\,d\,f^2\,g}{g^4\,\sqrt{f+g\,x}}+\sqrt{f+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(3*g^4) - (2*a*d*g^3 - 2*c*
e*f^3 - 2*a*e*f*g^2 - 2*b*d*f*g^2 + 2*b*e*f^2*g + 2*c*d*f^2*g)/(g^4*(f + g*
x)^(1/2)) + ((f + g*x)^(1/2)*(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g
 - 4*c*d*f*g))/g^4 + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

sympy [A] time = 34.53, size = 141, normalized size = 1.04

$$\frac{2ce\left(f+gx\right)^{\frac{5}{2}}}{5g^4}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2beg+2cdg-6cef\right)}{3g^4}+\frac{\sqrt{f+gx}\left(2aeg^2+2bdg^2-4befg-4cdfg+6cef^2\right)}{g^4}-\frac{2\left(dg-e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*b*e*g + 2*c*d*g - 6*c
*e*f)/(3*g**4) + sqrt(f + g*x)*(2*a*e*g**2 + 2*b*d*g**2 - 4*b*e*f*g - 4*c*d
*f*g + 6*c*e*f**2)/g**4 - 2*(d*g - e*f)*(a*g**2 - b*f*g + c*f**2)/(g**4*sqr
t(f + g*x))
```

3.829 $\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2-b*f*g+c*f^2)/g^3/(g*x+f)^(1/2)-2*(-b*g+2*c
*f)*(g*x+f)^(1/2)/g^3
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {698}

$$-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
+ g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx&=\int \left(\frac{cf^2-bfg+ag^2}{g^2(f+gx)^{3/2}}+\frac{-2cf+bg}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\&=-\frac{2\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.76

$$\frac{6g(-ag+2bf+bgx)+2c\left(-8f^2-4fgx+g^2x^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*Sqrt[
f + g*x])
```

fricas [A] time = 0.68, size = 63, normalized size = 0.89

$$\frac{2\left(cg^2x^2-8\,cf^2+6\,bfg-3\,ag^2-\left(4\,cfg-3\,bg^2\right)x\right)\sqrt{gx+f}}{3\left(g^4x+fg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)*sqrt(
g*x + f)/(g^4*x + f*g^3)
```

giac [A] time = 0.15, size = 74, normalized size = 1.04

$$-\frac{2\left(cf^2-bfg+ag^2\right)}{\sqrt{gx+f}\,g^3}+\frac{2\left(\left(gx+f\right)^{\frac{3}{2}}cg^6-6\sqrt{gx+f}\,cfg^6+3\sqrt{gx+f}\,bg^7\right)}{3\,g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6
 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

maple [A] time = 0.00, size = 53, normalized size = 0.75

$$-\frac{2\left(-c\,x^2g^2-3b\,g^2x+4cfgx+3a\,g^2-6bfg+8c\,f^2\right)}{3\sqrt{gx+f}\;g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/3/(g*x+f)^(1/2)*(-c*g^2*x^2-3*b*g^2*x+4*c*f*g*x+3*a*g^2-6*b*f*g+8*c*f^2)
/g^3
```

maxima [A] time = 0.44, size = 66, normalized size = 0.93

$$\frac{2\left(\frac{\left(gx+f\right)^{\frac{3}{2}}c-3\left(2\,cf-bg\right)\sqrt{gx+f}}{g^2}-\frac{3\left(cf^2-bfg+ag^2\right)}{\sqrt{gx+f}\,g^2}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 3*(2*c*f - b*g)*sqrt(g*x + f))/g^2 - 3*(c*f^2 - b
*f*g + a*g^2)/(sqrt(g*x + f)*g^2))/g
```

mupad [B] time = 0.06, size = 58, normalized size = 0.82

$$\frac{2\,c\,{\left(f+g\,x\right)}^2-6\,a\,g^2-6\,c\,f^2+6\,b\,g\,\left(f+g\,x\right)-12\,c\,f\,\left(f+g\,x\right)+6\,b\,f\,g}{3\,g^3\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] (2*c*(f + g*x)^2 - 6*a*g^2 - 6*c*f^2 + 6*b*g*(f + g*x) - 12*c*f*(f + g*x) +
 6*b*f*g)/(3*g^3*(f + g*x)^(1/2))
```

sympy [A] time = 13.12, size = 70, normalized size = 0.99

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3g^3}+\frac{\sqrt{f+gx}\,\left(2bg-4cf\right)}{g^3}-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*g**3) + sqrt(f + g*x)*(2*b*g - 4*c*f)/g**3 - 2*(a*g
**2 - b*f*g + c*f**2)/(g**3*sqrt(f + g*x))
```

3.830 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

```
[Out] -2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3
/2)/(-d*g+e*f)^(3/2)+2*(a*g^2-b*f*g+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c
*(g*x+f)^(1/2)/e/g^2
```

Rubi [A] time = 0.22, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {897, 1261, 208}

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f +
 g*x])/(e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2-bde+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}dx,\right.}{e(ef-dg)}$$

$$= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.29, size = 124, normalized size = 1.02

$$\frac{2\left(-\frac{g^2\left(cd^2-e(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{cf^2-g(bf-ag)}{\sqrt{f+gx}\,(ef-dg)}+\frac{c\sqrt{f+gx}}{e}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*((c*f^2 - g*(b*f - a*g))/((e*f - d*g)*Sqrt[f + g*x]) + (c*Sqrt[f + g*x])
/e - ((c*d^2 - e*(b*d - a*e))*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
- d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))))/g^2
```

fricas [B] time = 0.94, size = 540, normalized size = 4.43

$$\left[-\frac{\left(\left(cd^2-bde+ae^2\right)g^3x+\left(cd^2-bde+ae^2\right)fg^2\right)\sqrt{e^2f-deg}\,\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{e^2f-deg}\,\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^3-a\right.}{e^4f^3g^2-2\,de^3f^2g^3+d^2e^2fg^4+\left(e^4f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*d^2 - b*d*e + a*e^2)*g^3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(e^2
*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))
/(e*x + d)) - 2*(2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c*d*e^2 + b*e^3)*f^2*g + (c
*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*
g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^
4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x), 2*(((c*d^2 - b*d*e + a*e^2)*g^
3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*
f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c
*d*e^2 + b*e^3)*f^2*g + (c*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g -
2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2
*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

giac [A] time = 0.25, size = 112, normalized size = 0.92

$$-\frac{2\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge-fe^2\right)^{\frac{3}{2}}}+\frac{2\sqrt{gx+f}\,ce^{(-1)}}{g^2}-\frac{2\left(cf^2-bfg+ag^2\right)}{\left(dg^3-fg^2e\right)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g
*e - f*e^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 - b*f*g + a*g^2
)/((d*g^3 - f*g^2*e)*sqrt(g*x + f))
```

maple [B] time = 0.01, size = 237, normalized size = 1.94

$$-\frac{2ae\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}+\frac{2bd\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}-\frac{2c\,d^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}\,e}-\frac{2a}{(dg-ef)\sqrt{gx+f}}+\frac{2bf}{(dg-ef)\sqrt{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*c/e/g^2-2/(d*g-e*f)*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1
/2)/((d*g-e*f)*e)^(1/2)*e)*a+2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)
^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*d-2/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2-2/(d*g-e*f)/(g*x+f)^(1/2)*a+2/g/
(d*g-e*f)/(g*x+f)^(1/2)*b*f-2/g^2/(d*g-e*f)/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.21, size = 162, normalized size = 1.33

$$\frac{2c\sqrt{f+gx}}{e\,g^2}+\frac{2\,\mathrm{atan}\left(\frac{2\sqrt{f+gx}\,\left(e^2\,f-d\,e\,g\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{3/2}\left(2\,c\,d^2-2\,b\,d\,e+2\,a\,e^2\right)}\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{3/2}\left(d\,g-e\,f\right)^{3/2}}-\frac{2\left(c\,e\,f^2-b\,e\,f\,g+a\,e\,g^2\right)}{e\,g^2\sqrt{f+gx}\,\left(d\,g-e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*c*(f + g*x)^(1/2))/(e*g^2) + (2*atan((2*(f + g*x)^(1/2)*(e^2*f - d*e*g)*
(a*e^2 + c*d^2 - b*d*e))/(e^(1/2)*(d*g - e*f)^(3/2)*(2*a*e^2 + 2*c*d^2 - 2*
b*d*e)))*(a*e^2 + c*d^2 - b*d*e))/(e^(3/2)*(d*g - e*f)^(3/2)) - (2*(a*e*g^2
 + c*e*f^2 - b*e*f*g))/(e*g^2*(f + g*x)^(1/2)*(d*g - e*f))
```

sympy [A] time = 52.23, size = 116, normalized size = 0.95

$$\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\,\left(dg-ef\right)}-\frac{2\left(ae^2-bde+cd^2\right)\mathrm{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\,\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 - b*f*g + c*f**2)/(g**2*sqrt(f + g*x
)*(d*g - e*f)) - 2*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g -
 e*f)/e))/(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f))
```

3.831 $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2-bfg+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

```
[Out] (c*d*(-d*g+4*e*f)-e*(-3*a*e*g+b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2-b*f*g+c*f^2)/g/(-d*g+e
*f)^2/(g*x+f)^(1/2)-(a*e^2-b*d*e+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d
)
```

Rubi [A] time = 0.37, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {897, 1259, 453, 208}

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2-bfg+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 - b*
d*e + a*e^2)*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*
g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
```

```
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\,\mathrm{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(e(bd-ae)g^2+c\left(2e^2f^2-4def\right.\right.}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\right.}{e^2(ef-dg)^2}$$

$$= -\frac{2\left(cf^2-bfg+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(cd(4ef-dg)-e(2bef+}{}$$

$$= -\frac{2\left(cf^2-bfg+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(cd(4ef-dg)-e(2bef+}{e^{3/2}(}$$

Mathematica [A] time = 0.41, size = 176, normalized size = 1.07

$$-\frac{eg(2adg+ae(f+3gx)-b(3df+dgx+2efx))+c\left(d^2g(f+gx)+2def^2+2e^2f^2x\right)}{eg(d+ex)\sqrt{f+gx}\,(ef-dg)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] -((c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x)) + e*g*(2*a*d*g + a*e*(f +
3*g*x) - b*(3*d*f + 2*e*f*x + d*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f
+ g*x])) - ((c*d*(-4*e*f + d*g) + e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(S
qrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

fricas [B] time = 0.93, size = 1088, normalized size = 6.59

$$\left[\frac{\left(2\left(2\,cd^2e-bde^2\right)f^2g-\left(cd^3+bd^2e-3\,ade^2\right)fg^2+\left(2\left(2\,cde^2-be^3\right)fg^2-\left(cd^2e+bde^2-3\,ae^3\right)g^3\right)x^2+\left(2\left(2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*(2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2
+ (2*(2*c*d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (
2*(2*c*d*e^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3
+ b*d^2*e - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g
 + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a
*d^2*e^2*g^3 - (c*d^2*e^2 + 3*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e
```

```
^2 - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2
 + b*d*e^3 + 3*a*e^4)*f*g^2 - (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqr
t(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*
f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x), -((2*(
2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2 + (2*(2*c*
d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (2*(2*c*d*e
^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + b*d^2*e
- 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(
g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*d^2*e^2 + 3
*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*f*g^2 -
 (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*
d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^
5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g
^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

giac [A] time = 0.23, size = 282, normalized size = 1.71

$$\frac{\left(cd^2g - 4\,cdfe + bdge + 2\,bfe^2 - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge-fe^2}} - \frac{\left(gx+f\right)cd^2g^2 + 2\,cdf^2ge - \left(gx+f\right)bdg^2e - 2\,bdfg}{\left(d^2g^3e - 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e + b*d*g*e + 2*b*f*e^2 - 3*a*g*e^2)*arctan(sqrt(g*x + f
)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e -
f*e^2)) - ((g*x + f)*c*d^2*g^2 + 2*c*d*f^2*g*e - (g*x + f)*b*d*g^2*e - 2*b*
d*f*g^2*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^2 - 2*(g*x + f)
*b*f*g*e^2 + 2*b*f^2*g*e^2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g
^3*e - 2*d*f*g^2*e^2 + f^2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e -
sqrt(g*x + f)*f*e))
```

maple [B] time = 0.02, size = 418, normalized size = 2.53

$$-\frac{3aeg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}} + \frac{bdg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}} + \frac{2bef\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}} + \frac{c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}\,e} - \frac{4cd}{(dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

```
[Out] -1/(d*g-e*f)^2*(g*x+f)^(1/2)/(e*g*x+d*g)*a*e*g+g/(d*g-e*f)^2*(g*x+f)^(1/2)/
(e*g*x+d*g)*b*d-1/(d*g-e*f)^2*(g*x+f)^(1/2)/(e*g*x+d*g)*c*d^2/e*g-3/(d*g-e*
f)^2/((d*g-e*f)*e)^(1/2)*a*e*g*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)+
g/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*
e)*b*d+2/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2)*e)*b*f+1/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*c*d^2/e*g*arctan((g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2)*e)-4/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*c*d*f*arctan
((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)-2/(d*g-e*f)^2/(g*x+f)^(1/2)*a*g+2/(d*
g-e*f)^2/(g*x+f)^(1/2)*b*f-2/(d*g-e*f)^2/(g*x+f)^(1/2)*c*f^2/g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.30, size = 218, normalized size = 1.32

$$\frac{\operatorname{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{5/2}}\right)\left(2\,b\,e^2\,f-3\,a\,e^2\,g+c\,d^2\,g+b\,d\,e\,g-4\,c\,d\,e\,f\right)}{e^{3/2}\,\left(d\,g-e\,f\right)^{5/2}}-\frac{\frac{2\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)\left(c\,d^2\,g\ldots\right.}{\ldots}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))/(e^(1/2)*(d*g -
 e*f)^(5/2)))*(2*b*e^2*f - 3*a*e^2*g + c*d^2*g + b*d*e*g - 4*c*d*e*f))/(e^(
3/2)*(d*g - e*f)^(5/2)) - ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f +
g*x)*(3*a*e^2*g^2 + c*d^2*g^2 + 2*c*e^2*f^2 - b*d*e*g^2 - 2*b*e^2*f*g))/(e*
(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f + g*x)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{4e^{3/2}(ef-dg)^{7/2}}+\frac{2\left(ag^2\right.}{\sqrt{f+}}$$

```
[Out] -1/4*(c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2)+3*e*g*(5*a*e*g-b*(d*g+4*e*f)))*arcta
nh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^
2-b*f*g+c*f^2)/(-d*g+e*f)^3/(g*x+f)^(1/2)-1/2*(a*e^2-b*d*e+c*d^2)*(g*x+f)^(
1/2)/e/(-d*g+e*f)^2/(e*x+d)^2+1/4*(c*d*(-d*g+8*e*f)-e*(-7*a*e*g+3*b*d*g+4*b
*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

Rubi [A] time = 0.63, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {897, 1259, 456, 453, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{2\left(ag^2\right.}{\sqrt{f+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 - b*d*e
 + a*e^2)*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((c*d*(8*e*f - d
*g) - e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d
 + e*x)) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*
f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(3/2)*(e*
f - d*g)^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(3e(bd-ae)g^2+c\left(4e^2f^2-8de\right.\right.}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\right.}{2e^2(ef-dg)^2}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3b}{4e(ef-dg)^3(}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3b}{4e(ef-dg)^3(}$$

Mathematica [A] time = 1.10, size = 290, normalized size = 1.17

$$\frac{1}{4}\left(-\frac{2\sqrt{f+gx}\,\left(e(ae-bd)+cd^2\right)}{e(d+ex)^2(ef-dg)^2} + \frac{g\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-7aeg+3bdg+4bef)+cd(dg-8ef))}{e^{3/2}(ef-dg)^{7/2}} + \frac{8\left(g(ag\right.}{\sqrt{f+g}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((8*(c*f^2 + g*(-(b*f) + a*g)))/((e*f - d*g)^3*Sqrt[f + g*x]) - (2*(c*d^2 +
 e*(-(b*d) + a*e))*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)^2) - ((c*d*(-8
```

```
*e*f + d*g) + e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(e*(e*f - d*g
)^3*(d + e*x)) - (8*Sqrt[e]*(c*f^2 + g*(-(b*f) + a*g))*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/Sqrt[e*f - d*g]])/(e*f - d*g)^(7/2) + (g*(c*d*(-8*e*f + d*g) +
e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g]])/(e^(3/2)*(e*f - d*g)^(7/2)))/4
```

fricas [B] time = 1.06, size = 1883, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g - (c*d^4 + 3*b*
d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 - 3*b*e^4)*f*g^
2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 12*(2*c*d*
e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g^2 - 2*(c*d^3
*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*(c*d^2*e^2 -
 b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2 - (c*d^4 + 3*
b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*
g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a*d^3*e^2*g^3 -
2*(7*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a
*d*e^4)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
 12*b*e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2
+ 3*b*d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d
^2*e^3 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e
^4)*f*g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(
d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6
*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f
*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*
d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d
^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^
6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g -
(c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 -
3*b*e^4)*f*g^2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3
 + 12*(2*c*d*e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g
^2 - 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24
*(c*d^2*e^2 - b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2
- (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sq
rt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c
*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a*d*e^4
)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 12*b*
e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 + 3*b*
d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d^2*e^3
 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e^4)*f*
g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^
6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f
*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 +
 d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^
5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6
*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*
g^5)*x)]
```

giac [B] time = 0.26, size = 462, normalized size = 1.86

$$\frac{\left(cd^2g^2 - 8\,cdfge + 3\,bdg^2e - 8\,cf^2e^2 + 12\,bfge^2 - 15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4\right)\sqrt{dge - fe^2}} - \frac{2\left(cf^2 - bfg + ag^2\right)}{\left(d^3g^3 - 3\,d^2fg^2e + 3\,df^2ge^2 - f^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e + 3*b*d*g^2*e - 8*c*f^2*e^2 + 12*b*f*g*e^2 - 1
5*a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^
2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 - b*
f*g + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x
+ f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3 - (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt
(g*x + f)*c*d^2*f*g^2*e - 5*sqrt(g*x + f)*b*d^2*g^3*e + 8*(g*x + f)^(3/2)*c
*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 - 3*(g*x + f)^(3/2)*b*d*g^2*e^2
+ sqrt(g*x + f)*b*d*f*g^2*e^2 + 9*sqrt(g*x + f)*a*d*g^3*e^2 - 4*(g*x + f)^(
3/2)*b*f*g*e^3 + 4*sqrt(g*x + f)*b*f^2*g*e^3 + 7*(g*x + f)^(3/2)*a*g^2*e^3
- 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^
3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

maple [B] time = 0.03, size = 847, normalized size = 3.42

$$-\frac{9\sqrt{gx+f}\,ade\,g^{3}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}+\frac{9\sqrt{gx+f}\,a\,e^{2}f\,g^{2}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}+\frac{5\sqrt{gx+f}\,b\,d^{2}g^{3}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\frac{\sqrt{gx+f}\,bdef\,g^{2}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

```
[Out] -7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*a*e^2*g^2+3/4/(d*g-e*f)^3/(e*g
*x+d*g)^2*(g*x+f)^(3/2)*b*d*e*g^2+1/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)
*b*e^2*f*g+1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*c*d^2*g^2-2/(d*g-e*f
)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*c*d*e*f*g-9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3
*e*(g*x+f)^(1/2)*a*d+9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*e^2*(g*x+f)^(1/2)*a*
f+5/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3*(g*x+f)^(1/2)*b*d^2-1/4/(d*g-e*f)^3/(e*
g*x+d*g)^2*g^2*e*(g*x+f)^(1/2)*f*d*b-1/(d*g-e*f)^3/(e*g*x+d*g)^2*g*e^2*(g*x
+f)^(1/2)*b*f^2-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3/e*(g*x+f)^(1/2)*c*d^3-7/4
/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*(g*x+f)^(1/2)*c*d^2*f+2/(d*g-e*f)^3/(e*g*x+d
*g)^2*g*e*(g*x+f)^(1/2)*c*d*f^2-15/4/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arct
an((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a*g^2+3/4/(d*g-e*f)^3/((d*g-e*f)*e)
^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*d*g^2+3/(d*g-e*f)^3*e/
((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*f*g+1/4/(
d*g-e*f)^3/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e
)*c*d^2*g^2-2/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*
f)*e)^(1/2)*e)*c*d*f*g-2/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(
1/2)/((d*g-e*f)*e)^(1/2)*e)*c*f^2-2/(d*g-e*f)^3/(g*x+f)^(1/2)*a*g^2+2/(d*g-
e*f)^3/(g*x+f)^(1/2)*b*f*g-2/(d*g-e*f)^3/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.41, size = 363, normalized size = 1.46

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,\left(d\,g-e\,f\right)^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\ldots\right.}{4\,e^{3/2}\,\left(d\,g-e\,f\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 -
 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) -
 ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*
d^2*g^2 + 8*c*e^2*f^2 - 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*(d*g
- e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 - 5*b*d*e*g
^2 - 20*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2)
 - (f + g*x)^(3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2
 - 2*d*e*f*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.833 $\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx$

Optimal. Leaf size=91

$$\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{\sqrt{5}-2}\sqrt{x-1}}\right)-\cosh^{-1}(x)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2+\sqrt{5}}\sqrt{x-1}}\right)$$

```
[Out] -arccosh(x)+1/5*arctan((1+x)^(1/2)/(-1+x)^(1/2)/(-2+5^(1/2))^(1/2))*(-10+10
*5^(1/2))^(1/2)+1/5*arctanh((1+x)^(1/2)/(-1+x)^(1/2)/(2+5^(1/2))^(1/2))*(10
+10*5^(1/2))^(1/2)
```

Rubi [B] time = 0.14, antiderivative size = 191, normalized size of antiderivative = 2.10, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {901, 991, 217, 206, 1034, 725, 204}

$$\frac{\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\sqrt{x-1}\sqrt{x+1}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{x-1}\sqrt{x+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\sqrt{x-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[5])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTan[(2 - (1 - Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2] - (Sqrt[-1 + x
]*Sqrt[1 + x]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^2] - (Sqrt[(1 + Sqrt[5
])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 +
Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
```

```
 && EqQ[e*f + d*g, 0]
```

Rule 991

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx = \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{\sqrt{-1+x^2}}{1+x-x^2}\,dx}{\sqrt{-1+x^2}}$$

$$= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}} + \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{x}{\left(1+x-x^2\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}$$

$$= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} + \frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{}{(1-\sqrt{}}}{5\sqrt{-1+x^2}}$$

$$= -\frac{\sqrt{-1+x}\sqrt{1+x}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} - \frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{-4+\left(1-\sqrt{5}\right)^2}\right.}{5\sqrt{-1+x^2}}$$

$$= \frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\sqrt{-1+x}\sqrt{1+x}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} - \frac{\sqrt{-1+x}\sqrt{1+x}\,\tanh^{-1}\left(\right.}{\sqrt{-1+x^2}}$$

Mathematica [A] time = 0.21, size = 113, normalized size = 1.24

$$-\frac{1}{5}\sqrt{\sqrt{5}-2}\left(5+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\sqrt{5}-2}\sqrt{\frac{x-1}{x+1}}\right)-2\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{1}{5}\left(\sqrt{5}-5\right)\sqrt{2+\sqrt{5}}\,\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] -1/5*(Sqrt[-2 + Sqrt[5]]*(5 + Sqrt[5])*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 +
 x)/(1 + x)]]) - 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]] - ((-5 + Sqrt[5])*Sqrt[2
 + Sqrt[5]]*ArcTanh[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x)]])/5
```

fricas [B] time = 0.96, size = 214, normalized size = 2.35

$$\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\,\arctan\left(\frac{1}{8}\sqrt{-4\left(2x+\sqrt{5}-1\right)\sqrt{x+1}\sqrt{x-1}+8x^2+4\sqrt{5}x-4x}\sqrt{2\sqrt{5}-2}\left(\sqrt{5}+1\right)-\frac{1}{4}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/8*sqrt(-4*(2*x + sqrt(5) - 1)*sqrt
(x + 1)*sqrt(x - 1) + 8*x^2 + 4*sqrt(5)*x - 4*x)*sqrt(2*sqrt(5) - 2)*(sqrt(
5) + 1) - 1/4*(sqrt(x + 1)*sqrt(x - 1)*(sqrt(5) + 1) - sqrt(5)*x - x - 2)*s
qrt(2*sqrt(5) - 2)) + 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sq
rt(x - 1) - 2*x + sqrt(5) + sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt(5)*sqrt(2*
sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + sqrt(5) - sqrt(2*sqrt(5)
 + 2) + 1) + log(sqrt(x + 1)*sqrt(x - 1) - x)
```

giac [A] time = 0.20, size = 16, normalized size = 0.18

$$\log\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")
```

```
[Out] log((sqrt(x + 1) - sqrt(x - 1))^2)
```

maple [B] time = 0.09, size = 231, normalized size = 2.54

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{5}\,\left(-\sqrt{5}\,\sqrt{-2+2\sqrt{5}}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x+x-2}{\sqrt{2\sqrt{5}+2}\,\sqrt{x^2-1}}\right)-\sqrt{-2+2\sqrt{5}}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x+x-2}{\sqrt{2\sqrt{5}+2}\,\sqrt{x^2-1}}\right)-\right.}{5\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)*(x+1)^(1/2)/(-x^2+x+1),x)
```

```
[Out] -1/5*(x-1)^(1/2)*(x+1)^(1/2)*5^(1/2)*(5^(1/2)*ln(x+(x^2-1)^(1/2))*(2*5^(1/2
)+2)^(1/2)*(-2+2*5^(1/2))^(1/2)-5^(1/2)*arctan((x*5^(1/2)-x+2)/(-2+2*5^(1/2
))^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)+2)^(1/2)-5^(1/2)*arctanh((x*5^(1/2)+x-2)
/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))*(-2+2*5^(1/2))^(1/2)+arctan((x*5^(1/2)-
x+2)/(-2+2*5^(1/2))^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)+2)^(1/2)-arctanh((x*5^(
1/2)+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))*(-2+2*5^(1/2))^(1/2))/(x^2-1)^
(1/2)/(2*5^(1/2)+2)^(1/2)/(-2+2*5^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x+1}\,\sqrt{x-1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x + 1)*sqrt(x - 1)/(x^2 - x - 1), x)
```

mupad [B] time = 5.02, size = 916, normalized size = 10.07

$$-4\,\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)-\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{3408370\,\sqrt{10}\,\sqrt{\sqrt{5}+1}-\sqrt{10}\,\sqrt{\sqrt{5}+1}\,\sqrt{x-1}\,300730\mathrm{i}-3408370\,\sqrt{10}\,\sqrt{\sqrt{5}+1}\,\sqrt{x+1}-1771398}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(1/2)*(x + 1)^(1/2))/(x - x^2 + 1),x)
```

```
[Out] - 4*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) - (10^(1/2)*atan((34083
70*10^(1/2)*(5^(1/2) + 1)^(1/2) - 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2
)*300730i - 3408370*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 1771398*5^
(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2) + 7836865*10^(1/2)*x*(5^(1/2) + 1)^(1/2)
 + 3066340*10^(1/2)*x^2*(5^(1/2) + 1)^(1/2) - 1294942*5^(1/2)*10^(1/2)*x^2*
(5^(1/2) + 1)^(1/2) + 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1
/2)*300730i - 5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i +
1771398*5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(5^
(1/2) + 1)^(1/2)*(x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(5^(1/2) + 1)^(
1/2)*(x + 1)^(1/2) - 3475583*5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2) + 5^(1/
2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i + 10^(1
/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i - 5^(1/2)*10^
(1/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i + 2589884*5^(1/2)*10^(1/2
)*x*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) + 5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1
/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x + 1)^(1
/2) - 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) - 7067880*5^(1/2) - 39924
30*5^(1/2)*x^2 + 12033450*x^2 + 7067880*5^(1/2)*(x + 1)^(1/2) + 7984860*5^(
1/2)*x*(x + 1)^(1/2) + 10104760))*(5^(1/2) + 1)^(1/2)*1i)/5 - (10^(1/2)*ata
n((3408370*10^(1/2)*(1 - 5^(1/2))^(1/2) + 3066340*10^(1/2)*x^2*(1 - 5^(1/2)
)^(1/2) - 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*300730i - 3408370*10^(
1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) + 1771398*5^(1/2)*10^(1/2)*(1 - 5^(1
/2))^(1/2) + 7836865*10^(1/2)*x*(1 - 5^(1/2))^(1/2) + 3475583*5^(1/2)*10^(1
/2)*x*(1 - 5^(1/2))^(1/2) + 1294942*5^(1/2)*10^(1/2)*x^2*(1 - 5^(1/2))^(1/2
) + 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*300730i + 5^(1
/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 1771398*5^(1/2)*10
^(1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(
x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2)
 - 5^(1/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i
 + 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i + 5^(
1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 2589884*5^(1/2)
*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 5^(1/2)*10^(1/2)*x*(1 - 5^(
1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x
 + 1)^(1/2) + 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) + 7067880*5^(1/2)
 + 3992430*5^(1/2)*x^2 + 12033450*x^2 - 7067880*5^(1/2)*(x + 1)^(1/2) - 798
4860*5^(1/2)*x*(x + 1)^(1/2) + 10104760))*(1 - 5^(1/2))^(1/2)*1i)/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x-1}\,\sqrt{x+1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)*(1+x)**(1/2)/(-x**2+x+1),x)
```

```
[Out] -Integral(sqrt(x - 1)*sqrt(x + 1)/(x**2 - x - 1), x)
```

3.834 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+}{4e^2g^2}$$

```
[Out] 1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctanh
(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)
^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*(g*
x+f)^(1/2)/e^2/g^2
```

Rubi [A] time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+}{4e^2g^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

Mathematica [A] time = 0.77, size = 173, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)+e\sqrt{g}\sqrt{d+ex}\,(f+}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x))
 + Sqrt[e*f - d*g]*(c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g
- b*(e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d +
 e*x])/Sqrt[e*f - d*g]])/(4*e^3*g^(5/2)*Sqrt[f + g*x])
```

fricas [A] time = 1.11, size = 380, normalized size = 2.32

$$\left[\frac{\left(3\,ce^2f^2+2\left(cde-2\,be^2\right)fg+\left(3\,cd^2-4\,bde+8\,ae^2\right)g^2\right)\sqrt{eg}\,\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\left(2\,egx+\right.\right.}{16\,}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

giac [A] time = 0.26, size = 179, normalized size = 1.09

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\,\sqrt{xe+d}\left(\frac{2\,(xe+d)ce^{(-3)}}{g}-\frac{\left(5\,cdg^2e^5+3\,cfge^6-4\,bg^2e^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3\,cd^2g^2+2\,cdfg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6 - 4*b*g^2*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2
*g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)
*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g
*e + f*e^2)))/g^(5/2)
```

maple [B] time = 0.03, size = 425, normalized size = 2.59

$$\frac{\left(8a\,e^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4bde\,g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4b\,e^2fg\ln\left(\frac{2egx+dg+ef+}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*a*e^2*g^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1
/2))/(e*g)^(1/2))-4*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)
^(1/2))/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*e^2*f*g+3*c*d^2*g^2*ln(1/2*(2*e*g*x+d*g
+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+2*c*d*e*f*g*ln(1/2
*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+3*c*e
^2*f^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)
^(1/2))+4*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*g*x+8*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*b*e*g-6*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*d*g-6*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 22.38, size = 833, normalized size = 5.08

$$\frac{\frac{\left(2\,b\,d\,g+2\,b\,e\,f\right)\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{g^3\left(\sqrt{f+g\,x}-\sqrt{f}\right)}+\frac{\left(2\,b\,d\,g+2\,b\,e\,f\right)\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}-\frac{8\,b\,\sqrt{d}\,\sqrt{f}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.835 $\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=333

$$-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(ef-dg)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{64e^2g^4}+\frac{(d+ex)^{3/2}\sqrt{f+gx}\,(}{}$$

```
[Out] 1/64*(-d*g+e*f)^2*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*
g+5*e*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(
9/2)+1/96*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e*f)
))*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^3-1/24*(-8*b*e*g+9*c*d*g+7*c*e*f)*(e*x
+d)^(5/2)*(g*x+f)^(1/2)/e^2/g^2+1/4*c*(e*x+d)^(7/2)*(g*x+f)^(1/2)/e^2/g-1/6
4*(-d*g+e*f)*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e
*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^4
```

Rubi [A] time = 0.35, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{(d+ex)^{3/2}\sqrt{f+gx}\,\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{96e^2g^3}-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(ef-dg)\left(8e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] -((e*f - d*g)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b
*(5*e*f + d*g)))*Sqrt[d + e*x]*Sqrt[f + g*x])/(64*e^2*g^4) + ((c*(35*e^2*f^
2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(d + e*x)^
(3/2)*Sqrt[f + g*x])/(96*e^2*g^3) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*(d + e*x
)^(5/2)*Sqrt[f + g*x])/(24*e^2*g^2) + (c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(4*
e^2*g) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6
*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
+ g*x])])/(64*e^(5/2)*g^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(8ae^2g-cd(7ef+dg)\right)-\frac{1}{2}e(7cef+9cdg-8beg)x\right)}{\sqrt{f+gx}}\,dx}{4e^2g}$$

$$= -\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\left(c\left(35e^2f\right.\right.}{}$$

$$= \frac{\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}}{64e^2g^4}$$

Mathematica [A] time = 1.54, size = 313, normalized size = 0.94

$$\frac{3(ef-dg)^{5/2}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)-e\sqrt{g}\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-(e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(c*(9*d^3*g^3 + 3*d^2*e*g^2*(5*f - 2*g
*x) + d*e^2*g*(-145*f^2 + 92*f*g*x - 72*g^2*x^2) + e^3*(105*f^3 - 70*f^2*g*
x + 56*f*g^2*x^2 - 48*g^3*x^3)) - 8*e*g*(6*a*e*g*(-3*e*f + 5*d*g + 2*e*g*x)
 + b*(3*d^2*g^2 + 2*d*e*g*(-11*f + 7*g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*
x^2))))) + 3*(e*f - d*g)^(5/2)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8
*e*g*(6*a*e*g - b*(5*e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(
Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(192*e^3*g^(9/2)*Sqrt[f + g*x])
```

fricas [A] time = 1.32, size = 852, normalized size = 2.56

$$\left[\frac{3\left(35\,ce^4f^4-20\left(3\,cde^3+2\,be^4\right)f^3g+6\left(3\,cd^2e^2+12\,bde^3+8\,ae^4\right)f^2g^2+4\left(cd^3e-6\,bd^2e^2-24\,ade^3\right)fg^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*c*e^4*f^4 - 20*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 +
 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g
^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*g^4)*sqrt(e*g)*log(8*e^2*g^2*x^2
+ e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*
x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(48*c*e^4*g^4*x^3 - 105
*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (15*c*d^2*e^2 + 176*b*d*
e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*g^4 - 8*(
7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(35*c*e^4*f^2*g^2 - 2*(2
3*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*g^4)*x)
*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5), -1/384*(3*(35*c*e^4*f^4 - 20*(3*c*
d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 + 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4
*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d
^2*e^2)*g^4)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*
x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(
48*c*e^4*g^4*x^3 - 105*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (1
5*c*d^2*e^2 + 176*b*d*e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 -
 80*a*d*e^3)*g^4 - 8*(7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(3
5*c*e^4*f^2*g^2 - 2*(23*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e
^3 + 48*a*e^4)*g^4)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5)]
```

giac [A] time = 0.42, size = 448, normalized size = 1.35

$$\frac{1}{192}\sqrt{(xe+d)ge-dge+fe^2}\left(2\left(4\,(xe+d)\left(\frac{6\,(xe+d)ce^{(-3)}}{g}-\frac{\left(9\,cdg^6e^6+7\,cfg^5e^7-8\,bg^6e^7\right)e^{(-9)}}{g^7}\right)+\frac{\left(3\,cd^2g^6\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(4*(x*e + d)*(6*(x*e + d)*c*e^
(-3)/g - (9*c*d*g^6*e^6 + 7*c*f*g^5*e^7 - 8*b*g^6*e^7)*e^(-9)/g^7) + (3*c*d
```

```
^2*g^6*e^6 + 10*c*d*f*g^5*e^7 - 8*b*d*g^6*e^7 + 35*c*f^2*g^4*e^8 - 40*b*f*g
^5*e^8 + 48*a*g^6*e^8)*e^(-9)/g^7)*(x*e + d) + 3*(3*c*d^3*g^6*e^6 + 7*c*d^2
*f*g^5*e^7 - 8*b*d^2*g^6*e^7 + 25*c*d*f^2*g^4*e^8 - 32*b*d*f*g^5*e^8 + 48*a
*d*g^6*e^8 - 35*c*f^3*g^3*e^9 + 40*b*f^2*g^4*e^9 - 48*a*f*g^5*e^9)*e^(-9)/g
^7)*sqrt(x*e + d) - 1/64*(3*c*d^4*g^4 + 4*c*d^3*f*g^3*e - 8*b*d^3*g^4*e + 1
8*c*d^2*f^2*g^2*e^2 - 24*b*d^2*f*g^3*e^2 + 48*a*d^2*g^4*e^2 - 60*c*d*f^3*g*
e^3 + 72*b*d*f^2*g^2*e^3 - 96*a*d*f*g^3*e^3 + 35*c*f^4*e^4 - 40*b*f^3*g*e^4
 + 48*a*f^2*g^2*e^4)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt
((x*e + d)*g*e - d*g*e + f*e^2)))/g^(9/2)
```

maple [B] time = 0.03, size = 1207, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/384*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(9*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*
x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^4*g^4+105*ln(1/2*(2*e*g*x+d*g+e*f
+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*e^4*f^4-72*ln(1/2*(2
*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d^2*e^
2*f*g^3-30*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^2*e*f*g^2+224*(e*g)^(1/2
)*((e*x+d)*(g*x+f))^(1/2)*x*b*d*e^2*g^3-160*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(
1/2)*x*b*e^3*f*g^2+12*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*d^2*e*g^3+140
*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*e^3*f^2*g-112*x^2*c*e^3*f*g^2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-352*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*d
*e^2*f*g^2+290*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d*e^2*f^2*g+144*x^2*c*
d*e^2*g^3*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+144*ln(1/2*(2*e*g*x+d*g+e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*d^2*e^2*g^4-184*(e*g)^
(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*d*e^2*f*g^2+144*ln(1/2*(2*e*g*x+d*g+e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*e^4*f^2*g^2-120*ln(1/2
*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*e^4
*f^3*g-210*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^3*f^3-24*ln(1/2*(2*e*g*x
+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d^3*e*g^4-18
*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^3*g^3+216*ln(1/2*(2*e*g*x+d*g+e*f+
2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d*e^3*f^2*g^2+54*ln(1
/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d
^2*e^2*f^2*g^2-180*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^
(1/2))/(e*g)^(1/2))*c*d*e^3*f^3*g+480*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a
*d*e^2*g^3-288*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a*e^3*f*g^2+240*(e*g)^(1
/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^3*f^2*g-288*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x
+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*d*e^3*f*g^3+48*(e*g)^(1/2)*(
(e*x+d)*(g*x+f))^(1/2)*b*d^2*e*g^3+192*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*
x*a*e^3*g^3+96*x^3*c*e^3*g^3*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+128*x^2*b*
e^3*g^3*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+12*ln(1/2*(2*e*g*x+d*g+e*f+2*((
e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^3*e*f*g^3)/e^2/g^4/((e*
x+d)*(g*x+f))^(1/2)/(e*g)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}\,\left(c\,x^2+b\,x+a\right)}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Timed out
```

3.836 $\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(2eg(4aeg\right.}{8}$$

```
[Out] -1/8*(-d*g+e*f)*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*(4*a*e*g-b*(d*g+3*e*
f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(7/2)-1
/12*(-6*b*e*g+7*c*d*g+5*c*e*f)*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^2+1/3*c*(e
*x+d)^(5/2)*(g*x+f)^(1/2)/e^2/g+1/8*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*
(4*a*e*g-b*(d*g+3*e*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^3
```

Rubi [A] time = 0.26, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(2eg(4aeg\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] ((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*
Sqrt[d + e*x]*Sqrt[f + g*x])/(8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*(
d + e*x)^(3/2)*Sqrt[f + g*x])/(12*e^2*g^2) + (c*(d + e*x)^(5/2)*Sqrt[f + g*
x])/(3*e^2*g) - ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(
4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
 + g*x])])/(8*e^(5/2)*g^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(6ae^2g-cd(5ef+dg)\right)-\frac{1}{2}e(5cef+7cdg-6beg)x\right)}{\sqrt{f+gx}}\,dx}{3e^2g}$$

$$= -\frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\left(c\left(5e^2f\right.\right.}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5}{}$$

Mathematica [A] time = 1.01, size = 225, normalized size = 0.91

$$\frac{-e\sqrt{g}\sqrt{d+ex}(f+gx)\left(c\left(3d^2g^2-2deg(gx-2f)+e^2\left(-15f^2+10fgx-8g^2x^2\right)\right)-6eg(4aeg+b(dg-3ef+2\right.}{24e^3g^{7/2}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-(e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(-6*e*g*(4*a*e*g + b*(-3*e*f + d*g + 2
*e*g*x)) + c*(3*d^2*g^2 - 2*d*e*g*(-2*f + g*x) + e^2*(-15*f^2 + 10*f*g*x -
```

```
8*g^2*x^2)))) - 3*(e*f - d*g)^(3/2)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[
(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(24*e^3*g^(7/2)*Sqrt[f + g*x])
```

fricas [A] time = 1.15, size = 576, normalized size = 2.34

$$\left[-\frac{3\left(5\,ce^3f^3-3\left(cde^2+2\,be^3\right)f^2g-\left(cd^2e-4\,bde^2-8\,ae^3\right)fg^2-\left(cd^3-2\,bd^2e+8\,ade^2\right)g^3\right)\sqrt{eg}\,\log\left(8\,e^2g^2x^2+e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3)*f^2*g - (c*d^2*e - 4*b*d*e^2
 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*g^3)*sqrt(e*g)*log(8*e^
2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*
g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) - 4*(8*c*e^3*g^3*
x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*e - 2*b*d*e
^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e^3*g^4), 1/48*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3
)*f^2*g - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*
d*e^2)*g^3)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x
 + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) + 2*(8
*c*e^3*g^3*x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*
e - 2*b*d*e^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*
x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^4)]
```

giac [A] time = 0.35, size = 291, normalized size = 1.18

$$\frac{1}{24}\sqrt{(xe+d)ge-dge+fe^2}\left(2\,(xe+d)\left(\frac{4\,(xe+d)ce^{(-3)}}{g}-\frac{\left(7\,cdg^4e^6+5\,cfg^3e^7-6\,bg^4e^7\right)e^{(-9)}}{g^5}\right)+\frac{3\left(cd^2g^4e^6+2\,c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(x*e + d)*(4*(x*e + d)*c*e^(-3)
/g - (7*c*d*g^4*e^6 + 5*c*f*g^3*e^7 - 6*b*g^4*e^7)*e^(-9)/g^5) + 3*(c*d^2*g
^4*e^6 + 2*c*d*f*g^3*e^7 - 2*b*d*g^4*e^7 + 5*c*f^2*g^2*e^8 - 6*b*f*g^3*e^8
+ 8*a*g^4*e^8)*e^(-9)/g^5)*sqrt(x*e + d) - 1/8*(c*d^3*g^3 + c*d^2*f*g^2*e -
 2*b*d^2*g^3*e + 3*c*d*f^2*g*e^2 - 4*b*d*f*g^2*e^2 + 8*a*d*g^3*e^2 - 5*c*f^
3*e^3 + 6*b*f^2*g*e^3 - 8*a*f*g^2*e^3)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt
(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(7/2)
```

maple [B] time = 0.02, size = 763, normalized size = 3.10

$$\frac{\sqrt{ex+d}\,\sqrt{gx+f}\,\left(24ad\,e^2g^3\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-24a\,e^3f\,g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(16*x^2*c*e^2*g^2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2)+24*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2
```

```
))/(e*g)^(1/2))*a*d*e^2*g^3-24*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^
(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*e^3*f*g^2-6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e
*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d^2*e*g^3-12*ln(1/2*(2*e*g
*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d*e^2*f*g^
2+18*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(
1/2))*b*e^3*f^2*g+3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)
^(1/2))/(e*g)^(1/2))*c*d^3*g^3+3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^2*e*f*g^2+9*ln(1/2*(2*e*g*x+d*g+e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d*e^2*f^2*g-15*ln(1/2*
(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*e^3*
f^3+24*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*b*e^2*g^2+4*(e*g)^(1/2)*((e*x+
d)*(g*x+f))^(1/2)*x*c*d*e*g^2-20*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*e^
2*f*g+48*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a*e^2*g^2+12*(e*g)^(1/2)*((e*x
+d)*(g*x+f))^(1/2)*b*d*e*g^2-36*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^2*f
*g-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^2*g^2-8*(e*g)^(1/2)*((e*x+d)*(
g*x+f))^(1/2)*c*d*e*f*g+30*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^2*f^2)/g
^3/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*g)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g>0)', see `assume?` for more detai
ls)Is g positive, negative or zero?
```

mupad [B] time = 74.34, size = 1832, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] (((2*a*d*g + 2*a*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(g^2*((f + g*x)^(1/2)
- f^(1/2))^3) + ((2*a*e^2*f + 2*a*d*e*g)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*
((f + g*x)^(1/2) - f^(1/2))) - (8*a*d^(1/2)*e*f^(1/2)*((d + e*x)^(1/2) - d^
(1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/((((d + e*x)^(1/2) - d^(1/2))
^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2
))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((
c*d^3*e^3*g^3)/4 - (5*c*e^6*f^3)/4 + (3*c*d*e^5*f^2*g)/4 + (c*d^2*e^4*f*g^2
)/4))/(g^9*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/2))^5*((
33*c*e^4*f^3)/2 + (19*c*d^3*e*g^3)/2 + (313*c*d*e^3*f^2*g)/2 + (275*c*d^2*e
^2*f*g^2)/2))/(g^7*((f + g*x)^(1/2) - f^(1/2))^5) - (((d + e*x)^(1/2) - d^(
1/2))^7*((19*c*d^3*g^3)/2 + (33*c*e^3*f^3)/2 + (313*c*d*e^2*f^2*g)/2 + (275
*c*d^2*e*f*g^2)/2))/(g^6*((f + g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2)
 - d^(1/2))^3*((17*c*d^3*e^2*g^3)/12 - (85*c*e^5*f^3)/12 + (17*c*d*e^4*f^2*
g)/4 + (91*c*d^2*e^3*f*g^2)/4))/(g^8*((f + g*x)^(1/2) - f^(1/2))^3) + (((d
+ e*x)^(1/2) - d^(1/2))^11*((c*d^3*g^3)/4 - (5*c*e^3*f^3)/4 + (3*c*d*e^2*f^
2*g)/4 + (c*d^2*e*f*g^2)/4))/(e^2*g^4*((f + g*x)^(1/2) - f^(1/2))^11) - (((
d + e*x)^(1/2) - d^(1/2))^9*((17*c*d^3*g^3)/12 - (85*c*e^3*f^3)/12 + (17*c*
d*e^2*f^2*g)/4 + (91*c*d^2*e*f*g^2)/4))/(e*g^5*((f + g*x)^(1/2) - f^(1/2))^
9) + (d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6*(128*c*e^3*f^2 + 64*c*d
^2*e*g^2 + (704*c*d*e^2*f*g)/3))/(g^6*((f + g*x)^(1/2) - f^(1/2))^6) + (d^(
1/2)*f^(1/2)*(32*c*d^2*g + 96*c*d*e*f)*((d + e*x)^(1/2) - d^(1/2))^8)/(g^4*
((f + g*x)^(1/2) - f^(1/2))^8) + (d^(1/2)*f^(1/2)*(96*c*d*e^3*f + 32*c*d^2*
e^2*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^6*((f + g*x)^(1/2) - f^(1/2))^4))/
```

```
(((d + e*x)^(1/2) - d^(1/2))^12/((f + g*x)^(1/2) - f^(1/2))^12 + e^6/g^6 -
(6*e*((d + e*x)^(1/2) - d^(1/2))^10)/(g*((f + g*x)^(1/2) - f^(1/2))^10) - (
6*e^5*((d + e*x)^(1/2) - d^(1/2))^2)/(g^5*((f + g*x)^(1/2) - f^(1/2))^2) +
(15*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4)
- (20*e^3*((d + e*x)^(1/2) - d^(1/2))^6)/(g^3*((f + g*x)^(1/2) - f^(1/2))^6
) + (15*e^2*((d + e*x)^(1/2) - d^(1/2))^8)/(g^2*((f + g*x)^(1/2) - f^(1/2))
^8)) + ((((d + e*x)^(1/2) - d^(1/2))*((b*d^2*e^2*g^2)/2 - (3*b*e^4*f^2)/2 +
 b*d*e^3*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) + (((d + e*x)^(1/2) - d^(1
/2))^3*((11*b*e^3*f^2)/2 + (7*b*d^2*e*g^2)/2 + 23*b*d*e^2*f*g))/(g^5*((f +
g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^5*((7*b*d^2*g^2)/2
+ (11*b*e^2*f^2)/2 + 23*b*d*e*f*g))/(g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (
((d + e*x)^(1/2) - d^(1/2))^7*((b*d^2*g^2)/2 - (3*b*e^2*f^2)/2 + b*d*e*f*g)
)/(e*g^3*((f + g*x)^(1/2) - f^(1/2))^7) - (d^(1/2)*f^(1/2)*(32*b*e^2*f + 16
*b*d*e*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4
) - (8*b*d^(3/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6)/(g^2*((f + g*x)^(1/
2) - f^(1/2))^6) - (8*b*d^(3/2)*e^2*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^2)/
(g^4*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g
*x)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*(
(f + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*
((f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2
*((f + g*x)^(1/2) - f^(1/2))^4)) + (2*a*atanh((g^(1/2)*((d + e*x)^(1/2) - d
^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d*g - e*f))/(e^(1/2)*g^(3/
2)) - (b*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1
/2) - f^(1/2))))*(d*g - e*f)*(d*g + 3*e*f))/(2*e^(3/2)*g^(5/2)) + (c*atanh(
(g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2)))
)*(d*g - e*f)*(d^2*g^2 + 5*e^2*f^2 + 2*d*e*f*g))/(4*e^(5/2)*g^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.837 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+}{4e^2g^2}$$

[Out] 1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^2

Rubi [A] time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+}{4e^2g^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] || !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg\right.\right.}{}$$

Mathematica [**A**] time = 0.62, size = 173, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)+e\sqrt{g}\sqrt{d+ex}\,(f+}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x))
 + Sqrt[e*f - d*g]*(c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g
- b*(e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d +
 e*x])/Sqrt[e*f - d*g]])/(4*e^3*g^(5/2)*Sqrt[f + g*x])
```

fricas [**A**] time = 1.10, size = 380, normalized size = 2.32

$$\left[\frac{\left(3ce^2f^2+2\left(cde-2be^2\right)fg+\left(3cd^2-4bde+8ae^2\right)g^2\right)\sqrt{eg}\log\left(8e^2g^2x^2+e^2f^2+6defg+d^2g^2+4\left(2egx+\right.\right.}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

giac [**A**] time = 0.25, size = 179, normalized size = 1.09

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\,\sqrt{xe+d}\left(\frac{2\,(xe+d)ce^{(-3)}}{g}-\frac{\left(5\,cdg^2e^5+3\,cfge^6-4\,bg^2e^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3\,cd^2g^2+2\,cdfg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6 - 4*b*g^2*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2
*g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)
*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g
*e + f*e^2)))/g^(5/2)
```

maple [**B**] time = 0.00, size = 425, normalized size = 2.59

$$\frac{\left(8a\,e^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4bde\,g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4b\,e^2fg\ln\left(\frac{2egx+dg+ef+}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*a*e^2*g^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1
/2))/(e*g)^(1/2))-4*b*d*e*g^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(
1/2)*(e*g)^(1/2))/(e*g)^(1/2))-4*b*e^2*f*g*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+3*c*d^2*g^2*ln(1/2*(2*e*g*x+d*g
+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+2*c*d*e*f*g*ln(1/2
*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+3*c*e
^2*f^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)
^(1/2))+4*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*g*x+8*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*b*e*g-6*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*d*g-6*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/((e*x+d)*(g*x+f))^(1/2)/e^2/g^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 0.00, size = 833, normalized size = 5.08

$$\frac{\frac{\left(2\,b\,d\,g+2\,b\,e\,f\right)\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{g^3\left(\sqrt{f+g\,x}-\sqrt{f}\right)}+\frac{\left(2\,b\,d\,g+2\,b\,e\,f\right)\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}-\frac{8\,b\,\sqrt{d}\,\sqrt{f}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}}-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.838 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=129

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}\,(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g}$$

```
[Out] -(-2*b*e*g+3*c*d*g+c*e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/
2))/e^(5/2)/g^(3/2)-2*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
^(1/2)+c*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {949, 80, 63, 217, 206}

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}\,(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) +
(c*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*g^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
```

```
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} - \frac{2\int \frac{\frac{(cd-be)(ef-dg)}{2e^2}-\frac{c(ef-dg)x}{2e}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{ef-dg}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+g}}}{2e^2g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{f}}\right.}{e^3g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{g}{}}\right.}{e^3g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{}}{\sqrt{e}\sqrt{}}\right.}{e^{5/2}g^{3/2}}$$

Mathematica [C] time = 0.59, size = 222, normalized size = 1.72

$$-\frac{2\sqrt{f+gx}\left(e\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\left(g^2(ae-bd)+cf(2dg-ef)\right)+e\sqrt{g}\sqrt{d+ex}(2cf-bg)(ef-dg)\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d}}{\sqrt{ef-}}\right.\right.}{e^2g^2\sqrt{d+ex}(ef-dg)^{3/2}\sqrt{\frac{e(f+gx)}{ef-dg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(e*Sqrt[e*f - d*g]*((-(b*d) + a*e)*g^2 + c*f*(-(e*f) + 2*
d*g))*Sqrt[(e*(f + g*x))/(e*f - d*g)] + e*Sqrt[g]*(2*c*f - b*g)*(e*f - d*g)
*Sqrt[d + e*x]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]] + c*(e*f -
d*g)^(5/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (g*(d + e*x))/(-(e*f) + d*g)]
))/(e^2*g^2*(e*f - d*g)^(3/2)*Sqrt[d + e*x]*Sqrt[(e*(f + g*x))/(e*f - d*g)]
)
```

fricas [B] time = 3.10, size = 588, normalized size = 4.56

$$\left[-\frac{\left(cde^2f^2+2\left(cd^2e-bde^2\right)fg-\left(3\,cd^3-2\,bd^2e\right)g^2+\left(ce^3f^2+2\left(cde^2-be^3\right)fg-\left(3\,cd^2e-2\,bde^2\right)g^2\right)x\right)\sqrt{eg}\,\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^2
 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)*s
qrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e
*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x)
- 4*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g - c*d
*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (e^5
*f*g^2 - d*e^4*g^3)*x), 1/2*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*
c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e
- 2*b*d*e^2)*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x
)) + 2*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g -
c*d*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (
e^5*f*g^2 - d*e^4*g^3)*x)]
```

giac [**A**] time = 0.39, size = 201, normalized size = 1.56

$$\frac{\sqrt{(xe+d)ge-dge+fe^2}\,\sqrt{xe+d}\,ce^{(-3)}}{g}+\frac{4\left(cd^2\sqrt{g}\,e^{\frac{1}{2}}-bd\sqrt{g}\,e^{\frac{3}{2}}+a\sqrt{g}\,e^{\frac{5}{2}}\right)e^{(-2)}}{dge+\left(\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}-\sqrt{(xe+d)ge-dge+fe^2}\right)^2-fe^2}+\frac{\left(3\,cdg^{\frac{3}{2}}e^{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*c*e^(-3)/g + 4*(c*d^2*sqr
t(g)*e^(1/2) - b*d*sqrt(g)*e^(3/2) + a*sqrt(g)*e^(5/2))*e^(-2)/(d*g*e + (sq
rt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2 - f*e^
2) + 1/2*(3*c*d*g^(3/2)*e^(1/2) + c*f*sqrt(g)*e^(3/2) - 2*b*g^(3/2)*e^(3/2)
)*e^(-3)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e +
f*e^2))^2)/g^2
```

maple [**B**] time = 0.03, size = 697, normalized size = 5.40

$$\frac{\sqrt{gx+f}\,\left(2bd\,e^2g^2x\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-2b\,e^3fgx\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-3c\,d^2e\,g^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g
)^(1/2))/(e*g)^(1/2))*x*b*d*e^2*g^2-2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g
*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*b*e^3*f*g-3*ln(1/2*(2*e*g*x+d*g+e*
f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*c*d^2*e*g^2+2*ln(1/
2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*c*
d*e^2*f*g+ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e
*g)^(1/2))*x*c*e^3*f^2+2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2))/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*
(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d*e^2*f*g-3*ln(1/2*(2*e*g*x+d*g+
e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^3*g^2+2*ln(1/2*
(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^2*
e*f*g+ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^
(1/2))*c*d*e^2*f^2+2*x*c*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-2*x*c*e^
2*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((e*x+d)*(g*x+f))^(1/2)*(
e*g)^(1/2)-4*b*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+6*c*d^2*g*((e*x+d)
```

```
*(g*x+f))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/
(e*g)^(1/2)/g/(d*g-e*f)/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*x+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g>0)', see `assume?` for more detai
ls)Is g positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^{2}}{\left(d + ex\right)^{\frac{3}{2}} \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

3.839 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{f+gx}\left(c\left(6def-4d^2g\right)-e(-2aeg-bdg+3bef)\right)}{3e^2\sqrt{d+ex}\,(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

```
[Out] 2*c*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(1/2)-2/
3*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(3/2)+2/3*(c*(-4*d^
2*g+6*d*e*f)-e*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*
x+d)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {949, 78, 63, 217, 206}

$$\frac{2\sqrt{f+gx}\left(c\left(6def-4d^2g\right)-e(-2aeg-bdg+3bef)\right)}{3e^2\sqrt{d+ex}\,(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)
) + (2*(c*(6*d*e*f - 4*d^2*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x
])/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{cd(3ef-dg)-e(3bef-bdg-2aeg)}{2e^2}-\frac{3}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{3(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} +$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} +$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} +$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} +$$

Mathematica [C] time = 0.23, size = 173, normalized size = 1.08

$$\frac{2\sqrt{f+gx}\left(2g(d+ex)\left(g(ag-bf)+cf^2\right)-(ef-dg)\left(g(ag-bf)+cf^2\right)+(f+gx)(2cf-bg)(ef-dg)-\frac{c(ef-dg)}{\ldots}\right.}{3g^2(d+ex)^{3/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(-((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) + 2*g*(c*f^2 +
g*(-(b*f) + a*g))*(d + e*x) + (2*c*f - b*g)*(e*f - d*g)*(f + g*x) - (c*(e*f
 - d*g)^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (g*(d + e*x))/(-(e*f) + d*g)]
)/(e^2*Sqrt[(e*(f + g*x))/(e*f - d*g)])))/(3*g^2*(e*f - d*g)^2*(d + e*x)^(3
/2))
```

fricas [B] time = 7.06, size = 792, normalized size = 4.95

$$\left[\frac{3\left(cd^2e^2f^2-2\,cd^3efg+cd^4g^2+\left(ce^4f^2-2\,cde^3fg+cd^2e^2g^2\right)x^2+2\left(cde^3f^2-2\,cd^2e^2fg+cd^3eg^2\right)x\right)\sqrt{eg}\,\log\left(8\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g^2 + (c*e^4*f^2 - 2*c*d*e^3
*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c*d^2*e^2*f*g + c*d^3*e*g^2)
*x)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*
x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2
)*x) + 4*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*e^3)*g^2
 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*e^3*g^3
+ (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g - 2*d^2*e^
5*f*g^2 + d^3*e^4*g^3)*x), -1/3*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g
^2 + (c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c
*d^2*e^2*f*g + c*d^3*e*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*
sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g +
d*e*g^2)*x)) - 2*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*
e^3)*g^2 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*
e^3*g^3 + (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g -
2*d^2*e^5*f*g^2 + d^3*e^4*g^3)*x)]
```

giac [B] time = 0.68, size = 504, normalized size = 3.15

$$-\frac{ce^{\left(-\frac{5}{2}\right)}\log\left(\left(\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}-\sqrt{(xe+d)ge-dge+fe^2}\right)^2\right)}{\sqrt{g}}-\frac{4\left(4\,cd^3g^{\frac{5}{2}}e^{\frac{5}{2}}+6\left(\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}-\sqrt{(xe+d)ge-\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
 + f*e^2))^2)/sqrt(g) - 4/3*(4*c*d^3*g^(5/2)*e^(5/2) + 6*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*g^(3/2)*e^(3/2)
 + 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
4*c*d*sqrt(g)*e^(1/2) - 10*c*d^2*f*g^(3/2)*e^(7/2) - b*d^2*g^(5/2)*e^(7/2)
- 12*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
2*c*d*f*sqrt(g)*e^(5/2) - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)
*g*e - d*g*e + f*e^2))^4*b*sqrt(g)*e^(3/2) + 6*c*d*f^2*sqrt(g)*e^(9/2) + 4*
b*d*f*g^(3/2)*e^(9/2) - 2*a*d*g^(5/2)*e^(9/2) + 6*(sqrt(x*e + d)*sqrt(g)*e^
(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*f*sqrt(g)*e^(7/2) - 6*(sqr
t(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*g^(3/
2)*e^(7/2) - 3*b*f^2*sqrt(g)*e^(11/2) + 2*a*f*g^(3/2)*e^(11/2))*e^(-2)/(d*g
*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
2 - f*e^2)^3
```

maple [B] time = 0.03, size = 773, normalized size = 4.83

$$\frac{\sqrt{gx+f}\left(3c\,d^2e^2g^2x^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-6cd\,e^3fg\,x^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3c\,e^{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g
)^(1/2))/(e*g)^(1/2))*x^2*c*d^2*e^2*g^2-6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d
)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x^2*c*d*e^3*f*g+3*ln(1/2*(2*e*g*
```

```
x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x^2*c*e^4*f^2
+6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/
2))*x*c*d^3*e*g^2-12*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g
)^(1/2))/(e*g)^(1/2))*x*c*d^2*e^2*f*g+6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*
(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*c*d*e^3*f^2+3*ln(1/2*(2*e*g*x+d*
g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^4*g^2-6*ln(1/
2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^
3*e*f*g+3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e
*g)^(1/2))*c*d^2*e^2*f^2+4*x*a*e^3*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+2*
x*b*d*e^2*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-6*x*b*e^3*f*((e*x+d)*(g*x+f
))^(1/2)*(e*g)^(1/2)-8*x*c*d^2*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+12*x
*c*d*e^2*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+6*a*d*e^2*g*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2)-2*a*e^3*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-4*b*d*e^2
*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-6*c*d^3*g*((e*x+d)*(g*x+f))^(1/2)*(e
*g)^(1/2)+10*c*d^2*e*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2)/(d*
g-e*f)^2/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*x+d)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g>0)', see `assume?` for more detai
ls)Is g positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.840 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{f+gx}\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)}{15e^2\sqrt{d+ex}\,(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2\ldots}{\ldots}$$

```
[Out] -2/5*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(5/2)+2/15*(2*c*
d*(-3*d*g+5*e*f)-e*(-4*a*e*g-b*d*g+5*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2
/(e*x+d)^(3/2)+2/15*(2*e*g*(-4*a*e*g-b*d*g+5*b*e*f)-c*(3*d^2*g^2-10*d*e*f*g
+15*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {949, 78, 37}

$$\frac{2\sqrt{f+gx}\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)}{15e^2\sqrt{d+ex}\,(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)
) + (2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f + g*x
])/(15*e^2*(e*f - d*g)^2*(d + e*x)^(3/2)) + (2*(2*e*g*(5*b*e*f - b*d*g - 4*
a*e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(15*e^2*(e
*f - d*g)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} - \frac{2\int \frac{\frac{cd(5ef-dg)-e(5bef-bdg-4aeg)}{2e^2}-\frac{5}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{5/2}\sqrt{f+gx}}dx}{5(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}} - \ldots$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}} + \ldots$$

Mathematica [A] time = 0.23, size = 178, normalized size = 0.90

$$-\frac{2\sqrt{f+gx}\left(a\left(15d^2g^2-10deg(f-2gx)+e^2\left(3f^2-4fgx+8g^2x^2\right)\right)+b\left(5d^2g(gx-2f)+2de\left(f^2-13fgx+g^2\ldots\right.\right.\right.}{15(d+ex)^{5/2}(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(b*(5*e^2*f*x*(f - 2*g*x) + 5*d^2*g*(-2*f + g*x) + 2*d*e*
(f^2 - 13*f*g*x + g^2*x^2)) + c*(15*e^2*f^2*x^2 + 10*d*e*f*x*(2*f - g*x) +
d^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + a*(15*d^2*g^2 - 10*d*e*g*(f - 2*g*x) +
 e^2*(3*f^2 - 4*f*g*x + 8*g^2*x^2))))/(15*(e*f - d*g)^3*(d + e*x)^(5/2))
```

fricas [A] time = 25.37, size = 353, normalized size = 1.78

$$-\frac{2\left(15\,ad^2g^2+\left(8\,cd^2+2\,bde+3\,ae^2\right)f^2-10\left(bd^2+ade\right)fg+\left(15\,ce^2f^2-10\left(cde+be^2\right)fg+\left(3\,cd^2+2\,bde+8\,a\ldots\right.\right.\right.}{15\left(d^3e^3f^3-3\,d^4e^2f^2g+3\,d^5efg^2-d^6g^3+\left(e^6f^3-3\,de^5f^2g+3\,d^2e^4fg^2-d^3e^3g^3\right)x^3+3\left(de^5f^3\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*a*d^2*g^2 + (8*c*d^2 + 2*b*d*e + 3*a*e^2)*f^2 - 10*(b*d^2 + a*d*e
)*f*g + (15*c*e^2*f^2 - 10*(c*d*e + b*e^2)*f*g + (3*c*d^2 + 2*b*d*e + 8*a*e
^2)*g^2)*x^2 + (5*(4*c*d*e + b*e^2)*f^2 - 2*(2*c*d^2 + 13*b*d*e + 2*a*e^2)*
f*g + 5*(b*d^2 + 4*a*d*e)*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^3*e^3*f^3
- 3*d^4*e^2*f^2*g + 3*d^5*e*f*g^2 - d^6*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*
d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^3 + 3*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e
^3*f*g^2 - d^4*e^2*g^3)*x^2 + 3*(d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4*e^2*
f*g^2 - d^5*e*g^3)*x)
```

giac [B] time = 0.85, size = 1080, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(3*c*d^4*g^(9/2)*e^(9/2) + 30*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x
*e + d)*g*e - d*g*e + f*e^2))^4*c*d^2*g^(5/2)*e^(5/2) + 15*(sqrt(x*e + d)*s
qrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*sqrt(g)*e^(1/2) -
 16*c*d^3*f*g^(7/2)*e^(11/2) + 2*b*d^3*g^(9/2)*e^(11/2) - 20*(sqrt(x*e + d)
*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f*g^(5/2)*e
^(9/2) + 10*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^2*b*d^2*g^(7/2)*e^(9/2) - 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((
```

```
x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*f*g^(3/2)*e^(7/2) - 10*(sqrt(x*e + d)*
sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d*g^(5/2)*e^(7/2
) - 60*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2)
)^6*c*f*sqrt(g)*e^(5/2) + 30*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d
)*g*e - d*g*e + f*e^2))^6*b*g^(3/2)*e^(5/2) + 38*c*d^2*f^2*g^(5/2)*e^(13/2)
 - 14*b*d^2*f*g^(7/2)*e^(13/2) + 8*a*d^2*g^(9/2)*e^(13/2) + 80*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d*f^2*g^(3/2)
*e^(11/2) - 60*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
+ f*e^2))^2*b*d*f*g^(5/2)*e^(11/2) + 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sq
rt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d*g^(7/2)*e^(11/2) + 90*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*f^2*sqrt(g)*e
^(9/2) - 70*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^4*b*f*g^(3/2)*e^(9/2) + 80*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*
e + d)*g*e - d*g*e + f*e^2))^4*a*g^(5/2)*e^(9/2) - 40*c*d*f^3*g^(3/2)*e^(15
/2) + 22*b*d*f^2*g^(5/2)*e^(15/2) - 16*a*d*f*g^(7/2)*e^(15/2) - 60*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*f^3*sqrt(
g)*e^(13/2) + 50*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*
e + f*e^2))^2*b*f^2*g^(3/2)*e^(13/2) - 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) -
sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*f*g^(5/2)*e^(13/2) + 15*c*f^4*sqrt
(g)*e^(17/2) - 10*b*f^3*g^(3/2)*e^(17/2) + 8*a*f^2*g^(5/2)*e^(17/2))*e^(-2)
/(d*g*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e
^2))^2 - f*e^2)^5
```

maple [A] time = 0.01, size = 238, normalized size = 1.20

$$\frac{2\sqrt{gx+f}\,\left(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fg\right.}{15\,(ex+d)^{\frac{5}{2}}\left(g^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(8*a*e^2*g^2*x^2+2*b*d*e*g^2*x^2-10*b*e^2*f*g*x^2+3*c*d^
2*g^2*x^2-10*c*d*e*f*g*x^2+15*c*e^2*f^2*x^2+20*a*d*e*g^2*x-4*a*e^2*f*g*x+5*
b*d^2*g^2*x-26*b*d*e*f*g*x+5*b*e^2*f^2*x-4*c*d^2*f*g*x+20*c*d*e*f^2*x+15*a*
d^2*g^2-10*a*d*e*f*g+3*a*e^2*f^2-10*b*d^2*f*g+2*b*d*e*f^2+8*c*d^2*f^2)/(e*x
+d)^(5/2)/(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 4.30, size = 260, normalized size = 1.31

$$\frac{\sqrt{f+g\,x}\,\left(\frac{16\,c\,d^2\,f^2-20\,b\,d^2\,f\,g+30\,a\,d^2\,g^2+4\,b\,d\,e\,f^2-20\,a\,d\,e\,f\,g+6\,a\,e^2\,f^2}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}+\frac{x\,\left(-8\,c\,d^2\,f\,g+10\,b\,d^2\,g^2+40\,c\,d\,e\,f^2-52\,b\,d\,e\,f\,g+40\,a\,d\,e\,g^2+10\right.}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}\right.}{x^2\,\sqrt{d+e\,x}+\frac{d^2\,\sqrt{d+e\,x}}{e^2}+\frac{2\,d\,x\,\sqrt{d+e\,x}}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(7/2)),x)
```

[Out] ((f + g*x)^(1/2)*((30*a*d^2*g^2 + 6*a*e^2*f^2 + 16*c*d^2*f^2 + 4*b*d*e*f^2 - 20*b*d^2*f*g - 20*a*d*e*f*g)/(15*e^2*(d*g - e*f)^3) + (x*(10*b*d^2*g^2 + 10*b*e^2*f^2 + 40*a*d*e*g^2 + 40*c*d*e*f^2 - 8*a*e^2*f*g - 8*c*d^2*f*g - 52 *b*d*e*f*g))/(15*e^2*(d*g - e*f)^3) + (x^2*(16*a*e^2*g^2 + 6*c*d^2*g^2 + 30 *c*e^2*f^2 + 4*b*d*e*g^2 - 20*b*e^2*f*g - 20*c*d*e*f*g))/(15*e^2*(d*g - e*f)^3)))/(x^2*(d + e*x)^(1/2) + (d^2*(d + e*x)^(1/2))/e^2 + (2*d*x*(d + e*x)^ (1/2))/e)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)

[Out] Timed out

3.841 $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=281

$$-\frac{4g\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2\sqrt{d+ex}\,(ef-dg)^4}+\frac{2\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef\right.}{105e^2(d+ex)^3}$$

```
[Out] -2/7*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(7/2)+2/35*(2*c*
d*(-4*d*g+7*e*f)-e*(-6*a*e*g-b*d*g+7*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2
/(e*x+d)^(5/2)+2/105*(4*e*g*(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*
g+35*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(3/2)-4/105*g*(4*e*g*
(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*g+35*e^2*f^2))*(g*x+f)^(1/2)
/e^2/(-d*g+e*f)^4/(e*x+d)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {949, 78, 45, 37}

$$-\frac{4g\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2\sqrt{d+ex}\,(ef-dg)^4}+\frac{2\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef\right.}{105e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2
) + (2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f + g*x
])/(35*e^2*(e*f - d*g)^2*(d + e*x)^(5/2)) + (2*(4*e*g*(7*b*e*f - b*d*g - 6*
a*e*g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(
e*f - d*g)^3*(d + e*x)^(3/2)) - (4*g*(4*e*g*(7*b*e*f - b*d*g - 6*a*e*g) - c
*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(e*f - d*g)
^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} - \frac{2\int \frac{\frac{cd(7ef-dg)-e(7bef-bdg-6aeg)}{2e^2}-\frac{7}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx}{7(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} - \cdots$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} + \cdots$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} + \cdots$$

Mathematica [A] time = 0.35, size = 332, normalized size = 1.18

$$\frac{2\sqrt{f+gx}\left(3a\left(35d^3g^3-35d^2eg^2(f-2gx)+7de^2g\left(3f^2-4fgx+8g^2x^2\right)+e^3\left(-5f^3+6f^2gx-8fg^2x^2+16g^3x^3\right)\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(-35*e^3*f^2*x^2*(f - 2*g*x) + 7*d^3*g*(8*f^2 - 4*f*g*x
 + 3*g^2*x^2) - 7*d*e^2*f*x*(4*f^2 - 37*f*g*x + 4*g^2*x^2) + d^2*e*(-8*f^3
+ 200*f^2*g*x - 101*f*g^2*x^2 + 6*g^3*x^3)) + b*(35*d^3*g^2*(-2*f + g*x) +
7*d^2*e*g*(4*f^2 - 37*f*g*x + 4*g^2*x^2) - 7*e^3*f*x*(3*f^2 - 4*f*g*x + 8*g
^2*x^2) + d*e^2*(-6*f^3 + 101*f^2*g*x - 200*f*g^2*x^2 + 8*g^3*x^3)) + 3*a*(
35*d^3*g^3 - 35*d^2*e*g^2*(f - 2*g*x) + 7*d*e^2*g*(3*f^2 - 4*f*g*x + 8*g^2*
x^2) + e^3*(-5*f^3 + 6*f^2*g*x - 8*f*g^2*x^2 + 16*g^3*x^3))))/(105*(e*f - d
*g)^4*(d + e*x)^(7/2))
```

fricas [B] time = 99.86, size = 641, normalized size = 2.28

$$\frac{2\left(105\,ad^3g^3-\left(8\,cd^2e+6\,bde^2+15\,ae^3\right)f^3+7\left(8\,cd^3+4\,bd^2e+9\,ade^2\right)f^2g-35\left(2\,bd^3+3\,ad^2e\right)fg^2+2\left(35\,ce^3\cdots\right.\right.}{105\left(d^4e^4f^4-4\,d^5e^3f^3g+6\,d^6e^2f^2g^2-4\,d^7efg^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(105*a*d^3*g^3 - (8*c*d^2*e + 6*b*d*e^2 + 15*a*e^3)*f^3 + 7*(8*c*d^3
+ 4*b*d^2*e + 9*a*d*e^2)*f^2*g - 35*(2*b*d^3 + 3*a*d^2*e)*f*g^2 + 2*(35*c*e
^3*f^2*g - 14*(c*d*e^2 + 2*b*e^3)*f*g^2 + (3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3
)*g^3)*x^3 - (35*c*e^3*f^3 - 7*(37*c*d*e^2 + 4*b*e^3)*f^2*g + (101*c*d^2*e
+ 200*b*d*e^2 + 24*a*e^3)*f*g^2 - 7*(3*c*d^3 + 4*b*d^2*e + 24*a*d*e^2)*g^3)
*x^2 - (7*(4*c*d*e^2 + 3*b*e^3)*f^3 - (200*c*d^2*e + 101*b*d*e^2 + 18*a*e^3
)*f^2*g + 7*(4*c*d^3 + 37*b*d^2*e + 12*a*d*e^2)*f*g^2 - 35*(b*d^3 + 6*a*d^2
*e)*g^3)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^4*e^4*f^4 - 4*d^5*e^3*f^3*g + 6*
d^6*e^2*f^2*g^2 - 4*d^7*e*f*g^3 + d^8*g^4 + (e^8*f^4 - 4*d*e^7*f^3*g + 6*d^
2*e^6*f^2*g^2 - 4*d^3*e^5*f*g^3 + d^4*e^4*g^4)*x^4 + 4*(d*e^7*f^4 - 4*d^2*e
^6*f^3*g + 6*d^3*e^5*f^2*g^2 - 4*d^4*e^4*f*g^3 + d^5*e^3*g^4)*x^3 + 6*(d^2*
e^6*f^4 - 4*d^3*e^5*f^3*g + 6*d^4*e^4*f^2*g^2 - 4*d^5*e^3*f*g^3 + d^6*e^2*g
^4)*x^2 + 4*(d^3*e^5*f^4 - 4*d^4*e^4*f^3*g + 6*d^5*e^3*f^2*g^2 - 4*d^6*e^2*
f*g^3 + d^7*e*g^4)*x)
```

giac [B] time = 1.27, size = 1868, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(3*c*d^5*g^(13/2)*e^(11/2) + 21*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt
((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^4*g^(11/2)*e^(9/2) - 42*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d^3*g^(9/2)*e
^(7/2) + 210*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e +
f*e^2))^6*c*d^2*g^(7/2)*e^(5/2) - 105*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt
((x*e + d)*g*e - d*g*e + f*e^2))^8*c*d*g^(5/2)*e^(3/2) + 105*(sqrt(x*e + d)
*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^10*c*g^(3/2)*e^(1/2
) - 23*c*d^4*f*g^(11/2)*e^(13/2) + 4*b*d^4*g^(13/2)*e^(13/2) - 140*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^3*f*g^(
9/2)*e^(11/2) + 28*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*
g*e + f*e^2))^2*b*d^3*g^(11/2)*e^(11/2) - 42*(sqrt(x*e + d)*sqrt(g)*e^(1/2)
 - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d^2*f*g^(7/2)*e^(9/2) + 84*(sqr
t(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d^2*g
^(9/2)*e^(9/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e -
d*g*e + f*e^2))^6*c*d*f*g^(5/2)*e^(7/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2
) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*b*d*g^(7/2)*e^(7/2) - 455*(sqrt(
x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*f*g^(3/
2)*e^(5/2) + 280*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*
e + f*e^2))^8*b*g^(5/2)*e^(5/2) + 86*c*d^3*f^2*g^(9/2)*e^(15/2) - 40*b*d^3*
f*g^(11/2)*e^(15/2) + 24*a*d^3*g^(13/2)*e^(15/2) + 462*(sqrt(x*e + d)*sqrt(
g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f^2*g^(7/2)*e^(13
/2) - 252*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e
^2))^2*b*d^2*f*g^(9/2)*e^(13/2) + 168*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt
((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d^2*g^(11/2)*e^(13/2) + 714*(sqrt(x*e
+ d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*f^2*g^(5/
2)*e^(11/2) - 672*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g
*e + f*e^2))^4*b*d*f*g^(7/2)*e^(11/2) + 504*(sqrt(x*e + d)*sqrt(g)*e^(1/2)
- sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*d*g^(9/2)*e^(11/2) + 770*(sqrt(x
*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c*f^2*g^(3
/2)*e^(9/2) - 700*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g
*e + f*e^2))^6*b*f*g^(5/2)*e^(9/2) + 840*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - s
qrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*g^(7/2)*e^(9/2) - 150*c*d^2*f^3*g^(
7/2)*e^(17/2) + 96*b*d^2*f^2*g^(9/2)*e^(17/2) - 72*a*d^2*f*g^(11/2)*e^(17/2
) - 588*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2
))^2*c*d*f^3*g^(5/2)*e^(15/2) + 420*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((
x*e + d)*g*e - d*g*e + f*e^2))^2*b*d*f^2*g^(7/2)*e^(15/2) - 336*(sqrt(x*e +
 d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d*f*g^(9/2)*
e^(15/2) - 630*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
```

```
+ f*e^2))^4*c*f^3*g^(3/2)*e^(13/2) + 588*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - s
qrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*f^2*g^(5/2)*e^(13/2) - 504*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*f*g^(7/2)
*e^(13/2) + 119*c*d*f^4*g^(5/2)*e^(19/2) - 88*b*d*f^3*g^(7/2)*e^(19/2) + 72
*a*d*f^2*g^(9/2)*e^(19/2) + 245*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e
+ d)*g*e - d*g*e + f*e^2))^2*c*f^4*g^(3/2)*e^(17/2) - 196*(sqrt(x*e + d)*sq
rt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*f^3*g^(5/2)*e^(17/
2) + 168*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^
2))^2*a*f^2*g^(7/2)*e^(17/2) - 35*c*f^5*g^(3/2)*e^(21/2) + 28*b*f^4*g^(5/2)
*e^(21/2) - 24*a*f^3*g^(7/2)*e^(21/2))*e^(-1)/(d*g*e + (sqrt(x*e + d)*sqrt(
g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2 - f*e^2)^7
```

maple [A] time = 0.01, size = 468, normalized size = 1.67

$$2\sqrt{gx+f}\,\left(48a\,e^{3}g^{3}x^{3}+8bd\,e^{2}g^{3}x^{3}-56b\,e^{3}f\,g^{2}x^{3}+6c\,d^{2}e\,g^{3}x^{3}-28cd\,e^{2}f\,g^{2}x^{3}+70c\,e^{3}f^{2}g\,x^{3}+168ad\,e^{2}g^{3}x^{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3+
6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3*x
^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*f^2
*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*c*e^
3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d^3*g^
3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f*g^2*x
+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^2+63*a*
d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*f^3+56*c
*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^
2*g^2-4*d*e^3*f^3*g+e^4*f^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 4.65, size = 452, normalized size = 1.61

$$\sqrt{f+g\,x}\,\left(\frac{x^3\,\left(12\,c\,d^2\,e\,g^3-56\,c\,d\,e^2\,f\,g^2+16\,b\,d\,e^2\,g^3+140\,c\,e^3\,f^2\,g-112\,b\,e^3\,f\,g^2+96\,a\,e^3\,g^3\right)}{105\,e^3\,{\left(d\,g-e\,f\right)}^4}-\frac{-112\,c\,d^3\,f^2\,g+140\,b\,d^3\,f\,g^2-210\,a\,d^3\,g^3+16\,c\,d^2\,e\,f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((x^3*(96*a*e^3*g^3 + 16*b*d*e^2*g^3 + 12*c*d^2*e*g^3 - 11
2*b*e^3*f*g^2 + 140*c*e^3*f^2*g - 56*c*d*e^2*f*g^2))/(105*e^3*(d*g - e*f)^4
) - (30*a*e^3*f^3 - 210*a*d^3*g^3 + 12*b*d*e^2*f^3 + 16*c*d^2*e*f^3 + 140*b
*d^3*f*g^2 - 112*c*d^3*f^2*g - 126*a*d*e^2*f^2*g + 210*a*d^2*e*f*g^2 - 56*b
*d^2*e*f^2*g)/(105*e^3*(d*g - e*f)^4) + (x*(70*b*d^3*g^3 - 42*b*e^3*f^3 + 4
```

```
20*a*d^2*e*g^3 - 56*c*d*e^2*f^3 + 36*a*e^3*f^2*g - 56*c*d^3*f*g^2 - 168*a*d
*e^2*f*g^2 + 202*b*d*e^2*f^2*g - 518*b*d^2*e*f*g^2 + 400*c*d^2*e*f^2*g))/(1
05*e^3*(d*g - e*f)^4) + (2*x^2*(7*d*g - e*f)*(24*a*e^2*g^2 + 3*c*d^2*g^2 +
35*c*e^2*f^2 + 4*b*d*e*g^2 - 28*b*e^2*f*g - 14*c*d*e*f*g))/(105*e^3*(d*g -
e*f)^4)))/(x^3*(d + e*x)^(1/2) + (d^3*(d + e*x)^(1/2))/e^3 + (3*d*x^2*(d +
e*x)^(1/2))/e + (3*d^2*x*(d + e*x)^(1/2))/e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.842 $\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}} dx$

Optimal. Leaf size=249

$$\frac{\sqrt{d+ex}\sqrt{e+fx}\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4ef^3\left(e^2-df\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)\left(4ef\left(-2aef-bd\right.\right.}{4e^{3/}}$$

```
[Out] -1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*e^4))*arctanh
(f^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(f*x+e)^(1/2))/e^(3/2)/f^(7/2)+2*(a+e*(-b*f+
c*e)/f^2)*(e*x+d)^(3/2)/(-d*f+e^2)/(f*x+e)^(1/2)+1/2*c*(e*x+d)^(3/2)*(f*x+e
)^(1/2)/e/f^2+1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*
e^4))*(e*x+d)^(1/2)*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)
```

Rubi [A] time = 0.28, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {949, 80, 50, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{e+fx}\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4ef^3\left(e^2-df\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)\left(4ef\left(-2aef-bd\right.\right.}{4e^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x]) +
 ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sq
rt[d + e*x]*Sqrt[e + f*x])/(4*e*f^3*(e^2 - d*f)) + (c*(d + e*x)^(3/2)*Sqrt[
e + f*x])/(2*e*f^2) - ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d
*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])]
)/(4*e^(3/2)*f^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{f\left(3be^2-bdf-2aef\right)-c\left(3e^3-def\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e^2}{f}\right)x\right)}{\sqrt{e+fx}}\,dx}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)\right.}{}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2\right.\right.}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2\right.\right.}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2\right.\right.}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2\right.\right.}{4ef^3\left(e^2-df\right)}$$

Mathematica [A] time = 1.09, size = 196, normalized size = 0.79

$$\frac{\frac{\sqrt{e^2-df}\sqrt{\frac{e(e+fx)}{e^2-df}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e^2-df}}\right)\left(4ef\left(2aef+bdf-3be^2\right)+c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{e}+\sqrt{f}\sqrt{d+ex}\left(4ef(-2af+3be+bfx)+c\left(\right.\right.}{4ef^{7/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (Sqrt[f]*Sqrt[d + e*x]*(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*
f*x + d*f^2*x + e*f*(d + 2*f*x^2))) + (Sqrt[e^2 - d*f]*(4*e*f*(-3*b*e^2 + b
*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sqrt[(e*(e + f*x))/(e^2
 - d*f)]*ArcSinh[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[e^2 - d*f]])/e)/(4*e*f^(7/2)*
Sqrt[e + f*x])
```

fricas [A] time = 3.23, size = 580, normalized size = 2.33

$$\left[\frac{\left(15\,ce^5-\left(cd^2e-4\,bde^2-8\,ae^3\right)f^2-6\left(cde^3+2\,be^4\right)f+\left(15\,ce^4f-\left(cd^2-4\,bde-8\,ae^2\right)f^3-6\left(cde^2+2\,be^3\right)f^2\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f^2 - 6*(c*d*e^3 + 2*b*e
^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*f^3 - 6*(c*d*e^2 + 2*b*e^
3)*f^2)*x)*sqrt(e*f)*log(8*e^2*f^2*x^2 + e^4 + 6*d*e^2*f + d^2*f^2 + 4*(2*e
*f*x + e^2 + d*f)*sqrt(e*f)*sqrt(e*x + d)*sqrt(f*x + e) + 8*(e^3*f + d*e*f^
2)*x) + 4*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2 + 12*b*e^3
)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sqrt(f*x + e
))/(e^2*f^5*x + e^3*f^4), -1/8*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)
*f^2 - 6*(c*d*e^3 + 2*b*e^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*
f^3 - 6*(c*d*e^2 + 2*b*e^3)*f^2)*x)*sqrt(-e*f)*arctan(1/2*(2*e*f*x + e^2 +
d*f)*sqrt(-e*f)*sqrt(e*x + d)*sqrt(f*x + e)/(e^2*f^2*x^2 + d*e^2*f + (e^3*f
 + d*e*f^2)*x)) - 2*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2
+ 12*b*e^3)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sq
rt(f*x + e))/(e^2*f^5*x + e^3*f^4)]
```

giac [A] time = 0.44, size = 237, normalized size = 0.95

$$\frac{\left((xe+d)\left(\frac{2\,(xe+d)ce^{(-1)}}{f}-\frac{\left(3\,cdf^4e^2-4\,bf^4e^3+5\,cf^3e^4\right)e^{(-3)}}{f^5}\right)+\frac{\left(cd^2f^4e^2-4\,bdf^4e^3+6\,cdf^3e^4-8\,af^4e^4+12\,bf^3e^5-15\,cf^2e^6\right)e^{(-3)}}{f^5}\right)\sqrt{xe+d}}{4\sqrt{(xe+d)fe-dfe+e^3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((x*e + d)*(2*(x*e + d)*c*e^(-1)/f - (3*c*d*f^4*e^2 - 4*b*f^4*e^3 + 5*c
*f^3*e^4)*e^(-3)/f^5) + (c*d^2*f^4*e^2 - 4*b*d*f^4*e^3 + 6*c*d*f^3*e^4 - 8*
a*f^4*e^4 + 12*b*f^3*e^5 - 15*c*f^2*e^6)*e^(-3)/f^5)*sqrt(x*e + d)/sqrt((x*
e + d)*f*e - d*f*e + e^3) + 1/4*(c*d^2*f^2 - 4*b*d*f^2*e + 6*c*d*f*e^2 - 8*
a*f^2*e^2 + 12*b*f*e^3 - 15*c*e^4)*e^(-3/2)*log(abs(-sqrt(x*e + d)*sqrt(f)*
e^(1/2) + sqrt((x*e + d)*f*e - d*f*e + e^3)))/f^(7/2)
```

maple [B] time = 0.04, size = 834, normalized size = 3.35

$$\frac{\sqrt{ex+d}\,\left(8a\,e^2f^3x\ln\left(\frac{2efx+df+e^2+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}}{2\sqrt{ef}}\right)+4bde\,f^3x\ln\left(\frac{2efx+df+e^2+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}}{2\sqrt{ef}}\right)-12b\,e^3f^2x\ln\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

```
[Out] 1/8*(e*x+d)^(1/2)*(8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+
d*f+e^2)/(e*f)^(1/2))*x*a*e^2*f^3+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/
2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*b*d*e*f^3-12*ln(1/2*(2*e*f*x+2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*b*e^3*f^2-ln(1/2*(2*
e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*c*d^2*f
^3-6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(
1/2))*x*c*d*e^2*f^2+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2
)+d*f+e^2)/(e*f)^(1/2))*x*c*e^4*f+4*x^2*c*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*
f)^(1/2)+8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(
e*f)^(1/2))*a*e^3*f^2+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/
2)+d*f+e^2)/(e*f)^(1/2))*b*d*e^2*f^2-12*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))
^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^4*f-ln(1/2*(2*e*f*x+2*((e*x+d)
*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*e*f^2-6*ln(1/2*(2*e
*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d*e^3*f+
15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/
2))*c*e^5+8*x*b*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+2*x*c*d*f^2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-10*x*c*e^2*f*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(
1/2)-16*a*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+24*b*e^2*f*((e*x+d)*(f*
x+e))^(1/2)*(e*f)^(1/2)+2*c*d*e*f*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-30*c*
e^3*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2))/(e*f)^(1/2)/e/((e*x+d)*(f*x+e))^(1
/2)/f^3/(f*x+e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*f-e^2>0)', see `assume?` for more
 details)Is d*f-e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\left(c\,x^2+b\,x+a\right)}{{\left(e+f\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex} \left(a + bx + cx^{2}\right)}{\left(e + fx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)
```

3.843 $\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=240

$$\frac{(bd-ae)^2(35a^2e^2-90abde+73b^2d^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}(bd-ae)(35a^2e^2-90abde+73b^2d^2}{8b^4}$$

```
[Out] 2*e*(b*x+a)^(3/2)*(e*x+d)^(5/2)/b^2+1/8*(-a*e+b*d)^2*(35*a^2*e^2-90*a*b*d*e
+73*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(9/2)/e
^(1/2)+1/12*(35*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/
b^3+1/3*(-13*a*e+17*b*d)*(e*x+d)^(5/2)*(b*x+a)^(1/2)/b^2+1/8*(-a*e+b*d)*(35
*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^4
```

Rubi [A] time = 0.23, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx}(d+ex)^{3/2}(35a^2e^2-90abde+73b^2d^2)}{12b^3}+\frac{\sqrt{a+bx}\sqrt{d+ex}(bd-ae)(35a^2e^2-90abde+73b^2d^2)}{8b^4}+\frac{(bd-}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((b*d - a*e)*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d +
e*x])/(8*b^4) + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*(d +
e*x)^(3/2))/(12*b^3) + ((17*b*d - 13*a*e)*Sqrt[a + b*x]*(d + e*x)^(5/2))/(3
*b^2) + (2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + ((b*d - a*e)^2*(73*b^2*
d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{\int \frac{(d+ex)^{3/2}\left(4e\left(15b^2d^2-3abde-5a^2e^2\right)+4be^2(17bd-1\ldots}{\sqrt{a+bx}}}{4b^2e}$$

$$= \frac{(17bd-13ae)\sqrt{a+bx}\,(d+ex)^{5/2}}{3b^2}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{(73\ldots}{\ldots}$$

$$= \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,(d+ex)^{3/2}}{12b^3}+\frac{(17bd-13ae)\sqrt{\ldots}}{3\ldots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4}+\frac{\left(73b^2d^2\ldots\right.}{\ldots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4}+\frac{\left(73b^2d^2\ldots\right.}{\ldots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4}+\frac{\left(73b^2d^2\ldots\right.}{\ldots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4}+\frac{\left(73b^2d^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.85, size = 204, normalized size = 0.85

$$\frac{\sqrt{d+ex}\left(\frac{3\left(35a^2e^2-90abde+73b^2d^2\right)(bd-ae)^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}+\sqrt{a+bx}\left(-105a^3e^3+5a^2be^2(89d+14ex)-ab^2e\left(725\ldots\right.\right.}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x
]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) -
 a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x +
232*d*e^2*x^2 + 48*e^3*x^3)) + (3*(b*d - a*e)^(3/2)*(73*b^2*d^2 - 90*a*b*d*
e + 35*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*
Sqrt[(b*(d + e*x))/(b*d - a*e)])))/(24*b^4)
```

fricas [A] time = 2.02, size = 546, normalized size = 2.28

$$\left[\frac{3\left(73\,b^4d^4 - 236\,ab^3d^3e + 288\,a^2b^2d^2e^2 - 160\,a^3bde^3 + 35\,a^4e^4\right)\sqrt{be}\,\log\left(8\,b^2e^2x^2 + b^2d^2 + 6\,abde + a^2e^2 + 4\,(2\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/96*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 160*a^3*b*d*
e^3 + 35*a^4*e^4)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e
^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2
*d*e + a*b*e^2)*x) + 4*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^2
+ 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2 +
2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*sqr
t(e*x + d))/(b^5*e), -1/48*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d
^2*e^2 - 160*a^3*b*d*e^3 + 35*a^4*e^4)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d
 + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^
2*d*e + a*b*e^2)*x)) - 2*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^
2 + 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2
+ 2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*s
qrt(e*x + d))/(b^5*e)]
```

giac [B] time = 0.52, size = 717, normalized size = 2.99

$$-\frac{360\left(\frac{\left(b^2d-abe\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}}-\sqrt{b^2d+(bx+a)be-abe}\,\sqrt{bx+a}\right)d^3|b|}{b^2}-\frac{28\left(\sqrt{b^2d+(bx+a)be-abe}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/24*(360*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2)
 + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b
*e - a*b*e)*sqrt(b*x + a))*d^3*abs(b)/b^2 - 28*(sqrt(b^2*d + (b*x + a)*b*e
- a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^
5*e^4)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4
)/b^7) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log
(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))
/b^(3/2))*d*abs(b)*e^2/b^2 - 210*((b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*e^(
-3/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e -
 a*b*e)))/sqrt(b) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + (b*d*e - 5
*a*e^2)*e^(-2) + 2*a)*sqrt(b*x + a))*d^2*abs(b)*e/b^3 - (sqrt(b^2*d + (b*x
+ a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*d*e^5
- 25*a*b^11*e^6)*e^(-6)/b^14) - (5*b^13*d^2*e^4 + 14*a*b^12*d*e^5 - 163*a^2
*b^11*e^6)*e^(-6)/b^14) + 3*(5*b^14*d^3*e^3 + 9*a*b^13*d^2*e^4 + 15*a^2*b^1
2*d*e^5 - 93*a^3*b^11*e^6)*e^(-6)/b^14)*sqrt(b*x + a) + 3*(5*b^4*d^4 + 4*a*
b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 - 35*a^4*e^4)*e^(-7/2)*log(a
```

```
bs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b
^(5/2))*abs(b)*e^3/b^2)/b
```

maple [B] time = 0.04, size = 571, normalized size = 2.38

$$\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(105a^4e^4\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-480a^3bd\,e^3\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+8\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(96*x^3*b^3*e^3*((b*x+a)*(e*x+d))^(1/2)*(b
*e)^(1/2)-112*x^2*a*b^2*e^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+464*x^2*b^3
*d*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+105*ln(1/2*(2*b*e*x+2*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^4*e^4-480*ln(1/2*(2*b*e*x
+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^3*b*d*e^3+86
4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2
))*a^2*b^2*d^2*e^2-708*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2
)+a*e+b*d)/(b*e)^(1/2))*a*b^3*d^3*e+219*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))
^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^4*d^4+140*(b*e)^(1/2)*((b*x+a)*(
e*x+d))^(1/2)*x*a^2*b*e^3-584*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*a*b^2*d
*e^2+932*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*b^3*d^2*e-210*((b*x+a)*(e*x+
d))^(1/2)*(b*e)^(1/2)*a^3*e^3+890*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a^2*b
*d*e^2-1450*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a*b^2*d^2*e+1002*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)*b^3*d^3)/b^4/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^{3/2}\,\left(15\,d^2+20\,d\,e\,x+8\,e^2\,x^2\right)}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

```
[Out] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.844 $\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=176

$$\frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}\left(5a^2e^2-13abde+11b^2d^2\right)}{b^3}+\frac{8e(a+bx)}{}$$

[Out] 8/3*e*(b*x+a)^(3/2)*(e*x+d)^(3/2)/b^2+(-a*e+b*d)*(5*a^2*e^2-13*a*b*d*e+11*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(7/2)/e^(1/2)+2*(-3*a*e+4*b*d)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/b^2+(5*a^2*e^2-13*a*b*d*e+11*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^3

Rubi [A] time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\sqrt{d+ex}\left(5a^2e^2-13abde+11b^2d^2\right)}{b^3}+\frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{8e(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^3 + (
2*(4*b*d - 3*a*e)*Sqrt[a + b*x]*(d + e*x)^(3/2))/b^2 + (8*e*(a + b*x)^(3/2)
*(d + e*x)^(3/2))/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e
^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(7/2)*Sqrt
[e])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{d+ex}\left(3e(3bd-2ae)(5bd+2ae)+12be^2(4bd-3ae)x\right)}{\sqrt{a+bx}}\,d}{3b^2e}$$

$$= \frac{2(4bd-3ae)\sqrt{a+bx}\,(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\left(11b^2d\right.}{}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}}{b^2}$$

Mathematica [A] time = 0.44, size = 163, normalized size = 0.93

$$\frac{\sqrt{d+ex}\left(\sqrt{a+bx}\left(15a^2e^2-abe(49d+10ex)+b^2\left(57d^2+32dex+8e^2x^2\right)\right)+\frac{3\sqrt{bd-ae}\left(5a^2e^2-13abde+11b^2d^2\right)\sinh^{-1}(}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}\right.}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(15*a^2*e^2 - a*b*e*(49*d + 10*e*x) + b^2*(57
*d^2 + 32*d*e*x + 8*e^2*x^2)) + (3*Sqrt[b*d - a*e]*(11*b^2*d^2 - 13*a*b*d*e
 + 5*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*Sq
rt[(b*(d + e*x))/(b*d - a*e)])))/(3*b^3)
```

fricas [A] time = 1.30, size = 414, normalized size = 2.35

$$\left[-\frac{3\left(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3\right)\sqrt{be}\,\log\left(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2-4\,(2\,bex+bd+ae)\sqrt{be}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/12*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(b
*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(2*b*e*x + b*d +
a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) - 4*(
8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*
e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^4*e), -1/6*(3*(11*b^3
*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(-b*e)*arctan(1/2*(
2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 +
a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*
b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*
sqrt(e*x + d))/(b^4*e)]
```

giac [B] time = 0.38, size = 441, normalized size = 2.51

$$-\frac{45\left(\frac{\left(b^2d-abe\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}}-\sqrt{b^2d+(bx+a)be-abe}\,\sqrt{bx+a}\right)d^2|b|}{b^2}-\frac{\left(\sqrt{b^2d+(bx+a)be-abe}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{(b^{\cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/3*(45*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) +
 sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b*e
 - a*b*e)*sqrt(b*x + a))*d^2*abs(b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b
*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4
)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4)/b^7
) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log(abs(
-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b^(3
/2))*abs(b)*e^2/b^2 - 15*((b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*e^(-3/2)*lo
g(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e))
)/sqrt(b) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + (b*d*e - 5*a*e^2)*
e^(-2) + 2*a)*sqrt(b*x + a))*d*abs(b)*e/b^3)/b
```

maple [B] time = 0.02, size = 392, normalized size = 2.23

$$-\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(15a^3e^3\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-54a^2bd\,e^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+72a\,b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x)
```

```
[Out] -1/6*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(-16*x^2*b^2*e^2*((b*x+a)*(e*x+d))^(1/2)*(
b*e)^(1/2)+15*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)
)/(b*e)^(1/2))*a^3*e^3-54*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)
```

```
*(b*e)^(1/2))/(b*e)^(1/2))*a^2*b*d*e^2+72*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a
)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a*b^2*d^2*e-33*ln(1/2*(2*b*e*x+a
*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*b^3*d^3+20*(b*e)
^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*a*b*e^2-64*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(
1/2)*x*b^2*d*e-30*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*e^2+98*(b*e)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*a*b*d*e-114*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*b
^2*d^2)/b^3/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B] time = 73.15, size = 1797, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))^3*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e
))/(e^3*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))*(10*b
^3*d^3 + 20*a*b^2*d^2*e - 30*a^2*b*d*e^2))/(e^4*((d + e*x)^(1/2) - d^(1/2))
) - (160*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2)
 - d^(1/2))^6) + (((a + b*x)^(1/2) - a^(1/2))^7*(10*b^2*d^3 - 30*a^2*d*e^2
+ 20*a*b*d^2*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) + (((a + b*x)^(1/2)
- a^(1/2))^5*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e))/(b*e^2*((d + e*x
)^(1/2) - d^(1/2))^5) - (a^(1/2)*d^(1/2)*(320*b*d^2 + 640*a*d*e)*((a + b*x)
^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (160*a^(1/2)*b^2
*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2)
)/(((a + b*x)^(1/2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 -
(4*b^3*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) +
 (6*b^2*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4)
- (4*b*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) -
((((a + b*x)^(1/2) - a^(1/2))*(2*b^5*d^3 - 10*a^3*b^2*e^3 + 6*a^2*b^3*d*e^2
 + 2*a*b^4*d^2*e))/(e^6*((d + e*x)^(1/2) - d^(1/2))) - (((a + b*x)^(1/2) -
a^(1/2))^5*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*b^2*d^2*e + 1252*a^2*b*d*e^2)
)/(e^4*((d + e*x)^(1/2) - d^(1/2))^5) - (((a + b*x)^(1/2) - a^(1/2))^3*((34
*b^4*d^3)/3 - (170*a^3*b*e^3)/3 + 34*a^2*b^2*d*e^2 + 182*a*b^3*d^2*e))/(e^5
*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^11*(2*b^3*d^
3 - 10*a^3*e^3 + 2*a*b^2*d^2*e + 6*a^2*b*d*e^2))/(b^3*e*((d + e*x)^(1/2) -
d^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^9*((34*b^3*d^3)/3 - (170*a^3*e^
3)/3 + 182*a*b^2*d^2*e + 34*a^2*b*d*e^2))/(b^2*e^2*((d + e*x)^(1/2) - d^(1/
2))^9) - (((a + b*x)^(1/2) - a^(1/2))^7*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*
b^2*d^2*e + 1252*a^2*b*d*e^2))/(b*e^3*((d + e*x)^(1/2) - d^(1/2))^7) + (a^(
1/2)*d^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*(1024*a^2*e^2 + 512*b^2*d^2 + (5
632*a*b*d*e)/3))/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (a^(1/2)*d^(1/2)*(25
6*b*d^2 + 768*a*d*e)*((a + b*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) -
 d^(1/2))^8) + (a^(1/2)*d^(1/2)*(256*b^3*d^2 + 768*a*b^2*d*e)*((a + b*x)^(1
/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/2) -
a^(1/2))^12/((d + e*x)^(1/2) - d^(1/2))^12 + b^6/e^6 - (6*b^5*((a + b*x)^(1
/2) - a^(1/2))^2)/(e^5*((d + e*x)^(1/2) - d^(1/2))^2) + (15*b^4*((a + b*x)^
```

```
(1/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4) - (20*b^3*((a + b*x
)^(1/2) - a^(1/2))^6)/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (15*b^2*((a + b
*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) - d^(1/2))^8) - (6*b*((a + b*
x)^(1/2) - a^(1/2))^10)/(e*((d + e*x)^(1/2) - d^(1/2))^10)) + (((30*b*d^3 +
 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2) - d^(1/2)))
 - (120*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d + e*x)^(1/2)
- d^(1/2))^2) + ((30*b*d^3 + 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*
e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/((d + e*x)
^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d
+ e*x)^(1/2) - d^(1/2))^2)) - (2*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2))
)/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d)*(5*a^2*e^2 + b^2*d^2 +
 2*a*b*d*e))/(b^(7/2)*e^(1/2)) - (30*d^2*atanh((e^(1/2)*((a + b*x)^(1/2) -
a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d))/(b^(3/2)*e^(1
/2)) + (10*d*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x
)^(1/2) - d^(1/2))))*(a*e - b*d)*(3*a*e + b*d))/(b^(5/2)*e^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.845 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx$

Optimal. Leaf size=122

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\,\sqrt{d+ex}\,(7bd-5ae)}{b^2}$$

```
[Out] 2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*
x+d)^(1/2))/b^(5/2)/e^(1/2)+4*e*(b*x+a)^(3/2)*(e*x+d)^(1/2)/b^2+2*(-5*a*e+7
*b*d)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^2
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {951, 80, 63, 217, 206}

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\,\sqrt{d+ex}\,(7bd-5ae)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^2 + (4*e*(a + b*x)^(3/2)*
Sqrt[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e
]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(5/2)*Sqrt[e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
```

```
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\int \frac{2e\left(15b^2d^2-6abde-2a^2e^2\right)+4be^2(7bd-5ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{2b^2e}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(8b^2d^2-8abde+3a^2e^2\right)}{b^2}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(2\left(8b^2d^2-8abde+3a^2\right.\right.}{}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(2\left(8b^2d^2-8abde+3a^2\right.\right.}{}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2\left(8b^2d^2-8abde+3a^2e\right.}{b^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.41, size = 135, normalized size = 1.11

$$\frac{2\left(\frac{\sqrt{bd-ae}\left(3a^2e^2-8abde+8b^2d^2\right)\sqrt{\frac{b(d+ex)}{bd-ae}}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}}+b\sqrt{a+bx}(d+ex)(-3ae+7bd+2bex)\right)}{b^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(b*Sqrt[a + b*x]*(d + e*x)*(7*b*d - 3*a*e + 2*b*e*x) + (Sqrt[b*d - a*e]*
(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*Sqrt[(b*(d + e*x))/(b*d - a*e)]*ArcSinh
[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/Sqrt[e]))/(b^3*Sqrt[d + e*x])
```

fricas [A] time = 1.42, size = 308, normalized size = 2.52

$$\left[\frac{\left(8\,b^2d^2-8\,abde+3\,a^2e^2\right)\sqrt{be}\,\log\left(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2+4\,(2\,bex+bd+ae)\sqrt{be}\,\sqrt{bx+a}\,\sqrt{ex+d}\,+\right.}{2\,b^3e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*((8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2
*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a
)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + 4*(2*b^2*e^2*x + 7*b^2*d*e - 3
*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e), -((8*b^2*d^2 - 8*a*b*d*e +
```

```
3*a^2*e^2)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x
+ a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(2*
b^2*e^2*x + 7*b^2*d*e - 3*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e)]
```

giac [A] time = 0.24, size = 145, normalized size = 1.19

$$\frac{2\left(\sqrt{b^2d+(bx+a)be-abe}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)e}{b^3}+\frac{\left(7\,b^6de^2-5\,ab^5e^3\right)e^{(-2)}}{b^8}\right)-\frac{\left(8\,b^2d^2-8\,abde+3\,a^2e^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2\cdots}\right.\right.}{b^{\frac{5}{2}}}\right.}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*e/b^3 + (
7*b^6*d*e^2 - 5*a*b^5*e^3)*e^(-2)/b^8) - (8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2
)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*
b*e - a*b*e)))/b^(5/2))*b/abs(b)
```

maple [B] time = 0.03, size = 247, normalized size = 2.02

$$\frac{\left(3a^2e^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-8abde\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+8b^2d^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a\cdots}}{2\sqrt{be}}\right.\right.}{\sqrt{be}\,\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (3*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/
2))*a^2*e^2-8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)
)/(b*e)^(1/2))*a*b*d*e+8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*
(b*e)^(1/2))/(b*e)^(1/2))*b^2*d^2+4*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*b
*e-6*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*e+14*(b*e)^(1/2)*((b*x+a)*(e*x+d
))^(1/2)*b*d)*(e*x+d)^(1/2)*(b*x+a)^(1/2)/(b*e)^(1/2)/b^2/((b*x+a)*(e*x+d))
^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B] time = 20.64, size = 893, normalized size = 7.32

$$\frac{\frac{\left(40\,b\,d^2+40\,a\,e\,d\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}-\frac{160\,\sqrt{a}\,d^{3/2}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}+\frac{\left(40\,b\,d^2+40\,a\,e\,d\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{b\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}+\frac{b^2}{e^2}-\frac{2\,b\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\left(12\,a^2\,b\,e^2+8\,a\,b^2\,d\,e+12\cdots\right.}{e^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2)
- d^(1/2))) - (160*a^(1/2)*d^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d +
e*x)^(1/2) - d^(1/2))^2) + ((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2
))^3)/(b*e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/(
(d + e*x)^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2
)/(e*((d + e*x)^(1/2) - d^(1/2))^2)) - ((((a + b*x)^(1/2) - a^(1/2))*(12*b^
3*d^2 + 12*a^2*b*e^2 + 8*a*b^2*d*e))/(e^4*((d + e*x)^(1/2) - d^(1/2))) - ((
(a + b*x)^(1/2) - a^(1/2))^3*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(e^3*
((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^7*(12*a^2*e^2
 + 12*b^2*d^2 + 8*a*b*d*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) - (((a +
b*x)^(1/2) - a^(1/2))^5*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(b*e^2*((d
 + e*x)^(1/2) - d^(1/2))^5) + (a^(1/2)*d^(1/2)*(256*a*e + 256*b*d)*((a + b*
x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/
2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 - (4*b^3*((a + b*x)
^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) + (6*b^2*((a + b*x
)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (4*b*((a + b*x)
^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) - (60*d^2*atan((b*(
(d + e*x)^(1/2) - d^(1/2)))/((-b*e)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(-
b*e)^(1/2) - (2*log((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2)
- d^(1/2)) - b^(1/2))*(3*a^2*e^2 + 3*b^2*d^2 + 2*a*b*d*e))/(b^(5/2)*e^(1/2)
) + (log(b^(1/2) + (e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2) -
 d^(1/2)))*(6*a^2*e^2 + 6*b^2*d^2 + 4*a*b*d*e))/(b^(5/2)*e^(1/2)) - (40*d*a
tanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1
/2))))*(a*e + b*d))/(b^(3/2)*e^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*sqrt(d + e*x)),
x)
```

3.846 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}\,(bd-ae)}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}$$

```
[Out] 8*(-a*e+2*b*d)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(3/2)
/e^(1/2)+6*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(1/2)+8*(b*x+a)^(1/2)*(e*x+
d)^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {949, 80, 63, 217, 206}

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}\,(bd-ae)}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*Sqrt[a + b*x]*Sqrt[d
 + e*x])/b + (8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[d + e*x])])/(b^(3/2)*Sqrt[e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
```

```
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{2\int \frac{6d(bd-ae)+4e(bd-ae)x}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx}{bd-ae}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{(4(2bd-ae))\int \frac{1}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx}{b}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{(8(2bd-ae))\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx, x, \ldots\right.}{b^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{(8(2bd-ae))\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx, x, \frac{\sqrt{a+b}}{\sqrt{d+e}}\ldots\right.}{b^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.34, size = 134, normalized size = 1.24

$$\frac{2\left(\frac{b\sqrt{a+bx}\,(bd(7d+4ex)-4ae(d+ex))}{bd-ae} + \frac{4\sqrt{bd-ae}\,(2bd-ae)\sqrt{\frac{b(d+ex)}{bd-ae}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}}\right)}{b^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x
]
```

```
[Out] (2*((b*Sqrt[a + b*x]*(-4*a*e*(d + e*x) + b*d*(7*d + 4*e*x)))/(b*d - a*e) +
(4*Sqrt[b*d - a*e]*(2*b*d - a*e)*Sqrt[(b*(d + e*x))/(b*d - a*e)]*ArcSinh[(S
qrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/Sqrt[e]))/(b^2*Sqrt[d + e*x])
```

fricas [B] time = 2.03, size = 463, normalized size = 4.29

$$\left[-\frac{2\left(\left(2b^2d^3-3abd^2e+a^2de^2+\left(2b^2d^2e-3abde^2+a^2e^3\right)x\right)\sqrt{be}\,\log\left(8b^2e^2x^2+b^2d^2+6abde+a^2e^2-4\left(2bex\ldots\right.\right.}{b^3d^2e-ab^2de^2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-2*((2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2 + a^
2*e^3)*x)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(
```

```
2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a
*b*e^2)*x) - (7*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b
*x + a)*sqrt(e*x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x
), -2*(2*(2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2
+ a^2*e^3)*x)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b
*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7
*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b*x + a)*sqrt(e*
x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x)]
```

giac [B] time = 0.36, size = 193, normalized size = 1.79

$$-\frac{8\,(2\,bd-ae)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}\,|b|}+\frac{2\,\sqrt{bx+a}\left(\frac{4\left(b^3de^3-ab^2e^4\right)(bx+a)}{b^3d|b|e^2-ab^2|b|e^3}+\frac{7\,b^4d^2e^2}{b^3}\right.}{\sqrt{b^2d+(bx+a)be-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -8*(2*b*d - a*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2
*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b)*abs(b)) + 2*sqrt(b*x + a)*(4*(b^3*d*
e^3 - a*b^2*e^4)*(b*x + a)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^3) + (7*b^4*d
^2*e^2 - 8*a*b^3*d*e^3 + 4*a^2*b^2*e^4)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^
3))/sqrt(b^2*d + (b*x + a)*b*e - a*b*e)
```

maple [B] time = 0.03, size = 438, normalized size = 4.06

$$-\frac{2\sqrt{bx+a}\,\left(2a^2e^3x\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-6abd\,e^2x\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4b^2d^2ex\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)*(2*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)
^(1/2))/(b*e)^(1/2))*x*a^2*e^3-6*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d)
)^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x*a*b*d*e^2+4*ln(1/2*(2*b*e*x+a*e+b*d+2*(
(b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x*b^2*d^2*e+2*ln(1/2*(2*b*
e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a^2*d*e^2-6
*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)
)*a*b*d^2*e+4*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)
)/(b*e)^(1/2))*b^2*d^3-4*x*a*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+4*x*b*
d*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)-4*a*d*e*((b*x+a)*(e*x+d))^(1/2)*(b*
e)^(1/2)+7*b*d^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/b/(b*e)^(1/2)/(a*e-b*
d)/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{15\,d^2+20\,d\,e\,x+8\,e^2\,x^2}{\sqrt{a+b\,x}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)),x)
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(3/2)/(b*x+a)**(1/2),x)
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(3/2)
), x)
```

3.847 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(3bd-2ae)}{\sqrt{d+ex}\,(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{b}\,\sqrt{e}}$$

```
[Out] 16*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(1/2)/e^(1/2)+2*d
^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(3/2)+4*d*(-2*a*e+3*b*d)*(b*x+a)^(1/2)/
(-a*e+b*d)^2/(e*x+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {949, 78, 63, 217, 206}

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(3bd-2ae)}{\sqrt{d+ex}\,(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{b}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x]
```

```
[Out] (2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (4*d*(3*b*d - 2*a*e)*
Sqrt[a + b*x])/((b*d - a*e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
```

```
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx &= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{2\int \frac{3d(7bd-6ae)+12e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx}{3(bd-ae)}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+8\int \frac{1}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx,x,\sqrt{a+
\end{aligned}$$

$$\begin{aligned}
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{b}\,\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 128, normalized size = 1.10

$$\frac{2\left(\frac{8(bd-ae)^{3/2}\left(\frac{b(d+ex)}{bd-ae}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{b^2\sqrt{e}}+\frac{d\sqrt{a+bx}\,(bd(7d+6ex)-ae(5d+4ex))}{(bd-ae)^2}\right)}{(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x
]
```

```
[Out] (2*((d*Sqrt[a + b*x]*(-(a*e*(5*d + 4*e*x)) + b*d*(7*d + 6*e*x)))/(b*d - a*e
)^2 + (8*(b*d - a*e)^(3/2)*((b*(d + e*x))/(b*d - a*e))^(3/2)*ArcSinh[(Sqrt[
e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(b^2*Sqrt[e])))/(d + e*x)^(3/2)
```

fricas [B] time = 1.59, size = 665, normalized size = 5.73

$$\left[\frac{2\left(2\left(b^2d^4-2\,abd^3e+a^2d^2e^2+\left(b^2d^2e^2-2\,abde^3+a^2e^4\right)x^2+2\left(b^2d^3e-2\,abd^2e^2+a^2de^3\right)x\right)\sqrt{be}\,\log\left(8\,b^2e^2x^2+\right.}{b^3d^4e-2\,ab^2d^3e^2+a^2bd^2e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [2*(2*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a
^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(b*e)*log(8*
```

```
b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(
b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + (7*b^2*d^3*e
- 5*a*b*d^2*e^2 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x
 + d))/(b^3*d^4*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^
2*d*e^4 + a^2*b*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*
x), -2*(4*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3
 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(-b*e)*a
rctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2
*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7*b^2*d^3*e - 5*a*b*d^2*e^2
 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*d^4
*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^2*d*e^4 + a^2*b
*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 601, normalized size = 5.18

$$\frac{2\sqrt{bx+a}\,\left(4a^2e^4x^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-8abd\,e^3x^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4b^2d^2e^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)*(4*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^
(1/2))/(b*e)^(1/2))*x^2*a^2*e^4-8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d
))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x^2*a*b*d*e^3+4*ln(1/2*(2*b*e*x+a*e+b*d+
2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x^2*b^2*d^2*e^2+8*ln(1/
2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x*a^
2*d*e^3-16*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(
b*e)^(1/2))*x*a*b*d^2*e^2+8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/
2)*(b*e)^(1/2))/(b*e)^(1/2))*x*b^2*d^3*e+4*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+
a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a^2*d^2*e^2-8*ln(1/2*(2*b*e*x+a
*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a*b*d^3*e+4*ln(1
/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*b^2
*d^4-4*x*a*d*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+6*x*b*d^2*e*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)-5*a*d^2*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+7*b
*d^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)/(a*e-b*d)^2/((b*x+a)*
(e*x+d))^(1/2)/(e*x+d)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{15\, d^2 + 20\, d\, e\, x + 8\, e^2\, x^2}{\sqrt{a + b\, x}\, \left(d + e\, x\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)),x)
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(5/2)/(b*x+a)**(1/2),x)
[Out] Timed out
```

3.848 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{16\sqrt{a+bx}\,\left(15a^2e^2-35abde+23b^2d^2\right)}{15\sqrt{d+ex}\,(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}\,(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

```
[Out] 6/5*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(5/2)+8/15*d*(-5*a*e+8*b*d)*(b*x+a
)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(3/2)+16/15*(15*a^2*e^2-35*a*b*d*e+23*b^2*d^2)
*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {949, 78, 37}

$$\frac{16\sqrt{a+bx}\,\left(15a^2e^2-35abde+23b^2d^2\right)}{15\sqrt{d+ex}\,(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}\,(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (8*d*(8*b*d - 5*a*e
)*Sqrt[a + b*x])/(15*(b*d - a*e)^2*(d + e*x)^(3/2)) + (16*(23*b^2*d^2 - 35*
a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{2\int \frac{6d(6bd-5ae)+20e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx}{5(bd-ae)}$$
$$= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{\left(8\left(23b^2d^2-35abde+15a^2e^2\right)\right)}{15(bd-ae)^2}$$
$$= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{16\left(23b^2d^2-35abde+15a^2e^2\right)\sqrt{}}{15(bd-ae)^3\sqrt{d+ex}}$$

Mathematica [A] time = 0.08, size = 110, normalized size = 0.83

$$\frac{2\sqrt{a+bx}\left(a^2e^2\left(149d^2+260dex+120e^2x^2\right)-2abde\left(175d^2+306dex+140e^2x^2\right)+b^2d^2\left(225d^2+400dex+184\right.\right.}{15(d+ex)^{5/2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(a^2*e^2*(149*d^2 + 260*d*e*x + 120*e^2*x^2) - 2*a*b*d*e*(
175*d^2 + 306*d*e*x + 140*e^2*x^2) + b^2*d^2*(225*d^2 + 400*d*e*x + 184*e^2
*x^2)))/(15*(b*d - a*e)^3*(d + e*x)^(5/2))
```

fricas [B] time = 3.27, size = 293, normalized size = 2.20

$$\frac{2\left(225\,b^2d^4-350\,abd^3e+149\,a^2d^2e^2+8\left(23\,b^2d^2e^2-35\,abde^3+15\,a^2e^4\right)x^2+4\left(100\,b^2d^3e\right.\right.}{15\left(b^3d^6-3\,ab^2d^5e+3\,a^2bd^4e^2-a^3d^3e^3+\left(b^3d^3e^3-3\,ab^2d^2e^4+3\,a^2bde^5-a^3e^6\right)x^3+3\left(b^3d^4e^2-3\,ab^2d^3e^3+3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/15*(225*b^2*d^4 - 350*a*b*d^3*e + 149*a^2*d^2*e^2 + 8*(23*b^2*d^2*e^2 - 3
5*a*b*d*e^3 + 15*a^2*e^4)*x^2 + 4*(100*b^2*d^3*e - 153*a*b*d^2*e^2 + 65*a^2
*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d)/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*b*d
^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3
*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d*e^5)
*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 150, normalized size = 1.13

$$-\frac{2\sqrt{bx+a}\,\left(120a^2e^4x^2-280abd\,e^3x^2+184b^2d^2e^2x^2+260a^2d\,e^3x-612ab\,d^2e^2x+400b^2d^3ex+149a^2d^2e^2-35\right.}{15\,(ex+d)^{\frac{5}{2}}\left(a^3e^3-3a^2bd\,e^2+3a\,b^2d^2e-b^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(120*a^2*e^4*x^2-280*a*b*d*e^3*x^2+184*b^2*d^2*e^2*x^2+
260*a^2*d*e^3*x-612*a*b*d^2*e^2*x+400*b^2*d^3*e*x+149*a^2*d^2*e^2-350*a*b*d
^3*e+225*b^2*d^4)/(e*x+d)^(5/2)/(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^
3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B] time = 4.30, size = 268, normalized size = 2.02

$$-\frac{\sqrt{d+e\,x}\,\left(\frac{x^2\,\left(240\,a^3\,e^4-40\,a^2\,b\,d\,e^3-856\,a\,b^2\,d^2\,e^2+800\,b^3\,d^3\,e\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{x\,\left(520\,a^3\,d\,e^3-926\,a^2\,b\,d^2\,e^2+100\,a\,b^2\,d^3\,e+450\,b^3\,d^4\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{2\,a\,d^2\,\left(149\,a^2\,e^2\cdots\right.}{15\,e^3\cdots}\cdots\right)}{x^3\,\sqrt{a+b\,x}+\frac{d^3\,\sqrt{a+b\,x}}{e^3}+\frac{3\,d\,x^2\,\sqrt{a+b\,x}}{e}+\frac{3\,d^2\,x\,\sqrt{a+b\,x}}{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] -((d + e*x)^(1/2)*((x^2*(240*a^3*e^4 + 800*b^3*d^3*e - 856*a*b^2*d^2*e^2 -
40*a^2*b*d*e^3))/(15*e^3*(a*e - b*d)^3) + (x*(450*b^3*d^4 + 520*a^3*d*e^3 -
 926*a^2*b*d^2*e^2 + 100*a*b^2*d^3*e))/(15*e^3*(a*e - b*d)^3) + (2*a*d^2*(1
49*a^2*e^2 + 225*b^2*d^2 - 350*a*b*d*e))/(15*e^3*(a*e - b*d)^3) + (16*b*x^3
*(15*a^2*e^2 + 23*b^2*d^2 - 35*a*b*d*e))/(15*e*(a*e - b*d)^3)))/(x^3*(a + b
*x)^(1/2) + (d^3*(a + b*x)^(1/2))/e^3 + (3*d*x^2*(a + b*x)^(1/2))/e + (3*d^
2*x*(a + b*x)^(1/2))/e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.849 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=189

$$\frac{32b\sqrt{a+bx}\,\left(35a^2e^2-84abde+58b^2d^2\right)}{105\sqrt{d+ex}\,(bd-ae)^4}+\frac{16\sqrt{a+bx}\,\left(35a^2e^2-84abde+58b^2d^2\right)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a}}{35(a}$$

```
[Out] 6/7*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(7/2)+4/35*d*(-14*a*e+23*b*d)*(b*x
+a)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(5/2)+16/105*(35*a^2*e^2-84*a*b*d*e+58*b^2*d
^2)*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(3/2)+32/105*b*(35*a^2*e^2-84*a*b*d*
e+58*b^2*d^2)*(b*x+a)^(1/2)/(-a*e+b*d)^4/(e*x+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {949, 78, 45, 37}

$$\frac{32b\sqrt{a+bx}\,\left(35a^2e^2-84abde+58b^2d^2\right)}{105\sqrt{d+ex}\,(bd-ae)^4}+\frac{16\sqrt{a+bx}\,\left(35a^2e^2-84abde+58b^2d^2\right)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a}}{35(a}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (4*d*(23*b*d - 14*a
*e)*Sqrt[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^(5/2)) + (16*(58*b^2*d^2 - 8
4*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^(3/2))
+ (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a
*e)^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
```

```
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{9/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{2\int \frac{3d(17bd-14ae)+28e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx}{7(bd-ae)}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{\left(8\left(58b^2d^2-84abde+35a\right.\right.}{35(bd-}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16\left(58b^2d^2-84abde+35a\right.}{105(bd-ae)^3(d+}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16\left(58b^2d^2-84abde+35a\right.}{105(bd-ae)^3(d+}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.92

$$\frac{2\sqrt{a+bx}\left(-a^3e^3\left(409d^2+644dex+280e^2x^2\right)+a^2be^2\left(1953d^3+3890d^2ex+2632de^2x^2+560e^3x^3\right)-ab^2de\left(\right.\right.}{105(d+ex)^{7/2}(bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(-(a^3*e^3*(409*d^2 + 644*d*e*x + 280*e^2*x^2)) + a^2*b*e^
2*(1953*d^3 + 3890*d^2*e*x + 2632*d*e^2*x^2 + 560*e^3*x^3) + b^3*d^2*(1575*
d^3 + 3850*d^2*e*x + 3248*d*e^2*x^2 + 928*e^3*x^3) - a*b^2*d*e*(2975*d^3 +
6664*d^2*e*x + 5168*d*e^2*x^2 + 1344*e^3*x^3)))/(105*(b*d - a*e)^4*(d + e*x
)^(7/2))
```

fricas [B] time = 6.62, size = 487, normalized size = 2.58

$$\frac{2\left(1575\,b^3d^5-2975\,ab^2d^4e+1953\,a^2bd^3e^2-409\,a^3d^2e^3+16\left(58\,b^3d^2e^3-84\,ab^2\right.\right.}{105\left(b^4d^8-4\,ab^3d^7e+6\,a^2b^2d^6e^2-4\,a^3bd^5e^3+a^4d^4e^4+\left(b^4d^4e^4-4\,ab^3d^3e^5+6\,a^2b^2d^2e^6-4\,a^3bde^7+a^4e^8\right)x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/105*(1575*b^3*d^5 - 2975*a*b^2*d^4*e + 1953*a^2*b*d^3*e^2 - 409*a^3*d^2*e
^3 + 16*(58*b^3*d^2*e^3 - 84*a*b^2*d*e^4 + 35*a^2*b*e^5)*x^3 + 8*(406*b^3*d
^3*e^2 - 646*a*b^2*d^2*e^3 + 329*a^2*b*d*e^4 - 35*a^3*e^5)*x^2 + 2*(1925*b^
3*d^4*e - 3332*a*b^2*d^3*e^2 + 1945*a^2*b*d^2*e^3 - 322*a^3*d*e^4)*x)*sqrt(
b*x + a)*sqrt(e*x + d)/(b^4*d^8 - 4*a*b^3*d^7*e + 6*a^2*b^2*d^6*e^2 - 4*a^3
*b*d^5*e^3 + a^4*d^4*e^4 + (b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e
^6 - 4*a^3*b*d*e^7 + a^4*e^8)*x^4 + 4*(b^4*d^5*e^3 - 4*a*b^3*d^4*e^4 + 6*a^
2*b^2*d^3*e^5 - 4*a^3*b*d^2*e^6 + a^4*d*e^7)*x^3 + 6*(b^4*d^6*e^2 - 4*a*b^3
*d^5*e^3 + 6*a^2*b^2*d^4*e^4 - 4*a^3*b*d^3*e^5 + a^4*d^2*e^6)*x^2 + 4*(b^4*
```

```
d^7*e - 4*a*b^3*d^6*e^2 + 6*a^2*b^2*d^5*e^3 - 4*a^3*b*d^4*e^4 + a^4*d^3*e^5
)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 248, normalized size = 1.31

$$-\frac{2\sqrt{bx+a}\,\left(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-324\cdots\right.}{105\,(ex+d)^{\frac{7}{2}}\left(e^4a^4-4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-560*a^2*b*e^5*x^3+1344*a*b^2*d*e^4*x^3-928*b^3*d^2*e
^3*x^3+280*a^3*e^5*x^2-2632*a^2*b*d*e^4*x^2+5168*a*b^2*d^2*e^3*x^2-3248*b^3
*d^3*e^2*x^2+644*a^3*d*e^4*x-3890*a^2*b*d^2*e^3*x+6664*a*b^2*d^3*e^2*x-3850
*b^3*d^4*e*x+409*a^3*d^2*e^3-1953*a^2*b*d^3*e^2+2975*a*b^2*d^4*e-1575*b^3*d
^5)/(e*x+d)^(7/2)/(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^
4*d^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B] time = 4.51, size = 389, normalized size = 2.06

$$\frac{\sqrt{d+e\,x}\,\left(\frac{-818\,a^4\,d^2\,e^3+3906\,a^3\,b\,d^3\,e^2-5950\,a^2\,b^2\,d^4\,e+3150\,a\,b^3\,d^5}{105\,e^4\,(a\,e-b\,d)^4}+\frac{x\left(-1288\,a^4\,d\,e^4+6962\,a^3\,b\,d^2\,e^3-9422\,a^2\,b^2\,d^3\,e^2+1750\,a\,b^3\,d^4\,e+3150\,b^4\,\cdots\right.}{105\,e^4\,(a\,e-b\,d)^4}\cdots\right.}{x^4\,\sqrt{a+b\,x}+\frac{d^4\,\sqrt{a+\cdots}}{e^4}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((d + e*x)^(1/2)*((3150*a*b^3*d^5 - 818*a^4*d^2*e^3 - 5950*a^2*b^2*d^4*e +
3906*a^3*b*d^3*e^2)/(105*e^4*(a*e - b*d)^4) + (x*(3150*b^4*d^5 - 1288*a^4*d
*e^4 + 6962*a^3*b*d^2*e^3 - 9422*a^2*b^2*d^3*e^2 + 1750*a*b^3*d^4*e))/(105*
e^4*(a*e - b*d)^4) - (x^2*(560*a^4*e^5 - 7700*b^4*d^4*e + 6832*a*b^3*d^3*e^
2 + 2556*a^2*b^2*d^2*e^3 - 3976*a^3*b*d*e^4))/(105*e^4*(a*e - b*d)^4) + (32
*b^2*x^4*(35*a^2*e^2 + 58*b^2*d^2 - 84*a*b*d*e))/(105*e*(a*e - b*d)^4) + (1
6*b*x^3*(35*a^3*e^3 + 406*b^3*d^3 - 530*a*b^2*d^2*e + 161*a^2*b*d*e^2))/(10
```

```
5*e^2*(a*e - b*d)^4)))/(x^4*(a + b*x)^(1/2) + (d^4*(a + b*x)^(1/2))/e^4 + (
6*d^2*x^2*(a + b*x)^(1/2))/e^2 + (4*d*x^3*(a + b*x)^(1/2))/e + (4*d^3*x*(a
+ b*x)^(1/2))/e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.850 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=417

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\mathrm{ta}}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{}}$$

```
[Out] 2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)-2*
arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/
(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(e*(-b*e+2*c*d)+(2*c^2*d^2+b^2*e^2-
2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-2*arctanh((e*x+d)^(1/2)*(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*c+b^2)^
(1/2))/c/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2)))^(1/2)
```

Rubi [A] time = 3.14, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {909, 63, 217, 206, 6728, 93, 208}

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\mathrm{ta}}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e
*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(c*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) - (
2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(c*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 909

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{cd^2-ae^2+e(2cd-be)x}{c\sqrt{d+ex}\,\sqrt{f+gx}\,\left(a+bx+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+bx+cx^2)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{e(2cd-be)+\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{e(2cd-be)-\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right)dx}{c} + \frac{(2e)\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{\ldots}{\left(b+\sqrt{b^2 \ldots}\right.}}{c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{\ldots}{-2cd+\ldots}\right.}{c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2cf-(b\ldots}}{\sqrt{2cd-(b\ldots}}\right.}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{2cf-\left(b-\sqrt{b^2\ldots}\right.}}$$

Mathematica [A] time = 1.79, size = 401, normalized size = 0.96

$$\frac{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)^{3/2}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\,\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)-\left(e\left(\sqrt{b^2-4ac}+b\right)\ldots\right.}{c\sqrt{b^2-4ac}\,\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}\,\sqrt{g\left(\sqrt{b^2-4ac}\ldots\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)^(3/2)*((e*(f + g*x))/(e*f - d*g))^(3/2)*ArcSinh[(Sqrt[g]*Sqr
t[d + e*x])/Sqrt[e*f - d*g]])/(c*Sqrt[g]*(f + g*x)^(3/2)) + ((-2*c*d + (b -
 Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]*ArcTa
nh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d +
(b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])] - (-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e)^(3/2)*Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*ArcTanh[(Sqrt[-2*c*f
+ (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 -
4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*f + (b - Sqrt[b^
2 - 4*a*c])*g]*Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.18, size = 11688, normalized size = 28.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Timed out
```

3.851 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=285

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/
2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+2*arctanh(
(e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*
a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {909, 93, 208}

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g])
 + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 909

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e+\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{e-\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+}\right.$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx+\left(e+\frac{2cd}{\sqrt{b^2}}\right.$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b}\right.\right.}\right.$$

$$= -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}+\frac{2\sqrt{2cd-}}{}$$

Mathematica [A] time = 0.87, size = 266, normalized size = 0.93

$$\frac{2\left(\frac{\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}-\frac{\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)}{\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(-((Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b -
 Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c]
)*e]*Sqrt[f + g*x])])/Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]) + (Sqrt[-2*
c*d + (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a
*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g
*x])])/Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 100.73, size = 4471, normalized size = 15.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^
2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g
+ d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 -
2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4
*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^
2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 + sqrt(2
)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*f^3 - (
```

b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b
^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3
)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2
 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*s
qrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f
^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 -
4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a
^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)*g^2)*x - (2*(b^2
*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g
^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2
)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*
f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2
*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b
^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqrt(((
2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)
*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^
2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c
^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^
2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*
b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 - sqrt(2)*((b^2 - 4*a*c)*e*
f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*
sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c -
4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2
*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt((
(2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c
)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c
^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b
^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^
2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)*g^2)*x - (2*(b^2*c - 4*a*c^2)*d*f^3
 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*
c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*
c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2
*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2
)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g
^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) + 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b
*d - 2*a*e)*g - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a
^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 -
2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*
b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2
 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2
g^2 - 2(b*d*e - a*e^2)*f^2 + sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)
*d*f*g - ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3
*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d
*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*
b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (
b*d - 2*a*e)*g - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*
a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 -
 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a
*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2
 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g
 - (b*d^2 - 4*a*d*e)*g^2)*x + (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c
)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2
 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g +

```
d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*
a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a
^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - ((b
^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^
2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^
2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*
g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f
*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a
*e^2)*f^2 - sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g - ((b^3*c -
4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c
)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
(b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c -
8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - ((
b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e
^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c
^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f
*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*
f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e
)*g^2)*x + (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^
2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*
b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2
- 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2
)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 5482, normalized size = 19.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.852 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=287

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{\cdot}\right.}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{\cdot}\right.}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2c\cdot}}$$

```
[Out] -4*c*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+4*c*arct
anh((e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {911, 93, 208}

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{\cdot}\right.}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{\cdot}\right.}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2c\cdot}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
```

IntegerQ[n]

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} - \frac{}{\sqrt{b^2-4ac}\,\left(\right.}\right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+g}}}{\sqrt{b^2-4ac}}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}} - \frac{(4}{}$$

$$= -\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + -$$

Mathematica [A] time = 0.75, size = 267, normalized size = 0.93

$$\frac{4c\left(\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)}{\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*(ArcTanh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]/(Sqrt[-2*c*d + (b - S
qrt[b^2 - 4*a*c])*e]*Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]) - ArcTanh[(S
qrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c]
)*e]*Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g])))/Sqrt[b^2 - 4*a*c]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 5507, normalized size = 19.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)\sqrt{ex + d}\,\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex}\,\sqrt{f + gx}\,\left(a + bx + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.853 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=429

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g(}}{\sqrt{f+gx}\sqrt{2cd-e(}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}$$

```
[Out] 4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2
)^(1/2)/(e*x+d)^(1/2)-4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(-4*a*c+b^2)^(1/2)/(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)^(1/2)-8*c^2*arctanh((e*x+d)^(1/2)*(2*
c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(2
*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+8*c^2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = $\{911, 96, 93, 208\}$

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g(}}{\sqrt{f+gx}\sqrt{2cd-e(}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*ArcTan
h[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (8*c^
2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
```

```
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}} - \frac{}{\sqrt{b^2-\cdots}}\right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex\cdots}}{\sqrt{b^2-4ac}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{}{\sqrt{b^2-4\cdots}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{}{\sqrt{b^2-4\cdots}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{}{\sqrt{b^2-4\cdots}}$$

Mathematica [A] time = 2.06, size = 334, normalized size = 0.78

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)}{\sqrt{b^2-4ac}\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)^{3/2}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}} + \frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{\cdots}\right.}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{\cdots}\right.}}\right)}{\sqrt{b^2-4ac}\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)^{3/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*Sqrt[d + e
*x]) - (8*c^2*ArcTanh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*
```

```
x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 -
4*a*c]*(-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[-2*c*f + (b - Sqrt[b
^2 - 4*a*c])*g]) + (8*c^2*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]
*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/
(Sqrt[b^2 - 4*a*c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[-2*c*f +
 (b + Sqrt[b^2 - 4*a*c])*g])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.34, size = 47351, normalized size = 110.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)(ex + d)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f + g\,x}\,{\left(d + e\,x\right)}^{3/2}\,\left(c\,x^2 + b\,x + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^{\frac{3}{2}}\sqrt{f + gx}\,\left(a + bx + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.854 $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=532

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(2cd-be)\left(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3\right)-2g\left(-2ce(bd-ae)\right.\right.}{128c^{7/2}e^5}$$

```
[Out] 1/24*g^2*(-5*b*e*g-14*c*d*g+24*c*e*f)*(c*x^2+b*x+a)^(3/2)/c^2/e^2+1/4*g^3*(
e*x+d)*(c*x^2+b*x+a)^(3/2)/c/e^2-1/128*(4*c*e*(-b*e+2*c*d)*(16*c^2*e^2*f^3+
5*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-2*b*d*g+6*b*e*f))-2*(4*c^2*d^2-1/2*b^2*e^2-2
*c*e*(-a*e+b*d))*g*(5*b^2*e^2*g^2-4*c*e*g*(a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d
^2*g^2-3*d*e*f*g+3*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(7/2)/e^5+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^5+1/6
4*(5*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-2*b*d*g+6*b*e*f)+
16*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(5*b^2*e^2*g^2-4*c*e*g*(
a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x^2+b*x+
a)^(1/2)/c^3/e^4
```

Rubi [A] time = 1.70, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bd\right.}{64c^3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g +
 a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e
^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*e^4) + (g^2*(24*c*e*f - 14*
c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(24*c^2*e^2) + (g^3*(d + e*x)*(a
+ b*x + c*x^2)^(3/2))/(4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5
*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2
*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g
+ a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*e^5) + (Sqrt[c*d^2 - b*d*e
+ a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}e\left(8ce^2f^3-d(3bd+2ae)g^3\right)-eg\left(e(4bd+ae)g^2-3c\left(\right.\right.}{d+ex}}{4ce^3}$$

$$= \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \int$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\right.\right.}{}$$

Mathematica [A] time = 1.01, size = 559, normalized size = 1.05

$$\frac{-\frac{24e^2g(bg-2cf)(ef-dg)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}} - \frac{48eg\left(b^2-4ac\right)(ef-dg)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{e^3g\left(3\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (384*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)] + (96*e*g*(e*f - d*g)^2*(b + 2*c*x
)*Sqrt[a + x*(b + c*x)])/c + (128*e^2*g^2*(e*f - d*g)*(a + x*(b + c*x))^(3/
2))/c + (96*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(3/2))/c - (48*(b^2 - 4*a*c
)*e*g*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])]
/c^(3/2) - (24*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))/c^(5/2) + (e^3*g*(80*c^(3/2)*g*(2*c*f - b*g)*(a + x*(b + c*
x))^(3/2) + 3*(16*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])))/c^(7/2) + (192*(e*f - d*g)^3*((-2*c*d + b*e)*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*e))/(384*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.04, size = 3941, normalized size = 7.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] -1/e^4*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^
3*d^3+1/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*f^3+3/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*d^2*f*g^2-1/16*g^3/e/c^2*a*(c*x^2+b*x+a)^(1/2)*b+1/8*g^3/e^2*b^2/c^2*(c*x
^2+b*x+a)^(1/2)*d-3/8*g^2/e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*f-3/8*g/e*f^2/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-3/2/e^2*ln((1/2*(b*e-2*c
*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/c^(1/2)*b*d*f^2*g-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))*a*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*
d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+
d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d
*f^2*g+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^3*f*g^2-3/e^3/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/
e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^2*f^2*g-3/e^5/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^4*f*g^2+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))*c*d^3*f^2*g-3/4*g^2/e^2*d*f/c*(c*x^2+b*x+a)^(1/2)*b-3/2*g
^2/e^2*d*f/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/8*g^2/e^
2*d*f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-3/4*g^2/e*b/c
*x*(c*x^2+b*x+a)^(1/2)*f+1/4*g^3/e^2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a*d-1/2/e^4*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/
e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*g^3*
d^3-3/e^4*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3*f*g^2+3/e^3*ln((1/2*
(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*f^2*g+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))*a*g^3*d^3-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e
))*b*d^4*g^3+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d*f^3+1/e^
6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)
/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^5*g^3-1/e^3/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
```

```
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2*f^3+3/16*g^2/e*b^3/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-3/2*g^2/e^2*d*f*x*(c*x^2+b*x+a)^(1/2)-1/
8*g^3/e/c*a*x*(c*x^2+b*x+a)^(1/2)-3/e^2*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f^2*g+1/2/e*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*
c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/c^(1/2)*b*f^3+1/e^5*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^4*g^3-1/e
^2*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*f^3-1/e/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x+d/e))*a*f^3+5/32*g^3/e*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)+3/16*
g^3/e*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/4*g^3/e^3
*d^2/c*(c*x^2+b*x+a)^(1/2)*b+1/2*g^3/e^3*d^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a-1/8*g^3/e^3*d^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*b^2+3/4*g/e*f^2/c*(c*x^2+b*x+a)^(1/2)*b+3/2*g/e*f^2/c^(1/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/16*g^3/e^2*b^3/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-3/4*g^2/e*b/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f+1/4*g^3/e^2*b/c*x*(c*x^2+b*x+a)^(1/2)*
d+3/2/e^3*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d^2*f*g^2+3/2*g/e*f^2*
x*(c*x^2+b*x+a)^(1/2)+1/2*g^3/e^3*d^2*x*(c*x^2+b*x+a)^(1/2)+1/4*g^3/e*x*(c*
x^2+b*x+a)^(3/2)/c-5/24*g^3/e*b/c^2*(c*x^2+b*x+a)^(3/2)+5/64*g^3/e*b^3/c^3*
(c*x^2+b*x+a)^(1/2)-5/128*g^3/e*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))-1/8*g^3/e/c^(3/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))-1/3*g^3/e^2*(c*x^2+b*x+a)^(3/2)/c*d+g^2/e*(c*x^2+b*x+a)^(3/2)/c*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^3\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.855 $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt{a+bx+cx^2}\left(b^2e^2g^2-2cegx(-beg-2cdg+4cef)-2bceg(2ef-dg)-8c^2(ef-dg)^2\right)}{8c^2e^3}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{}$$

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/c/e+1/16*((8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*
g*(-b*e*g-2*c*d*g+4*c*e*f)-4*c*e*(-b*e+2*c*d)*(-b*d*g^2+2*c*e*f^2))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^4+(-d*g+e*f)^2*arctan
h(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4-1/8*(b^2*e^2*g^2-8*c^2*(-d*g+e*f)^2-2*b*
c*e*g*(-d*g+2*e*f)-2*c*e*g*(-b*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(1/2)/
c^2/e^3
```

Rubi [A] time = 0.71, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right.}{16c^{5/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] -((b^2*e^2*g^2 - 8*c^2*(e*f - d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*g*(4
*c*e*f - 2*c*d*g - b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*e^3) + (g^2*(a +
 b*x + c*x^2)^(3/2))/(3*c*e) + (((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*
g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*e^4) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} + \frac{\int \frac{\left(\frac{3}{2}e\left(2cef^2-bdg^2\right)+\frac{3}{2}eg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{3ce^2}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+}}{8c^2e^3}$$

Mathematica [A] time = 0.41, size = 372, normalized size = 1.14

$$-\frac{6eg\left(b^2-4ac\right)(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} - \frac{3e^2g(bg-2cf)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}} + \frac{24(ef-dg)^2\Big(2\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (48*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)] + (12*e*g*(e*f - d*g)*(b + 2*c*x)*S
qrt[a + x*(b + c*x)])/c + (16*e^2*g^2*(a + x*(b + c*x))^(3/2))/c - (6*(b^2
- 4*a*c)*e*g*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/c^(3/2) - (3*e^2*g*(-2*c*f + b*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(
b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])]))/c^(5/2) + (24*(e*f - d*g)^2*((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*e))/(48*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 2602, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/c/e+1/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2*g^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x+d/e))*c*d^3*f*g+1/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*f^2-1/e^2*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e
)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d*f*g
+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d*f*g-2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^2*f*g-2/e^2*((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f*g+1/2/e*ln((1/2*(b*e-2*c*d)/e+
(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/c^(1/2)*b*f^2-1/e^4*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x
+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3
*g^2-1/e^2*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*f^2-1/e/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*f^2-1/2*g^2/e^2*d*(c*x^2+b*x+a)^(1/2)*x-1/
```

```
4*g^2/e*b/c*(c*x^2+b*x+a)^(1/2)*x-1/4*g^2/e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*a-1/4*g^2/e^2*d/c*(c*x^2+b*x+a)^(1/2)*b-1/2*g^2/e^2*
d/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8*g^2/e^2*d/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2/e^3*ln((1/2*(b*e-2*c
*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/c^(1/2)*b*d^2*g^2+2/e^3*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c
^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c
^(1/2)*d^2*f*g-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d^2*g^2+
1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^3*g^2+1/e^2/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d*f^2-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))*c*d^4*g^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e
))*c*d^2*f^2+1/2*g/e*f/c*(c*x^2+b*x+a)^(1/2)*b+g/e*f/c^(1/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/4*g/e*f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*b^2+g/e*f*(c*x^2+b*x+a)^(1/2)*x-1/8*g^2/e*b^2/c^2*(c*x^2
+b*x+a)^(1/2)+1/16*g^2/e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{2} \sqrt{a + b x + c x^{2}}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.856 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=219

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}}{e^3}$$

```
[Out] -1/8*(b^2*e^2*g+8*c^2*d*(-d*g+e*f)-4*c*e*(a*e*g-b*d*g+b*e*f))*arctanh(1/2*(
2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^3+(-d*g+e*f)*arctanh(1/2*(b
*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*
e^2-b*d*e+c*d^2)^(1/2)/e^3+1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(c*x^2+b*x
+a)^(1/2)/c/e^2
```

Rubi [A] time = 0.32, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*e^3) + (Sqrt[c*
d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)f+4acdeg-bd(4cd-be)g)+\frac{1}{2}}{(d+ex)\sqrt{a}}}{4c}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)\right)\int \frac{}{(d+}}{e^3}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)(ef-dg)\right)\mathrm{Sul}}{}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef\right.}{8c}$$

Mathematica [A] time = 0.35, size = 216, normalized size = 0.99

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4ce(aeg-bdg+bef)-b^2e^2g+8c^2d(dg-ef)\right)+2\sqrt{c}\left(4c(dg-ef)\sqrt{e(ae-bd)+cd^2}\tan\right.}{8c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((-(b^2*e^2*g) + 8*c^2*d*(-(e*f) + d*g) + 4*c*e*(b*e*f - b*d*g + a*e*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a
+ x*(b + c*x)]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x)) + 4*c*Sqrt[c*d^2 + e*(
-(b*d) + a*e)]*(-(e*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2
*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1559, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*g*(c*x^2+b*x+a)^(1/2)*x+1/4/e*g/c*(c*x^2+b*x+a)^(1/2)*b+1/2/e*g/c^(1/
2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8/e*g/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e
)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*g+1/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f-1/2/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e
)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/c^(1/2)*b*d*g+1/2/e*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c
+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*f+1/e^3*ln
(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*g-1/e^2*ln(((x+d/e)*c+1/2*(b*e-2
*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))*c^(1/2)*d*f+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d*
g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*
d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(
x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*f-1/e^3/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^2*g+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x+d/e))*b*d*f+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^3*g
-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b
*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*
(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.857 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex} dx$

Optimal. Leaf size=152

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

```
[Out] -1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/c^
(1/2)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+(c*x^2+b*x+a)^(1/2)/e
```

Rubi [A] time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\sqrt{a+bx+cx}}}{2e^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Sub}}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2} + \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2}{2\sqrt{cd^2-bc}}\right.}{e^2}$$

Mathematica [A] time = 0.15, size = 145, normalized size = 0.95

$$\frac{-2\sqrt{e(ae-bd)+cd^2}\,\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) + \frac{(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + 2e\sqrt{a+x(b+cx)}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] (2*e*Sqrt[a + x*(b + c*x)] + ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])])/Sqrt[c] - 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])])/(2*e^2)
```

fricas [A] time = 5.70, size = 992, normalized size = 6.53

$$\left[\frac{4\sqrt{cx^2+bx+a}\,ce - (2\,cd-be)\sqrt{c}\,\log\left(-8\,c^2x^2 - 8\,bcx - b^2 - 4\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c} - 4\,ac\right) + 2\sqrt{cd^2 -}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
- (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2),
 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2
 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2
*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
```

```
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2), 1/4*(4*
sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sq
rt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*
e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b
*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c*e^2)
, 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arcta
n(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)
*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/(c*e
^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 715, normalized size = 4.70

$$-\frac{a\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\;e}+\frac{bd\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+2\cdots}{\cdots}\right)}{\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2/e
*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/e^2*ln(((x+d/e)*c+1/2*(b*e-2*
c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(1/2)*d-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/
e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a+1/e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d
^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(x+d/e))*c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                                 +c*d^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.858 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)} \, dx$

Optimal. Leaf size=228

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\sqrt{c}\ \mathrm{t}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/e/g+arctanh(1/2*
(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(
a*e^2-b*d*e+c*d^2)^(1/2)/e/(-d*g+e*f)-arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x
)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/
g/(-d*g+e*f)
```

Rubi [A] time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {895, 724, 206, 843, 621}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\sqrt{c}\ \mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(e*g) + (S
qrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (Sqrt[c*f
^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
 b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)}$$

$$= \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{eg} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)}{\sqrt{a+bx+cx^2}}\right.}{e(ef-dg)}$$

$$= \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right.}{eg}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg} + \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} - \frac{\sqrt{cf}}{}$$

Mathematica [A] time = 0.34, size = 218, normalized size = 0.96

$$\frac{g\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+bd-bex+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{ae^2-bde+cd^2}}\right) + \sqrt{c}\,(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - e\sqrt{ag^2-bfg+cf^2}}{eg(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
 + Sqrt[c*d^2 - b*d*e + a*e^2]*g*ArcTanh[(b*d - 2*a*e + 2*c*d*x - b*e*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + x*(b + c*x)])] - e*Sqrt[c*f^2 - b*f*g
 + a*g^2]*ArcTanh[(b*f - 2*a*g + 2*c*f*x - b*g*x)/(2*Sqrt[c*f^2 - b*f*g + a
*g^2]*Sqrt[a + x*(b + c*x)])])/(e*g*(e*f - d*g))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.03, size = 1529, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)+1/2/(d*g-e*f)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)*b-1/(d*g-e*f)/g*l
n((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f-1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/(x+f/g))*a+1/(d*g-e*f)/g/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((
2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(
x+f/g))*b*f-1/(d*g-e*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*
f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f
^2-1/(d*g-e*f)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)-1/2/(d*g-e*f)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/(d*g-e*f)/
e*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d+1/(d*g-e*f)/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/(x+d/e))*a-1/(d*g-e*f)/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x+d/e))*b*d+1/(d*g-e*f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c
*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*
c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\left(f + gx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

3.859 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\,\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2c}{}$$

```
[Out] -1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+
e*f)^2/c^(1/2)+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/g/(-d*g+e*f)^2/c^(1/2)-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))*c^(1/2)/g/(-d*g+e*f)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2
-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e*
f)^2+1/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c
*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(1/2)-e*a
rctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^2+(c*x^2+b*x+a)^(1/2)/(-d*
g+e*f)/(g*x+f)
```

Rubi [A] time = 0.70, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {960, 734, 843, 621, 206, 724, 732}

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\,\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (e*
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d
- 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g
)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g*(e*f - d*g)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}dx = \int \left(\frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^2(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^2} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)}\right)dx$$

$$= \frac{e^2\int \frac{\sqrt{a+bx+cx^2}}{d+ex}dx}{(ef-dg)^2} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}dx}{ef-dg}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}dx}{2(ef-dg)^2} + \frac{e\int \frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}dx}{2(ef-dg)^2} - \frac{\int \frac{b+2cx}{(f+gx)\sqrt{a+bx}}}{2(ef-d}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}dx}{2(ef-dg)^2} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}}{(ef-dg)^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} - \frac{\left(2\left(cd^2-bde+a\right.\right.}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2} + \frac{e(2cf-bg)\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{}}\right.}{2\sqrt{c}\,g(ef-dg)^2}$$

Mathematica [A] time = 0.52, size = 222, normalized size = 0.45

$$\frac{2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) - \frac{(2aeg-b(dg+ef)+2cdf)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}} + \frac{2\sqrt{a+x(}}{}}{2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + 2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*Sqrt[a + x*(b + c*x)])] - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*Arc
Tanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqr
t[a + x*(b + c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)])/(2*(e*f - d*g)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 3162, normalized size = 6.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x)
```

```
[Out] -g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g
+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*((x+f/g)^2*
c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-1/(d*g-e*f)/(a*g^2
-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2)*c*f-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c
^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c
^(1/2)*f*b+1/g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/
g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))*c^(3/2)*f^2-1/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g
+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g
^2)^(1/2))/(x+f/g))*a*b+1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))/(x+f/g))*a*c*f+1/2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2
*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-
b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2*f-3/2/g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)
/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c
*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g
)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*f^2*c+1/g^2/(d*g-e*f)/(a*g^2
-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(
a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c^2*f^3+g/(d*g-e*
f)*c/(a*g^2-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)*x+g/(d*g-e*f)*c^(1/2)/(a*g^2-b*f*g+c*f^2)*ln(((x+f/g)*c+1/2
*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))*a-1/(d*g-e*f)^2*e*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)-1/2/(d*g-e*f)^2*e*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)
/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))
/c^(1/2)*b+1/(d*g-e*f)^2*e/g*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+((x+f
/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f+1/(
d*g-e*f)^2*e/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a
*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a-1/(d*g-e*f)^2*e/
g/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+
c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/
g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*f+1/(d*g-e*f)^2*e/g^2/((a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/
g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f^2+1/(d*g-e*f)^2*e*((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2/(d*g-e*f)^2*e*ln(((
x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/(d*g-e*f)^2*ln(((x+d/e)*c+1/2*(b*e-
2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))*c^(1/2)*d-1/(d*g-e*f)^2*e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*
e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d
/e))*a+1/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e
)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d-1/(d*g
-e*f)^2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^
2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**2), x)
```

3.860 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3} \, dx$

Optimal. Leaf size=673

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bf\ldots}}{}$$

```
[Out] 1/8*(-4*a*c+b^2)*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*e*(-
b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/
c^(1/2)+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/g/(-d*g+e*f)^3/c^(1/2)-e*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))*c^(1/2)/g/(-d*g+e*f)^2+e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e
*f)^3+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*
g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(1/2
)-e^2*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^3+e*(c*x^2+b*x+a)^(
1/2)/(-d*g+e*f)^2/(g*x+f)-1/4*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1
/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2
```

Rubi [A] time = 0.86, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {960, 734, 843, 621, 206, 724, 720, 732}

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bf\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] (e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2*
c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)
*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3) - (Sq
rt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d
*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^3
+ ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g
^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^2*Sqrt[
c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2
])])/(g*(e*f - d*g)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\sqrt{a+bx+cx^2}}{(ef-dg)^3(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^3} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\sqrt{a+bx+cx}}{(ef-dg)^3(f+g}\right.$$

$$= \frac{e^3\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\,dx}{ef-dg}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e^2\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}}{2(ef-dg)^3}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{(e(2cd-be))\int \frac{1}{\sqrt{a+b}}}{2(ef-dg)^3}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{\left(b^2-4ac\right)g\tanh^{-1}(}{8(ef-dg)\left(c\right.}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e(2cd-be)\tanh^{-1}\left(\right.}{2\sqrt{c}\,(ef-}$$

Mathematica [A] time = 1.37, size = 609, normalized size = 0.90

$$\frac{\frac{g\left(b^2-4ac\right)(ef-dg)^2\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}} + 8e\sqrt{e(ae-bd)+cd^2}\,\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) + \frac{2g\sqrt{a+x(b+cx)}}{(f+g}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*e*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (2*g*(e*f - d*g)^2*(-(
b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f)
+ a*g))*(f + g*x)^2) + (4*e*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + x*(b + c*x)])])/Sqrt[c] + 8*e*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])] + ((b^2 - 4*a*c)*g*(e*f - d*g)^2*ArcTanh[(-2*a*g + 2*c*f*
x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])
/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (4*e*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (4*e^2*((2*c*f - b*g)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(8*(e*f - d*g)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 4.64, size = 1844, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")

```
[Out] -1/4*(b^2*d^2*g^3 - 4*a*c*d^2*g^3 - 8*c^2*d*f^3*e + 12*b*c*d*f^2*g*e - 6*b^
2*d*f*g^2*e + 4*a*b*d*g^3*e + 4*b*c*f^3*e^2 - 3*b^2*f^2*g*e^2 - 12*a*c*f^2*
g*e^2 + 12*a*b*f*g^2*e^2 - 8*a^2*g^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c*d^3*f^2*g^3 -
b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f^2*g^3*e - 3*a*d^2*f
*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^
3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*(c*d^2*e
 - b*d*e^2 + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c
)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^
2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*c^2*d*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*
d*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^4 + 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a*c*d*g^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b*c*f^2*g^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^3*e + 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a*b*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)
*d*f^3*g - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^3 - 4*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^3 + 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^(5/2)*f^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*
f^3*g*e + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^2*e + 1
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^2*e - 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^3*e - 8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^2*sqrt(c)*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
^2*d*f^3*g - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^2 - 8*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^2 - (sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^3*d*f*g^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^
3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^4 + 4*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*c*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*
f^4*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g*e - 16*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^3*f^2*g^2*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^2*
e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^3*e - 28*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*c*f*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^2*b*g^4*e + 2*b^2*c^(3/2)*d*f^3*g - b^3*sqrt(c)*d*f^2*g^2 - 4*a*b*c^(3/2
)*d*f^2*g^2 + a*b^2*sqrt(c)*d*f*g^3 + 4*a^2*c^(3/2)*d*f*g^3 + 2*b^2*c^(3/2)
*f^4*e - 3*b^3*sqrt(c)*f^3*g*e - 8*a*b*c^(3/2)*f^3*g*e + 15*a*b^2*sqrt(c)*f
^2*g^2*e + 4*a^2*c^(3/2)*f^2*g^2*e - 20*a^2*b*sqrt(c)*f*g^3*e + 8*a^3*sqrt(
c)*g^4*e)/((c*d^2*f^2*g^3 - b*d^2*f*g^4 + a*d^2*g^5 - 2*c*d*f^3*g^2*e + 2*b
*d*f^2*g^3*e - 2*a*d*f*g^4*e + c*f^4*g*e^2 - b*f^3*g^2*e^2 + a*f^2*g^3*e^2)
*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*sqrt(c)*f + b*f - a*g)^2)
```

maple [B] time = 0.02, size = 6714, normalized size = 9.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}}{(ex+d)(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\left(f+gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**3), x)
```

3.861 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

Optimal. Leaf size=933

$$\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}\,g(ef-dg)^4}-\frac{\sqrt{cf^2-bgf+ag^2}\,\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\cdots}\right.}{g(ef-\cdots}$$

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^3+1/16*(
-4*a*c+b^2)*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*
f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)+
1/8*(-4*a*c+b^2)*e*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*
f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*
e^2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e
*f)^4/c^(1/2)+1/2*e^3*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/g/(-d*g+e*f)^4/c^(1/2)-e^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))*c^(1/2)/g/(-d*g+e*f)^3+e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d
)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/
2)/(-d*g+e*f)^4+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)
/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^3/(a*g^2-b*f*g
+c*f^2)^(1/2)-e^3*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^4+e^2*(
c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^3/(g*x+f)-1/8*g*(-b*g+2*c*f)*(b*f-2*a*g+(-b*g
+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2-1
/4*e*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b
*f*g+c*f^2)/(g*x+f)^2
```

Rubi [A] time = 1.22, antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {960, 734, 843, 621, 206, 724, 730, 720, 732}

$$\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}\,g(ef-dg)^4}-\frac{\sqrt{cf^2-bgf+ag^2}\,\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\cdots}\right.}{g(ef-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] (e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b
*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt
[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) +
(g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) - (Sqrt[c]
*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)
^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^4 +
 ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2
 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g)*ArcTanh[(b*f -
 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^3*Sqrt[c*f^2 -
```

```
b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*
g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \left(\frac{e^4\sqrt{a+bx+cx^2}}{(ef-dg)^4(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^4} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^3} - \frac{e^2g\sqrt{a+bx}}{(ef-dg)^3(f}\right.$$

$$= \frac{e^4\int\frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^4} - \frac{\left(e^3g\right)\int\frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^4} - \frac{\left(e^2g\right)\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{(ef-dg)^3} - \frac{(eg)\int\frac{\sqrt{a+bx+cx}}{(f+gx)^3}}{(ef-dg)^2}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{g^2\left(a+\right.}{3(ef-dg)\left(cf^2\right.}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf}{4(ef}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf}{4(ef}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf}{4(ef}$$

Mathematica [A] time = 4.16, size = 858, normalized size = 0.92

$$\frac{24\left((2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\sqrt{cf^2+g(ag-bf)}\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(ag-bf)}\sqrt{a+x(b+cx)}}\right)\right)e^3}{\sqrt{c}g} + 24\left(\frac{(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] ((48*e^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (12*e*g*(e*f - d*g)
^2*(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-
```

```
(b*f) + a*g))*(f + g*x)^2) - (16*g^2*(-(e*f) + d*g)^3*(a + x*(b + c*x))^(3/
2))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^3) + 24*e^2*(((-2*c*d + b*e)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]) + (6*(b^2 - 4*a*c)*e*g*(e*f -
d*g)^2*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) +
 a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (3*g*(2*
c*f - b*g)*(e*f - d*g)^3*((2*Sqrt[a + x*(b + c*x)]*(-2*a*g + 2*c*f*x + b*(f
 - g*x)))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^2) + ((-b^2 + 4*a*c)*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(c*f^2 + g*(-(b*f) +
a*g)) - (24*e^2*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^2 + g
*(-(b*f) + a*g)]))/g + (24*e^3*((2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])]))/(Sqrt[c]*g))/(48*(e*f - d*g)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 11995, normalized size = 12.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{{\left(f + g\,x\right)}^4\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\left(f+gx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**4), x)
```

3.862 $\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=1098

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+\frac{(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}g^2}{60c^2e^2}+\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)\right.}{}$$

```
[Out] 1/192*(7*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-3*b*d*g+9*b*e
*f)+24*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(7*b^2*e^2*g^2-4*c*e
*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x^2+
b*x+a)^(3/2)/c^3/e^4+1/60*g^2*(-7*b*e*g-22*c*d*g+36*c*e*f)*(c*x^2+b*x+a)^(5
/2)/c^2/e^2+1/6*g^3*(e*x+d)*(c*x^2+b*x+a)^(5/2)/c/e^2+1/3072*(4*c*e*(-b*e+2
*c*d)*(8*c*e*(-2*a*e+b*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*
b*d*g+9*b*e*f))-d*(-4*a*c*e-3*b^2*e+8*b*c*d)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*
g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))-2*(4*c^2*d^2-1/2*
b^2*e^2-2*c*e*(-a*e+b*d))*(8*c*e*(-b*e+2*c*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3
-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*
c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d
*e*f*g+3*e^2*f^2))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
9/2)/e^7+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*
e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^7-1/1536*(21*b
^5*e^5*g^3-1536*c^5*d^2*(-d*g+e*f)^3+384*c^4*e*(-4*a*e+5*b*d)*(-d*g+e*f)^3-
12*b^3*c*e^4*g^2*(8*a*e*g-3*b*d*g+9*b*e*f)+24*b*c^2*e^3*g*(2*a^2*e^2*g^2+6*
a*b*e*g*(-d*g+3*e*f)+3*b^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))-96*b*c^3*e^2*(2*b
*(-d*g+e*f)^3+3*a*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))+2*c*e*(8*c*e*(-b*e+2*c
*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c
^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b
*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))*x)*(c*x^2+b*x+a)^(1/2)
/c^4/e^6
```

Rubi [A] time = 3.86, antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+\frac{(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}g^2}{60c^2e^2}+\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)
*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^
3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g
 + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e
*f*g + d^2*g^2))) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*
g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (
3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a
*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x + c*x^2]
)/(1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(
9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)
 + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3
*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3*e^4)
+ (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(60*c^2*e^2
) + (g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((4*c*e*(2*c*d - b*
e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*
f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 -
```

```
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e
)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g))
- 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2
)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/
e^7
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(\frac{1}{2}e\left(12ce^2f^3-d(5bd+2ae)g^3\right)-eg\left(e(6bd+ae)\ldots\right.\right.}{d+e\ldots}}{6ce\ldots}$$

$$= \frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \ldots$$

$$= \frac{\left(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg\left(3e^2f^2-\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2\left(\ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2\left(\ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2\left(\ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2\left(\ldots\right.\right.\right.}{\ldots}$$

Mathematica [A] time = 2.38, size = 743, normalized size = 0.68

$$\frac{-\frac{60e^2g(bg-2cf)(ef-dg)\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{7/2}} + \frac{360eg\left(b^2-4ac\right)(ef-dg)^2\left(\left(b^2-\ldots\right.\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (5120*(e*f - d*g)^3*(a + x*(b + c*x))^(3/2) + (1920*e*g*(e*f - d*g)^2*(b +
2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3072*e^2*g^2*(e*f - d*g)*(a + x*(b + c
*x))^(5/2))/c + (2560*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(5/2))/c + (360*(
b^2 - 4*a*c)*e*g*(e*f - d*g)^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/
c^(5/2) - (60*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[
a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)
^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(7/2) + (e^3*
g*(1792*g*(2*c*f - b*g)*(a + x*(b + c*x))^(5/2) + 5*(24*c^2*f^2 + 7*b^2*g^2
 - 4*c*g*(6*b*f + a*g))*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b
^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c^2 + (9
60*(e*f - d*g)^3*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*
a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*Sqrt[c]*(
e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*e*(-5*b*d
```

```
+ 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
c*x)])])))/(c^(3/2)*e^3))/(15360*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 10058, normalized size = 9.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{3} \left(a + bx + cx^{2}\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.863 $\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=662

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3e^2\left(-a^2e^2g(2ef-dg)-2abe(ef-dg)^2+b^2d(ef-dg)^2\right)+16bc^2e^3\left(3a^2e^2g^2+3abe\right.\right.}{}$$

```
[Out] -1/48*(3*b^2*e^2*g^2-16*c^2*(-d*g+e*f)^2-6*b*c*e*g*(-d*g+2*e*f)-6*c*e*g*(-b
*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(3/2)/c^2/e^3+1/5*g^2*(c*x^2+b*x+a)^
(5/2)/c/e-1/256*(3*b^5*e^5*g^2+256*c^5*d^3*(-d*g+e*f)^2-384*c^4*d*e*(-a*e+b
*d)*(-d*g+e*f)^2-6*b^3*c*e^4*g*(4*a*e*g-b*d*g+2*b*e*f)+16*b*c^2*e^3*(3*a^2*
e^2*g^2+b^2*(-d*g+e*f)^2+3*a*b*e*g*(-d*g+2*e*f))+96*c^3*e^2*(b^2*d*(-d*g+e*
f)^2-2*a*b*e*(-d*g+e*f)^2-a^2*e^2*g*(-d*g+2*e*f)))*arctanh(1/2*(2*c*x+b)/c^
(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/e^6+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)
^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/e^6+1/128*(3*b^4*e^4*g^2+128*c^4*d^2*(-d*g+e*f)^2-32*c^3*e*(-
4*a*e+5*b*d)*(-d*g+e*f)^2-6*b^2*c*e^3*g*(2*a*e*g-b*d*g+2*b*e*f)+8*b*c^2*e^2
*(2*b*(-d*g+e*f)^2+3*a*e*g*(-d*g+2*e*f))+2*c*e*((16*c^2*d^2-3*b^2*e^2-4*c*e
*(-3*a*e+2*b*d))*g*(-b*e*g-2*c*d*g+4*c*e*f)-8*c*e*(-b*e+2*c*d)*(-b*d*g^2+2*
c*e*f^2))*x)*(c*x^2+b*x+a)^(1/2)/c^3/e^5
```

Rubi [A] time = 1.55, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16bc^2e^3\left(3a^2e^2g^2+3abeg(2ef-dg)+b^2(ef-dg)^2\right)+96c^3e^2\left(-a^2e^2g(2ef-dg)-2abe\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f
 - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e
*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c
*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*
c*e*f^2 - b*d*g^2))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*e^5) - ((3*b^2*e^2*g
^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*
c*d*g - b*e*g)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*e^3) + (g^2*(a + b*x + c
*x^2)^(5/2))/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^
4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g)
 + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g
)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2
*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*
c^(7/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^2*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/e^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce} + \frac{\int \frac{\left(\frac{5}{2}e\left(2cef^2-bdg^2\right)+\frac{5}{2}eg(4cef-2cdg-beg)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex}dx}{5ce^2}$$

$$= -\frac{\left(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x\right)\left(\right.}{48c^2e^3}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

Mathematica [A] time = 1.25, size = 536, normalized size = 0.81

$$\frac{\frac{90eg\left(b^2-4ac\right)(ef-dg)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}} + \frac{15e^2g(2cf-bg)\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{\right.}{c^{5/2}}\right.}{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (1280*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2) + (480*e*g*(e*f - d*g)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (768*e^2*g^2*(a + x*(b + c*x))^(5/2))/c +
(90*(b^2 - 4*a*c)*e*g*(e*f - d*g)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2) + (15*e^2*g*(2*c*f - b*g)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/
2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2
)))/c + (240*(e*f - d*g)^2*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-
2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2
*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*
e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[
(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])]))))/(c^(3/2)*e^3))/(3840*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 6860, normalized size = 10.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{2} \left(a + bx + cx^{2}\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.864 $\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=441

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3de}{128c^{5/2}e^5}$$

```
[Out] 1/24*(6*c*e*g*x+3*b*e*g-8*c*d*g+8*c*e*f)*(c*x^2+b*x+a)^(3/2)/c/e^2+1/128*(3
*b^4*e^4*g-128*c^4*d^3*(-d*g+e*f)+192*c^3*d*e*(-a*e+b*d)*(-d*g+e*f)-8*b^2*c
*e^3*(3*a*e*g-b*d*g+b*e*f)+48*c^2*e^2*(a^2*e^2*g-b^2*d*(-d*g+e*f)+2*a*b*e*(
-d*g+e*f)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^5+
(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5-1/64*(3*b^3*e^3*g-64*c^
3*d^2*(-d*g+e*f)+16*c^2*e*(-4*a*e+5*b*d)*(-d*g+e*f)-4*b*c*e^2*(3*a*e*g-2*b*
d*g+2*b*e*f)+2*c*e*(3*b^2*e^2*g+16*c^2*d*(-d*g+e*f)-4*c*e*(3*a*e*g-2*b*d*g+
2*b*e*f))*x)*(c*x^2+b*x+a)^(1/2)/c^2/e^4
```

Rubi [A] time = 0.85, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3de}{128c^{5/2}e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*(e*f - d
*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 16*c^2
*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a + b*x + c*x
^2])/(64*c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c
*x^2)^(3/2))/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c^3
*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c
^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*e^5) + ((c*d^2 - b*
d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2} - \int \frac{\left(\frac{1}{2}\left(8ce(bd-2ae)f+4acdeg-2b\right.\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2b\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2b\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2b\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2b\right.}{}$$

Mathematica [A] time = 1.13, size = 420, normalized size = 0.95

$$\frac{3\left(\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2d(dg-ef)\right)-8b^2ce^3(3aeg-bdg+bef)-192c^3de(bd-ae)(dg-ef)+3b^4e^4g+128c^4d^3(dg-ef)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*(3*b*e*g + c*(8*e*f - 8*d*g + 6*e*g*x)) + (3*(2*Sq
rt[c]*e*Sqrt[a + x*(b + c*x)]*(-3*b^3*e^3*g - 32*c^3*d*(e*f - d*g)*(-2*d +
e*x) + 2*b*c*e^2*(6*a*e*g + b*(4*e*f - 4*d*g - 3*e*g*x)) + 8*c^2*e*(2*b*(e*
f - d*g)*(-5*d + e*x) + a*e*(8*e*f - 8*d*g + 3*e*g*x))) + (3*b^4*e^4*g + 12
8*c^4*d^3*(-(e*f) + d*g) - 192*c^3*d*e*(b*d - a*e)*(-(e*f) + d*g) - 8*b^2*c
```

```
*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g
) + b^2*d*(-(e*f) + d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c
*x)])] + 128*c^(5/2)*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*(-(e*f) + d*g)*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])]))/(16*c^(3/2)*e^3))/(24*c*e^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 4188, normalized size = 9.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*f+1
/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a*f+1/
4/e*g*(c*x^2+b*x+a)^(3/2)*x-1/3/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(3/2)*d*g-1/e^4*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/
2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/
2)*d^3*f-1/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*a*d*g-1/16/e/c^(3/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e
)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3*f-5/4/e^2*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d*f+1/4/
e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*f-1
/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e
+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a^2*f+1/8/e*g/c*(c*x^2+b*x+a
)^(3/2)*b+3/8/e*g*(c*x^2+b*x+a)^(1/2)*x*a-3/64/e*g/c^2*(c*x^2+b*x+a)^(1/2)*
b^3+3/8/e*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128/e
*g/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/e^4*((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^3*g+1/e^3*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2*f+1/e^5*
ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^4*g+1/8/e/c*((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f+5/4/e^3*((x+d/e)^2*c+(b
*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d^2*g+1/16/e^2/c^(3/2)
*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3*d*g-3/2/e^4*ln(((x+d/e)*c+1/2*(b*e-2*
c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(1/2)*d^3*b*g-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*
d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
```

```
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b
^2*d^2*f-3/8/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d*f-2/e^3/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2
)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*b*d^2*g-2/e^5/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/(x+d/e))*b*d^4*c*g+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x+d/e))*b*d^3*c*f+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*b*
d*f+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^
2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*c*d^3*g-2/e^3/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/
e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*c*d^2*f-3/4/e^2/c^(1/2)*ln(((x+d/e
)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))*a*b*d*g+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b
*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+
d/e))*b^2*d^3*g+3/2/e^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*a*g-3/
2/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a*f-1/e^5/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x+d/e))*c^2*d^4*f+1/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x+d/e))*c^2*d^5*g-1/2/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*x*c*d*f+3/4/e/c^(1/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e
)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*a*b*f-3/16/e*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+
3/16/e*g/c*(c*x^2+b*x+a)^(1/2)*b*a-3/32/e*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/2
/e^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b*f+3/8/e^3*ln(((x+d/e)*c
+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d^2*g-1/8/e^2/c*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*g+1/2/e^3*((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d^2*g-1/4/e^2*((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*d*g+1/e^2/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e
^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a^2*d*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right) \left(a + bx + cx^{2}\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
[Out] Integral((f + g*x)*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.865 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{16c^{3/2}e^4}$$

```
[Out] 1/3*(c*x^2+b*x+a)^(3/2)/e-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*
e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^4+(a
*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e
+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e
+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^3
```

Rubi [A] time = 0.35, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{16c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
```

```
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} +$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} +$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} -$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} -$$

Mathematica [A] time = 0.41, size = 236, normalized size = 0.94

$$\frac{2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(2ce(16ae-15bd+7bex)+3b^2e^2+4c^2\left(6d^2-3dex+2e^2x^2\right)\right)-24c\left(e(ae-bd)+cd^2\right)^{3/2}\right.}{48c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a + x*(b +
 c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x) + 4*c^2*(6*d^2 - 3*d
```

```
*e*x + 2*e^2*x^2)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d)
+ 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b
+ c*x)])]))/(48*c^(3/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1946, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/4
/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-1/
2/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c
*d+1/8/e/c*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b^2-5/4/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*b*d+3/4/e/c^(1/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b-3/2/e^2*ln(((x+
d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a-1/16/e/c^(3/2)*ln(((x+d/e)*c+1/2*(b*e
-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))*b^3-3/8/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d+1/e*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a+1/e^3*((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2+3/2/e
^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e
)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b-1/e^4*ln(((x+d/e)*c+1/2*(
b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))*c^(3/2)*d^3-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*
d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a
^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2
-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d
)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*b*d-2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*c*d^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(x+d/e))*b^2*d^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c
```

```
*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*
b*d^3*c-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-
2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c^2*d^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                            +c*d^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.866 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=491

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}\,eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\ (}{e^2(ef}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d
*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(3/
2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/g^3/(-d*g+e*f)-1/2*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)/c^(1/2)+1/8*(8*c^2
*e*f^3+b*g^2*(-4*a*e*g+b*d*g+3*b*e*f)-4*c*g*(3*b*e*f^2-a*g*(-d*g+3*e*f)))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/g^3/(-d*g+e*f)/c^(1/2)+
(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(1/2)/e^2/(-d*g+e*f)-1/4*(4*c*e*f^2-g*(-4
*a*e*g-b*d*g+5*b*e*f)-2*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(1/2)/e/g^2/(-d*g+e
*f)
```

Rubi [A] time = 0.84, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {895, 734, 843, 621, 206, 724, 814}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}\,eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\ (}{e^2(ef}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e
*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a + b*x +
c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*e^3*(e*f - d
*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2
 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(8*Sqrt[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
 + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g^3*(e*f - d*g))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{e(ef-dg)} + \frac{(cd^2-bde+ae^2)\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)}{4eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)}{4eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)}{4eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)}{4eg^2(ef-dg)}$$

Mathematica [A] time = 1.07, size = 323, normalized size = 0.66

$$\frac{\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-12ceg(-aeg+bdg+bef)+3b^2e^2g^2+8c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{\sqrt{c}} + 2\left(-4g^3\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+c\ldots}}\right.\right.}{8e^3g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (((3*b^2*e^2*g^2 - 12*c*e*g*(b*e*f + b*d*g - a*e*g) + 8*c^2*(e^2*f^2 + d*e*
f*g + d^2*g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqr
t[c] + (2*(e*g*(e*f - d*g)*Sqrt[a + x*(b + c*x)]*(5*b*e*g + c*(-4*e*f - 4*d
*g + 2*e*g*x)) - 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*g^3*ArcTanh[(-(b*d) + 2
*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c
*x)])] + 4*e^3*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g - 2
*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))
/(e*f - d*g))/(8*e^3*g^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 4226, normalized size = 8.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] -1/4/(d*g-e*f)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*x*b-1/8/(d*g-e*f)/c*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*b^2+1/16/(d*g-e*f)/c^(3/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e
)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*b^3+1/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e
+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a^2+1/4/(d*g-
e*f)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b+
1/8/(d*g-e*f)/c*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)*b^2-1/16/(d*g-e*f)/c^(3/2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+
((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*b^3-1/(d
*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-
b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a^2-1/3/(d*g-e*f)*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/3/(d*g-e*f)*
((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/(d*g-e*
f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b
*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*
(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b^2*d^2+1/(d*g-e*f)/e^4/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c^2*d^4+1/2/(d*g-e*f)/e*((x+d/e
)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d+3/2/(d*g-e
*f)/e*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a+3/8/(d*g-e*f)/e*ln(((x+d
/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d-3/2/(d*g-e*f)/e^2*ln(((x+d/e)*c+1/2*
(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))*c^(1/2)*d^2*b-1/2/(d*g-e*f)/g*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/
g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c*f-3/2/(d*g-e*f)/g*ln(((x+f/g)*c+1/2
*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))*c^(1/2)*f*a-3/8/(d*g-e*f)/g*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g
)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
)/c^(1/2)*b^2*f-1/(d*g-e*f)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*a+1/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*a+2/(d*g-e*f)/g^3/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*l
n(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
)/(x+f/g))*b*f^3*c-2/(d*g-e*f)/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2
*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)
)*a*b*d-2/(d*g-e*f)/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^3*c
-2/(d*g-e*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+
2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a*c*f^2+3/2/(d
*g-e*f)/g^2*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*
f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f^2*b-1/(d*g-e*f)/g^2/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*
f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)
/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2*f^2-1/(d*g-e*f)/g^4/((a*g^2
```

```
-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^
2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c^2*f^4-5/4/(d*g-e*f)/g*((x+f/g)^2*c+(
b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*f+3/4/(d*g-e*f)/c^(1/
2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g
)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*a*b+2/(d*g-e*f)/g/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/(x+f/g))*a*b*f+2/(d*g-e*f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x+d/e))*a*c*d^2+1/(d*g-e*f)/g^2*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f^2-1/(d*g-e*f)/g^3*ln(((x+f/g)*c+1/2*(b*g-2
*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2))*c^(3/2)*f^3+5/4/(d*g-e*f)/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d-3/4/(d*g-e*f)/c^(1/2)*ln(((x+d/e)*c+1/2*(b*
e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))*a*b-1/(d*g-e*f)/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*c*d^2+1/(d*g-e*f)/e^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/
e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))*c^(3/2)*d^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{(d+ex)\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)
```

3.867 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2} dx$

Optimal. Leaf size=787

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce(ef-dg)^2}-\frac{e\sqrt{a+bx+cx^2}\left(-2cg(5bf-4ag)+b^2g^2-2\right.}{8cg^2(ef-dg)^2}$$

```
[Out] (c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2
-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c
^(3/2)/e^2/(-d*g+e*f)^2+1/16*e*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*
g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-
d*g+e*f)^2+(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(-d*g+e*f)^2-e*(a*g^2-b
*f*g+c*f^2)^(3/2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*
(-a*g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+
e*f)/c^(1/2)+3/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2
-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g^3/(-d*
g+e*f)+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c
*x^2+b*x+a)^(1/2)/c/e/(-d*g+e*f)^2+3/4*(-2*c*g*x-3*b*g+4*c*f)*(c*x^2+b*x+a)
^(1/2)/g^2/(-d*g+e*f)-1/8*e*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+5*b*f)-2*c*g*(
-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^2
```

Rubi [A] time = 1.38, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {960, 734, 814, 843, 621, 206, 724, 732}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce(ef-dg)^2}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh}{16c^{3/2}e^2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(
5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2*(e*
f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b
*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f -
 b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2
*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*Sqrt[c*f^2
 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b
*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*
f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \int \left(\frac{e^2\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(d+ex)} - \frac{g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)^2} - \frac{eg\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)}\right)dx$$

$$= \frac{e^2\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^2}\,dx}{ef-dg}$$

$$= \frac{\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2(ef-dg)^2} + \frac{e\int \frac{(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{f+gx}\,d}{2(ef-dg)^2}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cg}{4g^2(ef}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cg}{4g^2(ef}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cg}{4g^2(ef}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cg}{4g^2(ef}$$

Mathematica [A] time = 1.40, size = 357, normalized size = 0.45

$$\frac{-2g^3(f+gx)\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+e\left(2g\sqrt{a+x(b+cx)}(dg-ef)(eg(bf-ag)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (-(Sqrt[c]*(e*f - d*g)^2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(f + g*x)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*(c*d^2 + e*(-(b*d) + a*e)
)^(3/2)*g^3*(f + g*x)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])] + e*(2*g*(-(e*f) + d*g)*Sqr
t[a + x*(b + c*x)]*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x))
- e*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(2*c*f*(2*e*f - 3*d*g) + g*(-(b*e*f) + 3
*b*d*g - 2*a*e*g))*(f + g*x)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*
Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(2*e^2*g^3*(e*f -
d*g)^2*(f + g*x))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 7959, normalized size = 10.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)\left(gx + f\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + b\,x + a\right)^{3/2}}{\left(f + g\,x\right)^2\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}}}{(d + ex)\left(f + gx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2,x)
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)**2), x)
```

3.868 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

Optimal. Leaf size=1066

$$\frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}-\frac{\left(cf^2-bgf+ag^2\right)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(}{2\sqrt{cf^2-bgf+a}}\right.}{g^3(ef-dg)^3}$$

```
[Out] 1/2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)^2+e*(c*x^2+b*x+a)^(3/2)/(-d*g+e*
f)^2/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arc
tanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/(-d*g+e*f)^3+1/16
*e^2*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*g+2*b*f))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-d*g+e*f)^3+(a*e^2-b*d*e+c*
d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^2+b*x+a)^(1/2))/e/(-d*g+e*f)^3-e^2*(a*g^2-b*f*g+c*f^2)^(3/2)*arctanh(
1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2
))/g^3/(-d*g+e*f)^3-3/8*e*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2/c^(1/2)+3/2*(-b*g
+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/g^3/(-d*
g+e*f)-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-
b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)
/(a*g^2-b*f*g+c*f^2)^(1/2)+3/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+
2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2
)^(1/2)/g^3/(-d*g+e*f)^2+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*
(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/(-d*g+e*f)^3+3/4*e*(-2*c*g*x-3*b*g+4*
c*f)*(c*x^2+b*x+a)^(1/2)/g^2/(-d*g+e*f)^2-3/4*(2*c*g*x-b*g+4*c*f)*(c*x^2+b*
x+a)^(1/2)/g^2/(-d*g+e*f)/(g*x+f)-1/8*e^2*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+
5*b*f)-2*c*g*(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^3
```

Rubi [A] time = 1.71, antiderivative size = 1066, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {960, 734, 814, 843, 621, 206, 724, 732, 812}

$$\frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}-\frac{\left(cf^2-bgf+ag^2\right)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(}{2\sqrt{cf^2-bgf+a}}\right.}{g^3(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*Sqrt
[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^
2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(
8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f + g*x)^2
) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*
(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[c]*(2*c*f
 - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f
 - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e
*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)^2)
+ ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)^3) + (
```

```
3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(
e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^
3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(
b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x
+ c*x^2])])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^3(d+ex)} - \frac{g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)^3} - \frac{eg\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\left(a+bx+c\right.}{(ef-dg)^3(f+}\right.$$

$$= \frac{e^3\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^2}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^3}}{ef-dg}$$

$$= \frac{\left(a+bx+cx^2\right)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)} - \frac{e^2\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2(ef-dg)^3} + \frac{e^2\int}{}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef}$$

Mathematica [A] time = 3.27, size = 1036, normalized size = 0.97

$$\frac{1}{4}\left(-\frac{\left((2cf-bg)\left(8c^2f^2-b^2g^2+4cg(3ag-2bf)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(8c\tanh^{-1}\left(\frac{-bf-2cxf+2ag\dots}{2\sqrt{cf^2+g(ag-bf)}\sqrt{a\dots}}\right)\dots\right.\right.}{4c^{3/2}g^3(d\dots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((2*(a + x*(b + c*x))^(3/2))/((e*f - d*g)*(f + g*x)^2) + (4*e*(a + x*(b + c
*x))^(3/2))/((e*f - d*g)^2*(f + g*x)) + (-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e
^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d
+ e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^(3/2)*e*(e*f - d*g)^3) - (3*e*((8*c^2
*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] + 2*Sqrt[c]*(g*(-4*c*f + 3*b*g + 2*c*g*x)*Sqrt[a + x*(b
+ c*x)] + 2*(2*c*f - b*g)*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[(-(b*f) +
2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b +
c*x)])])))/(2*Sqrt[c]*g^3*(e*f - d*g)^2) + (3*(((-2*c*f + b*g)*(a + x*(b +
c*x))^(3/2))/(f + g*x) - (Sqrt[a + x*(b + c*x)]*(b^2*g^2 + 2*c^2*f*(2*f - g
*x) + c*g*(-5*b*f + 2*a*g + b*g*x)))/g^2 + (4*Sqrt[c]*(2*c*f - b*g)*(c*f^2
+ g*(-(b*f) + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ (8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*Sqrt[c*f^2 + g*(-(b*f) + a*g
)]*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a
*g)]*Sqrt[a + x*(b + c*x)])])/(2*g^3)))/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a
*g))) - (e^2*((2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 + 4*c*g*(-2*b*f + 3*a*g))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(g*Sqrt[
a + x*(b + c*x)]*(-(b^2*g^2) + 4*c^2*f*(-2*f + g*x) - 2*c*g*(-5*b*f + 4*a*g
 + b*g*x)) + 8*c*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g -
 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]
)))/(4*c^(3/2)*g^3*(-(e*f) + d*g)^3))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Evaluation time: 0.6Error: Bad Argument Type
```

maple [B] time = 0.02, size = 15927, normalized size = 14.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + b\,x + a\right)^{3/2}}{\left(f + g\,x\right)^3\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}}}{(d + ex)\left(f + gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)**3), x)
```

3.869 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=886

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)\left(cd^2-bed+ae^2\right)^{5/2}}{e^5(ef-dg)}+\frac{\left(cx^2+bx+a\right)^{3/2}\left(cd^2-bed+ae^2\right)}{3e^2(ef-dg)}-\frac{(2cd-be)\left(8c^2d^2-\right.}{}$$

```
[Out] 1/3*(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(3/2)/e^2/(-d*g+e*f)-1/24*(8*c*e*f^2-
g*(-8*a*e*g-3*b*d*g+11*b*e*f)-6*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(3/2)/e/g^2
/(-d*g+e*f)-1/16*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2-b^2*e^2-4*c*e*
(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/
e^5/(-d*g+e*f)+1/128*(128*c^4*e*f^5-320*c^3*e*f^3*g*(-a*g+b*f)-b^3*g^4*(-8*
a*e*g+3*b*d*g+5*b*e*f)+48*c^2*g^2*(5*b^2*e*f^3-10*a*b*e*f^2*g+a^2*g^2*(-d*g
+5*e*f))-8*b*c*g^3*(5*b^2*e*f^2+12*a^2*e*g^2-3*a*b*g*(d*g+5*e*f)))*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/g^5/(-d*g+e*f)+(a*e^2-
b*d*e+c*d^2)^(5/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(5/2)*arct
anh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^
(1/2))/g^5/(-d*g+e*f)+1/8*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*
a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^4/(-d*g+e*f)-1/64*
(64*c^3*e*f^4-16*c^2*e*f^2*g*(-8*a*g+9*b*f)-b^2*g^3*(-8*a*e*g+3*b*d*g+5*b*e
*f)+4*c*g^2*(22*b^2*e*f^2+16*a^2*e*g^2-3*a*b*g*(-d*g+13*e*f))-2*c*g*(16*c^2
*e*f^3+b*g^2*(-8*a*e*g+3*b*d*g+5*b*e*f)-4*c*g*(6*b*e*f^2-a*g*(-3*d*g+7*e*f)
))*x)*(c*x^2+b*x+a)^(1/2)/c/e/g^4/(-d*g+e*f)
```

Rubi [A] time = 1.80, antiderivative size = 886, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {895, 734, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)\left(cd^2-bed+ae^2\right)^{5/2}}{e^5(ef-dg)}+\frac{\left(cx^2+bx+a\right)^{3/2}\left(cd^2-bed+ae^2\right)}{3e^2(ef-dg)}-\frac{(2cd-be)\left(8c^2d^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*e^4*(e*f - d*g)) - ((64*c^3
*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*
e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c
*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 -
 a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(64*c*e*g^4*(e*f - d*g)) +
 ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(e*f - d*g)) - ((
8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*
x + c*x^2)^(3/2))/(24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*
e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g)
 + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*
c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e*g^5*(e*f - d*g)) +
((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^5*(e*f - d*g)) - ((c
*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^5*(e*f - d*g))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}{3e^2(ef-dg)} - \frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-d\right.}{24eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

Mathematica [A] time = 2.54, size = 647, normalized size = 0.73

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(240c^2e^2g^2(ef-dg)\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)-40b^2ce^3g\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (3*(5*b^4*e^4*g^4*(-(e*f) + d*g) - 40*b^2*c*e^3*g^3*(e*f - d*g)*(b*e*f + b*
d*g - 3*a*e*g) + 320*c^3*e*g*(-(b*e^4*f^4) + a*e^4*f^3*g + b*d^4*g^4 - a*d^
3*e*g^4) + 128*c^4*(e^5*f^5 - d^5*g^5) + 240*c^2*e^2*g^2*(e*f - d*g)*(a^2*e
^2*g^2 - 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(-(e*g*(-(e*f)
 + d*g)*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 + 2*b*c*e^2*g^2*(278*a*e*g +
b*(-132*e*f - 132*d*g + 59*e*g*x)) - 16*c^3*(12*d^3*g^3 - 6*d^2*e*g^2*(-2*f
 + g*x) + 2*d*e^2*g*(6*f^2 - 3*f*g*x + 2*g^2*x^2) + e^3*(12*f^3 - 6*f^2*g*x
 + 4*f*g^2*x^2 - 3*g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-56*e*f - 56*d*g + 27*e*g*
x) + b*(54*d^2*g^2 + 2*d*e*g*(27*f - 13*g*x) + e^2*(54*f^2 - 26*f*g*x + 17*
g^2*x^2))))) - 192*c*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*g^5*ArcTanh[(-(b*d) +
 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
 c*x)])] + 192*c*e^5*(c*f^2 + g*(-(b*f) + a*g))^(5/2)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x
)])]))/(384*c^(3/2)*e^5*g^5*(e*f - d*g))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 9052, normalized size = 10.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

3.870 $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=431

$$-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2aeg-bdg+4bef)+\cdots\right.}{16c^{7/2}e^4}$$

```
[Out] -1/16*g*(5*b^3*e^3*g^3-6*b*c*e^2*g^2*(2*a*e*g-b*d*g+4*b*e*f)-16*c^3*(-d^3*g
^3+4*d^2*e*f*g^2-6*d*e^2*f^2*g+4*e^3*f^3)+8*c^2*e*g*(a*e*g*(-d*g+4*e*f)+b*(
d^2*g^2-4*d*e*f*g+6*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(7/2)/e^4+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/2)+1/
24*g^2*(15*b^2*e^2*g^2-4*c*e*g*(4*a*e*g-7*b*d*g+18*b*e*f)+4*c^2*(11*d^2*g^2
-36*d*e*f*g+36*e^2*f^2))*(c*x^2+b*x+a)^(1/2)/c^3/e^3+1/12*g^3*(-5*b*e*g-14*
c*d*g+24*c*e*f)*(e*x+d)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/3*g^4*(e*x+d)^2*(c*x^
2+b*x+a)^(1/2)/c/e^3
```

Rubi [A] time = 1.37, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4ceg(4aeg-7bdg+18bef)+15b^2e^2g^2+4c^2\left(11d^2g^2-36defg+36e^2f^2\right)\right)}{24c^3e^3}-\frac{g\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e
^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/(24*c^3*e^3) + (g
^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(12*c^2
*e^3) + (g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - (g*(5*b^3*e^3*g
^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^
2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*
e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*S
qrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{2}e\left(6ce^3f^4-d^2(bd+4ae)g^4\right)-\frac{1}{2}eg\left(de(7bd+8ae)g^3-c\left(24e^3f^3-2d^3g^3\right)\right.}{(d\cdots}}{\cdots}$$

$$= \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \cdots$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\cdots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\cdots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\cdots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\cdots}}{24c^3e^3}$$

Mathematica [A] time = 0.89, size = 553, normalized size = 1.28

$$\frac{\frac{6e^2g(ef-dg)\left(\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+6\sqrt{c}\,g\sqrt{a+x(b+cx)}\,(2cf-bg)\right)}{c^{5/2}} + \frac{e^3g\left(\frac{2g\sqrt{a+x(b+cx)}\left(-2cg(8ag+27bf+5bgx)+15b^2g^2+\cdots\right.}{c^2}\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((48*e*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/c + (24*e^2*g^2*(e*f - d*g)
*(f + g*x)*Sqrt[a + x*(b + c*x)])/c + (16*e^3*g^2*(f + g*x)^2*Sqrt[a + x*(b
 + c*x)])/c + (24*e*g*(2*c*f - b*g)*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (48*g*(e*f - d*g)^3*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (6*e^2*g*(e*f - d*g)*(6
*Sqrt[c]*g*(2*c*f - b*g)*Sqrt[a + x*(b + c*x)] + (8*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/c^(5/2) + (e^3*g*((2*g*Sqrt[a + x*(b + c*x)]*(15*b^2*g^2 + 4*c^2*f*(16*f
+ 5*g*x) - 2*c*g*(27*b*f + 8*a*g + 5*b*g*x)))/c^2 + (3*(2*c*f - b*g)*(8*c^2
*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])])/c^(5/2)))/c + (48*(e*f - d*g)^4*ArcTanh[(-2*a*e + 2*
c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(48*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 1597, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^4+2*g^3/e^2*b/c^(3/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f+2*g^3/e*x/c*(c*x^2+b*x+a)^(1/
2)*f+3/4*g^4/e^2*b/c^2*(c*x^2+b*x+a)^(1/2)*d-3*g^3/e*b/c^2*(c*x^2+b*x+a)^(1
/2)*f-3/8*g^4/e^2*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d
+3/2*g^3/e*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f+1/2*g^
4/e^2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-2*g^3/e*a/c^(
3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-4*g^3/e^2/c*(c*x^2+b*x+a
)^(1/2)*d*f-1/2*g^4/e^3*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*d^2-3*g^2/e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f^2+4*
g^3/e^3*d^2*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-6*g^2/e^2
*d*f^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+4/e^4/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*f*d^3-6/e^3/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x+d/e))*d^2*f^2*g^2+4/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b
```

```
*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+
d/e))*d*f^3*g-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c
+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^4*d^4+1/3
*g^4/e*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*g^4/e*b/c^2*x*(c*x^2+b*x+a)^(1/2)+3/4
*g^4/e*b/c^(5/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g^4/e^2*
x/c*(c*x^2+b*x+a)^(1/2)*d+5/8*g^4/e*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*g^4/e*
b^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*g^4/e/c^2*a*(c*
x^2+b*x+a)^(1/2)+g^4/e^3/c*(c*x^2+b*x+a)^(1/2)*d^2+6*g^2/e/c*(c*x^2+b*x+a)^
(1/2)*f^2-g^4/e^4*d^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+4
*g/e*f^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^4}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{4}}{\left(d + e x\right) \sqrt{a + b x + c x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.871 $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+c}}{}$$

```
[Out] 1/8*g*(3*b^2*e^2*g^2-4*c*e*g*(a*e*g-b*d*g+3*b*e*f)+8*c^2*(d^2*g^2-3*d*e*f*g
+3*e^2*f^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^3
+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1
/2)/(c*x^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+c*d^2)^(1/2)+3/4*g^2*(-b*e*g-2*c*
d*g+4*c*e*f)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*g^3*(e*x+d)*(c*x^2+b*x+a)^(1/2
)/c/e^2
```

Rubi [A] time = 0.71, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + (g^
3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*
(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^3) + ((e*f -
 d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{2}e\left(4ce^2f^3-d(bd+2ae)g^3\right)-eg\left(e(2bd+ae)g^2-c\left(6e^2f^2-d^2g^2\right)\right)x+\frac{3}{2}e^2g\cdots}{(d+ex)\sqrt{a+bx+cx^2}}}{2ce^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{4}e^3\left(8c^2e^2f\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{(ef-dg)^3\cdots}{\cdots}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} - \frac{\left(2(ef-dg)\cdots\right.}{\cdots}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{g\left(3b^2e^2g^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.38, size = 358, normalized size = 1.33

$$\frac{\frac{e^2g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}} + \frac{4eg(2cf-bg)(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{6e^2g^2\sqrt{a+x(b+cx)}\,(2cf-bg)}{c^2} + \frac{8(ef\cdots}{\cdots}}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((6*e^2*g^2*(2*c*f - b*g)*Sqrt[a + x*(b + c*x)])/c^2 + (8*e*g^2*(e*f - d*g)
*Sqrt[a + x*(b + c*x)])/c + (4*e^2*g^2*(f + g*x)*Sqrt[a + x*(b + c*x)])/c +
 (4*e*g*(2*c*f - b*g)*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/c^(3/2) + (8*g*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (e^2*g*(8*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(2
*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2
) + (8*(e*f - d*g)^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)
])/(8*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1007, normalized size = 3.73

$$\frac{d^3 g^3 \ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\; e^4}-\frac{3d^2 f g^2 \ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde\cdots}{e^2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*g^3/e*x/c*(c*x^2+b*x+a)^(1/2)-3/4*g^3/e*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*g
^3/e*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g^3/e*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-g^3/e^2/c*(c*x^2+b*x+a)^(
1/2)*d+3*g^2/e/c*(c*x^2+b*x+a)^(1/2)*f+1/2*g^3/e^2*b/c^(3/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-3/2*g^2/e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*f+g^3/e^3*d^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))/c^(1/2)-3*g^2/e^2*d*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/
2)+3*g/e*f^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^4/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)
/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*d^3-3/e^3/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x+d/e))*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
(b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x+d/e))*d*f^2*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c
+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^3}{\left(d+ex\right)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.872 $\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=176

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

```
[Out] 1/2*g*(-b*e*g-2*c*d*g+4*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(3/2)/e^2+(-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+g^
2*(c*x^2+b*x+a)^(1/2)/c/e
```

Rubi [A] time = 0.30, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) + ((e*f - d
*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x + c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{\int \frac{\frac{1}{2}e(2cef^2-bdg^2)+\frac{1}{2}eg(4cef-2cdg-beg)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ce^2}\\
&= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^2}+\frac{(g(4cef-2cdg-beg))\int -}{2ce^2}\\
&= \frac{g^2\sqrt{a+bx+cx^2}}{ce}-\frac{\left(2(ef-dg)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right.}{e^2}\\
&= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{g(4cef-2cdg-beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\,\text{ta}}{e^2}\end{aligned}$$

Mathematica [A] time = 0.48, size = 170, normalized size = 0.97

$$\frac{-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(beg+2cdg-4cef)}{c^{3/2}}+\frac{2(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{2eg^2\sqrt{a+x(b+cx)}}{c}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)])/c - (g*(-4*c*e*f + 2*c*d*g + b*e*g)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (2*(e*f - d*g)^
2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)
]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 613, normalized size = 3.48

$$-\frac{d^2 g^2 \ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e^3}+\frac{2dfg\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+}{e^2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g^2*(c*x^2+b*x+a)^(1/2)/c/e-1/2*g^2/e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))-g^2/e^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1
/2)+2*g/e*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/e^3/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/
e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*g^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(x+d/e))*d*f*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*
c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f+g\,x\right)}^2}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{2}}{\left(d + e x\right) \sqrt{a + b x + c x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.873 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=131

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e}$$

```
[Out] g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/c^(1/2)+(-d*g+e*f)*a
rctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {843, 621, 206, 724}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + ((
e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e}$$
$$= \frac{(2g)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e} - \frac{(2(ef-dg))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,\right.}{e}$$
$$= \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e} + \frac{(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.15, size = 126, normalized size = 0.96

$$\frac{\frac{(dg-ef)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + ((-(e
*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(
b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/e
```

fricas [B] time = 66.10, size = 1071, normalized size = 8.18

$$\left[\frac{\left(cd^2-bde+ae^2\right)\sqrt{c}\,g\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)-\sqrt{cd^2-bde+ae^2}\,\left(cef\right.}{2\left(c^2d^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - sqrt(c*d^2 - b*d*e + a
*e^2)*(c*e*f - c*d*g)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c
^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)
*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a
*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b
*c*d*e^2 + a*c*e^3), 1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2
- 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*
sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f - c*d*g)*arctan(-1/2*sqrt(-c*d^2 + b*d*
e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 -
 a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(c*d^2 - b*d*e
+ a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*(c*e*f - c*d*g)*log((
8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
 - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -((c*d
^2 - b*d*e + a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f -
c*d*g)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
```

```
- 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*
e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^
2 + a*c*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 349, normalized size = 2.66

$$\frac{dg\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\; e^2}-\frac{f\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\cdots}}{\cdots}\right)}{\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/e*g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^2/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.874 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

```
[Out] arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 0.99

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)])
```

fricas [B] time = 1.08, size = 343, normalized size = 4.34

$$\left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-\left(b^2+4\,ac\right)d^2-\left(8\,c^2d^2-8\,bcde+\left(b^2+4\,ac\right)e^2\right)x^2-4\,\sqrt{cd^2-bde+ae^2}\,\sqrt{cx^2+bx+a}\,(bd-2\,ae+(2\,cd-be)x)-2\,\left(4\,bcd^2+4\,abe^2-\left(3\,b^2+4\,ac\right)\right.}{e^2x^2+2\,dex+d^2}\right.}{2\,\sqrt{cd^2-bde+ae^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
 + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), sqr
t(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x
^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2
+ (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x))/(c*
d^2 - b*d*e + a*e^2)]
```

giac [A] time = 0.23, size = 72, normalized size = 0.91

$$\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

maple [B] time = 0.01, size = 157, normalized size = 1.99

$$-\frac{\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
```

```
more details)Is a*e^2-b*d*e                              +c*d^2    positive,
negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d + e\,x)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.875 $\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

```
[Out] e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(-d*g+e*f)/(a*e^2-b*d*e+c*d^2)^(1/2)-g*arctanh(1/2*(b*f-2*a*g+
(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(
a*g^2-b*f*g+c*f^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {960, 724, 206}

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*ArcT
anh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
 b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)\sqrt{a+bx+cx^2}}\right.$$
$$= \frac{e\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$
$$= -\frac{(2e)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{ef-dg} + \frac{(2g)\,\text{Subst}\left(\right.}{}$$
$$= \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}\,(ef-dg)} - \frac{g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

Mathematica [A] time = 0.25, size = 169, normalized size = 0.93

$$\frac{\frac{g\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}} - \frac{e\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}}{dg-ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((e*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)]) + (g*ArcTanh
[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
+ x*(b + c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)])/(-(e*f) + d*g)
```

fricas [B] time = 176.23, size = 1952, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((8*a*b*f*g
 - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 + 4*a*c)*g
^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g
+ (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)/(g^
2*x^2 + 2*f*g*x + f^2)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^2 - b*d*e
+ a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*
c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 +
 b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c*d*e^2 + a
*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^
2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*g^3),
-1/2*(2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*g*arctan(-1/2*
sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g + (2*c*f -
b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a*c*g^2)*x^2 +
(b*c*f^2 - b^2*f*g + a*b*g^2)*x)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*
d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sq
rt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*
e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c
*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*
```

```
c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e
^2)*g^3), -1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((
8*a*b*f*g - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 +
 4*a*c)*g^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f
 - 2*a*g + (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*
g)*x)/(g^2*x^2 + 2*f*g*x + f^2)) - 2*(c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(-c*
d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 +
b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^
2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/((c^2*d
^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f
^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e
 + a^2*d*e^2)*g^3), -((c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*
g*arctan(-1/2*sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a
*g + (2*c*f - b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a
*c*g^2)*x^2 + (b*c*f^2 - b^2*f*g + a*b*g^2)*x)) - (c*e*f^2 - b*e*f*g + a*e*
g^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*
sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e +
a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)
*x)))/((c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 -
a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^
3 - a*b*d^2*e + a^2*d*e^2)*g^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [A] time = 0.02, size = 327, normalized size = 1.80

$$\frac{\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(dg-ef\right)\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}-\frac{\ln\left(\frac{\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{2a g^2-2bfg+2c f^2}{g^2}+2\sqrt{\frac{a g^2-bfg+\cdots}{g^2}}\cdots}{x\cdots}\right)}{\left(dg-ef\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a
*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+1/(d*g-e*f)/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e
^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,(ex+d)\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(f+g\,x\right)\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.876 $\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=340

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\ldots$$

```
[Out] -1/2*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)+e^2*arc
tanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/(-d*g+e*f)^2/(a*e^2-b*d*e+c*d^2)^(1/2)-e*g*arctanh(1/2*(b*f-2*a*g+(
-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/
(a*g^2-b*f*g+c*f^2)^(1/2)+g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c
*f^2)/(g*x+f)
```

Rubi [A] time = 0.39, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {960, 724, 206, 730}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x))
 + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) -
(g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*
f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2
)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f
*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f*g + a*
g^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^2\sqrt{a+bx+}}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^2\sqrt{a+bx}}}{ef-dg}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-}\right.}{(ef-d}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+b}}\right.}{\sqrt{cd^2-bde+ae^2}(ef-d}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+b}}\right.}{\sqrt{cd^2-bde+ae^2}(ef-d}$$

Mathematica [A] time = 0.95, size = 256, normalized size = 0.75

$$-\frac{-\frac{2e^2\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{2g^2\sqrt{a+x(b+cx)}(dg-ef)}{(f+gx)\left(g(ag-bf)+cf^2\right)} + \frac{g(g(2aeg+bdg-3bef)+2cf(2ef-dg))\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}}}{2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((2*g^2*(-(e*f) + d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*
g))*(f + g*x)) - (2*e^2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a
*e)] + (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(-
2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x
*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d*g)^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 788, normalized size = 2.32

$$\frac{bg\ln\left(\frac{\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{2ag^2-2bfg+2cf^2}{g^2}+2\sqrt{\frac{ag^2-bfg+cf^2}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{ag^2-bfg+cf^2}{g^2}}}{x+\frac{f}{g}}\right)}{2\left(dg-ef\right)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{ag^2-bfg+cf^2}{g^2}}}-\frac{cf\ln\left(\frac{\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{2ag^2-2bfg+2cf^2}{g^2}+2}{}\right)}{\left(dg-ef\right)\left(ag^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^
2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)
/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f+e/(d*g-e*f)^2/((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2))/(x+f/g))-e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**2*sqrt(a + b*x + c*x**2)), x)
```

3.877 $\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=587

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g}{}$$

```
[Out] -1/2*e*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c
*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1/8
*g*(8*c^2*f^2+3*b^2*g^2-4*c*g*(a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c
*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f
*g+c*f^2)^(5/2)+e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/(a*e^2-b*d*e+c*d^2)^(1/2)-e^2*g
*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(1/2)+1/2*g^2*(c*x^2+b*x+a)^(1
/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2+3/4*g^2*(-b*g+2*c*f)*(c*x^2+b*
x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)+e*g^2*(c*x^2+b*x+a)^(1/
2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)
```

Rubi [A] time = 0.81, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {960, 724, 206, 744, 806, 730}

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x
)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 -
b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2
- b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g +
 (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(
2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*f - 2*a*
g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]
)/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3*b^2*g^2 -
 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^3\sqrt{a+bx+cx}}\right.$$

$$= \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{1}{(f+gx)^2\sqrt{a+b}}}{(ef-dg)}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)}$$

Mathematica [A] time = 2.43, size = 549, normalized size = 0.94

$$\frac{g(ef-dg)^2\left(\frac{6g\sqrt{a+x(b+cx)}\,(2cf-bg)}{(f+gx)\left(g(ag-bf)+cf^2\right)^2} - \frac{\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{5/2}}\right) + \frac{8e^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2c}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-b}}\right)}{\sqrt{e(ae-bd)+cd^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f
 + g*x)^2) + (8*e*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f
) + a*g))*(f + g*x)) + (8*e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*S
qrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*
d) + a*e)] + (4*e*g*(-2*c*f + b*g)*(e*f - d*g)*ArcTanh[(-2*a*g + 2*c*f*x +
b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*
f^2 + g*(-(b*f) + a*g))^(3/2) - (8*e^2*g*ArcTanh[(-2*a*g + 2*c*f*x + b*(f -
 g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^
2 + g*(-(b*f) + a*g)] + g*(e*f - d*g)^2*((6*g*(2*c*f - b*g)*Sqrt[a + x*(b +
 c*x)])/((c*f^2 + g*(-(b*f) + a*g))^2*(f + g*x)) - ((8*c^2*f^2 + 3*b^2*g^2
- 4*c*g*(2*b*f + a*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f
^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^
(5/2)))/(8*(e*f - d*g)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 3.26, size = 2256, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d^2*f^2*g^3 - 8*b*c*d^2*f*g^4 + 3*b^2*d^2*g^5 - 4*a*c*d^2*g^5 -
24*c^2*d*f^3*g^2*e + 28*b*c*d*f^2*g^3*e - 10*b^2*d*f*g^4*e + 4*a*b*d*g^5*e
+ 24*c^2*f^4*g*e^2 - 36*b*c*f^3*g^2*e^2 + 15*b^2*f^2*g^3*e^2 + 20*a*c*f^2*g
^3*e^2 - 20*a*b*f*g^4*e^2 + 8*a^2*g^5*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^2*d^3*f^4*g^3
 - 2*b*c*d^3*f^3*g^4 + b^2*d^3*f^2*g^5 + 2*a*c*d^3*f^2*g^5 - 2*a*b*d^3*f*g^
6 + a^2*d^3*g^7 - 3*c^2*d^2*f^5*g^2*e + 6*b*c*d^2*f^4*g^3*e - 3*b^2*d^2*f^3
*g^4*e - 6*a*c*d^2*f^3*g^4*e + 6*a*b*d^2*f^2*g^5*e - 3*a^2*d^2*f*g^6*e + 3*
c^2*d*f^6*g*e^2 - 6*b*c*d*f^5*g^2*e^2 + 3*b^2*d*f^4*g^3*e^2 + 6*a*c*d*f^4*g
^3*e^2 - 6*a*b*d*f^3*g^4*e^2 + 3*a^2*d*f^2*g^5*e^2 - c^2*f^7*e^3 + 2*b*c*f^
6*g*e^3 - b^2*f^5*g^2*e^3 - 2*a*c*f^5*g^2*e^3 + 2*a*b*f^4*g^3*e^3 - a^2*f^3
*g^4*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 2*arctan(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^3/((d^3*g^3 - 3
*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^3 - 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b*c*d*f*g^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^5 - 16*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f^3*g^2*e + 20*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b*c*f^2*g^3*e - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f
*g^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^4*e + 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a*b*g^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*c^(5/2)*d*f^3*g^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2
)*d*f^2*g^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^4 -
 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^4 - 40*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g*e + 44*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*b*c^(3/2)*f^3*g^2*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b^2*sqrt(c)*f^2*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f
^2*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^4*e + 8*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*g^5*e + 24*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^2 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b^2*c*d*f^2*g^3 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^
3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^4 + 28*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*d*f*g^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*g^5 - 40*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^2*c*f^3*g^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3
*g^2*e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f^2*g^3*e - 72*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^3*e + 13*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a*b^2*f*g^4*e + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^4
*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*g^5*e + 6*b^2*c^(3/2)*d*f^
3*g^2 - 3*b^3*sqrt(c)*d*f^2*g^3 - 20*a*b*c^(3/2)*d*f^2*g^3 + 11*a*b^2*sqrt(
c)*d*f*g^4 + 12*a^2*c^(3/2)*d*f*g^4 - 8*a^2*b*sqrt(c)*d*g^5 - 10*b^2*c^(3/2
)*f^4*g*e + 7*b^3*sqrt(c)*f^3*g^2*e + 32*a*b*c^(3/2)*f^3*g^2*e - 27*a*b^2*s
qrt(c)*f^2*g^3*e - 20*a^2*c^(3/2)*f^2*g^3*e + 28*a^2*b*sqrt(c)*f*g^4*e - 8*
a^3*sqrt(c)*g^5*e)/((c^2*d^2*f^4*g^2 - 2*b*c*d^2*f^3*g^3 + b^2*d^2*f^2*g^4
+ 2*a*c*d^2*f^2*g^4 - 2*a*b*d^2*f*g^5 + a^2*d^2*g^6 - 2*c^2*d*f^5*g*e + 4*b
*c*d*f^4*g^2*e - 2*b^2*d*f^3*g^3*e - 4*a*c*d*f^3*g^3*e + 4*a*b*d*f^2*g^4*e
- 2*a^2*d*f*g^5*e + c^2*f^6*e^2 - 2*b*c*f^5*g*e^2 + b^2*f^4*g^2*e^2 + 2*a*c
*f^4*g^2*e^2 - 2*a*b*f^3*g^3*e^2 + a^2*f^2*g^4*e^2)*((sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*f + b*f -
 a*g)^2)
```

maple [B] time = 0.02, size = 1817, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/
g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+3/4*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2
/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
b-3/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x
+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f-3/8*g^2/(d*g-e*f)/(a*g^2-b*f*g+c
*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-
b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2+3/2*g/(d*g-e*f)/(a*
g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/
g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+
(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*c*f-3/2/(d
*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*
f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((
x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c
^2*f^2+1/2/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))/(x+f/g))+g*e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/2*g*e/(d*g-e*f)^2/(a*g^2-b
*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*
g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*
c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b+e/(d*g-e*f)^2/(a*
g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+
2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f-e^2/(d*g-e
*f)^3/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*
f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(
x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+e^2/(d*g-e*f)^3/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f + g\,x\right)^3\,\left(d + e\,x\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**3*sqrt(a + b*x + c*x**2)), x)
```

3.878 $\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=496

$$-\frac{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4\right.\right.\right.}{}$$

```
[Out] 1/2*g^3*(-3*b*e*g-2*c*d*g+8*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(5/2)/e^2+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(3/2
)-2*(a*b^3*d*g^4-b^2*(a^2*e*g^4+4*a*c*d*f*g^3+c^2*e*f^4)+2*a*c*(a^2*e*g^4+c
^2*f^3*(-4*d*g+e*f)-2*a*c*f*g^2*(-2*d*g+3*e*f))+b*c*(c^2*d*f^4+a^2*g^3*(-3*
d*g+4*e*f)+2*a*c*f^2*g*(3*d*g+2*e*f))+(2*c^4*d*f^4+b^3*(-a*e+b*d)*g^4-b*c*g
^3*(4*b^2*d*f-3*a^2*e*g-4*a*b*(-d*g+e*f))+2*c^2*g^2*(3*b^2*d*f^2-3*a*b*f*(-
2*d*g+e*f)-a^2*g*(-d*g+4*e*f))+c^3*f^2*(4*a*g*(-3*d*g+2*e*f)-b*f*(4*d*g+e*f
)))*x)/c^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)+g^4*(c*x^2+
b*x+a)^(1/2)/c^2/e
```

Rubi [A] time = 1.20, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1646, 1653, 843, 621, 206, 724}

$$-\frac{2\left(x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab(ef-dg)+4b^2df\right)+b^3g^4(bd-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2
*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*
f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4
 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)
) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) +
 c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^
2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

Mathematica [A] time = 2.46, size = 587, normalized size = 1.18

$$-\frac{2e\left(b^2\left(3a^2e^2g^4+acg^3\left(d^2g+4de(2f+3gx)+e^2x(gx-8f)\right)+c^2\left(d^2g^4x^2-12def^2g^2x+2e^2f^4\right)\right)-2bc\left(a^2eg^3(-5dg+4ef+5egx)+2acg\left(d^2g^3x+deg\left(3f^2+6fgx-g\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8*e*f + d*g
 - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d*e*f^2*g^2*x + d^2*g
^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) + 4*d*e*(2*f + 3*g*x))) - 2*b
*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e*g*x) + c^2*e*f^3*(-(e*f*x) + d*(f - 4*g*
x)) + 2*a*c*g*(d^2*g^3*x + e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*g*x -
 g^2*x^2))) - 4*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2 - 2*d
*e*f^2*g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 + d*e*g*
(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2)))))/(c^2*(b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*(e*f - d*g)^4*Log[d + e*x
])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*Lo
g[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2) - (2*(e*f - d*g)^4*
Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqr
t[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 4453, normalized size = 8.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)

[Out] e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f^4+6/e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2*f^2*g^2-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^4-1/e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^4*d^4-2*g^4/e^4*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+4/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*f^3*g+4*g^3/e^2/c/(c*x^2+b*x+a)^(1/2)*d*f+3/2*g^4/e*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/4*g^4/e*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*g/e*f^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+g^4/e^2*x/c/(c*x^2+b*x+a)^(1/2)*d-4*g^3/e*x/c/(c*x^2+b*x+a)^(1/2)*f-1/2*g^4/e^2*b/c^2/(c*x^2+b*x+a)^(1/2)*d+2*g^3/e*b/c^2/(c*x^2+b*x+a)^(1/2)*f-4/e^2/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^3*f*d^3-e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^4+1/e^3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^4*d^4-4/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f^3*g+g^4/e*x^2/c/(c*x^2+b*x+a)^(1/2)-3/4*g^4/e*b^2/c^3/(c*x^2+b*x+a)^(1/2)-3/2*g^4/e*b/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*g^4/e/c^2*a/(c*x^2+b*x+a)^(1/2)-g^4/e^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d+4*g^3/e/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-g^4/e^3/c/(c*x^2+b*x+a)^(1/2)*d^2-6*g^2/e/c/(c*x^2+b*x+a)^(1/2)*f^2+12/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^3*f^2*g^2-2/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*g^4*d^4-16/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^4*g^3*f+8/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*f^3*g-8/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^2*f^3*g+24/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*f^2*g^2-16/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f^3*g-8/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^4*g^3*f-6/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*f^2*g^2+2/e^4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^5*g^4-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^4+4/e^4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^5*g^4+4/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/

```
e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*g^3*f*d^3+4
*g^4/e/c*a*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-g^4/e^2*b^2/c/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)*x*d+4*g^3/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f+8*g
^3/e^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f+4*g^3/e^2*b^2/c/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*d*f-24*g^2/e^2*d*f^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*
x+16*g^3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+8/e^2/(a*e^2-b*d*e+c
*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*b*c*g^3*f*d^3-12/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d^2*f^2*g^2+8*g^
3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-12*g^2/e^2*d*f^2/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*b-3/2*g^4/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2
*g^4/e/c^2*a*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*g/e*f^3/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*c*x-4*g^4/e^4*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x-g^4/
e^3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d^2-6*g^2/e*b^2/c/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*f^2+2*g^3/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f-2*g^
4/e^3*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d^2-12*g^2/e*b/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*f^2-1/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*g^4*d^4+4/e^2/(a*e^2-b
*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*f*d^3-6/e/(a*e^
2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*
(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*f^2*g^2-1/2*
g^4/e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d+4/(a*e^2-b*d*e+c*d^2)/(4*
a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
b^2*d*f^3*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f^4+4/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c
^2*d*f^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^4}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{4}}{\left(d + e x\right) \left(a + b x + c x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.879 $\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=357

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left(ae^2\right.}$$

```
[Out] g^3*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e+(-d*g+e*f)
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*(a*d*g^3+c*e*f^3)-2*a*c*(c
*f^2*(-3*d*g+e*f)-a*g^2*(-d*g+3*e*f))-b*(c^2*d*f^3+a^2*e*g^3+3*a*c*f*g*(d*g
+e*f))-(2*c^3*d*f^3-b^2*(-a*e+b*d)*g^3+c*g^2*(-2*a^2*e*g+3*a*b*d*g-3*a*b*e*
f+3*b^2*d*f)+c^2*f*(6*a*g*(-d*g+e*f)-b*f*(3*d*g+e*f)))*x)/c/(-4*a*c+b^2)/(a
*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1646, 843, 621, 206, 724}

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left(ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d
*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b
^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g)
+ c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^3*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*e) + ((e*f - d*g)^3*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

Mathematica [A] time = 1.04, size = 373, normalized size = 1.04

$$\frac{2\left(b\left(a^2eg^3+3acg(dg(f+gx)+ef(f-gx))+c^2f^2(d(f-3gx)-efx)\right)+2c\left(a^2g^2(dg-e(3f+gx))+acf(ef(\right.\right.}{c\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\left(e(ae-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) +
b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x)
+ d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a*c*
f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x)))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b
*d) + a*e))*Sqrt[a + x*(b + c*x)]) + ((e*f - d*g)^3*Log[d + e*x])/(e*(c*d^2
 + e*(-(b*d) + a*e))^(3/2)) + (g^3*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)]])/(c^(3/2)*e) + ((-(e*f) + d*g)^3*Log[-(b*d) + 2*a*e - 2*c*d*x + b*
e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(e*(c*d^2 +
e*(-(b*d) + a*e))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 3127, normalized size = 8.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*f^3+g^3/e/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+
6/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^3*f*g^2+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f
^3-6*g^2/e^2*d*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+12*g/e*f^2/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*c*x+1/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*g^3*d^3+3/(a*e^2-b*
d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*b^2*d*f^2*g-3/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*f*g^2-2/e^3/(a
*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*b*c*d^4*g^3-12*g^2/e^2*d*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*c*x-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/
e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^3-4/e^3/(a*e^2-b*d*e+c*d^2)/(4*
a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
x*c^2*d^4*g^3+2/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d
)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*g^3*d^3-6/e/(a*e^2-b*d*e+c
*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*b*c*d^2*f*g^2-6/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^2*f^2*g-3*g^2/e*b
^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f+4*g^3/e^3*d^2/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)*c*x+1/2*g^3/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*g^3/e^3*d
^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+6*g/e*f^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)*b+3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+
d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*f
^2*g-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^3+1/e^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*d^3+g^3/e^2/c/(c*x^2+b*x+a)^(1/2)*d-3*
g^2/e/c/(c*x^2+b*x+a)^(1/2)*f-1/e^2/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2
```

```
*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^3*d^3+g^3/e*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+2*g^3/e^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d-6
*g^2/e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f+g^3/e^2*b^2/c/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*d+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d
)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^3-3/e/(a*e^2-b*d*e+c*d
^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*
e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*f*g^2+3/e/(a*e^2-b*d*e+
c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^
2*f*g^2-3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*d*f^2*g-e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e))*f^3-g^3/e*x/c/(c*x^2+b*x+a)^(1/2)+1/2*g^3/e*b/c^2/(c*x^
2+b*x+a)^(1/2)+6/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(
x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*f^2*g+12/e^2/(a*e^2-b*d*e+c
*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*c^2*d^3*f*g^2-12/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f^2*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{3}}{\left(d + ex\right) \left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**3/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.880 $\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*e*f^2+2*a*(a*e*g^2
-c*f*(-2*d*g+e*f))-b*(c*d*f^2+a*g*(d*g+2*e*f))-(2*c^2*d*f^2+b*(-a*e+b*d)*g^
2+c*(2*a*g*(-d*g+2*e*f)-b*f*(2*d*g+e*f)))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^
2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1646, 12, 724, 206}

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*f
 + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*f*
(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c
*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

Mathematica [A] time = 0.63, size = 265, normalized size = 1.10

$$\frac{2\left(-2a^2eg^2+abg(dg+2ef-egx)-2acdg(2f+gx)+2acef(f+2gx)+b^2\left(dg^2x-ef^2\right)+bcf(d(f-2gx)-e\right.}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-2*a^2*e*g^2 + 2*c^2*d*f^2*x - 2*a*c*d*g*(2*f + g*x) + 2*a*c*e*f*(f + 2
*g*x) + a*b*g*(2*e*f + d*g - e*g*x) + b^2*(-(e*f^2) + d*g^2*x) + b*c*f*(-(e
*f*x) + d*(f - 2*g*x))))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a +
 x*(b + c*x)]) + ((e*f - d*g)^2*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^(3
/2) - ((e*f - d*g)^2*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 +
e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)
```

fricas [B] time = 10.42, size = 2023, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4
*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g
+ (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*
b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log(
(8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*
d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d
*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
```

```
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x), (((a
*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4*a^2*c)*d
^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g + (b^2*c
- 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*b*c)*d*e*
f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

giac [B] time = 0.35, size = 757, normalized size = 3.15

$$-\frac{2\left(\frac{\left(2\,c^3d^3f^2-2\,bc^2d^3fg+b^2cd^3g^2-2\,ac^2d^3g^2-3\,bc^2d^2f^2e+2\,b^2cd^2fge+4\,ac^2d^2fge-b^3d^2g^2e+abcd^2g^2e+b^2cdf^2e^2+2\,ac^2df^2e^2-6\,abcdfge^2+2\,ab^2dg^2e^2 \cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3 \cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f^2 - 2*b*c^2*d^3*f*g + b^2*c*d^3*g^2 - 2*a*c^2*d^3*g^2 - 3*
b*c^2*d^2*f^2*e + 2*b^2*c*d^2*f*g*e + 4*a*c^2*d^2*f*g*e - b^3*d^2*g^2*e + a
*b*c*d^2*g^2*e + b^2*c*d*f^2*e^2 + 2*a*c^2*d*f^2*e^2 - 6*a*b*c*d*f*g*e^2 +
2*a*b^2*d*g^2*e^2 - 2*a^2*c*d*g^2*e^2 - a*b*c*f^2*e^3 + 4*a^2*c*f*g*e^3 - a
^2*b*g^2*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*
e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8
*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f^2 - 4*a*c^2*d^3*
f*g + a*b*c*d^3*g^2 - 2*b^2*c*d^2*f^2*e + 2*a*c^2*d^2*f^2*e + 6*a*b*c*d^2*f
*g*e - a*b^2*d^2*g^2*e - 2*a^2*c*d^2*g^2*e + b^3*d*f^2*e^2 - a*b*c*d*f^2*e^
2 - 2*a*b^2*d*f*g*e^2 - 4*a^2*c*d*f*g*e^2 + 3*a^2*b*d*g^2*e^2 - a*b^2*f^2*e
^3 + 2*a^2*c*f^2*e^3 + 2*a^2*b*f*g*e^3 - 2*a^3*g^2*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*arctan(
-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e -
a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

maple [B] time = 0.02, size = 2123, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/
e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*f*g-1/e
/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e
)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*g^2-e/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*b^2*f^2+e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f^2-g^2/e/c/(c*x^2+b*x+a)^(1/2)
+1/e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*d^2*g^2+4/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*g^2+2/e^2/(a
*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*b*c*d^3*g^2-2/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f*g-e/(a*e^2-b*d*e+c*d^2)/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d
^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^2-4/e/(a*e^2-b*d*e+c*d^2)/(4*a
*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b
*c*d^2*f*g-2/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d^2*g^2+4/(a*e^2-b*d*e+c*d^2)/(
4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*x*b*c*d*f*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*f*g+2/(a*e^2-b*d*e+c*d^2)/(4*a
*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b
*c*d*f^2-4*g^2/e^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+8*g/e*f/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)*c*x+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b
*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^2-1/e/(a*e^2-b
*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*b^2*d^2*g^2-2*g^2/e^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+4*
g/e*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-2*g^2/e*b/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)*x-g^2/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-8/e/(a*e^2-b*d*e+c*d^2
)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*x*c^2*d^2*f*g-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^2}{\left(d+ex\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**2/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.881 $\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] e*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d*f-b^2*e*f+2*a*c*
e*f-2*a*c*d*g+a*b*e*g+c*(2*c*d*f+2*a*e*g-b*(d*g+e*f))*x)/(-4*a*c+b^2)/(a*e^
2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {822, 12, 724, 206}

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a
*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^
(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - $$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + $$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - $$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + $$

Mathematica [A] time = 0.16, size = 183, normalized size = 0.98

$$\frac{2b(aeg+cd(f-gx)-cefx)+4c(-adg+ae(f+gx)+cdfx)-2b^2ef}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}+\frac{e(dg-ef)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*f
*x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(
b + c*x)]) + (e*(-(e*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(
2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d
) + a*e))^(3/2)
```

fricas [B] time = 9.56, size = 1663, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c
^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 -
4*a*b*c)*d*e*g)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 -
 (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sq
rt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e
)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*
x + d^2)) + 4*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e +
 (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f - (2*a*c^2*d^3 - 3*a*b*c*d^
2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((2*c^3*d^3 - 3*b*c^2*d^2*e
- a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a
^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
```

```
- 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a
*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^
3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)
*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c
)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/
2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2
+ (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 -
 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f -
 (2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((
2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2
*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^
2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^
4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*
c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a
^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*
a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a
^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

giac [B] time = 0.32, size = 568, normalized size = 3.04

$$-\frac{2\left(\frac{\left(2\,c^3d^3f-bc^2d^3g-3\,bc^2d^2fe+b^2cd^2ge+2\,ac^2d^2ge+b^2cdfe^2+2\,ac^2dfe^2-3\,abcdge^2-abcfe^3+2\,a^2cge^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3f-2\,ac^2d^3g-2\,b^2cd^2fe+2\ldots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+\ldots}\ldots}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f - b*c^2*d^3*g - 3*b*c^2*d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*
d^2*g*e + b^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 - a*b*c*f*e^3 +
 2*a^2*c*g*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^
3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 +
 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f - 2*a*c^2*d^3*
g - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e + b^3*d*f*e^2 - a*b
*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3
+ a^2*b*g*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e
 + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*
a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a) - 2*(d*g*
e - f*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*
e^2))
```

maple [B] time = 0.01, size = 1261, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/e*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/(a*e^2-b*d*e+c*d^2)/((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*g+e/(a*e^2-b
*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*f+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*g-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)
```

```
/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f-
4/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*g+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f+1
/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*g-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^
2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f-2/e/(a*e^2-b
*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*b*c*d^2*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*
e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f+1/(a*e^2-b*d*e+c*
d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*g-e/(a*e^2-b*d*e+c*d^2)/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f+g\,x}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.882 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c
*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {740, 12, 724, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\right.}{cd^2-bde+}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+}}\right.}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.26, size = 162, normalized size = 1.05

$$\frac{\frac{e^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} + \frac{4c(ae+cdx)-2b^2e+2bc(d-ex)}{\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)}}{4ac-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*b^2*e + 4*c*(a*e + c*d*x) + 2*b*c*(d - e*x))/((c*d^2 + e*(-(b*d) + a*e
))*Sqrt[a + x*(b + c*x)]) + ((b^2 - 4*a*c)*e^2*ArcTanh[(-(b*d) + 2*a*e - 2*
c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(
c*d^2 + e*(-(b*d) + a*e))^(3/2))/(-b^2 + 4*a*c)
```

fricas [B] time = 1.40, size = 1349, normalized size = 8.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c
)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*
c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b
*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) -
4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a
^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^
2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 -
 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*
(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*
e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a
^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*
```

```
d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 -
 (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c +
2*a*c^2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 -
 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*
c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3
)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*
x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
- 4*a^3*b*c)*e^4)*x)]
```

giac [B] time = 0.32, size = 447, normalized size = 2.88

$$-\frac{2\left(\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3-\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc\cdots}\right.}{\sqrt{cx^2+bx+a}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^2 + b*x + a) + 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*e^2)
*sqrt(-c*d^2 + b*d*e - a*e^2))
```

maple [B] time = 0.01, size = 603, normalized size = 3.89

$$-\frac{2bcex}{\left(a\,e^2-bde+c\,d^2\right)\left(4ac-b^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}+\frac{4c^2\cdots}{\left(a\,e^2-bde+c\,d^2\right)\left(4ac-b^2\right)\sqrt{\left(x+\cdots\right.}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(
x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-
b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^
2*d-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)
^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d-e/(a*e^2-b*
d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e
^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c
*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                                +c*d^2    positive,
negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.883 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=352

$$-\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ae^2-bde+cd^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

```
[Out] e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)-g^3*arctanh(1/2*(b*f-2*
a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*
f)/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2
*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(c
*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {960, 740, 12, 724, 206}

$$-\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ae^2-bde+cd^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g +
 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e +
 a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2]])])/((e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)\left(a+bx+\right.}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{ef-dg}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

Mathematica [A] time = 1.22, size = 317, normalized size = 0.90

$$\frac{-\frac{2e\left(-2c(ae+cdx)+b^2e+bc(ex-d)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)} + \frac{2g\left(-2c(ag+cfx)+b^2g+bc(gx-f)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(g(bf-ag)-cf^2\right)} + \frac{e^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} - \frac{g^3\tanh^{-1}\left(\frac{-2ag+b}{2\sqrt{a+x(b+cx}}\right.}{\left(g(ag-bf)+\right.}}{ef-dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*g*(b^2*g - 2*c*(a*g + c*f*x
) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(
```

```
b + c*x)]) + (e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 +
e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)
 - (g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f)
+ a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d
*g)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1343, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)-2/(d*g-e*f)*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/
((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b*c+4/(
d*g-e*f)*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)
/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^2*f-1/(d*g-e*f)*g^2/(a*g^2-b*f*g+c*f^
2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)*b^2+2/(d*g-e*f)*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*c*f-1/(d*g-e*f)/(a*g^2-b*f
*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(
a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))-1/(d*g-e*f)/(a*e^
2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)+2/(d*g-e*f)*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c-4/(d*g-e*f)*e/(a*e^2
-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*x*c^2*d+1/(d*g-e*f)*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-2/(d*g-
e*f)*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d+1/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)*e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2
)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)\,\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.884 $\int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=642

$$\frac{g^2\sqrt{a+bx+cx^2}\,\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)^2\left(ae^2-bde+cd^2\right)}+$$

```
[Out] e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^2-3/2*g^3*(-b*g+2*c*f)*
arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)-e*g^3*arctanh(1/2*(b*f-2*a*
g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)
^2/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(
-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c
*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c
*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*
c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)+g^2*(4*c^
2*f^2+3*b^2*g^2-4*c*g*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e
*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)
```

Rubi [A] time = 0.91, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {960, 740, 12, 724, 206, 806}

$$\frac{g^2\sqrt{a+bx+cx^2}\,\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)^2\left(ae^2-bde+cd^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b
^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 -
b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*
g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^(3/2)*
(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^2\left(a+\right.}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{}{(f+gx)}}{e}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{\left(b^2-4a\right.}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{\left(b^2-4a\right.}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{\left(b^2-4a\right.}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{\left(b^2-4a\right.}$$

Mathematica [A] time = 5.08, size = 623, normalized size = 0.97

$$\frac{g^2\left(\frac{3g(bg-2cf)\tanh^{-1}\left(\frac{2ag-bf+bgx-2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{5/2}} - \frac{2\sqrt{a+x(b+cx)}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)\left(g(ag-bf)+cf^2\right)^2}\right)}{2(dg-ef)} - \frac{2e^2\left(-2c(ae+cdx)+\right.}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,(ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g -
 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2
) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c*f*x)
 + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f - a*g)
)*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (g^2*((-2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c
*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f)
+ a*g))^2*(f + g*x)) + (3*g*(-2*c*f + b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*
x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^
2 + g*(-(b*f) + a*g))^(5/2)))/(2*(-(e*f) + d*g)) + (e^4*ArcTanh[(-2*a*e + 2
*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x
)])])/((c*d^2 + e*(-(b*d) + a*e))^(3/2)*(e*f - d*g)^2) - (e*g^3*ArcTanh[(-2
*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*
(b + c*x)])])/((e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 2807, normalized size = 4.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/(d*g-e*f)^2*e/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g
^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*
g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))-1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*b^2+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^3+3*g^2/(d*g-e*f)/(a*g
^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^
2)^(1/2)*c*f-1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e))-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)/((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+3*g^3/(d*g-e*f)/(a*g^2-b
*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)*x*b^2*c+12*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/(
(x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^3*f^2-
6*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*
(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2*c*f+6*g/(d*g-e*f)/(a*g^2-b*f*g
+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*b*c^2*f^2-2/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4/(d*
g-e*f)^2*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/
e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d+2/(d*g-e*f)^2*e^2/(a*e^2-b*d*e+
c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*b*c*d-12*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)
^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b*c^2*f-2/(d*g-
e*f)^2*e*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)
/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*c*f+2/(d*g-e*f)^2*e*g^2/(a*g^2-b*f*g+c*
f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)*x*b*c-3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c
*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-1/(d*g-e*f)^2*e/(a*g^2-b*f*g
+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)-4/(d*g-e*f)^2*e*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*
f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^2*f-3*g^2/(d*g-e*f)/(a*g^2-
b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*
(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g
-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f-8*g/(d*g-e*f
)*c^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x-4*g/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/(4*a*c-b
^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b+1/(
```

```
d*g-e*f)^2*e*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(
x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+
c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2
-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f
)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)^2\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**2*(a + b*x + c*x**2)**(3/2)), x)
```

3.885 $\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=1064

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\,\sqrt{cx^2+bx+a}}\right)e^5}{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3}-\frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)e^3}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)(ef-dg)^3\sqrt{cx^2+bx+a}}-\frac{g^3\tanh^{-1}\left(\frac{bf-2ag}{2\sqrt{cf^2-bgf+}}\right.}{(ef-dg)^3\left(cf^2-\right.}$$

```
[Out] e^5*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^3-3/2*e*g^3*(-b*g+2*c*f
)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(5/2)-e^2*g^3*arctanh(1/2*(b*
f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*
g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(3/2)-3/8*g^3*(16*c^2*f^2+5*b^2*g^2-4*c*g*(a*g
+4*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(7/2)-2*e^3*(b*c*d-b^2*e
+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^3/(c
*x^2+b*x+a)^(1/2)+2*e^2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^
2)/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*
a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)
^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c
+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)+1/2*g^2*
(8*c^2*f^2+5*b^2*g^2-4*c*g*(3*a*g+2*b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/
(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2+e*g^2*(4*c^2*f^2+3*b^2*g^2-4*c*g
*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*
f^2)^2/(g*x+f)+1/4*g^2*(-b*g+2*c*f)*(8*c^2*f^2+15*b^2*g^2-4*c*g*(13*a*g+2*b
*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^3/(g*x
+f)
```

Rubi [A] time = 1.90, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {960, 740, 12, 724, 206, 834, 806}

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\,\sqrt{cx^2+bx+a}}\right)e^5}{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3}-\frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)e^3}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)(ef-dg)^3\sqrt{cx^2+bx+a}}-\frac{g^3\tanh^{-1}\left(\frac{bf-2ag}{2\sqrt{cf^2-bgf+}}\right.}{(ef-dg)^3\left(cf^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f^2
- b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c
*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f
- b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)*S
qrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g)
)*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*g))*Sq
rt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^2
*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 1
3*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqr
t[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2]])])/((c*d^2 - b*d*e + a*e^2)^
(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
```

```
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2]])]/(2*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2]])]/((e
*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*(16*c^2*f^2 + 5*b^2*g^2
 - 4*c*g*(4*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f
^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2]])]/(8*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^3\left(a+bx\right.}\right.$$

$$= \frac{e^3\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{}{(f+g}}{(e}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

Mathematica [A] time = 5.74, size = 1013, normalized size = 0.95

$$-\frac{\tanh^{-1}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(ae-bd)}\sqrt{a+x(b+cx)}}\right)e^5}{\left(cd^2+e(ae-bd)\right)^{3/2}(dg-ef)^3} - \frac{2\left(eb^2+c(ex-d)b-2c(ae+cdx)\right)e^3}{\left(b^2-4ac\right)\left(e(bd-ae)-cd^2\right)(ef-dg)^3\sqrt{a+x(b+cx)}} - \frac{2g\left(gb^2+\right.}{\left(b^2-4ac\right)(dg-e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)]) - (2*e^2*g*(b^2*g
 - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)^3*(-(
c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c
*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f -
 a*g))*(f + g*x)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g - 2*c*(a*g + c*f*
x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2) + g*(b*f - a*g
))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (e*g^2*((2*(4*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f
```

```
) + a*g))^2*(f + g*x)) + (3*g*(2*c*f - b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f
*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f
^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(e*f - d*g)^2) - (g^2*((4*(8*c^2*f^2 + 5*
b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*Sqrt[a + x*(b + c*x)])/(f + g*x)^2 + (2*(2
*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 13*a*g))*Sqrt[a + x*(b
 + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)) + (3*(b^2 - 4*a*c)*g*(16*c
^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x
 + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2
 + g*(-(b*f) + a*g))^(3/2)))/(8*(b^2 - 4*a*c)*(-(e*f) + d*g)*(c*f^2 + g*(-(
b*f) + a*g))^2) - (e^5*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((c*d^2 + e*(-(b*d) + a*e))
^(3/2)*(-(e*f) + d*g)^3) - (e^2*g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/((e*f - d*g)^3
*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 21.91, size = 14731, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^9*d^3*f^9 - 9*b*c^8*d^3*f^8*g + 18*b^2*c^7*d^3*f^7*g^2 - 21*b^3*c^
6*d^3*f^6*g^3 + 15*b^4*c^5*d^3*f^5*g^4 + 6*a*b^2*c^6*d^3*f^5*g^4 - 12*a^2*c
^7*d^3*f^5*g^4 - 6*b^5*c^4*d^3*f^4*g^5 - 15*a*b^3*c^5*d^3*f^4*g^5 + 30*a^2*
b*c^6*d^3*f^4*g^5 + b^6*c^3*d^3*f^3*g^6 + 12*a*b^4*c^4*d^3*f^3*g^6 - 18*a^2
*b^2*c^5*d^3*f^3*g^6 - 16*a^3*c^6*d^3*f^3*g^6 - 3*a*b^5*c^3*d^3*f^2*g^7 - 3
*a^2*b^3*c^4*d^3*f^2*g^7 + 24*a^3*b*c^5*d^3*f^2*g^7 + 3*a^2*b^4*c^3*d^3*f*g
^8 - 6*a^3*b^2*c^4*d^3*f*g^8 - 6*a^4*c^5*d^3*f*g^8 - a^3*b^3*c^3*d^3*g^9 +
3*a^4*b*c^4*d^3*g^9 - 3*b*c^8*d^2*f^9*e + 15*b^2*c^7*d^2*f^8*g*e - 6*a*c^8*
d^2*f^8*g*e - 33*b^3*c^6*d^2*f^7*g^2*e + 24*a*b*c^7*d^2*f^7*g^2*e + 41*b^4*
c^5*d^2*f^6*g^3*e - 34*a*b^2*c^6*d^2*f^6*g^3*e - 16*a^2*c^7*d^2*f^6*g^3*e -
 30*b^5*c^4*d^2*f^5*g^4*e + 9*a*b^3*c^5*d^2*f^5*g^4*e + 66*a^2*b*c^6*d^2*f^
5*g^4*e + 12*b^6*c^3*d^2*f^4*g^5*e + 24*a*b^4*c^4*d^2*f^4*g^5*e - 96*a^2*b^
2*c^5*d^2*f^4*g^5*e - 12*a^3*c^6*d^2*f^4*g^5*e - 2*b^7*c^2*d^2*f^3*g^6*e -
23*a*b^5*c^3*d^2*f^3*g^6*e + 49*a^2*b^3*c^4*d^2*f^3*g^6*e + 48*a^3*b*c^5*d^
2*f^3*g^6*e + 6*a*b^6*c^2*d^2*f^2*g^7*e + 3*a^2*b^4*c^3*d^2*f^2*g^7*e - 54*
a^3*b^2*c^4*d^2*f^2*g^7*e - 6*a^2*b^5*c^2*d^2*f*g^8*e + 15*a^3*b^3*c^3*d^2*
f*g^8*e + 9*a^4*b*c^4*d^2*f*g^8*e + 2*a^3*b^4*c^2*d^2*g^9*e - 7*a^4*b^2*c^3
*d^2*g^9*e + 2*a^5*c^4*d^2*g^9*e + b^2*c^7*d*f^9*e^2 + 2*a*c^8*d*f^9*e^2 -
6*b^3*c^6*d*f^8*g*e^2 - 3*a*b*c^7*d*f^8*g*e^2 + 15*b^4*c^5*d*f^7*g^2*e^2 -
6*a*b^2*c^6*d*f^7*g^2*e^2 - 20*b^5*c^4*d*f^6*g^3*e^2 + 13*a*b^3*c^5*d*f^6*g
^3*e^2 + 16*a^2*b*c^6*d*f^6*g^3*e^2 + 15*b^6*c^3*d*f^5*g^4*e^2 - 48*a^2*b^2
*c^5*d*f^5*g^4*e^2 - 12*a^3*c^6*d*f^5*g^4*e^2 - 6*b^7*c^2*d*f^4*g^5*e^2 - 1
5*a*b^5*c^3*d*f^4*g^5*e^2 + 51*a^2*b^3*c^4*d*f^4*g^5*e^2 + 42*a^3*b*c^5*d*f
^4*g^5*e^2 + b^8*c*d*f^3*g^6*e^2 + 12*a*b^6*c^2*d*f^3*g^6*e^2 - 19*a^2*b^4*
c^3*d*f^3*g^6*e^2 - 50*a^3*b^2*c^4*d*f^3*g^6*e^2 - 16*a^4*c^5*d*f^3*g^6*e^2
 - 3*a*b^7*c*d*f^2*g^7*e^2 - 3*a^2*b^5*c^2*d*f^2*g^7*e^2 + 27*a^3*b^3*c^3*d
*f^2*g^7*e^2 + 24*a^4*b*c^4*d*f^2*g^7*e^2 + 3*a^2*b^6*c*d*f*g^8*e^2 - 6*a^3
*b^4*c^2*d*f*g^8*e^2 - 9*a^4*b^2*c^3*d*f*g^8*e^2 - 6*a^5*c^4*d*f*g^8*e^2 -
a^3*b^5*c*d*g^9*e^2 + 3*a^4*b^3*c^2*d*g^9*e^2 + a^5*b*c^3*d*g^9*e^2 - a*b*c
```

^7*f^9*e^3 + 6*a*b^2*c^6*f^8*g*e^3 - 6*a^2*c^7*f^8*g*e^3 - 15*a*b^3*c^5*f^7*g^2*e^3 + 24*a^2*b*c^6*f^7*g^2*e^3 + 20*a*b^4*c^4*f^6*g^3*e^3 - 34*a^2*b^2*c^5*f^6*g^3*e^3 - 16*a^3*c^6*f^6*g^3*e^3 - 15*a*b^5*c^3*f^5*g^4*e^3 + 15*a^2*b^3*c^4*f^5*g^4*e^3 + 54*a^3*b*c^5*f^5*g^4*e^3 + 6*a*b^6*c^2*f^4*g^5*e^3 + 9*a^2*b^4*c^3*f^4*g^5*e^3 - 66*a^3*b^2*c^4*f^4*g^5*e^3 - 12*a^4*c^5*f^4*g^5*e^3 - a*b^7*c*f^3*g^6*e^3 - 11*a^2*b^5*c^2*f^3*g^6*e^3 + 31*a^3*b^3*c^3*f^3*g^6*e^3 + 32*a^4*b*c^4*f^3*g^6*e^3 + 3*a^2*b^6*c*f^2*g^7*e^3 - 30*a^4*b^2*c^3*f^2*g^7*e^3 - 3*a^3*b^5*c*f*g^8*e^3 + 9*a^4*b^3*c^2*f*g^8*e^3 + 3*a^5*b*c^3*f*g^8*e^3 + a^4*b^4*c*g^9*e^3 - 4*a^5*b^2*c^2*g^9*e^3 + 2*a^6*c^3*g^9*e^3)*x/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24*a*b*c^8*d^4*f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3*c^7*d^3*f^12*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2*c^7*d^3*f^11*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*e + 48*a^2*b*c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*d^3*f^9*g^3*e - 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e + 450*a^2*b^3*c^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^3*f^7*g^5*e + 72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e - 480*a^3*b^2*c^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^6*g^6*e + 60*a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 160*a^4*b*c^5*d^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f^5*g^7*e - 360*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e - 30*a^2*b^7*c*d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*d^3*f^4*g^8*e + 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e - 240*a^5*b^2*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3*c^2*d^3*f^2*g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11*e - 48*a^6*b^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*d^3*g^12*e + 8*a^7*b*c^2*d^3*g^12*e + b^4*c^6*d^2*f^12*e^2 - 2*a*b^2*c^7*d^2*f^12*e^2 - 8*a^2*c^8*d^2*f^12*e^2 - 6*b^5*c^5*d^2*f^11*g*e^2 + 12*a*b^3*c^6*d^2*f^11*g*e^2 + 48*a^2*b*c^7*d^2*f^11*g*e^2 + 15*b^6*c^4*d^2*f^10*g^2*e^2 - 24*a*b^4*c^5*d^2*f^10*g^2*e^2 - 132*a^2*b^2*c^6*d^2*f^10*g^2*e^2 - 48*a^3*c^7*d^2*f^10*g^2*e^2 - 20*b^7*c^3*d^2*f^9*g^3*e^2 + 10*a*b^5*c^4*d^2*f^9*g^3*e^2 + 220*a^2*b^3*c^5*d^2*f^9*g^3*e^2 + 240*a^3*b*c^6*d^2*f^9*g^3*e^2 + 15*b^8*c^2*d^2*f^8*g^4*e^2 + 30*a*b^6*c^3*d^2*f^8*g^4*e^2 - 225*a^2*b^4*c^4*d^2*f^8*g^4*e^2 - 510*a^3*b^2*c^5*d^2*f^8*g^4*e^2 - 120*a^4*c^6*d^2*f^8*g^4*e^2 - 6*b^9*c*d^2*f^7*g^5*e^2 - 48*a*b^7*c^2*d^2*f^7*g^5*e^2 + 108*a^2*b^5*c^3*d^2*f^7*g^5*e^2 + 600*a^3*b^3*c^4*d^2*f^7*g^5*e^2 + 480*a^4*b*c^5*d^2*f^7*g^5*e^2 + b^10*d^2*f^6*g^6*e^2 + 28*a*b^8*c*d^2*f^6*g^6*e^2 + 22*a^2*b^6*c^2*d^2*f^6*g^6*e^2 - 400*a^3*b^4*c^3*d^2*f^6*g^6*e^2 - 760*a^4*b^2*c^4*d^2*f^6*g^6*e^2 - 160*a^5*c^5*d^2*f^6*g^6*e^2 - 6*a*b^9*d^2*f^5*g^7*e^2 - 48*a^2*b^7*c*d^2*f^5*g^7*e^2 + 108*a^3*b^5*c^2*d^2*f^5*g^7*e^2 + 600*a^4*b^3*c^3*d^2*f^5*g^7*e^2 + 480*a^5*b*c^4*d^2*f^5*g^7*e^2 + 15*a^2*b^8*d^2*f^4*g^8*e^2 + 30*a^3*b^6*c*d^2*f^4*g^8*e^2 - 225*a^4*b^4*c^2*d^2*f^4*g^8*e^2 - 510*a^5*b^2*c^3*d^2*f^4*g^8*e^2 - 120*a^6*c^4*d^2*f^4*g^8*e^2 - 20*a^3*b^7*d^2*f^3*g^9*e^2 + 10*a^4*b^5*c*d^2*f^3*g^9*e^2 + 220*a^5*b^3*c^2*d^2*f^3*g^9*e^2 + 240*a^6*b*c^3*d^2*f^3*g^9*e^2 + 15*a^4*b^6*d^2*f^2*g^10*e^2 - 24*a^5*b^4*c*d^2*f^2*g^10*e^2 - 132*a^6*b^2*c^2*d^2*f^2*g^10*e^2 - 48*a^7*c^3*d^2*f^2*g^10*e^2 - 6*a^5*b^5*d^2*f*g^11*e^2 + 12*a^6*b^3*c*d^2*f*g^11*e^2 + 48*a^7*b*c^2*d^2*f*g^11*e^2

+ a^6*b^4*d^2*g^12*e^2 - 2*a^7*b^2*c*d^2*g^12*e^2 - 8*a^8*c^2*d^2*g^12*e^2 - 2*a*b^3*c^6*d*f^12*e^3 + 8*a^2*b*c^7*d*f^12*e^3 + 12*a*b^4*c^5*d*f^11*g*e^3 - 48*a^2*b^2*c^6*d*f^11*g*e^3 - 30*a*b^5*c^4*d*f^10*g^2*e^3 + 108*a^2*b^3*c^5*d*f^10*g^2*e^3 + 48*a^3*b*c^6*d*f^10*g^2*e^3 + 40*a*b^6*c^3*d*f^9*g^3*e^3 - 100*a^2*b^4*c^4*d*f^9*g^3*e^3 - 240*a^3*b^2*c^5*d*f^9*g^3*e^3 - 30*a*b^7*c^2*d*f^8*g^4*e^3 + 450*a^3*b^3*c^4*d*f^8*g^4*e^3 + 120*a^4*b*c^5*d*f^8*g^4*e^3 + 12*a*b^8*c*d*f^7*g^5*e^3 + 72*a^2*b^6*c^2*d*f^7*g^5*e^3 - 360*a^3*b^4*c^3*d*f^7*g^5*e^3 - 480*a^4*b^2*c^4*d*f^7*g^5*e^3 - 2*a*b^9*d*f^6*g^6*e^3 - 52*a^2*b^7*c*d*f^6*g^6*e^3 + 60*a^3*b^5*c^2*d*f^6*g^6*e^3 + 680*a^4*b^3*c^3*d*f^6*g^6*e^3 + 160*a^5*b*c^4*d*f^6*g^6*e^3 + 12*a^2*b^8*d*f^5*g^7*e^3 + 72*a^3*b^6*c*d*f^5*g^7*e^3 - 360*a^4*b^4*c^2*d*f^5*g^7*e^3 - 480*a^5*b^2*c^3*d*f^5*g^7*e^3 - 30*a^3*b^7*d*f^4*g^8*e^3 + 450*a^5*b^3*c^2*d*f^4*g^8*e^3 + 120*a^6*b*c^3*d*f^4*g^8*e^3 + 40*a^4*b^6*d*f^3*g^9*e^3 - 100*a^5*b^4*c*d*f^3*g^9*e^3 - 240*a^6*b^2*c^2*d*f^3*g^9*e^3 - 30*a^5*b^5*d*f^2*g^10*e^3 + 108*a^6*b^3*c*d*f^2*g^10*e^3 + 48*a^7*b*c^2*d*f^2*g^10*e^3 + 12*a^6*b^4*d*f*g^11*e^3 - 48*a^7*b^2*c*d*f*g^11*e^3 - 2*a^7*b^3*d*g^12*e^3 + 8*a^8*b*c*d*g^12*e^3 + a^2*b^2*c^6*f^12*e^4 - 4*a^3*c^7*f^12*e^4 - 6*a^2*b^3*c^5*f^11*g*e^4 + 24*a^3*b*c^6*f^11*g*e^4 + 15*a^2*b^4*c^4*f^10*g^2*e^4 - 54*a^3*b^2*c^5*f^10*g^2*e^4 - 24*a^4*c^6*f^10*g^2*e^4 - 20*a^2*b^5*c^3*f^9*g^3*e^4 + 50*a^3*b^3*c^4*f^9*g^3*e^4 + 120*a^4*b*c^5*f^9*g^3*e^4 + 15*a^2*b^6*c^2*f^8*g^4*e^4 - 225*a^4*b^2*c^4*f^8*g^4*e^4 - 60*a^5*c^5*f^8*g^4*e^4 - 6*a^2*b^7*c*f^7*g^5*e^4 - 36*a^3*b^5*c^2*f^7*g^5*e^4 + 180*a^4*b^3*c^3*f^7*g^5*e^4 + 240*a^5*b*c^4*f^7*g^5*e^4 + a^2*b^8*f^6*g^6*e^4 + 26*a^3*b^6*c*f^6*g^6*e^4 - 30*a^4*b^4*c^2*f^6*g^6*e^4 - 340*a^5*b^2*c^3*f^6*g^6*e^4 - 80*a^6*c^4*f^6*g^6*e^4 - 6*a^3*b^7*f^5*g^7*e^4 - 36*a^4*b^5*c*f^5*g^7*e^4 + 180*a^5*b^3*c^2*f^5*g^7*e^4 + 240*a^6*b*c^3*f^5*g^7*e^4 + 15*a^4*b^6*f^4*g^8*e^4 - 225*a^6*b^2*c^2*f^4*g^8*e^4 - 60*a^7*c^3*f^4*g^8*e^4 - 20*a^5*b^5*f^3*g^9*e^4 + 50*a^6*b^3*c*f^3*g^9*e^4 + 120*a^7*b*c^2*f^3*g^9*e^4 + 15*a^6*b^4*f^2*g^10*e^4 - 54*a^7*b^2*c*f^2*g^10*e^4 - 24*a^8*c^2*f^2*g^10*e^4 - 6*a^7*b^3*f*g^11*e^4 + 24*a^8*b*c*f*g^11*e^4 + a^8*b^2*g^12*e^4 - 4*a^9*c*g^12*e^4)
+ (b*c^8*d^3*f^9 - 6*b^2*c^7*d^3*f^8*g + 6*a*c^8*d^3*f^8*g + 15*b^3*c^6*d^3*f^7*g^2 - 24*a*b*c^7*d^3*f^7*g^2 - 20*b^4*c^5*d^3*f^6*g^3 + 34*a*b^2*c^6*d^3*f^6*g^3 + 16*a^2*c^7*d^3*f^6*g^3 + 15*b^5*c^4*d^3*f^5*g^4 - 15*a*b^3*c^5*d^3*f^5*g^4 - 54*a^2*b*c^6*d^3*f^5*g^4 - 6*b^6*c^3*d^3*f^4*g^5 - 9*a*b^4*c^4*d^3*f^4*g^5 + 66*a^2*b^2*c^5*d^3*f^4*g^5 + 12*a^3*c^6*d^3*f^4*g^5 + b^7*c^2*d^3*f^3*g^6 + 11*a*b^5*c^3*d^3*f^3*g^6 - 31*a^2*b^3*c^4*d^3*f^3*g^6 - 32*a^3*b*c^5*d^3*f^3*g^6 - 3*a*b^6*c^2*d^3*f^2*g^7 + 30*a^3*b^2*c^4*d^3*f^2*g^7 + 3*a^2*b^5*c^2*d^3*f*g^8 - 9*a^3*b^3*c^3*d^3*f*g^8 - 3*a^4*b*c^4*d^3*f*g^8 - a^3*b^4*c^2*d^3*g^9 + 4*a^4*b^2*c^3*d^3*g^9 - 2*a^5*c^4*d^3*g^9 - 2*b^2*c^7*d^2*f^9*e + 2*a*c^8*d^2*f^9*e + 12*b^3*c^6*d^2*f^8*g*e - 21*a*b*c^7*d^2*f^8*g*e - 30*b^4*c^5*d^2*f^7*g^2*e + 66*a*b^2*c^6*d^2*f^7*g^2*e + 40*b^5*c^4*d^2*f^6*g^3*e - 89*a*b^3*c^5*d^2*f^6*g^3*e - 32*a^2*b*c^6*d^2*f^6*g^3*e - 30*b^6*c^3*d^2*f^5*g^4*e + 45*a*b^4*c^4*d^2*f^5*g^4*e + 114*a^2*b^2*c^5*d^2*f^5*g^4*e - 12*a^3*c^6*d^2*f^5*g^4*e + 12*b^7*c^2*d^2*f^4*g^5*e + 12*a*b^5*c^3*d^2*f^4*g^5*e - 147*a^2*b^3*c^4*d^2*f^4*g^5*e + 6*a^3*b*c^5*d^2*f^4*g^5*e - 2*b^8*c*d^2*f^3*g^6*e - 21*a*b^6*c^2*d^2*f^3*g^6*e + 74*a^2*b^4*c^3*d^2*f^3*g^6*e + 46*a^3*b^2*c^4*d^2*f^3*g^6*e - 16*a^4*c^5*d^2*f^3*g^6*e + 6*a*b^7*c*d^2*f^2*g^7*e - 3*a^2*b^5*c^2*d^2*f^2*g^7*e - 63*a^3*b^3*c^3*d^2*f^2*g^7*e + 24*a^4*b*c^4*d^2*f^2*g^7*e - 6*a^2*b^6*c*d^2*f*g^8*e + 21*a^3*b^4*c^2*d^2*f*g^8*e - 6*a^5*c^4*d^2*f*g^8*e + 2*a^3*b^5*c*d^2*g^9*e - 9*a^4*b^3*c^2*d^2*g^9*e + 7*a^5*b*c^3*d^2*g^9*e + b^3*c^6*d*f^9*e^2 - a*b*c^7*d*f^9*e^2 - 6*b^4*c^5*d*f^8*g*e^2 + 9*a*b^2*c^6*d*f^8*g*e^2 + 6*a^2*c^7*d*f^8*g*e^2 + 15*b^5*c^4*d*f^7*g^2*e^2 - 27*a*b^3*c^5*d*f^7*g^2*e^2 - 24*a^2*b*c^6*d*f^7*g^2*e^2 - 20*b^6*c^3*d*f^6*g^3*e^2 + 35*a*b^4*c^4*d*f^6*g^3*e^2 + 50*a^2*b^2*c^5*d*f^6*g^3*e^2 + 16*a^3*c^6*d*f^6*g^3*e^2 + 15*b^7*c^2*d*f^5*g^4*e^2 - 15*a*b^5*c^3*d*f^5*g^4*e^2 - 75*a^2*b^3*c^4*d*f^5*g^4*e^2 - 42*a^3*b*c^5*d*f^5*g^4*e^2 - 6*b^8*c*d*f^4*g^5*e^2 - 9*a*b^6*c^2*d*f^4*g^5*e^2 + 72*a^2*b^4*c^3*d*f^4*g^5*e^2 + 48*a^3*b^2*c^4*d*f^4*g^5*e^2 + 12*a^4*c^5*d*f^4*g^5*e^2 + b^9*d*f^3*g^6*e^2 + 11*a*b^7*c*d*f^3*g^6*e^2 - 32*a^2*b^5

*c^2*d*f^3*g^6*e^2 - 45*a^3*b^3*c^3*d*f^3*g^6*e^2 - 16*a^4*b*c^4*d*f^3*g^6*e^2 - 3*a*b^8*d*f^2*g^7*e^2 + 33*a^3*b^4*c^2*d*f^2*g^7*e^2 + 6*a^4*b^2*c^3*d*f^2*g^7*e^2 + 3*a^2*b^7*d*f*g^8*e^2 - 9*a^3*b^5*c*d*f*g^8*e^2 - 6*a^4*b^3*c^2*d*f*g^8*e^2 + 3*a^5*b*c^3*d*f*g^8*e^2 - a^3*b^6*d*g^9*e^2 + 4*a^4*b^4*c*d*g^9*e^2 - a^5*b^2*c^2*d*g^9*e^2 - 2*a^6*c^3*d*g^9*e^2 - a*b^2*c^6*f^9*e^3 + 2*a^2*c^7*f^9*e^3 + 6*a*b^3*c^5*f^8*g*e^3 - 15*a^2*b*c^6*f^8*g*e^3 - 15*a*b^4*c^4*f^7*g^2*e^3 + 42*a^2*b^2*c^5*f^7*g^2*e^3 + 20*a*b^5*c^3*f^6*g^3*e^3 - 55*a^2*b^3*c^4*f^6*g^3*e^3 - 16*a^3*b*c^5*f^6*g^3*e^3 - 15*a*b^6*c^2*f^5*g^4*e^3 + 30*a^2*b^4*c^3*f^5*g^4*e^3 + 60*a^3*b^2*c^4*f^5*g^4*e^3 - 12*a^4*c^5*f^5*g^4*e^3 + 6*a*b^7*c*f^4*g^5*e^3 + 3*a^2*b^5*c^2*f^4*g^5*e^3 - 81*a^3*b^3*c^3*f^4*g^5*e^3 + 18*a^4*b*c^4*f^4*g^5*e^3 - a*b^8*f^3*g^6*e^3 - 10*a^2*b^6*c*f^3*g^6*e^3 + 43*a^3*b^4*c^2*f^3*g^6*e^3 + 14*a^4*b^2*c^3*f^3*g^6*e^3 - 16*a^5*c^4*f^3*g^6*e^3 + 3*a^2*b^7*f^2*g^7*e^3 - 3*a^3*b^5*c*f^2*g^7*e^3 - 33*a^4*b^3*c^2*f^2*g^7*e^3 + 24*a^5*b*c^3*f^2*g^7*e^3 - 3*a^3*b^6*f*g^8*e^3 + 12*a^4*b^4*c*f*g^8*e^3 - 3*a^5*b^2*c^2*f*g^8*e^3 - 6*a^6*c^3*f*g^8*e^3 + a^4*b^5*g^9*e^3 - 5*a^5*b^3*c*g^9*e^3 + 5*a^6*b*c^2*g^9*e^3)/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24*a*b*c^8*d^4*f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3*c^7*d^3*f^12*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2*c^7*d^3*f^11*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*e + 48*a^2*b*c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*d^3*f^9*g^3*e - 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e + 450*a^2*b^3*c^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^3*f^7*g^5*e + 72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e - 480*a^3*b^2*c^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^6*g^6*e + 60*a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 160*a^4*b*c^5*d^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f^5*g^7*e - 360*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e - 30*a^2*b^7*c*d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*d^3*f^4*g^8*e + 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e - 240*a^5*b^2*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3*c^2*d^3*f^2*g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11*e - 48*a^6*b^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*d^3*g^12*e + 8*a^7*b*c^2*d^3*g^12*e + b^4*c^6*d^2*f^12*e^2 - 2*a*b^2*c^7*d^2*f^12*e^2 - 8*a^2*c^8*d^2*f^12*e^2 - 6*b^5*c^5*d^2*f^11*g*e^2 + 12*a*b^3*c^6*d^2*f^11*g*e^2 + 48*a^2*b*c^7*d^2*f^11*g*e^2 + 15*b^6*c^4*d^2*f^10*g^2*e^2 - 24*a*b^4*c^5*d^2*f^10*g^2*e^2 - 132*a^2*b^2*c^6*d^2*f^10*g^2*e^2 - 48*a^3*c^7*d^2*f^10*g^2*e^2 - 20*b^7*c^3*d^2*f^9*g^3*e^2 + 10*a*b^5*c^4*d^2*f^9*g^3*e^2 + 220*a^2*b^3*c^5*d^2*f^9*g^3*e^2 + 240*a^3*b*c^6*d^2*f^9*g^3*e^2 + 15*b^8*c^2*d^2*f^8*g^4*e^2 + 30*a*b^6*c^3*d^2*f^8*g^4*e^2 - 225*a^2*b^4*c^4*d^2*f^8*g^4*e^2 - 510*a^3*b^2*c^5*d^2*f^8*g^4*e^2 - 120*a^4*c^6*d^2*f^8*g^4*e^2 - 6*b^9*c*d^2*f^7*g^5*e^2 - 48*a*b^7*c^2*d^2*f^7*g^5*e^2 + 108*a^2*b^5*c^3*d^2*f^7*g^5*e^2 + 600*a^3*b^3*c^4*d^2*f^7*g^5*e^2 + 480*a^4*b*c^5*d^2*f^7*g^5*e^2 + b^10*d^2*f^6*g^6*e^2 + 28*a*b^8*c*d^2*f^6*g^6*e^2 + 22*a^2*b^6*c^2*d^2*f^6*g^6*e^2 - 400*a^3*b^4*c^3*d^2*f^6*g^6*e^2 - 760*a^4*b^2*c^4*d^2*f^6*g^6*e^2 - 160*a^5*c^5*d^2*f^6*g^6*e^2 - 6*a*b^9*d^2*f^5*g^7*e^2 - 48*a^2*b^7*c*d^2*f^5*g^7*e^2 + 108*a^3*b^5*c^2*d^2*f^5*g^7*e^2 + 600*a^4*b^3*c^3*d^2*f^5*g^7*e^2 + 480*a^5*b*c^4*d

^2*f^5*g^7*e^2 + 15*a^2*b^8*d^2*f^4*g^8*e^2 + 30*a^3*b^6*c*d^2*f^4*g^8*e^2 - 225*a^4*b^4*c^2*d^2*f^4*g^8*e^2 - 510*a^5*b^2*c^3*d^2*f^4*g^8*e^2 - 120*a^6*c^4*d^2*f^4*g^8*e^2 - 20*a^3*b^7*d^2*f^3*g^9*e^2 + 10*a^4*b^5*c*d^2*f^3*g^9*e^2 + 220*a^5*b^3*c^2*d^2*f^3*g^9*e^2 + 240*a^6*b*c^3*d^2*f^3*g^9*e^2 + 15*a^4*b^6*d^2*f^2*g^10*e^2 - 24*a^5*b^4*c*d^2*f^2*g^10*e^2 - 132*a^6*b^2*c^2*d^2*f^2*g^10*e^2 - 48*a^7*c^3*d^2*f^2*g^10*e^2 - 6*a^5*b^5*d^2*f*g^11*e^2 + 12*a^6*b^3*c*d^2*f*g^11*e^2 + 48*a^7*b*c^2*d^2*f*g^11*e^2 + a^6*b^4*d^2*g^12*e^2 - 2*a^7*b^2*c*d^2*g^12*e^2 - 8*a^8*c^2*d^2*g^12*e^2 - 2*a*b^3*c^6*d*f^12*e^3 + 8*a^2*b*c^7*d*f^12*e^3 + 12*a*b^4*c^5*d*f^11*g*e^3 - 48*a^2*b^2*c^6*d*f^11*g*e^3 - 30*a*b^5*c^4*d*f^10*g^2*e^3 + 108*a^2*b^3*c^5*d*f^10*g^2*e^3 + 48*a^3*b*c^6*d*f^10*g^2*e^3 + 40*a*b^6*c^3*d*f^9*g^3*e^3 - 100*a^2*b^4*c^4*d*f^9*g^3*e^3 - 240*a^3*b^2*c^5*d*f^9*g^3*e^3 - 30*a*b^7*c^2*d*f^8*g^4*e^3 + 450*a^3*b^3*c^4*d*f^8*g^4*e^3 + 120*a^4*b*c^5*d*f^8*g^4*e^3 + 12*a*b^8*c*d*f^7*g^5*e^3 + 72*a^2*b^6*c^2*d*f^7*g^5*e^3 - 360*a^3*b^4*c^3*d*f^7*g^5*e^3 - 480*a^4*b^2*c^4*d*f^7*g^5*e^3 - 2*a*b^9*d*f^6*g^6*e^3 - 52*a^2*b^7*c*d*f^6*g^6*e^3 + 60*a^3*b^5*c^2*d*f^6*g^6*e^3 + 680*a^4*b^3*c^3*d*f^6*g^6*e^3 + 160*a^5*b*c^4*d*f^6*g^6*e^3 + 12*a^2*b^8*d*f^5*g^7*e^3 + 72*a^3*b^6*c*d*f^5*g^7*e^3 - 360*a^4*b^4*c^2*d*f^5*g^7*e^3 - 480*a^5*b^2*c^3*d*f^5*g^7*e^3 - 30*a^3*b^7*d*f^4*g^8*e^3 + 450*a^5*b^3*c^2*d*f^4*g^8*e^3 + 120*a^6*b*c^3*d*f^4*g^8*e^3 + 40*a^4*b^6*d*f^3*g^9*e^3 - 100*a^5*b^4*c*d*f^3*g^9*e^3 - 240*a^6*b^2*c^2*d*f^3*g^9*e^3 - 30*a^5*b^5*d*f^2*g^10*e^3 + 108*a^6*b^3*c*d*f^2*g^10*e^3 + 48*a^7*b*c^2*d*f^2*g^10*e^3 + 12*a^6*b^4*d*f*g^11*e^3 - 48*a^7*b^2*c*d*f*g^11*e^3 - 2*a^7*b^3*d*g^12*e^3 + 8*a^8*b*c*d*g^12*e^3 + a^2*b^2*c^6*f^12*e^4 - 4*a^3*c^7*f^12*e^4 - 6*a^2*b^3*c^5*f^11*g*e^4 + 24*a^3*b*c^6*f^11*g*e^4 + 15*a^2*b^4*c^4*f^10*g^2*e^4 - 54*a^3*b^2*c^5*f^10*g^2*e^4 - 24*a^4*c^6*f^10*g^2*e^4 - 20*a^2*b^5*c^3*f^9*g^3*e^4 + 50*a^3*b^3*c^4*f^9*g^3*e^4 + 120*a^4*b*c^5*f^9*g^3*e^4 + 15*a^2*b^6*c^2*f^8*g^4*e^4 - 225*a^4*b^2*c^4*f^8*g^4*e^4 - 60*a^5*c^5*f^8*g^4*e^4 - 6*a^2*b^7*c*f^7*g^5*e^4 - 36*a^3*b^5*c^2*f^7*g^5*e^4 + 180*a^4*b^3*c^3*f^7*g^5*e^4 + 240*a^5*b*c^4*f^7*g^5*e^4 + a^2*b^8*f^6*g^6*e^4 + 26*a^3*b^6*c*f^6*g^6*e^4 - 30*a^4*b^4*c^2*f^6*g^6*e^4 - 340*a^5*b^2*c^3*f^6*g^6*e^4 - 80*a^6*c^4*f^6*g^6*e^4 - 6*a^3*b^7*f^5*g^7*e^4 - 36*a^4*b^5*c*f^5*g^7*e^4 + 180*a^5*b^3*c^2*f^5*g^7*e^4 + 240*a^6*b*c^3*f^5*g^7*e^4 + 15*a^4*b^6*f^4*g^8*e^4 - 225*a^6*b^2*c^2*f^4*g^8*e^4 - 60*a^7*c^3*f^4*g^8*e^4 - 20*a^5*b^5*f^3*g^9*e^4 + 50*a^6*b^3*c*f^3*g^9*e^4 + 120*a^7*b*c^2*f^3*g^9*e^4 + 15*a^6*b^4*f^2*g^10*e^4 - 54*a^7*b^2*c*f^2*g^10*e^4 - 24*a^8*c^2*f^2*g^10*e^4 - 6*a^7*b^3*f*g^11*e^4 + 24*a^8*b*c*f*g^11*e^4 + a^8*b^2*g^12*e^4 - 4*a^9*c*g^12*e^4))/sqrt(c*x^2 + b*x + a) + 1/4*(48*c^2*d^2*f^2*g^5 - 48*b*c*d^2*f*g^6 + 15*b^2*d^2*g^7 - 12*a*c*d^2*g^7 - 120*c^2*d*f^3*g^4*e + 132*b*c*d*f^2*g^5*e - 42*b^2*d*f*g^6*e + 12*a*b*d*g^7*e + 80*c^2*f^4*g^3*e^2 - 100*b*c*f^3*g^4*e^2 + 35*b^2*f^2*g^5*e^2 + 28*a*c*f^2*g^5*e^2 - 28*a*b*f*g^6*e^2 + 8*a^2*g^7*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^3*d^3*f^6*g^3 - 3*b*c^2*d^3*f^5*g^4 + 3*b^2*c*d^3*f^4*g^5 + 3*a*c^2*d^3*f^4*g^5 - b^3*d^3*f^3*g^6 - 6*a*b*c*d^3*f^3*g^6 + 3*a*b^2*d^3*f^2*g^7 + 3*a^2*c*d^3*f^2*g^7 - 3*a^2*b*d^3*f*g^8 + a^3*d^3*g^9 - 3*c^3*d^2*f^7*g^2*e + 9*b*c^2*d^2*f^6*g^3*e - 9*b^2*c*d^2*f^5*g^4*e - 9*a*c^2*d^2*f^5*g^4*e + 3*b^3*d^2*f^4*g^5*e + 18*a*b*c*d^2*f^4*g^5*e - 9*a*b^2*d^2*f^3*g^6*e - 9*a^2*c*d^2*f^3*g^6*e + 9*a^2*b*d^2*f^2*g^7*e - 3*a^3*d^2*f*g^8*e + 3*c^3*d*f^8*g*e^2 - 9*b*c^2*d*f^7*g^2*e^2 + 9*b^2*c*d*f^6*g^3*e^2 + 9*a*c^2*d*f^6*g^3*e^2 - 3*b^3*d*f^5*g^4*e^2 - 18*a*b*c*d*f^5*g^4*e^2 + 9*a*b^2*d*f^4*g^5*e^2 + 9*a^2*c*d*f^4*g^5*e^2 - 9*a^2*b*d*f^3*g^6*e^2 + 3*a^3*d*f^2*g^7*e^2 - c^3*f^9*e^3 + 3*b*c^2*f^8*g*e^3 - 3*b^2*c*f^7*g^2*e^3 - 3*a*c^2*f^7*g^2*e^3 + b^3*f^6*g^3*e^3 + 6*a*b*c*f^6*g^3*e^3 - 3*a*b^2*f^5*g^4*e^3 - 3*a^2*c*f^5*g^4*e^3 + 3*a^2*b*f^4*g^5*e^3 - a^3*f^3*g^6*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^5/((c*d^5*g^3 - 3*c*d^4*f*g^2*e - b*d^4*g^3*e + 3*c*d^3*f^2*g*e^2 + 3*b*d^3*f*g^2*e^2 + a*d^3*g^3*e^2 - c*d^2*f^3*e^3 - 3*b*d^2*f^2*g*e^3 - 3*a*d^2*f*g^2*e^3 + b*d*f^3*e^4 + 3*a*d*f^2*g*e^4 - a*f^3*e^5)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x

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+ a))^3*c^2*d*f^2*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^
6 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^7 - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a*c*d*g^7 - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c
^2*f^3*g^4*e + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f^2*g^5*e - 11*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^6*e - 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a*c*f*g^6*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*
g^7*e + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g^4 - 48*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*f^2*g^5 + 13*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^6 - 28*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*c^(3/2)*d*f*g^6 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*
sqrt(c)*d*g^7 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g^3*e
+ 68*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f^3*g^4*e - 17*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^5*e + 20*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^5*e - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a*b*sqrt(c)*f*g^6*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*
sqrt(c)*g^7*e + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^4 - 44
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^5 - 88*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d*f^2*g^5 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3*d*f*g^6 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^6 - 9*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*c*d*g^7 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g^3
*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g^4*e + 112*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g^4*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^3*f^2*g^5*e - 104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*
g^5*e + 17*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^6*e + 28*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^6*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^2*b*g^7*e + 14*b^2*c^(3/2)*d*f^3*g^4 - 7*b^3*sqrt(c)*d*f^2*g^5 - 44
*a*b*c^(3/2)*d*f^2*g^5 + 23*a*b^2*sqrt(c)*d*f*g^6 + 28*a^2*c^(3/2)*d*f*g^6
- 16*a^2*b*sqrt(c)*d*g^7 - 18*b^2*c^(3/2)*f^4*g^3*e + 11*b^3*sqrt(c)*f^3*g^
4*e + 56*a*b*c^(3/2)*f^3*g^4*e - 39*a*b^2*sqrt(c)*f^2*g^5*e - 36*a^2*c^(3/2
)*f^2*g^5*e + 36*a^2*b*sqrt(c)*f*g^6*e - 8*a^3*sqrt(c)*g^7*e)/((c^3*d^2*f^6
*g^2 - 3*b*c^2*d^2*f^5*g^3 + 3*b^2*c*d^2*f^4*g^4 + 3*a*c^2*d^2*f^4*g^4 - b^
3*d^2*f^3*g^5 - 6*a*b*c*d^2*f^3*g^5 + 3*a*b^2*d^2*f^2*g^6 + 3*a^2*c*d^2*f^2
*g^6 - 3*a^2*b*d^2*f*g^7 + a^3*d^2*g^8 - 2*c^3*d*f^7*g*e + 6*b*c^2*d*f^6*g^
2*e - 6*b^2*c*d*f^5*g^3*e - 6*a*c^2*d*f^5*g^3*e + 2*b^3*d*f^4*g^4*e + 12*a*
b*c*d*f^4*g^4*e - 6*a*b^2*d*f^3*g^5*e - 6*a^2*c*d*f^3*g^5*e + 6*a^2*b*d*f^2
*g^6*e - 2*a^3*d*f*g^7*e + c^3*f^8*e^2 - 3*b*c^2*f^7*g*e^2 + 3*b^2*c*f^6*g^
2*e^2 + 3*a*c^2*f^6*g^2*e^2 - b^3*f^5*g^3*e^2 - 6*a*b*c*f^5*g^3*e^2 + 3*a*b
^2*f^4*g^4*e^2 + 3*a^2*c*f^4*g^4*e^2 - 3*a^2*b*f^3*g^5*e^2 + a^3*f^2*g^6*e^
2)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*sqrt(c)*f + b*f - a*g)^2)
```

maple [B] time = 0.04, size = 5459, normalized size = 5.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex\right)\left(f+gx\right)^{3}\left(a+bx+cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**3*(a + b*x + c*x**2)**(3/2)), x)
```

3.886 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=1551

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}\,(d+ex)^4}{11e}+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2f^2\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3\right)c^5-g\left(bf\right.\right.}{}$$

```
[Out] 2/3465*(48*b^3*e^3*g^3+b*c*e^2*g^2*(-157*a*e*g-198*b*d*g+67*b*e*f)+c^3*(-56
7*d^3*g^3+1107*d^2*e*f*g^2-843*d*e^2*f^2*g+233*e^3*f^3)-c^2*e*g*(2*a*e*g*(-
231*d*g+74*e*f)-3*b*(99*d^2*g^2-88*d*e*f*g+24*e^2*f^2)))*(g*x+f)^(3/2)*(c*x
^2+b*x+a)^(1/2)/c^3/g^4-2/693*e*(8*b^2*e^2*g^2+c*e*g*(-18*a*e*g-33*b*d*g+19
*b*e*f)+c^2*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+b*x+a)
^(1/2)/c^2/g^4+2/99*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(7/2)*(c*x^2+b*x+a)^(
1/2)/c/g^4-2/3465*(64*b^4*e^4*g^4+4*b^2*c*e^3*g^3*(-69*a*e*g-66*b*d*g+7*b*e
*f)+c^4*(315*d^4*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+1
87*e^4*f^4)+3*c^2*e^2*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f)+3*b^2*(
44*d^2*g^2-11*d*e*f*g+e^2*f^2))-c^3*e*g*(6*a*e*g*(165*d^2*g^2-33*d*e*f*g+2*
e^2*f^2)+b*(231*d^3*g^3-99*d^2*e*f*g^2+8*e^3*f^3)))*(g*x+f)^(1/2)*(c*x^2+b*
x+a)^(1/2)/c^4/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3
465*(128*b^5*e^3*g^5-8*b^3*c*e^2*g^4*(87*a*e*g+66*b*d*g+7*b*e*f)+2*c^5*f^2*
(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+b*c^2*e*g^3*(771*
a^2*e^2*g^2+6*a*b*e*g*(396*d*g+43*e*f)-b^2*(-792*d^2*g^2-264*d*e*f*g+37*e^2
*f^2))-c^4*g*(b*f*(-462*d^3*g^3+495*d^2*e*f*g^2-264*d*e^2*f^2*g+56*e^3*f^3)
-18*a*g*(77*d^3*g^3+88*d^2*e*f*g^2-33*d*e^2*f^2*g+6*e^3*f^3))-c^3*g^2*(6*a^
2*e^2*g^2*(231*d*g+26*e*f)-9*a*b*e*g*(-319*d^2*g^2-110*d*e*f*g+15*e^2*f^2)+
b^2*(462*d^3*g^3+495*d^2*e*f*g^2-198*d*e^2*f^2*g+37*e^3*f^3)))*EllipticE(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^5/g^5/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3465*(a*
g^2-b*f*g+c*f^2)*(64*b^4*e^3*g^4+4*b^2*c*e^2*g^3*(-69*a*e*g-66*b*d*g+7*b*e*
f)-2*c^4*f*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*
e*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f)+3*b^2*(44*d^2*g^2-11*d*e*f*
g+e^2*f^2))-c^3*g*(6*a*e*g*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+b*(231*d^3*g^
3-99*d^2*e*f*g^2+8*e^3*f^3)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^5/g^5/(g
*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 8.09, antiderivative size = 1551, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {918, 1653, 843, 718, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}\,(d+ex)^4}{11e}+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2f^2\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3\right)c^5-g\left(bf\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*e*g) + c^4
*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 +
315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) +
 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^2 -
 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3465*c^4*e*g^4) + (2*(d + e*x)^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])/(11*e) + (2*(48*b^3*e^3*g^3 + b*c*e^2*g^2
*(67*b*e*f - 198*b*d*g - 157*a*e*g) + c^3*(233*e^3*f^3 - 843*d*e^2*f^2*g +
1107*d^2*e*f*g^2 - 567*d^3*g^3) - c^2*e*g*(2*a*e*g*(74*e*f - 231*d*g) - 3*b
*(24*e^2*f^2 - 88*d*e*f*g + 99*d^2*g^2)))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*
x^2])/(3465*c^3*g^4) - (2*e*(8*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 1
8*a*e*g) + c^2*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt
[a + b*x + c*x^2])/(693*c^2*g^4) + (2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(f + g*
x)^(7/2)*Sqrt[a + b*x + c*x^2])/(99*c*g^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(12
8*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 87*a*e*g) + 2*c^5*f^2
*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + b*c^2*e*g
^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43*e*f + 396*d*g) - b^2*(37*e^2*f^2 - 264*
d*e*f*g - 792*d^2*g^2)) - c^4*g*(b*f*(56*e^3*f^3 - 264*d*e^2*f^2*g + 495*d^
2*e*f*g^2 - 462*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*
g^2 + 77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f + 231*d*g) - 9*a*b*e*g*
(15*e^2*f^2 - 110*d*e*f*g - 319*d^2*g^2) + b^2*(37*e^3*f^3 - 198*d*e^2*f^2*
g + 495*d^2*e*f*g^2 + 462*d^3*g^3)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(3465*c^5*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 -
b*f*g + a*g^2)*(64*b^4*e^3*g^4 + 4*b^2*c*e^2*g^3*(7*b*e*f - 66*b*d*g - 69*a
*e*g) - 2*c^4*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g
^3) + 3*c^2*e*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) + 3*b^2*(e^2
*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g +
 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3465*c^5*g^5*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{\int \frac{(d+ex)^3\left(bdf-3aef+adg+2(cdf-bef+b\right.}{\sqrt{f+gx}\sqrt{a+bx}}}{11e}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2e^2(cef-3cdg+beg)(f+gx)^{7}}{99cg^4}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{2e\left(8b^2e^2g^2+ceg(19bef-33bd\right.}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2\left(48b^3e^3g^3+bce^2g^2(67bef-1\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732\right.\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732\right.\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732\right.\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732\right.\right.}{}$$

Mathematica [C] time = 17.75, size = 26600, normalized size = 17.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)*sq
rt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,(ex + d)^3 \sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

maple [B] time = 0.26, size = 32647, normalized size = 21.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,(ex + d)^3 \sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f + g\,x}\,(d + e\,x)^3\,\sqrt{c\,x^2 + b\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3\,\sqrt{f + gx}\,\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.887 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=1015

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}\,(d+ex)^3}{9e}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(f^2\left(8e^2f^2-24degf+21d^2g^2\right)c^4+g\left(3ag\left(3e^2f^2-16deg\right.\right.\right.}{}$$

```
[Out] -4/315*(3*b^2*e^2*g^2+c*e*g*(-7*a*e*g-9*b*d*g+4*b*e*f)+c^2*(21*d^2*g^2-24*d
*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/63*e*(b*e*g-
3*c*d*g+c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^3+2/315*(8*b^3*e^3*g^3
+3*b*c*e^2*g^2*(-9*a*e*g-8*b*d*g+b*e*f)+c^3*(-35*d^3*g^3+63*d^2*e*f*g^2-57*
d*e^2*f^2*g+19*e^3*f^3)-3*c^2*e*g^2*(2*a*e*(-10*d*g+e*f)+b*d*(-7*d*g+2*e*f)
))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/e/g^3+2/9*(e*x+d)^3*(g*x+f)^(1/2)*
(c*x^2+b*x+a)^(1/2)/e-2/315*(8*b^4*e^2*g^4-4*b^2*c*e*g^3*(9*a*e*g+6*b*d*g+b
*e*f)+c^4*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)+3*c^2*g^2*(7*a^2*e^2*g^2+a*
b*e*g*(29*d*g+5*e*f)-b^2*(-7*d^2*g^2-5*d*e*f*g+e^2*f^2))+c^3*g*(3*a*g*(-21*
d^2*g^2-16*d*e*f*g+3*e^2*f^2)-b*f*(21*d^2*g^2-15*d*e*f*g+4*e^2*f^2)))*Ellip
ticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/g^4
/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
15*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^2*g^3+3*b*c*e*g^2*(-9*a*e*g-8*b*d*g+b*e*f)-
2*c^3*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)-3*c^2*g^2*(2*a*e*(-10*d*g+e*f)+b*
d*(-7*d*g+2*e*f)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^4/(g*x+f)^(1/2)
/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 3.78, antiderivative size = 1015, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {918, 1653, 843, 718, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}\,(d+ex)^3}{9e}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(f^2\left(8e^2f^2-24degf+21d^2g^2\right)c^4+g\left(3ag\left(3e^2f^2-16deg\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c^3*(19*e^3
*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) - 3*c^2*e*g^2*(2*a*e*(
e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/
(315*c^3*e*g^3) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e)
 - (4*(3*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^2
 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(315*c^
2*g^3) + (2*e*(c*e*f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^
2])/(63*c*g^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4 - 4*b^2*c*e*g^
3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^
2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*(5*e*f + 29*d*g) - b^2*(e^2*f^2 - 5
```

```
*d*e*f*g - 7*d^2*g^2)) + c^3*g*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2)
 - b*f*(4*e^2*f^2 - 15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d*g - 9*a
*e*g) - 2*c^3*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*(2*a*e*(e
*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
```

```
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9e} - \frac{\int \frac{(d+ex)^2\left(bdf-3aef+adg+2(cdf-bef+b\right.}{\sqrt{f+gx}\,\sqrt{a+bx}}}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9e} + \frac{2e(cef-3cdg+beg)(f+gx)^{5/2}}{63cg^3}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9e} - \frac{4\left(3b^2e^2g^2+ceg(4bef-9bdg-\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+\right.\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+\right.\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+\right.\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+\right.\right.}{}$$

Mathematica [C] time = 15.51, size = 15781, normalized size = 15.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,(ex + d)^2\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

maple [B] time = 0.09, size = 20224, normalized size = 19.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,(ex + d)^2\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f + g\,x}\,(d + e\,x)^2\,\sqrt{c\,x^2 + b\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^2\,\sqrt{f + gx}\,\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.888 $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=652

$$-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(-cg(-5aeg+7bdg+2bef)+4b^2eg^2-3cgx(-4beg+7cdg+cef)+c^2f(4ef-7dg\right.}{105c^2g^2}$$

```
[Out] 2/7*e*(c*x^2+b*x+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(4*b^2*e*g^2+c^2*f*(-7*d*g+
4*e*f)-c*g*(-5*a*e*g+7*b*d*g+2*b*e*f)-3*c*g*(-4*b*e*g+7*c*d*g+c*e*f)*x)*(g*
x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+1/105*((-4*b*e*g+7*c*d*g+c*e*f)*(8*c
^2*f^2-2*b^2*g^2-3*c*g*(-2*a*g+b*f))-5*c*g*(-b*g+2*c*f)*(7*c*d*f-e*(a*g+3*b
*f)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)/c^3/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2)+2/105*(a*g^2-b*f*g+c*f^2)*(4*b^2*e*g^2-2*c^2*f*(-7*d*g+4*e*f)+c*g*(
-10*a*e*g-7*b*d*g+b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^
2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f
)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {832, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(cg(-10aeg-7bdg+bef)+4b^2eg^2-2c^2\right.}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b
*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*Sqrt[a + b*x + c*x^2
])/(105*c^2*g^2) + (2*e*Sqrt[f + g*x]*(a + b*x + c*x^2)^(3/2))/(7*c) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2
- 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*S
qrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^3*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2
*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(
4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(1
05*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-3bef-aeg)+\frac{1}{2}(cef+7cdg-4beg)x\right)\sqrt{}}{\sqrt{f+gx}}}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg\right.}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg\right.}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg\right.}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg\right.}{105c^2g^2}$$

Mathematica [C] time = 13.95, size = 8432, normalized size = 12.93

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{cx^2+bx+a}\,(ex+d)\sqrt{gx+f}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+bx+a}\,(ex+d)\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

maple [B] time = 0.06, size = 10711, normalized size = 16.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,(ex + d)\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f + g\,x}\;(d + e\,x)\;\sqrt{c\,x^2 + b\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)\sqrt{f + gx}\,\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.889 $\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=513

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2cf}\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/5*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/g-2/15*(-b*g+2*c*f)*(g*x+f)^(1/2)*(c*
x^2+b*x+a)^(1/2)/c/g-2/15*(c^2*f^2+b^2*g^2-c*g*(3*a*g+b*f))*EllipticE(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*
c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^
(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g^2/(c*x^2+b*
x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-b*g+2*
c*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {734, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2cf}\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c*g) + (2*(f + g
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*g) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(15*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^2*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx = \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\int \frac{\sqrt{f+gx}\,(bf-2ag+(2cf-bg)x)}{\sqrt{a+bx+cx^2}}\,dx}{5g}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\int \frac{\frac{1}{2}(b \cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} + \frac{\left((2cf - \cdots\right.}{\cdots}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\left(2\sqrt{2}\,\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\sqrt{2}\,\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 10.69, size = 697, normalized size = 1.36

$$\frac{-\frac{4g^2(a+x(b+cx))\left(-cg(3ag+bf)+b^2g^2+c^2f^2\right)}{\sqrt{f+gx}} + \frac{i(f+gx)\sqrt{1-\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}-bg+2cf\right)}}\sqrt{\frac{4\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}+bg-2cf\right)}+2}\left(\left(\sqrt{g^2(b^2-4ac)}-bg+2 \cdots\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((-4*g^2*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*(a + x*(b + c*x)))/Sqrt[f
+ g*x] + 2*c*g^2*Sqrt[f + g*x]*(a + x*(b + c*x))*(b*g + c*(f + 3*g*x)) + (I
*(f + g*x)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^
2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c
*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 -
 4*a*c)*g^2])*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*EllipticE[I*ArcSinh[(
Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*
g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (b^3*g^3 - b^2*g^2*(2*c*f + Sqrt[(b^2 - 4*
a*c)*g^2]) + b*c*g*(-4*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-(c*f^2*Sqrt
[(b^2 - 4*a*c)*g^2]) + a*g^2*(8*c*f + 3*Sqrt[(b^2 - 4*a*c)*g^2])))*Elliptic
F[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2
 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/Sqrt[(c*f^2 + g*(-(b*f) + a*g
))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/(15*c^2*g^3*Sqrt[a + x*(b + c
*x)])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+bx+a}\,\sqrt{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,\sqrt{gx + f}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

maple [B] time = 0.04, size = 4356, normalized size = 8.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/15*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2*(8*x*a*c^2*f*g^3+8*x^3*b*c^2*g^4
+8*x^3*c^3*f*g^3+6*x^2*a*c^2*g^4+2*x^2*b^2*c*g^4+2*x^2*c^3*f^2*g^2+2*a*c^2*
f^2*g^2+6*x^4*c^3*g^4+2*a*b*c*f*g^3-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2))*a*b^2*g^4+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)
))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
,(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
)*b^3*f*g^3-4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(
(-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2
*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ellipt
icE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*f*g^3+
12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4
*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*
a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)
*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*g^4-12*2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(
1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1
/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*g^4+4*2^(1/2)*(-(g*x+f)*c/
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2
*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g
+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*g^4-8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(
-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*a*c^2*f^2*g^2+12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
```

```
2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/
2)))^(1/2))*a*c^2*f^2*g^2-12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2))*a*b*c*f*g^3+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*E
llipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a
*c+b^2)^(1/2)*b*c*f^2*g^2-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/
2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/
2))*(-4*a*c+b^2)^(1/2)*a*c*f*g^3+8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*b*c*f*g^3+2*x*b^2*c*f*g^3+2*x*b*c^2*f^2*g^2+10*x^2*b*c^2*f*g^3
+2*x*a*b*c*g^4+8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*
((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ell
ipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*f
^2*g^2+2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x
+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+
(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2
)*a*b*g^4-8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-
2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c
*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Elliptic
E(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*f^3*g-
2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*
c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c
+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-
(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*f
*g^3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x
+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+
(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2
)*c^2*f^3*g-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(
(-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2
*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ellipt
icF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*f^2*
g^2+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+
(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(
-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(
```

```
1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^3*f^4)/(c*g*x^3+
b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/g^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\sqrt{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f + g\,x}\,\sqrt{c\,x^2 + b\,x + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{f + gx}\sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.890 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=764

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(eg(2aeg-3bdg+bef)+c\left(3d^2g^2-e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{\cdots}}{\sqrt{b^2-\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{3ce^3g\sqrt{f+gx}\sqrt{a+x(b+cx)}}\cdots$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3*(b*e*g-3*c*d*g+c*e*f)*EllipticE
(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*
(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c
+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/g/(c*
x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3*(e*
g*(2*a*e*g-3*b*d*g+b*e*f)+c*(3*d^2*g^2-e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)
^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/c/e^3/g/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-(a*e^2-b*d*e+c*d^2)*
EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(
1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*
c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)
/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/c^(1/2)/(c*
x^2+b*x+a)^(1/2)
```

Rubi [A] time = 4.10, antiderivative size = 969, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {918, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(3*c*e^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*(c*e*f - 3*c*d*g + b*
e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
- (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(3*c*d*(e*f - d*g) - e*(2*b*e*f - 3*b*d*g +
2*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
```

```
 - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f -
b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt
[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b
*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \frac{bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3cdg+beg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \left(\frac{3cd(ef-dg)-e(2bef-3bdg+2aeg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(cef-3cdg+beg)x}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{}{e^2}\right.}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{e^3}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{(cef-3cdg+beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3e^2g} - \frac{(f(cef-3cdg}{}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\,(3cd(ef-dg)-e(2bef-3bdg+2ae}{}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx}{b^2-}}}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4}\right.}}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx}{b^2-}}}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4}\right.}}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx}{b^2-}}}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4}\right.}}}$$

Mathematica [C] time = 15.17, size = 35245, normalized size = 46.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

maple [B] time = 0.06, size = 6812, normalized size = 8.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.891 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=743

$$\frac{\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,(cd(2ef-3dg)-e(aeg-2bdg+bef))\Pi}{\sqrt{2}\sqrt{c}\,e^3\sqrt{a+bx+cx^2}\,(ef-dg)}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)+3/2*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2
)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)
*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(
g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+(2*b*e*g-c*(3*d*g+e*f))*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*
f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+
1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/2)*c^(1/2)
*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(
-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))
^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/e^3/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

Rubi [A] time = 3.32, antiderivative size = 934, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {916, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$\frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\,e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{3\sqrt{2}\sqrt{b^2-4ac}\,f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + (3*Sqrt[b^2 - 4*a*
c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^2*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2])
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*e*f - 3*c*d*g + 2*b
*e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (
```

```
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticP
i[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2
]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqr
t[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*
Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{2cef-3cdg+2beg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2e\cdots}{e^2(d+\cdots}\right.}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2} + \frac{(2cef-3cdg+2b\cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,d\cdots}{2e^2}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx\cdots}{2cf-\left(b+\sqrt{b^2\cdots}\right.}}}{ce^3\sqrt{\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{\sqrt{2}\,e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{\sqrt{2}\,e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{\sqrt{2}\,e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a\cdots}}$$

Mathematica [C] time = 13.84, size = 16573, normalized size = 22.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

maple [B] time = 0.09, size = 16688, normalized size = 22.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2,x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

3.892 $\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=1034

$$-\frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)\left(cd(2ef-3dg)-e(bef-2b\right.}{4\sqrt{2}\,e^2\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{cx^2+bx+a}}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)^2+1/4*(c*d*(-3*d*g+2*e*f)-
e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c
*d^2)/(-d*g+e*f)/(e*x+d)-1/8*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c
+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-
b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(-c*d*(3*d*g+2*e*f)+e*(-5*a*e*g+4*b*d*g+b*e
*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1
/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))*2^(1/2)/(g*x+f)
^(1/2)/(a+x*(c*x+b))^(1/2)+1/8*(b^2*e^4*f^2+a^2*e^4*g^2+c^2*d^3*g*(-3*d*g+4
*e*f)-2*a*c*e^2*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2)-2*b*e*g*(a*e^3*f+c*d^2*(-2*
d*g+3*e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f)
,((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2
*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))/(-d*g+e*f)^2*
2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(1/2)
```

Rubi [A] time = 8.09, antiderivative size = 1705, normalized size of antiderivative = 1.65, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {916, 6742, 718, 419, 939, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*e*(d + e*x)^2) + ((c*d*(2*e*f - 3
*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(
4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(c*
d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f -
d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x +
 c*x^2]) + (3*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^3*Sqrt[f
 + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f*(c*d*(2*e*f - 3*d*g)
- e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
```

```
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b
^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c
*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(2*Sqrt[2]*e^3*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e
*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Elli
pticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(S
qrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b
- Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqr
t[2]*Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))^2*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g
+ Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f +
 g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (
2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^3*(c*d
^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e(bef-2bdg+aeg)}{e^2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \right.}{4e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e^3} + \frac{(cef-3cdg+beg)\int}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

Mathematica [C] time = 16.53, size = 33765, normalized size = 32.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\,\sqrt{gx + f}}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

maple [B] time = 0.21, size = 55368, normalized size = 53.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\,\sqrt{gx + f}}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3,x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.893 $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1098

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}\,(d+ex)^3}{9g}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(-2f\left(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3\right)c^4-g\left(\right.}{}$$

```
[Out] -2/315*e*(6*b^2*e^2*g^2+c*e*g*(-14*a*e*g-27*b*d*g+17*b*e*f)-2*c^2*(42*d^2*g
^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^4-2/63*
e^2*(-b*e*g-6*c*d*g+8*c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^4+2/315*
(8*b^3*e^3*g^3+3*b*c*e^2*g^2*(-9*a*e*g-12*b*d*g+5*b*e*f)-c^3*(-70*d^3*g^3+3
36*d^2*e*f*g^2-408*d*e^2*f^2*g+152*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*
f)-b*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c^3/g^4+2/9*(e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/315*(16*b^4*e^3
*g^4+8*b^2*c*e^2*g^3*(-9*a*e*g-9*b*d*g+2*b*e*f)-2*c^4*f*(-105*d^3*g^3+252*d
^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*e*g^2*(14*a^2*e^2*g^2-a*b*e*g*
(-87*d*g+19*e*f)+b^2*(42*d^2*g^2-27*d*e*f*g+7*e^2*f^2))-c^3*g*(6*a*e*g*(63*
d^2*g^2-39*d*e*f*g+10*e^2*f^2)-b*(-105*d^3*g^3+189*d^2*e*f*g^2-144*d*e^2*f^
2*g+40*e^3*f^3)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*
a*c+b^2))^(1/2)/c^4/g^5/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)-2/315*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^3*g^3+3*b*c*e^2*g^2*
(-9*a*e*g-12*b*d*g+5*b*e*f)+2*c^3*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f
^2*g+64*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*f)-b*(21*d^2*g^2-24*d*e*f*g
+8*e^2*f^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c
*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^5/(g*x+f)^(1/2)/(c*x
^2+b*x+a)^(1/2)
```

Rubi [A] time = 5.76, antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {920, 1653, 843, 718, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}\,(d+ex)^3}{9g}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(-2f\left(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3\right)c^4-g\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) - c^3*(152
*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3) - 3*c^2*e*g*(6*a
*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(315*c^3*g^4) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2])/(9*g) - (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*
g - 14*a*e*g) - 2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3
/2)*Sqrt[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g - b*e*
```

```
g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(63*c*g^4) - (Sqrt[2]*Sqrt[b^2 -
4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) - 2*
c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^
2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d
*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^
2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*Sqr
t[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sqrt[(c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2
*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g +
 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8
*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2])/(g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)*Simp[b*d*f + 2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e
*f - d*g)*(2*m + 1))*x - (b*e*g + 2*c*(d*g*m - e*f*(m + 1)))*x^2, x])/(Sqrt
[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
```

```
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2\left(bdf+6aef-8adg+(2cdf+7bef-7bdg-2ae\ldots\right.}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9g} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+\ldots}}{63cg^4}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9g} - \frac{2e\left(6b^2e^2g^2+ceg(17bef-27bdg-14ae\ldots\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336\ldots\right.\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336\ldots\right.\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336\ldots\right.\right.}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336\ldots\right.\right.}{}$$

Mathematica [C] time = 15.88, size = 17771, normalized size = 16.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)/sq
rt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\,(ex + d)^3}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

maple [B] time = 0.11, size = 22215, normalized size = 20.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\,(ex + d)^3}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(d + e\,x)}^3\,\sqrt{c\,x^2 + b\,x + a}}{\sqrt{f + g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
```

```
[Out] int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.894 $\int \frac{(d+ex)^2 \sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$

Optimal. Leaf size=755

$$-\frac{4\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2-\left(c^2\left(10d^2g^2-34defg+21e^2f^2\right)\right)\right)}{105c^2g^3}+\frac{4\sqrt{2}\sqrt{b^2}}{}$$

[Out] -2/35*e*(-b*e*g-4*c*d*g+6*c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/g^3-4/105*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)-c^2*(10*d^2*g^2-34*d*e*f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/7*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g+1/105*(8*b^3*e^2*g^3+b*c*e*g^2*(-29*a*e*g-28*b*d*g+9*b*e*f)-2*c^3*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2)-c^2*g*(2*a*e*g*(-42*d*g+13*e*f)-b*(35*d^2*g^2-42*d*e*f*g+16*e^2*f^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^4/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/105*(a*g^2-b*f*g+c*f^2)*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)+c^2*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^4/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A] time = 1.93, antiderivative size = 755, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {920, 1653, 843, 718, 424, 419}

$$-\frac{4\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2+c^2\left(-\left(10d^2g^2-34defg+21e^2f^2\right)\right)\right)}{105c^2g^3}+\frac{4\sqrt{2}\sqrt{b}}{}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]

[Out] (-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(21*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c^2*g^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*g) - (2*e*(6*c*e*f - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c*g^3) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g - 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2)

```
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)])/(105*c^3*g^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2])/(g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)*Simp[b*d*f + 2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e
*f - d*g)*(2*m + 1))*x - (b*e*g + 2*c*(d*g*m - e*f*(m + 1)))*x^2, x])/(Sqrt
[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(bdf+4aef-6adg+(2cdf+5bef-5bdg-2aeg)x+\right.}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7g} - \frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+}}{35cg^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+}}{105c^2g^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+}}{105c^2g^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+}}{105c^2g^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+}}{105c^2g^3}$$

Mathematica [C] time = 14.14, size = 10030, normalized size = 13.28

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx+a}}{\sqrt{gx+f}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\,(ex+d)^2}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

maple [B] time = 0.08, size = 12923, normalized size = 17.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\,(ex + d)^2}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x)^2\,\sqrt{c\,x^2 + b\,x + a}}{\sqrt{f + g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
```

```
[Out] int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.895 $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=519

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(beg-10cdg+8cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/15*(-3*c*e*g*x-b*e*g-5*c*d*g+4*c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c/g^2-1/15*(2*b^2*e*g^2-2*c^2*f*(-5*d*g+4*e*f)+c*g*(-6*a*e*g-5*b*d*g+3*b*e*
f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)/c^2/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)-2/15*(b*e*g-10*c*d*g+8*c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^
2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2
)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)
^(1/2))))^(1/2)/c^2/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {814, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(beg-10cdg+8cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*Sqrt[a + b*x + c*
x^2])/(15*c*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f
 - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
15*c^2*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{2\int \frac{\frac{1}{2}\left(5cdg(bf-2ag)-bef(4c\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\left((8cef-10cdg+beg)\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(2b^2eg^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(2b^2eg^2\cdots\right.}{\cdots}$$

Mathematica [C] time = 10.51, size = 911, normalized size = 1.76

$$\frac{2\sqrt{a+x(b+cx)}\left(\left(2f(4ef-5dg)c^2+g(-3bef+5bdg+6aeg)c-2b^2eg^2\right)\left(c\left(\frac{f}{f+gx}-1\right)^2+\frac{g\left(-\frac{fb}{f+gx}+b+\frac{ag}{f+gx}\right)}{f+gx}\right)+\frac{i\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] ((2*(-4*c*e*f + 5*c*d*g + b*e*g))/(15*c*g^2) + (2*e*x)/(5*g))*Sqrt[f + g*x]
*Sqrt[a + x*(b + c*x)] + (2*(f + g*x)^(3/2)*Sqrt[a + x*(b + c*x)]*((-2*b^2*
e*g^2 + 2*c^2*f*(4*e*f - 5*d*g) + c*g*(-3*b*e*f + 5*b*d*g + 6*a*e*g))*(c*(-
1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x))
 + ((I/2)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 -
4*a*c)*g^2])*(2*b^2*e*g^2 + 2*c^2*f*(-4*e*f + 5*d*g) + c*g*(3*b*e*f - 5*b*d
*g - 6*a*e*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-
2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (2*b^3
*e*g^3 - b^2*g^2*(-(c*e*f) + 5*c*d*g + 2*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g
*(-8*a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e*f + 5*d*g)) + 2*c*(c*f*Sqrt[(b
^2 - 4*a*c)*g^2]*(4*e*f - 5*d*g) + a*g^2*(-2*c*e*f + 10*c*d*g + 3*e*Sqrt[(b
^2 - 4*a*c)*g^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^
2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b
```

```
*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(
Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*
g^2])]*Sqrt[f + g*x])))/(15*c^2*g^4*Sqrt[a + b*x + c*x^2]*Sqrt[((f + g*x)^2
*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f +
 g*x)))/g^2])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\,(ex+d)}{\sqrt{gx+f}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\,(ex+d)}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

maple [B] time = 0.05, size = 6207, normalized size = 11.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\,(ex+d)}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.896 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=444

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/3*(-b*g+2*c*f)*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b
^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/
2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/g^2/(c*x^2+b*x+a)^
(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/3*(a*g^2-b*f*g+c
*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)
/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^
(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {734, 843, 718, 424, 419}

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*g) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*g
^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c
*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*c*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{\int \frac{bf-2ag+(2cf-bg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{(2cf-bg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3g^2} + \frac{\left(2\left(cf^2-bfg+ag^2\right)\right)\int \frac{}{\sqrt{f+gx}\,\sqrt{}}}{3g^2}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{\left(\sqrt{2}\,\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int\right.}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\,\sqrt{a+b}}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{}}{}\right.\right.}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx}}$$

Mathematica [C] time = 8.31, size = 936, normalized size = 2.11

$$\sqrt{f+gx}\left(4(a+x(b+cx))g^2+\frac{(f+gx)\left(\frac{4(bg-2cf)\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}}(a+x(b+cx))g^2}{(f+gx)^2}+\frac{i\sqrt{2}(2cf-bg)\left(2cf-bg+\sqrt{\left(b^2-4ac\right)g^2}\right)\sqrt{\frac{-2ag^2+2cfxg+\cdots}{\left(2\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*(4*g^2*(a + x*(b + c*x)) + ((f + g*x)*((4*g^2*(-2*c*f + b*g)
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g)*(2*c*f - b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g
*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*(b^2*g^2 - 4*a*c*g^2 + 2*c*f*Sqrt[(b
^2 - 4*a*c)*g^2] - b*g*Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^
2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/
((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt
[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g
*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f
 - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(c*Sqrt[(c*f^2 + g*(-(
b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])))/(6*g^3*Sqrt[a + x
*(b + c*x)])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}}{\sqrt{gx+f}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

maple [B] time = 0.03, size = 1854, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] -2/3*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/c*(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*g^3-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(
-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*f*g^2+2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*f^2*g+2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*b*g^3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*c*f*g^2-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2
*f*g^2+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c
*x+(-4*a*c+b^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+
b+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2
^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*f^2*g-2*2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-2*c*x+(-4*a*c+b
^2)^(1/2)-b)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((2*c*x+b+(-4*a*c+b^
2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*f^3-x^3*c^2*g^3-x^2*b*c
*g^3-x^2*c^2*f*g^2-x*a*c*g^3-x*b*c*f*g^2-a*c*f*g^2)/(c*g*x^3+b*g*x^2+c*f*x^
2+a*g*x+b*f*x+a*f)/g^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

3.897 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=700

$$-\frac{\sqrt{2}\left(ae^2-bde+cd^2\right)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4a}\right.}{2c(ef-dg)}\right.}{\sqrt{c}\,e^2\sqrt{a+bx+cx^2}\,(ef-dg)}$$

```
[Out] EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/
g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*
(-b*e*g+c*d*g+c*e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(
1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(
1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/g/(g*x+f)^(1/
2)/(c*x^2+b*x+a)^(1/2)-(a*e^2-b*d*e+c*d^2)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+
f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))
))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(-d*g+e*f)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.95, antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {922, 934, 169, 538, 537, 843, 718, 424, 419}

$$-\frac{\sqrt{2}\left(ae^2-bde+cd^2\right)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4a}\right.}{2c(ef-dg)}\right.}{\sqrt{c}\,e^2\sqrt{a+bx+cx^2}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(e*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
+ b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g - b*e*g)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2
*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2
 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/
(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^2*(e*f - d*g)*Sqrt[a + b*x
 + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 922

```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[1/((d
```

```
+ e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d
- b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d
^2 - b*d*e + a*e^2, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{\int \frac{cd-be-cex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{eg} - \frac{(cef+cdg-beg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2g} + \frac{\left(\left(cd^2-bde+ae^2\right)\sqrt{b-\cdots}\right.}{\cdots}$$

$$= -\frac{\left(2\left(cd^2-bde+ae^2\right)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\mathrm{Subst}\left(\int \frac{\cdots}{(ef-dg-\cdots}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \cdots$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \cdots$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \cdots$$

Mathematica [C] time = 13.92, size = 16471, normalized size = 23.53

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

maple [B] time = 0.04, size = 3126, normalized size = 4.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
[Out] (2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*
c^2*e^2*f^3-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)
^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)
)^(1/2))*b*c*e^2*f^2*g+EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^
(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^
2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d^2*g^3+2*EllipticE(2^(1/2)*(-(g*x
+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d*e*f*g^2-2*EllipticPi(
2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*
g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d*e*f*g^2-EllipticF(2^(1/2)*(-(g*
x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1
/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b^2*d*e*g^3+EllipticF(2^(1
/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*
c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b^2*e^2*f*g^2+Ell
ipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2
*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d^
2*g^3-2*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/
2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1
/2))*c^2*d^2*f*g^2-EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c
+b^2)^(1/2)*a*e^2*g^3-EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^
(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g
-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*
a*c+b^2)^(1/2)*c*d^2*g^3-EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^
```

```
2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(
b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a
*b*e^2*g^3+EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)
^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*
a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b^2*d*e*g^3-Ell
ipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d^2*g^3+2*EllipticPi(2^(1
/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*
a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*
c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d^2*f*g^2+2*EllipticF(2^(1/2)*(-(g*x+
f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2
)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d*e*f^2*g+EllipticPi(2^(
1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4
*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2
*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e*g^3+2*EllipticP
i(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*
f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-
b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-2*EllipticE(2^(1/2)*(
-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*d*e*g^3+2*Elliptic
E(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e^2*f*g
^2-2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),
(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2)
)*b*c*e^2*f^2*g-2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1
/2)*g))^(1/2))*c^2*d*e*f^2*g+2*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a
*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-
4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*d*e*g^3-2*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-
2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b
*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e^2*f*g^2-EllipticF(2^(1/2)*(-(g
*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e*
g^3+EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(
-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))
*(-4*a*c+b^2)^(1/2)*b*e^2*f*g^2-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*
a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(
-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^2*f^2*g)*((2*c*x+b+(-4*
a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((-2*c*x-b+(-4*a*
c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*2^(1/2)*(-(g*x+f)/
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)
/c/g^2/e^2/(d*g-e*f)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

3.898 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=736

$$-\frac{\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2(bf-ag)-cd(2ef-dg)\right)\Pi\left(\frac{e\left(2cf-b\right.}{2c}\right.}{\sqrt{2}\sqrt{c}\,e^2\sqrt{a+bx+cx^2}\,(ef-dg)^2}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)+1/2*EllipticE(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x
+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/(-d*g+e*f)*2^(1/2)/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+EllipticF(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-
4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/2*(e^2
*(-a*g+b*f)-c*d*(-d*g+2*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c
*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c
/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))
^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-
(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/e^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 3.23, antiderivative size = 957, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {924, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\,e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+\frac{\sqrt{2}\sqrt{b^2-4ac}\,(2ef-dg)\sqrt{\frac{\quad}{2cf-}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x))) + (Sqrt[b^
2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
e*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(
e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(2*e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g
```

```
) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Ell
ipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(
Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b
 - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sq
rt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 924

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)*Simp[b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m +
 2))*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{bf-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{cgx}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{e^2(bf-ag)-}{e^2(d+ex)\sqrt{f+g}}\right.}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+}}}{2e^2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{2e(ef-dg)} + \frac{((bf}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+\right.}{b^2-4}}}{e^2(ef-dg)\sqrt{f+gx}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+}}$$

Mathematica [C] time = 13.39, size = 6911, normalized size = 9.39

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^2 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

maple [B] time = 0.07, size = 13872, normalized size = 18.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^2 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\sqrt{f + g\,x}\,(d + e\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^2 \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

3.899 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1049

$$-\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)(cd(2ef+dg)-e(bef+2bd}{4\sqrt{2}\,e\left(cd^2-bed+ae^2\right)(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(d*g+2
*e*f)-e*(-3*a*e*g+2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-
b*d*e+c*d^2)/(-d*g+e*f)^2/(e*x+d)-1/8*(c*d*(d*g+2*e*f)-e*(-3*a*e*g+2*b*d*g+
b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/
(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(e^2*(-a*g+b*f)+c*d*(d*g-2*e*f))*E
llipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(
-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))/(-d*g+e*f)*2^(1/2)/(g
*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/8*(3*a^2*e^4*g^2+c^2*d^3*g*(-d*g+4*e*f)+b
^2*e^3*f*(4*d*g-e*f)+2*a*c*e^2*(3*d^2*g^2-2*d*e*f*g+2*e^2*f^2)-2*b*e^2*g*(3
*c*d^2*f+a*e*(2*d*g+e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*
c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))
/(-d*g+e*f)^3*2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(1/2)
```

Rubi [A] time = 8.18, antiderivative size = 1747, normalized size of antiderivative = 1.67, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {924, 6742, 718, 419, 939, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((c*d*
(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x +
 c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)) - (Sqrt[b^2 -
4*a*c]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sq
rt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)
*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt
[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^
2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f*(
c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
```

```
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2
*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2]) - (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g
 - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*(c*e*f + c*d*g - b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) +
 (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2
*b*d*g - 3*a*e*g))*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt
[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*
x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^2*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 924

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)*Simp[b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m +
 2))*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{bf-3ag+2(cf-bg)x-cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{-cd(2ef+dg)+e(bef+2bdg-3aeg)}{e^2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{\cdots}{e^2(d+\cdots}\right.}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(cef+cdg-beg)\int \frac{\cdots}{(d+ex)\sqrt{f+\cdots}}}{2e^2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx\cdots}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}$$

Mathematica [C] time = 16.68, size = 36616, normalized size = 34.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^3 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

maple [B] time = 0.22, size = 57841, normalized size = 55.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^3 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\sqrt{f + g\,x}\,{(d + e\,x)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)
```

3.900 $\int \frac{(d+ex)^3 \sqrt{f+gx}}{\sqrt{a+bx+cx^2}} \, dx$

Optimal. Leaf size=774

$$-\frac{2\sqrt{2}\, e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(ceg(-25aeg-84bdg+13bef)+24b^2e\right.}{105c^4g^3\sqrt{f+gx}\sqrt{a+bx+}}$$

```
[Out] 2/35*e^2*(-6*b*e*g+11*c*d*g+c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^
2+2/105*e*(24*b^2*e^2*g^2+c*e*g*(-25*a*e*g-84*b*d*g+13*b*e*f)-c^2*(-90*d^2*
g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/g^2+2/7*e*
(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c-1/105*(48*b^3*e^3*g^3-8*b*c*e
^2*g^2*(13*a*e*g+21*b*d*g+2*b*e*f)-c^3*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^
2*f^2*g+8*e^3*f^3)+c^2*e*g*(a*e*g*(189*d*g+19*e*f)-b*(-210*d^2*g^2-63*d*e*f
*g+9*e^2*f^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))
))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)/c^4/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)-2/105*e*(a*g^2-b*f*g+c*f^2)*(24*b^2*e^2*g^2+c*e*g*(-25*a*
e*g-84*b*d*g+13*b*e*f)+c^2*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)/c^4/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 2.11, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {941, 1653, 843, 718, 424, 419}

$$\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2\left(-\left(-90d^2g^2+12defg+7e^2f^2\right)\right)\right)}{105c^3g^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(7*e^2*
f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c
^3*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + (2*
e^2*(c*e*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35
*c^2*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b
*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f
*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2 - 63
*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(105*c^4*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^2*
f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
```

```
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^4*g^3*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 941

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)*Simp[e*(b*d*f + a*(d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1
) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(
2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f + d*g*(4*m - 1)))*x^2, x])/(Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+e(bdf+4aef+adg)-(cd(12ef+7dg)-e(5b\ldots}{\sqrt{f+gx}\,\sqrt{a+bx+c\ldots}}}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{a+bx\ldots}}{35c^2g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+\ldots}}{105c^3g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+\ldots}}{105c^3g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+\ldots}}{105c^3g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+\ldots}}{105c^3g^2}$$

Mathematica [C] time = 14.67, size = 10649, normalized size = 13.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(g*x + f)/sqrt(c*x^2
 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

maple [B] time = 0.09, size = 14978, normalized size = 19.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{(d+e\,x)}^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.901 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=567

$$\frac{4\sqrt{2}\,e\sqrt{b^2-4ac}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,(2beg-5cdg+cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{15c^3g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

```
[Out] 2/15*e*(-4*b*e*g+7*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g+2/5
*e*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/15*(8*b^2*e^2*g^2-c*e*g*(9
*a*e*g+20*b*d*g+3*b*e*f)-c^2*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f^2))*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^2/(c*x
^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/15*e*(
2*b*e*g-5*c*d*g+c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+
b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)/c^3/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {941, 1653, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2+c^2\left(-\left(-15d^2g^2-10defg+2e\right.\right.\right.}{15c^3g^2\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*(c*e*f + 7*c*d*g - 4*b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c
^2*g) + (2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(8*b^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) -
 c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c
*e*f - 5*c*d*g + 2*b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15
*c^3*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 941

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)*Simp[e*(b*d*f + a*(d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1
) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(
2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f + d*g*(4*m - 1)))*x^2, x])/(Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+e(bdf+2aef+adg)-(cd(8ef+5dg)-e(3bef+2bdg+}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c}$$

Mathematica [C] time = 11.44, size = 1002, normalized size = 1.77

$$\frac{\left(\frac{2e^2x}{5c} - \frac{2e(-cef-10cdg+4beg)}{15c^2g}\right)\sqrt{f+gx}\,\left(cx^2+bx+a\right)}{\sqrt{a+x(b+cx)}} - \frac{2(f+gx)^{3/2}\sqrt{cx^2+bx+a}\left(\left(\left(2e^2f^2-10degf-15d^2g^2\right)c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((-2*e*(-(c*e*f) - 10*c*d*g + 4*b*e*g))/(15*c^2*g) + (2*e^2*x)/(5*c))*Sqrt
[f + g*x]*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqr
t[a + b*x + c*x^2]*((-8*b^2*e^2*g^2 + c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g)
+ c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(c*(-1 + f/(f + g*x))^2 + (g*(
b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((I/2)*Sqrt[1 - (2*(c*
f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x)
)]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a
*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g
^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) + c^2*(-2*e^2*f^2 + 10*d*e*f*g +
15*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (-30*c^3
*d^2*f*g^2 + 8*b^2*e^2*g^2*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*e*g*(-17*a*b
```

```
*e*g^2 + 9*a*e*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*Sqrt[(b^2 - 4*a*c)*g^2]*(3*e*f
 + 20*d*g) - b^2*g*(11*e*f + 20*d*g)) - c^2*(-15*b*d*g^2*(2*e*f + d*g) - 2*
a*e*g^2*(7*e*f + 10*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(-2*e^2*f^2 + 10*d*e*f*g
 + 15*d^2*g^2)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqr
t[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*Sqrt[f + g*x])))/(15*c^3*g^3*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2 + 2\,dex + d^2\right)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

maple [B] time = 0.07, size = 8248, normalized size = 14.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.902 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=452

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}e\cdots}{\cdots}$$

```
[Out] 2/3*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/3*(-2*b*e*g+3*c*d*g+c*e*f)*Elli
pticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(
-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-
4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g/
(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*
e*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g/(g*x+f)^(1/2
)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {832, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3*c^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-e(bf+ag))+\frac{1}{2}(cef+3cdg-2beg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{(cef+3cdg-2beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3cg} - \frac{\left(e\left(cf^2-bfg+\right.\right.}{}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{\left(\sqrt{2}\,\sqrt{b^2-4ac}\,(cef+3cdg-2beg)\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx}{b^2-4}}\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-}}}}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef+3cdg-2beg)\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+}{b^2-4a}}}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)}}}$$

Mathematica [C] time = 7.14, size = 638, normalized size = 1.41

$$2\sqrt{f+gx}\left(ce(a+x(b+cx))+\frac{(f+gx)\left(\frac{g^2(a+x(b+cx))(-2beg+3cdg+cef)}{(f+gx)^2}+\frac{i\sqrt{1-\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}-bg+2cf\right)}}\sqrt{\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}+bg-2cf\right)}+1}\left(c\left(\cdots\right.\right.}{\cdots}\right)}{\cdots}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*(a + x*(b + c*x)) + ((f + g*x)*((g^2*(c*e*f + 3*c*d*g
 - 2*b*e*g)*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*Sqrt[1 - (2*(c*f^2 + g*
(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[
1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(2*b*e*g - c*(e*f +
3*d*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b
^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (6*c^2*d*f*g
+ 2*b*e*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 - 3*b*g*(e*f + d*
g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f + 3*d*g)))*EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt
[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/g^2))/(3*c^2*Sqrt[a + x*(b
 + c*x)])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

maple [B] time = 0.05, size = 3805, normalized size = 8.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*(3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*
a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a
*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+
b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^
(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^
2)^(1/2)*g))^(1/2))*a*b*e*g^3-6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1
/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)*g)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c
)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g
))^(1/2))*a*c*d*g^3-2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(
1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1
/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)
*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b
*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-
4*a*c+b^2)^(1/2)*a*e*g^3-3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*
g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g
)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g
)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/
2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1
/2))*b^2*e*f*g^2+6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1
/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/
2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*
EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c
*d*f*g^2+3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2
*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c
*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Elliptic
F(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*e*f^2*g
+2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4
*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a
*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^(1/2)*
(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^
2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*b*
e*f*g^2-6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*
c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*
x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF
(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(
-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d*f^2*g-
2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*
a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*
c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^(1/2)*(
-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e
*f^2*g-4*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c
*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x
+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(
2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-
4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*b*e*g^3+6*2
^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a
*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c
```

```
+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-
(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)
^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*d*g^3+2*2^(1/2)*(-(
g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1
/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2
))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b
*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/
(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e*f*g^2+4*2^(1/2)*(-(g*x+f)/(
b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b
*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-
2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f
+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*
c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b^2*e*f*g^2-6*2^(1/2)*(-(g*x+f)/(b*g-2*c*
f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f
+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-
4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*
a*c+b^2)^(1/2)*g))^(1/2))*b*c*d*f*g^2-6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*
c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c
+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^
2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1
/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)
^(1/2)*g))^(1/2))*b*c*e*f^2*g+6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1
/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c
)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g
))^(1/2))*c^2*d*f^2*g+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*
c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g
)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(
1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),
(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2)
)*c^2*e*f^3-2*c^2*e*g^3*x^3-2*x^2*b*c*e*g^3-2*c^2*e*f*g^2*x^2-2*a*c*e*g^3*x
-2*x*b*c*e*f*g^2-2*a*c*e*f*g^2)/c^2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*
f)/g^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\left(d+e\,x\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.903 $\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/
(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 424}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(c*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a +
b*x + c*x^2])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}\,gx^2}{2cf-bg-\sqrt{b^2-4ac}\,g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\cdots\right.}{c\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.78, size = 365, normalized size = 1.94

$$\frac{i\left(g\left(\sqrt{b^2-4ac}-b\right)+2cf\right)\sqrt{\frac{g\left(\sqrt{b^2-4ac}+b+2cx\right)}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\sqrt{1-\frac{2c(f+gx)}{g\left(\sqrt{b^2-4ac}-b\right)+2cf}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\left(b+\sqrt{b^2-4ac}\right)g-2cf}}\sqrt{f\cdots}\right.\right.\right.}{\sqrt{2}\,cg\sqrt{a+x(b+cx)}\sqrt{\frac{\cdots}{g\left(\sqrt{b^2-\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (I*(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 + (-b + Sqrt[b^2 - 4*a*c])*g)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f
 + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)] - EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]))/(Sqrt[2]*c*g
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + x*(b + c*x)])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

maple [B] time = 0.04, size = 747, normalized size = 3.97

$$\frac{\sqrt{gx+f}\,\sqrt{c\,x^{2}+bx+a}\,\left(bg-2cf+\sqrt{-4ac+b^{2}}\,g\right)\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{bg-2cf+\sqrt{-4ac+b^{2}}\,g}}\,\sqrt{\frac{\left(-2cx-b+\sqrt{-4ac+b^{2}}\right)g}{-bg+2cf+\sqrt{-4ac+b^{2}}\,g}}\,\sqrt{\frac{\left(2cx+b+\sqrt{-4\right.}{bg-2cf+\sqrt{-}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*2^(1
/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+
b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^
2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(EllipticF(2^(1/2)*(-(g
*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*g*b-2*f*EllipticF(2^(1/2)
*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b
^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c-EllipticF(2^(1/2)*
(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^
2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*g*(-4*a*c+b^2)^(1/2)-
EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*g
+2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*
c*f+EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(
-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))
*(-4*a*c+b^2)^(1/2)*g)/g/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.904 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=467

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{2cf-\ldots}}{\ldots}$$

[Out] 2*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/c^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A] time = 1.60, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {943, 718, 419, 934, 169, 538, 537}

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{2cf-\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 943

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{}}}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 1.62, size = 379, normalized size = 0.81

$$-\frac{i\sqrt{2}\sqrt{\frac{g\left(\sqrt{b^2-4ac}+b+2cx\right)}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\sqrt{1-\frac{2c(f+gx)}{g\left(\sqrt{b^2-4ac}-b\right)+2cf}}\left(F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\left(b+\sqrt{b^2-4ac}\right)g-2cf}}\sqrt{f+gx}\right)|\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(\sqrt{b^2-4ac}-b\right)g}\right.\right.}{e\sqrt{a+x(b+cx)}\sqrt{\frac{c}{g\left(\sqrt{b^2-4ac}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c]
)*g)]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)] - EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(2
*c*(e*f - d*g)), I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)]))/(e*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + x*(b + c*x)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

maple [B] time = 0.04, size = 834, normalized size = 1.79

$$\frac{\left(-bg\,\mathrm{EllipticF}\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{bg-2cf+\sqrt{-4ac+b^2}\,g}},\sqrt{-\frac{bg-2cf+\sqrt{-4ac+b^2}\,g}{-bg+2cf+\sqrt{-4ac+b^2}\,g}}\right)+bg\,\mathrm{EllipticPi}\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{bg-2cf+\sqrt{-4ac+b^2}\,g}},\frac{(bg-}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(
b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*g
*(-4*a*c+b^2)^(1/2)-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^
(1/2)*g))^(1/2))*g*b+2*f*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+
b^2)^(1/2)*g))^(1/2))*c+EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b
*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-
4*a*c+b^2)^(1/2)*g+EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*g-2*E
llipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(
b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*e/c/(d*g-e*f),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c*f)*(g*x+f)^(1/2)*(c*x^2+b
*x+a)^(1/2)/e*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*(
(-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((
2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)/c/(c*
g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.905 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=994

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}\,e}{\left(cd^2-bed+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)\ \sqrt{\cdots}}{\sqrt{2}\left(cd^2-bed+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\cdots$$

```
[Out] -e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)+1/2*Ellipt
icE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)
^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2)-f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*
x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(g*x+f)^(1
/2)/(c*x^2+b*x+a)^(1/2)+d*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c
+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d
*e+c*d^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+1/2*(e^2*(-a*g+b*f)-c*d*(-d*g+2
*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1
/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g
-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/
(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1
/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c
*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 3.70, antiderivative size = 994, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {945, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}\,e}{\left(cd^2-bed+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)\ \sqrt{\cdots}}{\sqrt{2}\left(cd^2-bed+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x
))) + (Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sq
rt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c
```

```
*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(c*d^2 - b
*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f +
 g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sq
rt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 945

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2])/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1
) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m +
2)))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\int \frac{-2cdf+bef-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} - \frac{cgx}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{e^2(\cdots}{e(d+\cdots}\right.}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+\cdots}}}{2e\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,d\cdots}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\left(a+bx+\cdots\right.}{b^2-4a\cdots}}}{e\left(cd^2-bde+ae^2\right)\sqrt{\cdots}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(f+g\cdots}{2cf-\left(b+\sqrt{b^2\cdots}\right.}}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(f+g\cdots}{2cf-\left(b+\sqrt{b^2\cdots}\right.}}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(f+g\cdots}{2cf-\left(b+\sqrt{b^2\cdots}\right.}}}$$

Mathematica [C] time = 13.73, size = 18563, normalized size = 18.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\,(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

maple [B] time = 0.08, size = 13017, normalized size = 13.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\,(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/((d + e*x)**2*sqrt(a + b*x + c*x**2)), x)
```

3.906 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1786

$$-\frac{(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\,\sqrt{cx^2+bx+a}\,e}{4\left(cd^2-bed+ae^2\right)^2(ef-dg)(d+ex)}-\frac{\sqrt{f+gx}\,\sqrt{cx^2+bx+a}\,e}{2\left(cd^2-bed+ae^2\right)(d+ex)^2}+\frac{\sqrt{b^2-4ac}\,(c}{}$$

```
[Out] -1/2*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2-1/4*
e*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+
a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)/(e*x+d)+1/8*(c*d*(-5*d*g+6*e*f)-e
*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/
2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/2*g*EllipticF(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*
c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
-1/4*f*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2
)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)
^(1/2)+1/4*d*g*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)
*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(c*
x^2+b*x+a)^(1/2)+1/2*(b*e*g-3*c*d*g+c*e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+
f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c
+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)
*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2
+b*x+a)^(1/2)-1/8*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(c*d*(-3*
d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(
1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f
-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A] time = 8.40, antiderivative size = 1786, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {945, 6742, 718, 419, 939, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*
x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*
x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*
g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*(c*d^
2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^
2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*
e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e*f
 - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]
, (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]
*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqr
t[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g +
Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g
*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*
c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b
*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[
b^2 - 4*a*c])*g]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f -
 d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 945

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2])/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1
) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m +
2)))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \frac{-4cdf+3bef-aeg+2(cef-2cdg+beg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \left(\frac{cg}{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{-cd(6ef-5dg)+e(3bef-2bdg \cdots}{e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+ \cdots}} \cdots\right.}{4\left(cd^2-bde+ \cdots\right.}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e\left(cd^2-bde+ae^2\right)} - \frac{(cef-3cdg+b \cdots}{2e \cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f \cdots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(\cdots}$$

Mathematica [C] time = 17.66, size = 36634, normalized size = 20.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

maple [B] time = 0.25, size = 59522, normalized size = 33.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.907 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=675

$$\frac{2\sqrt{2}\,g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\,(ef\ldots}{}$$

```
[Out] g*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
c/e/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+
2*g*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(
c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(g*x+f)^(1/2)/(c*x^
2+b*x+a)^(1/2)-(-d*g+e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g
*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d
*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2)/e^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.65, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {957, 718, 419, 934, 169, 538, 537, 424}

$$\frac{2\sqrt{2}\,g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\,(ef\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
```

```
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{g\sqrt{f+g}}{e\sqrt{a+bx+}}\right.$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{f+gx}}}{e^2}$$

$$= \frac{\left((ef-dg)^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+}}}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4a}\right.}\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4a}\right.}\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4a}\right.}\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4a}\right.}\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [B] time = 4.55, size = 1385, normalized size = 2.05

$$\frac{\sqrt{2}\sqrt{\frac{c(f+gx)}{2cf+\left(\sqrt{b^2-4ac}-b\right)g}}\left(-\frac{4\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\,\Pi\left(\frac{2\sqrt{b^2-4ac}\,e}{2cd-be+\sqrt{b^2-4ac}\,e};\sin^{-1}\left(\frac{\sqrt{\frac{-b-2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}\,g}{2cf-bg+\sqrt{b^2-4ac}\,g}\right)f^2}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}+\frac{2g\left(b+2cx-\sqrt{b\cdots\right.}{\cdots}\right)}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*((2*f*g*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g)])/(c*e*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]) - (d
*g^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2
- 4*a*c]*g)])/(c*e^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c
]]) + (g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*((-2*c*f + (b + Sqrt[b^2 - 4*
a*c])*g)*EllipticE[ArcSin[Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)] - (b + Sqrt[b^2 - 4*a*c])*g*EllipticF[ArcSin[Sqrt[2]*Sqrt[(
c*(f + g*x))/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2
 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]))/(2*c^2*e*Sqrt[(g*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]) - (4*Sq
rt[b^2 - 4*a*c]*f^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b
 + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e) + (8*Sqrt[b^2 - 4*a*c]*d*f*g*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)
, ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(e*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)) - (4*Sqrt[b^2 - 4*a*c]*d^2*g^2*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e
 + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4
*a*c]*g)])/(e^2*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))))/(Sqrt[f + g*x]*Sqrt
[a + x*(b + c*x)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

maple [B] time = 0.05, size = 1879, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
[Out] (g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)
^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^
(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)*g)^(1/2)/c*(EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1
/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*d*g^2-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g
+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*e*f*g+2*EllipticF(2
^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4
*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*e*g^2+Ellipt
icF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*
f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*d*g^2-3
*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b
*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*
e*f*g-2*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/
2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1
/2))*c*d*f*g+4*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)
*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)
*g))^(1/2))*c*e*f^2-2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2
)^(1/2)*g))^(1/2))*a*e*g^2+2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*
a*c+b^2)^(1/2)*g))^(1/2))*b*e*f*g-2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c
*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c*e*f^2-EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-
2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(
d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1
/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*d*g^2+EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2
*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d
*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/
2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*e*f*g-EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*
g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2
)*g))^(1/2))*b*d*g^2+EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-
2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*e*f
*g+2*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)
,1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^
2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c*d*f*g-2*EllipticPi(
2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*
g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c*e*f^2)/e^2/(c*g*x^3+b*g*x^2+c*f*x^2
+a*g*x+b*f*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^{\frac{3}{2}}}{\left(d+ex\right)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**(3/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.908 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1138

$$\frac{2\sqrt{f+gx}\,\sqrt{cx^2+bx+a}\,g^2}{3ce}+\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\,\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{}}{2cf-(b}\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{cx^2+bx+a}}$$

```
[Out] 2/3*g^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/3*g*(-b*g+2*c*f)*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/e/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(-d*g+e*
f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
/c/e^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)+2*g*(-d*g+e*f)^2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/
(c*x^2+b*x+a)^(1/2)-2/3*g*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)/c^2/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-(-d*g+e*f)^2*EllipticPi(2^(1/
2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*
f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b
-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x
+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 2.14, antiderivative size = 1138, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {957, 718, 419, 934, 169, 538, 537, 424, 742, 843}

$$\frac{2\sqrt{f+gx}\,\sqrt{cx^2+bx+a}\,g^2}{3ce}+\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\,\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{}}{2cf-(b}\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(3*c^2*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
 + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f + g*x]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
```

```
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a +
b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 - b*f*g + a*g^2)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^2*Sqrt[1 - (2*c*(f + g*
x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f -
(b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[
b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
```

```
)/(b^2 - 4*a*c))]])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{g(ef-\ldots}{e^2\sqrt{a-\ldots}} \right.$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+bx+cx^2}}dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx}{e^2} + \frac{\left(g(ef-dg)^2\right)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx\ldots}}}{e^3}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}\left(3cf^2-g(bf+ag)\right)+g(2cf-bg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}dx}{3ce} + \frac{\left((ef-dg\ldots\right.}{\ldots}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2\ldots}{b^2-4ac}}}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4a\ldots}\right.}}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2\ldots}{b^2-4ac}}}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4a\ldots}\right.}}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+\ldots}{b^2-4a\ldots}}}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-\ldots}\right.}}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+\ldots}{b^2-4a\ldots}}}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-\ldots}\right.}}}$$

Mathematica [C] time = 16.25, size = 37137, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

maple [B] time = 0.07, size = 7464, normalized size = 6.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{5/2}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^{\frac{5}{2}}}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**(5/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.909 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=631

$$\frac{\sqrt{2}\,e\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\left(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2\left(45d^2g^2-30defg+8e^2f^2\right.\right.}{15c^3g^3\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] -8/15*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+2
/5*e^2*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g+1/15*e*(8*b^2*e^2*g^2+
c*e*g*(-9*a*e*g-30*b*d*g+7*b*e*f)+c^2*(45*d^2*g^2-30*d*e*f*g+8*e^2*f^2))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/
g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/15*(4*b*e^3*g^2*(-a*g+b*f)+c^2*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*e^2*f^2*g
+8*e^3*f^3)-c*e^2*g*(a*g*(-15*d*g+7*e*f)-3*b*f*(-5*d*g+e*f)))*EllipticF(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*
a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {930, 1653, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\left(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf-ag)\right.}{15c^3g^3\sqrt{f+gx}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-8*e^2*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*g^2) + (2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c*g) +
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9
*a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*
a*c]*(4*b*e^3*g^2*(b*f - a*g) + c^2*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*
f*g^2 - 15*d^3*g^3) - c*e^2*g*(a*g*(7*e*f - 15*d*g) - 3*b*f*(e*f - 5*d*g)))
*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)*Simp[b*d*e^2*f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*
g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g
*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(m - 1)*x^2, x])/(Sqrt[f +
 g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} - \frac{\int \frac{bde^2f-5cd^3g+ae^2(2ef+dg)+e(cd(2ef-15dg)+e(3\cdots}{\sqrt{f+gx}\sqrt{a+\cdots}}}{5cg\cdots}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+\cdots}}{5cg}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+\cdots}}{5cg}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+\cdots}}{5cg}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+\cdots}}{5cg}$$

Mathematica [C] time = 13.69, size = 12746, normalized size = 20.20

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)*sq
rt(g*x + f)/(c*g*x^3 + (c*f + b*g)*x^2 + a*f + (b*f + a*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

maple [B] time = 0.08, size = 8755, normalized size = 13.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.910 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=479

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2g(bf-ag)+c\left(3d^2g^2-6defg+2e^2f^2\right)\right)F\left(\sin^{-1}\left(\sqrt{\frac{b+2cx+}{\sqrt{b}}}\right.\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g-2/3*e*(b*e*g-3*c*d*g+c*e*f)*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)
*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2
/g^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
+2/3*(e^2*g*(-a*g+b*f)+c*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2))*EllipticF(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {930, 24, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2g(bf-ag)+c\left(3d^2g^2-6defg+2e^2f^2\right)\right)F\left(\sin^{-1}\left(\sqrt{\frac{b+2cx+}{\sqrt{b}}}\right.\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)])/(3*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e^2*g*(b*f -
 a*g) + c*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2
*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)*Simp[b*d*e^2*f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*
g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g
*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(m - 1)*x^2, x])/(Sqrt[f +
 g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{d\left(be^2f-3cd^2g+ae^2g\right)+e(cd(2ef-9dg)+e(bef+2bdg+aeg))x-}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{e^2\left(be^2f-3cd^2g+ae^2g\right)+2e^3(cef-3cdg+beg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{(2e(cef-3cdg+beg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3cg^2} + \frac{\left(e^2g(\right.}{}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\,e(cef-3cdg+beg)\sqrt{f+gx}\right.}{3c^2g^2\sqrt{\frac{}{2cf}}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\,e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{}}{3c^2g^2\sqrt{\frac{c}{2cf-(b}}}$$

Mathematica [C] time = 12.79, size = 1080, normalized size = 2.25

$$\frac{2\sqrt{f+gx}\left(cx^2+bx+a\right)e^2}{3cg\sqrt{a+x(b+cx)}} + \frac{(f+gx)^{3/2}\sqrt{cx^2+bx+a}\left(-4e(cef-3cdg+beg)\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}}\left(c\left(\frac{f}{f+gx}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*(a + b*x + c*x^2))/(3*c*g*Sqrt[a + x*(b + c*x)]) + ((f
 + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[(c
*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 +
 f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) +
(I*Sqrt[2]*e*(c*e*f - 3*c*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^
2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f
+ g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f
+ g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I
*Sqrt[2]*(-3*c^2*d^2*g^2 + b*e^2*g*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]) + c*e
```

```
*(3*b*d*g^2 + a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - 3*d*g)))*Sqrt[(Sqrt[
(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(
-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[
(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g
- (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I
*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 -
4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(3*c^2*g^3*Sqrt[(
c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a
+ x*(b + c*x)]*Sqrt[((f + g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f
 + g*x) + (a*g)/(f + g*x))))/(f + g*x)))/g^2])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(c*g
*x^3 + (c*f + b*g)*x^2 + a*f + (b*f + a*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

maple [B] time = 0.06, size = 4295, normalized size = 8.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3/c^2*(2*c^2*e^2*g^3*x^3+2*x*b*c*e^2*f*g^2+2*c^2*e^2*f*g^2*x^2+2*x^2*b*c*
e^2*g^3-6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*
c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*
x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF
(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(
-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e^2*f*g^
2-2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-
4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*
a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^(1/2)
*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b
^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*b
*e^2*f*g^2+12*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*(
(-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((
2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Ellip
ticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c
*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d*e*
f*g^2+6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*
```

x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+
b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2
^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4
*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(
1/2)*c*d*e*f*g^2-6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1
/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/
2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*
EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c
*d*e*f*g^2+12*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*(
(-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((
2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Ellip
ticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c
*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*d*e*
g^3+2*a*c*e^2*g^3*x-12*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c
)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)
^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1
/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(
-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))
*a*c*d*e*g^3+4*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*
((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(
(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Elli
pticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e^2
*f*g^2-12*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*
c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*
x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE
(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(
-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d*e*f^2*
g-2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+
(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-
4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^(1/
2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)
*c*e^2*f^2*g+4*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*
((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(
(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Elli
pticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*e^2
*f^3+2*a*c*e^2*f*g^2-3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c
)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)
^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1
/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(
-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))
*a*b*e^2*g^3-4*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*
((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(
(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Elli
pticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b^2*e^2
*f*g^2+2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x
-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b
+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^
(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*
a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1
/2)*a*e^2*g^3-3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)
*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*
((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Ell
ipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2
*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*

```
c+b^2)^(1/2)*c*d^2*g^3+3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)
*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*
g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^
(1/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)
,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2
))*b^2*e^2*f*g^2-3*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1
/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/
2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*
EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c
*d^2*g^3+6*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2
*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c
*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Elliptic
F(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d^2*f*g
^2+4*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b
+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(
-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1
/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*
c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*b*e^2*g^3)*(g*x
+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g^3/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.911 $\int \frac{d+ex}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=393

$$\frac{\sqrt{2}\,e\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c(a+\cdots}{b^2\cdots}}\cdots}{\cdots}$$

```
[Out] e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
c/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*
(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g/(g*x+f)^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {843, 718, 424, 419}

$$\frac{\sqrt{2}\,e\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c(a+\cdots}{b^2\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(f + g
*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{g}$$

$$= \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}\,gx^2}{2cf-bg-\sqrt{b^2-4ac}\,g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{b+\cdots}}{\cdots}\right.}{cg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4\cdots}\right.}\right.}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 5.88, size = 814, normalized size = 2.07

$$-\frac{(f+gx)^{3/2}\left(-\frac{4e\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))g^2}{(f+gx)^2}+\frac{i\sqrt{2}e\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfxg+b(f-gx)g+\sqrt{(b^2-4ac)g^2}(f+gx)}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{\frac{2ag^2-2c\cdots}{(-2\cdots}}}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((f + g*x)^(3/2)*((-4*e*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]
*e*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g
*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 -
 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt
[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f
 + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
 + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[
(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x
] - (I*Sqrt[2]*(2*c*d*g + e*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]))*Sqrt[(-2*a*
g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*
f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x +
b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b
*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -(
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*
g^2]))])/Sqrt[f + g*x]))/(c*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

fricas [F] time = 1.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\,(ex+d)\sqrt{gx+f}}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)/(c*g*x^3 + (c*f + b*
g)*x^2 + a*f + (b*f + a*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

maple [B] time = 0.04, size = 1014, normalized size = 2.58

$$\left(-2ae\,g^2\,\text{EllipticE}\left(\sqrt{2}\,\sqrt{-\frac{\left(gx+f\right)c}{bg-2cf+\sqrt{-4ac+b^2}\,g}},\sqrt{-\frac{bg-2cf+\sqrt{-4ac+b^2}\,g}{-bg+2cf+\sqrt{-4ac+b^2}\,g}}\right)+2ae\,g^2\,\text{EllipticF}\left(\sqrt{2}\,\sqrt{-\frac{\left(gx+f\right)c}{bg-2cf+\sqrt{-4ac}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (2*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*
a*e*g^2-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/
2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1
/2))*b*d*g^2-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c
)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g
```

```
))^(1/2))*b*e*f*g+2*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/
2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^
(1/2)*g))^(1/2))*c*d*f*g-EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+
b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*d*g^2+EllipticF(2^(1/2)*(-(g*x+f)/
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g
)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*(-4*a*c+b^2)^(1/2)*e*f*g-2*Elli
pticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*e*g^2
+2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*
b*e*f*g-2*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(
1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^
(1/2))*c*e*f^2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)*g)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*
g)*g)^(1/2)*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*(g*
x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g^2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a
*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex + d}{\sqrt{cx^2 + bx + a}\,\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d + e\,x}{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{\sqrt{f + gx}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.912 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*Sqrt[f + g*x]*Sqrt[a
+ b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}\,g}{2cf-bg-\sqrt{b^2-}}}}\right.}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{}{2c}\right.}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.64, size = 308, normalized size = 1.63

$$\frac{i(f+gx)\sqrt{2-\frac{4\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2\left(b^2-4ac\right)}-bg+2cf\right)}}\sqrt{\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2\left(b^2-4ac\right)}+bg-2cf\right)}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bgf+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2c}{2c}\right.}{g\sqrt{a+x(b+cx)}\sqrt{\frac{g(ag-bf)+cf^2}{\sqrt{g^2\left(b^2-4ac\right)}+bg-2cf}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))])/(g*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{gx+f}}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(c*g*x^3 + (c*f + b*g)*x^2 + a
*f + (b*f + a*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

maple [A] time = 0.04, size = 287, normalized size = 1.52

$$\frac{\left(-bg+2cf-\sqrt{-4ac+b^2}\,g\right)\sqrt{\frac{\left(2cx+b+\sqrt{-4ac+b^2}\right)g}{bg-2cf+\sqrt{-4ac+b^2}\,g}}\,\sqrt{\frac{\left(-2cx-b+\sqrt{-4ac+b^2}\right)g}{-bg+2cf+\sqrt{-4ac+b^2}\,g}}\,\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{bg-2cf+\sqrt{-4ac+b^2}\,g}}\,\sqrt{gx+f}\,\sqrt{c}}{\left(cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+af\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-(-4*a*c+b^2)^(1/2)*g-b*g+2*c*f)/c*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c
*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(
-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-
4*a*c+b^2)^(1/2)*g)*g)^(1/2)*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2
)*g)*c)^(1/2)/g*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+
a*g*x+b*f*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.913 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{2cf-(\cdots}}\right.\right.}{\sqrt{c}\sqrt{a+bx+cx^2}\,(ef-dg)}$$

```
[Out] -EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^
(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a
*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)/c^(
1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {934, 169, 538, 537}

$$-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{2cf-(\cdots}}\right.\right.}{\sqrt{c}\sqrt{a+bx+cx^2}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))
/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b
- Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]))
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b \cdots}}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{(ef-dg-\cdots}\right.}{\sqrt{a+bx+\cdots}}$$

$$= -\frac{\left(2\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{(ef-dg-e\cdots}\right.}{\sqrt{a+bx+c\cdots}}$$

$$= -\frac{\left(2\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{(ef-dg-ex\cdots}\right.}{\sqrt{a+bx+cx^{\cdots}}}$$

$$= -\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f\cdots}{2cf-\left(b+\cdots\right.}}}{\sqrt{c}\,(ef-dg\cdots}$$

Mathematica [C] time = 1.75, size = 499, normalized size = 1.78

$$\frac{i(f+gx)\sqrt{2-\frac{4\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2\left(b^2-4ac\right)}-bg+2cf\right)}}\sqrt{\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2\left(b^2-4ac\right)}+bg-2cf\right)}+1}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bgf+ag^2}{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}}}{\sqrt{f+gx}}\right)\middle| \cdots\right.\right.}{\sqrt{a+x(b+cx)}\,(dg-e\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(EllipticF[I*ArcSinh[(Sqr
```

```
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))] - EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(
b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sq
rt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2]))]))/((-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

maple [A] time = 0.05, size = 330, normalized size = 1.18

$$\frac{\left(-bg+2cf-\sqrt{-4ac+b^2}\,g\right)\sqrt{\frac{\left(2cx+b+\sqrt{-4ac+b^2}\right)g}{bg-2cf+\sqrt{-4ac+b^2}\,g}}\,\sqrt{\frac{\left(-2cx-b+\sqrt{-4ac+b^2}\right)g}{-bg+2cf+\sqrt{-4ac+b^2}\,g}}\,\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{bg-2cf+\sqrt{-4ac+b^2}\,g}}\,\sqrt{c\,x^2+bx+a}\cdots}{\left(dg-ef\right)\left(cg\,x^3+bg\,x^2+cf\,x^2+ag\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-b*g+2*c*f-(-4*a*c+b^2)^(1/2)*g)*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(
-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f
)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))
^(1/2))*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(
1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1
/2)*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)/c*(c*x^2+b*
x+a)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.914 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1037

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}\,e^2}{\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2cf-\left(b+\sqrt{b^2-\right.}}\right.}{\sqrt{2}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

```
[Out] -e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(e*x+
d)+1/2*e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2
))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(
a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f
-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a
*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+d*g*Elliptic
F(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g
*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*
c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*
x^2+b*x+a)^(1/2)-1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticP
i(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*
e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1
/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*
a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+
f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2
*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 3.53, antiderivative size = 1037, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {939, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}\,e^2}{\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2cf-\left(b+\sqrt{b^2-\right.}}\right.}{\sqrt{2}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f -
 d*g)*(d + e*x))) + (Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[(c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sq
rt[2]*Sqrt[b^2 - 4*a*c]*e*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
```

```
- 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*
(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*
c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^
2*Sqrt[a + b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} - \frac{\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2\ldots}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+c\ldots}}}{2\left(cd^2-bde+ae^2\right)(e\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} - \frac{\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\ldots}{\sqrt{f+g\ldots}}\right.}{2\left(cd^2 \ldots\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} + \frac{(cdg)\int\frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)(ef-dg\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} + \frac{(ce)\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)(ef-dg\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+g\ldots}{2cf-\left(b+\sqrt{b\ldots}\right.}}}{\left(cd^2 \ldots\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+\ldots}{b\ldots}}}{\sqrt{2}\left(cd^2-bde+ae^2\ldots\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+\ldots}{b\ldots}}}{\sqrt{2}\left(cd^2-bde+ae^2\ldots\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+\ldots}{b\ldots}}}{\sqrt{2}\left(cd^2-bde+ae^2\ldots\right.}$$

Mathematica [C] time = 13.95, size = 10881, normalized size = 10.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)^2\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

maple [B] time = 0.10, size = 14048, normalized size = 13.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)^2\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex\right)^{2}\sqrt{f + gx}\,\sqrt{a + bx + cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.915 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1114

$$-\frac{3(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{cx^2+bx+a}\,e^2}{4\left(cd^2-bed+ae^2\right)^2(ef-dg)^2(d+ex)}-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}\,e^2}{2\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)^2}+-$$

3

```
[Out] -1/2*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(
e*x+d)^2-3/4*e^2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)
*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/8*e*(c*d*
(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-
g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(1/2)/(c
*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1/4*(c
*d*(7*d*g-6*e*f)+e*(a*e*g-4*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2
)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x
+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/
(c*d^2+e*(a*e-b*d))^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+
1/8*(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+8*e^2*f^2)+2*c*e*(b*d*(-10*d^2*g^2+11*d
*e*f*g-4*e^2*f^2)+a*e*(3*d^2*g^2+2*d*e*f*g-2*e^2*f^2))+e^2*(3*a^2*e^2*g^2+2
*a*b*e*g*(-4*d*g+e*f)+b^2*(8*d^2*g^2-8*d*e*f*g+3*e^2*f^2)))*EllipticPi(2^(1
/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b
*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^
(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^
(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/(c*d^2+e*(a*e-b*d))^2/(d*g-e*f)^3*2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(
1/2)
```

Rubi [A] time = 8.01, antiderivative size = 1762, normalized size of antiderivative = 1.58, number of steps used = 25, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {939, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(e*f
 - d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*
e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*
f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2 - 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(b*
e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*
a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*S
```

```
qrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (3*Sqrt[b^2 - 4*a*c]*e*f*(c*d*(2*e*f
- 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c
*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
+ (3*Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*
g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^
2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*
c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g))
, ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c
])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f
)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x
+ c*x^2]) - (3*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g)
 - e*(b*e*f - 2*b*d*g + a*e*g))^2*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)
), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*
c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*
f)/g)])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^3*Sqrt[a +
 b*x + c*x^2])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
```

```
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3e^2(bf+ag)-4d(cef-cdg+beg)+2e(}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a-}}}{4\left(cd^2-bde+ae^2\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{3(-cd(2e}{(d+e}\right.}{4\left(cd\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4\left(cd^2-bde+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-}{4\left(cd^2-bde+\right.}$$

Mathematica [C] time = 17.88, size = 40396, normalized size = 36.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)^3\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

maple [B] time = 0.34, size = 64947, normalized size = 58.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)^3\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f + g\,x}\,(d + e\,x)^3\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3 \sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.916 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=553

$$-\frac{\sqrt{2}\,g\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}\,(ef-dg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\sqrt{2}\,e\sqrt{2cf-g\,(b-\sqrt{}$$

```
[Out] 2*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(1/2)-g*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*
g+e*f)/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)-e*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*
(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*
g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2
))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*
(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/(-d*g+e*f)^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.61, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {957, 744, 21, 718, 424, 934, 169, 538, 537}

$$-\frac{\sqrt{2}\,g\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}\,(ef-dg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\sqrt{2}\,e\sqrt{2cf-g\,(b-\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f +
 g*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*e*Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d
*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
```

```
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}} + \frac{e}{(ef-dg)(d+ex)\sqrt{f+\ldots}}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} + \frac{(2g)\int \frac{-\frac{cf}{2}-\frac{cgx}{2}}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\ldots\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\ldots\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{\left(\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\ldots\right.}{(ef-dg)\left(c\ldots\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\ldots}{(ef-dg)\left(cf^2\ldots\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\ldots}{(ef-dg)\left(cf^2\ldots\right.}$$

Mathematica [C] time = 6.12, size = 950, normalized size = 1.72

$$2\left(\frac{(a+x(b+cx))g^2}{ef-dg}+\frac{(f+gx)^2\left(\frac{cf^2}{(f+gx)^2}-\frac{2cf}{f+gx}-\frac{bgf}{(f+gx)^2}+c-\frac{i\sqrt{1-\frac{2(cf^2+g(ag-bf))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{\frac{4(cf^2+g(ag-bf))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}+2}\left((ef-dg)\left(2cf-bg+\sqrt{(b^2-4a\cdots}\right.\right.}{\cdots}\right)}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*((g^2*(a + x*(b + c*x)))/(e*f - d*g) + ((f + g*x)^2*(c + (c*f^2)/(f + g*
x)^2 - (b*f*g)/(f + g*x)^2 + (a*g^2)/(f + g*x)^2 - (2*c*f)/(f + g*x) + (b*g
)/(f + g*x) - ((I/4)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g
)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((e*f - d*g)*(2*c
*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f
^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x
]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2]))] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]) - 2*e*
(c*f^2 + g*(-(b*f) + a*g))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))] + 2*e*(c*f^2 + g*(-(b*f) + a*g))*EllipticPi[((e*f - d*g)*(2*c*f - b*g -
 Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))]))/((e*f - d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/(-(e*f) + d*g)))
/((c*f^2 + g*(-(b*f) + a*g))*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

maple [B] time = 0.10, size = 4757, normalized size = 8.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-2*c^2*d*g^3*x^2-2*a*c*d*g^3+2*a*c*e*f*g^2-2*2^(1/2)*EllipticF(2^(1/2)*(-(
g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^
(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e*f*g^2*(-(g*x+f)/(b
*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*
g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2
*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)-2*2^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)/
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^
2)^(1/2)*g))^(1/2))*a*c*e*f*g^2*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*
c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g
)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(
1/2)+3*2^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g
)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+
(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*e*f^2*g
*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2
)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^
(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)-2^(1/2)*EllipticPi(2^(1/2)
*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f
+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*e*f*g^2*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^
(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(
1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2
)*g)*g)^(1/2)*(-4*a*c+b^2)^(1/2)+2^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-
2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(
d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1
/2)*g))^(1/2))*c*e*f^2*g*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2
)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)
*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(-
4*a*c+b^2)^(1/2)+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1
/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/
2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*
EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2
*e*f^3-2*x*b*c*d*g^3+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c
)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)
^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1
/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(
-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))
*b*c*e*f^2*g+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*
((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(
(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Elli
pticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*e*f
*g^2+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x
-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b
+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^
(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*
```

a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d*f*g^2+2*c
^2*e*f*g^2*x^2-2*2^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+
b^2)^(1/2)*g))^(1/2))*c^2*e*f^3*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*
c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g
)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(
1/2)-2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x
-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b
+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticF(2^
(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*
a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b*c*d*f*g^2-2*2
^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/
2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+
b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*e*f^3*(-(g*x+f)
/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(
-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*
g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)-2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a
*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*
c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b
^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(
1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2
)^(1/2)*g))^(1/2))*b*c*e*f^2*g+2^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*
c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*
g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2
)*g))^(1/2))*a*b*e*g^3*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*
((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(
(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)+2^(1
/2)*EllipticPi(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),
1/2*(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*e*g^3*(-(g*x+f)/(b*g
-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+
2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c
*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*(-4*a*c+b^2)^(1/2)-2^(1/2)*EllipticPi(2^(
1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),1/2*(b*g-2*c*f+(-4
*a*c+b^2)^(1/2)*g)/(d*g-e*f)/c*e,(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2
*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*b^2*e*f*g^2*(-(g*x+f)/(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+
b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2
)^(1/2)*g)*g)^(1/2)+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)
^(1/2)*((-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^
(1/2)*((2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/
2)*EllipticF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*
a*c*d*g^3+2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-
2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*
c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*Ellipti
cF(2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f
+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d*f^2*
g-2*2^(1/2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+
(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-
4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/
2)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*a*c*d*g^3-2*2^(1/2
)*(-(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2)*((-2*c*x-b+(-4*a*c+b^
2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*((2*c*x+b+(-4*a*c+b^2)
^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*g)^(1/2)*EllipticE(2^(1/2)*(-(g*x+
f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)*c)^(1/2),(-(b*g-2*c*f+(-4*a*c+b^2)^(1/2
)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g))^(1/2))*c^2*d*f^2*g+2*x*b*c*e*f*g^2)
*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/(c*g*x

```
^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)\left(f + gx\right)^{\frac{3}{2}}\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

3.917 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1125

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{2cf-(\cdots}}\right.\right.}{\sqrt{c}\,(ef-dg)^3\sqrt{cx^2+bx+a}}$$

```
[Out] 2/3*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(3/2)+4/
3*g^2*(-b*g+2*c*f)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*
x+f)^(1/2)+2*e*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*
x+f)^(1/2)-2/3*g*(-b*g+2*c*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2
+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(c*x^2+b*x+a)^
(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*g*EllipticE(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a
*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)^2/(a*
g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/3*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)/(a*g^2-b*f
*g+c*f^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-e^2*EllipticPi(2^(1/2)*c^(1/2)*
(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-
4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(
1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-
g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 2.31, antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {957, 744, 834, 843, 718, 424, 419, 21, 934, 169, 538, 537}

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{2cf-(\cdots}}\right.\right.}{\sqrt{c}\,(ef-dg)^3\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^(3/2)) + (4*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*
f^2 - b*f*g + a*g^2)^2*Sqrt[f + g*x]) + (2*e*g^2*Sqrt[a + b*x + c*x^2])/((e
*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
```

```
])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*e*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((e*f - d*g)^2
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]) - (Sqrt[2]*e^2*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x)
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sq
rt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)
/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^3*Sqrt[a + b*x
+ c*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a}}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{}{(f}}{}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+bx}}{(ef-dg)^2\left(cf^2-bfg+\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a}}{3(ef-dg)\left(cf^2-bfg+\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a}}{3(ef-dg)\left(cf^2-bfg+\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a}}{3(ef-dg)\left(cf^2-bfg+\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a}}{3(ef-dg)\left(cf^2-bfg+\right.}$$

Mathematica [C] time = 15.52, size = 14759, normalized size = 13.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

maple [B] time = 0.21, size = 27597, normalized size = 24.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f + g\,x\right)^{5/2}\,\left(d + e\,x\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)\left(f + gx\right)^{\frac{5}{2}}\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

3.918 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=475

$$\frac{\sqrt{2}\,(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\,\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\,\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\,\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+2a\right)(e\cdots}{(d+ex)\left(f\sqrt{b^2-4ac}-2ag\cdots\right.}}}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\,\sqrt{a+bx+cx^2}\,\sqrt{2\cdots}}$$

```
[Out] (e*x+d)*EllipticPi((g*x+f)^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x+d)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2),e*(2*c*f-g*(b+(-4*a*c+b^2
)^(1/2)))/g/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))),((b*d-2*a*e+d*(-4*a*c+b^2)^(1/
2))*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2))/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(b+2*c*x-(-4*a*c+b^2)^(1/2))^(1
/2)*((-d*g+e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*x+d)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-d*g+e*f)*(2*a+x
*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2)))^(1/2)/g/
(c*x^2+b*x+a)^(1/2)/(c*x+2*a*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {926}

$$\frac{\sqrt{2}\,(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\,\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\,\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\,\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+2a\right)(e\cdots}{(d+ex)\left(f\sqrt{b^2-4ac}-2ag\cdots\right.}}}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\,\sqrt{a+bx+cx^2}\,\sqrt{2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c]
 + 2*c*x]*Sqrt[((e*f - d*g)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a + (b + Sqrt[b^2 -
 4*a*c])*x))/((b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g)*(d + e*x))]*(d + e*x)*Ell
ipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)*g), ArcSin[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]
)/(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])], ((b*d + Sqrt[b^
2 - 4*a*c]*d - 2*a*e)*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)*(b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g))])/(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*g*Sqrt[(2*a*c)/(b + Sqrt[b^2 - 4*a*c]) + c*x]*Sqr
t[a + b*x + c*x^2])
```

Rule 926

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(Sqr
t[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[((e*f - d*g
)*(b + q + 2*c*x))/((2*c*f - g*(b + q))*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a
+ (b + q)*x))/((b*f + q*f - 2*a*g)*(d + e*x))]*EllipticPi[(e*(2*c*f - g*(b
+ q)))/(g*(2*c*d - e*(b + q))), ArcSin[(Sqrt[2*c*d - e*(b + q)]*Sqrt[f + g*
x])/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x])], ((b*d + q*d - 2*a*e)*(2*c*f -
 g*(b + q)))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q)))])/(g*Sqrt[2*c*d - e*
(b + q)]*Sqrt[(2*a*c)/(b + q) + c*x]*Sqrt[a + b*x + c*x^2]), x]] /; FreeQ[{
```

```
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}}{\sqrt{2cd-}}$$

Mathematica [B] time = 9.47, size = 1118, normalized size = 2.35

$$-\frac{\sqrt{2}\sqrt{-\frac{g\left(cf^2+g(ag-bf)\right)(d+ex)}{\left(-2aeg^2-2cdfg+b(ef+dg)g-d\sqrt{\left(b^2-4ac\right)g^2}\,g+ef\sqrt{\left(b^2-4ac\right)g^2}\right)(f+gx)}}\,(f+gx)^{3/2}\left(\frac{2ef\sqrt{\left(b^2-4ac\right)g^2}\sqrt{-\frac{\left(cf^2+g(ag-bf)\right)(a+x(b+cx))}{\left(b^2-4ac\right)(f+gx)^2}}F}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-((g*(c*f^2 + g*(-(b*f) + a*g))*(d + e*x))/((-2*c*d*f*g - 2
*a*e*g^2 + e*f*Sqrt[(b^2 - 4*a*c)*g^2] - d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*
(e*f + d*g))*(f + g*x)))]*(f + g*x)^(3/2)*((2*e*f*Sqrt[(b^2 - 4*a*c)*g^2]*S
qrt[-(((c*f^2 + g*(-(b*f) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*
x)^2))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[
(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]]
, (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*S
qrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(
c*f^2 + g*(-(b*f) + a*g)) + (d*g*(2*a*g^2 - f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x - g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))*Sqrt[(2*a*g^2 - 2*c*
f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*
a*c)*g^2]*(f + g*x))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f
- g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a
*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e
*f + d*g))])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)*Sqrt[(-2*a*g^2 + 2*c*f*g
*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)
*g^2]*(f + g*x))]) - (4*e*Sqrt[(b^2 - 4*a*c)*g^2]*Sqrt[-(((c*f^2 + g*(-(b*f
) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*x)^2))]*EllipticPi[(2*Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]), ArcSin[Sqrt
[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))
/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*
(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*
Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2])))/(g^2*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

maple [A] time = 0.20, size = 645, normalized size = 1.36

$$\frac{4\sqrt{ex+d}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,\sqrt{\frac{\left(be-2cd+\sqrt{-4ac+b^2}\,e\right)\left(gx+f\right)}{\left(bg-2cf+\sqrt{-4ac+b^2}\,g\right)\left(ex+d\right)}}\,\sqrt{\frac{\left(dg-ef\right)\left(-2cx-b+\sqrt{-4ac+b^2}\right)}{\left(-bg+2cf+\sqrt{-4ac+b^2}\,g\right)\left(ex+d\right)}}\,\sqrt{\frac{\left(dg-ef\right)\left(2cx+b+\sqrt{-4a\right.}{\left(bg-2cf+\sqrt{-4ac+b^2}\,g\right.}}}{\sqrt{-\frac{\left(gx+f\right)\left(ex+d\right)\left(-2cx-b+\sqrt{-4ac+b^2}\right.}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 4*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g*((e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)*(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(e*x+d))^(1/2)*((d*g-e*
f)*(-2*c*x-b+(-4*a*c+b^2)^(1/2))/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)/(e*x+d))
^(1/2)*((d*g-e*f)*(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(-4*a*c+b^2)^(1/2
)*g)/(e*x+d))^(1/2)*EllipticPi(((e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)*(g*x+f)/(b
*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(e*x+d))^(1/2),(b*g-2*c*f+(-4*a*c+b^2)^(1/2)
*g)*e/g/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d),((e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)*
(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d))^(1/2))*((-4*a*c+b^2)^(1/2)*x^2*e^2*g+b*e^2*g*x^2-
2*c*e^2*f*x^2+2*(-4*a*c+b^2)^(1/2)*x*d*e*g+2*b*d*e*g*x-4*c*d*e*f*x+(-4*a*c+
b^2)^(1/2)*d^2*g+b*d^2*g-2*c*d^2*f)/(-1/c*(g*x+f)*(e*x+d)*(-2*c*x-b+(-4*a*c
+b^2)^(1/2))*(2*c*x+b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)/(c*e*g*x^4+b*e*g*x^3+c*d*g*x^3+c*e*f*x^3+a*e*g*x^2+b*d*g*x^2+b*e*f*x
^2+c*d*f*x^2+a*d*g*x+a*e*f*x+b*d*f*x+a*d*f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.919 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=588

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2}{(d+ex)(ag^2-bfg+cf^2)}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}}{\sqrt{a+bx+cx^2}\,(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-}}$$

```
[Out] -(c*f^2-g*(-a*g+b*f))^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)
*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arc
tan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+
d)^(1/2)))*EllipticF(sin(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(
a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/2*(2+(2*c*d*f+2*a*e*g-b*(d*g+e*f
))/(c*d^2-e*(-a*e+b*d))^(1/2)/(c*f^2-g*(-a*g+b*f))^(1/2))^(1/2))*(1+(g*x+f)
*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))*((-d*g+e*f)^
2*(c*x^2+b*x+a)/(a*g^2-b*f*g+c*f^2)/(e*x+d)^2)^(1/2)*((1-(2*c*d*f+2*a*e*g-b
*(d*g+e*f))*(g*x+f)/(a*g^2-b*f*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)
^2/(c*f^2-g*(-a*g+b*f))/(e*x+d)^2)/(1+(g*x+f)*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*
x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/4)/(-d*g+e
*f)/(c*x^2+b*x+a)^(1/2)/(1-(2*c*d*f+2*a*e*g-b*(d*g+e*f))*(g*x+f)/(a*g^2-b*f
*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)^2/(c*f^2-g*(-a*g+b*f))/(e*x+d
)^2)^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {935, 1103}

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2}{(d+ex)(ag^2-bfg+cf^2)}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}}{\sqrt{a+bx+cx^2}\,(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((c*f^2 - g*(b*f - a*g))^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c
*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 - b*d*e + a*
e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*g)]*(d + e*x)))*Sqrt[(1 - ((2*c*d*
f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g + a*g^2)*(d + e*x))
 + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f - a*g))*(d + e*x)
^2))/(1 + (Sqrt[c*d^2 - b*d*e + a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*
g)]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 - b*d*e + a*e^2)^(1/4)*Sqrt[f
 + g*x])/((c*f^2 - b*f*g + a*g^2)^(1/4)*Sqrt[d + e*x])], (2 + (2*c*d*f + 2*
a*e*g - b*(e*f + d*g))/(Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[c*f^2 - g*(b*f - a
*g)]))/4])/((c*d^2 - b*d*e + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]
*Sqrt[1 - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g +
 a*g^2)*(d + e*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f
 - a*g))*(d + e*x)^2)]))
```

Rule 935

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2
*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sq
rt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 -
b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+bx+cx^2\right)}{\left(cf^2-bfg+ag^2\right)(d+ex)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(2cdf-bef-bdg+2aeg)x^2}{cf^2-bfg+ag^2}+\frac{(cd^2-bde}{cf^2-bf}}}\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt[4]{cf^2-g(bf-ag)}\,(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+bx+cx^2\right)}{\left(cf^2-bfg+ag^2\right)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2-bde+ae^2}\,(f+gx}{\sqrt{cf^2-g(bf-ag)}\,(d+e}\right.}{\sqrt[4]{cd^2-bde+ae^2}\,(ef-dg}$$

Mathematica [A] time = 3.52, size = 375, normalized size = 0.64

$$\frac{2\sqrt{2}\,e\sqrt{a+x(b+cx)}\sqrt{-\frac{e(f+gx)\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(-dg\sqrt{e^2\left(b^2-4ac\right)}+ef\sqrt{e^2\left(b^2-4ac\right)}-2ae^2g+be(dg+ef)-2cdef\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{2ae^2-2cdxe+b(ex-d)e+\sqrt{\left(b^2-4ac\right)e}}{\sqrt{\left(b^2-4ac\right)e^2}\,(d+ex)}}}{\sqrt{2}}\right.\right.}{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{e^2\left(b^2-4ac\right)}\sqrt{-\frac{(a+x(b+cx))\left(e(ae-bd)+cd^2\right)}{\left(b^2-4ac\right)(d+ex)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*e*Sqrt[-((e*(c*d^2 + e*(-(b*d) + a*e))*(f + g*x))/((-2*c*d*e*f +
 e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g + b*
e*(e*f + d*g))*(d + e*x)))]*Sqrt[a + x*(b + c*x)]*EllipticF[ArcSin[Sqrt[(2*
a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/(Sq
rt[(b^2 - 4*a*c)*e^2]*(d + e*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]*(e*f
 - d*g))/(-2*c*d*e*f + e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^
2 - 4*a*c)*e^2]*g + b*e*(e*f + d*g))])/(Sqrt[(b^2 - 4*a*c)*e^2]*Sqrt[d + e*
x]*Sqrt[f + g*x]*Sqrt[-(((c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x)))/((b^
2 - 4*a*c)*(d + e*x)^2))])
```

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\,\sqrt{ex+d}\,\sqrt{gx+f}}{cegx^4+\left(cef+(cd+be)g\right)x^3+adf+\left((cd+be)f+(bd+ae)g\right)x^2+\left(adg+(bd+ae)f\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*
e*f + (c*d + b*e)*g)*x^3 + a*d*f + ((c*d + b*e)*f + (b*d + a*e)*g)*x^2 + (a
*d*g + (b*d + a*e)*f)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

maple [A] time = 0.10, size = 605, normalized size = 1.03

$$\frac{4\left(b e^2 g x^2-2c e^2 f x^2+2bdegx-4cdefx+\sqrt{-4ac+b^2}\,e^2 g x^2+b d^2 g-2c d^2 f+2\sqrt{-4ac+b^2}\,degx+\sqrt{-4a\cdots}\right.}{\sqrt{-\frac{(gx+f)(ex+d)\left(-2cx-b+\sqrt{-4ac+b^2}\right)\left(2cx+b+\sqrt{-4ac+b\cdots}\right.}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 4*(b*e^2*g*x^2-2*c*e^2*f*x^2+2*b*d*e*g*x-4*c*d*e*f*x+(-4*a*c+b^2)^(1/2)*e^2
*g*x^2+b*d^2*g-2*c*d^2*f+2*(-4*a*c+b^2)^(1/2)*d*e*g*x+(-4*a*c+b^2)^(1/2)*d^
2*g)*EllipticF(((b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)*(g*x+f)/(b*g-2*c*f+(-4*a*c
+b^2)^(1/2)*g)/(e*x+d))^(1/2),((-b*e+2*c*d+(-4*a*c+b^2)^(1/2)*e)*(b*g-2*c*f
+(-4*a*c+b^2)^(1/2)*g)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)/(b*e-2*c*d+(-4*a*c
+b^2)^(1/2)*e))^(1/2))*((d*g-e*f)*(2*c*x+b+(-4*a*c+b^2)^(1/2))/(b*g-2*c*f+(
-4*a*c+b^2)^(1/2)*g)/(e*x+d))^(1/2)*((d*g-e*f)*(-2*c*x-b+(-4*a*c+b^2)^(1/2)
)/(-b*g+2*c*f+(-4*a*c+b^2)^(1/2)*g)/(e*x+d))^(1/2)*((b*e-2*c*d+(-4*a*c+b^2)
^(1/2)*e)*(g*x+f)/(b*g-2*c*f+(-4*a*c+b^2)^(1/2)*g)/(e*x+d))^(1/2)*(e*x+d)^(
1/2)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-(g*x+f)*(e*x+d)*(-2*c*x-b+(-4*a*c+
b^2)^(1/2))*(2*c*x+b+(-4*a*c+b^2)^(1/2))/c)^(1/2)/(d*g-e*f)/(b*e-2*c*d+(-4*
a*c+b^2)^(1/2)*e)/(c*e*g*x^4+b*e*g*x^3+c*d*g*x^3+c*e*f*x^3+a*e*g*x^2+b*d*g*
x^2+b*e*f*x^2+c*d*f*x^2+a*d*g*x+a*e*f*x+b*d*f*x+a*d*f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.920 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=220

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(e$$

```
[Out] (a*e^2-b*d*e+c*d^2)*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^5/(1+m)-(-d*g+e*f)*(2*c*d*
(-2*d*g+e*f)-e*(2*a*e*g-3*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^5/(2+m)+(e*g*(a*e*g
-3*b*d*g+2*b*e*f)+c*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))*(e*x+d)^(3+m)/e^5/(3+m)+
g*(b*e*g-4*c*d*g+2*c*e*f)*(e*x+d)^(4+m)/e^5/(4+m)+c*g^2*(e*x+d)^(5+m)/e^5/(
5+m)
```

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {947}

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (
(e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)
^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 -
6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*
c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))
/(e^5*(5 + m))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^m}{e^4}+\frac{(ef-dg)(-2cd(ef-2d\right.\\ &= \frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)}-\frac{(ef-dg)(2cd(ef-2dg)\end{aligned}$$

Mathematica [A] time = 0.27, size = 198, normalized size = 0.90

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)^2\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{m+3}+\frac{(ef-dg)^2\left(e(ae-bd)+cd^2\right)}{m+1}+\frac{(d+ex)(ef-dg)(e(2aeg-3bdg+bef)+2cd(2d\right.}{m+2}}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^2)/(1 + m) + ((
e*f - d*g)*(2*c*d*(-(e*f) + 2*d*g) + e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*
x))/(2 + m) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g +
6*d^2*g^2))*(d + e*x)^2)/(3 + m) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*x)
^3)/(4 + m) + (c*g^2*(d + e*x)^4)/(5 + m)))/e^5
```

fricas [B] time = 1.27, size = 1381, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g^2*m^2 + 5
0*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + 30*b*e^5*g^2 + (2*c*e^5
*f*g + (c*d*e^4 + b*e^5)*g^2)*m^4 + (22*c*e^5*f*g + (6*c*d*e^4 + 11*b*e^5)*
g^2)*m^3 + (82*c*e^5*f*g + (11*c*d*e^4 + 41*b*e^5)*g^2)*m^2 + (122*c*e^5*f*
g + (6*c*d*e^4 + 61*b*e^5)*g^2)*m)*x^4 - (2*a*d^2*e^3*f*g + (b*d^2*e^3 - 14
*a*d*e^4)*f^2)*m^3 + (40*c*e^5*f^2 + 80*b*e^5*f*g + 40*a*e^5*g^2 + (c*e^5*f
^2 + 2*(c*d*e^4 + b*e^5)*f*g + (b*d*e^4 + a*e^5)*g^2)*m^4 + 4*(3*c*e^5*f^2
+ 2*(2*c*d*e^4 + 3*b*e^5)*f*g - (c*d^2*e^3 - 2*b*d*e^4 - 3*a*e^5)*g^2)*m^3
+ (49*c*e^5*f^2 + 2*(17*c*d*e^4 + 49*b*e^5)*f*g - (12*c*d^2*e^3 - 17*b*d*e^
4 - 49*a*e^5)*g^2)*m^2 + 2*(39*c*e^5*f^2 + 2*(5*c*d*e^4 + 39*b*e^5)*f*g - (
4*c*d^2*e^3 - 5*b*d*e^4 - 39*a*e^5)*g^2)*m)*x^3 + 20*(2*c*d^3*e^2 - 3*b*d^2
*e^3 + 6*a*d*e^4)*f^2 - 20*(3*c*d^4*e - 4*b*d^3*e^2 + 6*a*d^2*e^3)*f*g + 2*
(12*c*d^5 - 15*b*d^4*e + 20*a*d^3*e^2)*g^2 + (2*a*d^3*e^2*g^2 + (2*c*d^3*e^
2 - 12*b*d^2*e^3 + 71*a*d*e^4)*f^2 + 4*(b*d^3*e^2 - 6*a*d^2*e^3)*f*g)*m^2 +
 (60*b*e^5*f^2 + 120*a*e^5*f*g + (a*d*e^4*g^2 + (c*d*e^4 + b*e^5)*f^2 + 2*(
b*d*e^4 + a*e^5)*f*g)*m^4 + ((10*c*d*e^4 + 13*b*e^5)*f^2 - 2*(3*c*d^2*e^3 -
 10*b*d*e^4 - 13*a*e^5)*f*g - (3*b*d^2*e^3 - 10*a*d*e^4)*g^2)*m^3 + ((29*c*
d*e^4 + 59*b*e^5)*f^2 - 2*(18*c*d^2*e^3 - 29*b*d*e^4 - 59*a*e^5)*f*g + (12*
c*d^3*e^2 - 18*b*d^2*e^3 + 29*a*d*e^4)*g^2)*m^2 + ((20*c*d*e^4 + 107*b*e^5)
*f^2 - 2*(15*c*d^2*e^3 - 20*b*d*e^4 - 107*a*e^5)*f*g + (12*c*d^3*e^2 - 15*b
*d^2*e^3 + 20*a*d*e^4)*g^2)*m)*x^2 + ((18*c*d^3*e^2 - 47*b*d^2*e^3 + 154*a*
d*e^4)*f^2 - 2*(6*c*d^4*e - 18*b*d^3*e^2 + 47*a*d^2*e^3)*f*g - 6*(b*d^4*e -
 3*a*d^3*e^2)*g^2)*m + (120*a*e^5*f^2 + (2*a*d*e^4*f*g + (b*d*e^4 + a*e^5)*
f^2)*m^4 - 2*(a*d^2*e^3*g^2 + (c*d^2*e^3 - 6*b*d*e^4 - 7*a*e^5)*f^2 + 2*(b*
d^2*e^3 - 6*a*d*e^4)*f*g)*m^3 - ((18*c*d^2*e^3 - 47*b*d*e^4 - 71*a*e^5)*f^2
 - 2*(6*c*d^3*e^2 - 18*b*d^2*e^3 + 47*a*d*e^4)*f*g - 6*(b*d^3*e^2 - 3*a*d^2
*e^3)*g^2)*m^2 - 2*((20*c*d^2*e^3 - 30*b*d*e^4 - 77*a*e^5)*f^2 - 10*(3*c*d^
3*e^2 - 4*b*d^2*e^3 + 6*a*d*e^4)*f*g + (12*c*d^4*e - 15*b*d^3*e^2 + 20*a*d^
2*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*
m^2 + 274*e^5*m + 120*e^5)
```

giac [B] time = 0.27, size = 2740, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g^2*m^4*x^5*e^5 + (x*e + d)^m*c*d*g^2*m^4*x^4*e^4 + 2*(x*e +
 d)^m*c*f*g*m^4*x^4*e^5 + (x*e + d)^m*b*g^2*m^4*x^4*e^5 + 10*(x*e + d)^m*c*
g^2*m^3*x^5*e^5 + 2*(x*e + d)^m*c*d*f*g*m^4*x^3*e^4 + (x*e + d)^m*b*d*g^2*m
^4*x^3*e^4 + 6*(x*e + d)^m*c*d*g^2*m^3*x^4*e^4 - 4*(x*e + d)^m*c*d^2*g^2*m^
3*x^3*e^3 + (x*e + d)^m*c*f^2*m^4*x^3*e^5 + 2*(x*e + d)^m*b*f*g*m^4*x^3*e^5
 + (x*e + d)^m*a*g^2*m^4*x^3*e^5 + 22*(x*e + d)^m*c*f*g*m^3*x^4*e^5 + 11*(x
*e + d)^m*b*g^2*m^3*x^4*e^5 + 35*(x*e + d)^m*c*g^2*m^2*x^5*e^5 + (x*e + d)^
m*c*d*f^2*m^4*x^2*e^4 + 2*(x*e + d)^m*b*d*f*g*m^4*x^2*e^4 + (x*e + d)^m*a*d
*g^2*m^4*x^2*e^4 + 16*(x*e + d)^m*c*d*f*g*m^3*x^3*e^4 + 8*(x*e + d)^m*b*d*g
```

```
^2*m^3*x^3*e^4 + 11*(x*e + d)^m*c*d*g^2*m^2*x^4*e^4 - 6*(x*e + d)^m*c*d^2*f
*g*m^3*x^2*e^3 - 3*(x*e + d)^m*b*d^2*g^2*m^3*x^2*e^3 - 12*(x*e + d)^m*c*d^2
*g^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c*d^3*g^2*m^2*x^2*e^2 + (x*e + d)^m*b*f^2
*m^4*x^2*e^5 + 2*(x*e + d)^m*a*f*g*m^4*x^2*e^5 + 12*(x*e + d)^m*c*f^2*m^3*x
^3*e^5 + 24*(x*e + d)^m*b*f*g*m^3*x^3*e^5 + 12*(x*e + d)^m*a*g^2*m^3*x^3*e^
5 + 82*(x*e + d)^m*c*f*g*m^2*x^4*e^5 + 41*(x*e + d)^m*b*g^2*m^2*x^4*e^5 + 5
0*(x*e + d)^m*c*g^2*m*x^5*e^5 + (x*e + d)^m*b*d*f^2*m^4*x*e^4 + 2*(x*e + d)
^m*a*d*f*g*m^4*x*e^4 + 10*(x*e + d)^m*c*d*f^2*m^3*x^2*e^4 + 20*(x*e + d)^m*
b*d*f*g*m^3*x^2*e^4 + 10*(x*e + d)^m*a*d*g^2*m^3*x^2*e^4 + 34*(x*e + d)^m*c
*d*f*g*m^2*x^3*e^4 + 17*(x*e + d)^m*b*d*g^2*m^2*x^3*e^4 + 6*(x*e + d)^m*c*d
*g^2*m*x^4*e^4 - 2*(x*e + d)^m*c*d^2*f^2*m^3*x*e^3 - 4*(x*e + d)^m*b*d^2*f*
g*m^3*x*e^3 - 2*(x*e + d)^m*a*d^2*g^2*m^3*x*e^3 - 36*(x*e + d)^m*c*d^2*f*g*
m^2*x^2*e^3 - 18*(x*e + d)^m*b*d^2*g^2*m^2*x^2*e^3 - 8*(x*e + d)^m*c*d^2*g^
2*m*x^3*e^3 + 12*(x*e + d)^m*c*d^3*f*g*m^2*x*e^2 + 6*(x*e + d)^m*b*d^3*g^2*
m^2*x*e^2 + 12*(x*e + d)^m*c*d^3*g^2*m*x^2*e^2 - 24*(x*e + d)^m*c*d^4*g^2*m
*x*e + (x*e + d)^m*a*f^2*m^4*x*e^5 + 13*(x*e + d)^m*b*f^2*m^3*x^2*e^5 + 26*
(x*e + d)^m*a*f*g*m^3*x^2*e^5 + 49*(x*e + d)^m*c*f^2*m^2*x^3*e^5 + 98*(x*e
+ d)^m*b*f*g*m^2*x^3*e^5 + 49*(x*e + d)^m*a*g^2*m^2*x^3*e^5 + 122*(x*e + d)
^m*c*f*g*m*x^4*e^5 + 61*(x*e + d)^m*b*g^2*m*x^4*e^5 + 24*(x*e + d)^m*c*g^2*
x^5*e^5 + (x*e + d)^m*a*d*f^2*m^4*e^4 + 12*(x*e + d)^m*b*d*f^2*m^3*x*e^4 +
24*(x*e + d)^m*a*d*f*g*m^3*x*e^4 + 29*(x*e + d)^m*c*d*f^2*m^2*x^2*e^4 + 58*
(x*e + d)^m*b*d*f*g*m^2*x^2*e^4 + 29*(x*e + d)^m*a*d*g^2*m^2*x^2*e^4 + 20*(
x*e + d)^m*c*d*f*g*m*x^3*e^4 + 10*(x*e + d)^m*b*d*g^2*m*x^3*e^4 - (x*e + d)
^m*b*d^2*f^2*m^3*e^3 - 2*(x*e + d)^m*a*d^2*f*g*m^3*e^3 - 18*(x*e + d)^m*c*d
^2*f^2*m^2*x*e^3 - 36*(x*e + d)^m*b*d^2*f*g*m^2*x*e^3 - 18*(x*e + d)^m*a*d^
2*g^2*m^2*x*e^3 - 30*(x*e + d)^m*c*d^2*f*g*m*x^2*e^3 - 15*(x*e + d)^m*b*d^2
*g^2*m*x^2*e^3 + 2*(x*e + d)^m*c*d^3*f^2*m^2*e^2 + 4*(x*e + d)^m*b*d^3*f*g*
m^2*e^2 + 2*(x*e + d)^m*a*d^3*g^2*m^2*e^2 + 60*(x*e + d)^m*c*d^3*f*g*m*x*e^
2 + 30*(x*e + d)^m*b*d^3*g^2*m*x*e^2 - 12*(x*e + d)^m*c*d^4*f*g*m*e - 6*(x*
e + d)^m*b*d^4*g^2*m*e + 24*(x*e + d)^m*c*d^5*g^2 + 14*(x*e + d)^m*a*f^2*m^
3*x*e^5 + 59*(x*e + d)^m*b*f^2*m^2*x^2*e^5 + 118*(x*e + d)^m*a*f*g*m^2*x^2*
e^5 + 78*(x*e + d)^m*c*f^2*m*x^3*e^5 + 156*(x*e + d)^m*b*f*g*m*x^3*e^5 + 78
*(x*e + d)^m*a*g^2*m*x^3*e^5 + 60*(x*e + d)^m*c*f*g*x^4*e^5 + 30*(x*e + d)^
m*b*g^2*x^4*e^5 + 14*(x*e + d)^m*a*d*f^2*m^3*e^4 + 47*(x*e + d)^m*b*d*f^2*m
^2*x*e^4 + 94*(x*e + d)^m*a*d*f*g*m^2*x*e^4 + 20*(x*e + d)^m*c*d*f^2*m*x^2*
e^4 + 40*(x*e + d)^m*b*d*f*g*m*x^2*e^4 + 20*(x*e + d)^m*a*d*g^2*m*x^2*e^4 -
 12*(x*e + d)^m*b*d^2*f^2*m^2*e^3 - 24*(x*e + d)^m*a*d^2*f*g*m^2*e^3 - 40*(
x*e + d)^m*c*d^2*f^2*m*x*e^3 - 80*(x*e + d)^m*b*d^2*f*g*m*x*e^3 - 40*(x*e +
 d)^m*a*d^2*g^2*m*x*e^3 + 18*(x*e + d)^m*c*d^3*f^2*m*e^2 + 36*(x*e + d)^m*b
*d^3*f*g*m*e^2 + 18*(x*e + d)^m*a*d^3*g^2*m*e^2 - 60*(x*e + d)^m*c*d^4*f*g*
e - 30*(x*e + d)^m*b*d^4*g^2*e + 71*(x*e + d)^m*a*f^2*m^2*x*e^5 + 107*(x*e
+ d)^m*b*f^2*m*x^2*e^5 + 214*(x*e + d)^m*a*f*g*m*x^2*e^5 + 40*(x*e + d)^m*c
*f^2*x^3*e^5 + 80*(x*e + d)^m*b*f*g*x^3*e^5 + 40*(x*e + d)^m*a*g^2*x^3*e^5
+ 71*(x*e + d)^m*a*d*f^2*m^2*e^4 + 60*(x*e + d)^m*b*d*f^2*m*x*e^4 + 120*(x*
e + d)^m*a*d*f*g*m*x*e^4 - 47*(x*e + d)^m*b*d^2*f^2*m*e^3 - 94*(x*e + d)^m*
a*d^2*f*g*m*e^3 + 40*(x*e + d)^m*c*d^3*f^2*e^2 + 80*(x*e + d)^m*b*d^3*f*g*e
^2 + 40*(x*e + d)^m*a*d^3*g^2*e^2 + 154*(x*e + d)^m*a*f^2*m*x*e^5 + 60*(x*e
 + d)^m*b*f^2*x^2*e^5 + 120*(x*e + d)^m*a*f*g*x^2*e^5 + 154*(x*e + d)^m*a*d
*f^2*m*e^4 - 60*(x*e + d)^m*b*d^2*f^2*e^3 - 120*(x*e + d)^m*a*d^2*f*g*e^3 +
 120*(x*e + d)^m*a*f^2*x*e^5 + 120*(x*e + d)^m*a*d*f^2*e^4)/(m^5*e^5 + 15*m
^4*e^5 + 85*m^3*e^5 + 225*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

maple [B] time = 0.02, size = 1347, normalized size = 6.12

$$\frac{\left(c\,e^4g^2m^4x^4+b\,e^4g^2m^4x^3+2c\,e^4fg\,m^4x^3+10c\,e^4g^2m^3x^4+a\,e^4g^2m^4x^2+2b\,e^4fg\,m^4x^2+11b\,e^4g^2m^3x^3-4cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^4*g^2*m^4*x^4+b*e^4*g^2*m^4*x^3+2*c*e^4*f*g*m^4*x^3+10*c
*e^4*g^2*m^3*x^4+a*e^4*g^2*m^4*x^2+2*b*e^4*f*g*m^4*x^2+11*b*e^4*g^2*m^3*x^3
-4*c*d*e^3*g^2*m^3*x^3+c*e^4*f^2*m^4*x^2+22*c*e^4*f*g*m^3*x^3+35*c*e^4*g^2*
m^2*x^4+2*a*e^4*f*g*m^4*x+12*a*e^4*g^2*m^3*x^2-3*b*d*e^3*g^2*m^3*x^2+b*e^4*
f^2*m^4*x+24*b*e^4*f*g*m^3*x^2+41*b*e^4*g^2*m^2*x^3-6*c*d*e^3*f*g*m^3*x^2-2
4*c*d*e^3*g^2*m^2*x^3+12*c*e^4*f^2*m^3*x^2+82*c*e^4*f*g*m^2*x^3+50*c*e^4*g^
2*m*x^4-2*a*d*e^3*g^2*m^3*x+a*e^4*f^2*m^4+26*a*e^4*f*g*m^3*x+49*a*e^4*g^2*m
^2*x^2-4*b*d*e^3*f*g*m^3*x-24*b*d*e^3*g^2*m^2*x^2+13*b*e^4*f^2*m^3*x+98*b*e
^4*f*g*m^2*x^2+61*b*e^4*g^2*m*x^3+12*c*d^2*e^2*g^2*m^2*x^2-2*c*d*e^3*f^2*m^
3*x-48*c*d*e^3*f*g*m^2*x^2-44*c*d*e^3*g^2*m*x^3+49*c*e^4*f^2*m^2*x^2+122*c*
e^4*f*g*m*x^3+24*c*e^4*g^2*x^4-2*a*d*e^3*f*g*m^3-20*a*d*e^3*g^2*m^2*x+14*a*
e^4*f^2*m^3+118*a*e^4*f*g*m^2*x+78*a*e^4*g^2*m*x^2+6*b*d^2*e^2*g^2*m^2*x-b*
d*e^3*f^2*m^3-40*b*d*e^3*f*g*m^2*x-51*b*d*e^3*g^2*m*x^2+59*b*e^4*f^2*m^2*x+
156*b*e^4*f*g*m*x^2+30*b*e^4*g^2*x^3+12*c*d^2*e^2*f*g*m^2*x+36*c*d^2*e^2*g^
2*m*x^2-20*c*d*e^3*f^2*m^2*x-102*c*d*e^3*f*g*m*x^2-24*c*d*e^3*g^2*x^3+78*c*
e^4*f^2*m*x^2+60*c*e^4*f*g*x^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2-58*a*
d*e^3*g^2*m*x+71*a*e^4*f^2*m^2+214*a*e^4*f*g*m*x+40*a*e^4*g^2*x^2+4*b*d^2*e
^2*f*g*m^2+36*b*d^2*e^2*g^2*m*x-12*b*d*e^3*f^2*m^2-116*b*d*e^3*f*g*m*x-30*b
*d*e^3*g^2*x^2+107*b*e^4*f^2*m*x+80*b*e^4*f*g*x^2-24*c*d^3*e*g^2*m*x+2*c*d^
2*e^2*f^2*m^2+72*c*d^2*e^2*f*g*m*x+24*c*d^2*e^2*g^2*x^2-58*c*d*e^3*f^2*m*x-
60*c*d*e^3*f*g*x^2+40*c*e^4*f^2*x^2+18*a*d^2*e^2*g^2*m-94*a*d*e^3*f*g*m-40*
a*d*e^3*g^2*x+154*a*e^4*f^2*m+120*a*e^4*f*g*x-6*b*d^3*e*g^2*m+36*b*d^2*e^2*
f*g*m+30*b*d^2*e^2*g^2*x-47*b*d*e^3*f^2*m-80*b*d*e^3*f*g*x+60*b*e^4*f^2*x-1
2*c*d^3*e*f*g*m-24*c*d^3*e*g^2*x+18*c*d^2*e^2*f^2*m+60*c*d^2*e^2*f*g*x-40*c
*d*e^3*f^2*x+40*a*d^2*e^2*g^2-120*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+
80*b*d^2*e^2*f*g-60*b*d*e^3*f^2+24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^
2)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)
```

maxima [B] time = 0.58, size = 684, normalized size = 3.11

$$\frac{\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m bf^2}{\left(m^2+3m+2\right)e^2}+\frac{2\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m afg}{\left(m^2+3m+2\right)e^2}+\frac{(ex+d)^{m+1}af^2}{e(m+1)}+\frac{\left(\left(m^2+3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f^2/((m^2 + 3*m + 2)*e^2) +
 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*f*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 +
 m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f^2/((m^3 + 6*m^2 + 11*m
 + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x
 + 2*d^3)*(e*x + d)^m*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m +
2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*g^2/(
(m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3
+ 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(
e*x + d)^m*c*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^3 + 6*m^2
+ 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x
^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 +
11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*
d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c*g^2/((m^5 + 15*m^4 + 85*
m^3 + 225*m^2 + 274*m + 120)*e^5)
```

mupad [B] time = 3.94, size = 1354, normalized size = 6.15

$$\frac{(d+e\,x)^m\,\left(24\,c\,d^5\,g^2-12\,c\,d^4\,e\,f\,g\,m-60\,c\,d^4\,e\,f\,g-6\,b\,d^4\,e\,g^2\,m-30\,b\,d^4\,e\,g^2+2\,c\,d^3\,e^2\,f^2\,m^2+18\,c\,d^3\,e^2\,f^2\,m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*c*d^5*g^2 + 40*a*d^3*e^2*g^2 - 60*b*d^2*e^3*f^2 + 40*c*d^3
*e^2*f^2 + 120*a*d*e^4*f^2 - 30*b*d^4*e*g^2 - 120*a*d^2*e^3*f*g + 80*b*d^3*
e^2*f*g + 154*a*d*e^4*f^2*m - 6*b*d^4*e*g^2*m + 71*a*d*e^4*f^2*m^2 + 14*a*d
*e^4*f^2*m^3 + a*d*e^4*f^2*m^4 + 18*a*d^3*e^2*g^2*m - 47*b*d^2*e^3*f^2*m +
18*c*d^3*e^2*f^2*m - 60*c*d^4*e*f*g + 2*a*d^3*e^2*g^2*m^2 - 12*b*d^2*e^3*f^
2*m^2 - b*d^2*e^3*f^2*m^3 + 2*c*d^3*e^2*f^2*m^2 - 12*c*d^4*e*f*g*m - 94*a*d
^2*e^3*f*g*m + 36*b*d^3*e^2*f*g*m - 24*a*d^2*e^3*f*g*m^2 - 2*a*d^2*e^3*f*g*
m^3 + 4*b*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 +
 120)) + (x*(d + e*x)^m*(120*a*e^5*f^2 + 71*a*e^5*f^2*m^2 + 14*a*e^5*f^2*m^
3 + a*e^5*f^2*m^4 + 154*a*e^5*f^2*m + 60*b*d*e^4*f^2*m - 24*c*d^4*e*g^2*m -
 40*a*d^2*e^3*g^2*m + 47*b*d*e^4*f^2*m^2 + 12*b*d*e^4*f^2*m^3 + b*d*e^4*f^2
*m^4 + 30*b*d^3*e^2*g^2*m - 40*c*d^2*e^3*f^2*m - 18*a*d^2*e^3*g^2*m^2 - 2*a
*d^2*e^3*g^2*m^3 + 6*b*d^3*e^2*g^2*m^2 - 18*c*d^2*e^3*f^2*m^2 - 2*c*d^2*e^3
*f^2*m^3 + 120*a*d*e^4*f*g*m + 94*a*d*e^4*f*g*m^2 + 24*a*d*e^4*f*g*m^3 + 2*
a*d*e^4*f*g*m^4 - 80*b*d^2*e^3*f*g*m + 60*c*d^3*e^2*f*g*m - 36*b*d^2*e^3*f*
g*m^2 - 4*b*d^2*e^3*f*g*m^3 + 12*c*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2
+ 85*m^3 + 15*m^4 + m^5 + 120)) + (c*g^2*x^5*(d + e*x)^m*(50*m + 35*m^2 + 1
0*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (x^2*(
m + 1)*(d + e*x)^m*(60*b*e^3*f^2 + 12*b*e^3*f^2*m^2 + b*e^3*f^2*m^3 + 120*a
*e^3*f*g + 47*b*e^3*f^2*m + 12*c*d^3*g^2*m + 20*a*d*e^2*g^2*m - 15*b*d^2*e*
g^2*m + 20*c*d*e^2*f^2*m + 24*a*e^3*f*g*m^2 + 2*a*e^3*f*g*m^3 + 9*a*d*e^2*g
^2*m^2 + a*d*e^2*g^2*m^3 - 3*b*d^2*e*g^2*m^2 + 9*c*d*e^2*f^2*m^2 + c*d*e^2*
f^2*m^3 + 94*a*e^3*f*g*m + 40*b*d*e^2*f*g*m - 30*c*d^2*e*f*g*m + 18*b*d*e^2
*f*g*m^2 + 2*b*d*e^2*f*g*m^3 - 6*c*d^2*e*f*g*m^2))/(e^3*(274*m + 225*m^2 +
85*m^3 + 15*m^4 + m^5 + 120)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(20*a*e^2*
g^2 + 20*c*e^2*f^2 + a*e^2*g^2*m^2 + c*e^2*f^2*m^2 + 40*b*e^2*f*g + 9*a*e^2
*g^2*m - 4*c*d^2*g^2*m + 9*c*e^2*f^2*m + b*d*e*g^2*m^2 + 2*b*e^2*f*g*m^2 +
5*b*d*e*g^2*m + 18*b*e^2*f*g*m + 2*c*d*e*f*g*m^2 + 10*c*d*e*f*g*m))/(e^2*(2
74*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (g*x^4*(d + e*x)^m*(11*m +
 6*m^2 + m^3 + 6)*(5*b*e*g + 10*c*e*f + b*e*g*m + c*d*g*m + 2*c*e*f*m))/(e*
(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))
```

sympy [A] time = 14.70, size = 15757, normalized size = 71.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*f**2*x**2/2 + 2*
b*f*g*x**3/3 + b*g**2*x**4/4 + c*f**2*x**3/3 + c*f*g*x**4/2 + c*g**2*x**5/5
), Eq(e, 0)), (-a*d**2*e**2*g**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*a*d*e**3*f*g/(12*d**4*e**5 +
48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*a*d
*e**3*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) - 3*a*e**4*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d
**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*a*e**4*f*g*x/(12*d**4*e*
*5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) -
6*a*e**4*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*
d*e**8*x**3 + 12*e**9*x**4) - 3*b*d**3*e*g**2/(12*d**4*e**5 + 48*d**3*e**6*
x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*b*d**2*e**2*f*g/
(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e*
*9*x**4) - 12*b*d**2*e**2*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - b*d*e**3*f**2/(12*d**4*e**5 + 4
8*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*b*d*
e**3*f*g*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
**3 + 12*e**9*x**4) - 18*b*d*e**3*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x
+ 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*b*e**4*f**2*x/(12*
```

d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
4) - 12*b*e4*f*g*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**
2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 12*b*e**4*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*d*
*4*g**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) + 25*c*d**4*g**2/(12*d**4*e**5 + 48*d**3*e**6
*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*d**3*e*f*g/(1
2*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) + 48*c*d**3*e*g**2*x*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 7
2*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 88*c*d**3*e*g**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
4) - c*d2*e**2*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) - 24*c*d**2*e**2*f*g*x/(12*d**4*e**5 + 48*
d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 72*c*d**
2*e**2*g**2*x**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7
*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 108*c*d**2*e**2*g**2*x**2/(12*d**4
*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
 - 4*c*d*e**3*f**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) - 36*c*d*e**3*f*g*x**2/(12*d**4*e**5 + 48*d**
3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3
*g**2*x**3*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*e**
4*f**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*
x**3 + 12*e**9*x**4) - 24*c*e**4*f*g*x**3/(12*d**4*e**5 + 48*d**3*e**6*x +
72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*e**4*g**2*x**4*lo
g(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
3 + 12*e9*x**4), Eq(m, -5)), (-2*a*d**2*e**2*g**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*d*e**3*f*g/(6*d**3*e**5 + 18
*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*d*e**3*g**2*x/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*e**4*f**2/(6*d**3
*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*f*g*x/(6*
d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*g**2*
x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*d*
*3*e*g**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e
8*x3) + 11*b*d**3*e*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2
 + 6*e**8*x**3) - 4*b*d**2*e**2*f*g/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*
*7*x**2 + 6*e**8*x**3) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d**3*e**5 +
18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 27*b*d**2*e**2*g**2*x/(6*d
3*e5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - b*d*e**3*f**2/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*d*e**3*
f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 18*b*
d*e**3*g**2*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
2 + 6*e**8*x**3) + 18*b*d*e**3*g**2*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18
*d*e**7*x**2 + 6*e**8*x**3) - 3*b*e**4*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*e**4*f*g*x**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*e**4*g**2*x**3*log(d/e + x)/
(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 24*c*d**4*g
2*log(d/e + x)/(6*d3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x*
*3) - 44*c*d**4*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**
8*x**3) + 12*c*d**3*e*f*g*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d
*e**7*x**2 + 6*e**8*x**3) + 22*c*d**3*e*f*g/(6*d**3*e**5 + 18*d**2*e**6*x +
 18*d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**3*e*g**2*x*log(d/e + x)/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 108*c*d**3*e*g**2*x/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*c*d**2*e**
2*f**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 36*c
*d**2*e**2*f*g*x*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
2 + 6*e**8*x**3) + 54*c*d**2*e**2*f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*
d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**2*e**2*g**2*x**2*log(d/e + x)/(6*d**3*

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e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**2*e**2*g**2
*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*c*d
*e**3*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3)
+ 36*c*d*e**3*f*g*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*
*7*x**2 + 6*e**8*x**3) + 36*c*d*e**3*f*g*x**2/(6*d**3*e**5 + 18*d**2*e**6*x
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 24*c*d*e**3*g**2*x**3*log(d/e + x)/(6*d*
*3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*c*e**4*f**2*x*
*2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 12*c*e**
4*f*g*x**3*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*
e**8*x**3) + 6*c*e**4*g**2*x**4/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x
**2 + 6*e**8*x**3), Eq(m, -4)), (2*a*d**2*e**2*g**2*log(d/e + x)/(2*d**2*e*
*5 + 4*d*e**6*x + 2*e**7*x**2) + 3*a*d**2*e**2*g**2/(2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) - 2*a*d*e**3*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*a*d*e**3*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
+ 4*a*d*e**3*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - a*e**4*f**2/
(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*a*e**4*f*g*x/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) + 2*a*e**4*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) - 6*b*d**3*e*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e
**6*x + 2*e**7*x**2) - 9*b*d**3*e*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x
**2) + 4*b*d**2*e**2*f*g*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) + 6*b*d**2*e**2*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*d**
2*e**2*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*
d**2*e**2*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*d*e**3*f**2/(
2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x*log(d/e + x)/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2) - 6*b*d*e**3*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4
*d*e**6*x + 2*e**7*x**2) - 2*b*e**4*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e*
*7*x**2) + 4*b*e**4*f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**
7*x**2) + 2*b*e**4*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*
c*d**4*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 18*c*d*
*4*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d**3*e*f*g*log(d/e
+ x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 18*c*d**3*e*f*g/(2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x*log(d/e + x)/(2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x/(2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) + 2*c*d**2*e**2*f**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) + 3*c*d**2*e**2*f**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) - 24*c*d**2*e**2*f*g*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*
x**2) - 24*c*d**2*e**2*f*g*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*
c*d**2*e**2*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*c*d*e**3*f**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
+ 4*c*d*e**3*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d*e**3*
f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c*d*e**3
*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*c*e**4*f**2*x**2*lo
g(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*c*e**4*f*g*x**3/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c*e**4*g**2*x**4/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-12*a*d**2*e**2*g**2*log(d/e + x)/(6*d
*e**5 + 6*e**6*x) - 12*a*d**2*e**2*g**2/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3
*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3*f*g/(6*d*e**5 + 6*e**
6*x) - 12*a*d*e**3*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 6*a*e**4*f**
2/(6*d*e**5 + 6*e**6*x) + 12*a*e**4*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x
) + 6*a*e**4*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2/(6*d*e**5 + 6*e**6*x) - 24*b*d**
2*e**2*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e**2*f*g/(6*d*e**
5 + 6*e**6*x) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) +
6*b*d*e**3*f**2*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 6*b*d*e**3*f**2/(6*d*e
**5 + 6*e**6*x) - 24*b*d*e**3*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 9*
b*d*e**3*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*b*e**4*f**2*x*log(d/e + x)/(6*
d*e**5 + 6*e**6*x) + 12*b*e**4*f*g*x**2/(6*d*e**5 + 6*e**6*x) + 3*b*e**4*g*
*2*x**3/(6*d*e**5 + 6*e**6*x) - 24*c*d**4*g**2*log(d/e + x)/(6*d*e**5 + 6*e
```

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**6*x) - 24*c*d**4*g**2/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g/(6*d*e**5 + 6*e**6*x) - 24*c*d**3
*e*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2*log(d/e
+ x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2/(6*d*e**5 + 6*e**6*x) + 36
*c*d**2*e**2*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*c*d**2*e**2*g**2
*x**2/(6*d*e**5 + 6*e**6*x) - 12*c*d*e**3*f**2*x*log(d/e + x)/(6*d*e**5 + 6
*e**6*x) - 18*c*d*e**3*f*g*x**2/(6*d*e**5 + 6*e**6*x) - 4*c*d*e**3*g**2*x**
3/(6*d*e**5 + 6*e**6*x) + 6*c*e**4*f**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*c*e*
*4*f*g*x**3/(6*d*e**5 + 6*e**6*x) + 2*c*e**4*g**2*x**4/(6*d*e**5 + 6*e**6*x
), Eq(m, -2)), (a*d**2*g**2*log(d/e + x)/e**3 - 2*a*d*f*g*log(d/e + x)/e**2
 - a*d*g**2*x/e**2 + a*f**2*log(d/e + x)/e + 2*a*f*g*x/e + a*g**2*x**2/(2*e
) - b*d**3*g**2*log(d/e + x)/e**4 + 2*b*d**2*f*g*log(d/e + x)/e**3 + b*d**2
*g**2*x/e**3 - b*d*f**2*log(d/e + x)/e**2 - 2*b*d*f*g*x/e**2 - b*d*g**2*x**
2/(2*e**2) + b*f**2*x/e + b*f*g*x**2/e + b*g**2*x**3/(3*e) + c*d**4*g**2*lo
g(d/e + x)/e**5 - 2*c*d**3*f*g*log(d/e + x)/e**4 - c*d**3*g**2*x/e**4 + c*d
**2*f**2*log(d/e + x)/e**3 + 2*c*d**2*f*g*x/e**3 + c*d**2*g**2*x**2/(2*e**3
) - c*d*f**2*x/e**2 - c*d*f*g*x**2/e**2 - c*d*g**2*x**3/(3*e**2) + c*f**2*x
**2/(2*e) + 2*c*f*g*x**3/(3*e) + c*g**2*x**4/(4*e), Eq(m, -1)), (2*a*d**3*e
**2*g**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 18*a*d**3*e**2*g**2*m*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 40*a*d**3*e**2*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*a*d**2*e**3*f*g*m**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) - 24*a*d**2*e**3*f*g*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 94*a*d**2*e*
*3*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) - 120*a*d**2*e**3*f*g*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*
a*d**2*e**3*g**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 18*a*d**2*e**3*g**2*m**2*x*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) - 40*a*d**2*e**3*g**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*d*e**4*
f**2*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 14*a*d*e**4*f**2*m**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 71*a*d*e**4*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a*d*e**4*f**2*m*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 120*a*d*e**4*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*d*e**4*f*g*m**4*x*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*a*d*e**4*f*g*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 94*a*d*e*
*4*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*d*e**4*f*g*m*x*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + a*d*e**4*g**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*a*d*e**4*g**2*m**3*x**
2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5) + 29*a*d*e**4*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20
*a*d*e**4*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*e**5*f**2*m**4*x*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) + 14*a*e**5*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a*e**5*f**2*m**2*x
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
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4*e**5*m + 120*e**5) + 154*a*e**5*f**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5
*f**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 2*a*e**5*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
26*a*e**5*f*g*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 118*a*e**5*f*g*m**2*x**2*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 214*a*e**5*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5*f*
g*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) + a*e**5*g**2*m**4*x**3*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12
*a*e**5*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 49*a*e**5*g**2*m**2*x**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 78*a*e**5*g**2*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*a*e**5*g**2
*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
 + 274*e**5*m + 120*e**5) - 6*b*d**4*e*g**2*m*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*b*d*
*4*e*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 4*b*d**3*e**2*f*g*m**2*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 36*b*d**3*e**2*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*b*d**3*e**2*f*g*(d + e*x)**
m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 1
20*e**5) + 6*b*d**3*e**2*g**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 30*b*d**3*e**2*g
**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) - b*d**2*e**3*f**2*m**3*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 1
2*b*d**2*e**3*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 47*b*d**2*e**3*f**2*m*(d + e*
x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
 + 120*e**5) - 60*b*d**2*e**3*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*b*d**2*e**3*f*g*
m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) - 36*b*d**2*e**3*f*g*m**2*x*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 80*b*d**2*e**3*f*g*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 3*b*d**2*e**3*g**2*m**3*x**2
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) - 18*b*d**2*e**3*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) -
15*b*d**2*e**3*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*f**2*m**4*x*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 12*b*d*e**4*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 47*b*d*e**4*
f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + 60*b*d*e**4*f**2*m*x*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 2*b*d*e**4*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*b*d*e**4*f*g*m**3*x**2*
(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274
*e**5*m + 120*e**5) + 58*b*d*e**4*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b*
d*e**4*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 2
```

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25*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*g**2*m**4*x**3*(d + e*x)**
m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 1
20*e**5) + 8*b*d*e**4*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 17*b*d*e**4*g**2
*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 10*b*d*e**4*g**2*m*x**3*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + b*e**5*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*
m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 13*b*e**5*f**2*m**3*x**2*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 59*b*e**5*f**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 107*b*e*
*5*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 60*b*e**5*f**2*x**2*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + 2*b*e**5*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*b*e**5*f*g*m**3*x**3*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 98*b*e**5*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 156*b*e**
5*f*g*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) + 80*b*e**5*f*g*x**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 b*e**5*g**2*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*b*e**5*g**2*m**3*x**4*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 41*b*e**5*g**2*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 61*b*e**5*g
**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 30*b*e**5*g**2*x**4*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
24*c*d**5*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) - 12*c*d**4*e*f*g*m*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 60*c*d**4*e*f*g*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 2
25*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*c*d**4*e*g**2*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 2*c*d**3*e**2*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 18*c*d**3*e**2*f**2*m
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 40*c*d**3*e**2*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*
e**2*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*c*d**3*e**2*f*g*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 12*c*d**3*e**2*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*e**2*
g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) - 2*c*d**2*e**3*f**2*m**3*x*(d + e*x)**m/(e
**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e
**5) - 18*c*d**2*e**3*f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 40*c*d**2*e**3*f**2
*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) - 6*c*d**2*e**3*f*g*m**3*x**2*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 36*c*d**2*e**3*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*c*d**2*e**3*f*g*m*x
**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
 274*e**5*m + 120*e**5) - 4*c*d**2*e**3*g**2*m**3*x**3*(d + e*x)**m/(e**5*m
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**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 12*c*d**2*e**3*g**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8*c*d**2*e**3*g**2*m*
x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) + c*d*e**4*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10
*c*d*e**4*f**2*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 29*c*d*e**4*f**2*m**2*x**2*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 20*c*d*e**4*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*d*e*
*4*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22
5*e**5*m**2 + 274*e**5*m + 120*e**5) + 16*c*d*e**4*f*g*m**3*x**3*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 34*c*d*e**4*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*c*d*e**4*f*g
*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + c*d*e**4*g**2*m**4*x**4*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 6*c*d*e**4*g**2*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*c*d*e**4*g**2*m**2*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 6*c*d*e**4*g**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*
f**2*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*e**5*f**2*m**3*x**3*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) + 49*c*e**5*f**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 78*c*e**5*f**2*m*x*
*3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + 40*c*e**5*f**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*e**5
*f*g*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 22*c*e**5*f*g*m**3*x**4*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 82*c*e**5*f*g*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 122*c*e**5*f*g*m*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 60*c*e**5*f*g*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*g**2
*m**4*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 10*c*e**5*g**2*m**3*x**5*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 35*c*e**5*g**2*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 50*c*e**5*g**2*m*x**5*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*c*e**5*g**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), True))
```

3.921 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=144

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-3c}{e^4(m+3)}$$

[Out] (a*e^2-b*d*e+c*d^2)*(-d*g+e*f)*(e*x+d)^(1+m)/e^4/(1+m)-(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^4/(2+m)+(b*e*g-3*c*d*g+c*e*f)*(e*x+d)^(3+m)/e^4/(3+m)+c*g*(e*x+d)^(4+m)/e^4/(4+m)

Rubi [A] time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {771}

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-3c}{e^4(m+3)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]

[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4*(2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*g*(d + e*x)^(4 + m))/(e^4*(4 + m))

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx=\int \left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^m}{e^3}+\frac{(-cd(2ef-3dg)+e(bef-2}{e^3}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)}-\frac{(cd(2ef-3dg)-e(bef-2bdg}{e^4(2+m)}$$

Mathematica [A] time = 0.33, size = 180, normalized size = 1.25

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)\left(ce(2aeg(m+3)+bdg(m-2)+bef(m+4))-b^2e^2g(m+2)+2c^2d(3dg-ef(m+4))\right)}{e^2(m+2)}-\frac{\left(e(ae-bd)+cd^2\right)(beg(m+1)+6cdg-2cef(m+4))}{e^2(m+1)}+\right.}{ce^2(m+3)(m+4)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]

[Out] ((d + e*x)^(1 + m)*(-(((c*d^2 + e*(-(b*d) + a*e))*(6*c*d*g + b*e*g*(1 + m) - 2*c*e*f*(4 + m)))/(e^2*(1 + m))) + ((-(b^2*e^2*g*(2 + m)) + 2*c^2*d*(3*d*g - e*f*(4 + m)) + c*e*(b*d*g*(-2 + m) + 2*a*e*g*(3 + m) + b*e*f*(4 + m)))*(d + e*x))/(e^2*(2 + m)) + (a + x*(b + c*x))*(b*e*g + c*(-3*d*g + e*f*(4 + m) + e*g*(3 + m)*x))))/(c*e^2*(3 + m)*(4 + m))

fricas [B] time = 0.97, size = 613, normalized size = 4.26

$$\frac{\left(ade^3fm^3 + \left(ce^4gm^3 + 6\,ce^4gm^2 + 11\,ce^4gm + 6\,ce^4g\right)x^4 + \left(8\,ce^4f + 8\,be^4g + \left(ce^4f + \left(cde^3 + be^4\right)g\right)m^3 + \left(7\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^3*f*m^3 + (c*e^4*g*m^3 + 6*c*e^4*g*m^2 + 11*c*e^4*g*m + 6*c*e^4*g)*x
^4 + (8*c*e^4*f + 8*b*e^4*g + (c*e^4*f + (c*d*e^3 + b*e^4)*g)*m^3 + (7*c*e^
4*f + (3*c*d*e^3 + 7*b*e^4)*g)*m^2 + 2*(7*c*e^4*f + (c*d*e^3 + 7*b*e^4)*g)*
m)*x^3 - (a*d^2*e^2*g + (b*d^2*e^2 - 9*a*d*e^3)*f)*m^2 + (12*b*e^4*f + 12*a
*e^4*g + ((c*d*e^3 + b*e^4)*f + (b*d*e^3 + a*e^4)*g)*m^3 + ((5*c*d*e^3 + 8*
b*e^4)*f - (3*c*d^2*e^2 - 5*b*d*e^3 - 8*a*e^4)*g)*m^2 + ((4*c*d*e^3 + 19*b*
e^4)*f - (3*c*d^2*e^2 - 4*b*d*e^3 - 19*a*e^4)*g)*m)*x^2 + 4*(2*c*d^3*e - 3*
b*d^2*e^2 + 6*a*d*e^3)*f - 2*(3*c*d^4 - 4*b*d^3*e + 6*a*d^2*e^2)*g + ((2*c*
d^3*e - 7*b*d^2*e^2 + 26*a*d*e^3)*f + (2*b*d^3*e - 7*a*d^2*e^2)*g)*m + (24*
a*e^4*f + (a*d*e^3*g + (b*d*e^3 + a*e^4)*f)*m^3 - ((2*c*d^2*e^2 - 7*b*d*e^3
 - 9*a*e^4)*f + (2*b*d^2*e^2 - 7*a*d*e^3)*g)*m^2 - 2*((4*c*d^2*e^2 - 6*b*d*
e^3 - 13*a*e^4)*f - (3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)*g)*m)*x)*(e*x + d
)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

giac [B] time = 0.20, size = 1162, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g*m^3*x^4*e^4 + (x*e + d)^m*c*d*g*m^3*x^3*e^3 + (x*e + d)^m*
c*f*m^3*x^3*e^4 + (x*e + d)^m*b*g*m^3*x^3*e^4 + 6*(x*e + d)^m*c*g*m^2*x^4*e
^4 + (x*e + d)^m*c*d*f*m^3*x^2*e^3 + (x*e + d)^m*b*d*g*m^3*x^2*e^3 + 3*(x*e
 + d)^m*c*d*g*m^2*x^3*e^3 - 3*(x*e + d)^m*c*d^2*g*m^2*x^2*e^2 + (x*e + d)^m
*b*f*m^3*x^2*e^4 + (x*e + d)^m*a*g*m^3*x^2*e^4 + 7*(x*e + d)^m*c*f*m^2*x^3*
e^4 + 7*(x*e + d)^m*b*g*m^2*x^3*e^4 + 11*(x*e + d)^m*c*g*m*x^4*e^4 + (x*e +
 d)^m*b*d*f*m^3*x*e^3 + (x*e + d)^m*a*d*g*m^3*x*e^3 + 5*(x*e + d)^m*c*d*f*m
^2*x^2*e^3 + 5*(x*e + d)^m*b*d*g*m^2*x^2*e^3 + 2*(x*e + d)^m*c*d*g*m*x^3*e^
3 - 2*(x*e + d)^m*c*d^2*f*m^2*x*e^2 - 2*(x*e + d)^m*b*d^2*g*m^2*x*e^2 - 3*(
x*e + d)^m*c*d^2*g*m*x^2*e^2 + 6*(x*e + d)^m*c*d^3*g*m*x*e + (x*e + d)^m*a*
f*m^3*x*e^4 + 8*(x*e + d)^m*b*f*m^2*x^2*e^4 + 8*(x*e + d)^m*a*g*m^2*x^2*e^4
 + 14*(x*e + d)^m*c*f*m*x^3*e^4 + 14*(x*e + d)^m*b*g*m*x^3*e^4 + 6*(x*e + d
)^m*c*g*x^4*e^4 + (x*e + d)^m*a*d*f*m^3*e^3 + 7*(x*e + d)^m*b*d*f*m^2*x*e^3
 + 7*(x*e + d)^m*a*d*g*m^2*x*e^3 + 4*(x*e + d)^m*c*d*f*m*x^2*e^3 + 4*(x*e +
 d)^m*b*d*g*m*x^2*e^3 - (x*e + d)^m*b*d^2*f*m^2*e^2 - (x*e + d)^m*a*d^2*g*m
^2*e^2 - 8*(x*e + d)^m*c*d^2*f*m*x*e^2 - 8*(x*e + d)^m*b*d^2*g*m*x*e^2 + 2*
(x*e + d)^m*c*d^3*f*m*e + 2*(x*e + d)^m*b*d^3*g*m*e - 6*(x*e + d)^m*c*d^4*g
 + 9*(x*e + d)^m*a*f*m^2*x*e^4 + 19*(x*e + d)^m*b*f*m*x^2*e^4 + 19*(x*e + d
)^m*a*g*m*x^2*e^4 + 8*(x*e + d)^m*c*f*x^3*e^4 + 8*(x*e + d)^m*b*g*x^3*e^4 +
 9*(x*e + d)^m*a*d*f*m^2*e^3 + 12*(x*e + d)^m*b*d*f*m*x*e^3 + 12*(x*e + d)^
m*a*d*g*m*x*e^3 - 7*(x*e + d)^m*b*d^2*f*m*e^2 - 7*(x*e + d)^m*a*d^2*g*m*e^2
 + 8*(x*e + d)^m*c*d^3*f*e + 8*(x*e + d)^m*b*d^3*g*e + 26*(x*e + d)^m*a*f*m
*x*e^4 + 12*(x*e + d)^m*b*f*x^2*e^4 + 12*(x*e + d)^m*a*g*x^2*e^4 + 26*(x*e
+ d)^m*a*d*f*m*e^3 - 12*(x*e + d)^m*b*d^2*f*e^2 - 12*(x*e + d)^m*a*d^2*g*e^
2 + 24*(x*e + d)^m*a*f*x*e^4 + 24*(x*e + d)^m*a*d*f*e^3)/(m^4*e^4 + 10*m^3*
e^4 + 35*m^2*e^4 + 50*m*e^4 + 24*e^4)
```

maple [B] time = 0.01, size = 503, normalized size = 3.49

$$-\frac{\left(-c\,e^3g\,m^3x^3 - b\,e^3g\,m^3x^2 - c\,e^3f\,m^3x^2 - 6c\,e^3g\,m^2x^3 - a\,e^3g\,m^3x - b\,e^3f\,m^3x - 7b\,e^3g\,m^2x^2 + 3cd\,e^2g\,m^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x)
```

```
[Out] -(e*x+d)^(m+1)*(-c*e^3*g*m^3*x^3-b*e^3*g*m^3*x^2-c*e^3*f*m^3*x^2-6*c*e^3*g*
m^2*x^3-a*e^3*g*m^3*x-b*e^3*f*m^3*x-7*b*e^3*g*m^2*x^2+3*c*d*e^2*g*m^2*x^2-7
*c*e^3*f*m^2*x^2-11*c*e^3*g*m*x^3-a*e^3*f*m^3-8*a*e^3*g*m^2*x+2*b*d*e^2*g*m
^2*x-8*b*e^3*f*m^2*x-14*b*e^3*g*m*x^2+2*c*d*e^2*f*m^2*x+9*c*d*e^2*g*m*x^2-1
4*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^2*g*m^2-9*a*e^3*f*m^2-19*a*e^3*g*m*x+b*
d*e^2*f*m^2+10*b*d*e^2*g*m*x-19*b*e^3*f*m*x-8*b*e^3*g*x^2-6*c*d^2*e*g*m*x+1
0*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*c*e^3*f*x^2+7*a*d*e^2*g*m-26*a*e^3*f*m-12
*a*e^3*g*x-2*b*d^2*e*g*m+7*b*d*e^2*f*m+8*b*d*e^2*g*x-12*b*e^3*f*x-2*c*d^2*e
*f*m-6*c*d^2*e*g*x+8*c*d*e^2*f*x+12*a*d*e^2*g-24*a*e^3*f-8*b*d^2*e*g+12*b*d
*e^2*f+6*c*d^3*g-8*c*d^2*e*f)/e^4/(m^4+10*m^3+35*m^2+50*m+24)
```

maxima [B] time = 0.52, size = 352, normalized size = 2.44

$$\frac{\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m bf}{\left(m^2+3m+2\right)e^2}+\frac{\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m ag}{\left(m^2+3m+2\right)e^2}+\frac{(ex+d)^{m+1}af}{e(m+1)}+\frac{\left(\left(m^2+3m+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f/((m^2 + 3*m + 2)*e^2) + (
e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*g/((m^2 + 3*m + 2)*e^2) + (e
*x + d)^(m + 1)*a*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e
*x + d)^m*b*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^
4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m
*x - 6*d^4)*(e*x + d)^m*c*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

mupad [B] time = 3.59, size = 602, normalized size = 4.18

$$\frac{(d+e\,x)^m\,\left(24\,a\,d\,e^3\,f-6\,c\,d^4\,g+8\,b\,d^3\,e\,g+8\,c\,d^3\,e\,f-12\,a\,d^2\,e^2\,g-12\,b\,d^2\,e^2\,f+9\,a\,d\,e^3\,f\,m^2+a\,d\,e^3\,f\,m^3-7\right.}{e^4\,\left(m^4+10\,m^3+35\,m^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*a*d*e^3*f - 6*c*d^4*g + 8*b*d^3*e*g + 8*c*d^3*e*f - 12*a*d
^2*e^2*g - 12*b*d^2*e^2*f + 9*a*d*e^3*f*m^2 + a*d*e^3*f*m^3 - 7*a*d^2*e^2*g
*m - 7*b*d^2*e^2*f*m - a*d^2*e^2*g*m^2 - b*d^2*e^2*f*m^2 + 26*a*d*e^3*f*m +
 2*b*d^3*e*g*m + 2*c*d^3*e*f*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
+ (x*(d + e*x)^m*(24*a*e^4*f + 26*a*e^4*f*m + 9*a*e^4*f*m^2 + a*e^4*f*m^3 +
 7*a*d*e^3*g*m^2 + 7*b*d*e^3*f*m^2 + a*d*e^3*g*m^3 + b*d*e^3*f*m^3 - 8*b*d^
2*e^2*g*m - 8*c*d^2*e^2*f*m - 2*b*d^2*e^2*g*m^2 - 2*c*d^2*e^2*f*m^2 + 12*a*
d*e^3*g*m + 12*b*d*e^3*f*m + 6*c*d^3*e*g*m))/(e^4*(50*m + 35*m^2 + 10*m^3 +
 m^4 + 24)) + (x^2*(m + 1)*(d + e*x)^m*(12*a*e^2*g + 12*b*e^2*f + 7*a*e^2*g
*m + 7*b*e^2*f*m - 3*c*d^2*g*m + a*e^2*g*m^2 + b*e^2*f*m^2 + 4*b*d*e*g*m +
4*c*d*e*f*m + b*d*e*g*m^2 + c*d*e*f*m^2))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^
4 + 24)) + (c*g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 +
10*m^3 + m^4 + 24) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(4*b*e*g + 4*c*e*f +
b*e*g*m + c*d*g*m + c*e*f*m))/(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy [A] time = 5.86, size = 5930, normalized size = 41.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f*x + a*g*x**2/2 + b*f*x**2/2 + b*g*x**3/3 + c*f*x**3/3
+ c*g*x**4/4), Eq(e, 0)), (-a*d*e**2*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 2*a*e**3*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 3*a*e**3*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18
*d*e**6*x**2 + 6*e**7*x**3) - 2*b*d**2*e*g/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - b*d*e**2*f/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*d*e**2*g*x/(6*d**3*e**4 + 18*d**2*e**5*
x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*b*e**3*f*x/(6*d**3*e**4 + 18*d**2*e**
5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*e**3*g*x**2/(6*d**3*e**4 + 18*d**
2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*d**3*g*log(d/e + x)/(6*d**3*
e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c*d**3*g/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 2*c*d**2*e*f/(6*d*
*3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d**2*e*g*x*
log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3)
+ 27*c*d**2*e*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x
**3) - 6*c*d*e**2*f*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e*
*7*x**3) + 18*c*d*e**2*g*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**2/(6*d**3*e**4 + 18*d**2*e
**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*c*e**3*f*x**2/(6*d**3*e**4 + 18*d
**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*e**3*g*x**3*log(d/e + x)/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -4)), (
-a*d*e**2*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - a*e**3*f/(2*d**2*e**
4 + 4*d*e**5*x + 2*e**6*x**2) - 2*a*e**3*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*
e**6*x**2) + 2*b*d**2*e*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x
**2) + 3*b*d**2*e*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - b*d*e**2*f/(
2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x*log(d/e + x)/(2*d*
*2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x/(2*d**2*e**4 + 4*d*e**
5*x + 2*e**6*x**2) - 2*b*e**3*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2)
+ 2*b*e**3*g*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6
*c*d**3*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 9*c*d**3*
g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*d**2*e*f*log(d/e + x)/(2*d
**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 3*c*d**2*e*f/(2*d**2*e**4 + 4*d*e**5
*x + 2*e**6*x**2) - 12*c*d**2*e*g*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x
+ 2*e**6*x**2) - 12*c*d**2*e*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) +
 4*c*d*e**2*f*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*c
*d*e**2*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d*e**2*g*x**2*lo
g(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*f*x**2*log(d
/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*g*x**3/(2*d**2*
e**4 + 4*d*e**5*x + 2*e**6*x**2), Eq(m, -3)), (2*a*d*e**2*g*log(d/e + x)/(2
*d*e**4 + 2*e**5*x) + 2*a*d*e**2*g/(2*d*e**4 + 2*e**5*x) - 2*a*e**3*f/(2*d*
e**4 + 2*e**5*x) + 2*a*e**3*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d*
*2*e*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*e*g/(2*d*e**4 + 2*e**5
*x) + 2*b*d*e**2*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*d*e**2*f/(2*d*e
**4 + 2*e**5*x) - 4*b*d*e**2*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e
**3*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e**3*g*x**2/(2*d*e**4 + 2*
e**5*x) + 6*c*d**3*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g/(2*d*e
**4 + 2*e**5*x) - 4*c*d**2*e*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d**
2*e*f/(2*d*e**4 + 2*e**5*x) + 6*c*d**2*e*g*x*log(d/e + x)/(2*d*e**4 + 2*e**
5*x) - 4*c*d*e**2*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 3*c*d*e**2*g*x**
2/(2*d*e**4 + 2*e**5*x) + 2*c*e**3*f*x**2/(2*d*e**4 + 2*e**5*x) + c*e**3*g*
x**3/(2*d*e**4 + 2*e**5*x), Eq(m, -2)), (-a*d*g*log(d/e + x)/e**2 + a*f*log
(d/e + x)/e + a*g*x/e + b*d**2*g*log(d/e + x)/e**3 - b*d*f*log(d/e + x)/e**
2 - b*d*g*x/e**2 + b*f*x/e + b*g*x**2/(2*e) - c*d**3*g*log(d/e + x)/e**4 +
c*d**2*f*log(d/e + x)/e**3 + c*d**2*g*x/e**3 - c*d*f*x/e**2 - c*d*g*x**2/(2
*e**2) + c*f*x**2/(2*e) + c*g*x**3/(3*e), Eq(m, -1)), (-a*d**2*e**2*g*m**2*
(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4
) - 7*a*d**2*e**2*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
 + 50*e**4*m + 24*e**4) - 12*a*d**2*e**2*g*(d + e*x)**m/(e**4*m**4 + 10*e**
```

```
4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*f*m**3*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*d*e*
*3*f*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
 + 24*e**4) + 26*a*d*e**3*f*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*d*e**3*f*(d + e*x)**m/(e**4*m**4 + 1
0*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*g*m**3*x*(d +
e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7
*a*d*e**3*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
50*e**4*m + 24*e**4) + 12*a*d*e**3*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*
m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*f*m**3*x*(d + e*x)**m/(
e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*e**4*f
*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
 24*e**4) + 26*a*e**4*f*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**
4*m**2 + 50*e**4*m + 24*e**4) + 24*a*e**4*f*x*(d + e*x)**m/(e**4*m**4 + 10*
e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*g*m**3*x**2*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*
a*e**4*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
50*e**4*m + 24*e**4) + 19*a*e**4*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*a*e**4*g*x**2*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*b*d**3
*e*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
24*e**4) + 8*b*d**3*e*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m*
*2 + 50*e**4*m + 24*e**4) - b*d**2*e**2*f*m**2*(d + e*x)**m/(e**4*m**4 + 10
*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 7*b*d**2*e**2*f*m*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 12
*b*d**2*e**2*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
**4*m + 24*e**4) - 2*b*d**2*e**2*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*b*d**2*e**2*g*m*x*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e
**3*f*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**
4*m + 24*e**4) + 7*b*d*e**3*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*d*e**3*f*m*x*(d + e*x)**m/(e*
*4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e**3*g*m
**3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
+ 24*e**4) + 5*b*d*e**3*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
+ 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 4*b*d*e**3*g*m*x**2*(d + e*x)**m/(e
**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*e**4*f*m*
*3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
 24*e**4) + 8*b*e**4*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 3
5*e**4*m**2 + 50*e**4*m + 24*e**4) + 19*b*e**4*f*m*x**2*(d + e*x)**m/(e**4*
m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*e**4*f*x**
2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e*
*4) + b*e**4*g*m**3*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m
**2 + 50*e**4*m + 24*e**4) + 7*b*e**4*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 +
 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 14*b*e**4*g*m*x**3*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 8*b*e**4*g*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 5
0*e**4*m + 24*e**4) - 6*c*d**4*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 3
5*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*c*d**3*e*f*m*(d + e*x)**m/(e**4*m**4
 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*d**3*e*f*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*
c*d**3*e*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
**4*m + 24*e**4) - 2*c*d**2*e**2*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*c*d**2*e**2*f*m*x*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 3*c*d
**2*e**2*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
+ 50*e**4*m + 24*e**4) - 3*c*d**2*e**2*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 1
0*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*f*m**3*x**2*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
```

```
+ 5*c*d*e**3*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m
**2 + 50*e**4*m + 24*e**4) + 4*c*d*e**3*f*m*x**2*(d + e*x)**m/(e**4*m**4 +
10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*g*m**3*x**3*(
d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
 + 3*c*d*e**3*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*
m**2 + 50*e**4*m + 24*e**4) + 2*c*d*e**3*g*m*x**3*(d + e*x)**m/(e**4*m**4 +
 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**4*f*m**3*x**3*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 7*c*e**4*f*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**
2 + 50*e**4*m + 24*e**4) + 14*c*e**4*f*m*x**3*(d + e*x)**m/(e**4*m**4 + 10*
e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*e**4*f*x**3*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**
4*g*m**3*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e*
*4*m + 24*e**4) + 6*c*e**4*g*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m*
*3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 11*c*e**4*g*m*x**4*(d + e*x)**m/
(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*c*e**4*
g*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
24*e**4), True))
```

3.922 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{f+gx}\,dx$

Optimal. Leaf size=129

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right){}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}-\frac{(d+ex)^{m+1}(-beg+cdg+cef)}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

```
[Out] -(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1+m)/e^2/g^2/(1+m)+c*(e*x+d)^(2+m)/e^2/g/(2+
m)+(a*g^2-b*f*g+c*f^2)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-
d*g+e*f))/g^2/(-d*g+e*f)/(1+m)
```

Rubi [A] time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {951, 80, 68}

$$\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right){}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}-\frac{(d+ex)^{m+1}(-beg+cdg+cef)}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] -(((c*e*f + c*d*g - b*e*g)*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m))) + (c*(d +
e*x)^(2 + m))/(e^2*g*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f
 - d*g)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{f+gx}\,dx = \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\int \frac{(d+ex)^m(-e(cdf-aeg)(2+m)-e(cef+cdg-beg)(2+m)x)}{f+gx}\,dx}{e^2g(2+m)}$$

$$= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)\int \frac{(d+ex)^m}{f+gx}}{g^2}$$

$$= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)(d+ex)}{g^2(ef}$$

Mathematica [A] time = 0.14, size = 111, normalized size = 0.86

$$\frac{(d+ex)^{m+1}\left(\frac{\left(g(ag-bf)+cf^2\right){}_2F_1\left(1,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)}+\frac{beg-c(dg+ef)}{e^2(m+1)}+\frac{cg(d+ex)}{e^2(m+2)}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*e*g - c*(e*f + d*g))/(e^2*(1 + m)) + (c*g*(d + e*x))
/(e^2*(2 + m)) + ((c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2
+ m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m))))/g^2
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)(ex+d)^m}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^2+bx+a\right)(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)(ex + d)^m}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d + e\,x)^m\,\left(c\,x^2 + b\,x + a\right)}{f + g\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x)
```

```
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^m \left(a + bx + cx^2\right)}{f + gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f),x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x), x)
```

3.923 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=157

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)(cf(2dg-ef(m+2))-g(aegm+b(dg-ef(m+1))))}{g^2(m+1)(ef-dg)^2}+\frac{(d+ex)^{m+1}}{(f+gx}$$

```
[Out] c*(e*x+d)^(1+m)/e/g^2/(1+m)+(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(
g*x+f)+(c*f*(2*d*g-e*f*(2+m))-g*(a*e*g*m+b*(d*g-e*f*(1+m))))*(e*x+d)^(1+m)*
hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^2/(1+m)
```

Rubi [A] time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {949, 80, 68}

$$-\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)(g(aegm+bdg-bef(m+1))-cf(2dg-ef(m+2)))}{g^2(m+1)(ef-dg)^2}+\frac{(d+ex)^{m+}}{(f+g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] (c*(d + e*x)^(1 + m))/(e*g^2*(1 + m)) + ((a + (f*(c*f - b*g))/g^2)*(d + e*x
)^(1 + m))/((e*f - d*g)*(f + g*x)) - ((g*(b*d*g + a*e*g*m - b*e*f*(1 + m))
- c*f*(2*d*g - e*f*(2 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} + \frac{\int \frac{(d+ex)^m \left(\frac{cdfg-aeg^2m-cef^2(1+m)-bg(dg-ef(1+m))}{g^2}-c\left(d-\frac{ef}{g}\right)x\right)}{f+gx} dx}{ef-dg}$$
$$= \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} - \frac{(g(bdg+aegm-bef(1+m))-cf(}{g^2(ef-}$$
$$= \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} - \frac{(g(bdg+aegm-bef(1+m))-cf(}{}$$

Mathematica [A] time = 0.15, size = 134, normalized size = 0.85

$$\frac{(d+ex)^{m+1}\left(e^2\left(g(ag-bf)+cf^2\right) {}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)-e(2cf-bg)(ef-dg)\, {}_2F_1\left(1,m+1;m+2;\frac{g(d+e}{dg-e}\right.\right.}{eg^2(m+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*(e*f - d*g)^2 - e*(2*c*f - b*g)*(e*f - d*g)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e^2*(c*f^2 + g*(-
(b*f) + a*g))*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*
g)]))/(e*g^2*(e*f - d*g)^2*(1 + m))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^2+bx+a)(ex+d)^m}{g^2x^2+2\,fgx+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2+bx+a)(ex+d)^m}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2+bx+a)\,(ex+d)^m}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)(ex + d)^m}{\left(gx + f\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d + e\,x)^m\, \left(c\,x^2 + b\,x + a\right)}{\left(f + g\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.924 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(f+gx)^3}\,dx$

Optimal. Leaf size=245

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(c\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)-egm(aeg(1-m)-b\right.}{2g^2(m+1)(ef-dg)^3}$$

```
[Out] 1/2*(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(g*x+f)^2+1/2*(c*f*(4*d*g
-e*f*(3+m))+g*(a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)/g^2/(-d*g+e*
f)^2/(g*x+f)+1/2*(c*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-e*g*m*(
a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g
*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^3/(1+m)
```

Rubi [A] time = 0.32, antiderivative size = 243, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {949, 78, 68}

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(egm(-aeg(1-m)+2bdg-bef(m+1))+c\left(2d^2g^2-4defg(m+1)+\right.\right.}{2g^2(m+1)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] ((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2) -
 ((g*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) - c*f*(4*d*g - e*f*(3 + m)))
*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((e*g*m*(2*b*d*g - a*
e*g*(1 - m) - b*e*f*(1 + m)) + c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(
2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*
(d + e*x))/(e*f - d*g))])/(2*g^2*(e*f - d*g)^3*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)}{(f+gx)^3}\,dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m\left(\frac{cf(2dg-ef(1+m))-g(2bdg-aeg(1-m)-bef(1+m))}{g^2}-2c\left(d-\frac{ef}{g}\right.\right.}{(f+gx)^2}}{2(ef-dg)}$$

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf(4dg-}{2g^2(ef-dg)^2(f+gx)}$$

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf(4dg-}{2g^2(ef-dg)^2(f+gx)}$$

Mathematica [A] time = 0.16, size = 157, normalized size = 0.64

$$-\frac{(d+ex)^{m+1}\left(e\left(e\left(g(ag-bf)+cf^2\right){}_2F_1\left(3,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)-(2cf-bg)(ef-dg){}_2F_1\left(2,m+1;m+2;\right.\right.\right.}{g^2(m+1)(dg-ef)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] -(((d + e*x)^(1 + m)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + m, 2 + m, (g
*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)*Hypergeometric
2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e*(c*f^2 + g*(-(b*f)
+ a*g))*Hypergeometric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])))
/(g^2*(-(e*f) + d*g)^3*(1 + m)))
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)(ex+d)^m}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x +
 f^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)(ex + d)^m}{\left(gx + f\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x)^m\,\left(c\,x^2 + b\,x + a\right)}{\left(f + g\,x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^m \left(a + bx + cx^2\right)}{\left(f + gx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**3,x)
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x)**3, x)
```

3.925 $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=525

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-\right.\right.}{e^7(m+3)}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^7/(1+m)-2*(a*e^2-b*d*e+c
*d^2)*(-d*g+e*f)*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)
/e^7/(2+m)+(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+6*e^2*f^2)+e^2*(a^2*e^2*g^2+2*a*
b*e*g*(-3*d*g+2*e*f)+b^2*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))+2*c*e*(a*e*(6*d^2*g
^2-6*d*e*f*g+e^2*f^2)-b*d*(10*d^2*g^2-12*d*e*f*g+3*e^2*f^2)))*(e*x+d)^(3+m)
/e^7/(3+m)+2*(b*e^2*g*(a*e*g-2*b*d*g+b*e*f)-2*c^2*d*(5*d^2*g^2-5*d*e*f*g+e^
2*f^2)+c*e*(2*a*e*g*(-2*d*g+e*f)+b*(10*d^2*g^2-8*d*e*f*g+e^2*f^2)))*(e*x+d)
^(4+m)/e^7/(4+m)+(b^2*e^2*g^2+2*c*e*g*(a*e*g-5*b*d*g+2*b*e*f)+c^2*(15*d^2*g
^2-10*d*e*f*g+e^2*f^2))*(e*x+d)^(5+m)/e^7/(5+m)+2*c*g*(b*e*g-3*c*d*g+c*e*f)
*(e*x+d)^(6+m)/e^7/(6+m)+c^2*g^2*(e*x+d)^(7+m)/e^7/(7+m)
```

Rubi [A] time = 0.61, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {947}

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-\right.\right.}{e^7(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) -
 (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2 -
20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b
^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*g + 6
*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3 + m))/
(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 -
5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*
g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*
g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d
 + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)
^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^m}{e^6} + \frac{2\left(cd^2-bde+ae^2\right)(ef-\right.$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^{1+m}}{e^7(1+m)} - \frac{2\left(cd^2-bde+ae^2\right)(ef-d\right.$$

Mathematica [A] time = 0.77, size = 492, normalized size = 0.94

$$(d+ex)^{m+1}\left(\frac{(d+ex)^2\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)+bd\left(-10d^2g^2+12defg-3e^2f^2\right)\right)+c^2d^2(15\right.}{m+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)^2)/(1 + m) -
(2*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*(c*d*(-2*e*f + 3*d*g) + e*(b*e
*f - 2*b*d*g + a*e*g))*(d + e*x))/(2 + m) + ((c^2*d^2*(6*e^2*f^2 - 20*d*e*f
*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b^2*(e^2*
f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(b*d*(-3*e^2*f^2 + 12*d*e*f*g - 10*d^
2*g^2) + a*e*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)))*(d + e*x)^2)/(3 + m) + (2*
(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 - 5*d*e*f*g + 5*d^2*g
^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*g + 10*d^2*g^2)))*(
d + e*x)^3)/(4 + m) + ((b^2*e^2*g^2 + 2*c*e*g*(2*b*e*f - 5*b*d*g + a*e*g) +
 c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^4)/(5 + m) + (2*c*g*(c*
e*f - 3*c*d*g + b*e*g)*(d + e*x)^5)/(6 + m) + (c^2*g^2*(d + e*x)^6)/(7 + m)
))/e^7
```

fricas [B] time = 1.38, size = 4747, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^6*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g^
2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m + 7
20*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + 1680*b*c*e^7*g^2 + (2*c^2*e^7*f*g
 + (c^2*d*e^6 + 2*b*c*e^7)*g^2)*m^6 + (44*c^2*e^7*f*g + (15*c^2*d*e^6 + 44*
b*c*e^7)*g^2)*m^5 + 5*(76*c^2*e^7*f*g + (17*c^2*d*e^6 + 76*b*c*e^7)*g^2)*m^
4 + 5*(328*c^2*e^7*f*g + (45*c^2*d*e^6 + 328*b*c*e^7)*g^2)*m^3 + 2*(1849*c^
2*e^7*f*g + (137*c^2*d*e^6 + 1849*b*c*e^7)*g^2)*m^2 + 4*(1019*c^2*e^7*f*g +
 (30*c^2*d*e^6 + 1019*b*c*e^7)*g^2)*m)*x^6 - (2*a^2*d^2*e^5*f*g + (2*a*b*d^
2*e^5 - 27*a^2*d*e^6)*f^2)*m^5 + (1008*c^2*e^7*f^2 + 4032*b*c*e^7*f*g + 100
8*(b^2 + 2*a*c)*e^7*g^2 + (c^2*e^7*f^2 + 2*(c^2*d*e^6 + 2*b*c*e^7)*f*g + (2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*g^2)*m^6 + (23*c^2*e^7*f^2 + 2*(17*c^2*d*e^
6 + 46*b*c*e^7)*f*g - (6*c^2*d^2*e^5 - 34*b*c*d*e^6 - 23*(b^2 + 2*a*c)*e^7)
*g^2)*m^5 + 3*(69*c^2*e^7*f^2 + 2*(35*c^2*d*e^6 + 138*b*c*e^7)*f*g - (20*c^
2*d^2*e^5 - 70*b*c*d*e^6 - 69*(b^2 + 2*a*c)*e^7)*g^2)*m^4 + 5*(185*c^2*e^7*
f^2 + 2*(59*c^2*d*e^6 + 370*b*c*e^7)*f*g - (42*c^2*d^2*e^5 - 118*b*c*d*e^6
- 185*(b^2 + 2*a*c)*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + (187*c^2*d*e^6 + 2
144*b*c*e^7)*f*g - (75*c^2*d^2*e^5 - 187*b*c*d*e^6 - 536*(b^2 + 2*a*c)*e^7)
*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 4*(7*c^2*d*e^6 + 201*b*c*e^7)*f*g - (12*c
^2*d^2*e^5 - 28*b*c*d*e^6 - 201*(b^2 + 2*a*c)*e^7)*g^2)*m)*x^5 + (2*a^2*d^3
*e^4*g^2 - (50*a*b*d^2*e^5 - 295*a^2*d*e^6 - 2*(b^2 + 2*a*c)*d^3*e^4)*f^2 +
```

2*(4*a*b*d^3*e^4 - 25*a^2*d^2*e^5)*f*g)*m^4 + (2520*b*c*e^7*f^2 + 2520*a*b
*e^7*g^2 + 2520*(b^2 + 2*a*c)*e^7*f*g + ((c^2*d*e^6 + 2*b*c*e^7)*f^2 + 2*(2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f*g + (2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*g^2
)*m^6 + ((19*c^2*d*e^6 + 48*b*c*e^7)*f^2 - 2*(5*c^2*d^2*e^5 - 38*b*c*d*e^6
- 24*(b^2 + 2*a*c)*e^7)*f*g - (10*b*c*d^2*e^5 - 48*a*b*e^7 - 19*(b^2 + 2*a*
c)*d*e^6)*g^2)*m^5 + ((131*c^2*d*e^6 + 452*b*c*e^7)*f^2 - 2*(65*c^2*d^2*e^5
- 262*b*c*d*e^6 - 226*(b^2 + 2*a*c)*e^7)*f*g + (30*c^2*d^3*e^4 - 130*b*c*d
^2*e^5 + 452*a*b*e^7 + 131*(b^2 + 2*a*c)*d*e^6)*g^2)*m^4 + ((401*c^2*d*e^6
+ 2112*b*c*e^7)*f^2 - 2*(265*c^2*d^2*e^5 - 802*b*c*d*e^6 - 1056*(b^2 + 2*a*
c)*e^7)*f*g + (180*c^2*d^3*e^4 - 530*b*c*d^2*e^5 + 2112*a*b*e^7 + 401*(b^2
+ 2*a*c)*d*e^6)*g^2)*m^3 + 10*((54*c^2*d*e^6 + 509*b*c*e^7)*f^2 - (83*c^2*d
^2*e^5 - 216*b*c*d*e^6 - 509*(b^2 + 2*a*c)*e^7)*f*g + (33*c^2*d^3*e^4 - 83*
b*c*d^2*e^5 + 509*a*b*e^7 + 54*(b^2 + 2*a*c)*d*e^6)*g^2)*m^2 + 12*(3*(7*c^2
*d*e^6 + 164*b*c*e^7)*f^2 - (35*c^2*d^2*e^5 - 84*b*c*d*e^6 - 492*(b^2 + 2*a
*c)*e^7)*f*g + (15*c^2*d^3*e^4 - 35*b*c*d^2*e^5 + 492*a*b*e^7 + 21*(b^2 + 2
*a*c)*d*e^6)*g^2)*m)*x^4 - ((12*b*c*d^4*e^3 + 490*a*b*d^2*e^5 - 1665*a^2*d*
e^6 - 44*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 2*(88*a*b*d^3*e^4 - 245*a^2*d^2*e^5 -
6*(b^2 + 2*a*c)*d^4*e^3)*f*g + 4*(3*a*b*d^4*e^3 - 11*a^2*d^3*e^4)*g^2)*m^3
+ (6720*a*b*e^7*f*g + 1680*a^2*e^7*g^2 + 1680*(b^2 + 2*a*c)*e^7*f^2 + ((2*
b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f^2 + 2*(2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*f*
g + (2*a*b*d*e^6 + a^2*e^7)*g^2)*m^6 - ((4*c^2*d^2*e^5 - 42*b*c*d*e^6 - 25*
(b^2 + 2*a*c)*e^7)*f^2 + 2*(8*b*c*d^2*e^5 - 50*a*b*e^7 - 21*(b^2 + 2*a*c)*d
*e^6)*f*g - (42*a*b*d*e^6 + 25*a^2*e^7 - 4*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^5
- ((64*c^2*d^2*e^5 - 326*b*c*d*e^6 - 247*(b^2 + 2*a*c)*e^7)*f^2 - 2*(20*c^2
*d^3*e^4 - 128*b*c*d^2*e^5 + 494*a*b*e^7 + 163*(b^2 + 2*a*c)*d*e^6)*f*g - (
40*b*c*d^3*e^4 + 326*a*b*d*e^6 + 247*a^2*e^7 - 64*(b^2 + 2*a*c)*d^2*e^5)*g^
2)*m^4 - ((332*c^2*d^2*e^5 - 1134*b*c*d*e^6 - 1219*(b^2 + 2*a*c)*e^7)*f^2 -
2*(200*c^2*d^3*e^4 - 664*b*c*d^2*e^5 + 2438*a*b*e^7 + 567*(b^2 + 2*a*c)*d*
e^6)*f*g + (120*c^2*d^4*e^3 - 400*b*c*d^3*e^4 - 1134*a*b*d*e^6 - 1219*a^2*e
^7 + 332*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^3 - 8*((76*c^2*d^2*e^5 - 211*b*c*d*e
^6 - 389*(b^2 + 2*a*c)*e^7)*f^2 - (115*c^2*d^3*e^4 - 304*b*c*d^2*e^5 + 1556
*a*b*e^7 + 211*(b^2 + 2*a*c)*d*e^6)*f*g + (45*c^2*d^4*e^3 - 115*b*c*d^3*e^4
- 211*a*b*d*e^6 - 389*a^2*e^7 + 76*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^2 - 4*((8
4*c^2*d^2*e^5 - 210*b*c*d*e^6 - 949*(b^2 + 2*a*c)*e^7)*f^2 - 2*(70*c^2*d^3*
e^4 - 168*b*c*d^2*e^5 + 1898*a*b*e^7 + 105*(b^2 + 2*a*c)*d*e^6)*f*g + (60*c
^2*d^4*e^3 - 140*b*c*d^3*e^4 - 210*a*b*d*e^6 - 949*a^2*e^7 + 84*(b^2 + 2*a*
c)*d^2*e^5)*g^2)*m)*x^3 + 168*(6*c^2*d^5*e^2 - 15*b*c*d^4*e^3 - 30*a*b*d^2*
e^5 + 30*a^2*d*e^6 + 10*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 168*(10*c^2*d^6*e - 24
*b*c*d^5*e^2 - 40*a*b*d^3*e^4 + 30*a^2*d^2*e^5 + 15*(b^2 + 2*a*c)*d^4*e^3)*
f*g + 24*(30*c^2*d^7 - 70*b*c*d^6*e - 105*a*b*d^4*e^3 + 70*a^2*d^3*e^4 + 42
*(b^2 + 2*a*c)*d^5*e^2)*g^2 + 2*((12*c^2*d^5*e^2 - 108*b*c*d^4*e^3 - 1175*a
*b*d^2*e^5 + 2552*a^2*d*e^6 + 179*(b^2 + 2*a*c)*d^3*e^4)*f^2 + (48*b*c*d^5*
e^2 + 716*a*b*d^3*e^4 - 1175*a^2*d^2*e^5 - 108*(b^2 + 2*a*c)*d^4*e^3)*f*g -
(108*a*b*d^4*e^3 - 179*a^2*d^3*e^4 - 12*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m^2 +
(5040*a*b*e^7*f^2 + 5040*a^2*e^7*f*g + (a^2*d*e^6*g^2 + (2*a*b*e^7 + (b^2 +
2*a*c)*d*e^6)*f^2 + 2*(2*a*b*d*e^6 + a^2*e^7)*f*g)*m^6 - ((6*b*c*d^2*e^5 -
52*a*b*e^7 - 23*(b^2 + 2*a*c)*d*e^6)*f^2 - 2*(46*a*b*d*e^6 + 26*a^2*e^7 -
3*(b^2 + 2*a*c)*d^2*e^5)*f*g + (6*a*b*d^2*e^5 - 23*a^2*d*e^6)*g^2)*m^5 + 3*
((4*c^2*d^3*e^4 - 38*b*c*d^2*e^5 + 180*a*b*e^7 + 67*(b^2 + 2*a*c)*d*e^6)*f^
2 + 2*(8*b*c*d^3*e^4 + 134*a*b*d*e^6 + 90*a^2*e^7 - 19*(b^2 + 2*a*c)*d^2*e^
5)*f*g - (38*a*b*d^2*e^5 - 67*a^2*d*e^6 - 4*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^4
+ ((168*c^2*d^3*e^4 - 750*b*c*d^2*e^5 + 2840*a*b*e^7 + 817*(b^2 + 2*a*c)*d
*e^6)*f^2 - 2*(60*c^2*d^4*e^3 - 336*b*c*d^3*e^4 - 1634*a*b*d*e^6 - 1420*a^2
e^7 + 375(b^2 + 2*a*c)*d^2*e^5)*f*g - (120*b*c*d^4*e^3 + 750*a*b*d^2*e^5
- 817*a^2*d*e^6 - 168*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^3 + 2*((330*c^2*d^3*e^4
- 951*b*c*d^2*e^5 + 3929*a*b*e^7 + 739*(b^2 + 2*a*c)*d*e^6)*f^2 - (480*c^2
*d^4*e^3 - 1320*b*c*d^3*e^4 - 2956*a*b*d*e^6 - 3929*a^2*e^7 + 951*(b^2 + 2*
a*c)*d^2*e^5)*f*g + (180*c^2*d^5*e^2 - 480*b*c*d^4*e^3 - 951*a*b*d^2*e^5 +
739*a^2*d*e^6 + 330*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^2 + 12*((42*c^2*d^3*e^4 -

```
 105*b*c*d^2*e^5 + 879*a*b*e^7 + 70*(b^2 + 2*a*c)*d*e^6)*f^2 - (70*c^2*d^4*
e^3 - 168*b*c*d^3*e^4 - 280*a*b*d*e^6 - 879*a^2*e^7 + 105*(b^2 + 2*a*c)*d^2
*e^5)*f*g + (30*c^2*d^5*e^2 - 70*b*c*d^4*e^3 - 105*a*b*d^2*e^5 + 70*a^2*d*e
^6 + 42*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m)*x^2 + 4*((78*c^2*d^5*e^2 - 321*b*c*d
^4*e^3 - 1377*a*b*d^2*e^5 + 2007*a^2*d*e^6 + 319*(b^2 + 2*a*c)*d^3*e^4)*f^2
 - (60*c^2*d^6*e - 312*b*c*d^5*e^2 - 1276*a*b*d^3*e^4 + 1377*a^2*d^2*e^5 +
321*(b^2 + 2*a*c)*d^4*e^3)*f*g - (60*b*c*d^6*e + 321*a*b*d^4*e^3 - 319*a^2*
d^3*e^4 - 78*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m + (5040*a^2*e^7*f^2 + (2*a^2*d*e
^6*f*g + (2*a*b*d*e^6 + a^2*e^7)*f^2)*m^6 - (2*a^2*d^2*e^5*g^2 - (50*a*b*d*
e^6 + 27*a^2*e^7 - 2*(b^2 + 2*a*c)*d^2*e^5)*f^2 + 2*(4*a*b*d^2*e^5 - 25*a^2
*d*e^6)*f*g)*m^5 + ((12*b*c*d^3*e^4 + 490*a*b*d*e^6 + 295*a^2*e^7 - 44*(b^2
 + 2*a*c)*d^2*e^5)*f^2 - 2*(88*a*b*d^2*e^5 - 245*a^2*d*e^6 - 6*(b^2 + 2*a*c
)*d^3*e^4)*f*g + 4*(3*a*b*d^3*e^4 - 11*a^2*d^2*e^5)*g^2)*m^4 - ((24*c^2*d^4
*e^3 - 216*b*c*d^3*e^4 - 2350*a*b*d*e^6 - 1665*a^2*e^7 + 358*(b^2 + 2*a*c)*
d^2*e^5)*f^2 + 2*(48*b*c*d^4*e^3 + 716*a*b*d^2*e^5 - 1175*a^2*d*e^6 - 108*(
b^2 + 2*a*c)*d^3*e^4)*f*g - 2*(108*a*b*d^3*e^4 - 179*a^2*d^2*e^5 - 12*(b^2
+ 2*a*c)*d^4*e^3)*g^2)*m^3 - 4*((78*c^2*d^4*e^3 - 321*b*c*d^3*e^4 - 1377*a*
b*d*e^6 - 1276*a^2*e^7 + 319*(b^2 + 2*a*c)*d^2*e^5)*f^2 - (60*c^2*d^5*e^2 -
 312*b*c*d^4*e^3 - 1276*a*b*d^2*e^5 + 1377*a^2*d*e^6 + 321*(b^2 + 2*a*c)*d^
3*e^4)*f*g - (60*b*c*d^5*e^2 + 321*a*b*d^3*e^4 - 319*a^2*d^2*e^5 - 78*(b^2
+ 2*a*c)*d^4*e^3)*g^2)*m^2 - 12*((84*c^2*d^4*e^3 - 210*b*c*d^3*e^4 - 420*a*
b*d*e^6 - 669*a^2*e^7 + 140*(b^2 + 2*a*c)*d^2*e^5)*f^2 - 14*(10*c^2*d^5*e^2
 - 24*b*c*d^4*e^3 - 40*a*b*d^2*e^5 + 30*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^
4)*f*g + 2*(30*c^2*d^6*e - 70*b*c*d^5*e^2 - 105*a*b*d^3*e^4 + 70*a^2*d^2*e^
5 + 42*(b^2 + 2*a*c)*d^4*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^7*m^7 + 28*e^7*m^6
+ 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 13132*e^7*m^2 + 13068*e^7*m +
 5040*e^7)
```

giac [B] time = 0.55, size = 10489, normalized size = 19.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g^2*m^6*x^7*e^7 + (x*e + d)^m*c^2*d*g^2*m^6*x^6*e^6 + 2*(x
*e + d)^m*c^2*f*g*m^6*x^6*e^7 + 2*(x*e + d)^m*b*c*g^2*m^6*x^6*e^7 + 21*(x*e
 + d)^m*c^2*g^2*m^5*x^7*e^7 + 2*(x*e + d)^m*c^2*d*f*g*m^6*x^5*e^6 + 2*(x*e
+ d)^m*b*c*d*g^2*m^6*x^5*e^6 + 15*(x*e + d)^m*c^2*d*g^2*m^5*x^6*e^6 - 6*(x*
e + d)^m*c^2*d^2*g^2*m^5*x^5*e^5 + (x*e + d)^m*c^2*f^2*m^6*x^5*e^7 + 4*(x*e
 + d)^m*b*c*f*g*m^6*x^5*e^7 + (x*e + d)^m*b^2*g^2*m^6*x^5*e^7 + 2*(x*e + d)
^m*a*c*g^2*m^6*x^5*e^7 + 44*(x*e + d)^m*c^2*f*g*m^5*x^6*e^7 + 44*(x*e + d)^
m*b*c*g^2*m^5*x^6*e^7 + 175*(x*e + d)^m*c^2*g^2*m^4*x^7*e^7 + (x*e + d)^m*c
^2*d*f^2*m^6*x^4*e^6 + 4*(x*e + d)^m*b*c*d*f*g*m^6*x^4*e^6 + (x*e + d)^m*b^
2*d*g^2*m^6*x^4*e^6 + 2*(x*e + d)^m*a*c*d*g^2*m^6*x^4*e^6 + 34*(x*e + d)^m*
c^2*d*f*g*m^5*x^5*e^6 + 34*(x*e + d)^m*b*c*d*g^2*m^5*x^5*e^6 + 85*(x*e + d)
^m*c^2*d*g^2*m^4*x^6*e^6 - 10*(x*e + d)^m*c^2*d^2*f*g*m^5*x^4*e^5 - 10*(x*e
 + d)^m*b*c*d^2*g^2*m^5*x^4*e^5 - 60*(x*e + d)^m*c^2*d^2*g^2*m^4*x^5*e^5 +
30*(x*e + d)^m*c^2*d^3*g^2*m^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f^2*m^6*x^4*e^7
+ 2*(x*e + d)^m*b^2*f*g*m^6*x^4*e^7 + 4*(x*e + d)^m*a*c*f*g*m^6*x^4*e^7 + 2
*(x*e + d)^m*a*b*g^2*m^6*x^4*e^7 + 23*(x*e + d)^m*c^2*f^2*m^5*x^5*e^7 + 92*
(x*e + d)^m*b*c*f*g*m^5*x^5*e^7 + 23*(x*e + d)^m*b^2*g^2*m^5*x^5*e^7 + 46*(
x*e + d)^m*a*c*g^2*m^5*x^5*e^7 + 380*(x*e + d)^m*c^2*f*g*m^4*x^6*e^7 + 380*
(x*e + d)^m*b*c*g^2*m^4*x^6*e^7 + 735*(x*e + d)^m*c^2*g^2*m^3*x^7*e^7 + 2*(
x*e + d)^m*b*c*d*f^2*m^6*x^3*e^6 + 2*(x*e + d)^m*b^2*d*f*g*m^6*x^3*e^6 + 4*
(x*e + d)^m*a*c*d*f*g*m^6*x^3*e^6 + 2*(x*e + d)^m*a*b*d*g^2*m^6*x^3*e^6 + 1
9*(x*e + d)^m*c^2*d*f^2*m^5*x^4*e^6 + 76*(x*e + d)^m*b*c*d*f*g*m^5*x^4*e^6
+ 19*(x*e + d)^m*b^2*d*g^2*m^5*x^4*e^6 + 38*(x*e + d)^m*a*c*d*g^2*m^5*x^4*e
^6 + 210*(x*e + d)^m*c^2*d*f*g*m^4*x^5*e^6 + 210*(x*e + d)^m*b*c*d*g^2*m^4*
x^5*e^6 + 225*(x*e + d)^m*c^2*d*g^2*m^3*x^6*e^6 - 4*(x*e + d)^m*c^2*d^2*f^2
```

*m^5*x^3*e^5 - 16*(x*e + d)^m*b*c*d^2*f*g*m^5*x^3*e^5 - 4*(x*e + d)^m*b^2*d^2*g^2*m^5*x^3*e^5 - 8*(x*e + d)^m*a*c*d^2*g^2*m^5*x^3*e^5 - 130*(x*e + d)^m*c^2*d^2*f*g*m^4*x^4*e^5 - 130*(x*e + d)^m*b*c*d^2*g^2*m^4*x^4*e^5 - 210*(x*e + d)^m*c^2*d^2*g^2*m^3*x^5*e^5 + 40*(x*e + d)^m*c^2*d^3*f*g*m^4*x^3*e^4 + 40*(x*e + d)^m*b*c*d^3*g^2*m^4*x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m^3*x^4*e^4 - 120*(x*e + d)^m*c^2*d^4*g^2*m^3*x^3*e^3 + (x*e + d)^m*b^2*f^2*m^6*x^3*e^7 + 2*(x*e + d)^m*a*c*f^2*m^6*x^3*e^7 + 4*(x*e + d)^m*a*b*f*g*m^6*x^3*e^7 + (x*e + d)^m*a^2*g^2*m^6*x^3*e^7 + 48*(x*e + d)^m*b*c*f^2*m^5*x^4*e^7 + 48*(x*e + d)^m*b^2*f*g*m^5*x^4*e^7 + 96*(x*e + d)^m*a*c*f*g*m^5*x^4*e^7 + 48*(x*e + d)^m*a*b*g^2*m^5*x^4*e^7 + 207*(x*e + d)^m*c^2*f^2*m^4*x^5*e^7 + 828*(x*e + d)^m*b*c*f*g*m^4*x^5*e^7 + 207*(x*e + d)^m*b^2*g^2*m^4*x^5*e^7 + 414*(x*e + d)^m*a*c*g^2*m^4*x^5*e^7 + 1640*(x*e + d)^m*c^2*f*g*m^3*x^6*e^7 + 1640*(x*e + d)^m*b*c*g^2*m^3*x^6*e^7 + 1624*(x*e + d)^m*c^2*g^2*m^2*x^7*e^7 + (x*e + d)^m*b^2*d*f^2*m^6*x^2*e^6 + 2*(x*e + d)^m*a*c*d*f^2*m^6*x^2*e^6 + 4*(x*e + d)^m*a*b*d*f*g*m^6*x^2*e^6 + (x*e + d)^m*a^2*d*g^2*m^6*x^2*e^6 + 42*(x*e + d)^m*b*c*d*f^2*m^5*x^3*e^6 + 42*(x*e + d)^m*b^2*d*f*g*m^5*x^3*e^6 + 84*(x*e + d)^m*a*c*d*f*g*m^5*x^3*e^6 + 42*(x*e + d)^m*a*b*d*g^2*m^5*x^3*e^6 + 131*(x*e + d)^m*c^2*d*f^2*m^4*x^4*e^6 + 524*(x*e + d)^m*b*c*d*f*g*m^4*x^4*e^6 + 131*(x*e + d)^m*b^2*d*g^2*m^4*x^4*e^6 + 262*(x*e + d)^m*a*c*d*g^2*m^4*x^4*e^6 + 590*(x*e + d)^m*c^2*d*f*g*m^3*x^5*e^6 + 590*(x*e + d)^m*b*c*d*g^2*m^3*x^5*e^6 + 274*(x*e + d)^m*c^2*d*g^2*m^2*x^6*e^6 - 6*(x*e + d)^m*b*c*d^2*f^2*m^5*x^2*e^5 - 6*(x*e + d)^m*b^2*d^2*f*g*m^5*x^2*e^5 - 12*(x*e + d)^m*a*c*d^2*f*g*m^5*x^2*e^5 - 6*(x*e + d)^m*a*b*d^2*g^2*m^5*x^2*e^5 - 64*(x*e + d)^m*c^2*d^2*f^2*m^4*x^3*e^5 - 256*(x*e + d)^m*b*c*d^2*f*g*m^4*x^3*e^5 - 64*(x*e + d)^m*b^2*d^2*g^2*m^4*x^3*e^5 - 128*(x*e + d)^m*a*c*d^2*g^2*m^4*x^3*e^5 - 530*(x*e + d)^m*c^2*d^2*f*g*m^3*x^4*e^5 - 530*(x*e + d)^m*b*c*d^2*g^2*m^3*x^4*e^5 - 300*(x*e + d)^m*c^2*d^2*g^2*m^2*x^5*e^5 + 12*(x*e + d)^m*c^2*d^3*f^2*m^4*x^2*e^4 + 48*(x*e + d)^m*b*c*d^3*f*g*m^4*x^2*e^4 + 12*(x*e + d)^m*b^2*d^3*g^2*m^4*x^2*e^4 + 24*(x*e + d)^m*a*c*d^3*g^2*m^4*x^2*e^4 + 400*(x*e + d)^m*c^2*d^3*f*g*m^3*x^3*e^4 + 400*(x*e + d)^m*b*c*d^3*g^2*m^3*x^3*e^4 + 330*(x*e + d)^m*c^2*d^3*g^2*m^2*x^4*e^4 - 120*(x*e + d)^m*c^2*d^4*f*g*m^3*x^2*e^3 - 120*(x*e + d)^m*b*c*d^4*g^2*m^3*x^2*e^3 - 360*(x*e + d)^m*c^2*d^4*g^2*m^2*x^3*e^3 + 360*(x*e + d)^m*c^2*d^5*g^2*m^2*x^2*e^2 + 2*(x*e + d)^m*a*b*f^2*m^6*x^2*e^7 + 2*(x*e + d)^m*a^2*f*g*m^6*x^2*e^7 + 25*(x*e + d)^m*b^2*f^2*m^5*x^3*e^7 + 50*(x*e + d)^m*a*c*f^2*m^5*x^3*e^7 + 100*(x*e + d)^m*a*b*f*g*m^5*x^3*e^7 + 25*(x*e + d)^m*a^2*g^2*m^5*x^3*e^7 + 452*(x*e + d)^m*b*c*f^2*m^4*x^4*e^7 + 452*(x*e + d)^m*b^2*f*g*m^4*x^4*e^7 + 904*(x*e + d)^m*a*c*f*g*m^4*x^4*e^7 + 452*(x*e + d)^m*a*b*g^2*m^4*x^4*e^7 + 925*(x*e + d)^m*c^2*f^2*m^3*x^5*e^7 + 3700*(x*e + d)^m*b*c*f*g*m^3*x^5*e^7 + 925*(x*e + d)^m*b^2*g^2*m^3*x^5*e^7 + 1850*(x*e + d)^m*a*c*g^2*m^3*x^5*e^7 + 3698*(x*e + d)^m*c^2*f*g*m^2*x^6*e^7 + 3698*(x*e + d)^m*b*c*g^2*m^2*x^6*e^7 + 1764*(x*e + d)^m*c^2*g^2*m*x^7*e^7 + 2*(x*e + d)^m*a*b*d*f^2*m^6*x*e^6 + 2*(x*e + d)^m*a^2*d*f*g*m^6*x*e^6 + 23*(x*e + d)^m*b^2*d*f^2*m^5*x^2*e^6 + 46*(x*e + d)^m*a*c*d*f^2*m^5*x^2*e^6 + 92*(x*e + d)^m*a*b*d*f*g*m^5*x^2*e^6 + 23*(x*e + d)^m*a^2*d*g^2*m^5*x^2*e^6 + 326*(x*e + d)^m*b*c*d*f^2*m^4*x^3*e^6 + 326*(x*e + d)^m*b^2*d*f*g*m^4*x^3*e^6 + 652*(x*e + d)^m*a*c*d*f*g*m^4*x^3*e^6 + 326*(x*e + d)^m*a*b*d*g^2*m^4*x^3*e^6 + 401*(x*e + d)^m*c^2*d*f^2*m^3*x^4*e^6 + 1604*(x*e + d)^m*b*c*d*f*g*m^3*x^4*e^6 + 401*(x*e + d)^m*b^2*d*g^2*m^3*x^4*e^6 + 802*(x*e + d)^m*a*c*d*g^2*m^3*x^4*e^6 + 748*(x*e + d)^m*c^2*d*f*g*m^2*x^5*e^6 + 748*(x*e + d)^m*b*c*d*g^2*m^2*x^5*e^6 + 120*(x*e + d)^m*c^2*d*g^2*m*x^6*e^6 - 2*(x*e + d)^m*b^2*d^2*f^2*m^5*x*e^5 - 4*(x*e + d)^m*a*c*d^2*f^2*m^5*x*e^5 - 8*(x*e + d)^m*a*b*d^2*f*g*m^5*x*e^5 - 2*(x*e + d)^m*a^2*d^2*g^2*m^5*x*e^5 - 114*(x*e + d)^m*b*c*d^2*f^2*m^4*x^2*e^5 - 114*(x*e + d)^m*b^2*d^2*f*g*m^4*x^2*e^5 - 228*(x*e + d)^m*a*c*d^2*f*g*m^4*x^2*e^5 - 114*(x*e + d)^m*a*b*d^2*g^2*m^4*x^2*e^5 - 332*(x*e + d)^m*c^2*d^2*f^2*m^3*x^3*e^5 - 1328*(x*e + d)^m*b*c*d^2*f*g*m^3*x^3*e^5 - 332*(x*e + d)^m*b^2*d^2*g^2*m^3*x^3*e^5 - 664*(x*e + d)^m*a*c*d^2*g^2*m^3*x^3*e^5 - 830*(x*e + d)^m*c^2*d^2*f*g*m^2*x^4*e^5 - 830*(x*e + d)^m*b*c*d^2*g^2*m^2*x^4*e^5 - 144*(x*e + d)^m*c^2*d^2*g^2*m*x^5*e^5 + 12*(x*e + d)^m*b*c*d^3*

f^2*m^4*x*e^4 + 12*(x*e + d)^m*b^2*d^3*f*g*m^4*x*e^4 + 24*(x*e + d)^m*a*c*d
^3*f*g*m^4*x*e^4 + 12*(x*e + d)^m*a*b*d^3*g^2*m^4*x*e^4 + 168*(x*e + d)^m*c
^2*d^3*f^2*m^3*x^2*e^4 + 672*(x*e + d)^m*b*c*d^3*f*g*m^3*x^2*e^4 + 168*(x*e
 + d)^m*b^2*d^3*g^2*m^3*x^2*e^4 + 336*(x*e + d)^m*a*c*d^3*g^2*m^3*x^2*e^4 +
 920*(x*e + d)^m*c^2*d^3*f*g*m^2*x^3*e^4 + 920*(x*e + d)^m*b*c*d^3*g^2*m^2*
x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m*x^4*e^4 - 24*(x*e + d)^m*c^2*d^4*f^
2*m^3*x*e^3 - 96*(x*e + d)^m*b*c*d^4*f*g*m^3*x*e^3 - 24*(x*e + d)^m*b^2*d^4
*g^2*m^3*x*e^3 - 48*(x*e + d)^m*a*c*d^4*g^2*m^3*x*e^3 - 960*(x*e + d)^m*c^2
*d^4*f*g*m^2*x^2*e^3 - 960*(x*e + d)^m*b*c*d^4*g^2*m^2*x^2*e^3 - 240*(x*e +
 d)^m*c^2*d^4*g^2*m*x^3*e^3 + 240*(x*e + d)^m*c^2*d^5*f*g*m^2*x*e^2 + 240*(
x*e + d)^m*b*c*d^5*g^2*m^2*x*e^2 + 360*(x*e + d)^m*c^2*d^5*g^2*m*x^2*e^2 -
720*(x*e + d)^m*c^2*d^6*g^2*m*x*e + (x*e + d)^m*a^2*f^2*m^6*x*e^7 + 52*(x*e
 + d)^m*a*b*f^2*m^5*x^2*e^7 + 52*(x*e + d)^m*a^2*f*g*m^5*x^2*e^7 + 247*(x*e
 + d)^m*b^2*f^2*m^4*x^3*e^7 + 494*(x*e + d)^m*a*c*f^2*m^4*x^3*e^7 + 988*(x*
e + d)^m*a*b*f*g*m^4*x^3*e^7 + 247*(x*e + d)^m*a^2*g^2*m^4*x^3*e^7 + 2112*(
x*e + d)^m*b*c*f^2*m^3*x^4*e^7 + 2112*(x*e + d)^m*b^2*f*g*m^3*x^4*e^7 + 422
4*(x*e + d)^m*a*c*f*g*m^3*x^4*e^7 + 2112*(x*e + d)^m*a*b*g^2*m^3*x^4*e^7 +
2144*(x*e + d)^m*c^2*f^2*m^2*x^5*e^7 + 8576*(x*e + d)^m*b*c*f*g*m^2*x^5*e^7
 + 2144*(x*e + d)^m*b^2*g^2*m^2*x^5*e^7 + 4288*(x*e + d)^m*a*c*g^2*m^2*x^5*
e^7 + 4076*(x*e + d)^m*c^2*f*g*m*x^6*e^7 + 4076*(x*e + d)^m*b*c*g^2*m*x^6*e
^7 + 720*(x*e + d)^m*c^2*g^2*x^7*e^7 + (x*e + d)^m*a^2*d*f^2*m^6*e^6 + 50*(
x*e + d)^m*a*b*d*f^2*m^5*x*e^6 + 50*(x*e + d)^m*a^2*d*f*g*m^5*x*e^6 + 201*(
x*e + d)^m*b^2*d*f^2*m^4*x^2*e^6 + 402*(x*e + d)^m*a*c*d*f^2*m^4*x^2*e^6 +
804*(x*e + d)^m*a*b*d*f*g*m^4*x^2*e^6 + 201*(x*e + d)^m*a^2*d*g^2*m^4*x^2*e
^6 + 1134*(x*e + d)^m*b*c*d*f^2*m^3*x^3*e^6 + 1134*(x*e + d)^m*b^2*d*f*g*m^
3*x^3*e^6 + 2268*(x*e + d)^m*a*c*d*f*g*m^3*x^3*e^6 + 1134*(x*e + d)^m*a*b*d
*g^2*m^3*x^3*e^6 + 540*(x*e + d)^m*c^2*d*f^2*m^2*x^4*e^6 + 2160*(x*e + d)^m
*b*c*d*f*g*m^2*x^4*e^6 + 540*(x*e + d)^m*b^2*d*g^2*m^2*x^4*e^6 + 1080*(x*e
+ d)^m*a*c*d*g^2*m^2*x^4*e^6 + 336*(x*e + d)^m*c^2*d*f*g*m*x^5*e^6 + 336*(x
*e + d)^m*b*c*d*g^2*m*x^5*e^6 - 2*(x*e + d)^m*a*b*d^2*f^2*m^5*e^5 - 2*(x*e
+ d)^m*a^2*d^2*f*g*m^5*e^5 - 44*(x*e + d)^m*b^2*d^2*f^2*m^4*x*e^5 - 88*(x*e
 + d)^m*a*c*d^2*f^2*m^4*x*e^5 - 176*(x*e + d)^m*a*b*d^2*f*g*m^4*x*e^5 - 44*
(x*e + d)^m*a^2*d^2*g^2*m^4*x*e^5 - 750*(x*e + d)^m*b*c*d^2*f^2*m^3*x^2*e^5
 - 750*(x*e + d)^m*b^2*d^2*f*g*m^3*x^2*e^5 - 1500*(x*e + d)^m*a*c*d^2*f*g*m
^3*x^2*e^5 - 750*(x*e + d)^m*a*b*d^2*g^2*m^3*x^2*e^5 - 608*(x*e + d)^m*c^2*
d^2*f^2*m^2*x^3*e^5 - 2432*(x*e + d)^m*b*c*d^2*f*g*m^2*x^3*e^5 - 608*(x*e +
 d)^m*b^2*d^2*g^2*m^2*x^3*e^5 - 1216*(x*e + d)^m*a*c*d^2*g^2*m^2*x^3*e^5 -
420*(x*e + d)^m*c^2*d^2*f*g*m*x^4*e^5 - 420*(x*e + d)^m*b*c*d^2*g^2*m*x^4*e
^5 + 2*(x*e + d)^m*b^2*d^3*f^2*m^4*e^4 + 4*(x*e + d)^m*a*c*d^3*f^2*m^4*e^4
+ 8*(x*e + d)^m*a*b*d^3*f*g*m^4*e^4 + 2*(x*e + d)^m*a^2*d^3*g^2*m^4*e^4 + 2
16*(x*e + d)^m*b*c*d^3*f^2*m^3*x*e^4 + 216*(x*e + d)^m*b^2*d^3*f*g*m^3*x*e^
4 + 432*(x*e + d)^m*a*c*d^3*f*g*m^3*x*e^4 + 216*(x*e + d)^m*a*b*d^3*g^2*m^3
*x*e^4 + 660*(x*e + d)^m*c^2*d^3*f^2*m^2*x^2*e^4 + 2640*(x*e + d)^m*b*c*d^3
*f*g*m^2*x^2*e^4 + 660*(x*e + d)^m*b^2*d^3*g^2*m^2*x^2*e^4 + 1320*(x*e + d)
^m*a*c*d^3*g^2*m^2*x^2*e^4 + 560*(x*e + d)^m*c^2*d^3*f*g*m*x^3*e^4 + 560*(x
*e + d)^m*b*c*d^3*g^2*m*x^3*e^4 - 12*(x*e + d)^m*b*c*d^4*f^2*m^3*e^3 - 12*(
x*e + d)^m*b^2*d^4*f*g*m^3*e^3 - 24*(x*e + d)^m*a*c*d^4*f*g*m^3*e^3 - 12*(x
*e + d)^m*a*b*d^4*g^2*m^3*e^3 - 312*(x*e + d)^m*c^2*d^4*f^2*m^2*x*e^3 - 124
8*(x*e + d)^m*b*c*d^4*f*g*m^2*x*e^3 - 312*(x*e + d)^m*b^2*d^4*g^2*m^2*x*e^3
 - 624*(x*e + d)^m*a*c*d^4*g^2*m^2*x*e^3 - 840*(x*e + d)^m*c^2*d^4*f*g*m*x^
2*e^3 - 840*(x*e + d)^m*b*c*d^4*g^2*m*x^2*e^3 + 24*(x*e + d)^m*c^2*d^5*f^2*
m^2*e^2 + 96*(x*e + d)^m*b*c*d^5*f*g*m^2*e^2 + 24*(x*e + d)^m*b^2*d^5*g^2*m
^2*e^2 + 48*(x*e + d)^m*a*c*d^5*g^2*m^2*e^2 + 1680*(x*e + d)^m*c^2*d^5*f*g*
m*x*e^2 + 1680*(x*e + d)^m*b*c*d^5*g^2*m*x*e^2 - 240*(x*e + d)^m*c^2*d^6*f*
g*m*e - 240*(x*e + d)^m*b*c*d^6*g^2*m*e + 720*(x*e + d)^m*c^2*d^7*g^2 + 27*
(x*e + d)^m*a^2*f^2*m^5*x*e^7 + 540*(x*e + d)^m*a*b*f^2*m^4*x^2*e^7 + 540*(
x*e + d)^m*a^2*f*g*m^4*x^2*e^7 + 1219*(x*e + d)^m*b^2*f^2*m^3*x^3*e^7 + 243
8*(x*e + d)^m*a*c*f^2*m^3*x^3*e^7 + 4876*(x*e + d)^m*a*b*f*g*m^3*x^3*e^7 +
1219*(x*e + d)^m*a^2*g^2*m^3*x^3*e^7 + 5090*(x*e + d)^m*b*c*f^2*m^2*x^4*e^7

+ 5090*(x*e + d)^m*b^2*f*g*m^2*x^4*e^7 + 10180*(x*e + d)^m*a*c*f*g*m^2*x^4
e^7 + 5090(x*e + d)^m*a*b*g^2*m^2*x^4*e^7 + 2412*(x*e + d)^m*c^2*f^2*m*x^
5*e^7 + 9648*(x*e + d)^m*b*c*f*g*m*x^5*e^7 + 2412*(x*e + d)^m*b^2*g^2*m*x^5
e^7 + 4824(x*e + d)^m*a*c*g^2*m*x^5*e^7 + 1680*(x*e + d)^m*c^2*f*g*x^6*e^
7 + 1680*(x*e + d)^m*b*c*g^2*x^6*e^7 + 27*(x*e + d)^m*a^2*d*f^2*m^5*e^6 + 4
90*(x*e + d)^m*a*b*d*f^2*m^4*x*e^6 + 490*(x*e + d)^m*a^2*d*f*g*m^4*x*e^6 +
817*(x*e + d)^m*b^2*d*f^2*m^3*x^2*e^6 + 1634*(x*e + d)^m*a*c*d*f^2*m^3*x^2*
e^6 + 3268*(x*e + d)^m*a*b*d*f*g*m^3*x^2*e^6 + 817*(x*e + d)^m*a^2*d*g^2*m^
3*x^2*e^6 + 1688*(x*e + d)^m*b*c*d*f^2*m^2*x^3*e^6 + 1688*(x*e + d)^m*b^2*d
*f*g*m^2*x^3*e^6 + 3376*(x*e + d)^m*a*c*d*f*g*m^2*x^3*e^6 + 1688*(x*e + d)^
m*a*b*d*g^2*m^2*x^3*e^6 + 252*(x*e + d)^m*c^2*d*f^2*m*x^4*e^6 + 1008*(x*e +
d)^m*b*c*d*f*g*m*x^4*e^6 + 252*(x*e + d)^m*b^2*d*g^2*m*x^4*e^6 + 504*(x*e
+ d)^m*a*c*d*g^2*m*x^4*e^6 - 50*(x*e + d)^m*a*b*d^2*f^2*m^4*e^5 - 50*(x*e +
d)^m*a^2*d^2*f*g*m^4*e^5 - 358*(x*e + d)^m*b^2*d^2*f^2*m^3*x*e^5 - 716*(x*
e + d)^m*a*c*d^2*f^2*m^3*x*e^5 - 1432*(x*e + d)^m*a*b*d^2*f*g*m^3*x*e^5 - 3
58*(x*e + d)^m*a^2*d^2*g^2*m^3*x*e^5 - 1902*(x*e + d)^m*b*c*d^2*f^2*m^2*x^2
e^5 - 1902(x*e + d)^m*b^2*d^2*f*g*m^2*x^2*e^5 - 3804*(x*e + d)^m*a*c*d^2*
f*g*m^2*x^2*e^5 - 1902*(x*e + d)^m*a*b*d^2*g^2*m^2*x^2*e^5 - 336*(x*e + d)^
m*c^2*d^2*f^2*m*x^3*e^5 - 1344*(x*e + d)^m*b*c*d^2*f*g*m*x^3*e^5 - 336*(x*e
+ d)^m*b^2*d^2*g^2*m*x^3*e^5 - 672*(x*e + d)^m*a*c*d^2*g^2*m*x^3*e^5 + 44*
(x*e + d)^m*b^2*d^3*f^2*m^3*e^4 + 88*(x*e + d)^m*a*c*d^3*f^2*m^3*e^4 + 176*
(x*e + d)^m*a*b*d^3*f*g*m^3*e^4 + 44*(x*e + d)^m*a^2*d^3*g^2*m^3*e^4 + 1284
*(x*e + d)^m*b*c*d^3*f^2*m^2*x*e^4 + 1284*(x*e + d)^m*b^2*d^3*f*g*m^2*x*e^4
+ 2568*(x*e + d)^m*a*c*d^3*f*g*m^2*x*e^4 + 1284*(x*e + d)^m*a*b*d^3*g^2*m^
2*x*e^4 + 504*(x*e + d)^m*c^2*d^3*f^2*m*x^2*e^4 + 2016*(x*e + d)^m*b*c*d^3*
f*g*m*x^2*e^4 + 504*(x*e + d)^m*b^2*d^3*g^2*m*x^2*e^4 + 1008*(x*e + d)^m*a*
c*d^3*g^2*m*x^2*e^4 - 216*(x*e + d)^m*b*c*d^4*f^2*m^2*e^3 - 216*(x*e + d)^m
*b^2*d^4*f*g*m^2*e^3 - 432*(x*e + d)^m*a*c*d^4*f*g*m^2*e^3 - 216*(x*e + d)^
m*a*b*d^4*g^2*m^2*e^3 - 1008*(x*e + d)^m*c^2*d^4*f^2*m*x*e^3 - 4032*(x*e +
d)^m*b*c*d^4*f*g*m*x*e^3 - 1008*(x*e + d)^m*b^2*d^4*g^2*m*x*e^3 - 2016*(x*e
+ d)^m*a*c*d^4*g^2*m*x*e^3 + 312*(x*e + d)^m*c^2*d^5*f^2*m*e^2 + 1248*(x*e
+ d)^m*b*c*d^5*f*g*m*e^2 + 312*(x*e + d)^m*b^2*d^5*g^2*m*e^2 + 624*(x*e +
d)^m*a*c*d^5*g^2*m*e^2 - 1680*(x*e + d)^m*c^2*d^6*f*g*e - 1680*(x*e + d)^m*
b*c*d^6*g^2*e + 295*(x*e + d)^m*a^2*f^2*m^4*x*e^7 + 2840*(x*e + d)^m*a*b*f^
2*m^3*x^2*e^7 + 2840*(x*e + d)^m*a^2*f*g*m^3*x^2*e^7 + 3112*(x*e + d)^m*b^2
*f^2*m^2*x^3*e^7 + 6224*(x*e + d)^m*a*c*f^2*m^2*x^3*e^7 + 12448*(x*e + d)^m
*a*b*f*g*m^2*x^3*e^7 + 3112*(x*e + d)^m*a^2*g^2*m^2*x^3*e^7 + 5904*(x*e + d
)^m*b*c*f^2*m*x^4*e^7 + 5904*(x*e + d)^m*b^2*f*g*m*x^4*e^7 + 11808*(x*e + d
)^m*a*c*f*g*m*x^4*e^7 + 5904*(x*e + d)^m*a*b*g^2*m*x^4*e^7 + 1008*(x*e + d)
^m*c^2*f^2*x^5*e^7 + 4032*(x*e + d)^m*b*c*f*g*x^5*e^7 + 1008*(x*e + d)^m*b^
2*g^2*x^5*e^7 + 2016*(x*e + d)^m*a*c*g^2*x^5*e^7 + 295*(x*e + d)^m*a^2*d*f^
2*m^4*e^6 + 2350*(x*e + d)^m*a*b*d*f^2*m^3*x*e^6 + 2350*(x*e + d)^m*a^2*d*f
*g*m^3*x*e^6 + 1478*(x*e + d)^m*b^2*d*f^2*m^2*x^2*e^6 + 2956*(x*e + d)^m*a*
c*d*f^2*m^2*x^2*e^6 + 5912*(x*e + d)^m*a*b*d*f*g*m^2*x^2*e^6 + 1478*(x*e +
d)^m*a^2*d*g^2*m^2*x^2*e^6 + 840*(x*e + d)^m*b*c*d*f^2*m*x^3*e^6 + 840*(x*e
+ d)^m*b^2*d*f*g*m*x^3*e^6 + 1680*(x*e + d)^m*a*c*d*f*g*m*x^3*e^6 + 840*(x
*e + d)^m*a*b*d*g^2*m*x^3*e^6 - 490*(x*e + d)^m*a*b*d^2*f^2*m^3*e^5 - 490*(
x*e + d)^m*a^2*d^2*f*g*m^3*e^5 - 1276*(x*e + d)^m*b^2*d^2*f^2*m^2*x*e^5 - 2
552*(x*e + d)^m*a*c*d^2*f^2*m^2*x*e^5 - 5104*(x*e + d)^m*a*b*d^2*f*g*m^2*x*
e^5 - 1276*(x*e + d)^m*a^2*d^2*g^2*m^2*x*e^5 - 1260*(x*e + d)^m*b*c*d^2*f^2
*m*x^2*e^5 - 1260*(x*e + d)^m*b^2*d^2*f*g*m*x^2*e^5 - 2520*(x*e + d)^m*a*c*
d^2*f*g*m*x^2*e^5 - 1260*(x*e + d)^m*a*b*d^2*g^2*m*x^2*e^5 + 358*(x*e + d)^
m*b^2*d^3*f^2*m^2*e^4 + 716*(x*e + d)^m*a*c*d^3*f^2*m^2*e^4 + 1432*(x*e + d
)^m*a*b*d^3*f*g*m^2*e^4 + 358*(x*e + d)^m*a^2*d^3*g^2*m^2*e^4 + 2520*(x*e +
d)^m*b*c*d^3*f^2*m*x*e^4 + 2520*(x*e + d)^m*b^2*d^3*f*g*m*x*e^4 + 5040*(x*
e + d)^m*a*c*d^3*f*g*m*x*e^4 + 2520*(x*e + d)^m*a*b*d^3*g^2*m*x*e^4 - 1284*
(x*e + d)^m*b*c*d^4*f^2*m*e^3 - 1284*(x*e + d)^m*b^2*d^4*f*g*m*e^3 - 2568*(
x*e + d)^m*a*c*d^4*f*g*m*e^3 - 1284*(x*e + d)^m*a*b*d^4*g^2*m*e^3 + 1008*(x
*e + d)^m*c^2*d^5*f^2*e^2 + 4032*(x*e + d)^m*b*c*d^5*f*g*e^2 + 1008*(x*e +

```
d)^m*b^2*d^5*g^2*e^2 + 2016*(x*e + d)^m*a*c*d^5*g^2*e^2 + 1665*(x*e + d)^m*
a^2*f^2*m^3*x*e^7 + 7858*(x*e + d)^m*a*b*f^2*m^2*x^2*e^7 + 7858*(x*e + d)^m
*a^2*f*g*m^2*x^2*e^7 + 3796*(x*e + d)^m*b^2*f^2*m*x^3*e^7 + 7592*(x*e + d)^
m*a*c*f^2*m*x^3*e^7 + 15184*(x*e + d)^m*a*b*f*g*m*x^3*e^7 + 3796*(x*e + d)^
m*a^2*g^2*m*x^3*e^7 + 2520*(x*e + d)^m*b*c*f^2*x^4*e^7 + 2520*(x*e + d)^m*b
^2*f*g*x^4*e^7 + 5040*(x*e + d)^m*a*c*f*g*x^4*e^7 + 2520*(x*e + d)^m*a*b*g^
2*x^4*e^7 + 1665*(x*e + d)^m*a^2*d*f^2*m^3*e^6 + 5508*(x*e + d)^m*a*b*d*f^2
*m^2*x*e^6 + 5508*(x*e + d)^m*a^2*d*f*g*m^2*x*e^6 + 840*(x*e + d)^m*b^2*d*f
^2*m*x^2*e^6 + 1680*(x*e + d)^m*a*c*d*f^2*m*x^2*e^6 + 3360*(x*e + d)^m*a*b*
d*f*g*m*x^2*e^6 + 840*(x*e + d)^m*a^2*d*g^2*m*x^2*e^6 - 2350*(x*e + d)^m*a*
b*d^2*f^2*m^2*e^5 - 2350*(x*e + d)^m*a^2*d^2*f*g*m^2*e^5 - 1680*(x*e + d)^m
*b^2*d^2*f^2*m*x*e^5 - 3360*(x*e + d)^m*a*c*d^2*f^2*m*x*e^5 - 6720*(x*e + d
)^m*a*b*d^2*f*g*m*x*e^5 - 1680*(x*e + d)^m*a^2*d^2*g^2*m*x*e^5 + 1276*(x*e
+ d)^m*b^2*d^3*f^2*m*e^4 + 2552*(x*e + d)^m*a*c*d^3*f^2*m*e^4 + 5104*(x*e +
 d)^m*a*b*d^3*f*g*m*e^4 + 1276*(x*e + d)^m*a^2*d^3*g^2*m*e^4 - 2520*(x*e +
d)^m*b*c*d^4*f^2*e^3 - 2520*(x*e + d)^m*b^2*d^4*f*g*e^3 - 5040*(x*e + d)^m*
a*c*d^4*f*g*e^3 - 2520*(x*e + d)^m*a*b*d^4*g^2*e^3 + 5104*(x*e + d)^m*a^2*f
^2*m^2*x*e^7 + 10548*(x*e + d)^m*a*b*f^2*m*x^2*e^7 + 10548*(x*e + d)^m*a^2*
f*g*m*x^2*e^7 + 1680*(x*e + d)^m*b^2*f^2*x^3*e^7 + 3360*(x*e + d)^m*a*c*f^2
*x^3*e^7 + 6720*(x*e + d)^m*a*b*f*g*x^3*e^7 + 1680*(x*e + d)^m*a^2*g^2*x^3*
e^7 + 5104*(x*e + d)^m*a^2*d*f^2*m^2*e^6 + 5040*(x*e + d)^m*a*b*d*f^2*m*x*e
^6 + 5040*(x*e + d)^m*a^2*d*f*g*m*x*e^6 - 5508*(x*e + d)^m*a*b*d^2*f^2*m*e^
5 - 5508*(x*e + d)^m*a^2*d^2*f*g*m*e^5 + 1680*(x*e + d)^m*b^2*d^3*f^2*e^4 +
 3360*(x*e + d)^m*a*c*d^3*f^2*e^4 + 6720*(x*e + d)^m*a*b*d^3*f*g*e^4 + 1680
*(x*e + d)^m*a^2*d^3*g^2*e^4 + 8028*(x*e + d)^m*a^2*f^2*m*x*e^7 + 5040*(x*e
 + d)^m*a*b*f^2*x^2*e^7 + 5040*(x*e + d)^m*a^2*f*g*x^2*e^7 + 8028*(x*e + d)
^m*a^2*d*f^2*m*e^6 - 5040*(x*e + d)^m*a*b*d^2*f^2*e^5 - 5040*(x*e + d)^m*a^
2*d^2*f*g*e^5 + 5040*(x*e + d)^m*a^2*f^2*x*e^7 + 5040*(x*e + d)^m*a^2*d*f^2
*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^7 + 1
3132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)
```

maple [B] time = 0.04, size = 5890, normalized size = 11.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.79, size = 2034, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f^2/((m^2 + 3*m + 2)*e^
2) + 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*f*g/((m^2 + 3*m +
2)*e^2) + (e*x + d)^(m + 1)*a^2*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3
+ (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f^2/((m^3 + 6*
m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2
*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*c*f^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 4*
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
+ d)^m*a*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m + 2)*e^3*x^3 +
(m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*g^2/((m^3 + 6*m^
2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2
*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*
b*c*f^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m
+ 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*
```

```
d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*e^4) + 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3
 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*f*g/((m^4
 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4
+ (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6
*d^4)*(e*x + d)^m*a*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4
 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^
4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d
^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2*f^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 +
274*m + 120)*e^5) + 4*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 +
 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(
m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*b*c*f*g/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + ((m^4 + 10*m^3 + 35*m^2 + 50
*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2
+ 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x
 + d)^m*b^2*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*(
(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*
d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 -
24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*a*c*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^
2 + 274*m + 120)*e^5) + 2*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*
e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^
3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^
2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*f*g/((m^6 + 2
1*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 2*((m^5 + 15*m^
4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m
^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3
 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 12
0*d^6)*(e*x + d)^m*b*c*g^2/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 +
1764*m + 720)*e^6) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
 + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6
*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*
m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360
*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^2*g^2/((m^7
 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7
)
```

mupad [B] time = 5.38, size = 4871, normalized size = 9.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(720*c^2*d^7*g^2 + 5040*a^2*d*e^6*f^2 + 1680*a^2*d^3*e^4*g^2 +
 1680*b^2*d^3*e^4*f^2 + 1008*b^2*d^5*e^2*g^2 + 1008*c^2*d^5*e^2*f^2 - 1680*
b*c*d^6*e*g^2 - 1680*c^2*d^6*e*f*g + 358*a^2*d^3*e^4*g^2*m^2 + 358*b^2*d^3*
e^4*f^2*m^2 + 44*a^2*d^3*e^4*g^2*m^3 + 44*b^2*d^3*e^4*f^2*m^3 + 2*a^2*d^3*e
^4*g^2*m^4 + 2*b^2*d^3*e^4*f^2*m^4 + 24*b^2*d^5*e^2*g^2*m^2 + 24*c^2*d^5*e^
2*f^2*m^2 - 5040*a*b*d^2*e^5*f^2 - 2520*a*b*d^4*e^3*g^2 + 3360*a*c*d^3*e^4*
f^2 + 2016*a*c*d^5*e^2*g^2 - 2520*b*c*d^4*e^3*f^2 - 5040*a^2*d^2*e^5*f*g -
2520*b^2*d^4*e^3*f*g + 8028*a^2*d*e^6*f^2*m + 5104*a^2*d*e^6*f^2*m^2 + 1665
*a^2*d*e^6*f^2*m^3 + 295*a^2*d*e^6*f^2*m^4 + 27*a^2*d*e^6*f^2*m^5 + a^2*d*e
^6*f^2*m^6 + 1276*a^2*d^3*e^4*g^2*m + 1276*b^2*d^3*e^4*f^2*m + 312*b^2*d^5*
e^2*g^2*m + 312*c^2*d^5*e^2*f^2*m - 2350*a*b*d^2*e^5*f^2*m^2 - 490*a*b*d^2*
e^5*f^2*m^3 - 50*a*b*d^2*e^5*f^2*m^4 - 2*a*b*d^2*e^5*f^2*m^5 - 216*a*b*d^4*
e^3*g^2*m^2 + 716*a*c*d^3*e^4*f^2*m^2 - 12*a*b*d^4*e^3*g^2*m^3 + 88*a*c*d^3
*e^4*f^2*m^3 + 4*a*c*d^3*e^4*f^2*m^4 + 48*a*c*d^5*e^2*g^2*m^2 - 216*b*c*d^4
*e^3*f^2*m^2 - 12*b*c*d^4*e^3*f^2*m^3 - 2350*a^2*d^2*e^5*f*g*m^2 - 490*a^2*
d^2*e^5*f*g*m^3 - 50*a^2*d^2*e^5*f*g*m^4 - 2*a^2*d^2*e^5*f*g*m^5 - 216*b^2*
d^4*e^3*f*g*m^2 - 12*b^2*d^4*e^3*f*g*m^3 + 6720*a*b*d^3*e^4*f*g - 5040*a*c*
```

d^4*e^3*f*g + 4032*b*c*d^5*e^2*f*g - 240*b*c*d^6*e*g^2*m - 240*c^2*d^6*e*f*g*m - 5508*a*b*d^2*e^5*f^2*m - 1284*a*b*d^4*e^3*g^2*m + 2552*a*c*d^3*e^4*f^2*m + 624*a*c*d^5*e^2*g^2*m - 1284*b*c*d^4*e^3*f^2*m - 5508*a^2*d^2*e^5*f*g*m - 1284*b^2*d^4*e^3*f*g*m + 1432*a*b*d^3*e^4*f*g*m^2 + 176*a*b*d^3*e^4*f*g*m^3 + 8*a*b*d^3*e^4*f*g*m^4 - 432*a*c*d^4*e^3*f*g*m^2 - 24*a*c*d^4*e^3*f*g*m^3 + 96*b*c*d^5*e^2*f*g*m^2 + 5104*a*b*d^3*e^4*f*g*m - 2568*a*c*d^4*e^3*f*g*m + 1248*b*c*d^5*e^2*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x*(d + e*x)^m*(5040*a^2*e^7*f^2 + 8028*a^2*e^7*f^2*m + 5104*a^2*e^7*f^2*m^2 + 1665*a^2*e^7*f^2*m^3 + 295*a^2*e^7*f^2*m^4 + 27*a^2*e^7*f^2*m^5 + a^2*e^7*f^2*m^6 - 1276*a^2*d^2*e^5*g^2*m^2 - 1276*b^2*d^2*e^5*f^2*m^2 - 358*a^2*d^2*e^5*g^2*m^3 - 358*b^2*d^2*e^5*f^2*m^3 - 44*a^2*d^2*e^5*g^2*m^4 - 44*b^2*d^2*e^5*f^2*m^4 - 2*a^2*d^2*e^5*g^2*m^5 - 2*b^2*d^2*e^5*f^2*m^5 - 312*b^2*d^4*e^3*g^2*m^2 - 312*c^2*d^4*e^3*f^2*m^2 - 24*b^2*d^4*e^3*g^2*m^3 - 24*c^2*d^4*e^3*f^2*m^3 - 720*c^2*d^6*e*g^2*m - 1680*a^2*d^2*e^5*g^2*m - 1680*b^2*d^2*e^5*f^2*m - 1008*b^2*d^4*e^3*g^2*m - 1008*c^2*d^4*e^3*f^2*m + 1284*a*b*d^3*e^4*g^2*m^2 - 2552*a*c*d^2*e^5*f^2*m^2 + 216*a*b*d^3*e^4*g^2*m^3 - 716*a*c*d^2*e^5*f^2*m^3 + 12*a*b*d^3*e^4*g^2*m^4 - 88*a*c*d^2*e^5*f^2*m^4 - 4*a*c*d^2*e^5*f^2*m^5 - 624*a*c*d^4*e^3*g^2*m^2 + 1284*b*c*d^3*e^4*f^2*m^2 - 48*a*c*d^4*e^3*g^2*m^3 + 216*b*c*d^3*e^4*f^2*m^3 + 12*b*c*d^3*e^4*f^2*m^4 + 240*b*c*d^5*e^2*g^2*m^2 + 1284*b^2*d^3*e^4*f*g*m^2 + 216*b^2*d^3*e^4*f*g*m^3 + 12*b^2*d^3*e^4*f*g*m^4 + 240*c^2*d^5*e^2*f*g*m^2 + 5040*a*b*d*e^6*f^2*m + 5040*a^2*d*e^6*f*g*m + 5508*a*b*d*e^6*f^2*m^2 + 2350*a*b*d*e^6*f^2*m^3 + 490*a*b*d*e^6*f^2*m^4 + 50*a*b*d*e^6*f^2*m^5 + 2*a*b*d*e^6*f^2*m^6 + 2520*a*b*d^3*e^4*g^2*m - 3360*a*c*d^2*e^5*f^2*m - 2016*a*c*d^4*e^3*g^2*m + 2520*b*c*d^3*e^4*f^2*m + 1680*b*c*d^5*e^2*g^2*m + 5508*a^2*d*e^6*f*g*m^2 + 2350*a^2*d*e^6*f*g*m^3 + 490*a^2*d*e^6*f*g*m^4 + 50*a^2*d*e^6*f*g*m^5 + 2*a^2*d*e^6*f*g*m^6 + 2520*b^2*d^3*e^4*f*g*m + 1680*c^2*d^5*e^2*f*g*m - 5104*a*b*d^2*e^5*f*g*m^2 - 1432*a*b*d^2*e^5*f*g*m^3 - 176*a*b*d^2*e^5*f*g*m^4 - 8*a*b*d^2*e^5*f*g*m^5 + 2568*a*c*d^3*e^4*f*g*m^2 + 432*a*c*d^3*e^4*f*g*m^3 + 24*a*c*d^3*e^4*f*g*m^4 - 1248*b*c*d^4*e^3*f*g*m^2 - 96*b*c*d^4*e^3*f*g*m^3 - 6720*a*b*d^2*e^5*f*g*m + 5040*a*c*d^3*e^4*f*g*m - 4032*b*c*d^4*e^3*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(840*a^2*e^4*g^2 + 840*b^2*e^4*f^2 + 638*a^2*e^4*g^2*m + 638*b^2*e^4*f^2*m - 120*c^2*d^4*g^2*m + 179*a^2*e^4*g^2*m^2 + 179*b^2*e^4*f^2*m^2 + 22*a^2*e^4*g^2*m^3 + 22*b^2*e^4*f^2*m^3 + a^2*e^4*g^2*m^4 + b^2*e^4*f^2*m^4 + 1680*a*c*e^4*f^2 + 1276*a*c*e^4*f^2*m - 52*b^2*d^2*e^2*g^2*m^2 - 52*c^2*d^2*e^2*f^2*m^2 - 4*b^2*d^2*e^2*g^2*m^3 - 4*c^2*d^2*e^2*f^2*m^3 + 358*a*c*e^4*f^2*m^2 + 44*a*c*e^4*f^2*m^3 + 2*a*c*e^4*f^2*m^4 + 3360*a*b*e^4*f*g - 168*b^2*d^2*e^2*g^2*m - 168*c^2*d^2*e^2*f^2*m + 2552*a*b*e^4*f*g*m - 104*a*c*d^2*e^2*g^2*m^2 - 8*a*c*d^2*e^2*g^2*m^3 + 420*a*b*d*e^3*g^2*m + 420*b*c*d*e^3*f^2*m + 280*b*c*d^3*e*g^2*m + 716*a*b*e^4*f*g*m^2 + 88*a*b*e^4*f*g*m^3 + 4*a*b*e^4*f*g*m^4 + 420*b^2*d*e^3*f*g*m + 280*c^2*d^3*e*f*g*m + 214*a*b*d*e^3*g^2*m^2 + 36*a*b*d*e^3*g^2*m^3 + 2*a*b*d*e^3*g^2*m^4 - 336*a*c*d^2*e^2*g^2*m + 214*b*c*d*e^3*f^2*m^2 + 36*b*c*d*e^3*f^2*m^3 + 2*b*c*d*e^3*f^2*m^4 + 40*b*c*d^3*e*g^2*m^2 + 214*b^2*d*e^3*f*g*m^2 + 36*b^2*d*e^3*f*g*m^3 + 2*b^2*d*e^3*f*g*m^4 + 40*c^2*d^3*e*f*g*m^2 - 208*b*c*d^2*e^2*f*g*m^2 - 16*b*c*d^2*e^2*f*g*m^3 + 840*a*c*d*e^3*f*g*m + 428*a*c*d*e^3*f*g*m^2 + 72*a*c*d*e^3*f*g*m^3 + 4*a*c*d*e^3*f*g*m^4 - 672*b*c*d^2*e^2*f*g*m))/(e^4*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(42*b^2*e^2*g^2 + 42*c^2*e^2*f^2 + 13*b^2*e^2*g^2*m - 6*c^2*d^2*g^2*m + 13*c^2*e^2*f^2*m + b^2*e^2*g^2*m^2 + c^2*e^2*f^2*m^2 + 84*a*c*e^2*g^2 + 26*a*c*e^2*g^2*m + 2*a*c*e^2*g^2*m^2 + 168*b*c*e^2*f*g + 14*b*c*d*e*g^2*m + 52*b*c*e^2*f*g*m + 14*c^2*d*e*f*g*m + 2*b*c*d*e*g^2*m^2 + 4*b*c*e^2*f*g*m^2 + 2*c^2*d*e*f*g*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^2*(m + 1)*(d + e*x)^m*(360*c^2*d^5*g^2*m + 5040*a*b*e^5*f^2 + 5040*a^2*e^5*f*g + 5508*a*b*e^5*f^2*m + 5508*a^2*e^5*f*g*m + 156*b^2*d^3*e^2*g^2*m^2 + 156*c^2*d^3*e^2*f^2*m^2 + 12*b^2*d^3*e^2*g^2*m^3 + 12*c^2*d^3*e^2*f^2*m^3 + 23

```
50*a*b*e^5*f^2*m^2 + 490*a*b*e^5*f^2*m^3 + 50*a*b*e^5*f^2*m^4 + 2*a*b*e^5*f
^2*m^5 + 840*a^2*d*e^4*g^2*m + 840*b^2*d*e^4*f^2*m + 2350*a^2*e^5*f*g*m^2 +
 490*a^2*e^5*f*g*m^3 + 50*a^2*e^5*f*g*m^4 + 2*a^2*e^5*f*g*m^5 + 638*a^2*d*e
^4*g^2*m^2 + 638*b^2*d*e^4*f^2*m^2 + 179*a^2*d*e^4*g^2*m^3 + 179*b^2*d*e^4*
f^2*m^3 + 22*a^2*d*e^4*g^2*m^4 + 22*b^2*d*e^4*f^2*m^4 + a^2*d*e^4*g^2*m^5 +
 b^2*d*e^4*f^2*m^5 + 504*b^2*d^3*e^2*g^2*m + 504*c^2*d^3*e^2*f^2*m - 642*a*
b*d^2*e^3*g^2*m^2 - 108*a*b*d^2*e^3*g^2*m^3 - 6*a*b*d^2*e^3*g^2*m^4 + 312*a
*c*d^3*e^2*g^2*m^2 - 642*b*c*d^2*e^3*f^2*m^2 + 24*a*c*d^3*e^2*g^2*m^3 - 108
*b*c*d^2*e^3*f^2*m^3 - 6*b*c*d^2*e^3*f^2*m^4 - 642*b^2*d^2*e^3*f*g*m^2 - 10
8*b^2*d^2*e^3*f*g*m^3 - 6*b^2*d^2*e^3*f*g*m^4 + 1680*a*c*d*e^4*f^2*m - 840*
b*c*d^4*e*g^2*m - 840*c^2*d^4*e*f*g*m - 1260*a*b*d^2*e^3*g^2*m + 1276*a*c*d
*e^4*f^2*m^2 + 358*a*c*d*e^4*f^2*m^3 + 44*a*c*d*e^4*f^2*m^4 + 2*a*c*d*e^4*f
^2*m^5 + 1008*a*c*d^3*e^2*g^2*m - 1260*b*c*d^2*e^3*f^2*m - 120*b*c*d^4*e*g^
2*m^2 - 1260*b^2*d^2*e^3*f*g*m - 120*c^2*d^4*e*f*g*m^2 - 1284*a*c*d^2*e^3*f
*g*m^2 - 216*a*c*d^2*e^3*f*g*m^3 - 12*a*c*d^2*e^3*f*g*m^4 + 624*b*c*d^3*e^2
*f*g*m^2 + 48*b*c*d^3*e^2*f*g*m^3 + 3360*a*b*d*e^4*f*g*m + 2552*a*b*d*e^4*f
*g*m^2 + 716*a*b*d*e^4*f*g*m^3 + 88*a*b*d*e^4*f*g*m^4 + 4*a*b*d*e^4*f*g*m^5
 - 2520*a*c*d^2*e^3*f*g*m + 2016*b*c*d^3*e^2*f*g*m))/(e^5*(13068*m + 13132*
m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c^2*g^2*x^7*
(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(
13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
+ (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*c^2*d^3*g^2*m + 420*a*b*e^3
*g^2 + 420*b*c*e^3*f^2 + 420*b^2*e^3*f*g + 214*a*b*e^3*g^2*m + 214*b*c*e^3*
f^2*m + 214*b^2*e^3*f*g*m + 36*a*b*e^3*g^2*m^2 + 2*a*b*e^3*g^2*m^3 + 36*b*c
*e^3*f^2*m^2 + 2*b*c*e^3*f^2*m^3 + 42*b^2*d*e^2*g^2*m + 42*c^2*d*e^2*f^2*m
+ 36*b^2*e^3*f*g*m^2 + 2*b^2*e^3*f*g*m^3 + 840*a*c*e^3*f*g + 13*b^2*d*e^2*g
^2*m^2 + 13*c^2*d*e^2*f^2*m^2 + b^2*d*e^2*g^2*m^3 + c^2*d*e^2*f^2*m^3 + 428
*a*c*e^3*f*g*m + 84*a*c*d*e^2*g^2*m - 70*b*c*d^2*e*g^2*m + 72*a*c*e^3*f*g*m
^2 + 4*a*c*e^3*f*g*m^3 - 70*c^2*d^2*e*f*g*m + 26*a*c*d*e^2*g^2*m^2 + 2*a*c*
d*e^2*g^2*m^3 - 10*b*c*d^2*e*g^2*m^2 - 10*c^2*d^2*e*f*g*m^2 + 168*b*c*d*e^2
*f*g*m + 52*b*c*d*e^2*f*g*m^2 + 4*b*c*d*e^2*f*g*m^3))/(e^3*(13068*m + 13132
*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c*g*x^6*(d
+ e*x)^m*(14*b*e*g + 14*c*e*f + 2*b*e*g*m + c*d*g*m + 2*c*e*f*m)*(274*m + 2
25*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 +
 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.926 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=311

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))\right.}{e^6(m}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)*(e*x+d)^(1+m)/e^6/(1+m)-(a*e^2-b*d*e+c*d^2
)*(c*d*(-5*d*g+4*e*f)-e*(a*e*g-3*b*d*g+2*b*e*f))*(e*x+d)^(2+m)/e^6/(2+m)+(2
*c^2*d^2*(-5*d*g+3*e*f)+b*e^2*(2*a*e*g-3*b*d*g+b*e*f)+2*c*e*(a*e*(-3*d*g+e*
f)-3*b*d*(-2*d*g+e*f)))*(e*x+d)^(3+m)/e^6/(3+m)+(b^2*e^2*g-2*c^2*d*(-5*d*g+
2*e*f)+2*c*e*(a*e*g-4*b*d*g+b*e*f))*(e*x+d)^(4+m)/e^6/(4+m)+c*(2*b*e*g-5*c*
d*g+c*e*f)*(e*x+d)^(5+m)/e^6/(5+m)+c^2*g*(e*x+d)^(6+m)/e^6/(6+m)
```

Rubi [A] time = 0.39, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {771}

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))\right.}{e^6(m}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - (
(c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g
))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(
b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)
))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g)
 + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(
c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e
*x)^(6 + m))/(e^6*(6 + m))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^m}{e^5}+\frac{\left(cd^2-bde+ae^2\right)(-cd(4e\right.}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)}-\frac{\left(cd^2-bde+ae^2\right)(cd(4ef-}{}$$

Mathematica [B] time = 1.52, size = 655, normalized size = 2.11

$$\frac{(d+ex)^{m+1}\left(2\left(\frac{(d+ex)\left(c^2e^2\left(4a^2e^2g\left(m^2+8m+15\right)+2abe\left(dg\left(4m^2+11m-18\right)+ef\left(2m^2+19m+42\right)\right)+b^2d\left(dg\left(m^2-13m+6\right)+2ef\left(m^2+5m-6\right)\right)\right)-b^2ce^3(m+2)(aeg(5m+21)+bdg(2m\right.}{e^2(m+2)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^2*(2*b*e*g + c*(-5*d*g + e*f*(6 + m)
+ e*g*(5 + m)*x)) + (2*(((c*d^2 + e*(-(b*d) + a*e))*(b^3*e^3*g*(3 + 4*m + m
^2) + 12*c^3*d^2*(-5*d*g + e*f*(6 + m)) - b*c*e^2*(1 + m)*(b*d*g*(-6 + m) +
 b*e*f*(6 + m) + 2*a*e*g*(9 + 2*m)) + 2*c^2*e*(-3*b*d*(d*g*(-9 + m) + 2*e*f
*(6 + m)) + 2*a*e*(d*g*(-15 + m + m^2) + e*f*(24 + 10*m + m^2)))))/(e^2*(1
+ m)) + ((b^4*e^4*g*(6 + 5*m + m^2) + 12*c^4*d^3*(5*d*g - e*f*(6 + m)) - b^
2*c*e^3*(2 + m)*(b*e*f*(6 + m) + b*d*g*(-3 + 2*m) + a*e*g*(21 + 5*m)) + 2*c
^3*d*e*(3*b*d*(d*g*(-14 + m) + 3*e*f*(6 + m)) - 2*a*e*(d*g*(-30 - 4*m + m^2
) + e*f*(42 + 19*m + 2*m^2))) + c^2*e^2*(4*a^2*e^2*g*(15 + 8*m + m^2) + b^2
*d*(d*g*(6 - 13*m + m^2) + 2*e*f*(-6 + 5*m + m^2)) + 2*a*b*e*(e*f*(42 + 19*
m + 2*m^2) + d*g*(-18 + 11*m + 4*m^2))))*(d + e*x))/(e^2*(2 + m)) - (c*e*(4
 + m)*(b*d*(-5*c*d + 2*b*e)*g - 2*a*c*d*e*g*m + a*b*e^2*g*(1 + m) + c*e*(b*
d - 2*a*e)*f*(6 + m)) - (3*c*d - b*e)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g
+ e*f*(6 + m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m))) +
c*e*(3 + m)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g + e*f*(6 + m)) - c*e*(b*d*
g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m)))*x)*(a + x*(b + c*x))))/(c*e^
2*(3 + m)*(4 + m))))/(c*e^2*(5 + m)*(6 + m))
```

fricas [B] time = 1.33, size = 2368, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^5*f*m^5 + (c^2*e^6*g*m^5 + 15*c^2*e^6*g*m^4 + 85*c^2*e^6*g*m^3 + 2
25*c^2*e^6*g*m^2 + 274*c^2*e^6*g*m + 120*c^2*e^6*g)*x^6 + (144*c^2*e^6*f +
288*b*c*e^6*g + (c^2*e^6*f + (c^2*d*e^5 + 2*b*c*e^6)*g)*m^5 + 2*(8*c^2*e^6*
f + (5*c^2*d*e^5 + 16*b*c*e^6)*g)*m^4 + 5*(19*c^2*e^6*f + (7*c^2*d*e^5 + 38
*b*c*e^6)*g)*m^3 + 10*(26*c^2*e^6*f + (5*c^2*d*e^5 + 52*b*c*e^6)*g)*m^2 + 1
2*(27*c^2*e^6*f + 2*(c^2*d*e^5 + 27*b*c*e^6)*g)*m)*x^5 - (a^2*d^2*e^4*g + 2
*(a*b*d^2*e^4 - 10*a^2*d*e^5)*f)*m^4 + (360*b*c*e^6*f + 180*(b^2 + 2*a*c)*e
^6*g + ((c^2*d*e^5 + 2*b*c*e^6)*f + (2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*g)*m^
5 + (2*(6*c^2*d*e^5 + 17*b*c*e^6)*f - (5*c^2*d^2*e^4 - 24*b*c*d*e^5 - 17*(b
^2 + 2*a*c)*e^6)*g)*m^4 + ((47*c^2*d*e^5 + 214*b*c*e^6)*f - (30*c^2*d^2*e^4
 - 94*b*c*d*e^5 - 107*(b^2 + 2*a*c)*e^6)*g)*m^3 + (2*(36*c^2*d*e^5 + 307*b*
c*e^6)*f - (55*c^2*d^2*e^4 - 144*b*c*d*e^5 - 307*(b^2 + 2*a*c)*e^6)*g)*m^2
+ 6*(6*(c^2*d*e^5 + 22*b*c*e^6)*f - (5*c^2*d^2*e^4 - 12*b*c*d*e^5 - 66*(b^2
 + 2*a*c)*e^6)*g)*m)*x^4 - ((36*a*b*d^2*e^4 - 155*a^2*d*e^5 - 2*(b^2 + 2*a*
c)*d^3*e^3)*f - 2*(2*a*b*d^3*e^3 - 9*a^2*d^2*e^4)*g)*m^3 + (480*a*b*e^6*g +
 240*(b^2 + 2*a*c)*e^6*f + ((2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*f + (2*a*b*e^
6 + (b^2 + 2*a*c)*d*e^5)*g)*m^5 - 2*((2*c^2*d^2*e^4 - 14*b*c*d*e^5 - 9*(b^2
 + 2*a*c)*e^6)*f + (4*b*c*d^2*e^4 - 18*a*b*e^6 - 7*(b^2 + 2*a*c)*d*e^5)*g)*
m^4 - ((36*c^2*d^2*e^4 - 130*b*c*d*e^5 - 121*(b^2 + 2*a*c)*e^6)*f - (20*c^2
*d^3*e^3 - 72*b*c*d^2*e^4 + 242*a*b*e^6 + 65*(b^2 + 2*a*c)*d*e^5)*g)*m^3 -
4*((20*c^2*d^2*e^4 - 56*b*c*d*e^5 - 93*(b^2 + 2*a*c)*e^6)*f - (15*c^2*d^3*e
^3 - 40*b*c*d^2*e^4 + 186*a*b*e^6 + 28*(b^2 + 2*a*c)*d*e^5)*g)*m^2 - 4*((12
*c^2*d^2*e^4 - 30*b*c*d*e^5 - 127*(b^2 + 2*a*c)*e^6)*f - (10*c^2*d^3*e^3 -
24*b*c*d^2*e^4 + 254*a*b*e^6 + 15*(b^2 + 2*a*c)*d*e^5)*g)*m)*x^3 - (2*(6*b*
c*d^4*e^2 + 119*a*b*d^2*e^4 - 290*a^2*d*e^5 - 15*(b^2 + 2*a*c)*d^3*e^3)*f -
 (60*a*b*d^3*e^3 - 119*a^2*d^2*e^4 - 6*(b^2 + 2*a*c)*d^4*e^2)*g)*m^2 + (720
*a*b*e^6*f + 360*a^2*e^6*g + ((2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*f + (2*a*b*
d*e^5 + a^2*e^6)*g)*m^5 - (2*(3*b*c*d^2*e^4 - 19*a*b*e^6 - 8*(b^2 + 2*a*c)*
d*e^5)*f - (32*a*b*d*e^5 + 19*a^2*e^6 - 3*(b^2 + 2*a*c)*d^2*e^4)*g)*m^4 + (
(12*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 274*a*b*e^6 + 89*(b^2 + 2*a*c)*d*e^5)*f
+ (24*b*c*d^3*e^3 + 178*a*b*d*e^5 + 137*a^2*e^6 - 36*(b^2 + 2*a*c)*d^2*e^4)
```

```
*g)*m^3 + (2*(42*c^2*d^3*e^3 - 123*b*c*d^2*e^4 + 461*a*b*e^6 + 97*(b^2 + 2*
a*c)*d*e^5)*f - (60*c^2*d^4*e^2 - 168*b*c*d^3*e^3 - 388*a*b*d*e^5 - 461*a^2
*e^6 + 123*(b^2 + 2*a*c)*d^2*e^4)*g)*m^2 + 6*(2*(6*c^2*d^3*e^3 - 15*b*c*d^2
*e^4 + 117*a*b*e^6 + 10*(b^2 + 2*a*c)*d*e^5)*f - (10*c^2*d^4*e^2 - 24*b*c*d
^3*e^3 - 40*a*b*d*e^5 - 117*a^2*e^6 + 15*(b^2 + 2*a*c)*d^2*e^4)*g)*m)*x^2 +
 24*(6*c^2*d^5*e - 15*b*c*d^4*e^2 - 30*a*b*d^2*e^4 + 30*a^2*d*e^5 + 10*(b^2
 + 2*a*c)*d^3*e^3)*f - 12*(10*c^2*d^6 - 24*b*c*d^5*e - 40*a*b*d^3*e^3 + 30*
a^2*d^2*e^4 + 15*(b^2 + 2*a*c)*d^4*e^2)*g + 2*(2*(6*c^2*d^5*e - 33*b*c*d^4*
e^2 - 171*a*b*d^2*e^4 + 261*a^2*d*e^5 + 37*(b^2 + 2*a*c)*d^3*e^3)*f + (24*b
*c*d^5*e + 148*a*b*d^3*e^3 - 171*a^2*d^2*e^4 - 33*(b^2 + 2*a*c)*d^4*e^2)*g)
*m + (720*a^2*e^6*f + (a^2*d*e^5*g + (2*a*b*d*e^5 + a^2*e^6)*f)*m^5 + 2*((1
8*a*b*d*e^5 + 10*a^2*e^6 - (b^2 + 2*a*c)*d^2*e^4)*f - (2*a*b*d^2*e^4 - 9*a^
2*d*e^5)*g)*m^4 + ((12*b*c*d^3*e^3 + 238*a*b*d*e^5 + 155*a^2*e^6 - 30*(b^2
+ 2*a*c)*d^2*e^4)*f - (60*a*b*d^2*e^4 - 119*a^2*d*e^5 - 6*(b^2 + 2*a*c)*d^3
*e^3)*g)*m^3 - 2*(2*(6*c^2*d^4*e^2 - 33*b*c*d^3*e^3 - 171*a*b*d*e^5 - 145*a
^2*e^6 + 37*(b^2 + 2*a*c)*d^2*e^4)*f + (24*b*c*d^4*e^2 + 148*a*b*d^2*e^4 -
171*a^2*d*e^5 - 33*(b^2 + 2*a*c)*d^3*e^3)*g)*m^2 - 12*((12*c^2*d^4*e^2 - 30
*b*c*d^3*e^3 - 60*a*b*d*e^5 - 87*a^2*e^6 + 20*(b^2 + 2*a*c)*d^2*e^4)*f - (1
0*c^2*d^5*e - 24*b*c*d^4*e^2 - 40*a*b*d^2*e^4 + 30*a^2*d*e^5 + 15*(b^2 + 2*
a*c)*d^3*e^3)*g)*m)*x)*(e*x + d)^m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 73
5*e^6*m^3 + 1624*e^6*m^2 + 1764*e^6*m + 720*e^6)
```

giac [B] time = 0.35, size = 4940, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g*m^5*x^6*e^6 + (x*e + d)^m*c^2*d*g*m^5*x^5*e^5 + (x*e + d
)^m*c^2*f*m^5*x^5*e^6 + 2*(x*e + d)^m*b*c*g*m^5*x^5*e^6 + 15*(x*e + d)^m*c^
2*g*m^4*x^6*e^6 + (x*e + d)^m*c^2*d*f*m^5*x^4*e^5 + 2*(x*e + d)^m*b*c*d*g*m
^5*x^4*e^5 + 10*(x*e + d)^m*c^2*d*g*m^4*x^5*e^5 - 5*(x*e + d)^m*c^2*d^2*g*m
^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f*m^5*x^4*e^6 + (x*e + d)^m*b^2*g*m^5*x^4*e^
6 + 2*(x*e + d)^m*a*c*g*m^5*x^4*e^6 + 16*(x*e + d)^m*c^2*f*m^4*x^5*e^6 + 32
*(x*e + d)^m*b*c*g*m^4*x^5*e^6 + 85*(x*e + d)^m*c^2*g*m^3*x^6*e^6 + 2*(x*e
+ d)^m*b*c*d*f*m^5*x^3*e^5 + (x*e + d)^m*b^2*d*g*m^5*x^3*e^5 + 2*(x*e + d)^
m*a*c*d*g*m^5*x^3*e^5 + 12*(x*e + d)^m*c^2*d*f*m^4*x^4*e^5 + 24*(x*e + d)^m
*b*c*d*g*m^4*x^4*e^5 + 35*(x*e + d)^m*c^2*d*g*m^3*x^5*e^5 - 4*(x*e + d)^m*c
^2*d^2*f*m^4*x^3*e^4 - 8*(x*e + d)^m*b*c*d^2*g*m^4*x^3*e^4 - 30*(x*e + d)^m
*c^2*d^2*g*m^3*x^4*e^4 + 20*(x*e + d)^m*c^2*d^3*g*m^3*x^3*e^3 + (x*e + d)^m
*b^2*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*c*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*b*g*
m^5*x^3*e^6 + 34*(x*e + d)^m*b*c*f*m^4*x^4*e^6 + 17*(x*e + d)^m*b^2*g*m^4*x
^4*e^6 + 34*(x*e + d)^m*a*c*g*m^4*x^4*e^6 + 95*(x*e + d)^m*c^2*f*m^3*x^5*e^
6 + 190*(x*e + d)^m*b*c*g*m^3*x^5*e^6 + 225*(x*e + d)^m*c^2*g*m^2*x^6*e^6 +
 (x*e + d)^m*b^2*d*f*m^5*x^2*e^5 + 2*(x*e + d)^m*a*c*d*f*m^5*x^2*e^5 + 2*(x
*e + d)^m*a*b*d*g*m^5*x^2*e^5 + 28*(x*e + d)^m*b*c*d*f*m^4*x^3*e^5 + 14*(x*
e + d)^m*b^2*d*g*m^4*x^3*e^5 + 28*(x*e + d)^m*a*c*d*g*m^4*x^3*e^5 + 47*(x*e
 + d)^m*c^2*d*f*m^3*x^4*e^5 + 94*(x*e + d)^m*b*c*d*g*m^3*x^4*e^5 + 50*(x*e
+ d)^m*c^2*d*g*m^2*x^5*e^5 - 6*(x*e + d)^m*b*c*d^2*f*m^4*x^2*e^4 - 3*(x*e +
 d)^m*b^2*d^2*g*m^4*x^2*e^4 - 6*(x*e + d)^m*a*c*d^2*g*m^4*x^2*e^4 - 36*(x*e
 + d)^m*c^2*d^2*f*m^3*x^3*e^4 - 72*(x*e + d)^m*b*c*d^2*g*m^3*x^3*e^4 - 55*(
x*e + d)^m*c^2*d^2*g*m^2*x^4*e^4 + 12*(x*e + d)^m*c^2*d^3*f*m^3*x^2*e^3 + 2
4*(x*e + d)^m*b*c*d^3*g*m^3*x^2*e^3 + 60*(x*e + d)^m*c^2*d^3*g*m^2*x^3*e^3
- 60*(x*e + d)^m*c^2*d^4*g*m^2*x^2*e^2 + 2*(x*e + d)^m*a*b*f*m^5*x^2*e^6 +
(x*e + d)^m*a^2*g*m^5*x^2*e^6 + 18*(x*e + d)^m*b^2*f*m^4*x^3*e^6 + 36*(x*e
+ d)^m*a*c*f*m^4*x^3*e^6 + 36*(x*e + d)^m*a*b*g*m^4*x^3*e^6 + 214*(x*e + d)
^m*b*c*f*m^3*x^4*e^6 + 107*(x*e + d)^m*b^2*g*m^3*x^4*e^6 + 214*(x*e + d)^m*
a*c*g*m^3*x^4*e^6 + 260*(x*e + d)^m*c^2*f*m^2*x^5*e^6 + 520*(x*e + d)^m*b*c
*g*m^2*x^5*e^6 + 274*(x*e + d)^m*c^2*g*m*x^6*e^6 + 2*(x*e + d)^m*a*b*d*f*m^
```

5*x*e^5 + (x*e + d)^m*a^2*d*g*m^5*x*e^5 + 16*(x*e + d)^m*b^2*d*f*m^4*x^2*e^5 + 32*(x*e + d)^m*a*c*d*f*m^4*x^2*e^5 + 32*(x*e + d)^m*a*b*d*g*m^4*x^2*e^5 + 130*(x*e + d)^m*b*c*d*f*m^3*x^3*e^5 + 65*(x*e + d)^m*b^2*d*g*m^3*x^3*e^5 + 130*(x*e + d)^m*a*c*d*g*m^3*x^3*e^5 + 72*(x*e + d)^m*c^2*d*f*m^2*x^4*e^5 + 144*(x*e + d)^m*b*c*d*g*m^2*x^4*e^5 + 24*(x*e + d)^m*c^2*d*g*m*x^5*e^5 - 2*(x*e + d)^m*b^2*d^2*f*m^4*x*e^4 - 4*(x*e + d)^m*a*c*d^2*f*m^4*x*e^4 - 4*(x*e + d)^m*a*b*d^2*g*m^4*x*e^4 - 72*(x*e + d)^m*b*c*d^2*f*m^3*x^2*e^4 - 36*(x*e + d)^m*b^2*d^2*g*m^3*x^2*e^4 - 72*(x*e + d)^m*a*c*d^2*g*m^3*x^2*e^4 - 80*(x*e + d)^m*c^2*d^2*f*m^2*x^3*e^4 - 160*(x*e + d)^m*b*c*d^2*g*m^2*x^3*e^4 - 30*(x*e + d)^m*c^2*d^2*g*m*x^4*e^4 + 12*(x*e + d)^m*b*c*d^3*f*m^3*x*e^3 + 6*(x*e + d)^m*b^2*d^3*g*m^3*x*e^3 + 12*(x*e + d)^m*a*c*d^3*g*m^3*x*e^3 + 84*(x*e + d)^m*c^2*d^3*f*m^2*x^2*e^3 + 168*(x*e + d)^m*b*c*d^3*g*m^2*x^2*e^3 + 40*(x*e + d)^m*c^2*d^3*g*m*x^3*e^3 - 24*(x*e + d)^m*c^2*d^4*f*m^2*x*e^2 - 48*(x*e + d)^m*b*c*d^4*g*m^2*x*e^2 - 60*(x*e + d)^m*c^2*d^4*g*m*x^2*e^2 + 120*(x*e + d)^m*c^2*d^5*g*m*x*e + (x*e + d)^m*a^2*f*m^5*x*e^6 + 38*(x*e + d)^m*a*b*f*m^4*x^2*e^6 + 19*(x*e + d)^m*a^2*g*m^4*x^2*e^6 + 121*(x*e + d)^m*b^2*f*m^3*x^3*e^6 + 242*(x*e + d)^m*a*c*f*m^3*x^3*e^6 + 242*(x*e + d)^m*a*b*g*m^3*x^3*e^6 + 614*(x*e + d)^m*b*c*f*m^2*x^4*e^6 + 307*(x*e + d)^m*b^2*g*m^2*x^4*e^6 + 614*(x*e + d)^m*a*c*g*m^2*x^4*e^6 + 324*(x*e + d)^m*c^2*f*m*x^5*e^6 + 648*(x*e + d)^m*b*c*g*m*x^5*e^6 + 120*(x*e + d)^m*c^2*g*x^6*e^6 + (x*e + d)^m*a^2*d*f*m^5*e^5 + 36*(x*e + d)^m*a*b*d*f*m^4*x*e^5 + 18*(x*e + d)^m*a^2*d*g*m^4*x*e^5 + 89*(x*e + d)^m*b^2*d*f*m^3*x^2*e^5 + 178*(x*e + d)^m*a*c*d*f*m^3*x^2*e^5 + 178*(x*e + d)^m*a*b*d*g*m^3*x^2*e^5 + 224*(x*e + d)^m*b*c*d*f*m^2*x^3*e^5 + 112*(x*e + d)^m*b^2*d*g*m^2*x^3*e^5 + 224*(x*e + d)^m*a*c*d*g*m^2*x^3*e^5 + 36*(x*e + d)^m*c^2*d*f*m*x^4*e^5 + 72*(x*e + d)^m*b*c*d*g*m*x^4*e^5 - 2*(x*e + d)^m*a*b*d^2*f*m^4*e^4 - (x*e + d)^m*a^2*d^2*g*m^4*e^4 - 30*(x*e + d)^m*b^2*d^2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*c*d^2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*b*d^2*g*m^3*x*e^4 - 246*(x*e + d)^m*b*c*d^2*f*m^2*x^2*e^4 - 123*(x*e + d)^m*b^2*d^2*g*m^2*x^2*e^4 - 246*(x*e + d)^m*a*c*d^2*g*m^2*x^2*e^4 - 48*(x*e + d)^m*c^2*d^2*f*m*x^3*e^4 - 96*(x*e + d)^m*b*c*d^2*g*m*x^3*e^4 + 2*(x*e + d)^m*b^2*d^3*f*m^3*e^3 + 4*(x*e + d)^m*a*c*d^3*f*m^3*e^3 + 4*(x*e + d)^m*a*b*d^3*g*m^3*e^3 + 132*(x*e + d)^m*b*c*d^3*f*m^2*x*e^3 + 66*(x*e + d)^m*b^2*d^3*g*m^2*x*e^3 + 132*(x*e + d)^m*a*c*d^3*g*m^2*x*e^3 + 72*(x*e + d)^m*c^2*d^3*f*m*x^2*e^3 + 144*(x*e + d)^m*b*c*d^3*g*m*x^2*e^3 - 12*(x*e + d)^m*b*c*d^4*f*m^2*e^2 - 6*(x*e + d)^m*b^2*d^4*g*m^2*e^2 - 12*(x*e + d)^m*a*c*d^4*g*m^2*e^2 - 144*(x*e + d)^m*c^2*d^4*f*m*x*e^2 - 288*(x*e + d)^m*b*c*d^4*g*m*x*e^2 + 24*(x*e + d)^m*c^2*d^5*f*m*e + 48*(x*e + d)^m*b*c*d^5*g*m*e - 120*(x*e + d)^m*c^2*d^6*g + 20*(x*e + d)^m*a^2*f*m^4*x*e^6 + 274*(x*e + d)^m*a*b*f*m^3*x^2*e^6 + 137*(x*e + d)^m*a^2*g*m^3*x^2*e^6 + 372*(x*e + d)^m*b^2*f*m^2*x^3*e^6 + 744*(x*e + d)^m*a*c*f*m^2*x^3*e^6 + 744*(x*e + d)^m*a*b*g*m^2*x^3*e^6 + 792*(x*e + d)^m*b*c*f*m*x^4*e^6 + 396*(x*e + d)^m*b^2*g*m*x^4*e^6 + 792*(x*e + d)^m*a*c*g*m*x^4*e^6 + 144*(x*e + d)^m*c^2*f*x^5*e^6 + 288*(x*e + d)^m*b*c*g*x^5*e^6 + 20*(x*e + d)^m*a^2*d*f*m^4*e^5 + 238*(x*e + d)^m*a*b*d*f*m^3*x*e^5 + 119*(x*e + d)^m*a^2*d*g*m^3*x*e^5 + 194*(x*e + d)^m*b^2*d*f*m^2*x^2*e^5 + 388*(x*e + d)^m*a*c*d*f*m^2*x^2*e^5 + 388*(x*e + d)^m*a*b*d*g*m^2*x^2*e^5 + 120*(x*e + d)^m*b*c*d*f*m*x^3*e^5 + 60*(x*e + d)^m*b^2*d*g*m*x^3*e^5 + 120*(x*e + d)^m*a*c*d*g*m*x^3*e^5 - 36*(x*e + d)^m*a*b*d^2*f*m^3*e^4 - 18*(x*e + d)^m*a^2*d^2*g*m^3*e^4 - 148*(x*e + d)^m*b^2*d^2*f*m^2*x*e^4 - 296*(x*e + d)^m*a*c*d^2*f*m^2*x*e^4 - 296*(x*e + d)^m*a*b*d^2*g*m^2*x*e^4 - 180*(x*e + d)^m*b*c*d^2*f*m*x^2*e^4 - 90*(x*e + d)^m*b^2*d^2*g*m*x^2*e^4 - 180*(x*e + d)^m*a*c*d^2*g*m*x^2*e^4 + 30*(x*e + d)^m*b^2*d^3*f*m^2*e^3 + 60*(x*e + d)^m*a*c*d^3*f*m^2*e^3 + 60*(x*e + d)^m*a*b*d^3*g*m^2*e^3 + 360*(x*e + d)^m*b*c*d^3*f*m*x*e^3 + 180*(x*e + d)^m*b^2*d^3*g*m*x*e^3 + 360*(x*e + d)^m*a*c*d^3*g*m*x*e^3 - 132*(x*e + d)^m*b*c*d^4*f*m*e^2 - 66*(x*e + d)^m*b^2*d^4*g*m*e^2 - 132*(x*e + d)^m*a*c*d^4*g*m*e^2 + 144*(x*e + d)^m*c^2*d^5*f*e + 288*(x*e + d)^m*b*c*d^5*g*e + 155*(x*e + d)^m*a^2*f*m^3*x*e^6 + 922*(x*e + d)^m*a*b*f*m^2*x^2*e^6 + 461*(x*e + d)^m*a^2*g*m^2*x^2*e^6 + 508*(x*e + d)^m*b^2*f*m*x^3*e^6 + 1016*(x*e + d)^m*a*c*f*m*x^3*e^6 + 1016*(x*e + d)^m*a*b*g*m*x^3*e^6 + 360*(x*e +

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d)^m*b*c*f*x^4*e^6 + 180*(x*e + d)^m*b^2*g*x^4*e^6 + 360*(x*e + d)^m*a*c*g*
x^4*e^6 + 155*(x*e + d)^m*a^2*d*f*m^3*e^5 + 684*(x*e + d)^m*a*b*d*f*m^2*x*e
^5 + 342*(x*e + d)^m*a^2*d*g*m^2*x*e^5 + 120*(x*e + d)^m*b^2*d*f*m*x^2*e^5
+ 240*(x*e + d)^m*a*c*d*f*m*x^2*e^5 + 240*(x*e + d)^m*a*b*d*g*m*x^2*e^5 - 2
38*(x*e + d)^m*a*b*d^2*f*m^2*e^4 - 119*(x*e + d)^m*a^2*d^2*g*m^2*e^4 - 240*
(x*e + d)^m*b^2*d^2*f*m*x*e^4 - 480*(x*e + d)^m*a*c*d^2*f*m*x*e^4 - 480*(x*
e + d)^m*a*b*d^2*g*m*x*e^4 + 148*(x*e + d)^m*b^2*d^3*f*m*e^3 + 296*(x*e + d
)^m*a*c*d^3*f*m*e^3 + 296*(x*e + d)^m*a*b*d^3*g*m*e^3 - 360*(x*e + d)^m*b*c
*d^4*f*e^2 - 180*(x*e + d)^m*b^2*d^4*g*e^2 - 360*(x*e + d)^m*a*c*d^4*g*e^2
+ 580*(x*e + d)^m*a^2*f*m^2*x*e^6 + 1404*(x*e + d)^m*a*b*f*m*x^2*e^6 + 702*
(x*e + d)^m*a^2*g*m*x^2*e^6 + 240*(x*e + d)^m*b^2*f*x^3*e^6 + 480*(x*e + d)
^m*a*c*f*x^3*e^6 + 480*(x*e + d)^m*a*b*g*x^3*e^6 + 580*(x*e + d)^m*a^2*d*f*
m^2*e^5 + 720*(x*e + d)^m*a*b*d*f*m*x*e^5 + 360*(x*e + d)^m*a^2*d*g*m*x*e^5
 - 684*(x*e + d)^m*a*b*d^2*f*m*e^4 - 342*(x*e + d)^m*a^2*d^2*g*m*e^4 + 240*
(x*e + d)^m*b^2*d^3*f*e^3 + 480*(x*e + d)^m*a*c*d^3*f*e^3 + 480*(x*e + d)^m
*a*b*d^3*g*e^3 + 1044*(x*e + d)^m*a^2*f*m*x*e^6 + 720*(x*e + d)^m*a*b*f*x^2
*e^6 + 360*(x*e + d)^m*a^2*g*x^2*e^6 + 1044*(x*e + d)^m*a^2*d*f*m*e^5 - 720
*(x*e + d)^m*a*b*d^2*f*e^4 - 360*(x*e + d)^m*a^2*d^2*g*e^4 + 720*(x*e + d)^
m*a^2*f*x*e^6 + 720*(x*e + d)^m*a^2*d*f*e^5)/(m^6*e^6 + 21*m^5*e^6 + 175*m^
4*e^6 + 735*m^3*e^6 + 1624*m^2*e^6 + 1764*m*e^6 + 720*e^6)
```

maple [B] time = 0.02, size = 2563, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x)

[Out] -(e*x+d)^(m+1)*(-c^2*e^5*g*m^5*x^5-2*b*c*e^5*g*m^5*x^4-c^2*e^5*f*m^5*x^4-15
*c^2*e^5*g*m^4*x^5-2*a*c*e^5*g*m^5*x^3-b^2*e^5*g*m^5*x^3-2*b*c*e^5*f*m^5*x^
3-32*b*c*e^5*g*m^4*x^4+5*c^2*d*e^4*g*m^4*x^4-16*c^2*e^5*f*m^4*x^4-85*c^2*e^
5*g*m^3*x^5-2*a*b*e^5*g*m^5*x^2-2*a*c*e^5*f*m^5*x^2-34*a*c*e^5*g*m^4*x^3-b^
2*e^5*f*m^5*x^2-17*b^2*e^5*g*m^4*x^3+8*b*c*d*e^4*g*m^4*x^3-34*b*c*e^5*f*m^4
*x^3-190*b*c*e^5*g*m^3*x^4+4*c^2*d*e^4*f*m^4*x^3+50*c^2*d*e^4*g*m^3*x^4-95*
c^2*e^5*f*m^3*x^4-225*c^2*e^5*g*m^2*x^5-a^2*e^5*g*m^5*x-2*a*b*e^5*f*m^5*x-3
6*a*b*e^5*g*m^4*x^2+6*a*c*d*e^4*g*m^4*x^2-36*a*c*e^5*f*m^4*x^2-214*a*c*e^5*
g*m^3*x^3+3*b^2*d*e^4*g*m^4*x^2-18*b^2*e^5*f*m^4*x^2-107*b^2*e^5*g*m^3*x^3+
6*b*c*d*e^4*f*m^4*x^2+96*b*c*d*e^4*g*m^3*x^3-214*b*c*e^5*f*m^3*x^3-520*b*c*
e^5*g*m^2*x^4-20*c^2*d^2*e^3*g*m^3*x^3+48*c^2*d*e^4*f*m^3*x^3+175*c^2*d*e^4
*g*m^2*x^4-260*c^2*e^5*f*m^2*x^4-274*c^2*e^5*g*m*x^5-a^2*e^5*f*m^5-19*a^2*e
^5*g*m^4*x+4*a*b*d*e^4*g*m^4*x-38*a*b*e^5*f*m^4*x-242*a*b*e^5*g*m^3*x^2+4*a
*c*d*e^4*f*m^4*x+84*a*c*d*e^4*g*m^3*x^2-242*a*c*e^5*f*m^3*x^2-614*a*c*e^5*g
*m^2*x^3+2*b^2*d*e^4*f*m^4*x+42*b^2*d*e^4*g*m^3*x^2-121*b^2*e^5*f*m^3*x^2-3
07*b^2*e^5*g*m^2*x^3-24*b*c*d^2*e^3*g*m^3*x^2+84*b*c*d*e^4*f*m^3*x^2+376*b*
c*d*e^4*g*m^2*x^3-614*b*c*e^5*f*m^2*x^3-648*b*c*e^5*g*m*x^4-12*c^2*d^2*e^3*
f*m^3*x^2-120*c^2*d^2*e^3*g*m^2*x^3+188*c^2*d*e^4*f*m^2*x^3+250*c^2*d*e^4*g
*m*x^4-324*c^2*e^5*f*m*x^4-120*c^2*e^5*g*x^5+a^2*d*e^4*g*m^4-20*a^2*e^5*f*m
^4-137*a^2*e^5*g*m^3*x+2*a*b*d*e^4*f*m^4+64*a*b*d*e^4*g*m^3*x-274*a*b*e^5*f
*m^3*x-744*a*b*e^5*g*m^2*x^2-12*a*c*d^2*e^3*g*m^3*x+64*a*c*d*e^4*f*m^3*x+39
0*a*c*d*e^4*g*m^2*x^2-744*a*c*e^5*f*m^2*x^2-792*a*c*e^5*g*m*x^3-6*b^2*d^2*e
^3*g*m^3*x+32*b^2*d*e^4*f*m^3*x+195*b^2*d*e^4*g*m^2*x^2-372*b^2*e^5*f*m^2*x
^2-396*b^2*e^5*g*m*x^3-12*b*c*d^2*e^3*f*m^3*x-216*b*c*d^2*e^3*g*m^2*x^2+390
*b*c*d*e^4*f*m^2*x^2+576*b*c*d*e^4*g*m*x^3-792*b*c*e^5*f*m*x^3-288*b*c*e^5*
g*x^4+60*c^2*d^3*e^2*g*m^2*x^2-108*c^2*d^2*e^3*f*m^2*x^2-220*c^2*d^2*e^3*g*
m*x^3+288*c^2*d*e^4*f*m*x^3+120*c^2*d*e^4*g*x^4-144*c^2*e^5*f*x^4+18*a^2*d*
e^4*g*m^3-155*a^2*e^5*f*m^3-461*a^2*e^5*g*m^2*x-4*a*b*d^2*e^3*g*m^3+36*a*b*
d*e^4*f*m^3+356*a*b*d*e^4*g*m^2*x-922*a*b*e^5*f*m^2*x-1016*a*b*e^5*g*m*x^2-
4*a*c*d^2*e^3*f*m^3-144*a*c*d^2*e^3*g*m^2*x+356*a*c*d*e^4*f*m^2*x+672*a*c*d
*e^4*g*m*x^2-1016*a*c*e^5*f*m*x^2-360*a*c*e^5*g*x^3-2*b^2*d^2*e^3*f*m^3-72*
b^2*d^2*e^3*g*m^2*x+178*b^2*d*e^4*f*m^2*x+336*b^2*d*e^4*g*m*x^2-508*b^2*e^5
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*f*m*x^2-180*b^2*e^5*g*x^3+48*b*c*d^3*e^2*g*m^2*x-144*b*c*d^2*e^3*f*m^2*x-4
80*b*c*d^2*e^3*g*m*x^2+672*b*c*d*e^4*f*m*x^2+288*b*c*d*e^4*g*x^3-360*b*c*e^
5*f*x^3+24*c^2*d^3*e^2*f*m^2*x+180*c^2*d^3*e^2*g*m*x^2-240*c^2*d^2*e^3*f*m*
x^2-120*c^2*d^2*e^3*g*x^3+144*c^2*d*e^4*f*x^3+119*a^2*d*e^4*g*m^2-580*a^2*e
^5*f*m^2-702*a^2*e^5*g*m*x-60*a*b*d^2*e^3*g*m^2+238*a*b*d*e^4*f*m^2+776*a*b
*d*e^4*g*m*x-1404*a*b*e^5*f*m*x-480*a*b*e^5*g*x^2+12*a*c*d^3*e^2*g*m^2-60*a
*c*d^2*e^3*f*m^2-492*a*c*d^2*e^3*g*m*x+776*a*c*d*e^4*f*m*x+360*a*c*d*e^4*g*
x^2-480*a*c*e^5*f*x^2+6*b^2*d^3*e^2*g*m^2-30*b^2*d^2*e^3*f*m^2-246*b^2*d^2*
e^3*g*m*x+388*b^2*d*e^4*f*m*x+180*b^2*d*e^4*g*x^2-240*b^2*e^5*f*x^2+12*b*c*
d^3*e^2*f*m^2+336*b*c*d^3*e^2*g*m*x-492*b*c*d^2*e^3*f*m*x-288*b*c*d^2*e^3*g
*x^2+360*b*c*d*e^4*f*x^2-120*c^2*d^4*e*g*m*x+168*c^2*d^3*e^2*f*m*x+120*c^2*
d^3*e^2*g*x^2-144*c^2*d^2*e^3*f*x^2+342*a^2*d*e^4*g*m-1044*a^2*e^5*f*m-360*
a^2*e^5*g*x-296*a*b*d^2*e^3*g*m+684*a*b*d*e^4*f*m+480*a*b*d*e^4*g*x-720*a*b
*e^5*f*x+132*a*c*d^3*e^2*g*m-296*a*c*d^2*e^3*f*m-360*a*c*d^2*e^3*g*x+480*a*
c*d*e^4*f*x+66*b^2*d^3*e^2*g*m-148*b^2*d^2*e^3*f*m-180*b^2*d^2*e^3*g*x+240*
b^2*d*e^4*f*x-48*b*c*d^4*e*g*m+132*b*c*d^3*e^2*f*m+288*b*c*d^3*e^2*g*x-360*
b*c*d^2*e^3*f*x-24*c^2*d^4*e*f*m-120*c^2*d^4*e*g*x+144*c^2*d^3*e^2*f*x+360*
a^2*d*e^4*g-720*a^2*e^5*f-480*a*b*d^2*e^3*g+720*a*b*d*e^4*f+360*a*c*d^3*e^2
*g-480*a*c*d^2*e^3*f+180*b^2*d^3*e^2*g-240*b^2*d^2*e^3*f-288*b*c*d^4*e*g+36
0*b*c*d^3*e^2*f+120*c^2*d^5*g-144*c^2*d^4*e*f)/e^6/(m^6+21*m^5+175*m^4+735*
m^3+1624*m^2+1764*m+720)
```

maxima [B] time = 0.67, size = 1118, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f/((m^2 + 3*m + 2)*e^2)
 + (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a^2*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 +
 m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f/((m^3 + 6*m^2 + 11*m
 + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x
 + 2*d^3)*(e*x + d)^m*a*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m
+ 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*b*g
/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^
3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)
*(e*x + d)^m*b*c*f/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^3 + 6*m^
2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2
*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*g/((m^4 + 10*m^3 + 35*m^2 + 50*
m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*
e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*g/
((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2
*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x +
d)^m*c^2*f/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*((m^4
+ 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4
*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^
4*e*m*x + 24*d^5)*(e*x + d)^m*b*c*g/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274
*m + 120)*e^5) + ((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 +
 (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^
2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^
4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*g/((m^6 + 21*m^5 + 175
*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6)
```

mupad [B] time = 4.38, size = 2307, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(240*b^2*d^3*e^3*f - 360*a^2*d^2*e^4*g - 120*c^2*d^6*g - 180*b
^2*d^4*e^2*g + 720*a^2*d*e^5*f + 144*c^2*d^5*e*f - 720*a*b*d^2*e^4*f + 480*
a*b*d^3*e^3*g + 480*a*c*d^3*e^3*f - 360*a*c*d^4*e^2*g - 360*b*c*d^4*e^2*f +
 1044*a^2*d*e^5*f*m + 24*c^2*d^5*e*f*m + 580*a^2*d*e^5*f*m^2 + 155*a^2*d*e^
5*f*m^3 + 20*a^2*d*e^5*f*m^4 + a^2*d*e^5*f*m^5 - 342*a^2*d^2*e^4*g*m + 148*
b^2*d^3*e^3*f*m - 66*b^2*d^4*e^2*g*m + 288*b*c*d^5*e*g - 119*a^2*d^2*e^4*g*
m^2 + 30*b^2*d^3*e^3*f*m^2 - 18*a^2*d^2*e^4*g*m^3 + 2*b^2*d^3*e^3*f*m^3 - a
^2*d^2*e^4*g*m^4 - 6*b^2*d^4*e^2*g*m^2 + 48*b*c*d^5*e*g*m - 684*a*b*d^2*e^4
*f*m + 296*a*b*d^3*e^3*g*m + 296*a*c*d^3*e^3*f*m - 132*a*c*d^4*e^2*g*m - 13
2*b*c*d^4*e^2*f*m - 238*a*b*d^2*e^4*f*m^2 - 36*a*b*d^2*e^4*f*m^3 - 2*a*b*d^
2*e^4*f*m^4 + 60*a*b*d^3*e^3*g*m^2 + 60*a*c*d^3*e^3*f*m^2 + 4*a*b*d^3*e^3*g
*m^3 + 4*a*c*d^3*e^3*f*m^3 - 12*a*c*d^4*e^2*g*m^2 - 12*b*c*d^4*e^2*f*m^2))/
(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x*(d
+ e*x)^m*(720*a^2*e^6*f + 580*a^2*e^6*f*m^2 + 155*a^2*e^6*f*m^3 + 20*a^2*e^
6*f*m^4 + a^2*e^6*f*m^5 + 1044*a^2*e^6*f*m + 360*a^2*d*e^5*g*m + 120*c^2*d^
5*e*g*m - 240*b^2*d^2*e^4*f*m + 342*a^2*d*e^5*g*m^2 + 119*a^2*d*e^5*g*m^3 +
 18*a^2*d*e^5*g*m^4 + a^2*d*e^5*g*m^5 + 180*b^2*d^3*e^3*g*m - 144*c^2*d^4*e
^2*f*m - 148*b^2*d^2*e^4*f*m^2 - 30*b^2*d^2*e^4*f*m^3 - 2*b^2*d^2*e^4*f*m^4
 + 66*b^2*d^3*e^3*g*m^2 - 24*c^2*d^4*e^2*f*m^2 + 6*b^2*d^3*e^3*g*m^3 + 720*
a*b*d*e^5*f*m + 684*a*b*d*e^5*f*m^2 + 238*a*b*d*e^5*f*m^3 + 36*a*b*d*e^5*f*
m^4 + 2*a*b*d*e^5*f*m^5 - 480*a*b*d^2*e^4*g*m - 480*a*c*d^2*e^4*f*m + 360*a
*c*d^3*e^3*g*m + 360*b*c*d^3*e^3*f*m - 288*b*c*d^4*e^2*g*m - 296*a*b*d^2*e^
4*g*m^2 - 296*a*c*d^2*e^4*f*m^2 - 60*a*b*d^2*e^4*g*m^3 - 60*a*c*d^2*e^4*f*m
^3 - 4*a*b*d^2*e^4*g*m^4 - 4*a*c*d^2*e^4*f*m^4 + 132*a*c*d^3*e^3*g*m^2 + 13
2*b*c*d^3*e^3*f*m^2 + 12*a*c*d^3*e^3*g*m^3 + 12*b*c*d^3*e^3*f*m^3 - 48*b*c*
d^4*e^2*g*m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
+ 720)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(120*b^2*e^3*f + 15*b^2*e^3*f*m^
2 + b^2*e^3*f*m^3 + 240*a*b*e^3*g + 240*a*c*e^3*f + 74*b^2*e^3*f*m + 20*c^2
*d^3*g*m + 30*a*b*e^3*g*m^2 + 30*a*c*e^3*f*m^2 + 2*a*b*e^3*g*m^3 + 2*a*c*e^
3*f*m^3 + 30*b^2*d*e^2*g*m - 24*c^2*d^2*e*f*m + 11*b^2*d*e^2*g*m^2 - 4*c^2*
d^2*e*f*m^2 + b^2*d*e^2*g*m^3 + 148*a*b*e^3*g*m + 148*a*c*e^3*f*m + 60*a*c*
d*e^2*g*m + 60*b*c*d*e^2*f*m - 48*b*c*d^2*e*g*m + 22*a*c*d*e^2*g*m^2 + 22*b
*c*d*e^2*f*m^2 + 2*a*c*d*e^2*g*m^3 + 2*b*c*d*e^2*f*m^3 - 8*b*c*d^2*e*g*m^2)
)/(e^3*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x^4
*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*b^2*e^2*g + b^2*e^2*g*m^2 + 60*a*
c*e^2*g + 60*b*c*e^2*f + 11*b^2*e^2*g*m - 5*c^2*d^2*g*m + 2*a*c*e^2*g*m^2 +
 2*b*c*e^2*f*m^2 + c^2*d*e*f*m^2 + 22*a*c*e^2*g*m + 22*b*c*e^2*f*m + 6*c^2*
d*e*f*m + 2*b*c*d*e*g*m^2 + 12*b*c*d*e*g*m))/(e^2*(1764*m + 1624*m^2 + 735*
m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (c^2*g*x^6*(d + e*x)^m*(274*m + 225*
m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4
+ 21*m^5 + m^6 + 720) + (x^2*(m + 1)*(d + e*x)^m*(360*a^2*e^4*g + 119*a^2*e
^4*g*m^2 + 18*a^2*e^4*g*m^3 + a^2*e^4*g*m^4 + 720*a*b*e^4*f + 342*a^2*e^4*g
*m - 60*c^2*d^4*g*m + 238*a*b*e^4*f*m^2 + 36*a*b*e^4*f*m^3 + 2*a*b*e^4*f*m^
4 + 120*b^2*d*e^3*f*m + 72*c^2*d^3*e*f*m + 74*b^2*d*e^3*f*m^2 + 15*b^2*d*e^
3*f*m^3 + b^2*d*e^3*f*m^4 - 90*b^2*d^2*e^2*g*m + 12*c^2*d^3*e*f*m^2 + 684*a
*b*e^4*f*m - 33*b^2*d^2*e^2*g*m^2 - 3*b^2*d^2*e^2*g*m^3 + 240*a*b*d*e^3*g*m
 + 240*a*c*d*e^3*f*m + 144*b*c*d^3*e*g*m + 148*a*b*d*e^3*g*m^2 + 148*a*c*d*
e^3*f*m^2 + 30*a*b*d*e^3*g*m^3 + 30*a*c*d*e^3*f*m^3 + 2*a*b*d*e^3*g*m^4 + 2
*a*c*d*e^3*f*m^4 - 180*a*c*d^2*e^2*g*m - 180*b*c*d^2*e^2*f*m + 24*b*c*d^3*e
*g*m^2 - 66*a*c*d^2*e^2*g*m^2 - 66*b*c*d^2*e^2*f*m^2 - 6*a*c*d^2*e^2*g*m^3
- 6*b*c*d^2*e^2*f*m^3))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^
5 + m^6 + 720)) + (c*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(1
2*b*e*g + 6*c*e*f + 2*b*e*g*m + c*d*g*m + c*e*f*m))/(e*(1764*m + 1624*m^2 +
 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.927 $\int \frac{(d+ex)^m(a+bx+cx^2)^2}{f+gx}\,dx$

Optimal. Leaf size=287

$$\frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4g^3(m+2)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(\right.}{e^4g^4}$$

```
[Out] (b*e*g-c*(d*g+e*f))*(c*(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*(e*x+d)
^(1+m)/e^4/g^4/(1+m)+(b^2*e^2*g^2+c^2*(3*d^2*g^2+2*d*e*f*g+e^2*f^2)+2*c*e*g
*(a*e*g-b*(2*d*g+e*f)))*(e*x+d)^(2+m)/e^4/g^3/(2+m)-c*(-2*b*e*g+3*c*d*g+c*e
*f)*(e*x+d)^(3+m)/e^4/g^2/(3+m)+c^2*(e*x+d)^(4+m)/e^4/g/(4+m)+(a*g^2-b*f*g+
c*f^2)^2*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^4/
(-d*g+e*f)/(1+m)
```

Rubi [A] time = 0.87, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {951, 1620, 68}

$$\frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4g^3(m+2)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(\right.}{e^4g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((b*e*g - c*(e*f + d*g))*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f + d
*g)))*(d + e*x)^(1 + m))/(e^4*g^4*(1 + m)) + ((b^2*e^2*g^2 + c^2*(e^2*f^2 +
 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x)^(2 +
 m))/(e^4*g^3*(2 + m)) - (c*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^(3 + m))/
(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2 - b*f
*g + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d
+ e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{f+gx}\,dx = \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int \frac{(d+ex)^m\left(-e\left(c^2d^3f-a^2e^3g\right)(4+m)+e\left(2abe^3g-c^2d^2(3ef+dg)\right)(4+m)x+e^2\left(b^2e \cdots\right.\right.}{f+gx}}{e^4g(4+ \cdots}$$

$$= \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int\left(\frac{e(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(4+m)(d+ex)^m}{g^3} + \frac{e(\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+m}}{e^4g^4(1+m)} + \cdots$$

$$= \frac{(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+m}}{e^4g^4(1+m)} + \cdots$$

Mathematica [A] time = 0.41, size = 265, normalized size = 0.92

$$\frac{(d+ex)^{m+1}\left(\frac{g(d+ex)\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4(m+2)} - \frac{(-beg+cdg+cef)\left(eg(2aeg-b(dg+ef))+c\left(d^2g^2+e^2f^2\right)\right)}{e^4(m+1)} + \frac{(g(\cdots}{\cdots}\right.}{g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g
*(2*a*e*g - b*(e*f + d*g))))/(e^4*(1 + m))) + (g*(b^2*e^2*g^2 + c^2*(e^2*f^
2 + 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x))/
(e^4*(2 + m)) - (c*g^2*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^2)/(e^4*(3 + m
)) + (c^2*g^3*(d + e*x)^3)/(e^4*(4 + m)) + ((c*f^2 + g*(-(b*f) + a*g))^2*Hy
pergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g
)*(1 + m))))/g^4
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)(ex+d)^m}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+bx+a\right)^{2} \left(ex+d\right)^{m}}{gx+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}+bx+a\right)^{2} \left(ex+d\right)^{m}}{gx+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d+e\,x\right)^{m} \left(c\,x^{2}+b\,x+a\right)^{2}}{f+g\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^{m} \left(a+bx+cx^{2}\right)^{2}}{f+gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f),x)
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x), x)
```

3.928 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx$

Optimal. Leaf size=298

$$\frac{(d+ex)^{m+1}\left(2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)\right)}{e^3g^4(m+1)}+\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)}{}$$

```
[Out] (b^2*e^2*g^2+c^2*(d^2*g^2+2*d*e*f*g+3*e^2*f^2)+2*c*e*g*(a*e*g-b*(d*g+2*e*f)
))*(e*x+d)^(1+m)/e^3/g^4/(1+m)-2*c*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(2+m)/e^3/g
^3/(2+m)+c^2*(e*x+d)^(3+m)/e^3/g^2/(3+m)+(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m
)/g^4/(-d*g+e*f)/(g*x+f)+(a*g^2-b*f*g+c*f^2)*(c*f*(4*d*g-e*f*(4+m))-g*(a*e*
g*m+b*(2*d*g-e*f*(2+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)
/(-d*g+e*f))/g^4/(-d*g+e*f)^2/(1+m)
```

Rubi [A] time = 1.14, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {949, 1620, 68}

$$\frac{(d+ex)^{m+1}\left(2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)\right)}{e^3g^4(m+1)}-\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*c*e*g*(a*e*g - b*
(2*e*f + d*g)))*(d + e*x)^(1 + m))/(e^3*g^4*(1 + m)) - (2*c*(c*e*f + c*d*g
- b*e*g)*(d + e*x)^(2 + m))/(e^3*g^3*(2 + m)) + (c^2*(d + e*x)^(3 + m))/(e^
3*g^2*(3 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f -
d*g)*(f + g*x)) - ((c*f^2 - b*f*g + a*g^2)*(g*(2*b*d*g + a*e*g*m - b*e*f*(2
 + m)) - c*f*(4*d*g - e*f*(4 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)^2*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx = \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\int \frac{(d+ex)^m\left(\frac{c^2f^3(dg-ef(1+m))-2cfg(bf-ag)(dg-ef(1+m))-g^2(a^2}{g^4}\right.}{}}{}$$

$$= \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\int\left(\frac{(ef-dg)\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2ceg(aeg-b\right.}{e^2g^4}\right.}{}$$

$$= \frac{\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2ceg(aeg-b(2ef+dg))\right)(d+ex)^{1+m}}{e^3g^4(1+m)} -$$

$$= \frac{\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2ceg(aeg-b(2ef+dg))\right)(d+ex)^{1+m}}{e^3g^4(1+m)} -$$

Mathematica [A] time = 0.39, size = 261, normalized size = 0.88

$$\frac{(d+ex)^{m+1}\left(\frac{2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)}{e^3(m+1)} + \frac{e\left(g(ag-bf)+cf^2\right)^2 {}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)^2} - \frac{2(2cf-bg)\left(g(ag-bf)+cf^2\right.}{(m+1)}\right.}{g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*c*e*g*(a*e*g - b*(2*e*f + d*g)))/(e^3*(1 + m)) - (2*c*g*(c*e*f + c*d*g -
b*e*g)*(d + e*x))/(e^3*(2 + m)) + (c^2*g^2*(d + e*x)^2)/(e^3*(3 + m)) - (2*
(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2 + m,
 (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m)) + (e*(c*f^2 + g*(-(b*
f) + a*g))^2*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g
)])/((e*f - d*g)^2*(1 + m))))/g^4
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)(ex+d)^m}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+bx+a\right)^{2}\left(ex+d\right)^{m}}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}+bx+a\right)^{2}\left(ex+d\right)^{m}}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d+e\,x\right)^{m}\,\left(c\,x^{2}+b\,x+a\right)^{2}}{\left(f+g\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.929 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$

Optimal. Leaf size=461

$$(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(m+1))-\left(b^2\left(2d^2g^2-4def\right.\right.\right.\right.$$

```
[Out] -c*(-2*b*e*g+c*d*g+3*c*e*f)*(e*x+d)^(1+m)/e^2/g^4/(1+m)+c^2*(e*x+d)^(2+m)/e
^2/g^3/(2+m)+1/2*(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m)/g^4/(-d*g+e*f)/(g*x+f)
^2+1/2*(a*g^2-b*f*g+c*f^2)*(c*f*(8*d*g-e*f*(7+m))+g*(a*e*g*(1-m)-b*(4*d*g-e
*f*(3+m))))*(e*x+d)^(1+m)/g^4/(-d*g+e*f)^2/(g*x+f)+1/2*(c^2*f^2*(12*d^2*g^2
-8*d*e*f*g*(3+m)+e^2*f^2*(m^2+7*m+12))-g^2*(a^2*e^2*g^2*(1-m)*m-2*a*b*e*g*m
*(2*d*g-e*f*(1+m))-b^2*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2)))+2*c
*g*(a*g*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-b*f*(6*d^2*g^2-6*d*
e*f*g*(2+m)+e^2*f^2*(m^2+5*m+6))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-
g*(e*x+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^3/(1+m)
```

Rubi [A] time = 1.49, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {949, 1621, 951, 80, 68}

$$(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(m+1))+b^2\left(-\left(2d^2g^2-4d\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] -((c*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^(1 + m))/(e^2*g^4*(1 + m))) + (c
^2*(d + e*x)^(2 + m))/(e^2*g^3*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e
*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)^2) - (((c*f^2 - b*f*g + a*g^2)*(g*
(4*b*d*g - a*e*g*(1 - m) - b*e*f*(3 + m)) - c*f*(8*d*g - e*f*(7 + m)))*(d +
 e*x)^(1 + m))/(2*g^4*(e*f - d*g)^2*(f + g*x)) + ((c^2*f^2*(12*d^2*g^2 - 8*
d*e*f*g*(3 + m) + e^2*f^2*(12 + 7*m + m^2)) - g^2*(a^2*e^2*g^2*(1 - m)*m -
2*a*b*e*g*m*(2*d*g - e*f*(1 + m)) - b^2*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^
2*f^2*(2 + 3*m + m^2))) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f
^2*(2 + 3*m + m^2)) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2 + m) + e^2*f^2*(6 + 5*m
 + m^2))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e
*x))/(e*f - d*g))])/(2*g^4*(e*f - d*g)^3*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx = \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m\left(\frac{c^2f^3(2dg-ef(1+m))-2cfg(bf-ag)(2dg-ef(1+m))}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{\left(cf^2-bfg+ag^2\right)(g(4bdg-aeg(1-m)\cdots}{2g^4(ef\cdots}$$

$$= \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{\left(cf^2-bfg+ag^2\right)(g(4b\cdots}{\cdots}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(\cdots}{2g^4(ef-dg)(f+\cdots}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(\cdots}{2g^4(ef-dg)(f+\cdots}$$

Mathematica [A] time = 0.34, size = 257, normalized size = 0.56

$$\frac{(d+ex)^{m+1}\left(\frac{\left(2cg(ag-3bf)+b^2g^2+6c^2f^2\right)\,_2F_1\left(1,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)} + \frac{e^2\left(g(ag-bf)+cf^2\right)^2\,_2F_1\left(3,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)^3} - \frac{2e(2cf-bg)\left(g(ag-bf)\cdots}{(m\cdots}\right.}{g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*(-((c*(3*c*e*f + c*d*g - 2*b*e*g))/(e^2*(1 + m))) + (c^2
*g*(d + e*x))/(e^2*(2 + m)) + ((6*c^2*f^2 + b^2*g^2 + 2*c*g*(-3*b*f + a*g))
*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f -
d*g)*(1 + m)) - (2*e*(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometri
c2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^2*(1 + m)
) + (e^2*(c*f^2 + g*(-(b*f) + a*g))^2*Hypergeometric2F1[3, 1 + m, 2 + m, (g
*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^3*(1 + m))))/g^4
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2x^4 + 2\,bcx^3 + 2\,abx + \left(b^2 + 2\,ac\right)x^2 + a^2\right)(ex+d)^m}{g^3x^3 + 3\,fg^2x^2 + 3\,f^2gx + f^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^2 (ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^2 + bx + a\right)^2 (ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^2 (ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m \left(c\,x^2 + b\,x + a\right)^2}{\left(f + g\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x)
```

```
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{\left(f+gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x)**3, x)
```

3.930 $\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=183

$$-\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1\right.}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] 3687/64*(1+4*x)^(1+m)/(1+m)+207/32*(1+4*x)^(2+m)/(2+m)+27/64*(1+4*x)^(3+m)/
(3+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2))
)*(5499-1631*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([
1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(5499+1631*13^(1/2))/(1+m)/(13+2*1
3^(1/2))
```

Rubi [A] time = 0.23, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1628, 68}

$$-\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1\right.}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3687*(1 + 4*x)^(1 + m))/(64*(1 + m)) + (207*(1 + 4*x)^(2 + m))/(32*(2 + m)
) + (27*(1 + 4*x)^(3 + m))/(64*(3 + m)) - (3*(5499 - 1631*Sqrt[13])*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13
])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(5499 + 1631*Sqrt[13])*(1 + 4*x)^(
1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])
/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx &= \int \left(\frac{3687}{16}(1+4x)^m+\frac{207}{8}(1+4x)^{1+m}+\frac{27}{16}(1+4x)^{2+m}+\frac{\left(1269+\frac{4893}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\right.\\
&= \frac{3687(1+4x)^{1+m}}{64(1+m)}+\frac{207(1+4x)^{2+m}}{32(2+m)}+\frac{27(1+4x)^{3+m}}{64(3+m)}+\frac{1}{13}\left(3\left(5499-1631\sqrt{13}\right)\right)\int\\
&= \frac{3687(1+4x)^{1+m}}{64(1+m)}+\frac{207(1+4x)^{2+m}}{32(2+m)}+\frac{27(1+4x)^{3+m}}{64(3+m)}-\frac{3\left(5499-1631\sqrt{13}\right)(1+4x\right.}{26\left(13-\right.}\end{aligned}$$

Mathematica [A] time = 0.33, size = 151, normalized size = 0.83

$$\frac{3}{832}(4x+1)^{m+1}\left(-\frac{32\left(1631\sqrt{13}-5499\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)}-\frac{32\left(5499+1631\sqrt{13}\right)\,{}_2F_1\left(1,m+1;\right.}{\left(13+2\sqrt{13}\right)(m+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*(15977/(1 + m) + (1794*(1 + 4*x))/(2 + m) + (117*(1 +
4*x)^2)/(3 + m) - (32*(-5499 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (32*(54
99 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*S
qrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/832
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(81\,x^{4}+216\,x^{3}+216\,x^{2}+96\,x+16\right)(4\,x+1)^{m}}{3\,x^{2}-5\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(3*x^2 - 5*x
+ 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(4\,x+1)^{m}(3\,x+2)^{4}}{3\,x^{2}-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int\frac{(3x+2)^{4}\,(4x+1)^{m}}{3x^{2}-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(4\,x+1)^{m}(3\,x+2)^{4}}{3\,x^{2}-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x+2)^4 (4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

```
[Out] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^4 (4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.931 $\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx$

Optimal. Leaf size=165

$$-\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;r\right.}{13\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] 123/16*(1+4*x)^(1+m)/(1+m)+9/16*(1+4*x)^(2+m)/(2+m)-3/13*(1+4*x)^(1+m)*hype
rgeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(416-135*13^(1/2))/(1+m)/(1
3-2*13^(1/2))-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*1
3^(1/2)))*(416+135*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

Rubi [A] time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1628, 68}

$$-\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;r\right.}{13\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (123*(1 + 4*x)^(1 + m))/(16*(1 + m)) + (9*(1 + 4*x)^(2 + m))/(16*(2 + m)) -
 (3*(416 - 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3*(4
16 + 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx &= \int \left(\frac{123}{4}(1+4x)^m+\frac{9}{4}(1+4x)^{1+m}+\frac{\left(192+\frac{810}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(192-\frac{810}{\sqrt{13}}\right)(1+4\right.}{-5+\sqrt{13}+6x}\\
&= \frac{123(1+4x)^{1+m}}{16(1+m)}+\frac{9(1+4x)^{2+m}}{16(2+m)}+\frac{1}{13}\left(6\left(416-135\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x} dx\\
&= \frac{123(1+4x)^{1+m}}{16(1+m)}+\frac{9(1+4x)^{2+m}}{16(2+m)}-\frac{3\left(416-135\sqrt{13}\right)(1+4x)^{1+m}{}_2F_1\left(1,1+m;2\right.}{13\left(13-2\sqrt{13}\right)(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.71

$$\frac{(4x+1)^{m+1}\left(16\left(71\sqrt{13}-146\right)(m+2)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)-16\left(146+71\sqrt{13}\right)(m+2)\,{}_2F_1\left(1,m+1;\right.\right.}{624\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(117*(85 + 12*x + 4*m*(11 + 3*x)) + 16*(-146 + 71*Sqrt[1
3])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
] - 16*(146 + 71*Sqrt[13])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 +
12*x)/(13 + 2*Sqrt[13])]))/(624*(2 + 3*m + m^2))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(27\,x^3+54\,x^2+36\,x+8\right)(4\,x+1)^m}{3\,x^2-5\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\frac{(3x+2)^3\,(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^3*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((3*x+2)^3*(4*x+1)^m/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(4\,x+1)^m(3\,x+2)^3}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(3\,x+2)^3\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

```
[Out] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^3 (4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.932 $\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=147

$$-\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2,\right.}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] 3/4*(1+4*x)^(1+m)/(1+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(117-47*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1
+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117+47*13^(1/2))/(
1+m)/(13+2*13^(1/2))
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1628, 68}

$$-\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2,\right.}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m))/(4*(1 + m)) - (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*
(13 - 2*Sqrt[13])*(1 + m)) - (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2
*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx &= \int \left(3(1+4x)^m+\frac{\left(27+\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(27-\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx\\
&= \frac{3(1+4x)^{1+m}}{4(1+m)}+\frac{1}{13}\left(3\left(117-47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx+\frac{1}{13}\left(3\left(117+47\sqrt{13}\right)\right.\\
&= \frac{3(1+4x)^{1+m}}{4(1+m)}-\frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}-\frac{3(117}{}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 91, normalized size = 0.62

$$\frac{(4x+1)^{m+1}\left(\left(58\sqrt{13}-46\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)-2\left(23+29\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)+\right.}{156(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(117 + (-46 + 58*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] - 2*(23 + 29*Sqrt[13])*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(156*(1 + m))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(9\,x^2+12\,x+4\right)(4\,x+1)^m}{3\,x^2-5\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^2\,(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^2*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((3*x+2)^2*(4*x+1)^m/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^2\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

```
[Out] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^2 (4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.933 $\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx$

Optimal. Leaf size=129

$$-\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] -3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13
-9*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2
+m],3*(1+4*x)/(13+2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

Rubi [A] time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {830, 68}

$$-\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (-3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13 +
 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4
*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx &= \int \left(\frac{\left(3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(3-\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx\\ &= \frac{1}{13}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx+\frac{1}{13}\left(3\left(13+9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx\\ &= -\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}-\frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.69

$$\frac{(4x+1)^{m+1}\left(\left(5+7\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)+\left(5-7\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)\right)}{78(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5 + 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
+ 12*x)/(13 - 2*Sqrt[13])] + (5 - 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(78*(1 + m))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(4x+1)^m(3x+2)}{3x^2-5x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((3*x+2)*(4*x+1)^m/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x+2)\,(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

```
[Out] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.934 $\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=117

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] 3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))/(1+m
)/(13-2*13^(1/2))*13^(1/2)-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1
+4*x)/(13+2*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {711, 68}

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 -
 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(1 + 4*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[
13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx &= \int \left(-\frac{6(1+4x)^m}{\sqrt{13}\left(5+\sqrt{13}-6x\right)}-\frac{6(1+4x)^m}{\sqrt{13}\left(-5+\sqrt{13}+6x\right)}\right)dx\\ &= -\frac{6\int \frac{(1+4x)^m}{5+\sqrt{13}-6x}\,dx}{\sqrt{13}}-\frac{6\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{\sqrt{13}}\\ &= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}-\frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

Mathematica [A] time = 0.12, size = 94, normalized size = 0.80

$$\frac{(4x+1)^{m+1}\left(\left(13+2\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)+\left(2\sqrt{13}-13\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)\right)}{39\sqrt{13}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])] + (-13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m
, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(39*Sqrt[13]*(1 + m))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(4x+1)^m}{3x^2-5x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((4*x+1)^m/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/(3*x^2 - 5*x + 1),x)
```

```
[Out] int((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

3.935 $\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)}\,dx$

Optimal. Leaf size=164

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{85(m+1)}+\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}+\frac{3}{}$$

```
[Out] 3/85*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/442*(1+4*x
)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))
/(1+m)/(13+2*13^(1/2))+3/442*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {960, 68, 830}

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{85(m+1)}+\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/
(85*(1 + m)) + (3*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(442*(13 - 2*Sqrt[13])*(1 +
 m)) + (3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(442*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)}\,dx = \int \left(\frac{3(1+4x)^m}{17(2+3x)} + \frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)}\right) dx$$
$$= \frac{1}{17}\int \frac{(7-3x)(1+4x)^m}{1-5x+3x^2}\,dx + \frac{3}{17}\int \frac{(1+4x)^m}{2+3x}\,dx$$
$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)} + \frac{1}{17}\int \left(\frac{\left(-3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}\right.$$
$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)} - \frac{1}{221}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+}{-5-}$$
$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)} + \frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1}{442\left(13-2\sqrt{1}\right.}$$

Mathematica [A] time = 0.19, size = 110, normalized size = 0.67

$$\frac{(4x+1)^{m+1}\left(234\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)+5\left(31+11\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)+5\left(31-11\right.\right.}{6630(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(234*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5
] + 5*(31 + 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
- 2*Sqrt[13])] + 5*(31 - 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])]))/(6630*(1 + m))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(4\,x+1)^m}{9\,x^3-9\,x^2-7\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(9*x^3 - 9*x^2 - 7*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x+2)/(3*x^2-5*x+1),x)
```

```
[Out] int((4*x+1)^m/(3*x+2)/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4\,x+1)^m}{(3\,x+2)\,\left(3\,x^2-5\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)),x)
```

```
[Out] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)
```

```
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)), x)
```

3.936 $\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)} dx$

Optimal. Leaf size=199

$$\frac{27(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{3}{}$$

```
[Out] 27/1445*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+12/425*(1
+4*x)^(1+m)*hypergeom([2, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/7514*(1+4*x)^(1+m
)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-47*13^(1/2))/(1+
m)/(13+2*13^(1/2))+3/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/
(13-2*13^(1/2)))*(117+47*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

Rubi [A] time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {960, 68, 830}

$$\frac{27(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (27*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])
/(1445*(1 + m)) + (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13]
)*(1 + m)) + (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1
+ m)) + (12*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5])/(425*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx = \int \left(\frac{3(1+4x)^m}{17(2+3x)^2}+\frac{27(1+4x)^m}{289(2+3x)}+\frac{(46-27x)(1+4x)^m}{289(1-5x+3x^2)}\right)dx$$
$$= \frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{1-5x+3x^2}\,dx+\frac{27}{289}\int \frac{(1+4x)^m}{2+3x}\,dx+\frac{3}{17}\int \frac{(1+4x)^m}{(2+3x)^2}\,dx$$
$$= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+\right.}{425(1+}$$
$$= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+\right.}{425(1+}$$
$$= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{3\left(117+47\sqrt{13}\right)(1+4x)}{7514\left(13\right.}$$

Mathematica [A] time = 0.16, size = 152, normalized size = 0.76

$$\frac{(4x+1)^{m+1}\left(10530\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)+25\left(211+65\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)-16\right.}{56}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(10530*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))
/5] + 25*(211 + 65*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/
(13 - 2*Sqrt[13])] + 5275*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
 + 2*Sqrt[13])] - 1625*Sqrt[13]*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*
x)/(13 + 2*Sqrt[13])] + 15912*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5]))/(563550*(1 + m))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(4\,x+1)^m}{27\,x^4-9\,x^3-39\,x^2-8\,x+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(27*x^4 - 9*x^3 - 39*x^2 - 8*x + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)(3\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)^2\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x+2)^2/(3*x^2-5*x+1),x)
[Out] int((4*x+1)^m/(3*x+2)^2/(3*x^2-5*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4x+1)^m}{(3x+2)^2\,(3x^2-5x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)),x)
[Out] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)^2\,(3x^2-5x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)), x)
```

3.937 $\int \frac{(2+3x)^4(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(13689-\sqrt{13}\left(-1570\sqrt{13}\,m+4474m+297\right)\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(m+1)}-\frac{\left(\sqrt{13}\left(1570\sqrt{13}\right.\right.}{}$$

```
[Out] 9/4*(1+4*x)^(1+m)/(1+m)+1/39*(844-2355*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-1/169
*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13689-1
3^(1/2)*(297+4474*m-1570*m*13^(1/2)))/(1+m)/(13-2*13^(1/2))-1/169*(1+4*x)^(
1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13689+13^(1/2)*(2
97+4474*m+1570*m*13^(1/2)))/(1+m)/(13+2*13^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1648, 1628, 68}

$$-\frac{\left(13689-\sqrt{13}\left(-1570\sqrt{13}\,m+4474m+297\right)\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(m+1)}-\frac{\left(\sqrt{13}\left(1570\sqrt{13}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] (9*(1 + 4*x)^(1 + m))/(4*(1 + m)) + ((844 - 2355*x)*(1 + 4*x)^(1 + m))/(39*
(1 - 5*x + 3*x^2)) - ((13689 - Sqrt[13]*(297 + 4474*m - 1570*Sqrt[13]*m))*(
1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*S
qrt[13])])/(169*(13 - 2*Sqrt[13])*(1 + m)) - ((13689 + Sqrt[13]*(297 + 4474
*m + 1570*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(169*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
```

```
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2+3x)^4(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m\left(13(4617+3376m)-39(1521+3140\right.}{1-5x+3x^2}$$
$$= \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int\left(-4563(1+4x)^m + \frac{\left(-82134-122460m-6\sqrt{13}\right.}{-5-\sqrt{1}}\right.$$
$$= \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\left(-82134-122460m+6\sqrt{13}\,(297\right.$$
$$= \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(13689-\sqrt{13}\left(297+4474m-1570\sqrt{1}\right.\right.}{169\,(13}$$

Mathematica [A] time = 0.44, size = 251, normalized size = 1.24

$$\frac{(4x+1)^{m+1}\left(-\frac{1053\left(128\sqrt{13}-117\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{1053\left(117+128\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(m+1)} - \frac{\left(2\left(5731+667\sqrt{13}\right)m-146\right.}{}\right.}{1521}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(13689/(4 + 4*m) + (39*(844 - 2355*x))/(1 - 5*x + 3*x^2)
 - (1053*(-117 + 128*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x
)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (1053*(117 + 128*Sqrt[
13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13
 + 2*Sqrt[13])*(1 + m)) - (-((-14679*(2 + Sqrt[13]) + 2*(-5731 + 667*Sqrt[1
3])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])]) +
(-14679*(-2 + Sqrt[13]) + 2*(5731 + 667*Sqrt[13])*m)*Hypergeometric2F1[1, 1
 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(1 + m)))/1521
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(81\,x^4+216\,x^3+216\,x^2+96\,x+16\right)(4\,x+1)^m}{9\,x^4-30\,x^3+31\,x^2-10\,x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(9*x^4 - 30*x
^3 + 31*x^2 - 10*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m(3\,x+2)^4}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^{4} (4x+1)^{m}}{\left(3x^{2}-5x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^4*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
[Out] int((3*x+2)^4*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^{m}(3\,x+2)^{4}}{\left(3\,x^{2}-5\,x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x+2)^{4}\,(4\,x+1)^{m}}{\left(3\,x^{2}-5\,x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^{4} (4x+1)^{m}}{\left(3x^{2}-5x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.938 $\int \frac{(2+3x)^3(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=181

$$-\frac{\left(\sqrt{13}\left(568\sqrt{13}m-1168m+1701\right)+1521\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\sqrt{13}(1701-1168m)-\right.}{}$$

```
[Out] 1/39*(209-426*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/338*(1+4*x)^(1+m)*hypergeom(
[1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(-1521-7384*m+(1701-1168*m)*13^(1
/2))/(1+m)/(13+2*13^(1/2))-1/338*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(
1+4*x)/(13-2*13^(1/2)))*(1521+13^(1/2)*(1701-1168*m+568*m*13^(1/2)))/(1+m)/
(13-2*13^(1/2))
```

Rubi [A] time = 0.28, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1648, 830, 68}

$$-\frac{\left(\sqrt{13}\left(568\sqrt{13}m-1168m+1701\right)+1521\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\sqrt{13}(1701-1168m)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((209 - 426*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((1521 + Sqrt[13
]*(1701 - 1168*m + 568*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(338*(13 - 2*Sqrt[13])*(1 +
 m)) + ((Sqrt[13]*(1701 - 1168*m) - 13*(117 + 568*m))*(1 + 4*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(338*(13
+ 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
```

```
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2+3x)^3(1+4x)^m}{\left(1-5x+3x^2\right)^2}dx = \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(13(1143+836m)-39(117+568m)}{1-5x+3x^2}$$

$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int\left(\frac{\left(-39(117+568m)-3\sqrt{13}(-1701+1168m)\right)}{-5-\sqrt{13}+6x}\right.$$

$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{169}\left(\sqrt{13}(1701-1168m)-13(117+568m)\right)\int \frac{(1}{-5-}$$

$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(\sqrt{13}(1701-1168m)+13(117+568m)\right)(1+4x)^{1+m}}{338\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.38, size = 252, normalized size = 1.39

$$\frac{(4x+1)^{m+1}\left(-\frac{12\left(\sqrt{13}(1215-292m)+1846m\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(m+1)} - \frac{351\left(27\sqrt{13}-13\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} + \frac{12\left(\sqrt{13}(1215-\right.}{}\right.}{1014}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5434 - 11076*x)/(1 - 5*x + 3*x^2) - (351*(-13 + 27*Sqr
t[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((
-13 + 2*Sqrt[13])*(1 + m)) - (12*(Sqrt[13]*(1215 - 292*m) + 1846*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 - 2*Sqrt[13
])*(1 + m)) - (351*(13 + 27*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (12*(Sqrt[13]*(1
215 - 292*m) - 1846*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 +
2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/1014
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(27x^3+54x^2+36x+8\right)(4x+1)^m}{9x^4-30x^3+31x^2-10x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2
 - 10*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m(3x+2)^3}{\left(3x^2-5x+1\right)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^{3} (4x+1)^{m}}{\left(3x^{2}-5x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^3*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
[Out] int((3*x+2)^3*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^{m}(3\,x+2)^{3}}{\left(3\,x^{2}-5\,x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^{3}\,(4\,x+1)^{m}}{\left(3\,x^{2}-5\,x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^{3} (4x+1)^{m}}{\left(3x^{2}-5x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.939 $\int \frac{(2+3x)^2(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=179

$$-\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(4x+1)^{m\cdot}}{13\sqrt{13}\left(13+2\sqrt{\right.}}$$

```
[Out] 1/39*(61-87*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-2/169*(1+4*x)^(1+m)*hypergeom([1
, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(153-m*(23-29*13^(1/2)))/(1+m)/(13-
2*13^(1/2))*13^(1/2)+2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)
/(13+2*13^(1/2)))*(153-m*(23+29*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

Rubi [A] time = 0.25, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1648, 830, 68}

$$-\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(4x+1)^{m\cdot}}{13\sqrt{13}\left(13+2\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((61 - 87*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(153 - (23 - 29
*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
4*x))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(153
 - (23 + 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
```

```
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2+3x)^2(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(26(153+122m)-4524mx)}{1-5x+3x^2}\,dx$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-4524m-12\sqrt{13}\,(-153+23m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{(-4$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} + \frac{\left(4\left(153-\left(23-29\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{13\sqrt{13}} - \frac{\left(4\left(153-(2\right.\right.}{}$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\right.}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.38, size = 156, normalized size = 0.87

$$\frac{1}{507}(4x+1)^{m+1}\left(-\frac{6\left(\left(23\sqrt{13}-377\right)m-153\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{6\left(\left(377+23\sqrt{13}\right)m-153\sqrt{\right.}{\left(13+2\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((793 - 1131*x)/(1 - 5*x + 3*x^2) - (6*(-153*Sqrt[13] +
(-377 + 23*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 -
 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (6*(-153*Sqrt[13] + (377 + 23
*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13
])])/((13 + 2*Sqrt[13])*(1 + m))))/507
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(9\,x^2+12\,x+4\right)(4\,x+1)^m}{9\,x^4-30\,x^3+31\,x^2-10\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m(3\,x+2)^2}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^2 (4x+1)^m}{\left(3x^2-5x+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x+2)^2*(4*x+1)^m/(3*x^2-5*x+1)^2,x)

[Out] int((3*x+2)^2*(4*x+1)^m/(3*x^2-5*x+1)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m (3\,x+2)^2}{\left(3\,x^2-5\,x+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")

[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^2\,(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)

[Out] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^2 (4x+1)^m}{\left(3x^2-5x+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1)**2,x)

[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)

3.940 $\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2} dx$

Optimal. Leaf size=179

$$-\frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m}$$

```
[Out] 1/39*(20-21*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/169*(1+4*x)^(1+m)*hypergeom([1
, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+2*m*(5-7*13^(1/2)))/(1+m)*13^(1
/2)/(13+2*13^(1/2))-1/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/
(13-2*13^(1/2)))*(81+2*m*(5+7*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {822, 830, 68}

$$-\frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((20 - 21*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((81 + 2*(5 + 7*Sq
rt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x
))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + ((81 + 2*(
5 - 7*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(2+3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(13(81+80m)-1092mx)}{1-5x+3x^2}\,dx$$
$$= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-1092m+6\sqrt{13}\,(81+10m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-109\right.}{}\right.$$
$$= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} + \frac{1}{169}\left(2\left(182m+\sqrt{13}\,(81+10m)\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx$$
$$= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(182m+\sqrt{13}\,(81+10m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+\right.}{169\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.24, size = 149, normalized size = 0.83

$$\frac{1}{507}(4x+1)^{m+1}\left(\frac{3\left(182m+\sqrt{13}\,(10m+81)\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{3\left(182m-\sqrt{13}\,(10m+81)\right)\,{}_2F_1}{\left(13+2\sqrt{13}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((260 - 273*x)/(1 - 5*x + 3*x^2) + (3*(182*m + Sqrt[13]*
(81 + 10*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13]
)])/((-13 + 2*Sqrt[13])*(1 + m)) - (3*(182*m - Sqrt[13]*(81 + 10*m))*Hyperg
eometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[1
3])*(1 + m))))/507
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(4\,x+1)^m(3\,x+2)}{9\,x^4-30\,x^3+31\,x^2-10\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m*(3*x + 2)/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m(3\,x+2)}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)\,(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((3*x+2)*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m(3\,x+2)}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)\,(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
```

```
[Out] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)\,(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.941 $\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2} dx$

Optimal. Leaf size=177

$$-\frac{2\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(2\left(2-\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,{}_2F_1\left(1,\ldots\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m\ldots}$$

```
[Out] 1/39*(7-6*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+2/169*(1+4*x)^(1+m)*hypergeom([1,
1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(9+2*m*(2-13^(1/2)))/(1+m)*13^(1/2)/(
13+2*13^(1/2))-2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2
*13^(1/2)))*(9+2*m*(2+13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {740, 830, 68}

$$-\frac{2\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(2\left(2-\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,{}_2F_1\left(1,\ldots\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((7 - 6*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(9 + 2*(2 + Sqrt[
13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/
(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(9 + 2*(2
- Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(26(9+14m)-312mx)}{1-5x+3x^2}\,dx$$
$$= \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-312m+12\sqrt{13}\,(9+4m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-312m-12\sqrt{1\right.}}{-5+}\right.$$
$$= \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(4\left(9+2\left(2-\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{13\sqrt{13}} + \frac{\left(4\left(9+2\left(2+\sqrt{13}\right)n\right.\right.}{13\sqrt{1}}$$
$$= \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.20, size = 150, normalized size = 0.85

$$\frac{1}{507}(4x+1)^{m+1}\left(\frac{6\left(26m+\sqrt{13}\,(4m+9)\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} + \frac{6\sqrt{13}\left(9-2\left(\sqrt{13}-2\right)m\right)\,{}_2F_1\left(1,m\right.}{\left(13+2\sqrt{13}\right)(m+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((91 - 78*x)/(1 - 5*x + 3*x^2) + (6*(26*m + Sqrt[13]*(9
+ 4*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/(
(-13 + 2*Sqrt[13])*(1 + m)) + (6*Sqrt[13]*(9 - 2*(-2 + Sqrt[13])*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13
])*(1 + m))))/507
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(4\,x+1)^m}{9\,x^4-30\,x^3+31\,x^2-10\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2,x)
```

```
[Out] int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

3.942 $\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=340

$$\frac{9(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}-\frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$

```
[Out] 1/663*(43-33*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+9/1445*(1+4*x)^(1+m)*hypergeom(
[1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[
2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/287
3*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+m*(
62-22*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/7514*(1+4*x)^(1+m)*hyperg
eom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*1
3^(1/2))-1/2873*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(
1/2)))*(81+m*(62+22*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {960, 68, 822, 830}

$$\frac{9(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}-\frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((43 - 33*x)*(1 + 4*x)^(1 + m))/(663*(1 - 5*x + 3*x^2)) + (9*(1 + 4*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1445*(1 + m)) +
(9*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (
3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13])*(1 + m)) - ((81 +
(62 + 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(221*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m))
+ (9*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1 + m)) + ((81
+ (62 - 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(221*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \int \left(\frac{9(1+4x)^m}{289(2+3x)} + \frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)^2} - \frac{3(-7+3x)(1+4x)^m}{289\left(1-5x+3x^2\right)}\right) dx$$

$$= -\left(\frac{3}{289}\int \frac{(-7+3x)(1+4x)^m}{1-5x+3x^2}\,dx\right) + \frac{9}{289}\int \frac{(1+4x)^m}{2+3x}\,dx + \frac{1}{17}\int \frac{(7-3x)(1\cdots}{(1-5x\cdots}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} - \cdots$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} - \cdots$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \cdots$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \cdots$$

Mathematica [A] time = 0.77, size = 274, normalized size = 0.81

$$\frac{(4x+1)^{m+1}\left(\frac{9126\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{m+1} + \frac{1755\left(13+9\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(m+1)} + \frac{1755\left(13-9\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{1\cdots}}\right)}{\left(13+2\sqrt{13}\right)(m+1)} \cdots\right)}{1465230}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((2210*(43 - 33*x))/(1 - 5*x + 3*x^2) + (9126*Hypergeome
tric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (1755*(13 + 9*Sqrt[13
```

```
])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 -
 2*Sqrt[13])*(1 + m)) + (1755*(13 - 9*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (510*S
qrt[13]*(((81 + (62 + 22*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])])/(-13 + 2*Sqrt[13]) + ((81 + (62 - 22*Sqrt[13])
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(13 +
 2*Sqrt[13])))/(1 + m)))/1465230
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(4\,x+1)^{m}}{27\,x^{5}-72\,x^{4}+33\,x^{3}+32\,x^{2}-17\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(27*x^5 - 72*x^4 + 33*x^3 + 32*x^2 - 17*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^{m}}{\left(3\,x^{2}-5\,x+1\right)^{2}(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^{m}}{(3x+2)\left(3x^{2}-5x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x+2)/(3*x^2-5*x+1)^2,x)
[Out] int((4*x+1)^m/(3*x+2)/(3*x^2-5*x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^{m}}{\left(3\,x^{2}-5\,x+1\right)^{2}(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(4\,x+1)^{m}}{(3\,x+2)\,\left(3\,x^{2}-5\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2),x)
```

```
[Out] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1)**2,x)
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)**2), x)
```

3.943 $\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2} \, dx$

Optimal. Leaf size=376

$$\frac{162(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{24565(m+1)}-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\right.}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$

```
[Out] 1/11271*(268-195*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+162/24565*(1+4*x)^(1+m)*hyp
ergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+36/7225*(1+4*x)^(1+m)*hypergeom([2
, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/63869*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2
+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-64*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/48
841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(423+
m*(422-130*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/63869*(1+4*x)^(1+m)*
hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(117+64*13^(1/2))/(1+m)
/(13-2*13^(1/2))-1/48841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(
13-2*13^(1/2)))*(423+2*m*(211+65*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {960, 68, 822, 830}

$$\frac{162(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{24565(m+1)}-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\right.}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((268 - 195*x)*(1 + 4*x)^(1 + m))/(11271*(1 - 5*x + 3*x^2)) + (162*(1 + 4*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(24565*(1 +
 m)) + (9*(117 + 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(63869*(13 - 2*Sqrt[13])*(1 + m))
 - ((423 + 2*(211 + 65*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(3757*Sqrt[13]*(13 - 2*Sqrt
[13])*(1 + m)) + (9*(117 - 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(63869*(13 + 2*Sqrt[13]
)*(1 + m)) + ((423 + (422 - 130*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(3757*Sqrt[13]*(13
 + 2*Sqrt[13])*(1 + m)) + (36*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m,
 2 + m, (-3*(1 + 4*x))/5])/(7225*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx = \int \left(\frac{9(1+4x)^m}{289(2+3x)^2}+\frac{162(1+4x)^m}{4913(2+3x)}+\frac{(46-27x)(1+4x)^m}{289\left(1-5x+3x^2\right)^2}-\frac{3(1+4x)^m(-10}{4913\left(1-5x\right.}$$

$$= -\frac{3\int \frac{(1+4x)^m(-109+54x)}{1-5x+3x^2}\,dx}{4913}+\frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx+\frac{9}{289}\int \frac{(1+4}{(2+3}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right.}{24565(1+m)}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right.}{24565(1+m)}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right.}{24565(1+m)}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right.}{24565(1+m)}$$

Mathematica [A] time = 0.64, size = 287, normalized size = 0.76

$$\frac{(4x+1)^{m+1}\left(\frac{1232010\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{m+1}+\frac{26325\left(117+64\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(m+1)}+\frac{26325\left(117-64\sqrt{13}\right){}_2F_1\left(1,m+1;m\right.}{\left(13+2\sqrt{13}\right)(m+1)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((16575*(268 - 195*x))/(1 - 5*x + 3*x^2) + (1232010*Hype
rgeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (26325*(117 + 6
```

```
4*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
])/((13 - 2*Sqrt[13])*(1 + m)) + (26325*(117 - 64*Sqrt[13])*Hypergeometric2
F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 +
m)) - (425*((423*(2 + Sqrt[13]) + (2534 + 682*Sqrt[13])*m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] + (-423*(-2 + Sqrt[13]) +
(2534 - 682*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
+ 2*Sqrt[13])]))/(1 + m) + (930852*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(
1 + 4*x))/5])/(1 + m)))/186816825
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(4\,x+1)^{m}}{81\,x^{6}-162\,x^{5}-45\,x^{4}+162\,x^{3}+13\,x^{2}-28\,x+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(81*x^6 - 162*x^5 - 45*x^4 + 162*x^3 + 13*x^2 - 28*x +
 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^{m}}{\left(3\,x^{2}-5\,x+1\right)^{2}(3\,x+2)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^{m}}{(3x+2)^{2}\left(3x^{2}-5x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x+2)^2/(3*x^2-5*x+1)^2,x)
[Out] int((4*x+1)^m/(3*x+2)^2/(3*x^2-5*x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,x+1)^{m}}{\left(3\,x^{2}-5\,x+1\right)^{2}(3\,x+2)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(4\,x+1)^{m}}{(3\,x+2)^{2}\left(3\,x^{2}-5\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2),x)
```

```
[Out] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)^2 \left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)**2), x)
```

3.944 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2\sqrt{e+fx}\,(d+ex)^m\left(-\frac{f(d+ex)}{e^2-df}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)\left(c\left(d^2f^2+4de^2f(m+1)-4e^4\left(m^2+3m+2\right)\right)-ef(2m+\cdots\right.}{ef^3(2m+3)\left(e^2-df\right)}$$

```
[Out] 2*(a+e*(-b*f+c*e)/f^2)*(e*x+d)^(1+m)/(-d*f+e^2)/(f*x+e)^(1/2)+2*c*(e*x+d)^(
1+m)*(f*x+e)^(1/2)/e/f^2/(3+2*m)+2*(c*(d^2*f^2+4*d*e^2*f*(1+m)-4*e^4*(m^2+3
*m+2))-e*f*(3+2*m)*(a*e*f*(1+2*m)+b*(d*f-2*e^2*(1+m))))*(e*x+d)^m*hypergeom
([1/2, -m],[3/2],e*(f*x+e)/(-d*f+e^2))*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)/(3+2*
m)/((-f*(e*x+d)/(-d*f+e^2))^m)
```

Rubi [A] time = 0.35, antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {949, 80, 70, 69}

$$-\frac{2\sqrt{e+fx}\,(d+ex)^m\left(-\frac{f(d+ex)}{e^2-df}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)\left(f\left(aef(2m+1)+bdf-2be^2(m+1)\right)-\frac{c\left(d^2f^2+4de^2f(m+1\cdots\right.}{e(2m\cdots}\right.}{f^3\left(e^2-df\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x])
 + (2*c*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*f^2*(3 + 2*m)) - (2*(f*(b*d*f -
 2*b*e^2*(1 + m) + a*e*f*(1 + 2*m)) - (c*(d^2*f^2 + 4*d*e^2*f*(1 + m) - 4*e
^4*(2 + 3*m + m^2)))/(e*(3 + 2*m)))*(d + e*x)^m*Sqrt[e + f*x]*Hypergeometri
c2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)])/(f^3*(e^2 - d*f)*(-((f*(d +
e*x))/(e^2 - d*f)))^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2\int \frac{(d+ex)^m\left(\frac{c\left(def-2e^3(1+m)\right)-f\left(bdf-2be^2(1+m)+aef(1+2m)\right)}{2f^2}-\frac{1}{2}c\ldots\right)}{\sqrt{e+fx}}}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{\left(f\left(bdf-2be^2(1+\ldots\right.\right.}{\ldots}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{\left(\left(f\left(bdf-2be^2(1+\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{2\left(f\left(bdf-2be^2(1+\ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 0.29, size = 171, normalized size = 0.72

$$\frac{2(d+ex)^m\left(\frac{f(d+ex)}{df-e^2}\right)^{-m}\left(-3\left(f(af-be)+ce^2\right){}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{e(e+fx)}{e^2-df}\right)-(e+fx)\left((6ce-3bf)\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e\ldots}{e^2\ldots}\right.\right.\right.}{3f^3\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(d + e*x)^m*(-3*(c*e^2 + f*(-(b*e) + a*f))*Hypergeometric2F1[-1/2, -m, 1
/2, (e*(e + f*x))/(e^2 - d*f)] - (e + f*x)*((6*c*e - 3*b*f)*Hypergeometric2
F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)] - c*(e + f*x)*Hypergeometric2F1
[3/2, -m, 5/2, (e*(e + f*x))/(e^2 - d*f)])))/(3*f^3*((f*(d + e*x))/(-e^2 +
d*f))^m*Sqrt[e + f*x])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)\sqrt{fx+e}\,(ex+d)^m}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*sqrt(f*x + e)*(e*x + d)^m/(f^2*x^2 + 2*e*f*x + e
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)(ex + d)^m}{\left(fx + e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^2 + bx + a\right)(ex + d)^m}{\left(fx + e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)(ex + d)^m}{\left(fx + e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x)^m\,\left(c\,x^2 + b\,x + a\right)}{\left(e + f\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.945 $\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\, dx$

Optimal. Leaf size=509

$$\frac{\sqrt{a+bx+cx^2}\,(d+ex)^{m+1} F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(eg^2(m+1)(bd-ae)+c \cdots\right.}{ce^3(m+1)(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/e/(4+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*(
3*d^2*g^2+e^2*f^2*(4+m)-2*d*e*f*g*(4+m)))*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-
1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3/(1+m)/(4+m)/(1-2*c*(e*x+d
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-1/2*g*(b*e*g*(5+2*m)+2*c*(3*d*g-2*e*f*(4+m)))*(e*x+d)
^(2+m)*AppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2
))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3
/(2+m)/(4+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 506, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1653, 843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}\,(d+ex)^{m+1} F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd-ae)+\frac{c\left(3d^2g^2-2d \cdots\right.}{\cdots}\right.}{ce^2(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*e*(4 + m)) + (((b*d - a*
e)*g^2 + (c*(3*d^2*g^2 + e^2*f^2*(4 + m) - 2*d*e*f*g*(4 + m)))/(e*(1 + m)))
*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(4 + m)*Sqrt[1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]) - (g*(6*c*d*g - 4*c*e*f*(4 + m) + b*e*g*(5 + 2*m))*(
d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (
2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(d +
e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
```

```
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{\int (d+ex)^m \left(\frac{1}{2}e\left(2cef^2(4+m)-g^2(3\right.\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{(g(6cdg-4cef(4+m)+beg(5+2m}{2ce^2(4}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{\left(g(6cdg-4cef(4+m)+beg(5+2m\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{\left(e(bd-ae)g^2(1+m)+c\left(3d^2g^2+e^2\right.\right.}{}$$

Mathematica [F] time = 1.39, size = 0, normalized size = 0.00

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2], x]
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^2x^2 + 2\,fgx + f^2\right)\sqrt{cx^2 + bx + a}\,(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\left(gx + f\right)^2 (ex + d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(gx + f\right)^2 \sqrt{c\,x^2 + bx + a}\;(ex + d)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\left(gx + f\right)^2 (ex + d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f + g\,x\right)^2 (d + e\,x)^m\,\sqrt{c\,x^2 + b\,x + a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^m \left(f + gx\right)^2 \sqrt{a + bx + cx^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)**2*sqrt(a + b*x + c*x**2), x)
```

3.946 $\int (d+ex)^m (f+gx)\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=388

$$\frac{\sqrt{a+bx+cx^2}\,(ef-dg)(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}\,\cdots}{\cdots}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+
b*x+a)^(1/2)/e^2/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/
2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(e*x+d)^(2+m)*A
ppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/e^2/(2+m)/(1-
2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}\,(ef-dg)(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}\,\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2,
-1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*(d
+ e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^
2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \frac{g\int (d+ex)^{1+m}\sqrt{a+bx+cx^2}\,dx}{e} + \frac{(ef-dg)\int (d+ex)^m\sqrt{a+bx+c}}{e}$$

$$= \frac{\left(g\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2}{2cd-\left(b+\right.}}\right.}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d}{2cd-\left(b-\right.}\right.}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-\right.}}}}$$

Mathematica [F] time = 0.75, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+bx+a}\left(gx+f\right)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+bx+a}\left(gx+f\right)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)\sqrt{c\,x^2+bx+a}\;(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+bx+a}\,(gx+f)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,(d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^m\,(f+gx)\,\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

3.947 $\int (d+ex)^m \sqrt{a+bx+cx^2}\, dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a+bx+cx^2}\,(d+ex)^{m+1} F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] (e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2
)/e/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*
x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {759, 133}

$$\frac{\sqrt{a+bx+cx^2}\,(d+ex)^{m+1} F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{a+bx+cx^2}\,\text{Subst}\left(\int x^m\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,dx, x, d\right.}{e\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4\right.}}\right.}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Mathematica [A] time = 0.02, size = 207, normalized size = 1.10

$$\frac{\sqrt{a+x(b+cx)}\,(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e(m+1)\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + x*(b + c*x)]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+bx+a}\,(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+bx+a}\,(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^2+bx+a}\;(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2 + bx + a}\,(ex + d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d + e\,x)^m\,\sqrt{c\,x^2 + b\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^m\,\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2), x)
```

3.948 $\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\sqrt{a+bx+cx^2}\,(d+ex)^m}{f+gx},x\right)$$

```
[Out] Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
[Out] Defer[Int][((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Rubi steps

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Mathematica [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
[Out] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

fricas [A] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\,(ex+d)^m}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\,(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^{2}+b x +a}\;\left(e x +d \right)^{m}}{g x +f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}\,(ex + d)^m}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(d+e\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}}{f+g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x),x)
```

```
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + e x\right)^{m} \sqrt{a + b x + c x^{2}}}{f + g x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2)/(g*x+f),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2)/(f + g*x), x)
```

3.949 $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=502

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)(}{ce^3(m+1)(m+2)\sqrt{a+bx+cx^2}}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(1/2)/c/e/(2+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*(
d^2*g^2+e^2*f^2*(2+m)-2*d*e*f*g*(2+m)))*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,
2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*
(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(1+m)/(2+m)/(c
*x^2+b*x+a)^(1/2)-1/2*g*(b*e*g*(3+2*m)+c*(2*d*g-4*e*f*(2+m)))*(e*x+d)^(2+m)
*AppellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)/c/e^3/(2+m)^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1653, 843, 759, 133}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)(}{ce^2(m+2)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2])/(c*e*(2 + m)) + (((b*d - a*e)
*g^2 + (c*(d^2*g^2 + e^2*f^2*(2 + m) - 2*d*e*f*g*(2 + m)))/(e*(1 + m)))*(d
+ e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[1
+ m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(2 + m)*Sqrt[a
 + b*x + c*x^2]) - (g*(2*c*d*g - 4*c*e*f*(2 + m) + b*e*g*(3 + 2*m))*(d + e*
x)^(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m,
 1/2, 1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)^2*Sqrt[a
 + b*x + c*x^2])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
```

```
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m (f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{\int \frac{(d+ex)^m\left(\frac{1}{2}e\left(2cef^2(2+m)-g^2(bd+2ae(1+m))\right)-\frac{1}{2}eg(2cdg-4cef(2 \cdots}{\sqrt{a+bx+cx^2}}}{ce^2(2+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{(g(2cdg-4cef(2+m)+beg(3+2m)))\int \frac{(d+ex)^{1+ \cdots}}{\sqrt{a+bx+c \cdots}}}{2ce^2(2+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{\left(g(2cdg-4cef(2+m)+beg(3+2m))\sqrt{1-\frac{a \cdots}{d-\frac{(b- \cdots}{\cdots}}} \cdots\right.}{\cdots}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{\left(e(bd-ae)g^2(1+m)+c\left(d^2g^2+e^2f^2(2+m)-2 \cdots\right.\right.}{\cdots}$$

Mathematica [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m (f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2], x]
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g^2x^2+2\,fgx+f^2\right)(ex+d)^m}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}(ex+d)^{m}}{\sqrt{cx^{2}+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}(ex+d)^{m}}{\sqrt{c\,x^{2}+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}(ex+d)^{m}}{\sqrt{cx^{2}+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{2}\,(d+e\,x)^{m}}{\sqrt{c\,x^{2}+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{m}\left(f+gx\right)^{2}}{\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)**2/sqrt(a + b*x + c*x**2), x)
```

3.950 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=388

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c}{2cd-\left(b\right.}\right.}{e^2(m+1)\sqrt{a+bx+cx^2}}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(
-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e
*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/e^2/(1+m)/(c*x^2+b*x+a)^(1/2)+g*(e*x+d)^(2+m)*App
ellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x
+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^
2/(2+m)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {843, 759, 133}

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c}{2cd-\left(b\right.}\right.}{e^2(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(1 + m)*Sqrt[a + b*x + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*
x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{(d+ex)^{1+m}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left(g\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^{1+m}}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m\right.}{e^2(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2], x]
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(gx+f\right)(ex+d)^m}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(ex+d\right)^m}{\sqrt{c\,x^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx + f\right)(ex + d)^m}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f + g\,x\right)\,{\left(d + e\,x\right)}^m}{\sqrt{c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^m \left(f + gx\right)}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

3.951 $\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

```
[Out] (e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/e/(1+m)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^m}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}\right.}{e\sqrt{a+bx+cx^2}}$$
$$= \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\right.}\right.}{e(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.10

$$\frac{(d+ex)^{m+1}\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e(m+1)\sqrt{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*
a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a + x*(b + c*x)])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{c\,x^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^m/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(a + b*x + c*x**2), x)
```

3.952 $\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Defer[Int][(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\,(ex+d)^m}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(c*g*x^3 + (c*f + b*g)*x^2 + a*f
 + (b*f + a*g)*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)\sqrt{c\,x^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(d+e\,x)^m}{(f+g\,x)\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m/((f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.953 $\int (d+ex)^m(f+gx)^n\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=265

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(g(m+n+2)\left(ae^2g(m+n+3)-cd(dg(n+\right.\right.}{e^3g^2(m+1)(m+n+2)(}$$

```
[Out] (b*e*g*(3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n)))*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e^2
/g^2/(2+m+n)/(3+m+n)+c*(e*x+d)^(2+m)*(g*x+f)^(1+n)/e^2/g/(3+m+n)+(g*(2+m+n)
*(a*e^2*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n)))-(e*f*(1+m)+d*g*(1+n))*(b*e*g*(
3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n))))*(e*x+d)^(1+m)*(g*x+f)^n*hypergeom([-n,
 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^3/g^2/(1+m)/(2+m+n)/(3+m+n)/((e*(g*x+f
)/(-d*g+e*f))^n)
```

Rubi [A] time = 0.35, antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {951, 80, 70, 69}

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(g(m+n+2)\left(ae^2g(m+n+3)-cd(dg(n+\right.\right.}{e^3g^2(m+1)(m+n+2)(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n))*(d + e*x)^(1 +
 m)*(f + g*x)^(1 + n))/(e^2*g^2*(2 + m + n)*(3 + m + n))) + (c*(d + e*x)^(2
 + m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + (((e*f*(1 + m) + d*g*(1 + n)
)*(c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n)) + g*(2 + m + n)
*(a*e^2*g*(3 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))))*(d + e*x)^(1 + m)
*(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g
))])/(e^3*g^2*(1 + m)*(2 + m + n)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n
)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{\int (d+ex)^m(f+gx)^n \left(ae^2g(3+m+n)-c\right.}{}$$

$$= -\frac{(cef(2+m)-beg(3+m+n)+cdg(4+m+2n))(d+ex)^{1+m}(f+gx}{e^2g^2(2+m+n)(3+m+n)}$$

$$= -\frac{(cef(2+m)-beg(3+m+n)+cdg(4+m+2n))(d+ex)^{1+m}(f+gx}{e^2g^2(2+m+n)(3+m+n)}$$

$$= -\frac{(cef(2+m)-beg(3+m+n)+cdg(4+m+2n))(d+ex)^{1+m}(f+gx}{e^2g^2(2+m+n)(3+m+n)}$$

Mathematica [A] time = 0.22, size = 187, normalized size = 0.71

$$\frac{(d+ex)^{m+1}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} \left(e\left(e\left(g(ag-bf)+cf^2\right) {}_2F_1\left(m+1,-n;m+2;\frac{g(d+ex)}{dg-ef}\right)-(2cf-bg)(ef-dg)\,{}_2F\right.\right.}{e^3g^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -2
 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)
*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e
*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x
))/(-(e*f) + d*g)])))/(e^3*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2+bx+a\right)(ex+d)^m\left(gx+f\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+bx+a\right)(ex+d)^m\left(gx+f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+bx+a\right)\left(ex+d\right)^{m}\left(gx+f\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2}+bx+a\right)\left(ex+d\right)^{m}\left(gx+f\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^n\,{\left(d+e\,x\right)}^m\,\left(c\,x^2+b\,x+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.954 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=525

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4\right.}\right.}\right.}{ce^3(m+1)(m+}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(1+p)/c/e/(3+m+2*p)+(e*(-a*e+b*d)*g^2*(1+m)
+c*(2*d^2*g^2*(1+p)+e^2*f^2*(3+m+2*p)-2*d*e*f*g*(3+m+2*p)))*(e*x+d)^(1+m)*(
c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c/e^3/(1+m)/(3+m+2*p)
/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^p)-g*(b*e*g*(2+m+p)+2*c*(d*g*(1+p)-e*f*(3+m+2*
p)))*(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c
/e^3/(2+m)/(3+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/(
(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] time = 0.70, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1653, 843, 759, 133}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4\right.}\right.}\right.}{ce^2(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) + (((
b*d - a*e)*g^2 + (c*(2*d^2*g^2*(1 + p) + e^2*f^2*(3 + m + 2*p) - 2*d*e*f*g*
(3 + m + 2*p)))/(e*(1 + m)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1
[1 + m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(3 + m + 2*p)
*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p) - (g*(2*c*d*g*(1 + p) + b*e*
g*(2 + m + p) - 2*c*e*f*(3 + m + 2*p))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^
p*AppellF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(2
+ m)*(3 + m + 2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
```

```
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{\int (d+ex)^m \left(e\left(cef^2(3+m+2p\right.\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{(g(2cdg(1+p)+beg(2+m+p}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{\left(g(2cdg(1+p)+beg(2+m+p\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{\left(e(bd-ae)g^2(1+m)+c\left(2d^2g^2\right.\right.}{}$$

Mathematica [F] time = 2.24, size = 0, normalized size = 0.00

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p, x]
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^2x^2+2\,fgx+f^2\right)\left(cx^2+bx+a\right)^p(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{2}\left(cx^{2}+bx+a\right)^{p}(ex+d)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

maple **[F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{2}(ex+d)^{m}\left(c\,x^{2}+bx+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{2}\left(cx^{2}+bx+a\right)^{p}(ex+d)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^m\,{\left(c\,x^2+b\,x+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

```
[Out] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

3.955 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=384

$$\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\right.}{e^2(m+1)}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))/e^2/(1+m)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c
*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)+g*(e*x+d)^(2+m)*(c*x^2+b*x+a)
^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e^2/(2+m)/((1-2*c*(e*x+d)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))^p)
```

Rubi [A] time = 0.34, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {843, 759, 133}

$$\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\right.}{e^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p,
2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e))^p) + (g*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[
2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*
c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \frac{g\int (d+ex)^{1+m}\left(a+bx+cx^2\right)^p\,dx}{e}+\frac{(ef-dg)\int (d+ex)^m\left(a+bx+c\right.}{e}$$

$$=\frac{\left(g\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\text{Subst}\left(\int\right.}{e^2}$$

$$=\frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{}{2cd-(}\right.}{e^2(}$$

Mathematica [F] time = 1.32, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x]
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx+f\right)\left(cx^2+bx+a\right)^p(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] integral((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)(ex+d)^m\left(c\,x^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (gx+f)(cx^2+bx+a)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
[Out] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**p,x)
[Out] Timed out
```

3.956 $\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=187

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4\right.}\right.}\right.}{e(m+1)}$$

```
[Out] (e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e/(1+m
)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4\right.}\right.}\right.}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p dx = \frac{\left(\left(a+bx+cx^2\right)^p \left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right) \text{Subst}\left(\int x^m \left(1\right.\right.}{e}$$

$$= \frac{(d+ex)^{1+m} \left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}}{e(1+m)}$$

Mathematica [A] time = 0.10, size = 205, normalized size = 1.10

$$\frac{(d+ex)^{m+1}(a+x(b+cx))^p \left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{-p} \left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\right.}{e(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
[Out] ((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)]])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)
)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2+bx+a\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(c\,x^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d + e\,x)^m \left(c\,x^2 + b\,x + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

```
[Out] int((d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

3.957 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx},x\right)$$

```
[Out] Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
[Out] Defer[Int][((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx = \int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Mathematica [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
[Out] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

fricas [A] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)^p(ex+d)^m}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+bx+a\right)^p(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{m} \left(c\,x^{2}+bx+a\right)^{p}}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}+bx+a\right)^{p} (ex+d)^{m}}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(d+e\,x)^{m} \left(c\,x^{2}+b\,x+a\right)^{p}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x)
```

```
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p/(g*x+f),x)
```

```
[Out] Timed out
```

3.958 $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\, x^2 \sqrt{d+ex}}\, dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] -2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e
*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1574, 933, 168, 538, 537}

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] (-2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx &= \frac{\sqrt{-\frac{1}{c^2}+x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\\
&= \frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\\
&= -\frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\\
&= -\frac{\left(2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\\
&= -\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\end{aligned}$$

Mathematica [C] time = 0.64, size = 188, normalized size = 2.11

$$-\frac{2i(d+ex)\sqrt{\frac{e(cx-1)}{c(d+ex)}}\sqrt{\frac{cex+e}{cd+cex}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)|\frac{cd-e}{cd+e}\right)-\Pi\left(\frac{cd}{cd+e};i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)|\frac{cd-e}{cd+e}\right)\right)}{dx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{cd+e}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] ((-2*I)*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)*Sqrt[(e + c*e*x)/(c*d
+ c*e*x)]*(EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d -
e)/(c*d + e)] - EllipticPi[(c*d)/(c*d + e), I*ArcSinh[Sqrt[-((c*d + e)/c)]/
Sqrt[d + e*x]], (c*d - e)/(c*d + e)]))/(d*Sqrt[-((c*d + e)/c)]*Sqrt[1 - 1/(
c^2*x^2)]*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+d}\, x^2 \sqrt{-\frac{1}{c^2x^2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

maple [A] time = 0.14, size = 148, normalized size = 1.66

$$-\frac{2\,(cd-e)\sqrt{-\frac{(cx+1)e}{cd-e}}\,\sqrt{-\frac{(cx-1)e}{cd+e}}\,\sqrt{\frac{(ex+d)c}{cd-e}}\;\mathrm{EllipticPi}\left(\sqrt{\frac{(ex+d)c}{cd-e}}, \frac{cd-e}{cd}, \sqrt{\frac{cd-e}{cd+e}}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\sqrt{ex+d}\; cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x)
[Out] -2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c*d-e)*EllipticPi(((e*x+d)*c/(c*d-e))^(1/
2),(c*d-e)/c/d,((c*d-e)/(c*d+e))^(1/2))*(-(c*x+1)*e/(c*d-e))^(1/2)*(-(c*x-1
)*e/(c*d+e))^(1/2)*((e*x+d)*c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/c/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+d}\, x^2 \sqrt{-\frac{1}{c^2x^2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{d+e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)),x)
[Out] int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-\left(-1+\frac{1}{cx}\right)\left(1+\frac{1}{cx}\right)}\,\sqrt{d+ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1-1/c**2/x**2)**(1/2)/(e*x+d)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x)))*sqrt(d + e*x)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```